Stochastic gravitational wave background and axion detection
随机引力波背景及轴子探测

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Based on: **JCAP 05, 049 (2021) and arXiv:2307.00746**

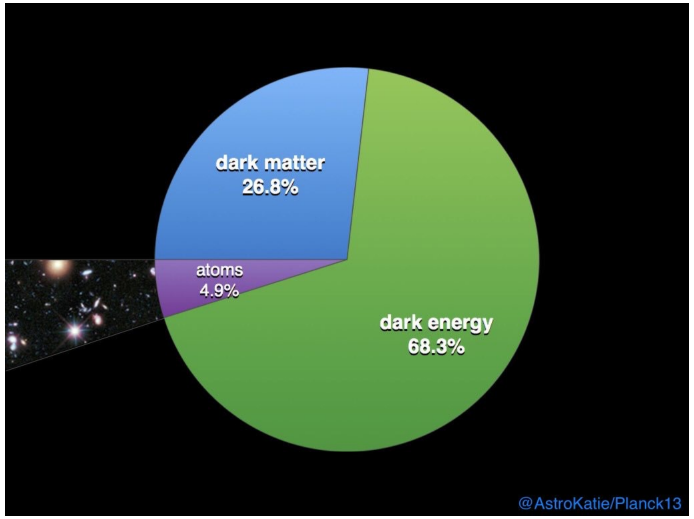
Talk @: **第二届轴子物理和实验国际会议 2023.7.24**

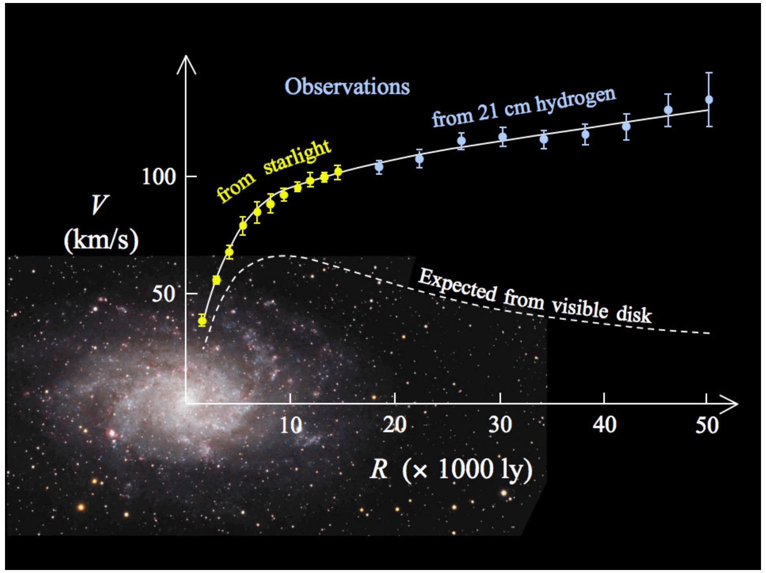
Outline

- Background
- QCD axion
- Clockwork axion
- GWs from domain wall annihilation
- GWs from first-order phase transition
- Summary

DM and evidence

1. Stellar rotation curve
2. CMB
3. Gravitational lens
4. Cosmic large-scale structure
5. …



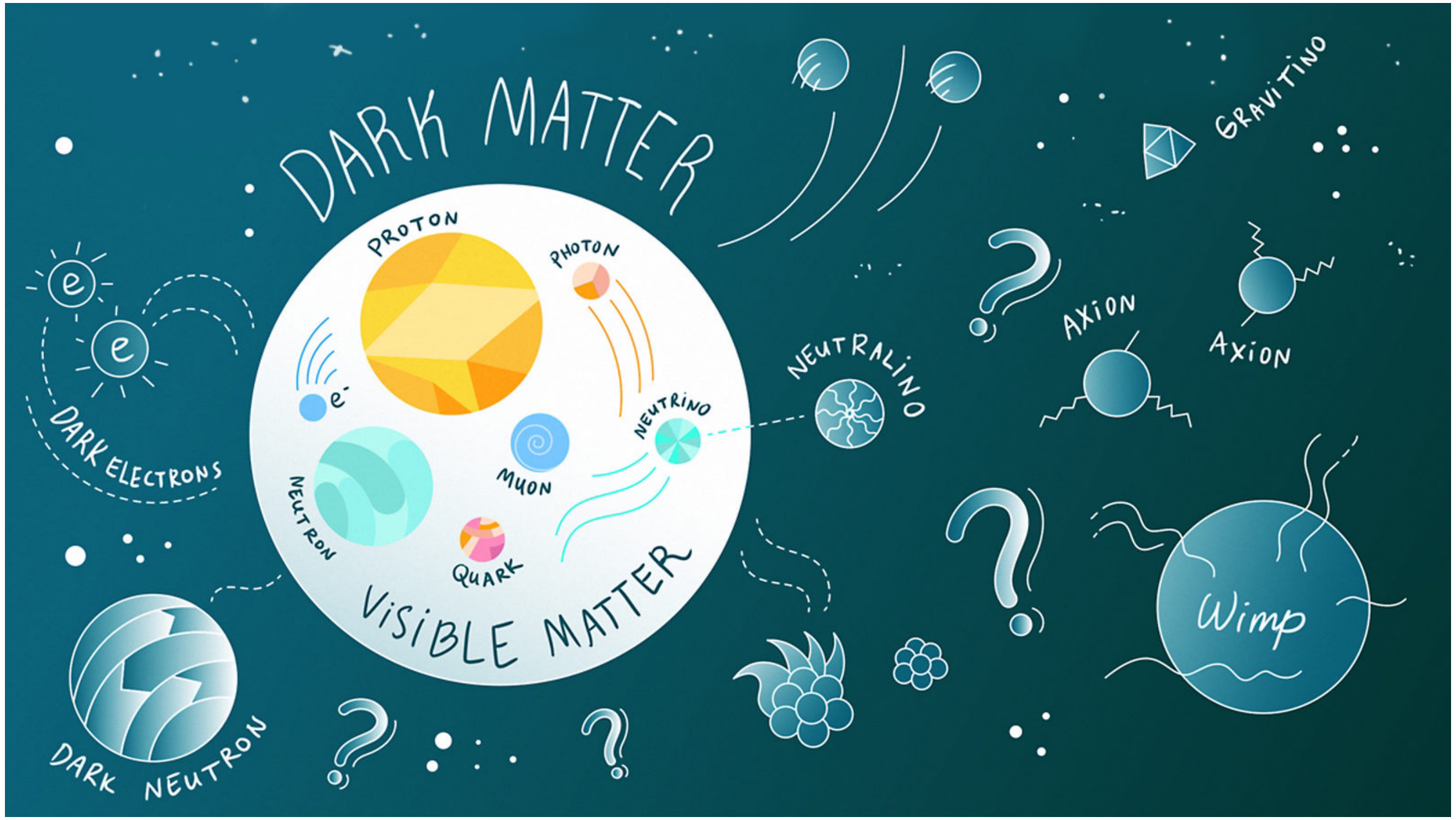


Stellar rotation curve

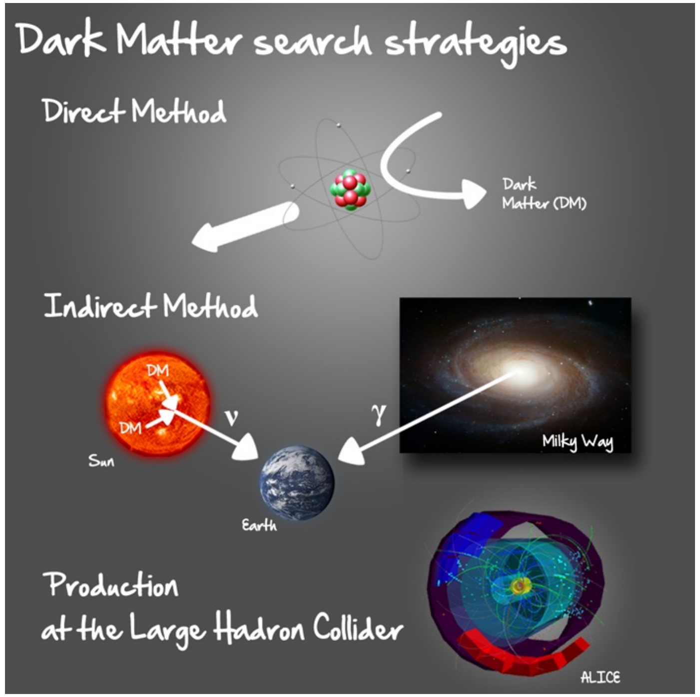
Bullet cluster X-ray and Gravitational lens imaging

