The Second International Conference on Axion Physics and Experiment

# ALP from $U(1)_L$ and its phenomenology

**WEI CHAO (BNU)** **2023.07.24**

**Based on arXiv:2210.13233, with Ying-Quan Peng、H.J. Li、M.J. Jin、Yue Wang**

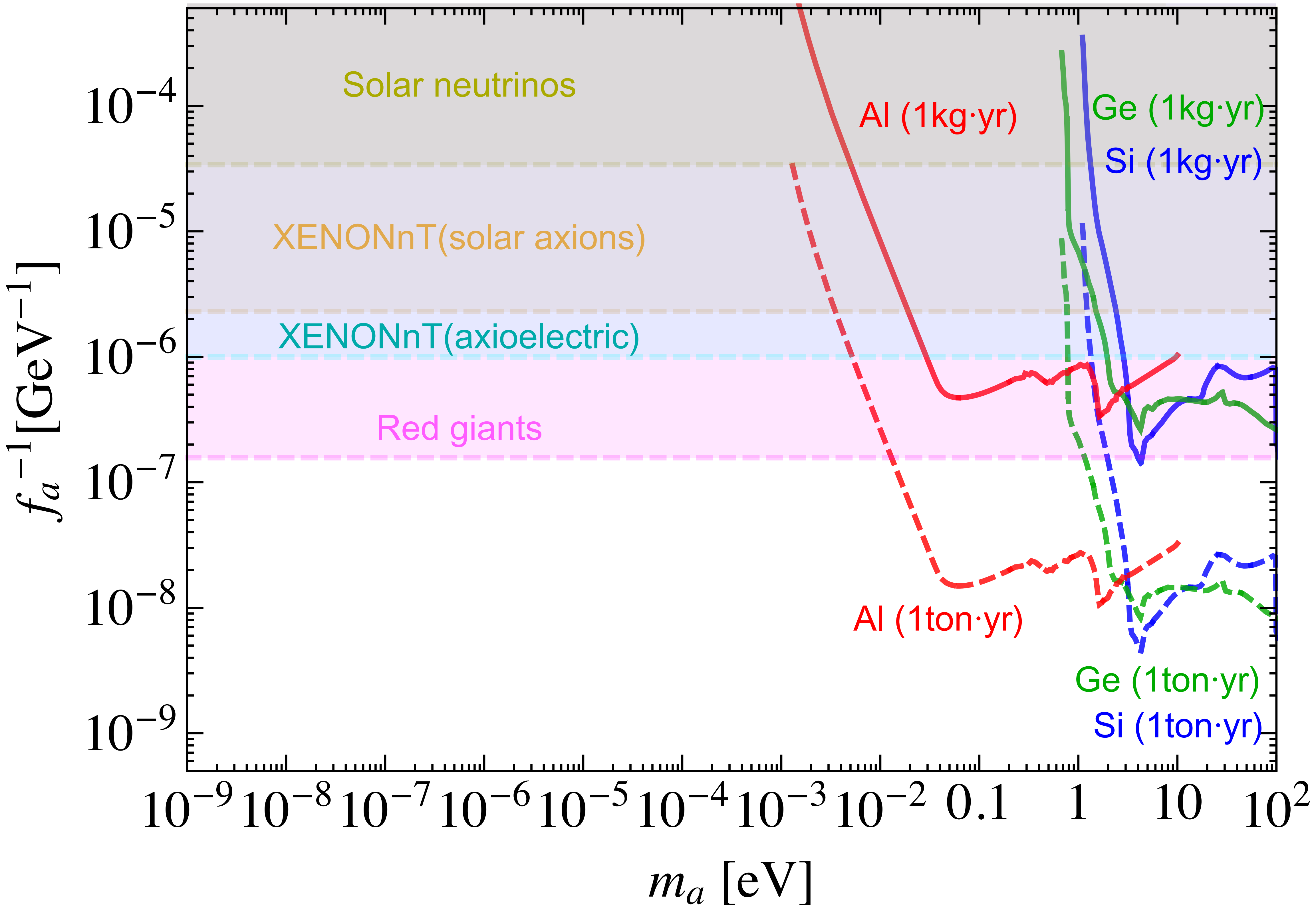
# Preview

## I: The origin of the ALP mass

**There is no well-known mechanism for the mass generation of ALP, except the QCD axion.**

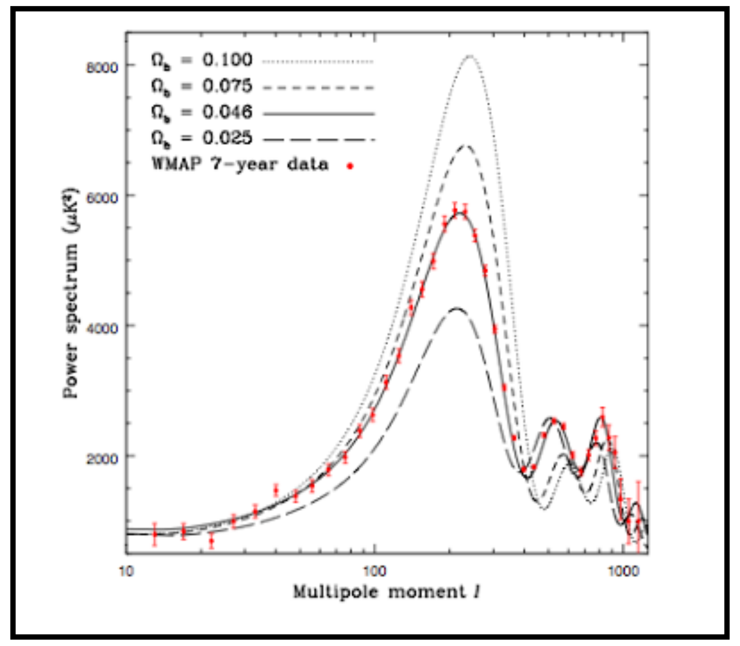
**We present a mechanism for the mass generation of Majoron dark matter via the type-II seesaw mechanism!**

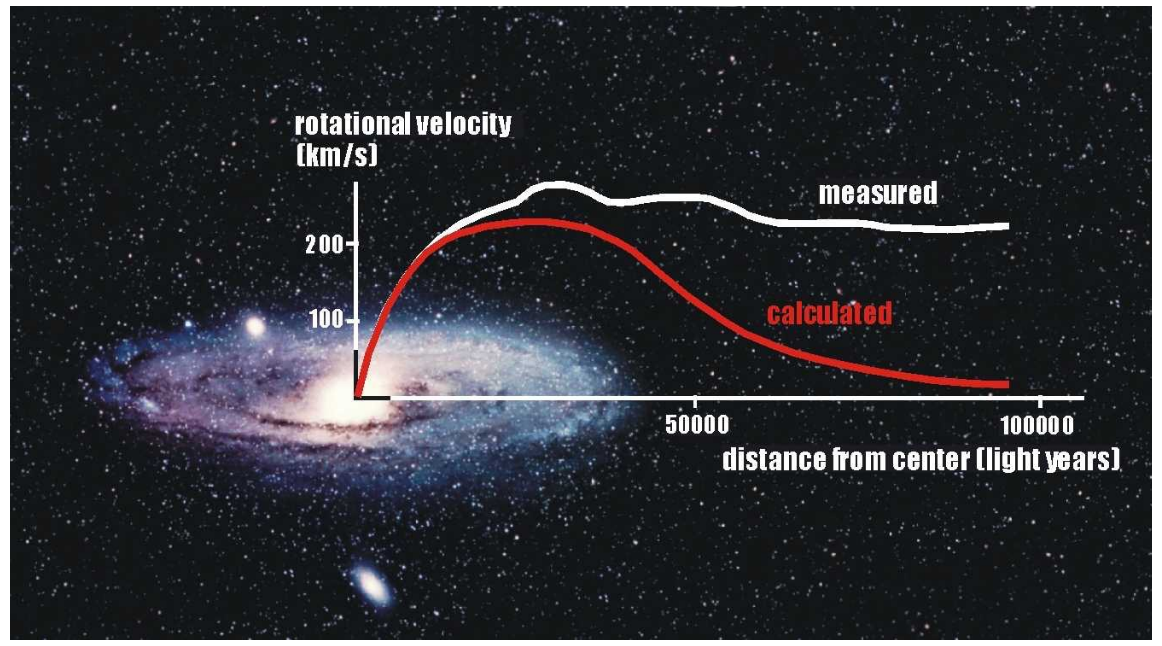
## II: Constraints on the ALP parameter space

