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No-scale inflation inspired from string compactification

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Based on Wu and Li, 2205.14639 (PRD); Wu, Gong and Li 2105.07694 (PRD)

Outline

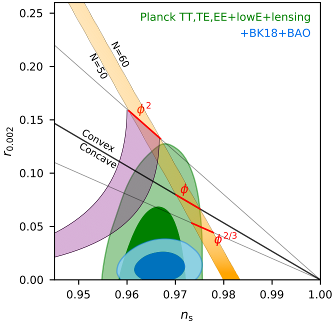
Inflation is one of leading candidates describing the evolution of the early Universe.

- The Universe undergoes a brief period of **exponential expansion** after the Big Bang;
- The generated scalar perturbations are supposed to be adiabatic, almost Gaussian and close to scale-invariant, which are consistent with the full-sky CMB measurements.

Planck 2018 data:
$n_s = 0.9649 \pm 0.0042$ (68% C.L.), $r_{0.002} \leq 0.10$ (95% C.L.), $A_s = 2.10 \times 10^{-9}$

- Combine with BK15: $r_{0.002} \leq 0.056$ (95% C.L.)
- Combine with BK18: $r_{0.05} \leq 0.032$ (95% C.L.)

Big challenge to inflationary models!



Planck Collaboration, 1807.06211; and BICEP/Keck Collaboration, 2110.00483, 2112.07961, 2203.16556

- Supergravity, low-energy effective theory derived from string theory, is a natural framework for inflationary model building.
- No-scale supergravity
  - can elegantly avoid η-problem
  - has a vanishing cosmological constant
  - evades the Anti-de Sitter(AdS) vacua
  - can be realized by string compactifications [1]
- The simple no-scale supergravity inspired from string compactifications is

$$K = -3\log(T + \bar{T} - 2|\varphi_i|^2)$$

where T is the Kähler moduli and φ_i denote the matter, Higgs and inflaton fields.

  - $R + R^2$ Starobinsky model can be obtained by considering a Wess-Zumino superpotential [2]
  - Detectable predictions: lower $r \sim 0.001$, PBH and SIGW formation, NANOGrav data ...

[1] E. Witten, PLB 1985; T. Li, J.L. Lopez and D. V. Nanopoulos, PRD 1997

[2] J. Ellis, M. A. G. Garcia, D. V. Nanopoulos and K. A. Olive, PRL 2013

- Generalizing the factor 3 to 3α, the unified no-scale attractors[3] are studied with

$$K = -3\alpha \log(T + \overline{T} - 2|\varphi|^2)$$

  - $\alpha < 1$, may occur if not all the complex Kähler moduli contribute to driving inflation;
  - $\alpha > 1$, may occur if complex structure moduli also contribute to driving inflation

- α-attractor inflation (E/T-model, etc.)[4] is bulit by introducing a parameter α related to the curvature of the inflaton Kähler manifold. The cosmological predictions are

$$n_s = 1 - \frac{2}{N}, \quad r = \alpha \frac{12}{N^2}.$$

[3] J. Ellis, D. V. Nanopoulos, K. A. Olive and S. Verner, JCAP 2019

[4] Kallosh, Linde, arXiv:2202.06492 and references in there

- The $\mathcal{N}=1$ supergravity Lagrangian can be written in the form

$$\mathcal{L} = -\frac{1}{2}R + K_i^{\bar{j}}\partial_\mu\varphi^i\partial^\mu\overline{\varphi}_{\bar{j}} - V,$$

where the Kähler metric is $K_i^{\bar{j}} \equiv \partial^2 K/(\partial\varphi^i\partial\overline{\varphi}_{\bar{j}})$.

- The effective scalar potential is

$$V = e^G\left[\frac{\partial G}{\partial\varphi^i}\left(K^{-1}\right)^i_{\bar{j}}\frac{\partial G}{\partial\overline{\varphi}_{\bar{j}}} - 3\right],$$

where the Kähler function is $G \equiv K + \ln|W|^2$, and $\left(K^{-1}\right)^i_{\bar{j}}$ is the inverse of the Kähler metric.

