The Realistic Scattering of Puffy Dark Matter

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- **Realistic Scattering of Puffy dark matter**

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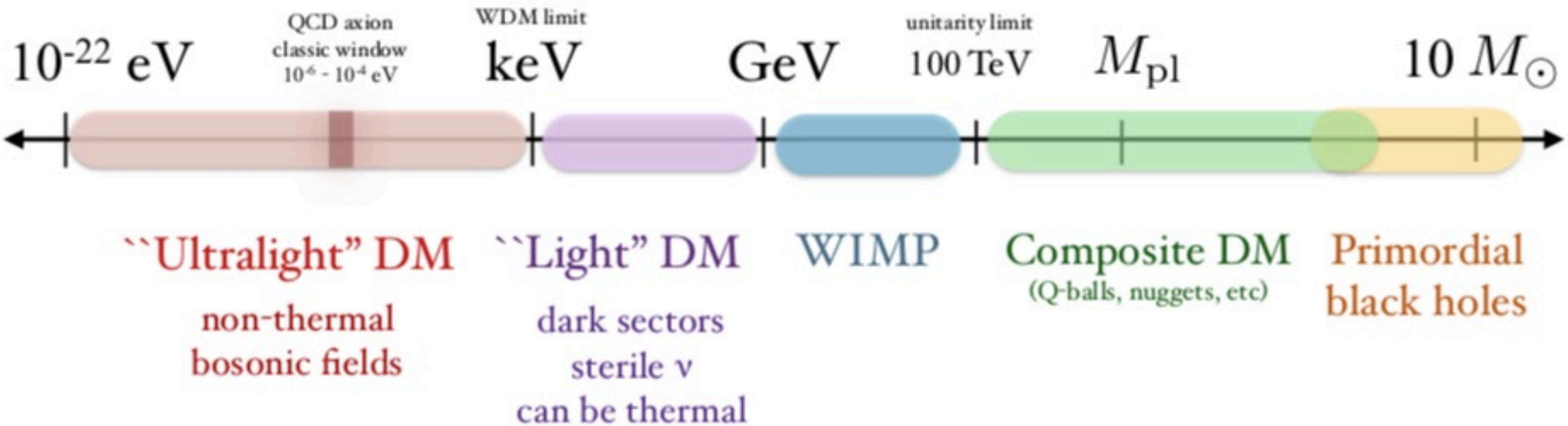
Based on
WYW, Wu-Long Xu, Bin Zhu, Phys. Rev. D 105 (2022) 7, 075013,
WYW, Wu-Long Xu, Jin Min Yang, Bin Zhu, JHEP06（2023）103

Mass scale of dark matter

(not to scale)