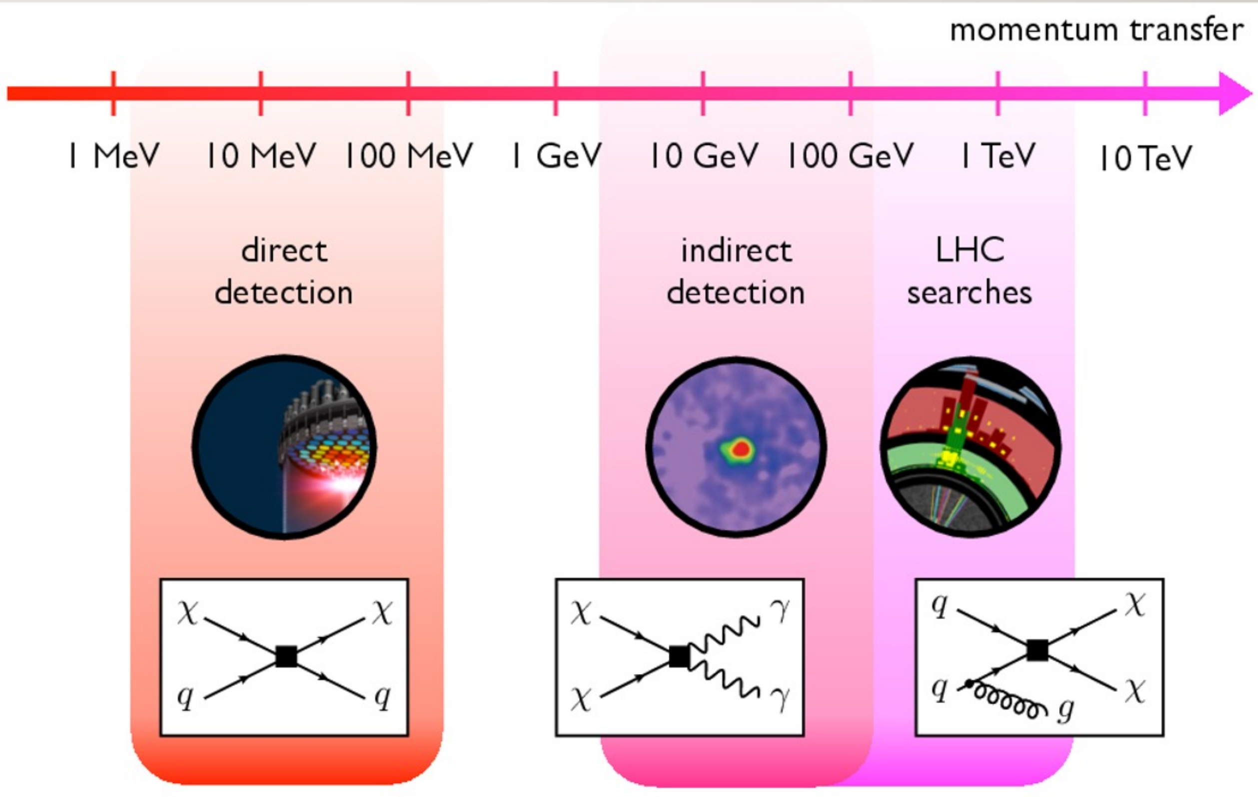
Dark matter candidate

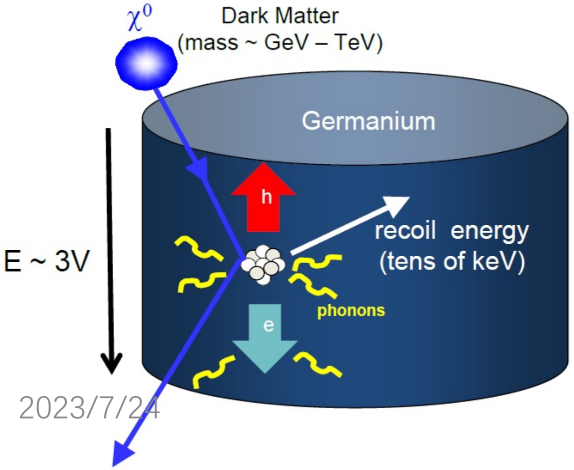
- DM candidate:
  1. Axion
  2. Dark photon
  3. Sterile neutrino
  4. ALP
  5. Sub-GeV DM
  6. WIMP
  7. Primordial black hole
  8. …

