ALP concurrent effect with photon and electron couplings within collider and beamdump searches

arXiv:2304.05435

Muyuan Song

Peking University, CHEP

July 24, 2023

The 2nd International Conference on Axion Physics and Experiment (AXION 2023)

ALP searches with mass region within $\mathcal{O}(1)$ MeV to 10 GeV

With Jia Liu and Yan Luo

Motivation:

- Axion$\rightarrow$ Strong CP problem (PQ symmetry, $\bar{\theta}$)

  [*Peccei, Quinn, 1977; Wilczek,1978; Weinberg,1978*]
- Axion-like particles (ALP) (independent f_a and m_a)
  (e.g., string theory [*P. Svrček, E.Witten, 2006; A. Arvanitaki,et al. 2009*])
- Concurrent effects (both couplings simultaneously) of ALP-electron and ALP-photon couplings would be essential.
- low masses ALPs (below eV scales) have covered the interplay of multiple couplings / concurrence (e.g., $g_{a\gamma}$ and g_{ae})
- Searches for ALPs (MeVs to GeVs) derive singly coupling bounds while multiple couplings exist in UV.

Past searches with relevant couplings interplay

- Majority focus on eV scales and cosmology/astrophysics constraints or ALP-gauge boson couplings

→ ALP-electron/photon$(g_{ae}, g_{a\gamma})$ [*C. Gao et al, 2020; M. Xiao et al,2022*]

→ ALP-nucleon/photon$(g_{aN}, g_{a\gamma})$ [*L. Di Luzio et al.,2022*]

→ ALP-EW gauge bosons$(g_{aWW}, g_{aBB}, ..)$

[*F. Ertas and F. Kahlhoefer, 2020; J. Bonilla et al, 2021; G. Alonso-Álvarez et al, 2022*]

- New Physics

→ $(g-2)_\mu$ $(g_{a\mu}, g_{a\gamma})$ [*D. Chang et al, 2001; M.A. Buen-Abad et al, 2021, J. Liu et al, 2023*]

→ CP-violation/LFV $(g_{ae}, g_{a\mu}, g_{aW}, ...)$

[*W.J. Marciano, 2016; C. Cornella, 2020; M. Bauer 2020*]

Outline

- ALP UV models with one of the couplings missing in high energy but generate radiatively at low energy.

$\rightarrow$ KSVZ-like and DFSZ-like.

- Collider and electron beamdump limit reinterpretation (ALP-electron and ALP-photon) for $m_a \rightarrow \mathcal{O}(1)$ MeV to 10 GeV.

Low energy effective Lagrangian ALPs with electrons and photon

$$\mathcal{L}_a^{D\leq 5} = \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_a^2 a^2 - \frac{1}{4} g_{a\gamma\gamma}^{\text{eff}} \times a F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{2} g_{a\bar{e}e}^{\text{eff}} \times \partial_\mu a \, \bar{e}\gamma^\mu \gamma_5 e$$

dual field strength tensor $\left(\frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}\right)$

- Decay of ALP

$$\Gamma_{a\to e\bar{e}} = \frac{(g_{a\bar{e}e}^{\text{eff}})^2 m_e^2 m_a}{8\pi}\left(1 - \frac{4m_e^2}{m_a^2}\right)^{\frac{1}{2}}, \Gamma_{a\to\gamma\gamma} = \frac{(g_{a\gamma\gamma}^{\text{eff}})^2 m_a^3}{64\pi}$$

$\rightarrow$ $\mathcal{B}(a\to\gamma\gamma)/\mathcal{B}(a\to e\bar{e}) \approx (g_{a\gamma\gamma}^{\text{eff}})^2 m_a^2 / 8 (g_{a\bar{e}e}^{\text{eff}})^2 m_e^2$

$\Longrightarrow$ Favor for diphoton at $\mathcal{O}(1)$ GeV, but dielectron could be comparable at lower mass (Beamdump).