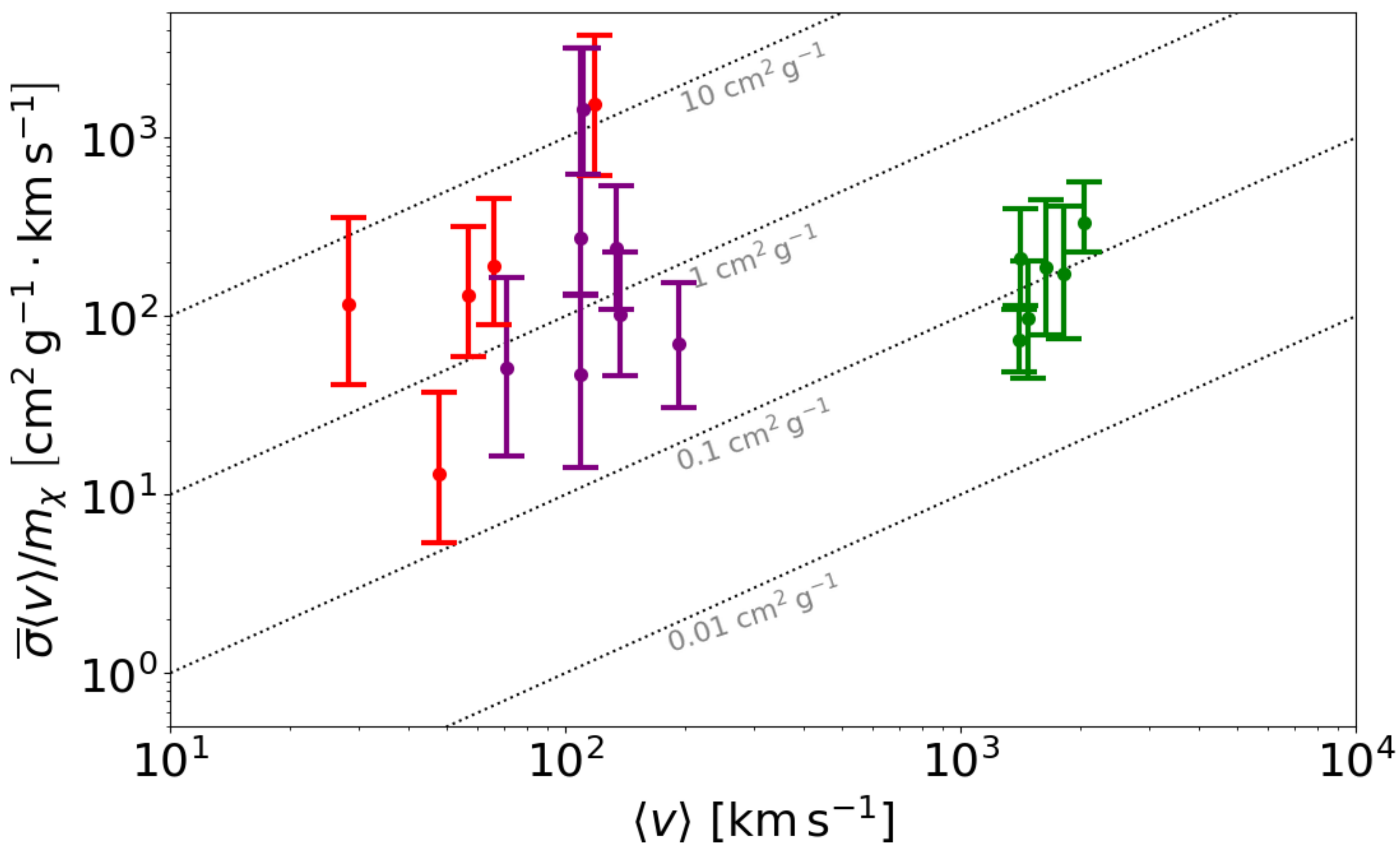
From Tongyan Lin, *PoS* 333 (2019) 009

Problems of CDM in small cosmological structure

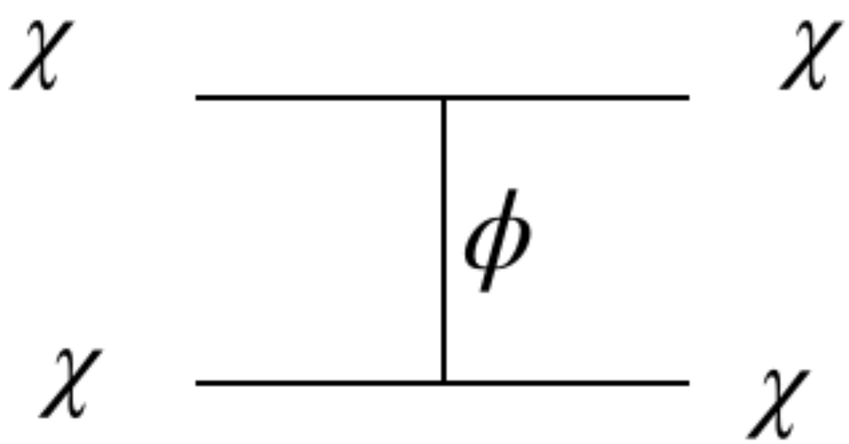
Though succeed in the large structure in cosmology, the CDM model still confronts with small structure problems which means that it gives inconsistent predictions in the small cosmological scales such as clusters, galaxies and dwarf galaxies. The problems are so-called

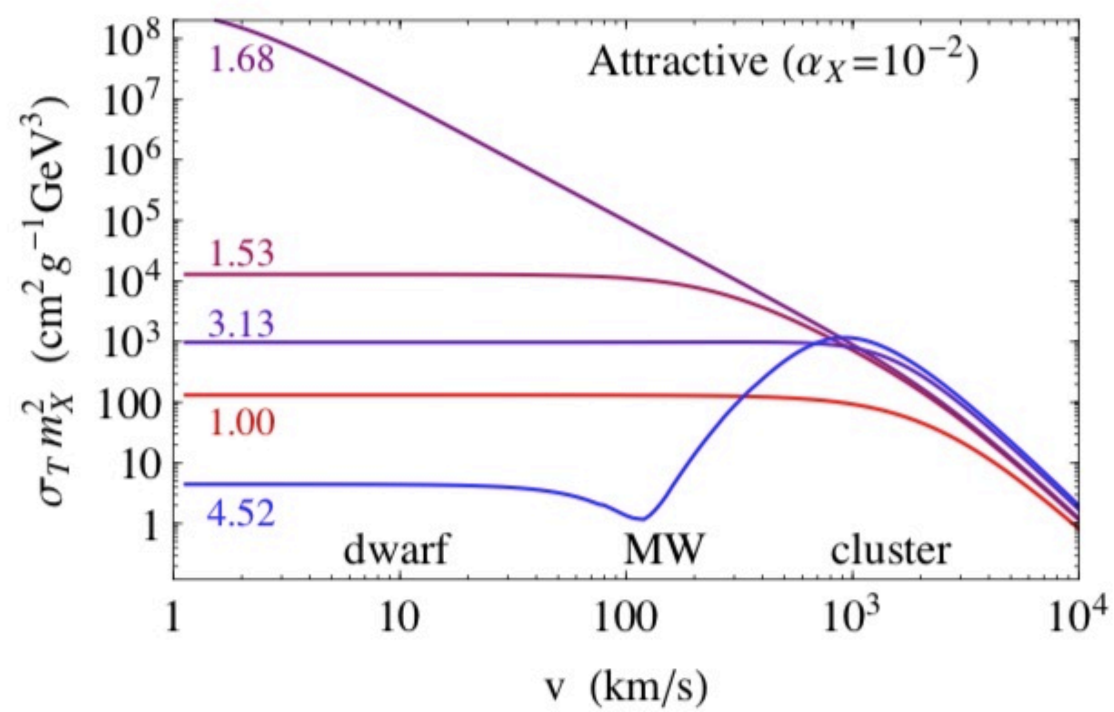
1. **Cusp vs. Core**
2. **Too big to fail**
3. **Missing satellite**
4. **Diversity problem**