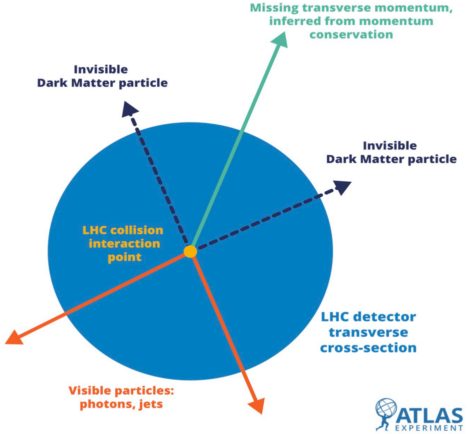


Dark matter detection

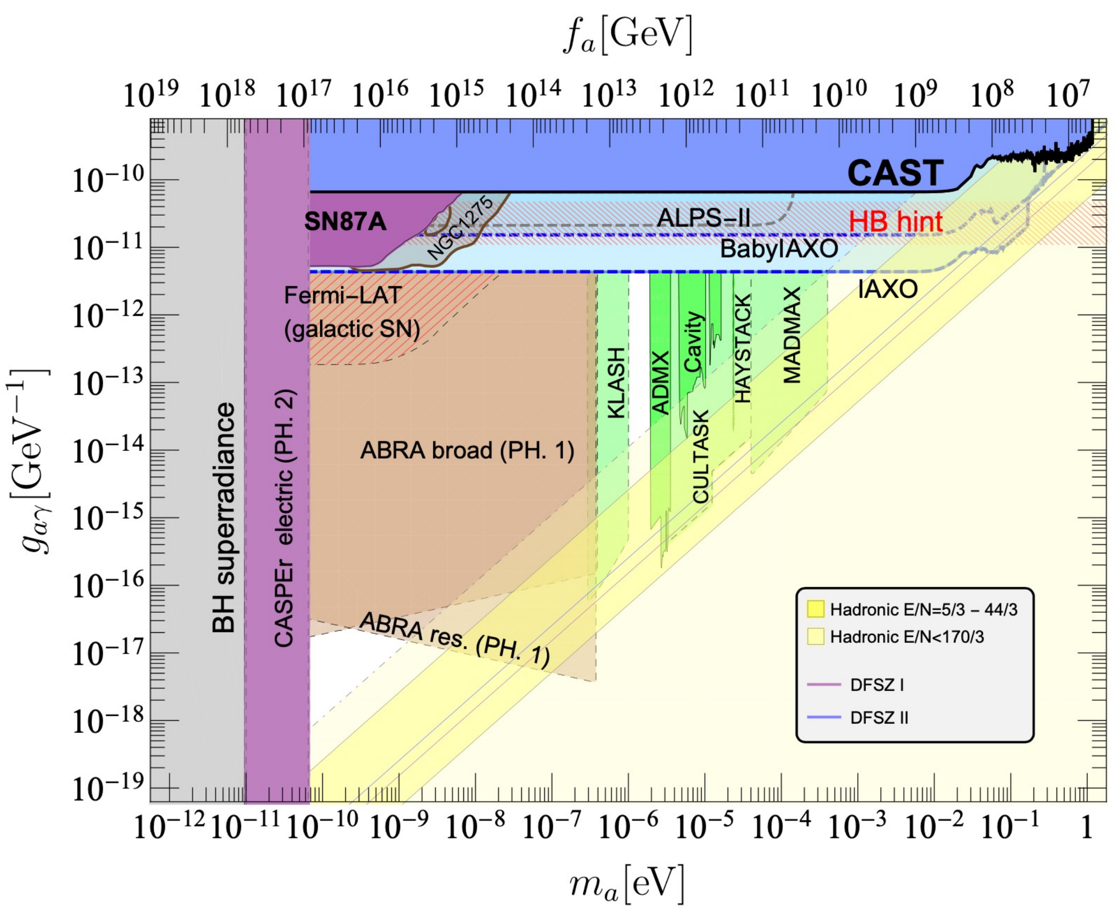


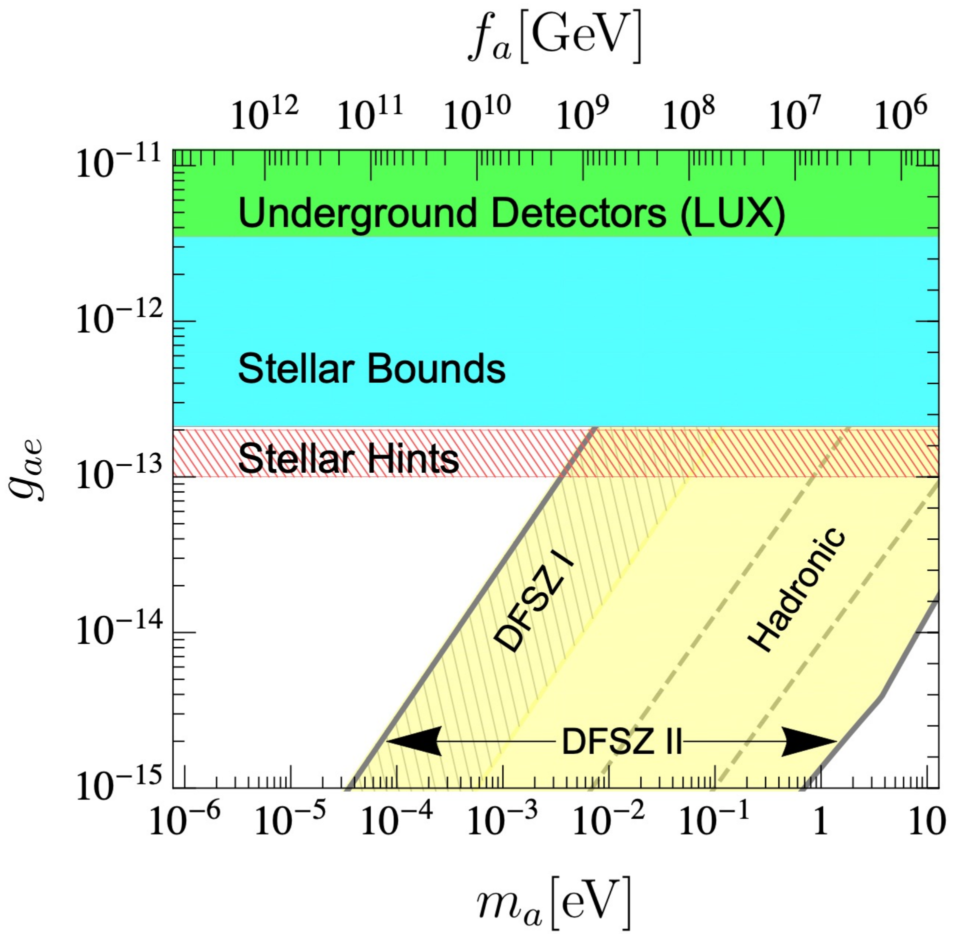






Experimental Searches for axion





Luzio et al., Phys. Rept. 870, 1-117 (2020).

QCD axion

- The QCD strong-CP problem: A topological term is allowed in the SM Lagrangian

$$\mathcal{L}_{\theta \mathrm{QCD}} = \frac{\theta_{\mathrm{QCD}}}{32\pi^2} \mathrm{Tr}\ G_{\mu\nu}\tilde{G}^{\mu\nu}$$

- The θ term is CP-violating and gives rise to an electric dipole moment (EDM) for the neutron

$$d_n \approx 3.6 \times 10^{-16} \theta_{\mathrm{QCD}}\, e\ \mathrm{cm}$$

- The dipole moment is constrained to $|d_n| < 2.9 \times 10^{-26}\, e\,\mathrm{cm}\ (90\%\ \mathrm{C.L.})$, which implies

$$\theta_{\mathrm{QCD}} \lesssim 10^{-10}$$

- This is a fine-tuning problem, since θ_{QCD} could obtain an O(1) contribution from the CP-violation in the electroweak (EW) sector.

QCD axion

David J. E. Marsh, Phys. Rept. 643, 1-79 (2016).

- The Peccei and Quinn (PQ) mechanism
  1. A global $U_{PQ}(1)$ symmetry is imposed on the classical action.
  2. Spontaneous $U_{PQ}(1)$ symmetry breaking at scale f leads to an angular degree of freedom (Goldstone boson), ϕ, with a shift symmetry $\phi \to \phi + const$.
  3. The $U_{PQ}(1)$ symmetry is anomalous at quantum level and explicit breaking is generated by quantum effects (instantons etc.), which emerge with some particular scale, Λ_a. The anomaly gives rise to the QCD topological term

$$S \to S + \int d^4x \frac{N_{DW}\phi}{32\pi^2 f} TrG_{\mu\nu}\tilde{G}_{\mu\nu}.$$

  4. Due to the shift symmetry, θ_{QCD} can be absorbed in a shift of ϕ. The strong CP problem is solved dynamically when the potential for the shifted field is minimized at $\phi/f = 0$ mod 2π.
  5. The non-perturbative effects break the axion shift symmetry down to the discrete shift symmetry, $\phi \to \phi + 2\pi f/N_{DW}$, and the axion is a pseudo Nambu-Goldstone boson (pNGB). The axion potential generated by QCD instantons is

$$V(\phi) = m_u\Lambda_{\mathrm{QCD}}^3\left[1 - \cos\left(\frac{N_{\mathrm{DW}}\phi}{f}\right)\right]$$

QCD axion

- The axion mass, self-interaction, and its interactions with SM fields are suppressed by powers of f_a since these terms all break the shift symmetry.
- The shift symmetry renders the axion a light, weakly interacting, long-lived particle, which can naturally serve as an DM candidate.
- In the conventional QCD axion scenarios, the axion decay constant f_a is at the same order as the PQ symmetry-breaking scale f, $f_a \sim f$. The axion decay constant has been restricted to the range

$$10^9 \lesssim f_a \lesssim 10^{12}\,\mathrm{GeV}$$

- The lower bound comes from the SN 1987A neutrino burst duration observations, while the upper bound is to ensure that the Universe is not over-closed by the axion DM. As a consequence, the classical QCD axion is nearly invisible.

Clockwork axion

- The clockwork model contains a number of N +1 complex scalars, Φ_i. The potential of these scalars is

$$V(\Phi)=\sum_{j=0}^{N}\left(-m^2\left|\Phi_j\right|^2+\lambda\left|\Phi_j\right|^4\right)-\varepsilon\sum_{j=0}^{N-1}\left(\Phi_j^{\dagger}\Phi_{j+1}^3+\mathrm{h.c.}\right)$$

- The first term respects a global $U(1)^{N+1}$ symmetry, which is explicitly broken by the ε-dependent term down to a global $U(1)$ symmetry.
- After the spontaneous symmetry breaking, we parametrize the scalar field as $\Phi_i = f e^{i\pi_i/f}$, then the potential is given by

$$V(\pi)=-\frac{1}{2}\varepsilon f^4\sum_{i=0}^{N-1}\cos\frac{\pi_i-q\pi_{i+1}}{f}\simeq\frac{\varepsilon f^2}{4}\sum_{i=0}^{N-1}\left(\pi_i-q\pi_{i+1}\right)^2=\frac{1}{2}\sum_{i,j=0}^{N}\pi_j\left(M_\pi^2\right)_{ji}\pi_i$$

- The spontaneous symmetry breaking of the potential leads to N massive pseudo-Goldstone bosons and one massless Goldstone boson, whose mass eigenstates $a_i \equiv (a, A_1, \ldots, A_N)$ are given by

$$\pi_i=\sum_{j=0}^{N}O_{ij}a_j\equiv O_{i0}a+\sum_{j=1}^{N}O_{ij}A_j$$

$$O_{i0}=\frac{\mathcal{N}_0}{q^i},\quad O_{ik}=\mathcal{N}_k\left[q\sin\frac{ik\pi}{N+1}-\sin\frac{(i+1)k\pi}{N+1}\right]\qquad\mathcal{N}_0\equiv\sqrt{\frac{q^2-1}{q^2-q^{-2N}}},\quad\mathcal{N}_k\equiv\sqrt{\frac{2}{(N+1)\eta_k}}.$$

The clockwork mechanism

Giudice et al., JHEP 02, 036 2017.