- Introducing Kähler covariant derivative $D_iW \equiv W_i + K_iW$, the scalar potential can be rewritten as[5]

$$V = e^K\left[D_iW\left(K^{-1}\right)^i_{\bar{j}}D^{\bar{j}}\overline{W} - 3|W|^2\right].$$

[5] J. Ellis etal., 2009.01709 and Refs. in there

We propose inflationary models in the general no-scale supergravity theories inspired by various compactifications of M-theory:

- **One Modulus Model**: Simple no-scale supergravity, Calabi-Yau compactification with standard embedding of the weakly coupled heterotic $E_8 \times E_8$ theory and M-theory on S^1/Z_2 [6],

$$K = -3\ln\left(T + \overline{T} - 2|\varphi|^2\right).$$

- **Two Moduli Model**: Orbifold compactification on T^6/Z_{12} by keeping singlets under $SU(2) \times U(1)$ symmetry and then the compactification on S^1/Z_2 [7]:

$$K = -2\ln\left(T_1 + \overline{T}_1 - 2|\varphi|^2\right) - \ln\left(T_2 + \overline{T}_2\right).$$

- **Three Moduli Model**: Orbifold compactification on T^6/Z_{12} and S^1/Z_2 [5]:

$$K = -\log(T_1 + \overline{T}_1 - 2|\varphi|^2) - \log(T_2 + \overline{T}_2) - \log(T_3 + \overline{T}_3).$$

[6] E. Witten, Nucl. Phys. B (1985); T. Li etal., hep-ph/9704247

[7] T. Li, hep-th/9801123

- **Generic Kähler potential**

  $$K = -N_X \log(T_1 + \overline{T}_1 - 2|\varphi|^2) - N_Y \log(T_2 + \overline{T}_2) - N_Z \log(T_3 + \overline{T}_3),$$

  where $N_X + N_Y + N_Z = 3$ and $N_{X,Y,Z}$ are integers.

- The **renormalizable superpotential** in ploynomial form

  $$W = \sum_{i=0}^{3} a_i(\sqrt{2}\varphi)^i, \quad \text{with} \quad W_T = 0.$$

- The general scalar potential can be written as

  $$\boxed{V = \frac{|W_\varphi|^2}{2N_X X^{N_X-1} Y^{N_Y} Z^{N_Z}}}$$

  where $X \equiv T_1 + \overline{T}_1 - 2|\varphi|^2$, $Y \equiv T_2 + \overline{T}_2$ and $Z \equiv T_3 + \overline{T}_3$.

- The inflationary model with $N_X = 1$ is similar to that with the global supersymmetry.

Stabilizing the muduli fields, $2\langle \mathrm{Re}(T_i)\rangle = c_i$ and $\langle \mathrm{Im}(T_i)\rangle = 0$, and choosing the inflationary trajectory along with $\overline{\varphi} = \varphi$, the inflation potential becomes

$$V = \frac{1}{2N_X c_2^{N_Y} c_3^{N_Z}} \frac{|W_\varphi|^2}{(c_1 - 2|\varphi|^2)^{N_X - 1}}.$$

The kinetic term in Lagrangian is **noncanonical**, so we need to define a new canonical field χ, which satisfies

$$\frac{1}{2}\partial_\mu \chi \partial^\mu \chi = K_{\varphi\bar{\varphi}} \partial_\mu \varphi \partial^\mu \bar{\varphi}$$

By integrating the above equation, we get the **field transformation**

$$\varphi = \sqrt{\frac{c_1}{2}} \tanh\left(\frac{\chi}{\sqrt{2N_X}}\right)$$

Then the potential in the Einstein Frame is

$$V = V_0 \mathrm{sech}^{2m}(b\chi)\left(a_1 + 2a_2\sqrt{c_1}\tanh(b\chi) + 3a_3 c_1 \tanh^2(b\chi)\right)^2$$

where $V_0 = N_X^{-1} c_1^m c_2^{-N_Y} c_3^{-N_Z}$, $b = 1/\sqrt{2N_X}$ and $m = 1 - N_X$.