Solutions: Self-interaction DM



Self-interaction DM with light mediator





$$V(r) = \pm \frac{\alpha_\chi e^{-m_\phi r}}{r}$$

long-range force

$$\sigma_T = \int d\Omega (1 - \cos\theta) \frac{d\sigma}{d\Omega}$$

Partial wave approach

$$\langle \sigma_T v \rangle = \int f(v) \sigma_T v dv \qquad f(v) = \frac{32 v^2 e^{-4v^2/\pi \langle v \rangle^2}}{\pi^2 \langle v \rangle^3}$$

Self-interacting DM candidates

- **Elastic SIDM model**
  - **Light mediator (partial wave analysis)**
  - **Composite DM**
    - **Atom DM**
    - **meson, nucleus, Bound state etc.**
  - **Resonant SIDM**
    - **Point particle DM**
    - **Dark QCD DM**
- **Inelastic SIDM model**
  - **Two mass splitting dark states**
  - **Excited state**

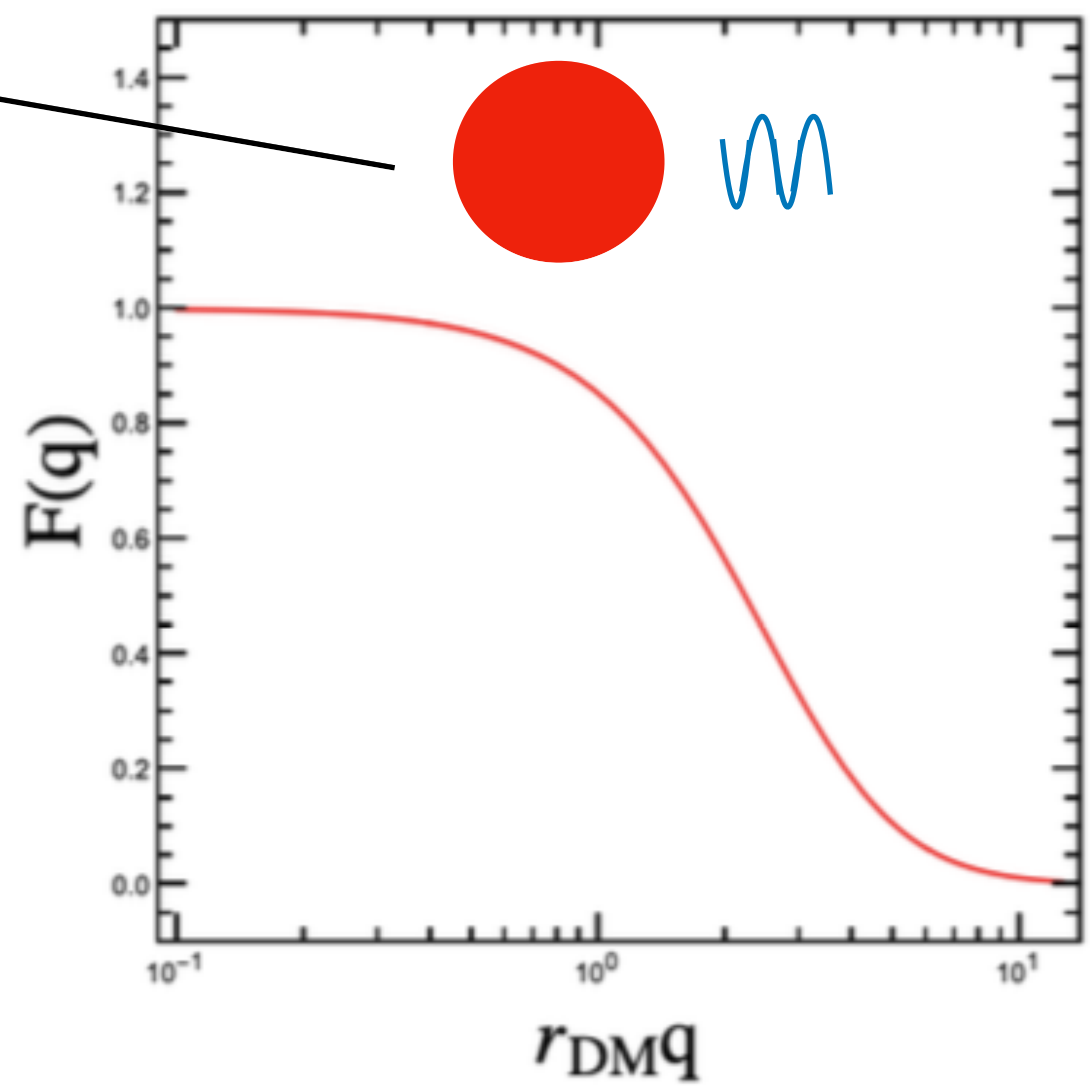
Puffy DM

If dark matter has a finite size that is larger than its Compton wavelength, the corresponding self-interaction cross section decreases with the velocity. The radius effect of DM from the form factor is another source of the velocity of dependence for cross section.