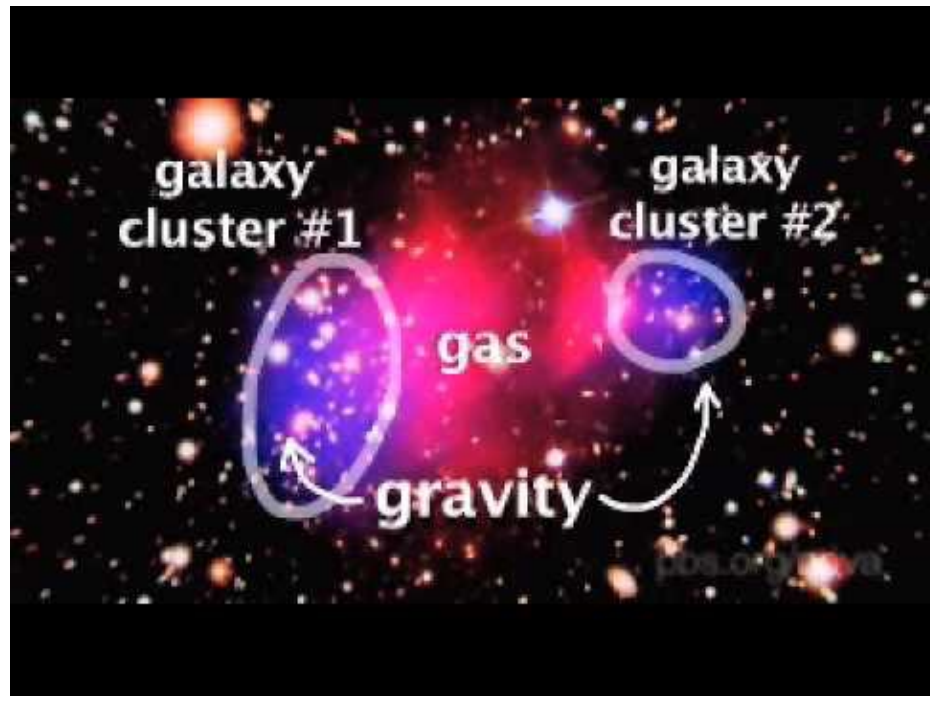


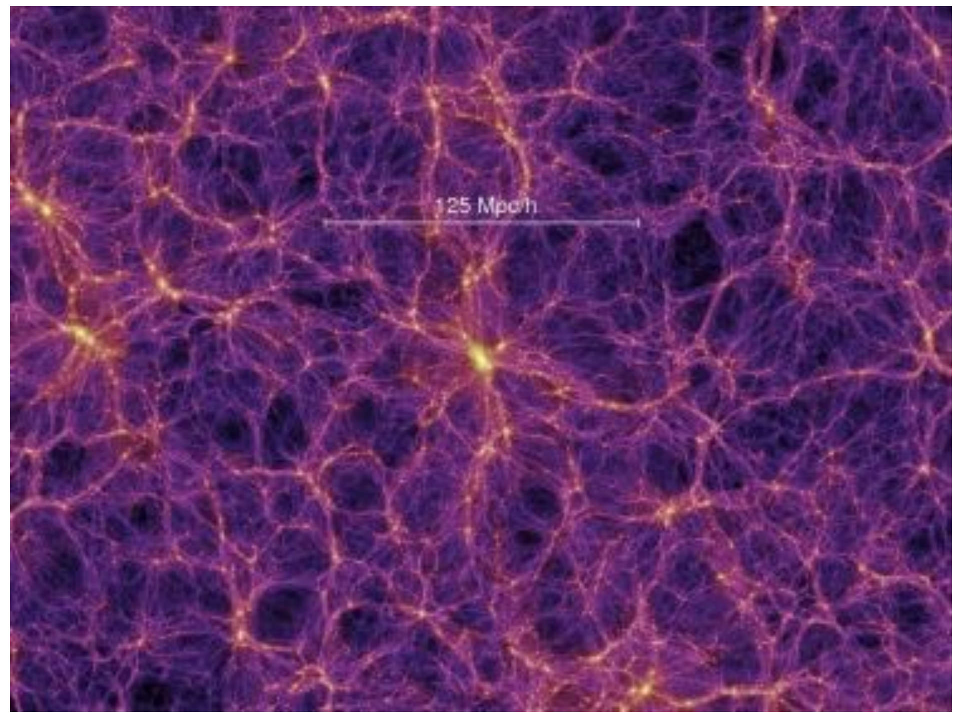
# New physics beyond the SM-dark matter

## Evidence of dark matter



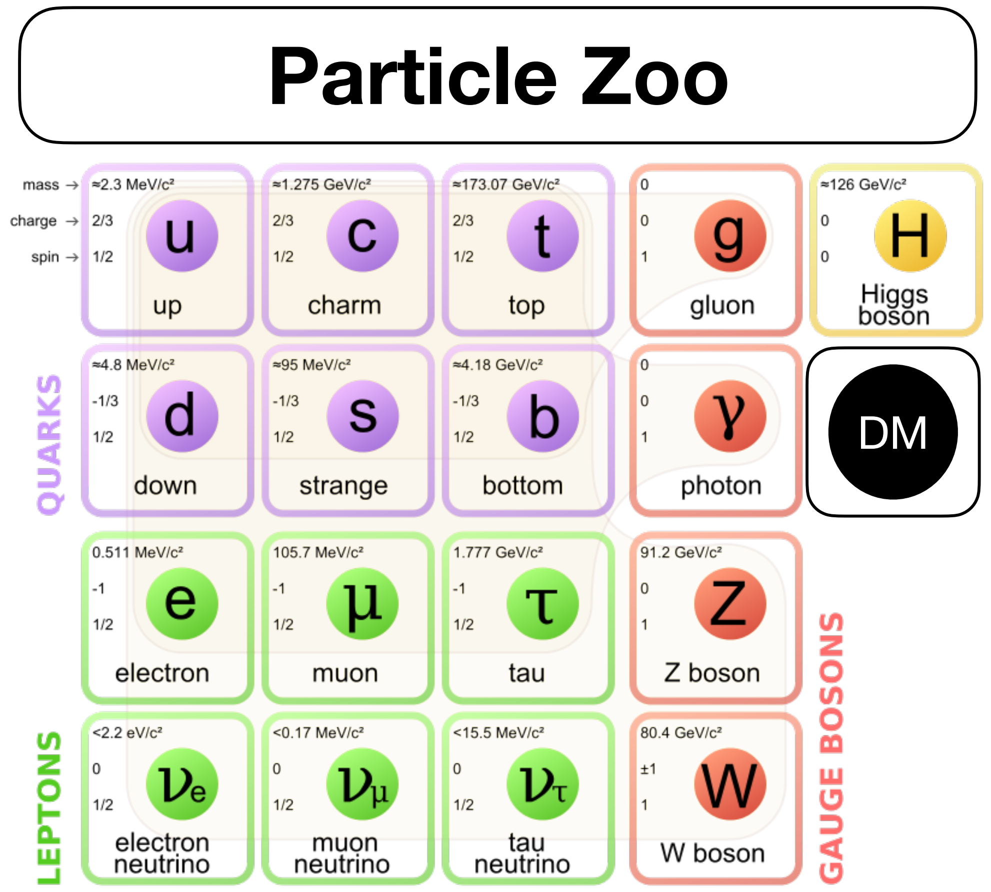



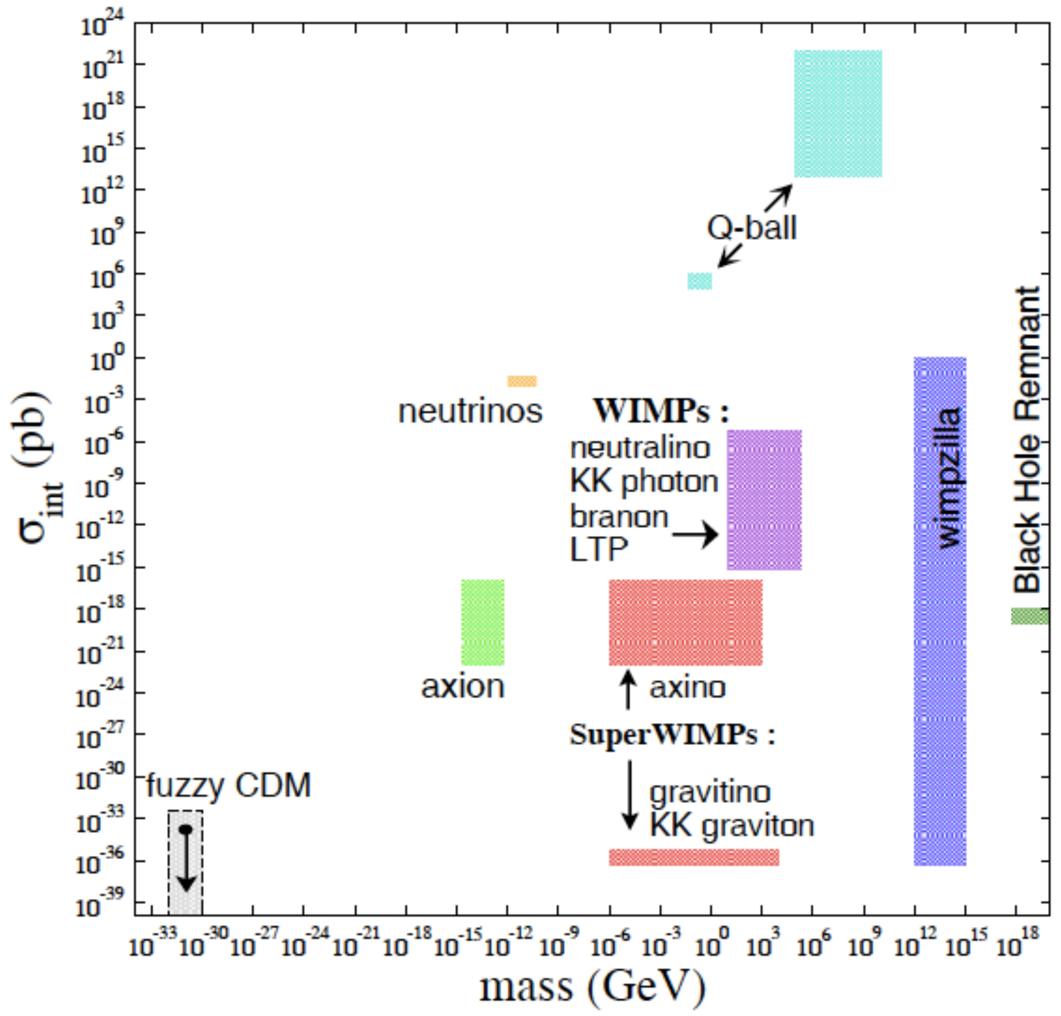




## What is dark matter?

~~Mass~~ ~~Spin~~ ~~Interactions~~

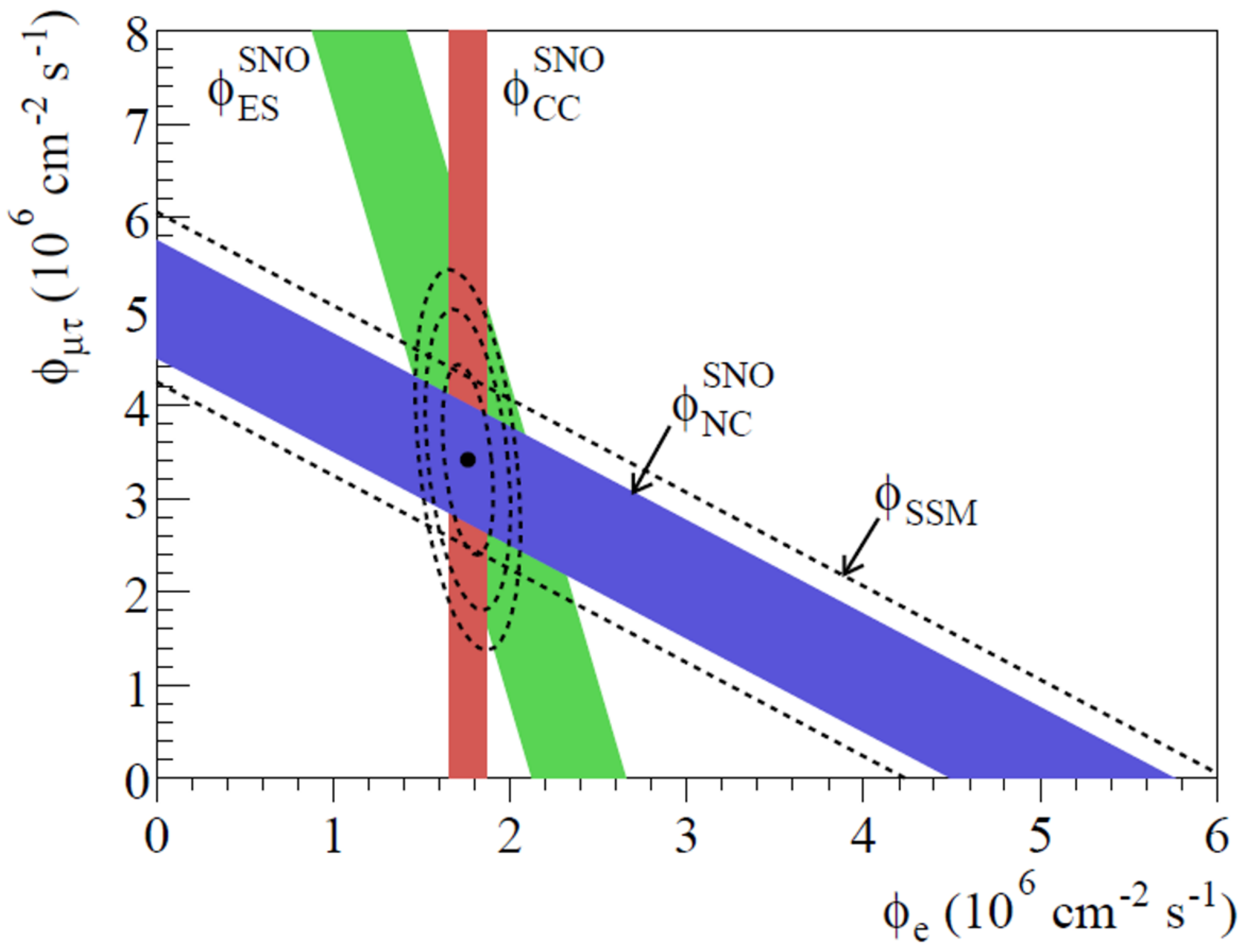
Neutral, non-baryonic, weakly interacting particle!



# New physics beyond the SM-neutrino physics

## Evidence of of neutrino oscillations



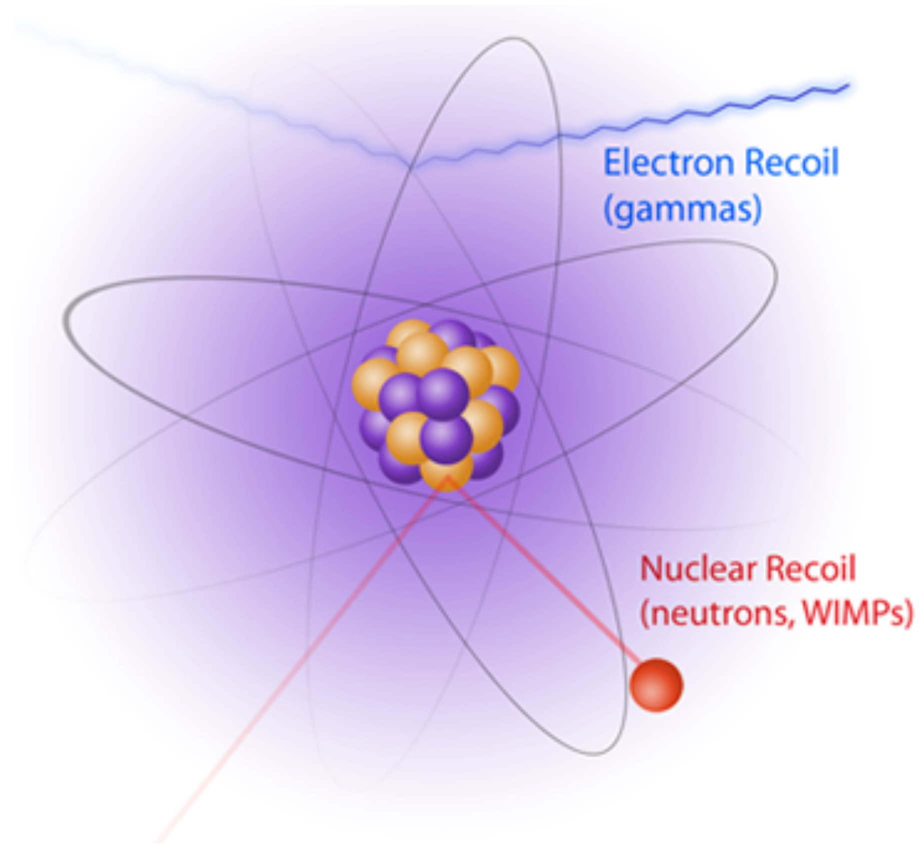

## Key issues for neutrino physics

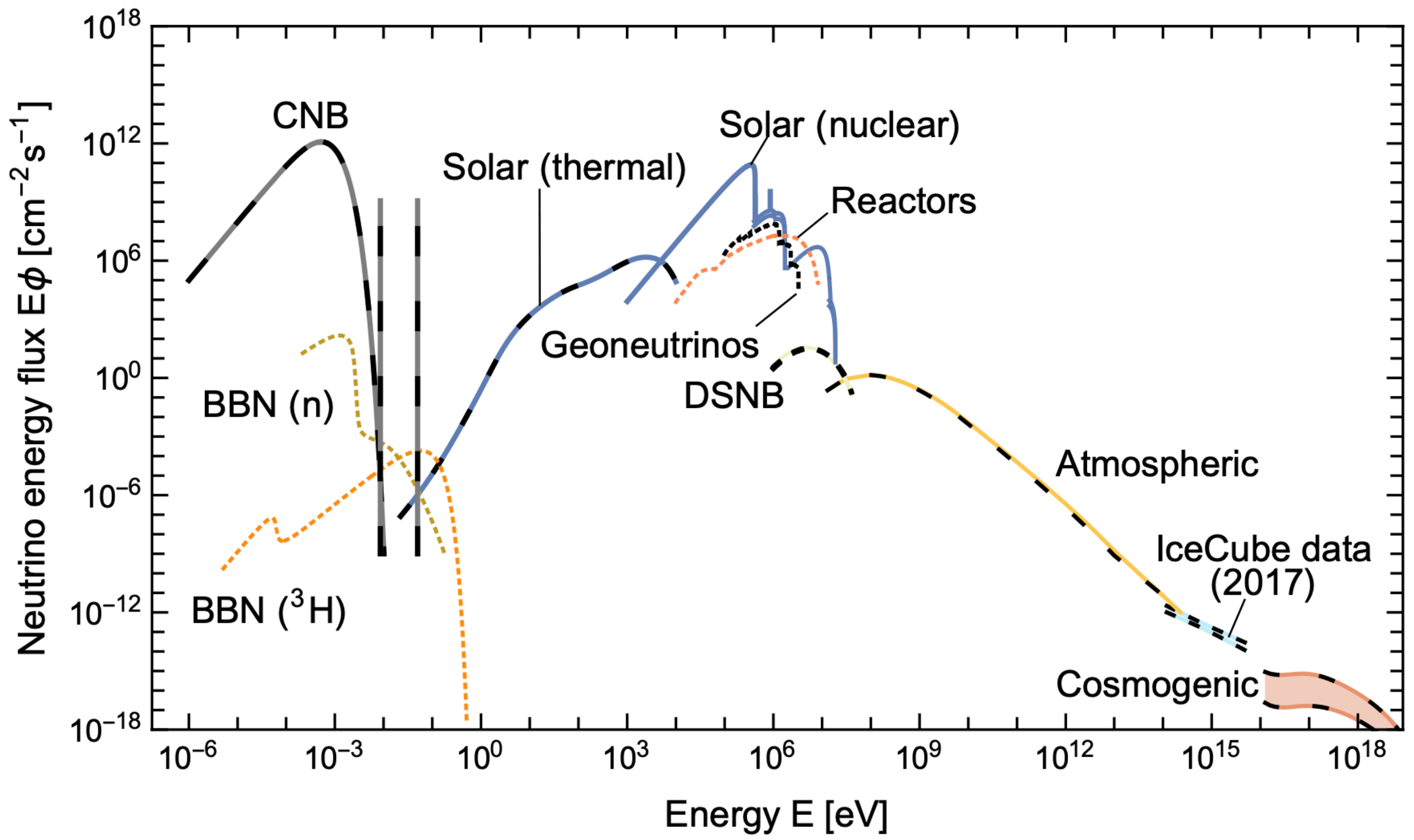
| |
|---|
| The origin of tiny but non-zero neutrino masses |
| Neutrino mass hierarchy |
| CP-violating phase in lepton mixing matrix |
| Is neutrino Majorana or Dirac particle? |
| …… |

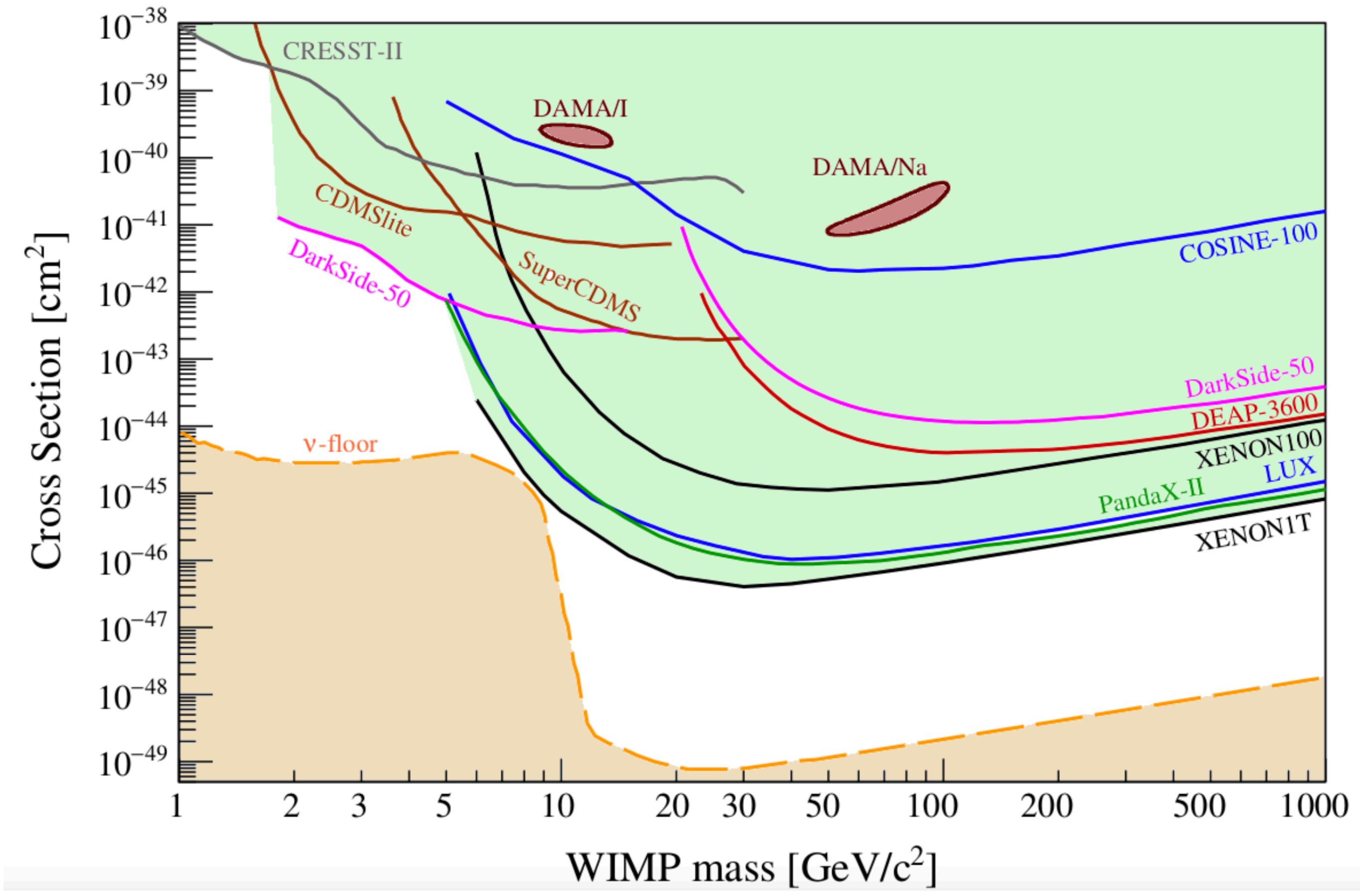
# Is neutrino physics correlated with DM?