UV models for ALPs (KSVZ-like and DFSZ-like)

KSVZ-like

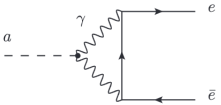
- $\rightarrow$ Vector-like fermion ($\mathcal{Q}_L$, $\mathcal{Q}_R$) carry SM gauge charge $\sim (1, 1, Y)$ for $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$.
- $\rightarrow$ Singlet (Φ) $\sim (1, 1, 0)$.
- $\rightarrow$ $\mathcal{Q}_L$, $\mathcal{Q}_R$ and Φ satisfy $U(1)_{\text{PQ}} \sim (+1,\ -1 \text{ and } +2)$.
- $\rightarrow$ a is the axion after global $U(1)_{\text{PQ}}$ breaking.

$$\Phi = \frac{1}{\sqrt{2}}(v_a + \rho_a)e^{i\frac{a}{v_a}}$$

- $\rightarrow$ After EWSB, chiral rotation of the $\mathcal{Q}_{L/R}$ leaves chiral anomaly (no N):

$$\delta\mathcal{L}_{\text{KSVZ-like}} \quad = \quad 3Y^2 \frac{e^2}{16\pi^2}\frac{a}{v_a}F\tilde{F} = E\frac{\alpha_{\text{QED}}}{4\pi}\frac{a}{v_a}F\tilde{F}$$

KSVZ-like



→ $g^{\text{eff}}_{a\bar{e}e}$ (cut off $\sim v_a \to f_a$)

[*M. Srednicki, 1985; M. Bauer et al, 2017*]

$$\begin{aligned} g^{\text{eff}}_{a\bar{e}e} &\to g^{0}_{a\bar{e}e}(=0) \\ &\approx \frac{3\alpha_{\text{QED}}}{4\pi} g^{\text{eff}}_{a\gamma\gamma}\left[\ln\left(\frac{f_a^2}{m_e^2}\right) + g\left(\tau_e\right)\right] \end{aligned}$$

$$g(\tau_e) = -\frac{1}{6}\left(\ln\left(\frac{m_a^2}{m_e^2}\right) - i\pi\right)^2 + \frac{2}{3}, \text{ for } m_a^2 \gg m_e^2$$

→ $g^{\text{eff}}_{a\gamma\gamma}$

$$g^{\text{eff}}_{a\gamma\gamma} = E\frac{\alpha_{\text{QED}}}{4\pi}\frac{1}{f_a}$$

→ $F\tilde{Z}$ and $Z\tilde{Z}$ contribute for large f_a, m_a.

DFSZ-like

- → $H_u \sim (1, 2, 1/2)$, $H_d \sim (1, 2, 1/2)$, and $\Phi \sim (1, 1, 0)$.
- → Quarks couple with H_u (Type-Lepton specific 2HDM).
- → PQ charges $X_{H_u} = -1$, $X_{H_d} = +1$, and $X_\Phi = -X_{H_u} + X_{H_d} = +2$
  - $c_u, c_d, c_\ell = \cos^2\beta, -\cos^2\beta, \sin^2\beta \ (v_u/v_d)$

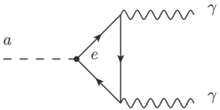
$$\begin{aligned}\mathcal{L} \quad &\supset \quad i(c_f \frac{a}{f_a}) m_f \bar{f}\gamma_5 f \supset \frac{c_\ell}{2}\frac{\partial_\mu a}{f_a}\bar{\ell}\gamma^\mu\gamma_5\ell + c_u(...) + c_d(...) \\ &- \quad E(N_c^f, N_g, c_u, c_d, c_\ell)\frac{e^2}{16\pi^2}\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}\end{aligned}$$

where f_a is broken scale ($\propto v_\phi$).

- → $E(N_c^f, N_g, c_u, c_d, c_\ell) \to 1$
- → $N \propto c_u + c_d = 0$ (no color anomaly coefficient for $G\tilde{G}$)

DFSZ-like

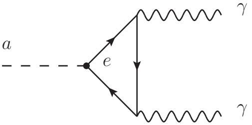
→ $g^{\text{eff}}_{a\gamma\gamma}$
($U(1)_{\text{EM}}$ anomaly and loop contribution)

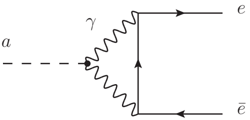


$$g^{\text{eff}}_{a\gamma\gamma} = \frac{e^2}{16\pi^2}\left(\sum_{i=e,\mu,\tau}(B_1(\tau_i)-1)g^{\text{eff}}_{a\bar{\ell}\ell}\right)$$

$$B(\tau_f) = 1 - \tau_f f^2(\tau_f), \;\; \tau_f \equiv \frac{4m_f^2}{m_a^2}, \;\; \tau_f \to \infty, B(\tau_f) \approx -\frac{m_a^2}{12m_f^2}; \quad \tau_f \to 0, B(\tau_f) \approx 1$$