Dipole

$$F(\mathbf{q}) = \int \mathbf{dr} e^{\mathbf{iqr}} \rho(\mathbf{r})$$

$$F(q) = \frac{1}{(1 + 1/24 r_{DM}^2 q^2)^2}$$



- [2] X. Chu, C. Garcia-Cely and H. Murayama, Phys. Rev. Lett. 124, no. 4, 041101 (2020)

In case of Born approximation

$$\mathbf{H}_{\text{int}} = \int \mathbf{d}\overrightarrow{\mathbf{x}}\,\overrightarrow{\mathbf{y}}\,\rho_{\mathbf{1}}(\overrightarrow{\mathbf{x}})\frac{\alpha\mathbf{e}^{-|\overrightarrow{\mathbf{x}}-\overrightarrow{\mathbf{y}}|/\lambda}}{|\overrightarrow{\mathbf{x}}-\overrightarrow{\mathbf{y}}|}\rho_{\mathbf{2}}(\overrightarrow{\mathbf{y}})$$

$$= \int \frac{d\overrightarrow{\mathbf{q}}}{(2\pi)^3}F_1(\overrightarrow{\mathbf{q}})\frac{4\pi\alpha}{\overrightarrow{\mathbf{q}}^2+\lambda^{-2}}F_2\overrightarrow{\mathbf{q}}$$

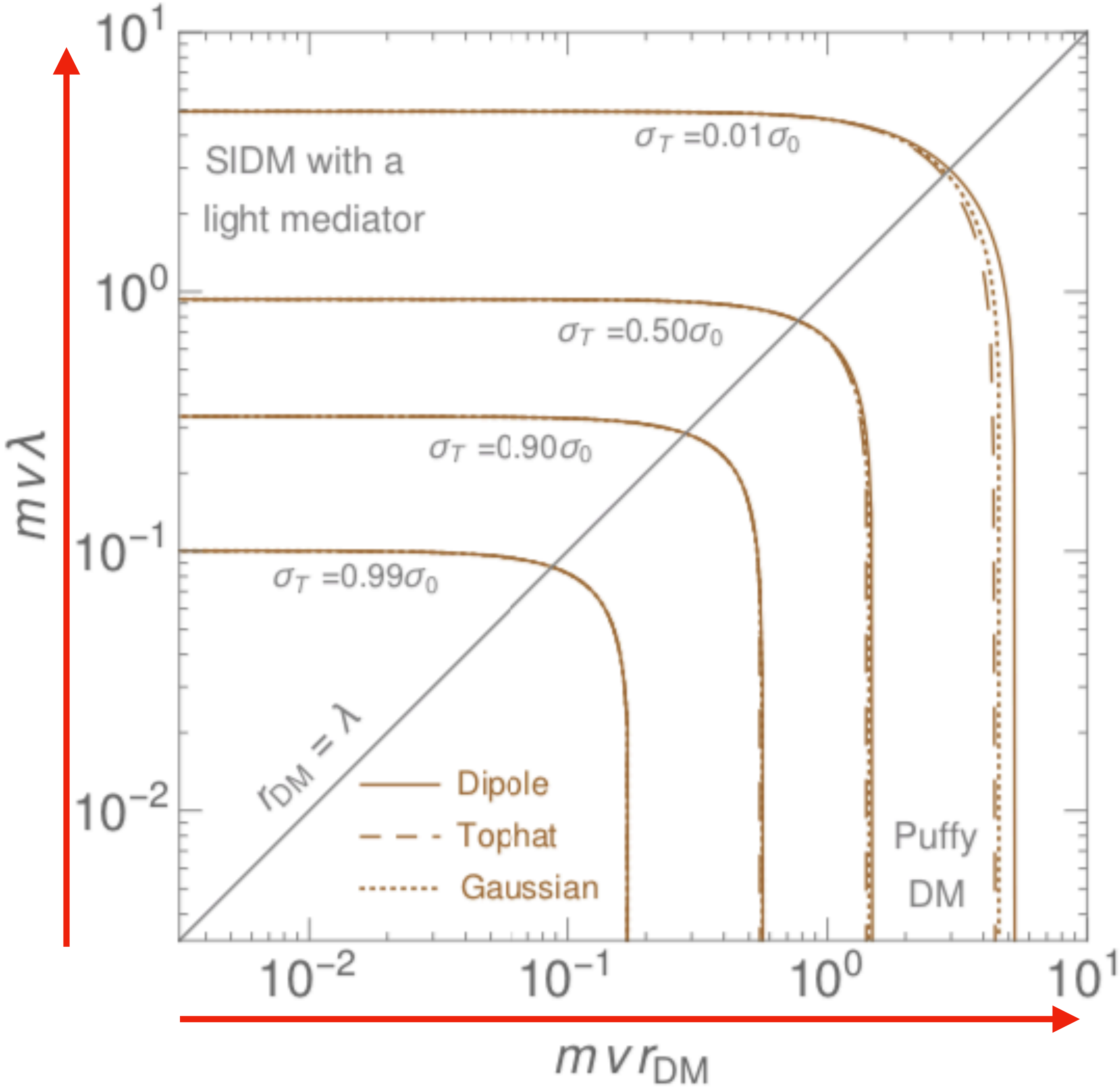
$$F(\overrightarrow{\mathbf{q}}) = \int d\overrightarrow{\mathbf{r}}\,e^{i\overrightarrow{\mathbf{q}}\cdot\overrightarrow{\mathbf{r}}}\rho(\overrightarrow{\mathbf{r}})$$

$$\frac{d\sigma_y}{d\Omega} = \sigma_0\frac{F^4(q)}{((mv)^2\lambda^2\frac{1-\cos\theta}{2}+1)^2}$$

$$\sigma_0 = 4\pi(m_{\text{DM}}\alpha\lambda^2)^2$$

Puffy DM

Even in the presence of a light particle mediating self-interactions, the finite-size effect may dominate the velocity dependence.