**Yes!**

- Properties of neutrinos are similar to these of dark matter
- Neutrino is a **hot** dark matter candidate
- Sterile neutrino is typical **warm/cold** dark matter candidate
- The signal of neutrino in direct detection experiments is similar to that of DM



# The issue we concern in this talk

Standard model:  Lepton number is accidental global U(1) symmetry

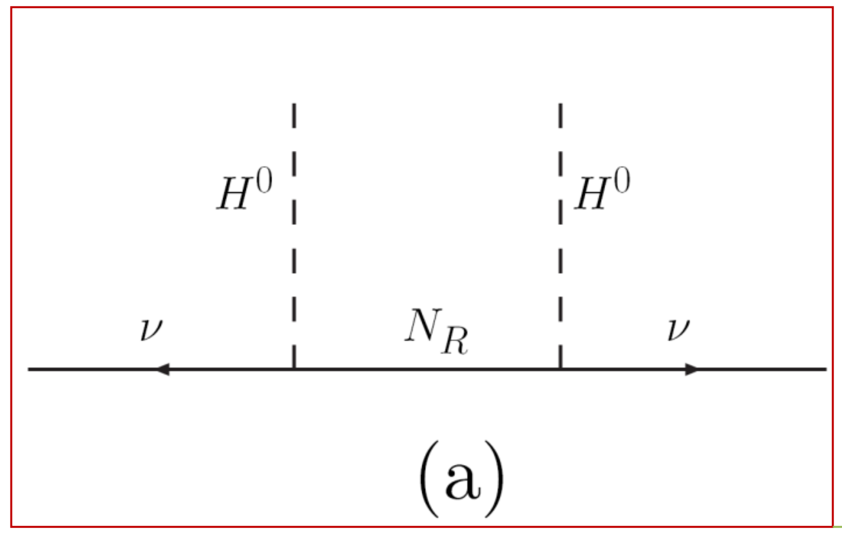
Traditional seesaw mechanisms: $U(1)_L$ is explicitly broken  at the tree-level

Is lepton number spontaneously broken, or explicitly broken, or both ?

Connections between neutrino mass and the ALP physics!

# Neutrino mass generations

**History: Majorana neutrino mass from the dim-5 Weinberg operator** $\kappa_{\alpha\beta}\overline{\ell_L^{\alpha}}\tilde{H}\tilde{H}^T\ell_L^{\beta C}$

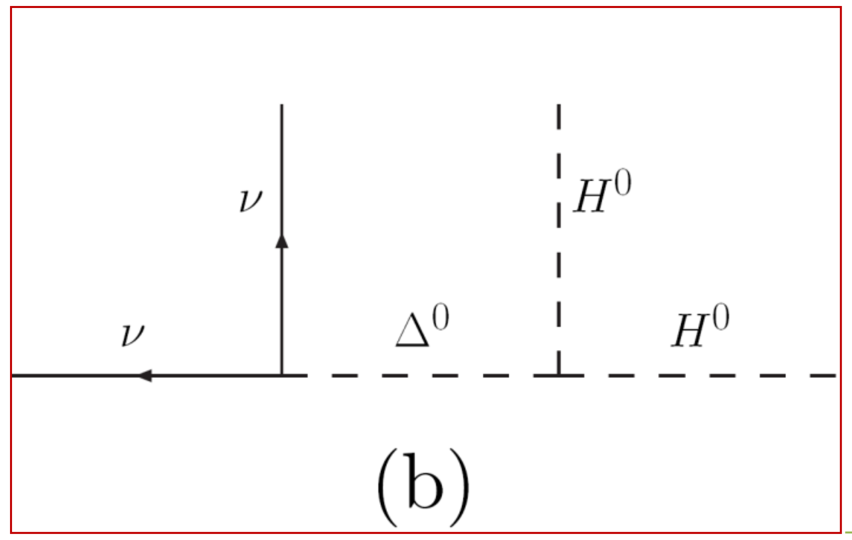



$$-\mathcal{L} = \overline{\ell_L}Y_\nu N_R\tilde{H} + \frac{1}{2}\overline{N_R^{\rm C}}M_R N_R + {\rm h.c.}$$

SU(2)_L fermion singlet

$$M_\nu = -M_D M_R^{-1} M_D^T$$

Minkowski（77）

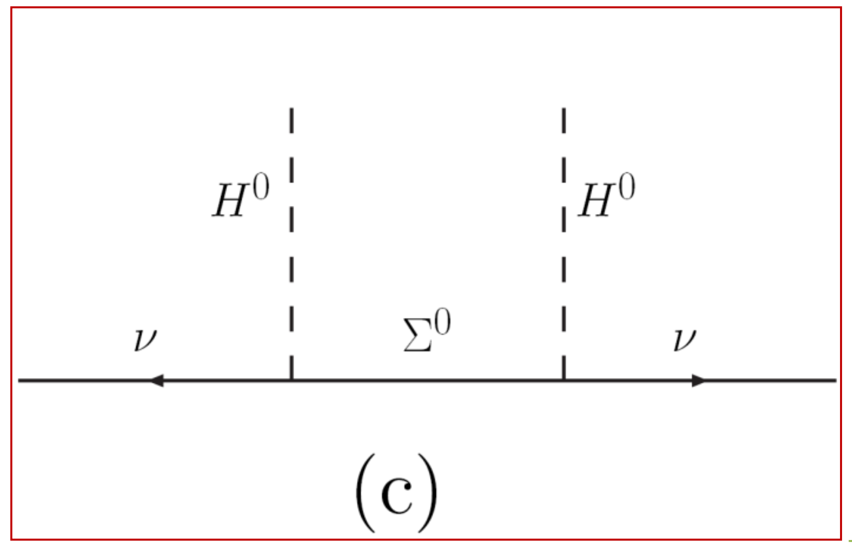



$$-\mathcal{L} = \frac{1}{2}\overline{\ell_L}Y_\Delta\Delta\varepsilon\ell_L^{\rm C} - \lambda_\Delta M_\Delta H^T\varepsilon\Delta H + {\rm h.c.}$$

SU(2)_L scalar triplet

$$M_\nu = Y_\Delta v_\Delta$$

Magg， Wetterich（80）

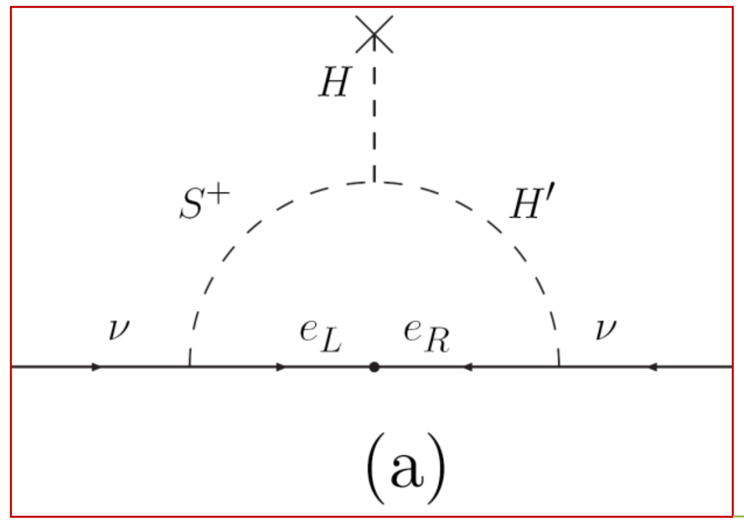



$$-\mathcal{L} = \frac{1}{2}{\rm Tr}\left[\Sigma M_\Sigma\Sigma^{\rm C}\right] + \sqrt{2}\overline{\ell_L}Y_\Sigma\Sigma\tilde{H} + {\rm h.c.}$$

SU(2)_L fermion triplet

$$M_\nu = -M_D M_\Sigma^{-1} M_D^T$$

Foot，Lew， He，Joshi（89）

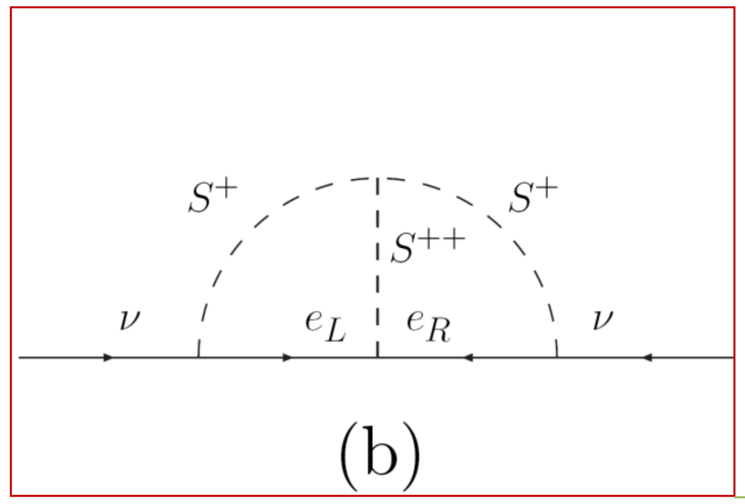



$$-\mathcal{L} = \ell_L^T Y_S\varepsilon\ell_L S^+ - \mu H^T\varepsilon H' S^- + {\rm h.c.}$$

Zee（80）

$$(M_\nu)_{\alpha\beta} = A\,(Y_S)_{\alpha\beta}\,(m_\alpha^2 - m_\beta^2)$$

Charged scalar singlet+ doublets

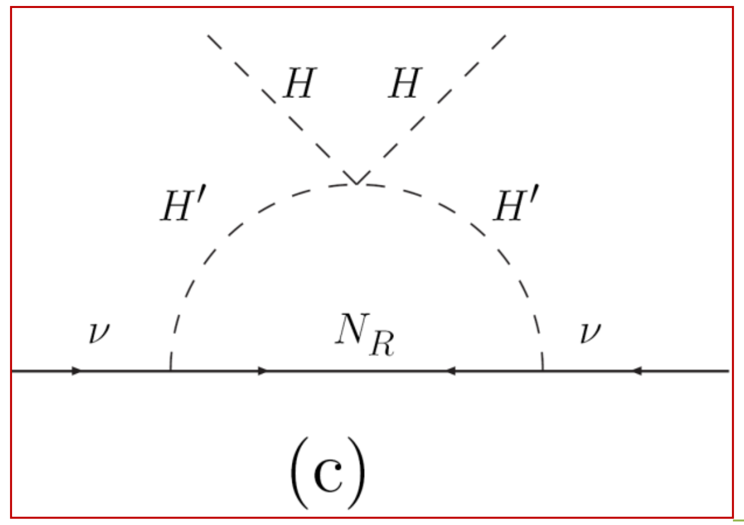



$$-\mathcal{L} = \ell_L^T Y_S\varepsilon\ell_L S^+ + e_R^T F_S e_R S^{++} + \mu S^- S^- S^{++} + {\rm h.c.}$$

Babu（88）

$$(M_\nu)_{\alpha\beta} = 8\mu\sum_{\kappa\lambda} m_\kappa m_\lambda\,(Y_S)_{\alpha\kappa}\,(F_S)_{\kappa\lambda}\,(Y_S)_{\lambda\beta}\,I_{\kappa\lambda}$$

Two charged scalar singlets




$$-\mathcal{L} = \overline{\ell_L}Y_\nu N_R\tilde{H}' + \frac{1}{2}\overline{N_R^{\rm C}}M_R N_R + \frac{1}{2}\lambda_5(H^\dagger H')^2 + {\rm h.c.}$$

Ma （98）

$$(M_\nu)_{\alpha\beta} = \sum_i (Y_\nu)_{\alpha i}M_i^{-1}(Y_\nu)_{\beta i}I(M_i^2/m_0^2)$$

Higgs doublets + Majorana neutrino+ Z2 symmetry

● ● ●

Y. Cai, T. Han, T. Li, 1711.02180

# Neutrino mass generations

## Neutrino mass from higher dimensional effective operators

**Majorana neutrino mass the tree-level:**

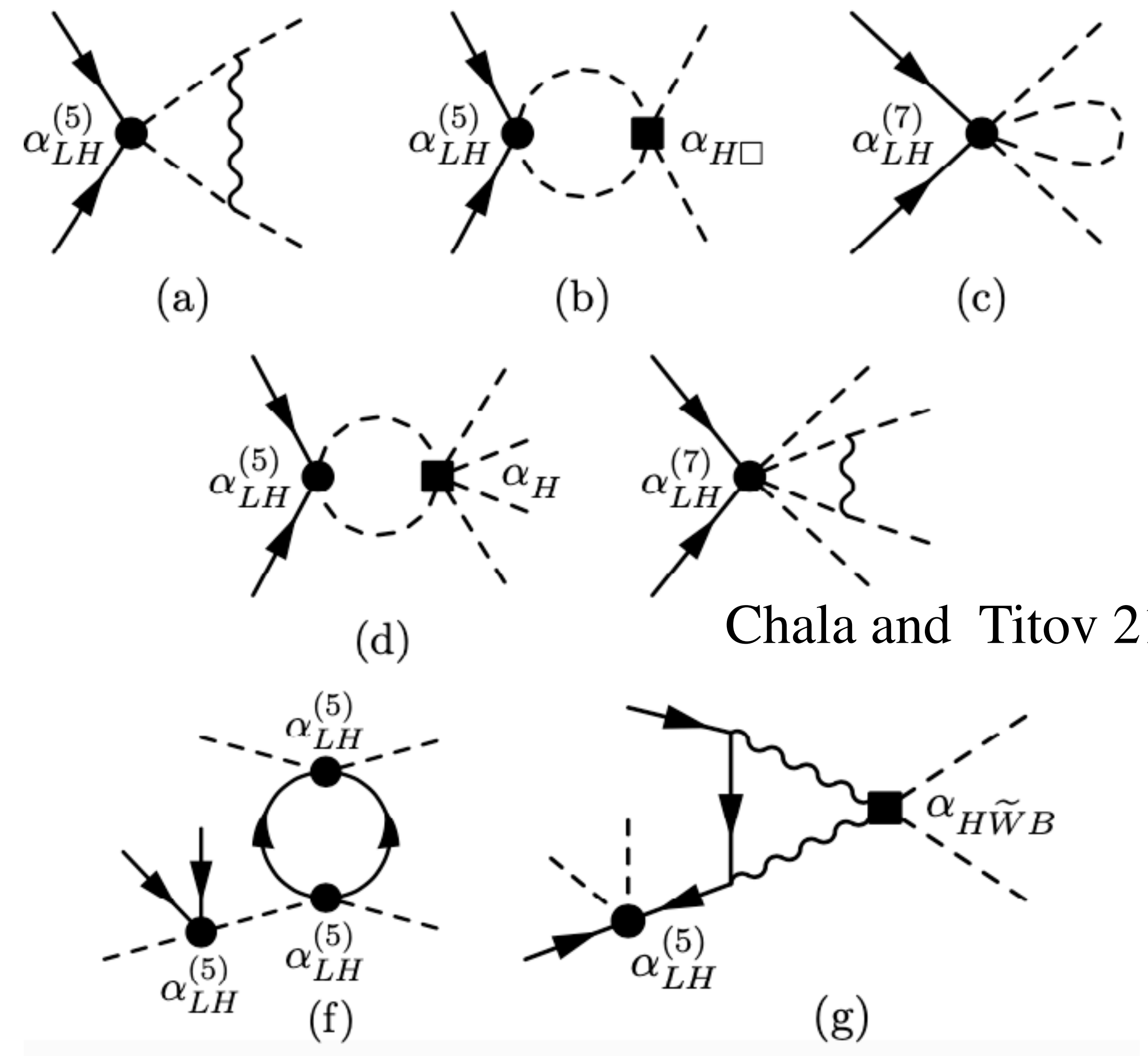
(F. Bonnet et al, 2009; Y. Liao, 2011)