Single field inflation

- Background evolution of the canonical scalar field

$$\ddot{\chi} + 3H\dot{\chi} + V'(\chi) = 0$$

$$3H^2 = \frac{1}{2}\dot{\chi} + V(\chi)$$

Here, we set $M_{\rm Pl} = 1$.

- Number of e-foldings

$$N = \int_{t_*}^{t_e} H(t)\mathrm{d}t \sim 50 - 60$$

- Hubble slow-roll parameters

$$\epsilon_H = -\frac{\dot{H}}{H^2}, \quad \eta_H = \frac{\dot{\epsilon}_H}{H\epsilon_H}$$

- Slow-roll conditions

$$\epsilon_H, \eta_H \ll 1$$

- CMB observations

$$n_s = 1 + 2\eta_H - 4\epsilon_H, \quad r = 16\epsilon_H$$

$$P_{\mathcal{R}} = \frac{1}{8\pi^2}\frac{H^2}{\epsilon_H}$$

One modulus models: $N_X = 3, N_Y = N_Z = 0$

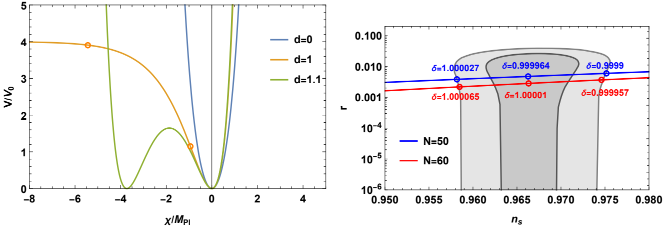


Figure 1: r vs n_s predicitons for the one modulus model with $a_1 = 0$.

Here the parameter $\delta = \pm 3a_3\sqrt{c_1}/2a_2$. And when $2a_2 = -3a_3\sqrt{c_1}$, the E-model for φ^2 or Starobinsky inflation model is realized. The potential is

$$V = V_0(1 - e^{\sqrt{2/3}\chi})^2 \simeq V_0(1 - 2e^{\sqrt{2/3}\chi} + ...).$$

The spectrum index and tensor-to-scalar ratio are

$$n_s \simeq 1 - \frac{2}{N}, \;\; r \simeq \frac{12}{N^2} \sim 10^{-3}.$$

Two moduli models: $N_X = 2, N_Y = 1, N_Z = 0$

For $a_1 = 0$ case, the scalar potential is

$$V = V_0 \frac{(1 - e^{\chi})^2 \left(1 + \frac{1+d}{1-d} e^{\chi}\right)^2}{e^{\chi} (1 + e^{\chi})^2}$$

- The potential remains the same after both parameter $d = 3a_3\sqrt{c_1}/2a_2$ and field χ become negative. Therefore, we only discuss inflation with $d \geq 0$.
- For $0 < d \leq \sqrt{27/32}$, one notes that the **higher order term** $\left(\frac{1+d}{1-d} e^{\chi}\right)^2$ in the numerator in potential is not small and **cannot be ignored**.
- When $d = \sqrt{27/32}$, the inflation initials at $\chi_* \simeq -2.26 M_{\rm Pl}$, and then the tensor-to-scalar ratio r is approximate to be

$$r \simeq \frac{8192\left(49 - 20\sqrt{6}\right)}{N^4} \simeq \frac{83.60}{N^4} \sim 10^{-5}$$