→ $g^{\text{eff}}_{a\bar{e}e} \propto \sin^2\beta$ (lepton couplings are same)





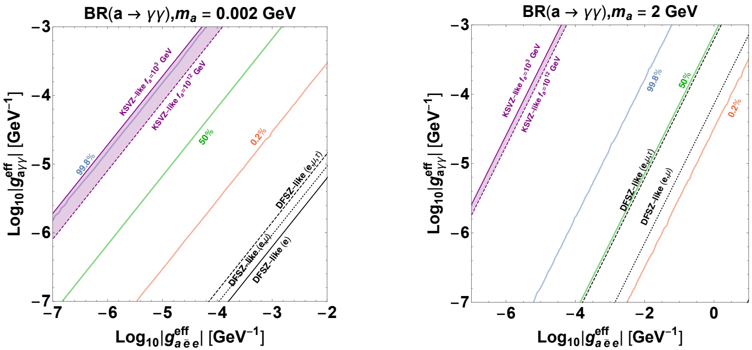
DFSZ-like **KSVZ-like**

$$\frac{|g^{\text{eff}}_{a\gamma\gamma}|}{|g^{\text{eff}}_{a\bar{e}e}|} = \frac{\sum_{\ell=e,\mu,\tau} \left(B(\tau_\ell) - 1\right) \frac{N_c Q_f^2 e^2}{16\pi^2} g^{\text{eff}}_{a\bar{\ell}\ell}}{g^{\text{eff}}_{a\bar{e}e}}$$

$$\frac{|g^{\text{eff}}_{a\gamma\gamma}|}{|g^{\text{eff}}_{a\bar{e}e}|} = \frac{8\pi}{\alpha_{\text{QED}} \left| \left(6 \ln\left(\frac{f_a^2}{m_e^2}\right) - \left(\ln\left(\frac{m_a^2}{m_e^2}\right) - i\pi\right)^2 + 4 \right) \right|}$$

- KSVZ-like favors for ALP-photon coupling while DFSZ-like favors for ALP-lepton/electron coupling.

$\mathcal{B}\ (a \to \bar{e}e/\gamma\gamma)$ with UV models in $[g^{\text{eff}}_{a\bar{e}e}, g^{\text{eff}}_{a\gamma\gamma}]$



→ $\mathcal{B}(a \to \bar{e}e)$ increases when m_a decreases due to $m_a^2/(8m_e^2)$.

→ KSVZ-like favors for large $g^{\text{eff}}_{a\gamma\gamma}$ while DFSZ-like favors for $g^{\text{eff}}_{a\bar{e}e}$.

Collider and beamdump (electron) experiments

- For $m_a \to 0.2 \sim 10$ GeV, we focus on two main experiments ($e^+ + e^- \to a + \gamma$ follows $a \to \bar{e}e/\gamma\gamma$)

$\to$ Belle-II ($\sqrt{s} = 10.58$ GeV) limits $g^{\text{eff}}_{a\gamma\gamma}$ [Belle-II,2020]

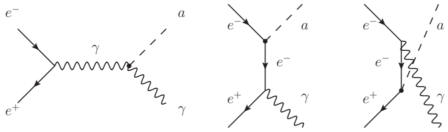
$\to$ BaBar limits $g^{\text{eff}}_{a\bar{e}e}$ [BaBar,2020]

- For $m_a \to \mathcal{O}(1)$ MeV ~ 1 GeV, NA64 and E137 beamdump experiments are investigated ($e + N \to e + N + a$).

$\to$ Both two experiments have constrained $g^{\text{eff}}_{a\bar{e}e}$ and $g^{\text{eff}}_{a\gamma\gamma}$ separately.

The concurrent effects of two couplings have a greater impact on electron beamdump searches.