**The unique operator of dim 2n+5, that can give neutrino masses at the tree level is**

$$\mathcal{O}^{2n+5} = \mathcal{O}_{\text{weinberg}} \times \frac{(H^\dagger H)^n}{\Lambda^{2n}}$$

**Neutrino mass from loop corrections:**

- *Dimension-5: Weinberg operator for neutrino-masses (S. Weinberg 1979)*
- *Dimension-6: W. Buchmuller and D. Wyler, 1986; B. Grzadkowski et al, 2010; ……*
- *Dimension-7: L. Lehman, 2014; Y. Liao and X.D. Ma, 2016; ……*
  *Dimension-8: C.W. Murphy, 2020; H.L. Li et al., 2020; … …*
- *Dimension-9: Y. Liao and X.D. Ma, 2020; H.L. Li et al, 2020, 2021; … …*




Chala and Titov 2104.08248

| | |
|---|---|
| $\mathcal{O}_{2B}$ | $-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$ |
| $\mathcal{O}_{2W}$ | $-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$ |
| $\mathcal{O}_{BDH}$ | $\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$ |
| $\mathcal{O}_{WDH}$ | $D_\nu W^{I\mu\nu}(H^\dagger i\overleftrightarrow{D}^I_\mu H)$ |
| $\mathcal{O}_{DH}$ | $(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$ |
| $\mathcal{O}'_{HD}$ | $(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$ |
| $\mathcal{O}''_{HD}$ | $(H^\dagger H)D_\mu(H^\dagger i\overleftrightarrow{D}^\mu H)$ |
| $\mathcal{O}_{LD}$ | $\frac{i}{2}\overline{L}\{D_\mu D^\mu, \not{D}\}L$ |
| $\mathcal{O}'^{(1)}_{HL}$ | $(H^\dagger H)(\overline{L}i\overleftrightarrow{\not{D}}L)$ |
| $\mathcal{O}''^{(1)}_{HL}$ | $\partial_\mu(H^\dagger H)(\overline{L}\gamma^\mu L)$ |
| $\mathcal{O}'^{(3)}_{HL}$ | $(H^\dagger\sigma^I H)(\overline{L}i\overleftrightarrow{\not{D}}^I L)$ |
| $\mathcal{O}''^{(3)}_{HL}$ | $D_\mu(H^\dagger\sigma^I H)(\overline{L}\gamma^\mu\sigma^I L)$ |
| $\mathcal{O}^{(R)}_{LHD}$ | $\epsilon_{ij}\epsilon_{mn}L^iCL^mH^j\Box H^n$ |

# Majoron & neutrino mass via type-II seesaw

**Type-II seesaw + spontaneous breaking $U(1)_L$ symmetry**

LNV term!

$$V(S,\Phi,\Delta) = V(\Phi,\Delta) - \mu_S^2(S^\dagger S) + \lambda_6(S^\dagger S)^2$$
$$+\lambda_7(S^\dagger S)(\Phi^\dagger\Phi) + \lambda_8(S^\dagger S)\mathrm{Tr}(\Delta^\dagger\Delta) + \mu\Phi^T i\tau_2\Delta^\dagger\Phi + \lambda S\Phi^T i\tau_2\Delta^\dagger\Phi + \mathrm{h.c.},$$

$$\Phi = \begin{pmatrix} \phi^+ \\ \dfrac{v_\phi + \phi + i\chi}{\sqrt{2}} \end{pmatrix} \qquad \Delta = \begin{pmatrix} \dfrac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \dfrac{v_\Delta + \delta + i\xi}{\sqrt{2}} & \dfrac{\Delta^+}{\sqrt{2}} \end{pmatrix} \qquad S = \frac{v_s + \tilde{s} + i\tilde{a}}{\sqrt{2}}$$

$\tilde{a}$ : **Majoron**

**Yukawa Interaction** $\quad -\mathscr{L}_\Delta = Y_{\alpha\beta}\overline{\ell_L^{\alpha C}} i\sigma^2\Delta\ell_L^\beta + h.c.$

**Key term:** $\quad \mu\Phi^T i\sigma^2\Delta\Phi + h.c.$

# Majoron & neutrino mass via type-II seesaw

## Gauge boson masses

$$m_W^2 = \frac{g^2}{4}\left(v_\phi^2 + 2v_\Delta^2\right), \quad m_Z^2 = \frac{g^2}{4\cos^2\theta_W}\left(v_\phi^2 + 4v_\Delta^2\right). \qquad \rho \equiv \frac{m_W^2}{m_Z^2\cos^2\theta_W} = \frac{1+\frac{2v_\Delta^2}{v_\phi^2}}{1+\frac{4v_\Delta^2}{v_\phi^2}}.$$

## Scalar mixings and masses

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \mathscr{R}(\beta)\begin{pmatrix} \phi^\pm \\ \Delta^\pm \end{pmatrix}, \quad \begin{pmatrix} G \\ A \\ a \end{pmatrix} = \mathscr{V}(\beta_1', \beta_2', \beta_3')\begin{pmatrix} \chi \\ \xi \\ \tilde{a} \end{pmatrix}, \quad \begin{pmatrix} h \\ H \\ s \end{pmatrix} = \mathscr{U}(\alpha_1, \alpha_2, \alpha_3)\begin{pmatrix} \phi \\ \delta \\ \tilde{s} \end{pmatrix},$$

## Mixing angl for pseudo-scalars

Majoron gets non-zero mass from the mixing!

$$\tan\beta = \frac{\sqrt{2}v_\Delta}{v_\phi}, \quad \tan\beta_1' = \frac{2v_\Delta}{v_\phi}, \quad \tan\beta_2' = 0, \quad \tan 2\beta_3' = \frac{2\lambda v_\Delta v_s v_\phi\sqrt{v_\phi^2 + 4v_\Delta^2}}{v_\phi^2\left(-\lambda v_\Delta^2 + \lambda v_s^2 + \sqrt{2}\mu v_s\right) + 4v_\Delta^2 v_s\left(\sqrt{2}\mu + \lambda v_s\right)}.$$

# Majoron & neutrino mass via type-II seesaw

## Sequential breaking of various symmetries

Majoron massive!

massless Majoron!

Neutrino massive

EWSB scale

fa $\cancel{U}(1)_L$

T

Temperature of Universe

$$(m_\nu)_{\alpha\beta} = y_{\alpha\beta} v_\Delta / \sqrt{2}\,.$$

$$m_a^2 = \frac{\sqrt{2}\mu v_\phi^2 v_\Delta (v_\phi^2 + 4v_\Delta^2)}{2v_\phi^2(v_\Delta^2 + v_s^2) + 8v_\Delta^2 v_s^2} \simeq \frac{\mu v_\phi^2 v_\Delta}{\sqrt{2} v_s^2}\,,$$

# For experts of axion physics

**Majoron mass should arise from cosine like potential!**

$$\ell_L \to e^{-\frac{ia}{2f}} \ell_L \qquad S \to e^{+\frac{ia}{f}} S$$

$$E_R \to e^{-\frac{ia}{2f}} E_R \qquad \Delta \to e^{-\frac{ia}{f}} \Delta$$

$$H \to H$$

$$-\mathscr{L}_{\rm int} \supset \mu e^{i\frac{a}{f_a}} \Phi^T i\tau_2 \Delta^\dagger \Phi + \mathrm{h.c.}.$$

**After electroweak symmetry breaking**

$$-\mathscr{L}_{\rm int} \supset \frac{\mu v_\Phi^2 v_\Delta}{\sqrt{2}} \cos\left(\frac{a}{f_a}\right)$$

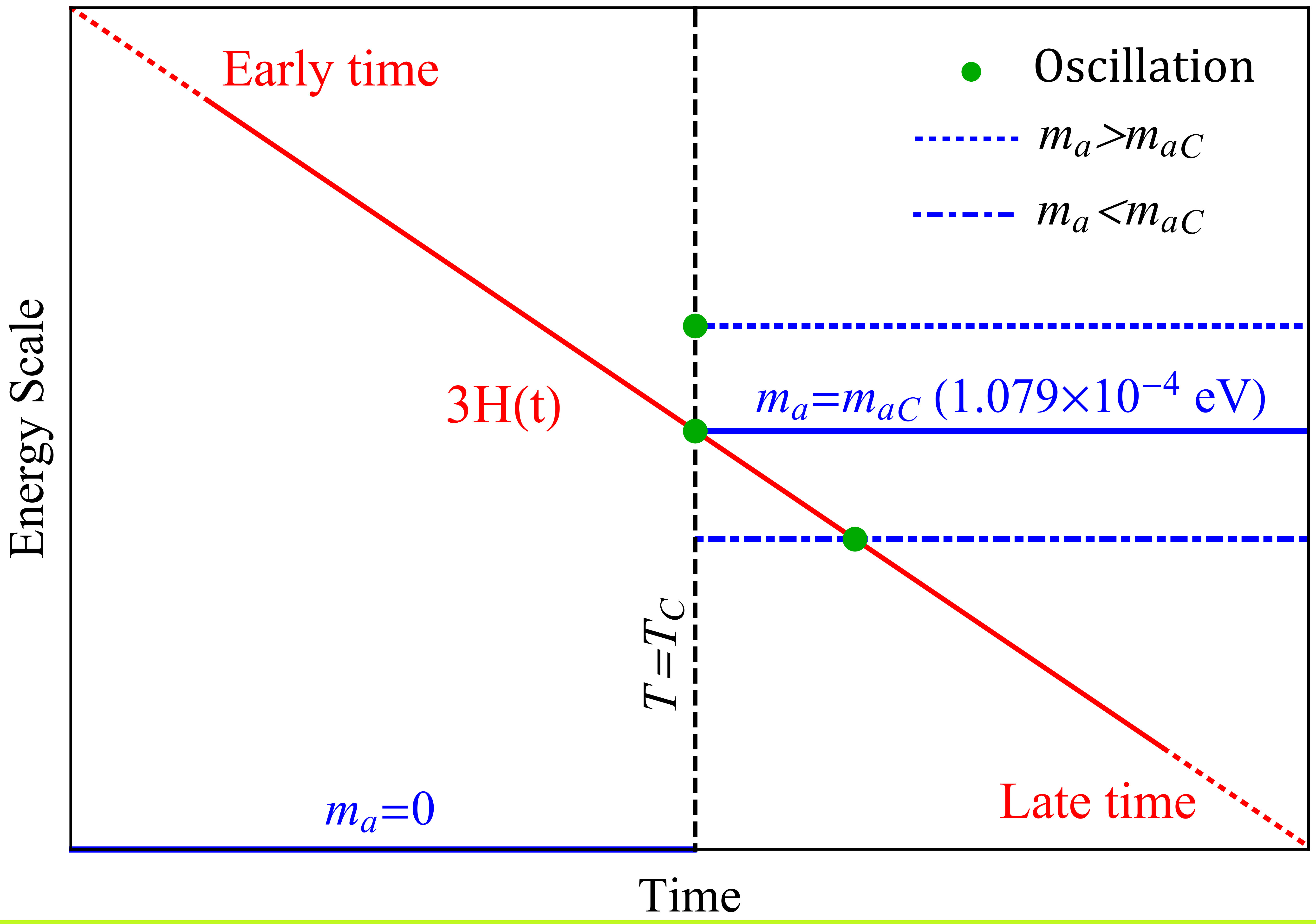
**Non-zero Majoron mass**

# Majoron DM—oscillation time

$$m_a^2(T) = \begin{cases} \dfrac{\mu v_\phi^2(T) v_\Delta(T)}{\sqrt{2} f_a^2} , & T \leq T_C \\ 0 , & T > T_C \end{cases}$$

$$T_{\rm osc} = \begin{cases} T_* , & m_a < m_{aC} \\ T_C , & m_a \geq m_{aC} \end{cases}$$