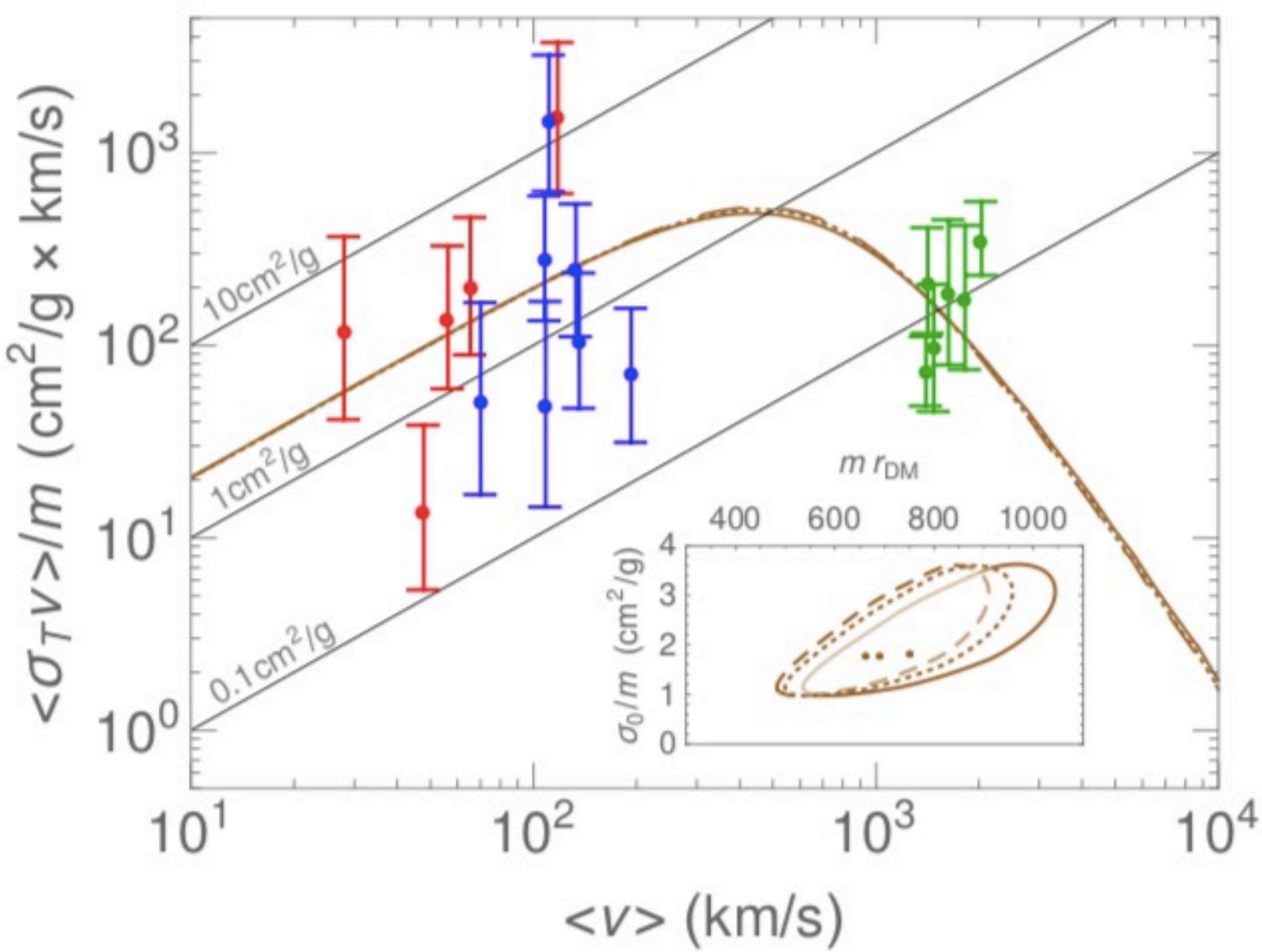
Transfer cross section as a function of the force range and Radius

$\sigma_0 = 4\pi(m_{\rm DM}\alpha\lambda^2)^2$ The reference cross section

- [2] X. Chu, C. Garcia-Cely and H. Murayama, Phys. Rev. Lett. 124, no. 4, 041101 (2020)

Self-interacting puffy DM

This puffy DM model can solve the small scale problems.