Two moduli models: $N_X = 2, N_Y = 1, N_Z = 0$

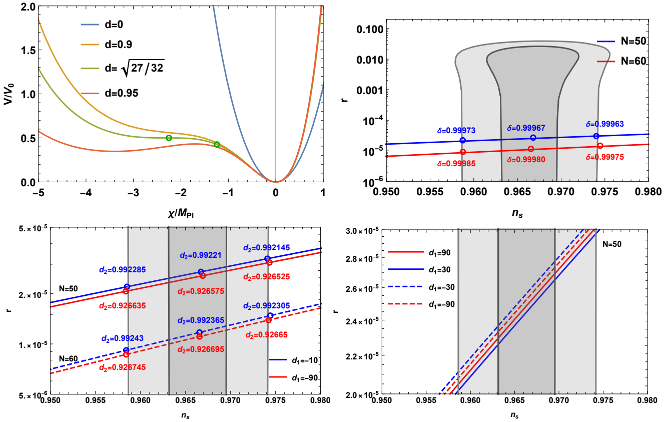


Figure 2: Top: r vs n_s predicitons for the two moduli model with $a_1 = 0$; Bottom: r vs n_s predicitons for the general case in two moduli model.

Here, $\delta = \sqrt{32/27} \times d$ and $d_1 = 2a_2\sqrt{c_1}/a_1$ and $d_2 = d = 3a_3\sqrt{c_1}/2a_2$.

Three moduli models: $N_X = N_Y = N_Z = 1$

T-models for φ^{2n} are realized:

Cases	$V \propto \lvert W_\varphi \rvert^2$		
$a_{1,3} = 0$	$V_J \propto \lvert\varphi\rvert^2$	$V_E \propto \tanh^2\left(\chi/\sqrt{2}\right)$	T-model for φ^2
$a_{1,2} = 0$	$V_J \propto \lvert\varphi^2\rvert^2$	$V_E \propto \tanh^4\left(\chi/\sqrt{2}\right)$	T-model for φ^4

- The slow-roll parameters are

$$\varepsilon = 4n^2 \mathrm{csch}^2\left(\sqrt{2}\chi\right), \quad \eta = 4n\left(2n - \cosh\left(\sqrt{2}\chi\right)\right)\mathrm{csch}^2\left(\sqrt{2}\chi\right).$$

- The inflation ends at $\chi_e = \sinh^{-1}(2n)/\sqrt{2}$ with $\varepsilon = 1$.
- The cosmological predictions n_s and r, and e-folding number are given by

$$n_s = 1 - \frac{2\left(n + \sqrt{4n^2+1} + 4nN\right)}{n + 2N\sqrt{4n^2+1} + 4nN^2} \sim 1 - \frac{2}{N},$$

$$r = \frac{16n}{n + 2N\sqrt{4n^2+1} + 4nN^2} \sim \frac{4}{N^2}$$

where $N = \frac{1}{4n}\cosh\left(\sqrt{2}\chi\right)\Big|_{\chi_e}^{\chi_*} = \frac{1}{4n}\left(\cosh\left(\sqrt{2}\chi_*\right) - \sqrt{4n^2+1}\right)$.

Connect T-model and E-model

For $a_2 = 0$ or $a_3 = 0$ case, the potentials are

$$V_1 = V_0 \left(1 + d_1 \tanh^2\left(\chi/\sqrt{2}\right)\right)^2, \quad V_2 = V_0 \left(1 + d_2 \tanh\left(\chi/\sqrt{2}\right)\right)^2,$$

where $V_0 = a_1^2/c_2 c_3$, $d_1 = 3a_3 c_1/a_1$ and $d_2 = 2a_2\sqrt{c_1}/a_1$.

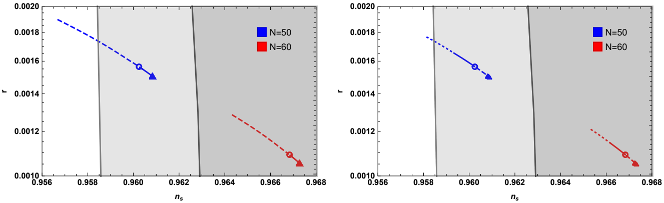


Figure 3: r versus n_s predictions for the $a_2 = 0$ (left) and $a_3 = 0$ (right) cases. The circles and triangles are corresponding to T- ($d_i \to \pm\infty$) and E-model ($d_i \to -1$), respectively.