$$m_{aC} = 1.079 \times 10^{-4}\,{\rm eV}$$



# Majoron DM—simulations

**Equation of motion**

$$\ddot{\theta} + 3H\dot{\theta} + m_a^2(T)\theta = 0$$

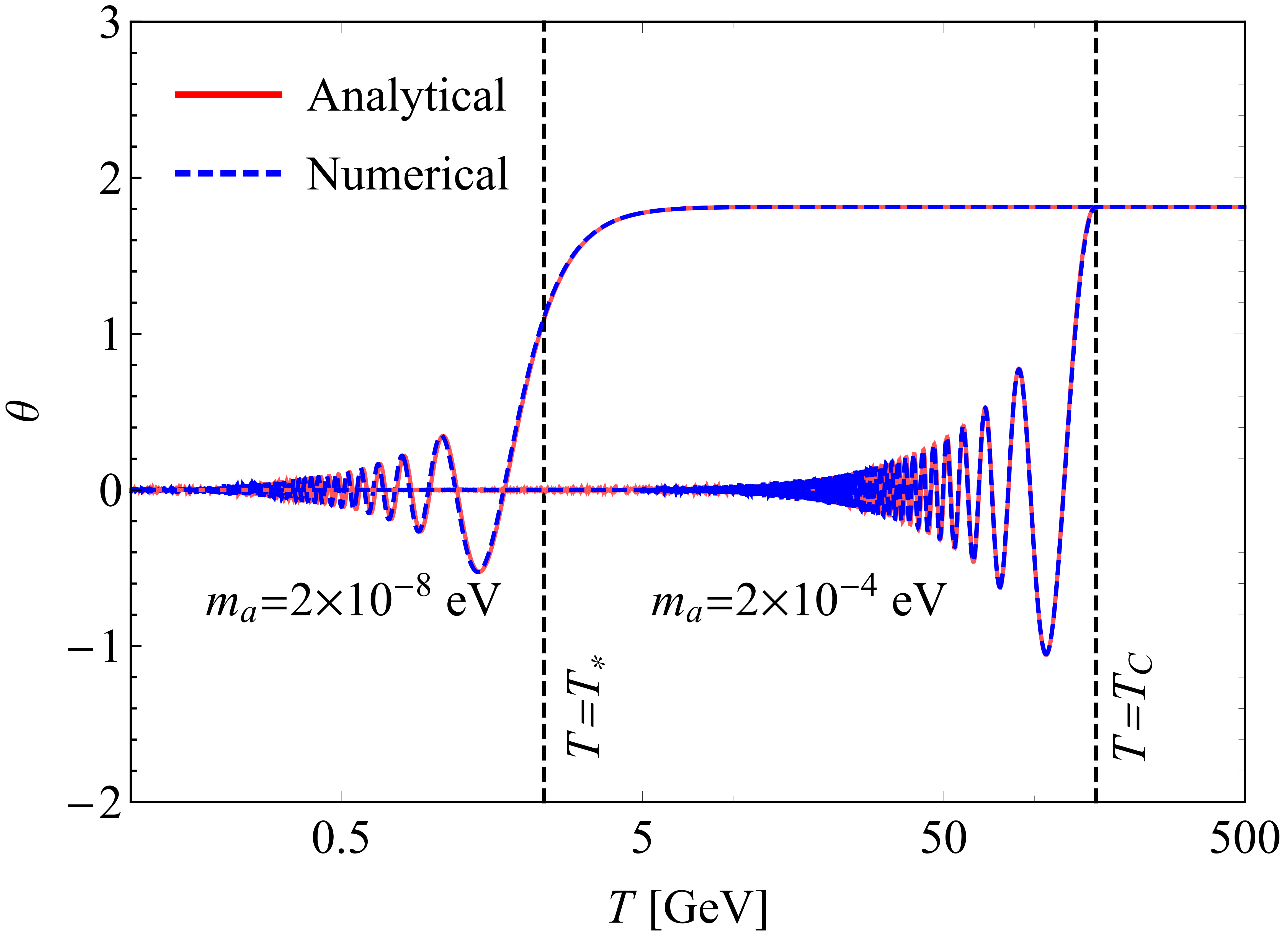
**Analytical results**

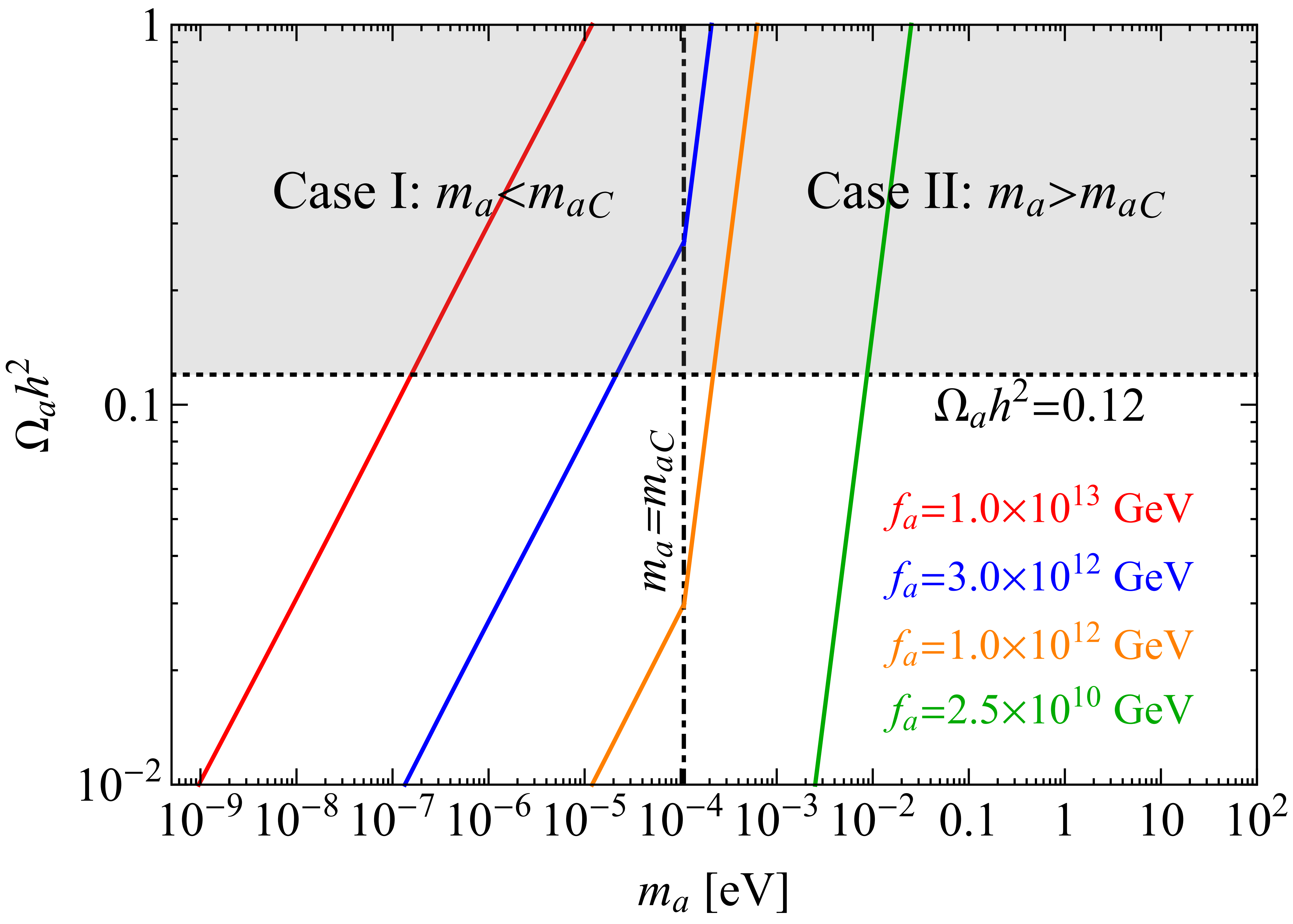
$$\begin{aligned}\theta(t) = -\pi\Big[ & -2m_at_iJ_{\frac{1}{4}}(m_at)Y_{-\frac{3}{4}}(m_at_i) + 2m_at_iY_{\frac{1}{4}}(m_at)J_{-\frac{3}{4}}(m_at_i) \\ & -Y_{\frac{1}{4}}(m_at)J_{\frac{1}{4}}(m_at_i) - 2m_at_iY_{\frac{1}{4}}(m_at)_{\frac{5}{4}}(m_at_i) + J_{\frac{1}{4}}(m_at)Y_{\frac{1}{4}}(m_at_i) \\ & +2m_at_iJ_{\frac{1}{4}}(m_at)Y_{\frac{5}{4}}(m_at_i)\Big] \Big/ \Big\{ 2\sqrt{3}\,t^{\frac{1}{4}}t_i^{\frac{3}{4}}\Big[J_{\frac{1}{4}}(m_at_i)Y_{-\frac{3}{4}}(m_at_i) - J_{-\frac{3}{4}}(m_at_i)Y_{\frac{1}{4}}(m_at_i) \\ & +J_{\frac{5}{4}}(m_at_i)Y_{\frac{1}{4}}(m_at_i) - J_{\frac{1}{4}}(m_at_i)Y_{\frac{5}{4}}(m_at_i)\Big]\end{aligned}$$

**Majoron energy density**

$$\rho_a(T_0) = \frac{1}{2}m_a^2f_a^2\langle\theta_{a,i}^2\rangle\frac{g_{*s}(T_0)}{g_{*s}(T_{\rm osc})}\left(\frac{T_0}{T_{\rm osc}}\right)^3$$

# Majoron DM—Relic Density



# Majoron interactions from mixings

## Interactions with scalars

| Vertices | Coefficients |
|---|---|
| $a^4$ | $\frac{1}{4}\lambda_1 V_{13}^4 + \frac{1}{4}\lambda_4 V_{13}^2 V_{23}^2 + \frac{1}{4}\lambda_5 V_{13}^2 V_{23}^2 + \frac{1}{2}\lambda V_{13}^2 V_{23} V_{33} + \frac{1}{4}\lambda_2 V_{23}^4 + \frac{1}{4}\lambda_3 V_{23}^4 + \frac{1}{4}\lambda_6 V_{33}^4$ |
| $a^3 G$ | $\lambda_1 V_{11} V_{13}^3 + \frac{1}{2}\lambda_4 V_{11} V_{13} V_{23}^2 + \frac{1}{2}\lambda_5 V_{11} V_{13} V_{23}^2 + \lambda V_{11} V_{13} V_{23} V_{33} + \frac{1}{2}\lambda_4 V_{13}^2 V_{21} V_{23}$ $+\frac{1}{2}\lambda_5 V_{13}^2 V_{21} V_{23} + \frac{1}{2}\lambda V_{13}^2 V_{21} V_{33} + \frac{1}{2}\lambda V_{13}^2 V_{23} V_{31} + \lambda_2 V_{21} V_{23}^3 + \lambda_3 V_{21} V_{23}^3 + \lambda_6 V_{31} V_{33}^3$ |
| $a^2 h^2$ | $\frac{1}{2}\lambda_1 U_{11}^2 V_{13}^2 + \frac{1}{4}\lambda_4 U_{11}^2 V_{23}^2 + \frac{1}{4}\lambda_5 U_{11}^2 V_{23}^2 - \frac{1}{2}\lambda U_{11}^2 V_{23} V_{33} + \lambda U_{11} U_{21} V_{13} V_{33}$ $-\lambda U_{11} U_{31} V_{13} V_{23} + \frac{1}{4}\lambda_4 U_{21}^2 V_{13}^2 + \frac{1}{4}\lambda_5 U_{21}^2 V_{13}^2 + \frac{1}{2}\lambda_2 U_{21}^2 V_{23}^2 + \frac{1}{2}\lambda_3 U_{21}^2 V_{23}^2 + \frac{1}{2}\lambda U_{21} U_{31} V_{13}^2 + \frac{1}{2}\lambda_6 U_{31}^2 V_{33}^2$ |
| $a^2 h$ | $\lambda_1 U_{11} v_\phi V_{13}^2 + \frac{1}{2}\lambda_4 U_{11} v_\phi V_{23}^2 + \frac{1}{2}\lambda_5 U_{11} v_\phi V_{23}^2 - \lambda U_{11} v_\phi V_{23} V_{33} - \sqrt{2}\mu U_{11} V_{13} V_{23}$ $-\lambda U_{11} V_{13} V_{23} v_s + \lambda U_{11} V_{13} V_{33} v_\Delta + \lambda U_{21} v_\phi V_{13} V_{33} + \frac{1}{\sqrt{2}}\mu U_{21} V_{13}^2 + \frac{1}{2}\lambda_4 U_{21} V_{13}^2 v_\Delta + \frac{1}{2}\lambda_5 U_{21} V_{13}^2 v_\Delta$ $+\frac{1}{2}\lambda U_{21} V_{13}^2 v_s + \lambda_2 U_{21} V_{23}^2 v_\Delta + \lambda_3 U_{21} V_{23}^2 v_\Delta - \lambda U_{31} v_\phi V_{13} V_{23} + \frac{1}{2}\lambda U_{31} V_{13}^2 v_\Delta + \lambda_6 U_{31} V_{33}^2 v_s$ |
| $a^2 G^2$ | $\frac{3}{2}\lambda_1 V_{11}^2 V_{13}^2 + \frac{1}{4}\lambda_4 V_{11}^2 V_{23}^2 + \frac{1}{4}\lambda_5 V_{11}^2 V_{23}^2 + \frac{1}{2}\lambda V_{11}^2 V_{23} V_{33} + \lambda_4 V_{11} V_{13} V_{21} V_{23} + \lambda_5 V_{11} V_{13} V_{21} V_{23}$ $+\lambda V_{11} V_{13} V_{21} V_{33} + \lambda V_{11} V_{13} V_{23} V_{31} + \frac{1}{4}\lambda_4 V_{13}^2 V_{21}^2 + \frac{1}{4}\lambda_5 V_{13}^2 V_{21}^2 + \frac{1}{2}\lambda V_{13}^2 V_{21} V_{31} + \frac{3}{2}\lambda_2 V_{21}^2 V_{23}^2 + \frac{3}{2}\lambda_3 V_{21}^2 V_{23}^2 + \frac{3}{2}\lambda_6 V_{31}^2 V_{33}^2$ |
| $a^2 G^+ G^-$ | $\lambda_1 V_{13}^2 \cos^2\beta + \frac{1}{2}\lambda_4 V_{13}^2 \sin^2\beta + \frac{1}{4}\lambda_5 V_{13}^2 \sin^2\beta + \frac{1}{\sqrt{2}}\lambda_5 V_{13} V_{23} \sin\beta\cos\beta$ $+\sqrt{2}\lambda V_{13} V_{33} \sin\beta\cos\beta + \lambda_2 V_{23}^2 \sin^2\beta + \frac{1}{2}\lambda_3 V_{23}^2 \sin^2\beta + \frac{1}{2}\lambda_4 V_{23}^2 \cos^2\beta$ |
| $a G^3$ | $\lambda_1 V_{11}^3 V_{13} + \frac{1}{2}\lambda_4 V_{11}^2 V_{21} V_{23} + \frac{1}{2}\lambda_5 V_{11}^2 V_{21} V_{23} + \frac{1}{2}\lambda V_{11}^2 V_{21} V_{33} + \frac{1}{2}\lambda V_{11}^2 V_{23} V_{31}$ $+\frac{1}{2}\lambda_4 V_{11} V_{13} V_{21}^2 + \frac{1}{2}\lambda_5 V_{11} V_{13} V_{21}^2 + \lambda V_{11} V_{13} V_{21} V_{31} + \lambda_2 V_{21}^3 V_{23} + \lambda_3 V_{21}^3 V_{23} + \lambda_6 V_{31}^3 V_{33}$ |
| $a h^2 G$ | $\lambda_1 U_{11}^2 V_{11} V_{13} + \frac{1}{2}\lambda_4 U_{11}^2 V_{21} V_{23} + \frac{1}{2}\lambda_5 U_{11}^2 V_{21} V_{23} - \frac{1}{2}\lambda U_{11}^2 V_{21} V_{33} - \frac{1}{2}\lambda U_{11}^2 V_{23} V_{31}$ $+\lambda U_{11} U_{21} V_{11} V_{33} + \lambda U_{11} U_{21} V_{13} V_{31} - \lambda U_{11} U_{31} V_{11} V_{23} - \lambda U_{11} U_{31} V_{13} V_{21} + \frac{1}{2}\lambda_4 U_{21}^2 V_{11} V_{13}$ $+\frac{1}{2}\lambda_5 U_{21}^2 V_{11} V_{13} + \lambda_2 U_{21}^2 V_{21} V_{23} + \lambda_3 U_{21}^2 V_{21} V_{23} + \lambda U_{21} U_{31} V_{11} V_{13} + \lambda_6 U_{31}^2 V_{31} V_{33}$ |
| $a\bar{\nu}\nu$ | $V_{23} m_\nu / v_\Delta$ |

## Interactions with fermions

$$\overline{\nu_L^C} i a \lambda_{a\bar{\nu}\nu} \nu_L + \mathrm{h.c.}$$

$$\to \lambda_{a\bar{\nu}\nu} : V_{23} m_\nu / v_\Delta ,$$

$$Y_E \overline{\ell_L} H E_R + \mathrm{h.c.} \to$$

$$\lambda_{aee} \bar{e} i \gamma_5 e$$

$$\to \lambda_{aee} : V_{13} \frac{m_e}{v_h} ,$$

# Majoron interactions from anomaly

### Schemas

$$-\mathscr{L}_{\rm int} \supset \frac{\lambda}{\sqrt{2}} f_a e^{i\frac{a}{f_a}} \Phi^T i\tau_2 \Delta^\dagger \Phi + {\rm h.c.}.$$

$$\Delta \to \Delta e^{-i\frac{a}{f_a}}$$

$$-\mathscr{L}_{\rm Yukawa} = y_{\alpha\beta} \overline{\ell_L^{\alpha c}} i\tau_2 \Delta' e^{i\frac{a}{f_a}} \ell_L^\beta + \text{h.c.}$$

$$-\mathscr{L}_{\rm Yukawa} = y^E_{\alpha\beta} \overline{\ell_L^{\alpha'}} H e^{\frac{ia}{2f_A}} E_R^\beta + \text{h.c.}$$

$$\left.\begin{aligned} \ell_L &\to e^{\frac{-ia}{2f}} \ell_L \\ E_R &\to e^{\frac{-ia}{2f}} E_R \end{aligned}\right\} \longrightarrow$$