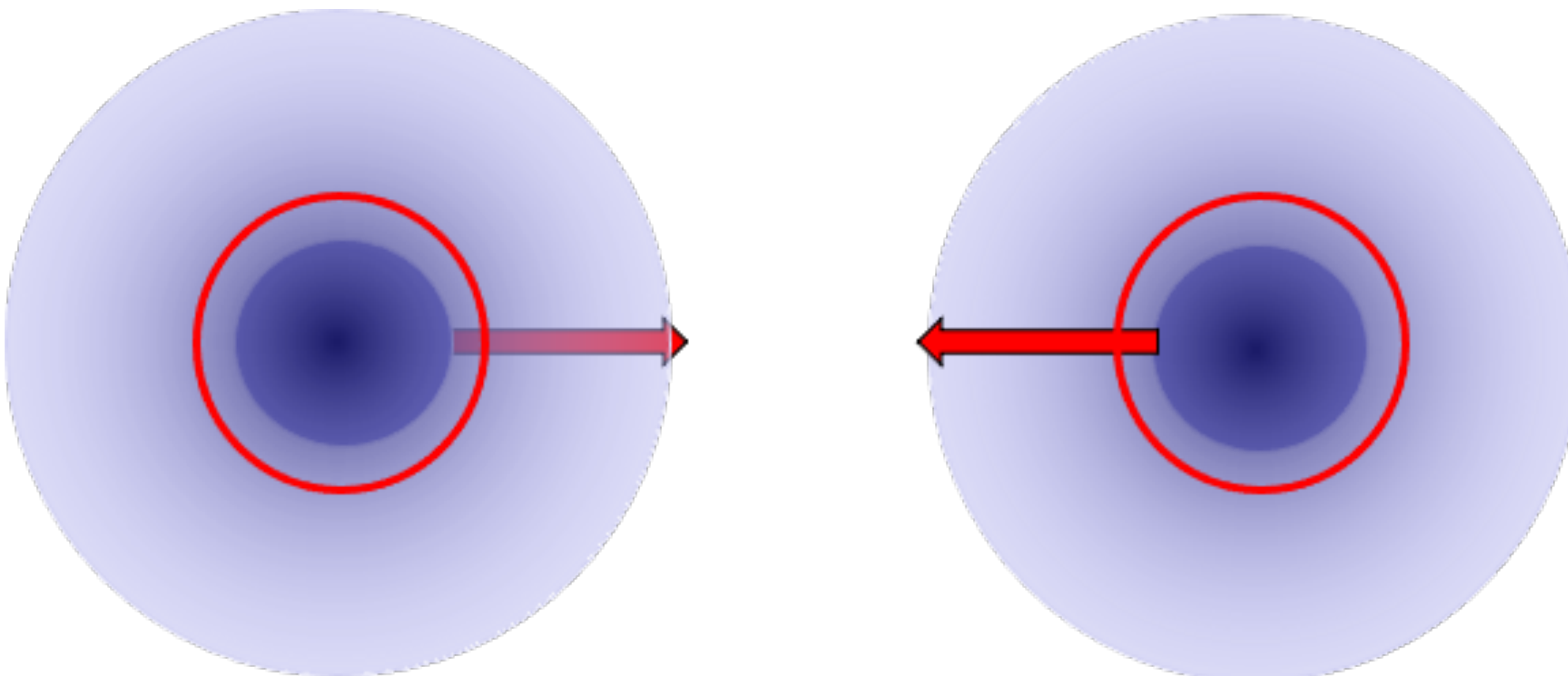


Two points need to be speculated on

- **The interaction to confine the matter to form the puffy dark matter**

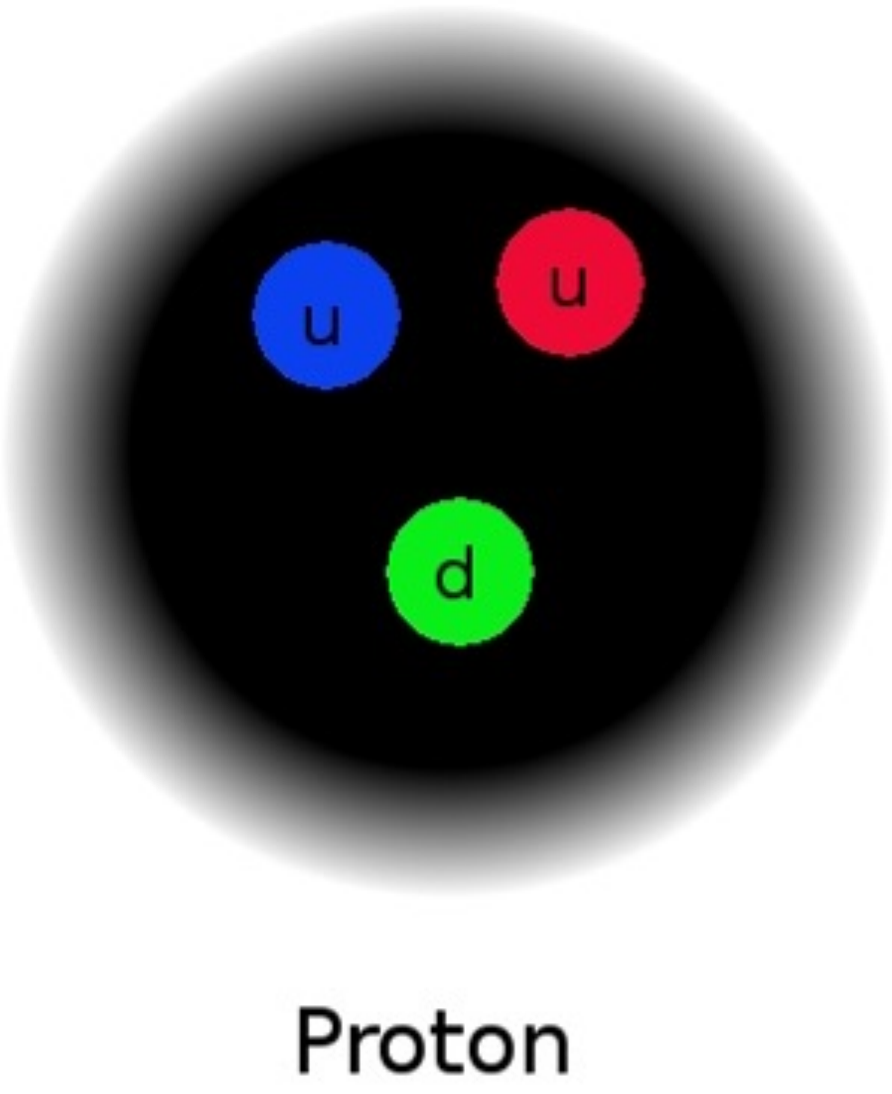
- **The validity of Born approximation**