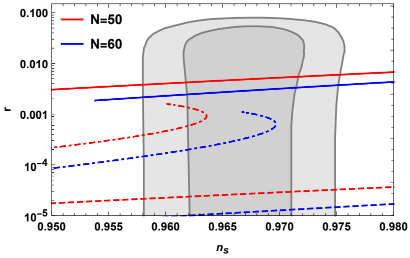
CMB observations with constraints from Planck2018 and BK18

Taking the Wess-Zumino superpotential,

$$W = \frac{M}{2}\varphi^2 - \frac{\lambda}{3}\varphi^3, \quad \text{with} \quad W_T = 0.$$

the inflation potentials become ($d = \lambda/M$)

$$V_1 = \frac{M^2\varphi^2(1-d\varphi)^2}{3(c-\varphi^2)^2}, \quad V_2 = \frac{M^2\varphi^2(1-d\varphi)^2}{2c_3(c_1-\varphi^2)}, \quad V_3 = \frac{M^2\varphi^2(1-d\varphi)^2}{c_2 c_3}.$$



Top: one modulus model $r \sim 10^{-3}$

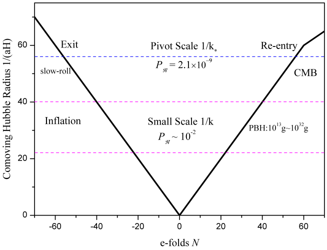
Middle: three moduli model $r \sim 10^{-4}$

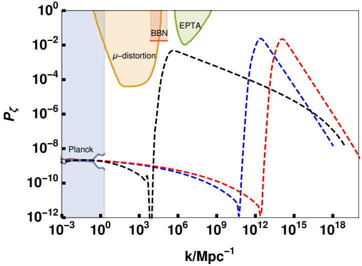
Bottom: two moduli model $r \sim 10^{-5}$

- The tensor-to-scalar ratios are much smaller than 0.032.

- PBH will give useful information for early Universe and produce various astrophysical consequences
  - primordial density perturbations, ...
  - seed for supermassive BHs, generation of large-scale structure, ...
- PBH as cold dark matter candidates.
- An important topic: formation of PBH during inflation:
  - Inflection inflation model
  - Multi-scalar inflation model
  - Framework of non-minimal derivative coupling
- Near the inflection point: the slow-roll conditions are violated ($\varepsilon_H \sim 10^{-7}$; $\eta_H \sim 3$), so the primordial power spectrum is enhanced, at the same time the number of e-folds also increases dramatically.

Inflection point model





Power spectrum at different scale:
(1) **Large scale**: Scale invariant, CMB
(2) **Small scale**: Enhancement, PBHs

Modified Kähler potential

An **exponential term** is added to bring an inflection point into the Kähler potential

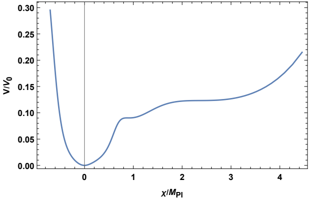
$$K = -2\log\left[T_1 + \overline{T}_1 - |\varphi|^2 + ae^{-b(\varphi^\alpha + \overline{\varphi}^\alpha)}\left(\varphi^\beta + \overline{\varphi}^\beta\right)\right] - \log[T_2 + \overline{T}_2]$$

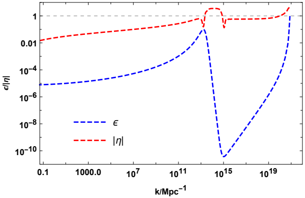
Here, $\alpha = \beta = 2$.
The scalar potential of the inflaton $\phi = \mathrm{Re}(\varphi)$ becomes

$$V = V_0 \frac{\varphi^2 e^{4b\varphi^2}(d\varphi - 1)^2}{\left(e^{2b\varphi^2} - 8ab\varphi^2\left(b\varphi^2 - 1\right)\right)\left(2a\varphi^2 + e^{2b\varphi^2}\left(c - \varphi^2\right)\right)},$$

with $c = c_1,\ V_0 = M^2/(2c_2)$ and $d = \lambda/M$.