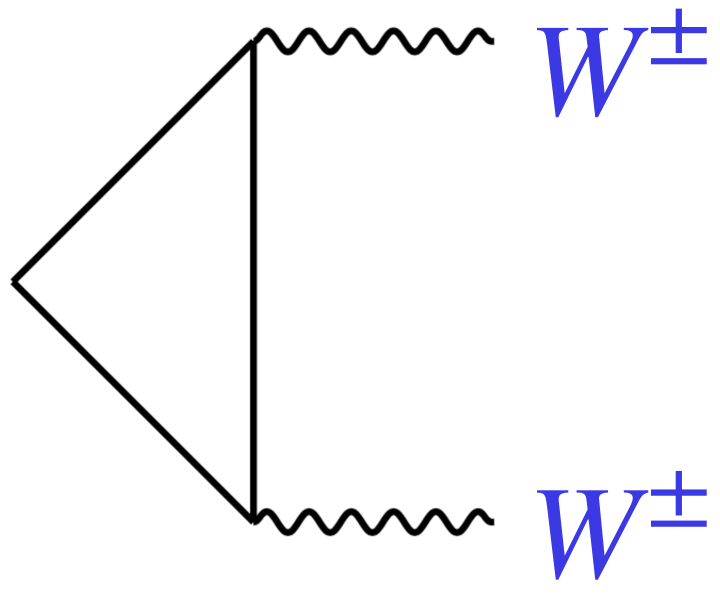
$$\mathscr{L} \to \mathscr{L} - \frac{a}{2f} \partial_\mu \left( \overline{\ell_L} \gamma^\mu \ell_L + \overline{E_R} \gamma^\mu E_R \right)$$

$$= \mathscr{L} - \frac{a}{2f} \partial_\mu J^L_\mu$$

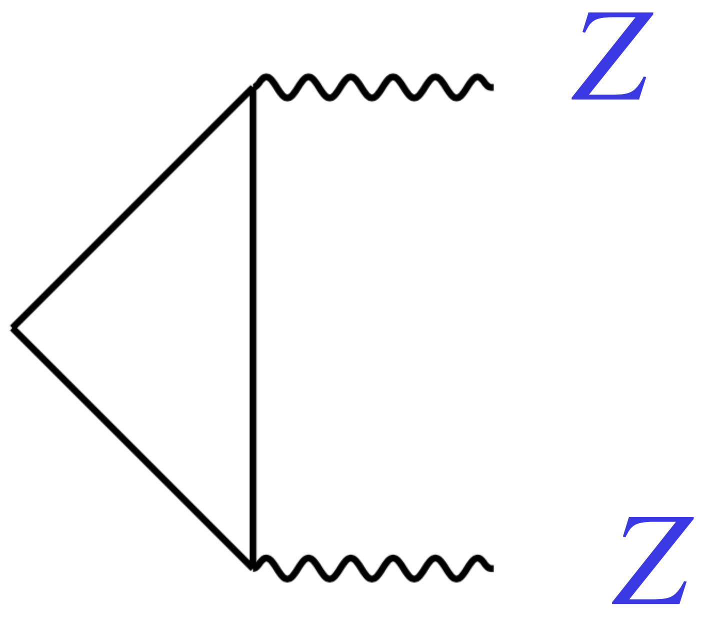
$$= \mathscr{L} + \frac{a}{2f} \frac{N_f}{32\pi^2} \left( g^2 W^a_{\mu\nu} \widetilde{W}^{\mu\nu,a} - g'^2 B_{\mu\nu} \widetilde{B}^{\mu\nu} \right)$$

# Majoron interactions from anomaly

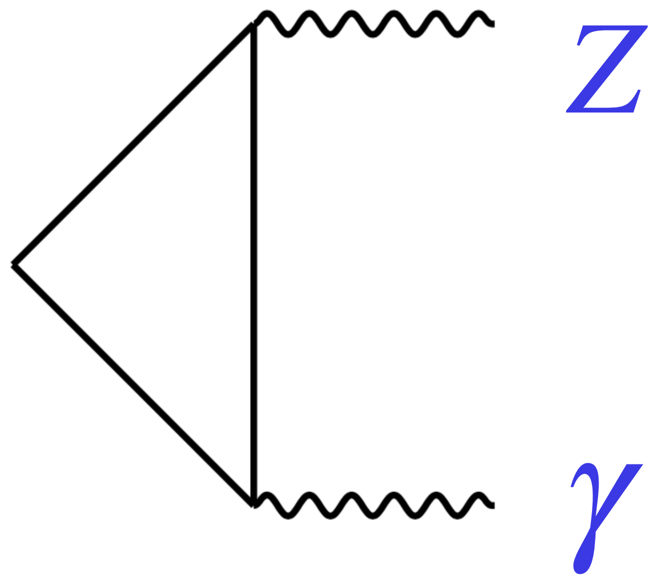
## Interactions in mass eigenstates




$$\frac{\alpha N_f}{16\pi f_a} a W_{\mu\nu} \widetilde{W}^{\mu\nu}$$

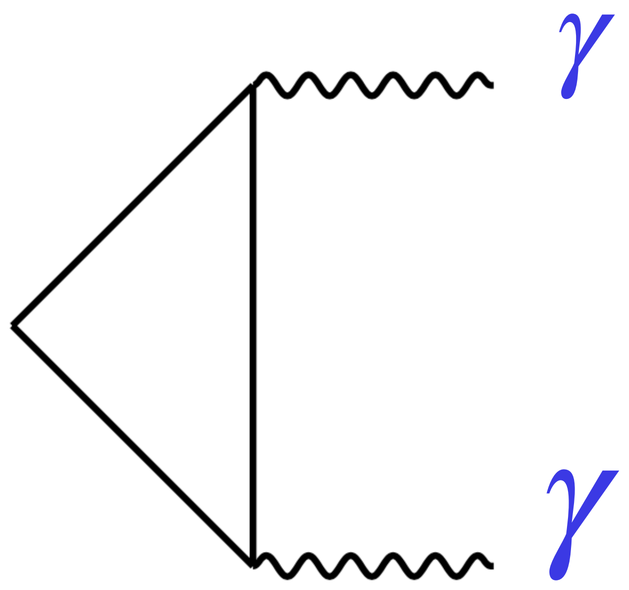



$$\frac{\alpha}{8\pi \cos^2\theta_w f_a}\left(\frac{1}{2} - \sin^2\theta_w\right) a Z_{\mu\nu}\widetilde{Z}^{\mu\nu}$$




$$\frac{\alpha \tan\theta_W}{32\pi f_a} a \left(Z_{\mu\nu}\widetilde{F}^{\mu\nu} + F_{\mu\nu}\widetilde{Z}^{\mu\nu}\right)$$

$$\boxed{\alpha = \frac{g^2}{4\pi}}$$




$$\boxed{0 \times a F_{\mu\nu}\widetilde{F}^{\mu\nu}}$$

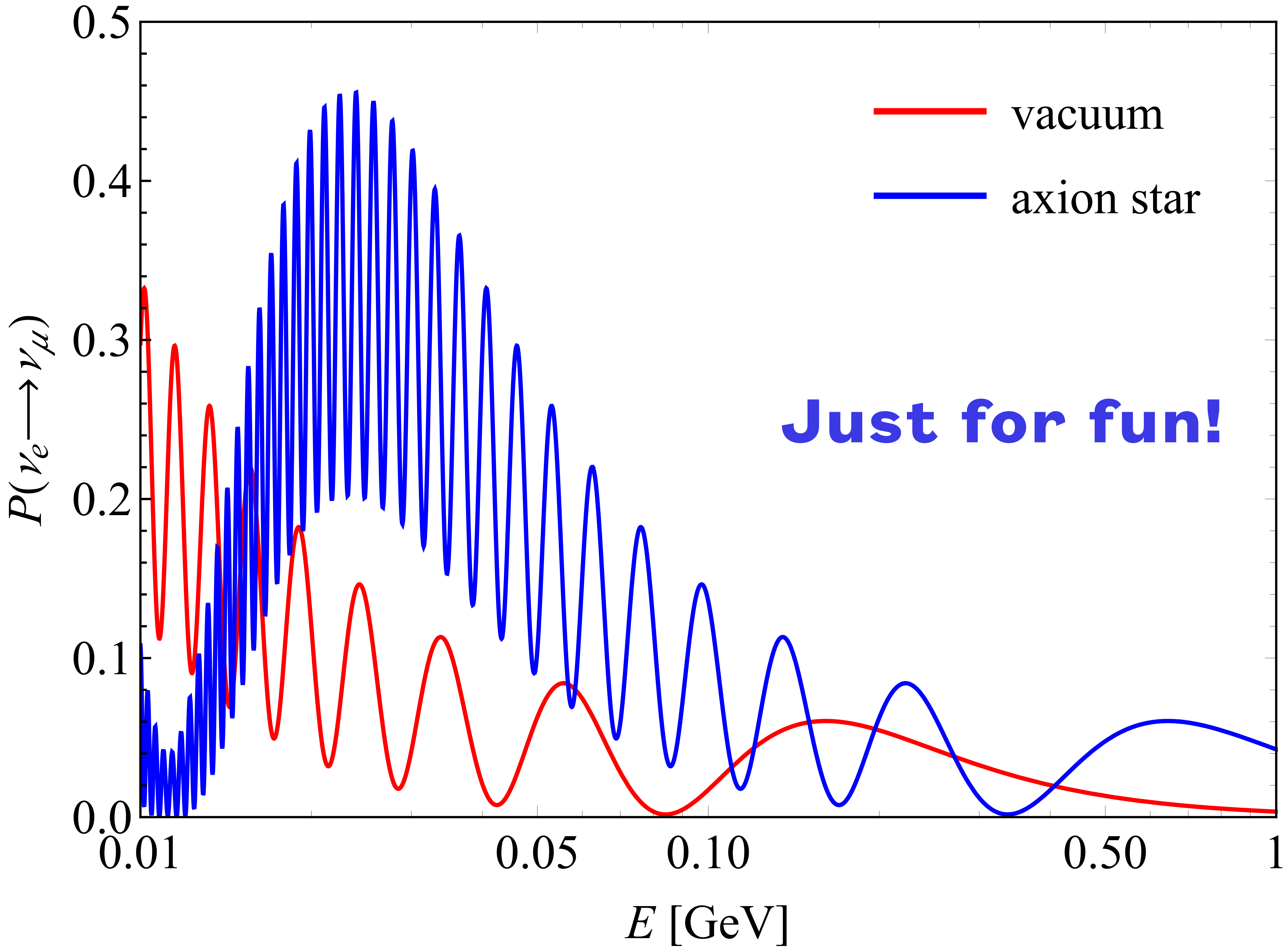
# Neutrino oscillation in Majoron star

## Effective potential

$$V_{\text{eff}} = i\sqrt{2\rho_a}V_{23}m_a^{-1}v_\Delta^{-1}\cos(m_a t)\overline{\nu_L^C}m_\nu \nu_L + \text{h.c.}$$

## Amplitude:

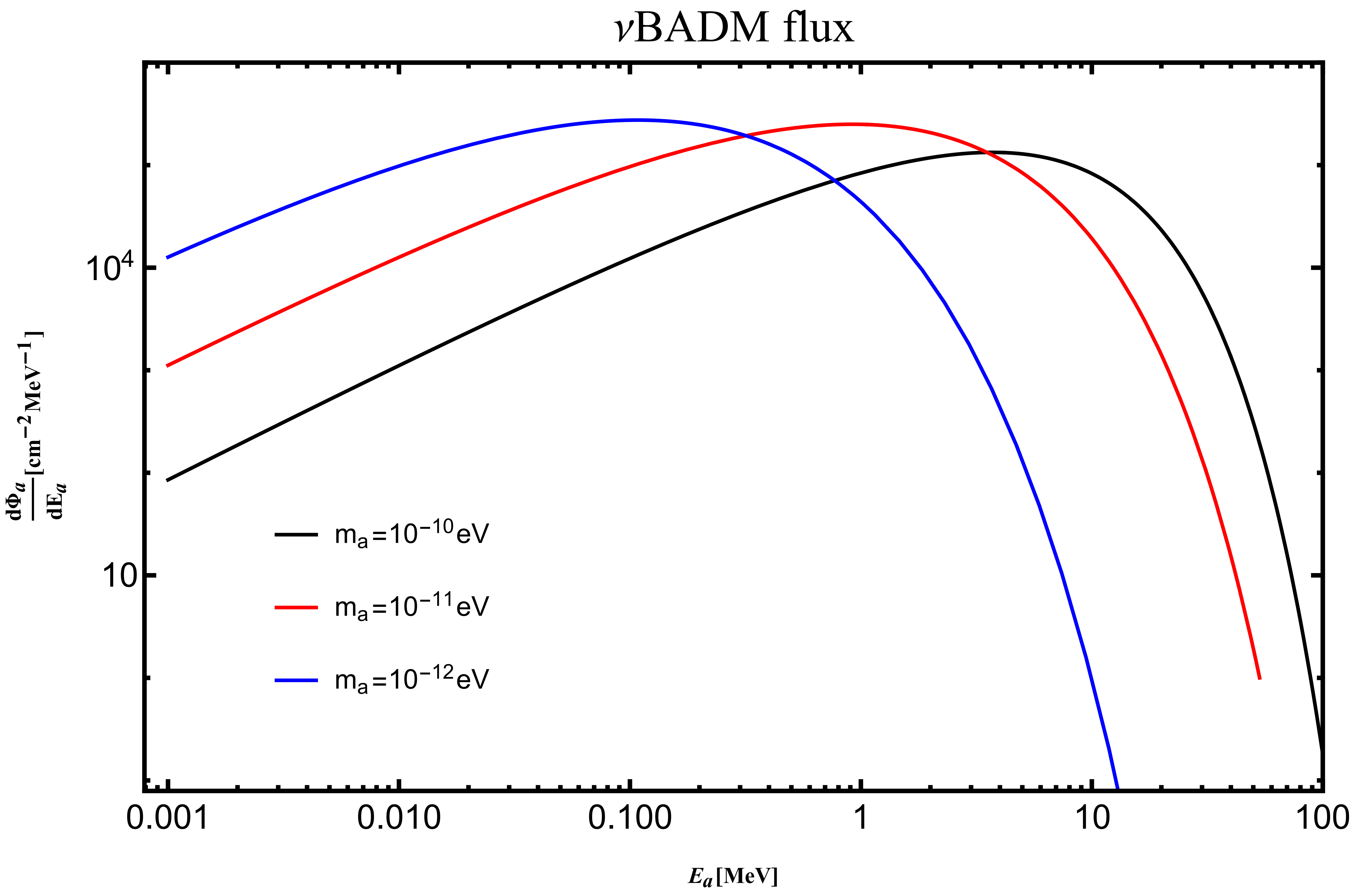
$$A_{\alpha\to\beta} = \sum_i \widehat{U}_{\beta i}\widehat{U}^*_{\alpha i}\exp\left[-i\frac{m_i^2 x}{2E}\left(1+\frac{\rho_a V_{23}^2}{m_a^2 v_\Delta^2}+\frac{\rho_a V_{23}^2\cos 2m_a x}{2x m_a^3 v_\Delta^2}\right)\right]$$