The interaction to confine the matter to form the puffy dark matter

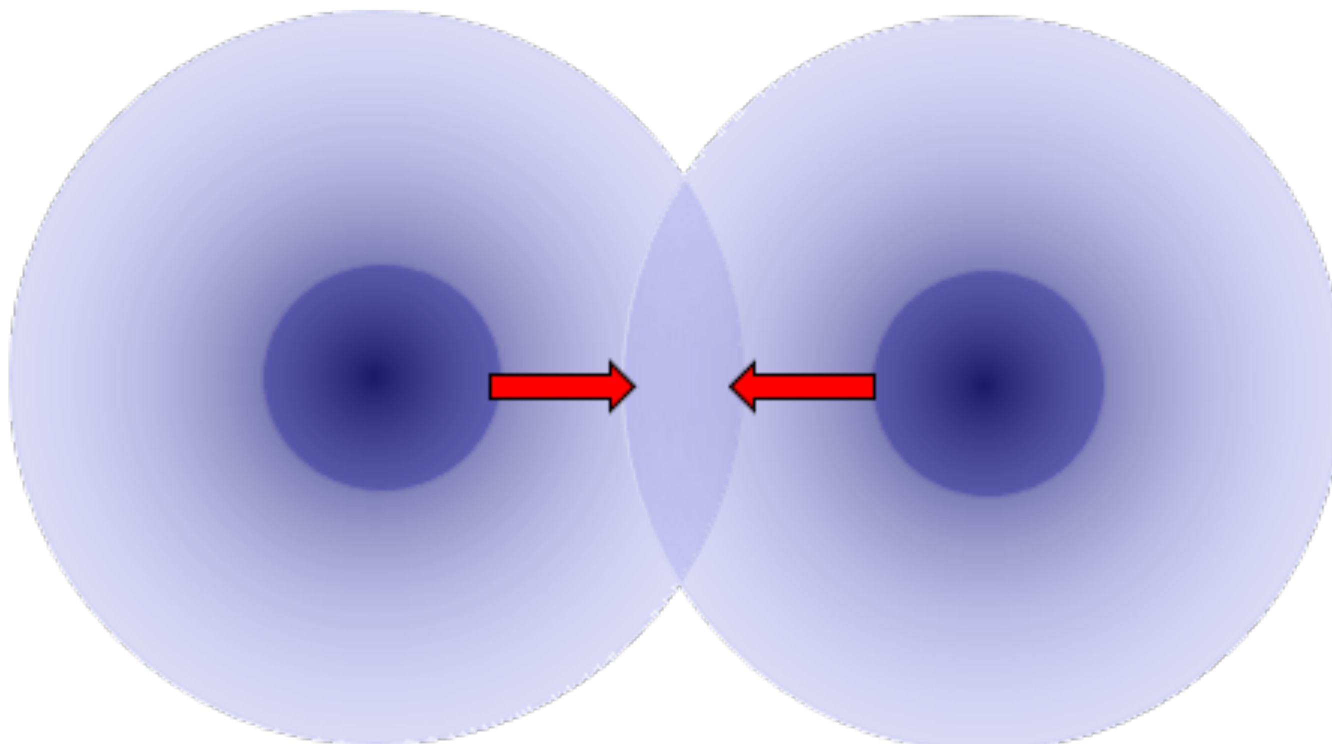


The interaction to confine the matter to form the puffy dark matter must exists!

Just like proton and proton collision!



High energy pp elastic scattering



$$\frac{d\sigma}{d\Omega} = |f_c(q)e^{i\alpha\phi(q)} + f_n(q)|^2 = \frac{d\sigma_c}{d\Omega} + \frac{d\sigma_{\text{int}}}{d\Omega} + \frac{d\sigma_n}{d\Omega},$$

Proton-Proton elastic scattering

$$\frac{d\sigma}{dt} = \left| f_c(t)e^{i\alpha\phi(t)} + f_n(t) \right|^2 = \frac{d\sigma_c}{dt} + \frac{d\sigma_{int}}{dt} + \frac{d\sigma_n}{dt}, \quad (1)$$