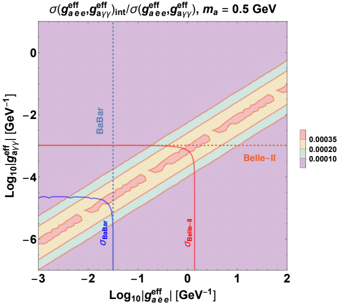
Cross section for collider approach



$$\sigma(ee \to a\gamma) \propto \alpha_{\text{qed}} \left[\frac{(g^{\text{eff}}_{a\gamma\gamma})^2 \left(s - m_a^2\right)^3}{24 s^3} + \frac{m_e^2}{(s - m_a^2)} \left((g^{\text{eff}}_{a\gamma\gamma})^2 + (g^{\text{eff}}_{a\bar{e}e})^2 + g^{\text{eff}}_{a\gamma\gamma} g^{\text{eff}}_{a\bar{e}e} \right) \right] + \dots$$

- $\rightarrow$ First term corresponds to $s-$channel/Primakoff effect (Left figure).
- $\rightarrow$ Second term suppressed by (m_e^2/s).

Collider scenario $[g^{\text{eff}}_{a\bar{e}e}, g^{\text{eff}}_{a\gamma\gamma}]$ with $m_a = 0.5$ GeV

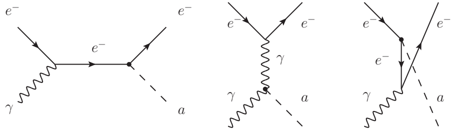


- Left: $\sigma_{\text{int}}/\sigma_{\text{total}}$. Right: $\sigma \times \mathcal{B}(a \to \bar{e}e/\gamma\gamma)$.
- Horizontal(Belle-II) and vertical(BaBar) dashed lines are existing bounds (left bottom region is survived).
- Concurrent effect:σ and $\mathcal{B}$.

$\rightarrow$ $\sigma \uparrow$ when the other coupling is large (survived region squeezed)

$\rightarrow$ $\mathcal{B} \downarrow$ when the other coupling is large (survived region relaxed)

Beamdump exclusion for ALPs ($e^- N \to e^- aN$ with Nuclei attached with photon)



- Improved Weizsäcker-Williams (IWW) approximation to reduce phase space integral ($2 \to 3 \Longrightarrow 2 \to 2$) ($\gamma_{\text{vir}} \to \gamma_{\text{real}}$)

[*C.F. Weizsäcker, 1934; E.J. Williams, 1934; Y.-S. Tsai and V. Whitis,1966; K.J. Kim and Y.-S. Tsai, 1973*]

$\to$ $m_a << E_{e(\text{in})}$.

$\to$ Final states are highly collinear.

$$\frac{d\sigma}{dx} = \frac{\alpha_{\text{QED}}}{4\pi^2} \frac{\sqrt{E_{e(\text{in})}^2 x^2 - m_a^2}}{E_{e(\text{in})}} \chi_\gamma \int d\tilde{u} \frac{\mathcal{A}}{\tilde{u}^2} \frac{1-x}{x}$$

$x = E_a/E_{e(\text{in})}$, $\mathcal{A}$ (Amplitude), $\tilde{u}$ (modified u),χ_γ (Photon flux)

Number of ALP events from electron beamdump

$$N_a \approx \frac{N_e X}{M_{\mathrm{target}}} \int_{E_{\mathrm{min}}}^{E_0} dE \int_{x_{\mathrm{min}}}^{x_{\mathrm{max}}} dx \int_0^T dt\, \mathcal{I}_e \frac{d\sigma}{dx} e^{-L_{\mathrm{sh}}\left(\frac{1}{L_a}+\frac{1}{l_\lambda}\right)} \left(1 - e^{-\frac{L_{\mathrm{dec}}}{L_a}}\right)$$

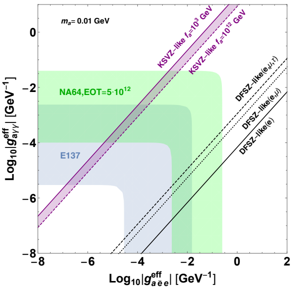
[*Y.-S. Tsai and V. Whitis, 1966*]

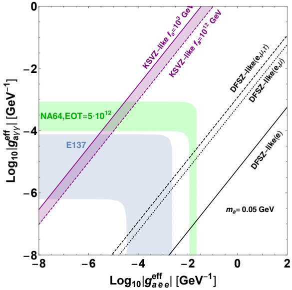
- $\mathcal{I}_e$ Energy distribution of electrons passing through radiation length t
- $N_a = 3$ (95% C.L) for E137 (Al). $N_a = 2.3$ (90% C.L) for NA64 (Pb).
- L_a decay length of ALP $\propto \Gamma_a$(ALP-photon/electron)

$$L_a = \frac{E_a}{m_a}\frac{1}{\Gamma_a}$$

$\rightarrow$ $L_a \downarrow$ when $\Gamma_a \uparrow$ (additional coupling add in).