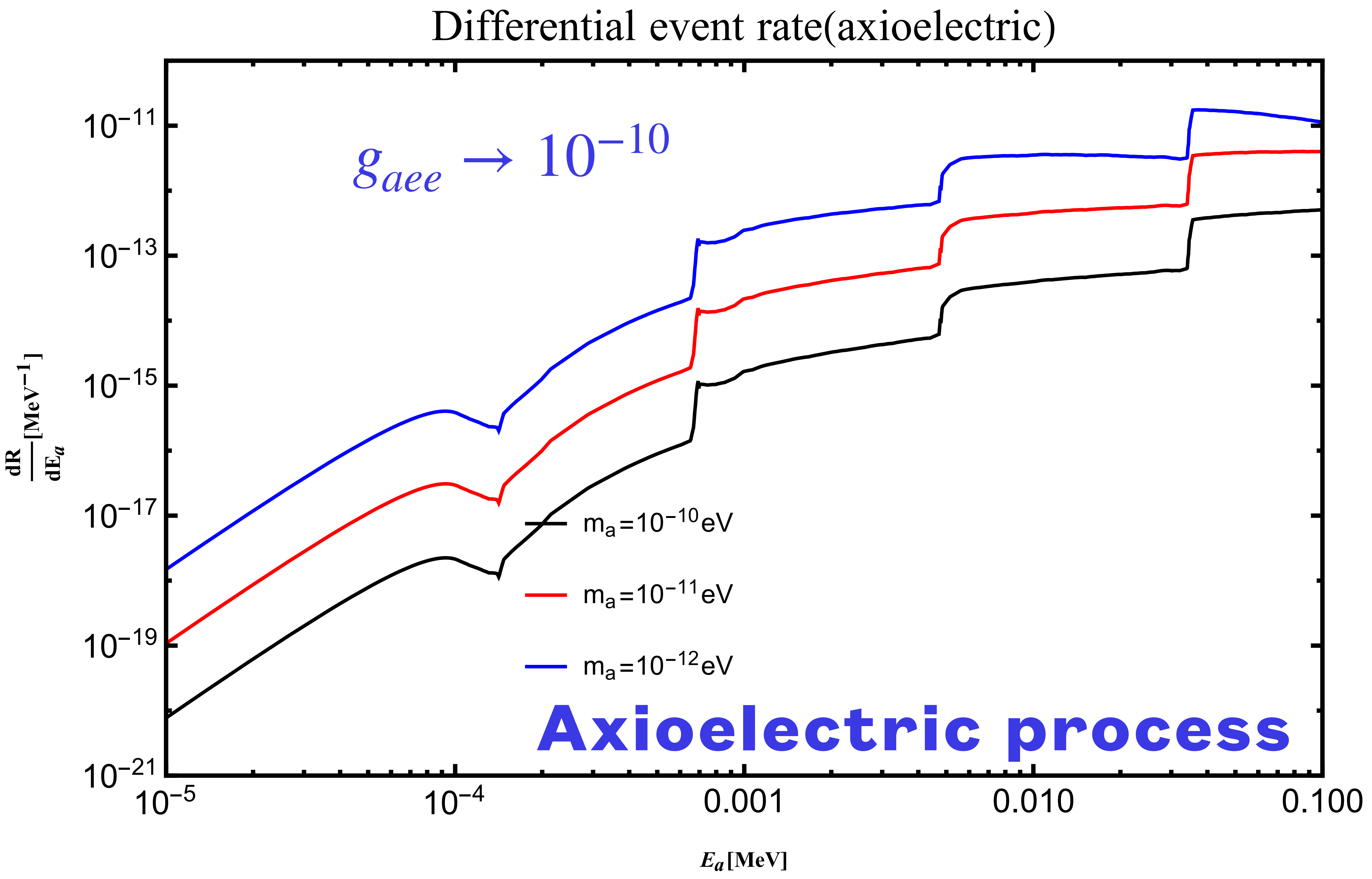

**Just for fun!**

# Direct detections of Majoron DM

## Boosted Majoron by supernova $\nu$



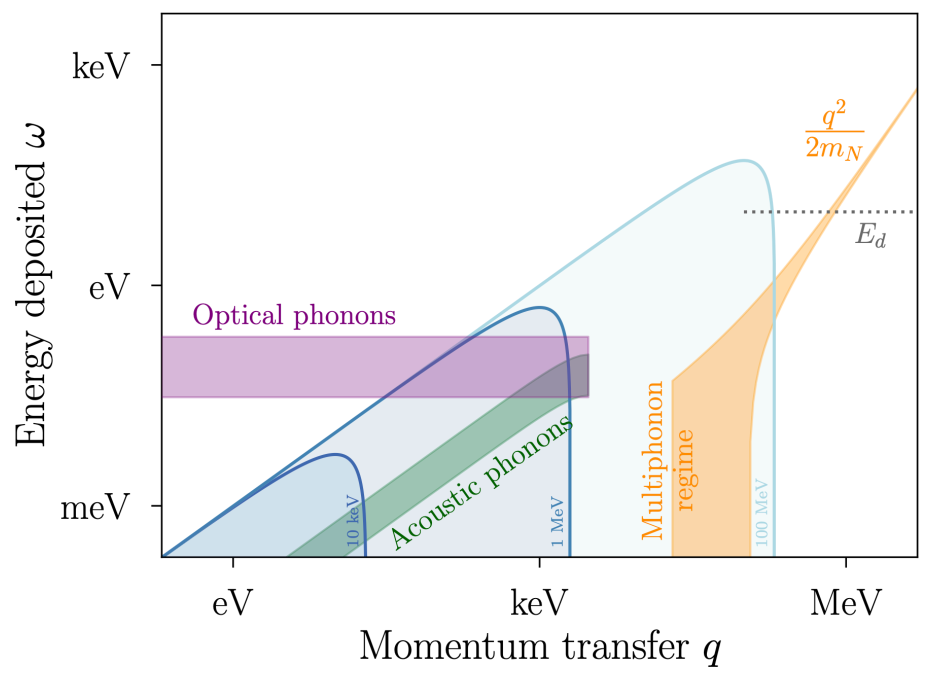

## Differential event rate



# Direct detections of Majoron DM

## Direct detections in condensed matter systems

| DM mass | DM energy or momentum | CM scale |
|---|---|---|
| 50 MeV | $p_\chi \sim 50$ keV | zero-point ion momentum in lattice |
| 20 MeV | $E_\chi \sim 10$ eV | atomic ionization energy |
| 2 MeV | $E_\chi \sim 1$ eV | semiconductor band gap |
| 100 keV | $E_\chi \sim 50$ meV | optical phonon energy |

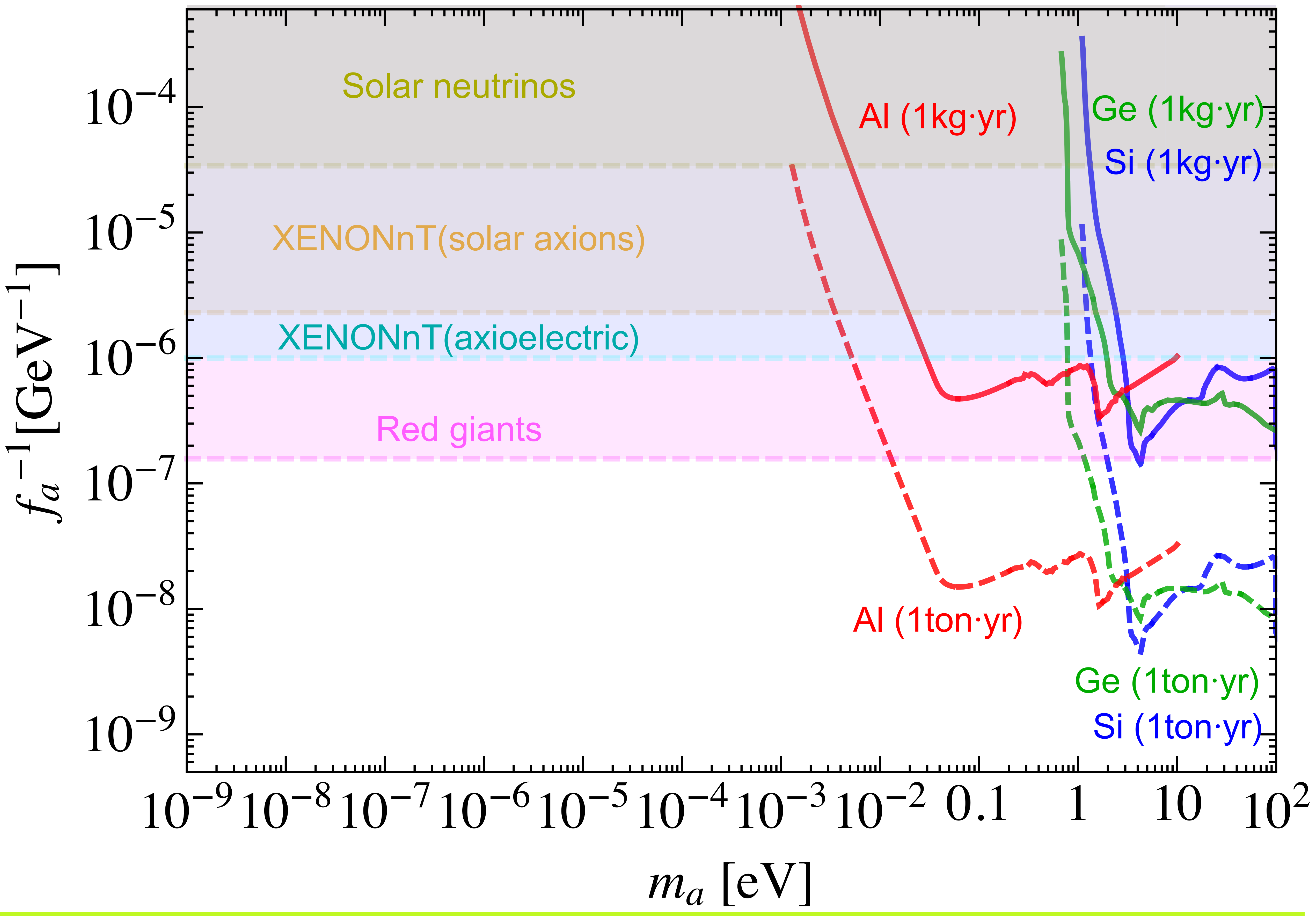



$$R \sim \frac{1}{\rho}\frac{\rho_a}{m_a}\frac{3m_a^2}{4m_e^2}\frac{g_{aee}^2}{e^2}\langle n_e \sigma_{abs} v_{rel}\rangle_\gamma$$

$$\langle n_e \sigma_{abs} v_{rel}\rangle_\gamma = -\frac{\mathrm{Im}\Pi(\omega)}{\omega}$$

## Absorption rate for photon in material

## Combined Constraints



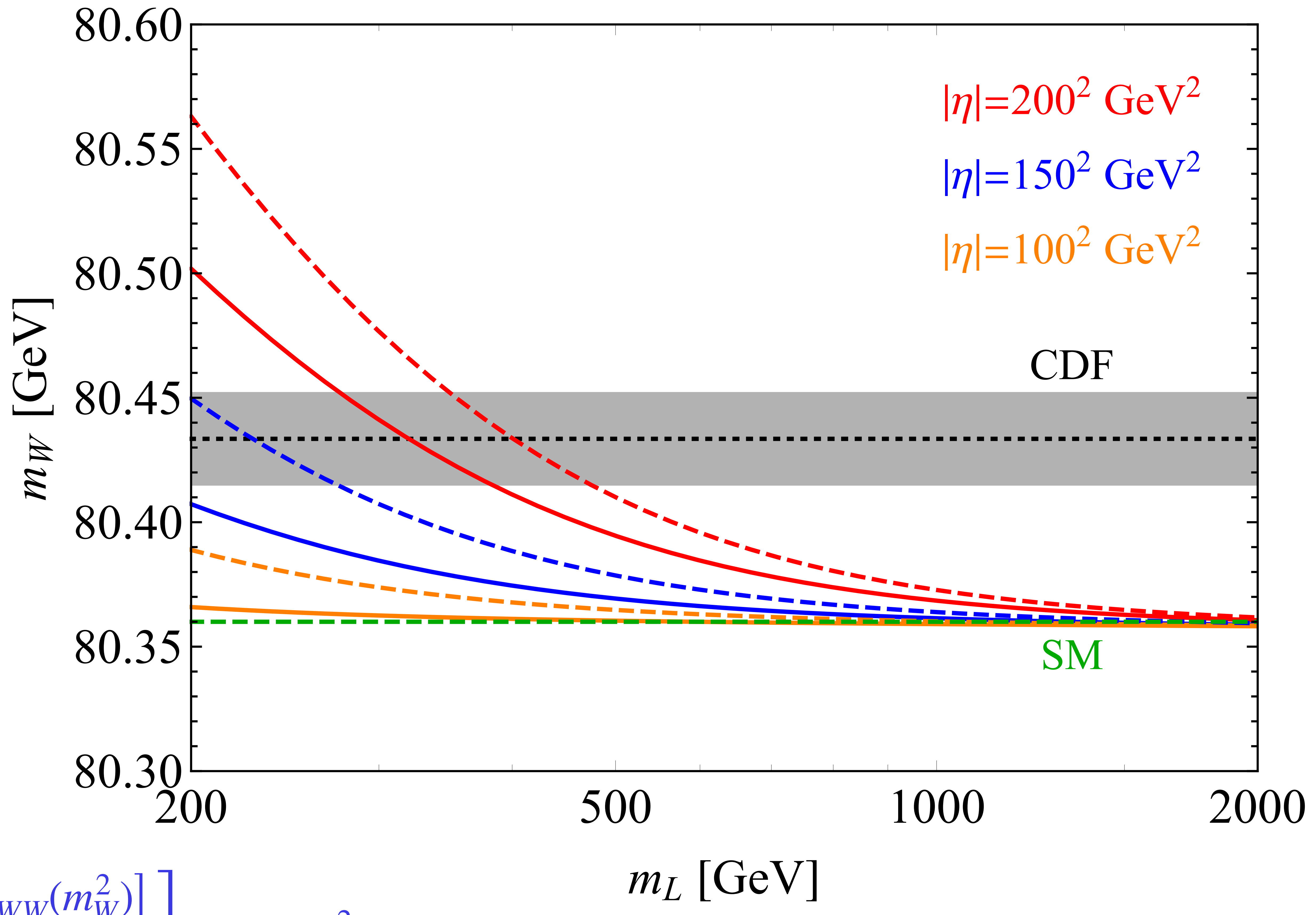
# Relevant pheno: W-boson mass anomaly

$$m_W^2 = \frac{m_Z^2}{2}\left[1+\sqrt{1-\frac{4\pi\alpha_{\rm em}}{\sqrt{2}G_F m_Z^2}(1+\Delta r)}\right],$$

$$\Delta r = \Delta\alpha_{\rm em} - c_W^2/s_W^2 \Delta\rho_{\rm loop} + \Delta r_{\rm rem}$$

$$\Delta\alpha_{\rm em} = \Pi'_{\gamma\gamma}(0) - \Pi'_{\gamma\gamma}(m_Z^2),$$

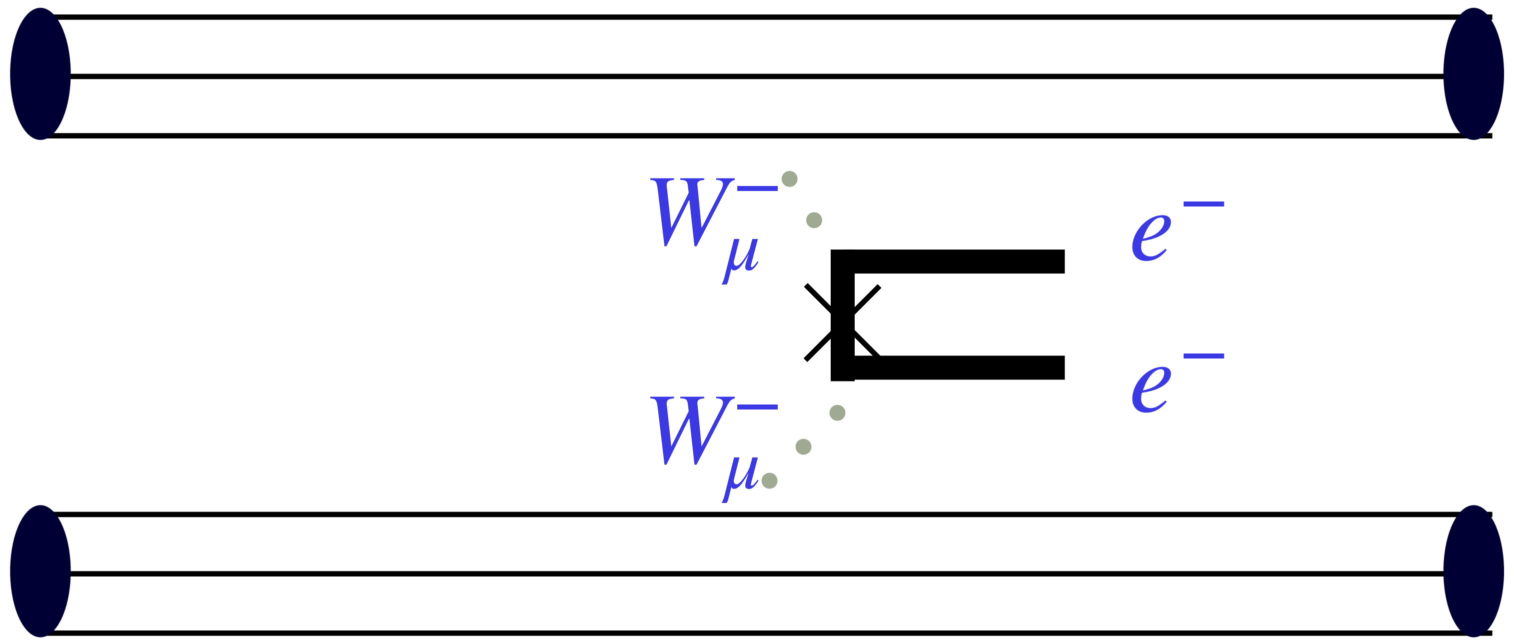
$$\Delta\rho_{\rm loop} = \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2} + \frac{2s_W}{c_W}\frac{\Pi_{Z\gamma}(0)}{m_Z^2},$$