- Consider the effective Lagrangian in which the N-th site π_N is coupled to the QCD topological term

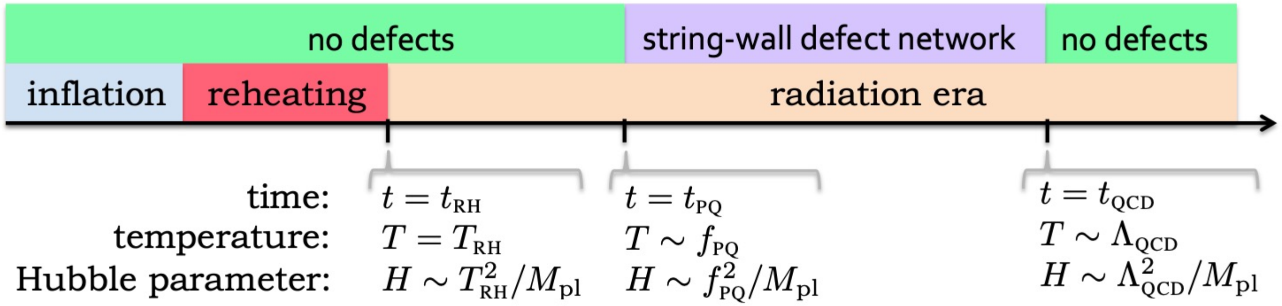
$$\mathcal{L} \supset \frac{\alpha_s}{8\pi} \frac{\pi_N}{f} G^a_{\mu\nu} \tilde{G}^{\mu\nu,a}.$$

- Rotating to the mass eigenstate, the coupling of the axion at `last` site to the topological term is then given by

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi} \frac{a}{f_a} G^a_{\mu\nu} \tilde{G}^{\mu\nu,a} \qquad f_a \equiv \frac{f}{O_{N0}} = \frac{q^N f}{\mathcal{N}_0} \simeq q^N f \, .$$

- We observe that the axion at `first` site has the largest coupling to gluon, while the coupling of the massless axion at `last` site to gluon is further suppressed by a factor q^N.
- With the clockwork mechanism, a low PQ symmetry-breaking scale f and a nearly invisible axion (a large axion decay constant f_a) can be simultaneously achieved in an axion model.

String-wall network



Long, JHEP 07, 066 2018.

1. When the $U(1)^{N+1}$ symmetries are spontaneously broken, there appear N kinds of cosmic strings corresponding to a non-trivial topological configuration of $\pi_{0,\ldots,N-1}$.
2. At a later time, the shift symmetry breaking terms generate discrete minima for the N massive axions, and domain walls are formed between two strings.
3. For large $N > 3$, no Isolated objects are produced, and the string-wall network can survive for a long time.
4. The QCD axion potential induced by the QCD phase transition can serve as an energy bias $V_b \sim \Lambda_{QCD}^4$ for the $W_{N,N-1}$ domain walls, leading to the domain walls $W_{N,N-1}$, $W_{N-1,N-2}$,⋯, $W_{1,0}$ annihilate one after another with attached cosmic strings.

Gravitational waves from domain wall annihilation

- In the scaling regime for long-lived domain walls, the evolution of the energy density of domain walls can be parameterized as

$$\rho_{\rm wall}(t) = \mathcal{A}\frac{\sigma}{t}$$

- The domain wall annihilation becomes significant when the tension of domain walls is comparable with the volume pressure, then the annihilation temperature of domain walls is determined as

$$T_{\rm ann} \simeq 7.15 \times 10^{-2}\ {\rm GeV}\ \varepsilon^{-1/4} \left(\frac{g_*(T_{\rm ann})}{10}\right)^{-1/4} \left(\frac{f}{100\ {\rm TeV}}\right)^{-3/2} \left(\frac{\Lambda_{\rm QCD}}{100\ {\rm MeV}}\right)^{2}$$

- The energy density of the Universe would eventually be dominated by the domain walls, i.e., $\rho_c = \rho_{wall}$, at

$$T_{\rm dom} = 5.44 \times 10^{-2}\ {\rm GeV}\ \varepsilon^{1/4} \left(\frac{g_*(T_{\rm ann})}{10}\right)^{-1/4} \left(\frac{f}{100\ {\rm TeV}}\right)^{3/2}$$

- This give the constraint

$$f \lesssim 100\ {\rm TeV}\ \varepsilon^{-1/6} \left(\frac{\Lambda_{\rm QCD}}{100\ {\rm MeV}}\right)^{2/3}$$

Gravitational waves from domain wall annihilation

- The peak amplitude of the GW spectrum is produced at the annihilation time of domain walls

$$\Omega_{\mathrm{GW}}\left(\nu_{\text{peak}}\left(t_{\text{ann}}\right)\right) \simeq \frac{8\pi\tilde{\epsilon}_{\text{gw}} G^2 A^2 \sigma^2}{3H_{\text{ann}}^2}$$

- Taking into account the expansion of the Universe, the peak amplitude of the GW spectrum today is given by

$$h^2\Omega_{\mathrm{GW}}^{\text{peak}}\left(t_0\right) = 6.45\times 10^{-6}\varepsilon\left(\frac{\tilde{\epsilon}_{\text{gw}}}{0.7}\right)\left(\frac{A}{10}\right)^2\left(\frac{g_{*s}\left(T_{\text{ann}}\right)}{10}\right)^{-4/3}\left(\frac{f}{100\ \mathrm{TeV}}\right)^6\left(\frac{T_{\text{ann}}}{0.1\ \mathrm{GeV}}\right)^{-4}$$

- With the peak frequency today is

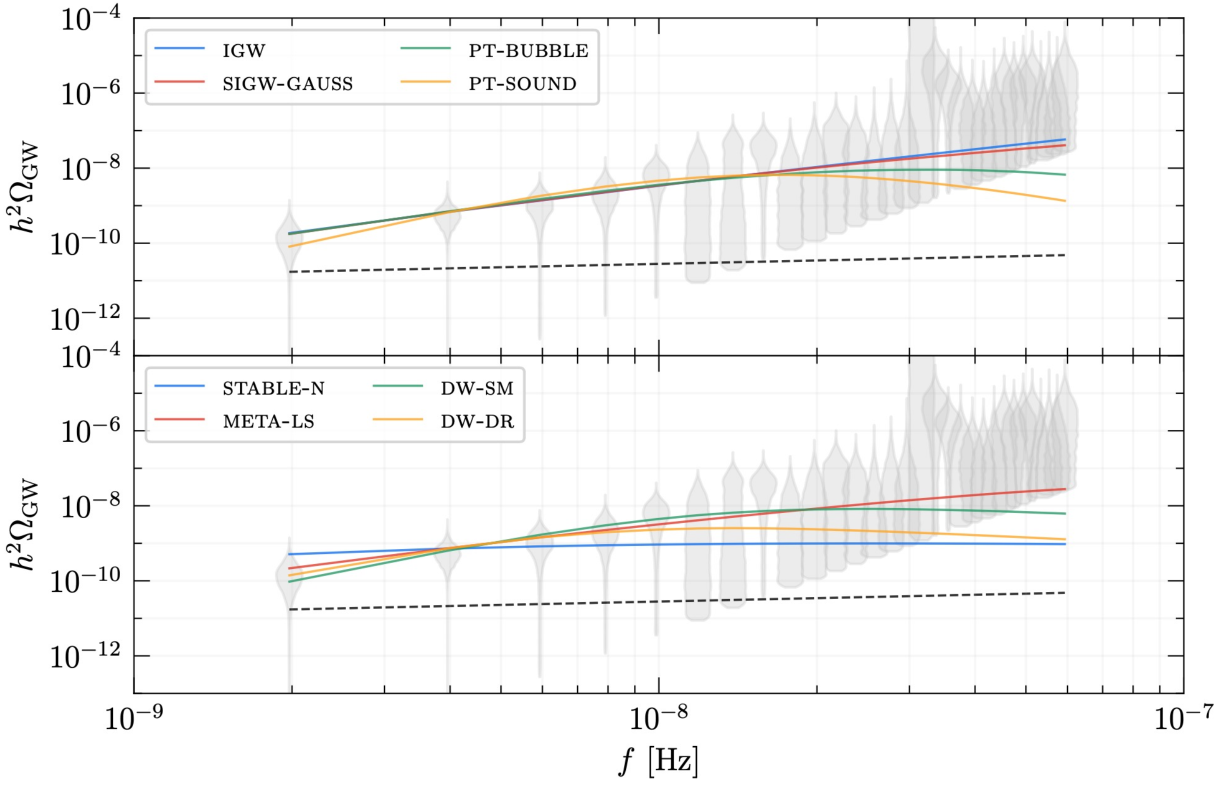
$$\nu_{\text{peak}}(t_0) \simeq 1.1\times 10^{-8}\ \mathrm{Hz}\left(\frac{g_*\left(T_{\text{ann}}\right)}{10}\right)^{1/2}\left(\frac{g_{*s}\left(T_{\text{ann}}\right)}{10}\right)^{-1/3}\left(\frac{T_{\text{ann}}}{0.1\ \mathrm{GeV}}\right)$$