Beamdump coupling contours [$g_{a\bar{e}e}^{\rm eff}$, $g_{a\gamma\gamma}^{\rm eff}$] (UV models included)





- Concurrent effect of $g_{a\bar{e}e}^{\rm eff}$, $g_{a\gamma\gamma}^{\rm eff}$ for Blue(E137), Green(NA64).
- Left: $m_a = 0.01$ GeV. Right: $m_a = 0.05$ GeV.

$\rightarrow$ (Right and Top region) couplings $\uparrow$ affects $L_a \downarrow$ (surpass $\sigma \uparrow$) caused by $e^{-L_{\rm sh}/L_a} \downarrow$ and $N_a \downarrow$.

$\rightarrow$ (Left bottom corner) excluded region shifts towards the lower coupling corner when $m_a \uparrow$.

$\Longrightarrow$ a mild increase for N_a.

Summary

- ALP searches with electrons and photons from eV$\rightarrow$ GeVs (cosmology/astrophysics, Beamdumps, and Colliders).
- Both Beamdumps and Colliders have less attention to the interplay of two couplings (concurrence).
- UV complete models would have both couplings generated non-trivially (one of the two generated radiately).

$\rightarrow$ (KSVZ/DFSZ)-like models.

- Three physical effects (concurrence): σ ($\uparrow$), $\mathcal{B}$ (relax bound) and $\Gamma_a \uparrow$ ($L_a \downarrow$).
- Beamdump searches have more impact with L_a.

$\rightarrow$ Large coupling regions survived because $N_a \downarrow$ when $L_a \downarrow$.

Thanks

List of References

- [*Peccei, Quinn, 1977; Wilczek,1978; Weinberg,1978*]
  [*Phys.Rev.Lett. 38 (1977) 1440; Phys.Rev.Lett. 40 (1978) 279; Phys.Rev.Lett. 40 (1978) 223*]
- [*A. Arvanitaki,et al. 2009; P Svrček, E.Witten, 2006*]
  [*0905.4720, hep-th/0605206*]
- [*J. Preskill, M. B. Wise and F. Wilczek, 1983*]
  [*Phys.Lett.B 120 (1983) 127-132*]
- [*C. Gao et al, 2020; M. Xiao et al,2022*]
  [*2204.03121; 2006.14598*]
- [*L. Di Luzio et al,2022*]
  [*2111.06407*]
- [*M. Srednicki, 1985*]
  [*Nucl. Phys. B 260(1985) 689*]
- [*F. Ertas and F. Kahlhoefer, 2020; J. Bonilla et al, 2021; G. Alonso-Álvarez et al, 2022*]
  [*2004.01193; 2107.11392; 1811.05466*]
- [*Y. Nomura et al, 2008; L, Darmé et al 2021*]
  [*0810.5397; 2012.07894*]
- [*D. Chang et al, 2001; M.A. Buen-Abad et al, 2021, J. Liu et al, 2023*]
  [*hep-ph/0009292; 2104.03267; 2210.09335*]
- [*W.J. Marciano, 2016; C. Cornella, 2020; M. Bauer 2020*]
  [*1607.01022; 1911.06279; 1908.00008*]
- [*Belle-II,2020; BaBar,2020*]
  [*2007.13071; 1406.2980*]
- [*C.F. Weizsäcker, 1934; E.J. Williams, 1934; Y.-S. Tsai and V. Whitis,1966; K.J. Kim and Y.-S. Tsai, 1973*]
  [*Z.Phys.88(1934)612; Phys.Rev.45(1934)729; Phys.Rev.149(1966)1248; Phys.Rev.D 8(1973)3109*]
- [*R.R.Dusaev et al, 2020*]
  [*2004.04469*]