The potential of the cannonical scalar field χ





- There is an inflection point at $\chi_p = 0.877\ \mathrm{M_{Pl}}$, where the slow-roll conditions are no longer satisfied: $\epsilon \sim 10^{-7}$ and $\eta \sim 3$.
- Near the inflection, the primordial curvature perturbations are enhanced, which cause gravitational collapse in the overdense region at the horizon re-entry during the radiation-dominated era.
- If the density fluctuation is large than a centain threshold $\delta_c(0.07 - 0.7)$, the gravity can overcome the pressure and hence PBH forms.

- Assuming the primordial perturbations obey Gaussian statistics, the fractional energy density of PBHs at their formation time is given by the Press-Schechter formalism

$$\beta(M) \equiv \frac{\rho_{PBH}}{\rho_{tot}} \simeq \sqrt{\frac{2}{\pi}} \frac{\sqrt{P_\zeta}}{\mu_c} \exp\left(-\frac{\mu_c^2}{2P_\zeta}\right), \text{ with } \mu_c = 9\delta_c/2\sqrt{2}$$

- The fractional energy density of PBHs with the mass M to DM is[8]

$$f_{PBH}(M) \equiv \frac{\Omega_{PBH} h^2}{\Omega_{DM} h^2}$$
$$= \frac{\beta(M)}{3.94 \times 10^{-9}} \left(\frac{\gamma}{0.2}\right)^{\frac{1}{2}} \left(\frac{g_*}{3.36}\right)^{-\frac{1}{4}} \left(\frac{0.12}{\Omega_{DM} h^2}\right) \left(\frac{M}{M_\odot}\right)^{-\frac{1}{2}},$$

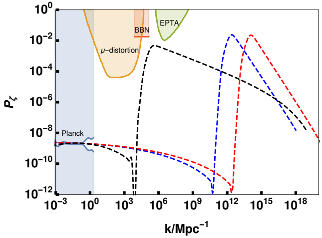
where $\gamma = 0.2$, $\Omega_{DM} h^2 = 0.12$ and $g_* = 106.75$.

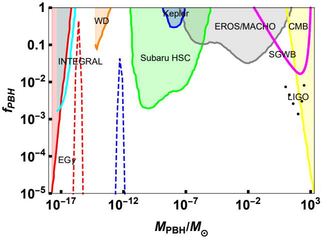
- The mass of PBHs is

$$\frac{M(k)}{M_\odot} = 3.68 \times \left(\frac{\gamma}{0.2}\right) \left(\frac{g_*}{3.36}\right)^{-\frac{1}{6}} \left(\frac{k}{10^6 \ \mathrm{Mpc}^{-1}}\right)^{-2}.$$

[8] B. Carr etal., 1607.06077; B.Carr etal., 2002.12778

Primordial curvature perturbations and PBH abundances





- Three benchmark points where the PBH mass is around $\mathcal{O}(10^{-16}M_\odot)$, $\mathcal{O}(10^{-12}M_\odot)$ and $\mathcal{O}(M_\odot)$.
- The PBHs with masses around $\mathcal{O}(10^{-16}M_\odot)$ and $\mathcal{O}(10^{-12}M_\odot)$ can make up almost all DM and the peak abundances are $f_{PBH} \simeq 1$.
- The PBH with the mass around $\mathcal{O}(M_\odot)$ only explains part of dark matter with $f_{PBH} \simeq 10^{-75}$.