- GW spectrum can be approximately parameterized as

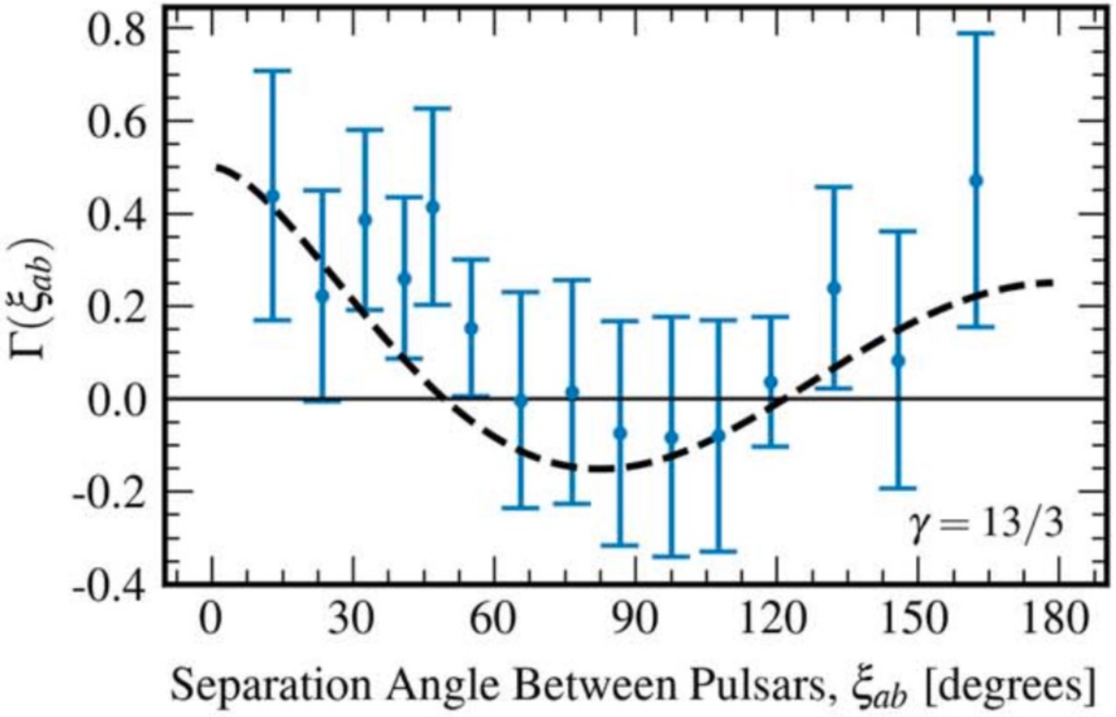
$$h^2\Omega_{\mathrm{GW}} = \begin{cases} h^2\Omega_{\mathrm{GW}}^{\text{peak}}\left(\dfrac{\nu}{\nu_{\text{peak}}}\right)^3 & \text{for } \nu < \nu_{\text{peak}} \\ h^2\Omega_{\mathrm{GW}}^{\text{peak}}\left(\dfrac{\nu_{\text{peak}}}{\nu}\right) & \text{for } \nu > \nu_{\text{peak}}. \end{cases}$$

NANOGrav 15-year data

NANOGrav Collaboration, arXiv: 2306.16219.

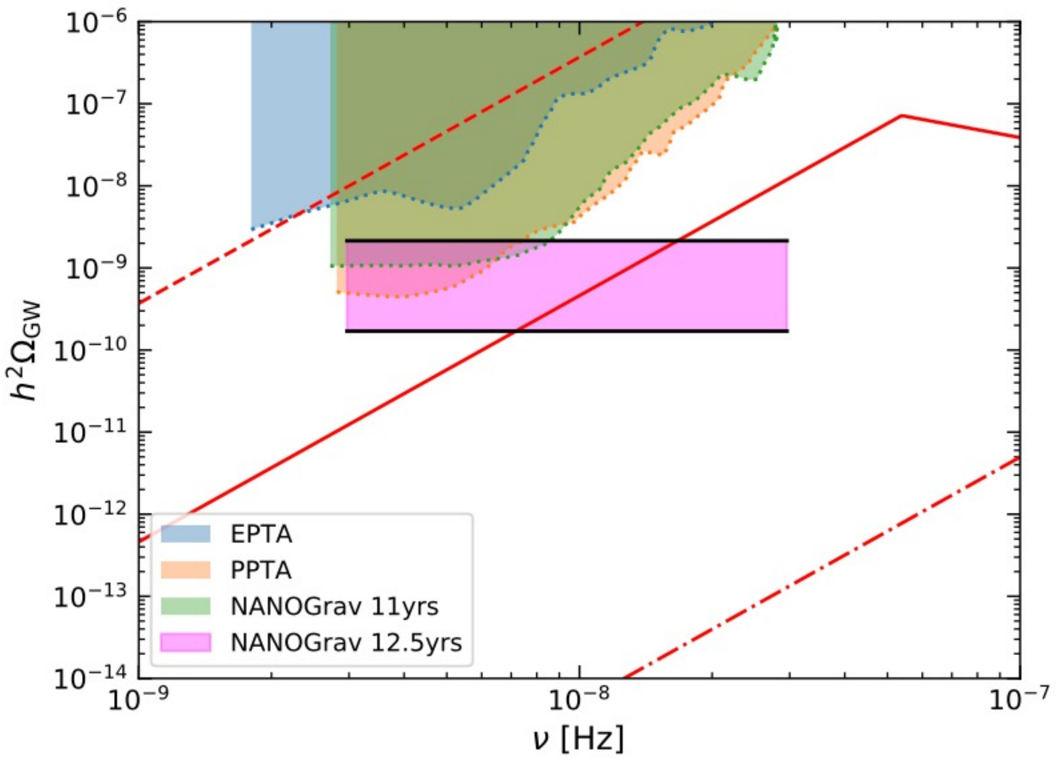


NANOGrav Collaboration, APJL 951, L8 2023.



NANOGrav 12.5-year data and 15-year data

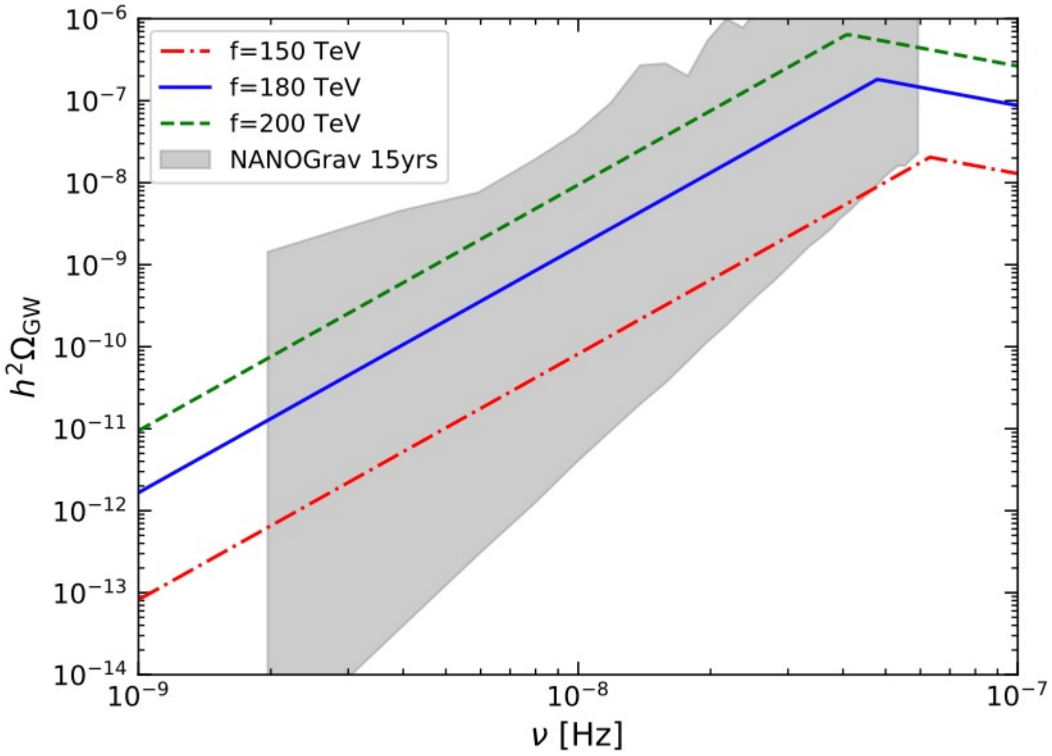
C.W. Chiang and B.Q. Lu, JCAP05(2021)049, arXiv: 2012.14071.



f = 300 TeV (Dashed),
f = 200 TeV (solid),
f = 100 TeV (dashed-dot).

$N = 10, \varepsilon = 0.1$

B.Q. Lu and C.W. Chiang, arXiv:2307.00746



f = 200 TeV (Dashed),
f = 180 TeV (solid),
f = 150 TeV (dashed-dot).