Back up slides

KSVZ-like model

$$\mathcal{L} \supset |\partial^\mu \Phi|^2 + i\bar{\mathcal{Q}}\not{D}\mathcal{Q} - (\mathcal{Y}_{\mathcal{Q}}\bar{\mathcal{Q}}_L \mathcal{Q}_R \Phi + \mathrm{h.c}) - V(\Phi)$$

$$V(\Phi) = \lambda_\Phi \left(\Phi^* \Phi - \frac{v_a^2}{2} \right)^2 .$$

one can perform the chiral rotation ($\mathcal{Q} \to e^{-i\gamma^5 \frac{a}{2v_a}} \mathcal{Q}$) as follows (remove ALP),

$$\mathcal{Q}_L \to e^{i\frac{a}{2v_a}} \mathcal{Q}_L, \;\; \mathcal{Q}_R \to e^{-i\frac{a}{2v_a}} \mathcal{Q}_R$$

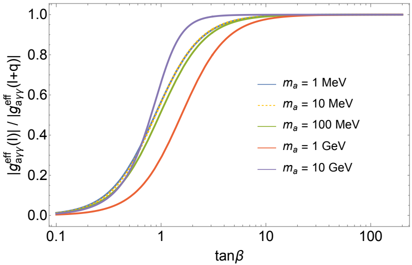
DFSZ-like model

- $SU(2)_Y \times U(1)_Y \times U(1)_{PQ} \to U(1)_{EM}$ (2 pseudoscalar Goldstone bosons)
- The broken scale f_a is defined by

$$\frac{1}{f_a} \equiv \frac{N_g v_{\rm SM}}{\sqrt{v_1^2 v_2^2 + v_{\rm SM}^2 v_s^2}}$$

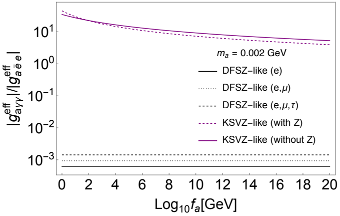
$$\begin{pmatrix} G_z \\ a \\ A_0 \end{pmatrix} = \begin{pmatrix} \frac{v_1}{v_{\rm SM}} & \frac{v_2}{v_{\rm SM}} & 0 \\ \frac{v_1 v_2^2}{v_{\rm SM}\sqrt{v_1^2 v_2^2 + v_{\rm SM}^2 v_s^2}} & -\frac{v_1^2 v_2}{v_{\rm SM}\sqrt{v_1^2 v_2^2 + v_{\rm SM}^2 v_s^2}} & -\frac{v_s v_{\rm SM}^2}{v_{\rm SM}\sqrt{v_1^2 v_2^2 + v_{\rm SM}^2 v_s^2}} \\ \frac{v_2 v_s}{\sqrt{v_1^2 v_2^2 + v_{\rm SM}^2 v_s^2}} & -\frac{v_1 v_s}{\sqrt{v_1^2 v_2^2 + v_{\rm SM}^2 v_s^2}} & \frac{v_1 v_2}{\sqrt{v_1^2 v_2^2 + v_{\rm SM}^2 v_s^2}} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_s \end{pmatrix}$$

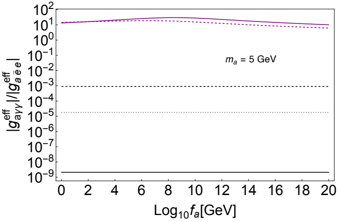
DFSZ-like model



- Large $\tan\beta > 10$, coupling between ALP and fermions/leptons are of similar magnitude.
- Neglect ALP-quark coupling for $\tan\beta > 10$.

KSVZ-like and DFSZ-like correspond with f_a (PQ scale)





- Purple for KSVZ-like model

$\rightarrow$ Solid for $F\tilde{F}$ while dashed include both $F\tilde{Z}$ and $Z\tilde{Z}$.