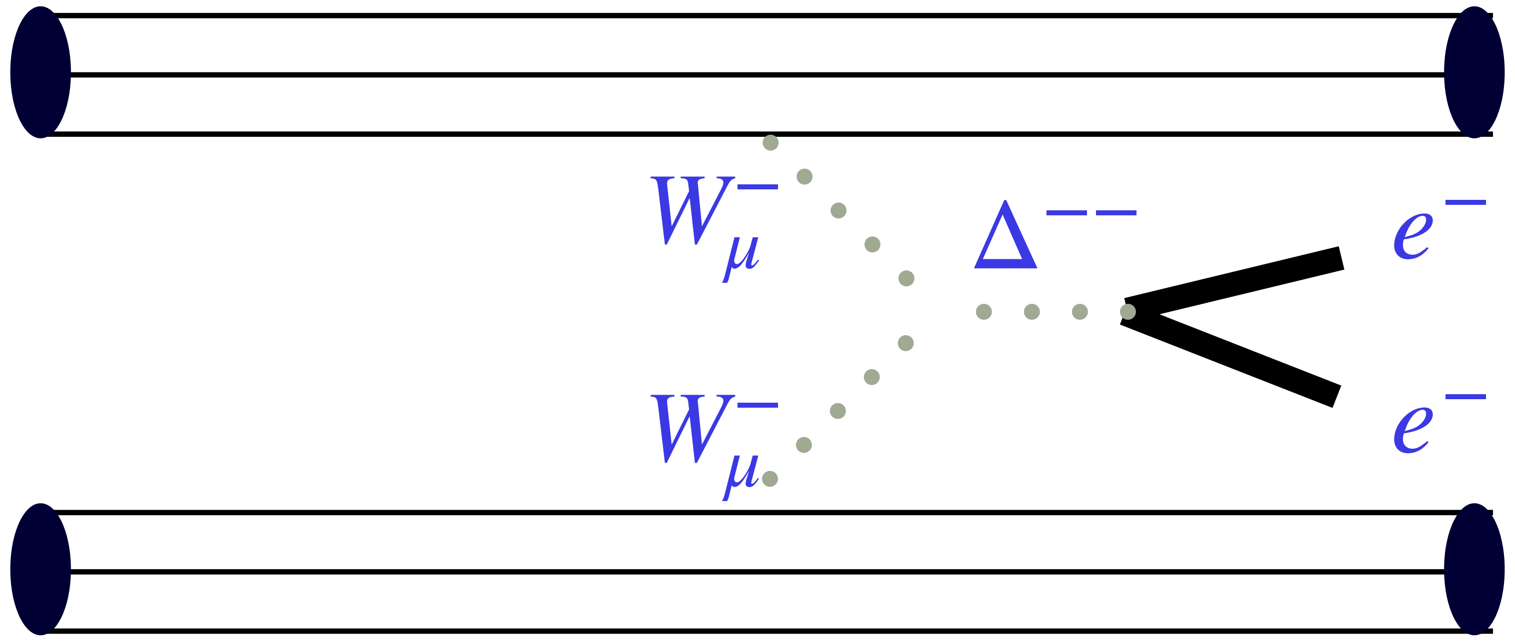



$$\Delta r_{\rm rem} = \frac{c_W^2}{s_W^2}\left[\frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{{\rm Re}\left[\Pi_{ZZ}(m_Z^2)\right]}{m_Z^2}\right] + \left(1-\frac{c_W^2}{s_W^2}\right)\left[\frac{\Pi_{WW}(0)}{m_W^2} - \frac{{\rm Re}\left[\Pi_{WW}(m_W^2)\right]}{m_W^2}\right] + \Pi'_{\gamma\gamma}(m_Z^2) + \delta_{\rm VB}\,.$$

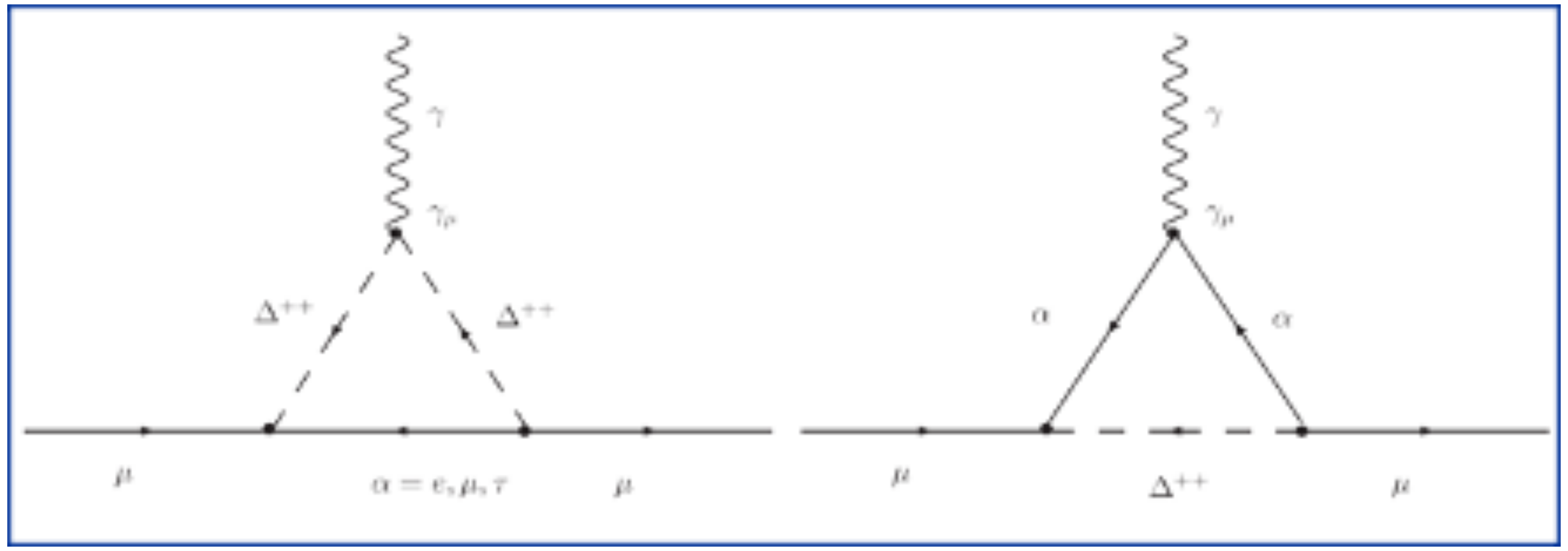
# Relevant pheno: $0\nu\beta\beta$ process and muon g-2

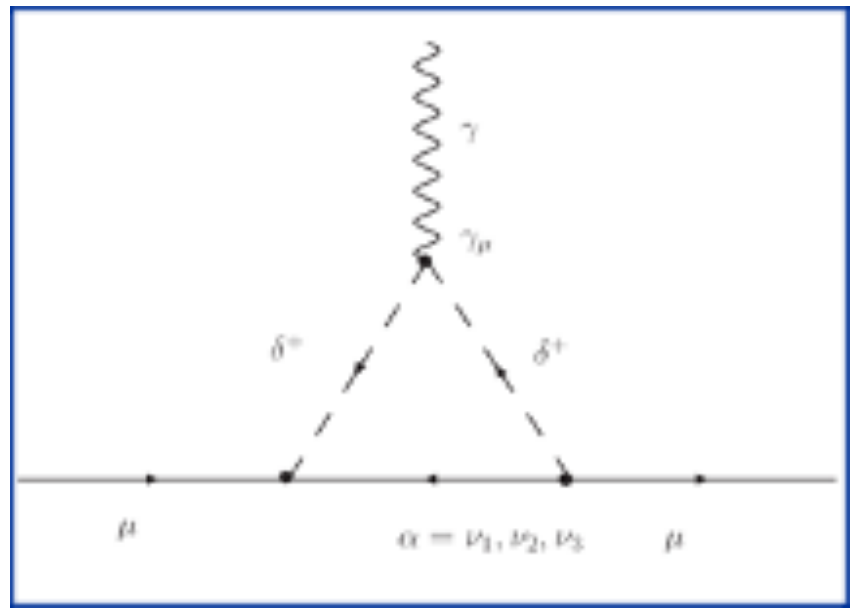
## $0\nu\beta\beta$ process in type-II seesaw

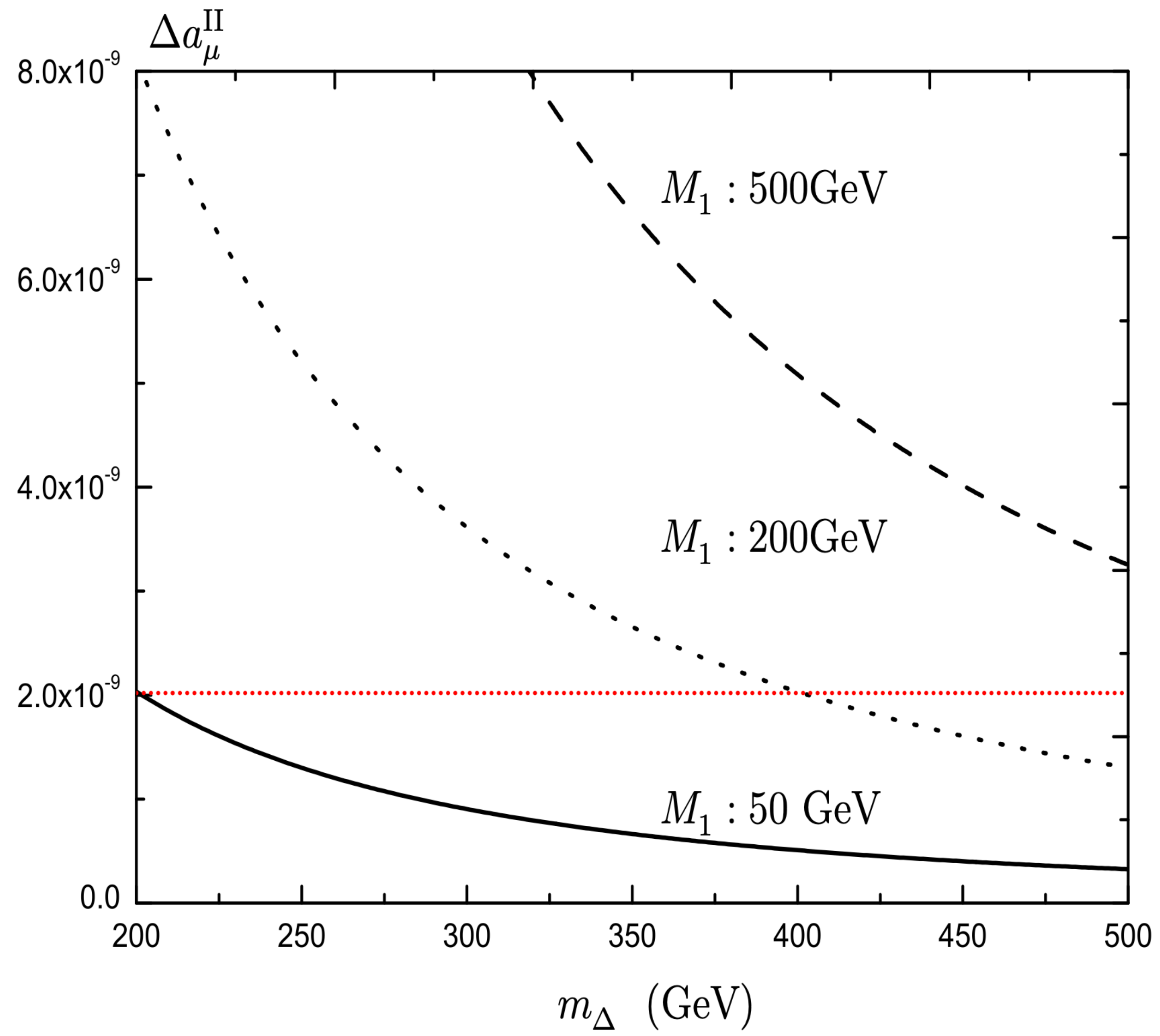


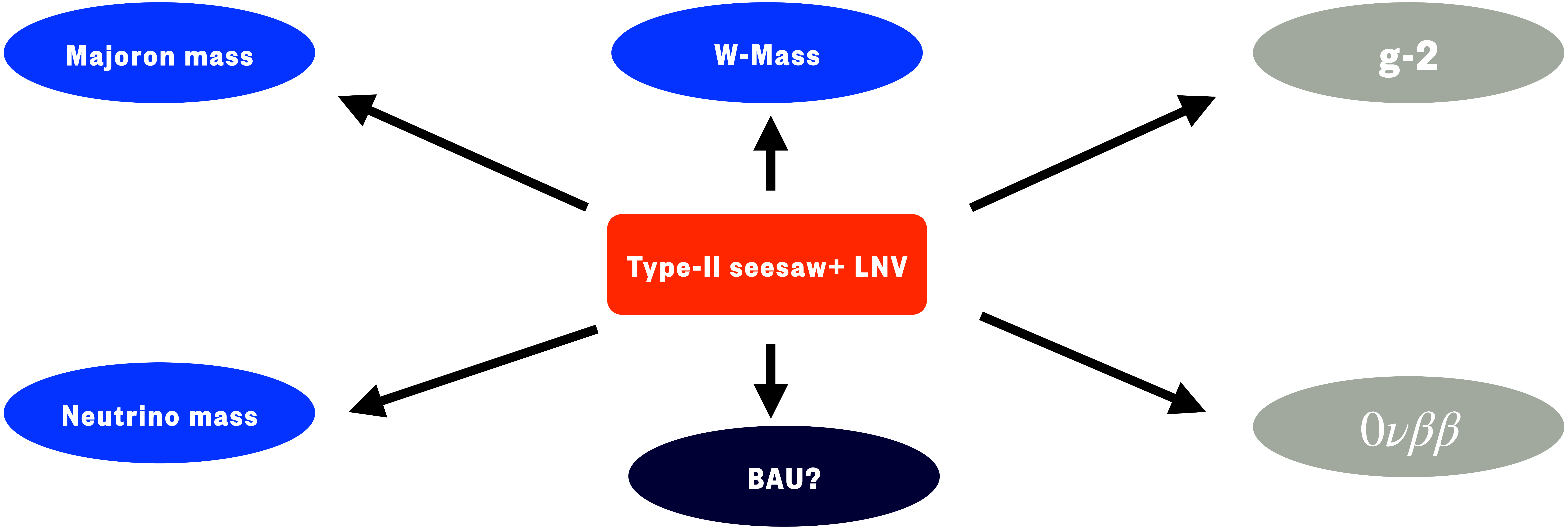

**Muon g-2**



# Concluding Remarks




**Thank you!**