$N = 10, \varepsilon = 0.3$

Gravitation wave from first-order phase transition

- The phase transition during PQ symmetry breaking can be first order if a dimension-6 operator is added to the scalar potential

$$V_\Lambda(\Phi) = V(\Phi) + \sum_{j=0}^{N} \frac{1}{\Lambda^2} |\Phi_j|^6$$

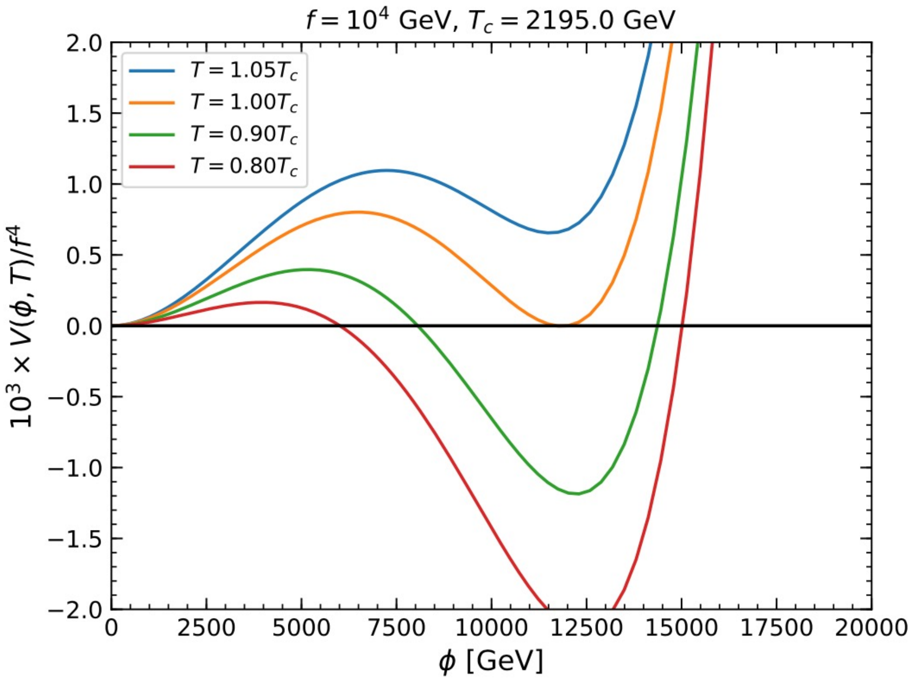
- The effective scalar potential at finite temperature

$$V_{\rm eff}(\phi, T) = V_0(\phi) + V_{\rm CW}(\phi) + V_T(\phi, T) + V_{\rm ring}(\phi, T)$$

$$V_{\rm CW}(\phi) = \sum_i \frac{n_i}{64\pi^2} \left[m_i^4(\phi) \left(\log \frac{m_i^2(\phi)}{m_i^2(f)} - \frac{3}{2} \right) + 2 m_i^2(\phi) m_i^2(f) - \frac{m_i^4(\phi)}{2} \right]$$

$$V_{\rm T}(\phi, T) = \frac{T^4}{2\pi^2} \sum_i n_i J_{B,F}\left(m_i^2(\phi)/T^2 \right)$$

$$V_{\rm ring}(\phi, T) = \sum_i \frac{n_i T}{12\pi} \left[m_i^3(\phi) - \left(m_i^2(\phi) + \Pi_i(T) \right)^{\frac{3}{2}} \right]$$



Gravitation wave from first-order phase transition

- There exist three processes that can produce the GWs:
  1. Bubble collision

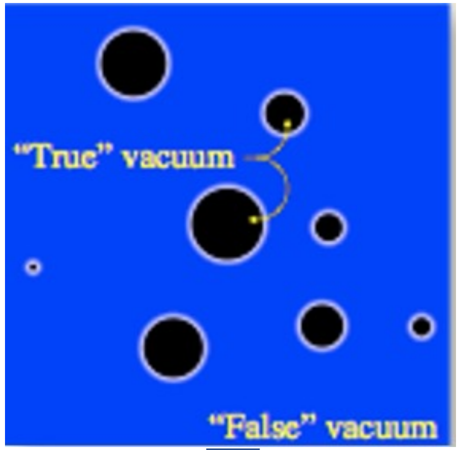
$$h^2\Omega_{\rm col}(f) = 1.67\times10^{-5}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa_{\rm col}\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{0.11v_w^3}{0.42+v_w^2}\right)S_{\rm col}(f).$$

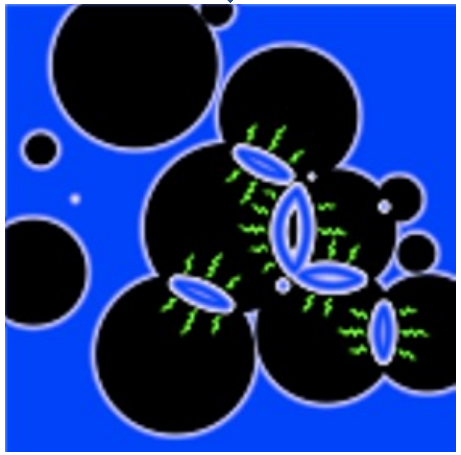
  2. Sound waves

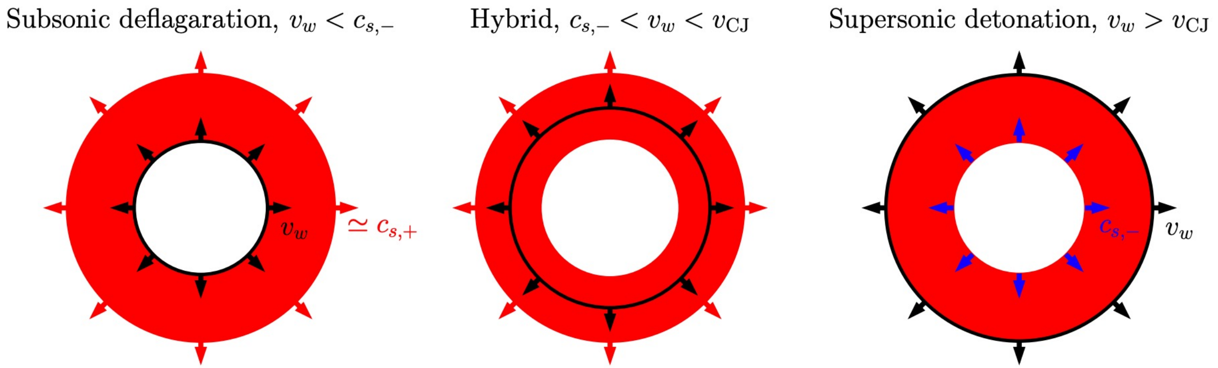
$$h^2\Omega_{\rm sw}(f) = 2.65\times10^{-6}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}v_wS_{\rm sw}(f).$$

  3. Turbulence

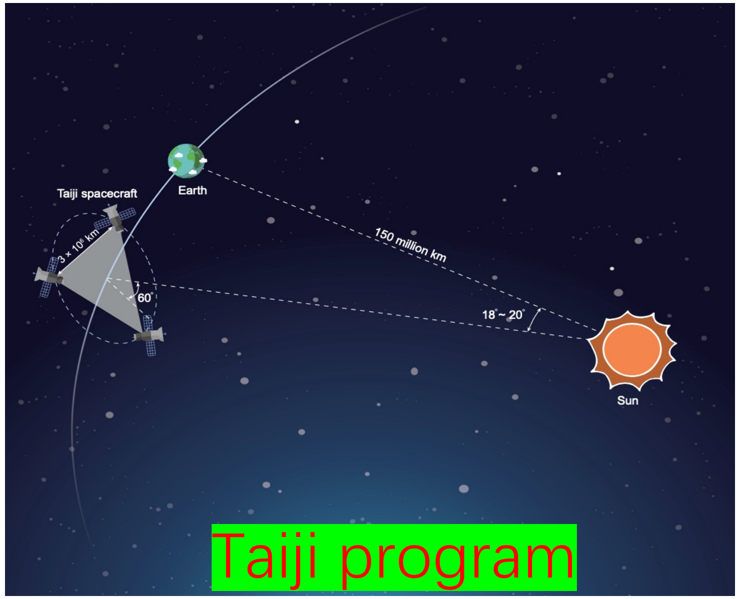
$$h^2\Omega_{\rm turb}(f) = 3.35\times10^{-4}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm turb}\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{100}{g_*}\right)^{1/3}v_wS_{\rm turb}(f).$$