- Black for DFSZ-like model

IWW approximation continued

- Effective photon flux χ_γ

$$\chi_\gamma = \int_{t_{\text{min}}}^{m_a^2+m_e^2} \frac{t - t_{\text{min}}}{t} F^2(t), \ \ t_{\text{min}} = \left(\frac{m_a^2}{2E_{e(\text{in})}}\right)^2$$

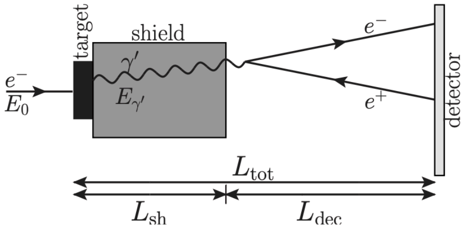
→ Elastic form factor $F(t)$

$$F(t) = \left(\frac{a^2 t}{1 + a^2 t}\right) \left(\frac{1}{1 + t/d}\right) Z$$

→ Elastic atomic form factor with $a = 111\, Z^{1/3}/m_e$

→ Elastic nuclear form factor (nuclear size) with $d = 0.164\,\text{GeV}^{-1}\, A^{-2/3}$

electron beamdump searches for ALPs (E137)



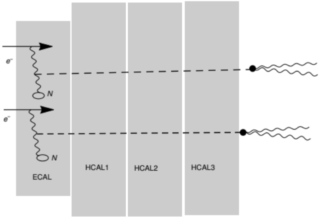
electron beamdump searches for ALPs continued

Experiment	E_e[GeV]	Target	$L_{\rm sh}$[m]	$L_{\rm dec}$[m]	Year
E137	20	Al	179	204	1988(SLAC)
NA64(Invis)	100	Pb	$\sim$ 4.35	∞	2020(CERN)
KEK	7	W	$\sim$ 0.25	1	2013(KEK linac)
E141	9.0	W	0.12	35	1987(SLAC)
E774	275	W	0.3	28	1989(Fermilab)
Orsay(Higgs)	1.6	W	1	2	1989(LAL)

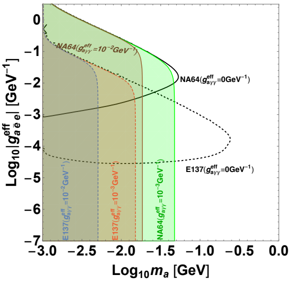
- NA64(Invis) represents the invisible signature configuration of NA64, where ALPs decay beyond all subdetectors of NA64.

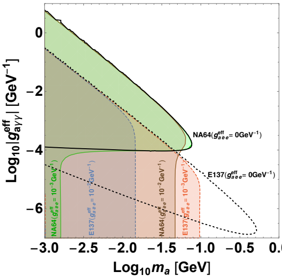
$\rightarrow$ Invisible $\rightarrow$ one ECAL and three HCAL ($L_{\rm sh}$) and $L_{\rm dec}$ as ∞

[*R.R.Dusaev et al, 2020*]



Beamdump (concurrence) scenarios $[g^{\rm eff}_{a\bar{e}e}, m_a]$, $[g^{\rm eff}_{a\gamma\gamma}, m_a]$





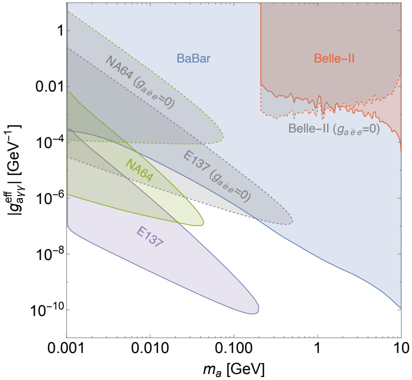
- Solid line (NA64), Dashed line (E137) are recent existing bounds (region inside excluded).
- (E137) Blue and orange with the other coupling equals to $(10^{-2}, 10^{-3}$ GeV$^{-2})$.
- (NA64) Brown and green with the other coupling equals to $(10^{-2}, 10^{-3}$ GeV$^{-2})$.

$\rightarrow$ $\Gamma_a \uparrow$ ($\tau_a \downarrow$) affects $L_a \downarrow$ and $N_a \downarrow$ in the large m_a region.

$\rightarrow$ Presence of two couplings affect $N_a \uparrow$ in the lower coupling region (hard to distinguish electron or photon in the final state).

$\Longrightarrow$ $g^{\rm eff}_{a\gamma\gamma}$ has a large impact (hard for $g^{\rm eff}_{a\bar{e}e}$ to yield the enhancement).

Modified existing limits for DFSZ/KSVZ-like model



- Downward shifts of beamdump limits due to $\sigma \uparrow$ and $\tau_a \downarrow$.
- The impact of $g^{\mathrm{eff}}_{a\gamma\gamma}$ greater than $g^{\mathrm{eff}}_{a\bar{e}e}$ in KSVZ-like model, the modification is negligible.