Primordial curvature perturbations and PBH abundances

Model	b	c	d	χ_*	χ_p	χ_e	n_s	r	N
I	17.331408	0.2498	1.8409	2.339	0.8772	0.5523	0.9681	1.3×10^{-4}	51.9
II	17.346393	0.2490	1.8435	2.339	0.8788	0.5528	0.9681	2.1×10^{-4}	48.3
III	17.770380	0.2293	1.9159	2.346	0.9223	0.5657	0.9650	4.2×10^{-3}	48.0

Table 1: The model parameters with $a = -2$ and the predicted CMB observables n_s and r.

Model	$k_{peak}(\mathrm{Mpc}^{-1})$	$\mathcal{P}_\zeta$	$M_{PBH}(\mathrm{M}_\odot)$	f_{PBH}	$f_{GW}(\mathrm{Hz})$
I	1.20×10^{14}	0.023	2.54×10^{-16}	0.44	1.17
II	2.72×10^{12}	0.024	4.50×10^{-13}	0.04	2.64×10^{-2}
III	5.44×10^{5}	4.33×10^{-3}	17.28	6.6×10^{-75}	5.28×10^{-9}

Table 2: The peak values of the power spectrum, the mass and abundance of PBH, and the frequency of SIGWs.

Scalar induced gravitational waves

Since the scalar perturbations and tensor perturbations are coupled at the second order, the large primordial curvature perturbation on small scales will induce second order tensor perturbations.

- The equation of motion for the tensor mode is

$$h''_{\mathbf{k}} + 2\mathcal{H}h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} = 4\mathcal{S}_{\mathbf{k}}$$

- Using Green's function, the power spectrum of the tensor perturbation can be written as

$$\mathcal{P}_h(k,\eta) = 4\int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2 I_{RD}^2(u,v,x)\mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)$$

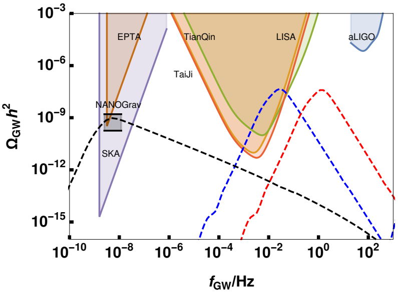
and the fractional energy density of the induced GWs[9] is

$$\Omega_{GW}(\eta,k) = \frac{\rho_{GW}(\eta,k)}{\rho_{tot}(\eta)} = \frac{1}{24}\frac{k^2}{\mathcal{H}^2}\overline{\mathcal{P}_h(\eta,k)},$$

where the overline denotes oscillation average.

[9] J.R. Espinosa etal., 1804.07732; F. Zhang etal., 2008.12961

The energy densities of SIGWs



- Model III: wide peak at $[10^{-10},\ 10^{2}]$ Hz
- Model II: peak at 10^{-2} Hz
- Model I: peak at 1 Hz

- The generated SIWGs will be tested by the space-based or ground-based GW detector.
- The wide band can be interpreted as the stochastic GW background observed by NANOGrav.

- We have studied three classes of no-scale inflation models with one, two, and three moduli which can be realized naturally via string compactifications.
- The E-model and T-model are emerged in the one modulus model and the three moduli model, respectively. They are connected by the three moduli model in the limits $2a_2\sqrt{c_1}/a_1 \to \pm 1$ and $2a_2\sqrt{c_1}/a_1 \to \pm\infty$.
- The detailed analyses of the spectral indices and the tensor-to-scalar ratio have been preformed, and they are consistent with the Planck and BICEP/Keck experimental data:
  - $n_s \simeq 1 - 2/N \sim 0.965$ for all models;
  - r is $r \simeq 12/N^2$, $83/N^4$ and $4/N^2$ for the one, two and three moduli models, respectively
- Formation of PBHs and SIGWs are investigated by introducing an exponential term into Kähler potential.

Thanks!