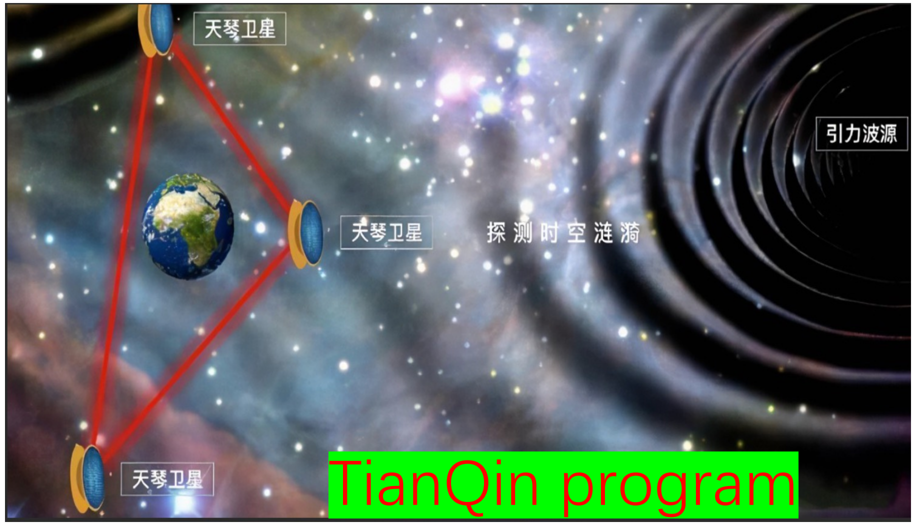


P. Athron, et al., arXiv:2305.02357

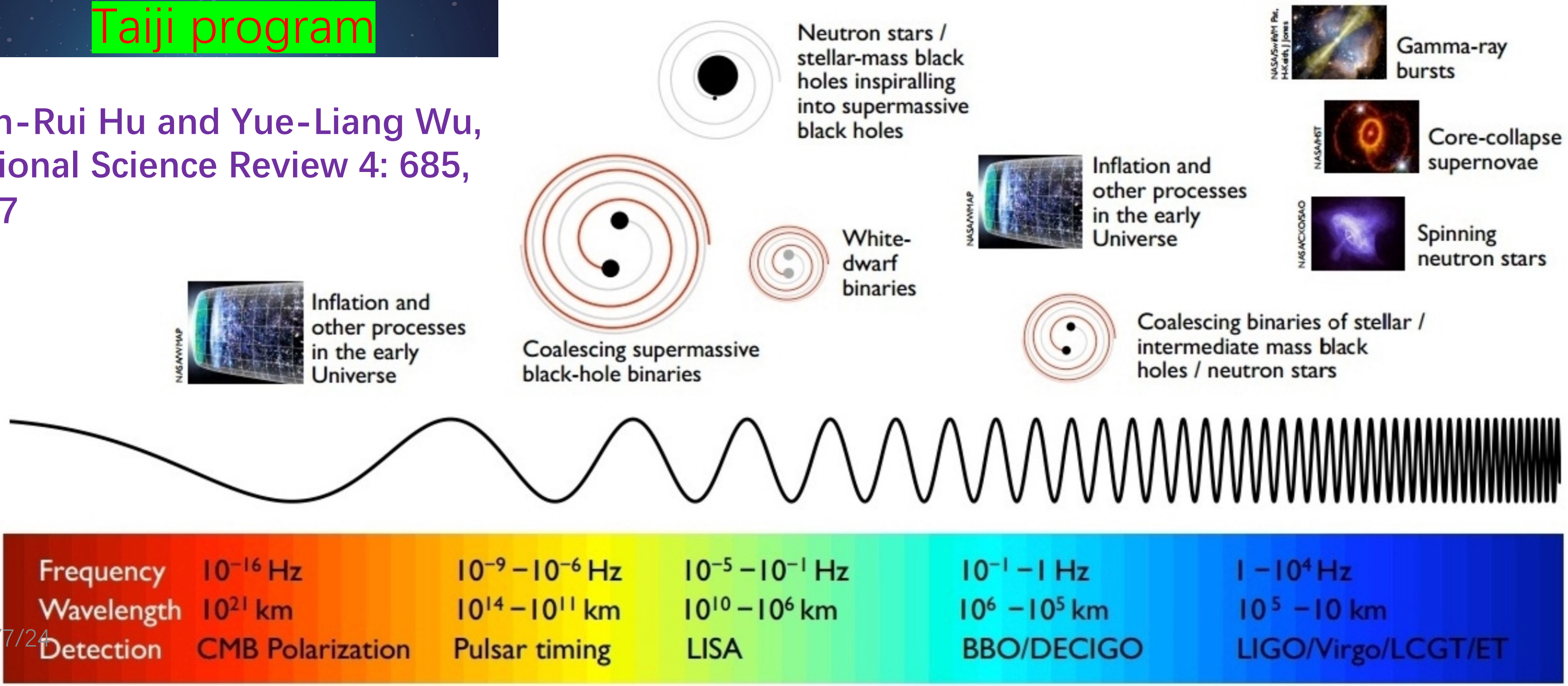


Space-based GW detections:

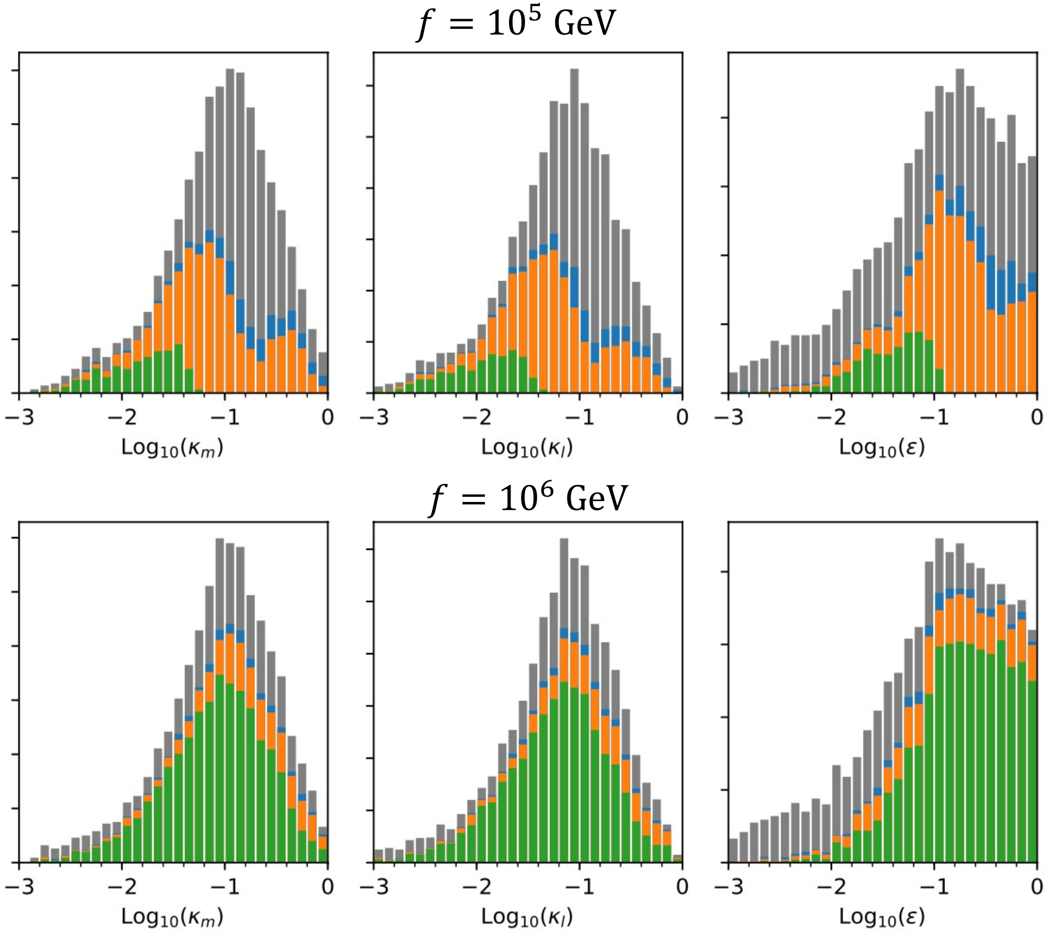
1. Taiji
2. TianQin
3. LISA
4. BBO…

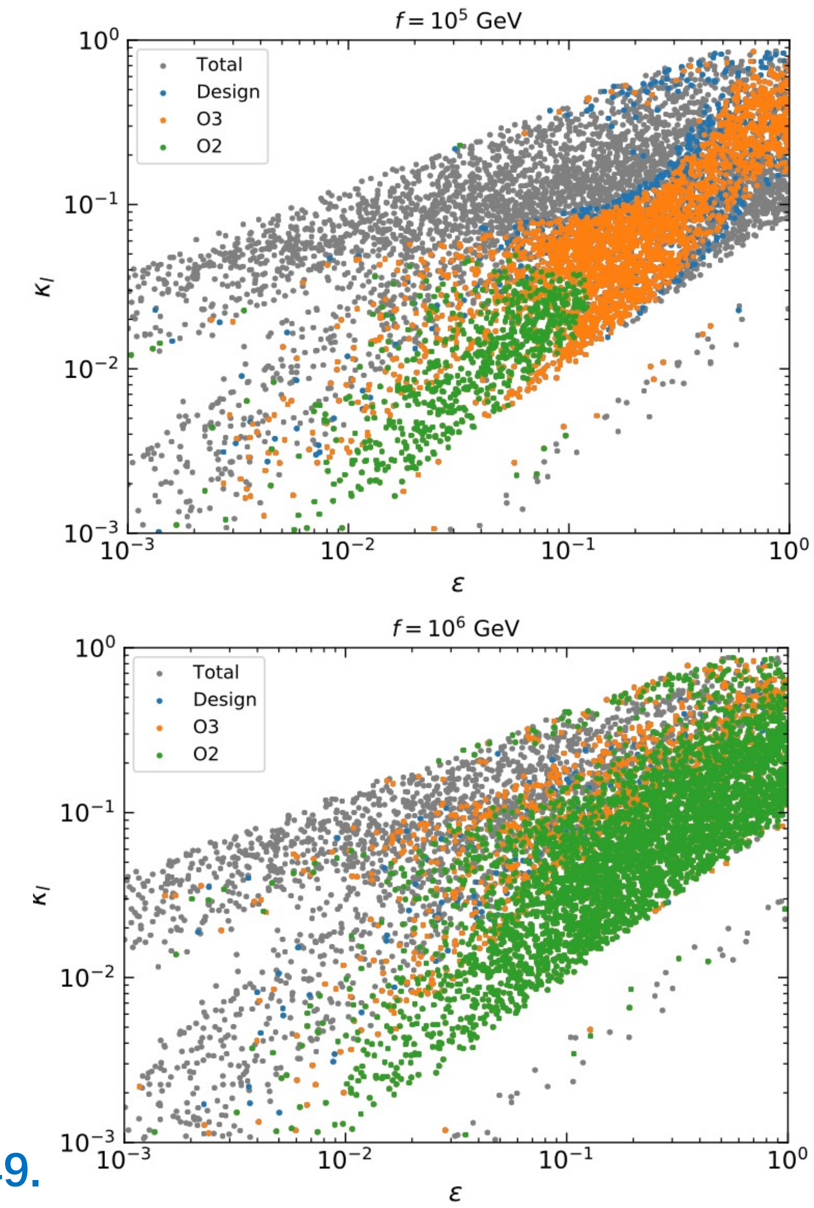


Wen-Rui Hu and Yue-Liang Wu, National Science Review 4: 685, 2017



GWs from first-order phase transition

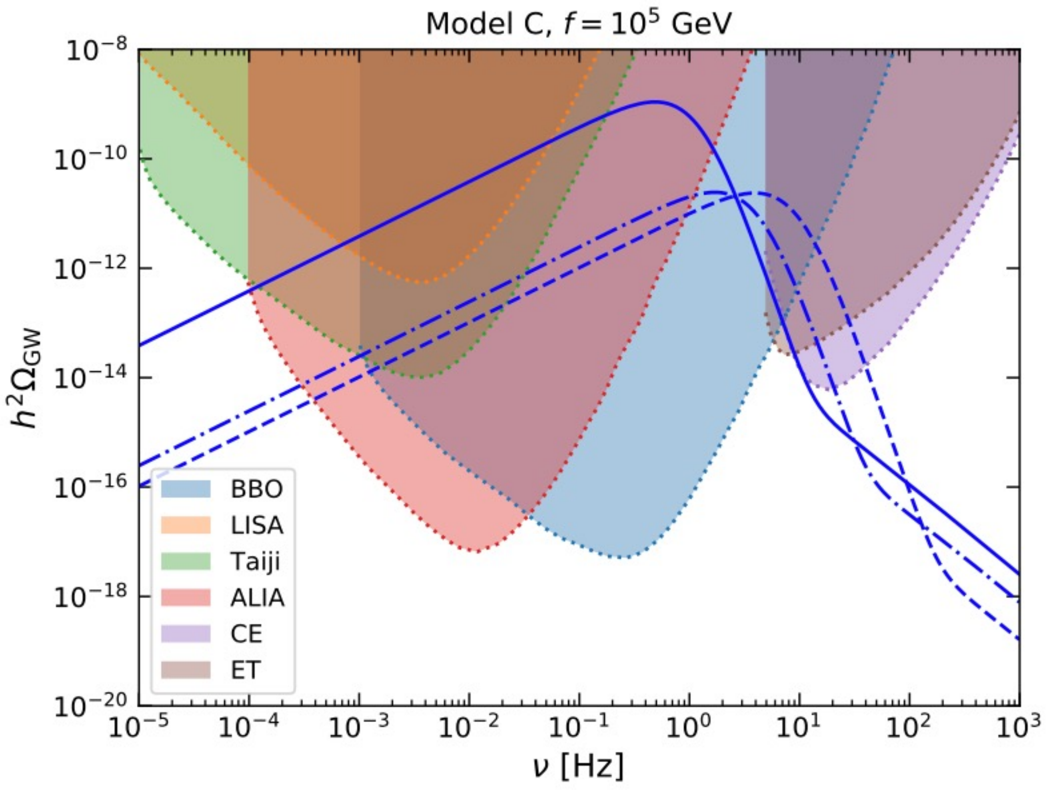


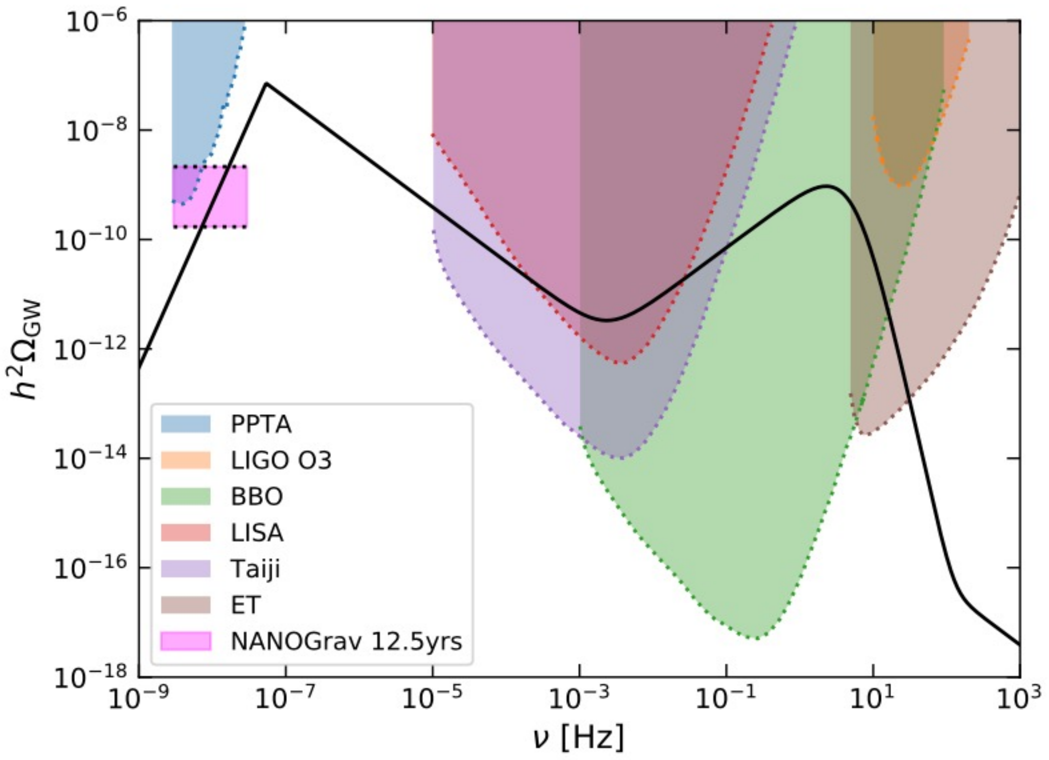


Bule: LIGO Design, Orange: O3, Green: O2

C.W. Chiang and B.Q. Lu, JCAP05(2021)049.

GWs from first-order phase transition





C.W. Chiang and B.Q. Lu, JCAP05(2021)049.

Summary

- The PQ mechanism provide an elegant solution to the strong CP problem. However, the PQ symmetry-breaking scale is so high that the axion is nearly invisible.
- A low PQ symmetry-breaking scale and a large axion decay constant can be simultaneously achieved with the help of clockwork mechanism.
- For large N, String-wall network formed after the PQ symmetry breaking can survive for a long time.
- The QCD axion potential induced by the QCD phase transition can serve as an energy bias, which leads to the annihilation of the domain walls.
- GWs from the annihilation of the domain wall can give rise to the GW signal in the NANOGrav 15-year data.
- The spontaneous PQ symmetry breaking at the scale about 100 TeV can lead to a cosmological first-order phase transition, whose accompanying GW emissions may be tested by the LIGO O3 and design phases.

Thank you for your attention!