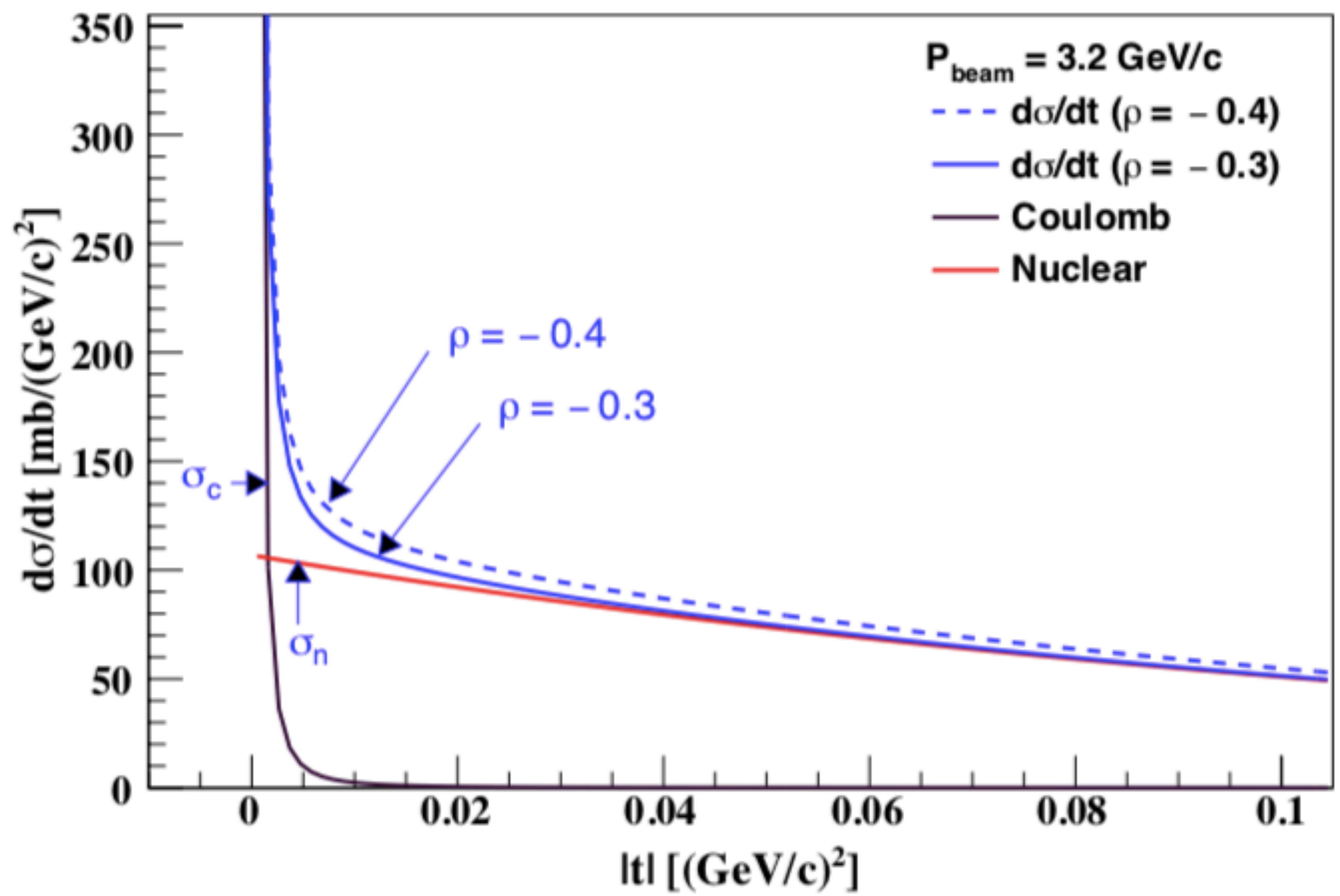
where

$$\frac{d\sigma_c}{dt} = \frac{4\pi\alpha^2 G^4(t)(\hbar c)^2}{\beta^2 t^2}, \quad (2)$$

$$\frac{d\sigma_{int}}{dt} = -\frac{\alpha\sigma_{tot}}{\beta|t|} G^2(t) e^{\frac{-B|t|}{2}} (\rho\cos(\alpha\phi(t)) + \sin(\alpha\phi(t))), \quad (3)$$

and

$$\frac{d\sigma_n}{dt} = \frac{\sigma_{tot}^2(1+\rho^2)e^{-B|t|}}{16\pi(\hbar c)^2}. \quad (4)$$



The total cross section σ_{tot}

The relative real amplitude ratio $\rho = Re f_n(0)/Im f_n(0)$

The slope parameter B

- [4] H. Xu, Y. Zhou, U. Bechstedt, J. Böker, A. Gillitzer, F. Goldenbaum, D. Grzonka, Q. Hu, A. Khoukaz and F. Klehr, et al. Phys. Lett. B 812 (2021), 136022

dark matter-dark matter elastic scattering

$$\frac{d\sigma}{dt} = |f_c(t)e^{i\alpha\phi(t)} + f_n(t)|^2 = \frac{d\sigma_c}{dt} + \frac{d\sigma_{int}}{dt} + \frac{d\sigma_n}{dt},$$

where

$$\frac{d\sigma_c}{dt} = \frac{\pi}{v^2}\frac{\alpha^2 F^4(t)}{(t+m_\phi^2)^2} \tag{3}$$

Mass of the mediator

$$\frac{d\sigma_{int}}{dt} = -\frac{\alpha\sigma_{tot}}{2v(t+m_\phi^2)}F^2(t)e^{-\frac{B\times|t|}{2}}(\rho\cos(\alpha\phi(t)) + \sin(\alpha\phi(t))) \tag{4}$$

$$\frac{d\sigma_n}{dt} = \frac{\sigma_{tot}^2(1+\rho^2)e^{-B\times|t|}}{16\pi}, \tag{5}$$

where $\phi(t) = -[\gamma + \ln(\frac{B\times|t|}{2}) + \ln(1+\frac{8\times r0^2}{B}) + \ln(4\times|t|\times r0^2)\cdot(4\times|t|\times r0^2) + 2|t|\times r0^2]$

Self-interaction puffy DM

The standard partial-wave expansion of the scattering amplitude is

$$f_{c.m.}(s,t) = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)a_l(k) \qquad a_l(k) = \frac{e^{2i\delta_i}-1}{2i}$$

$l \to \infty$ Polynomials $p_l(\cos\theta) \to J_0[(2l+1)\sin(\theta/2)]$

eikonal approximation condition: de Broglie wavelength of the heavy dark matter are much smaller than the size of the dark matter $mvr_{DM} \gg 1$

$$f_n(s,t) = 2k\int bdbJ_0(qb)a(b,s) \qquad bk = l + 1/2 \qquad J_0(t) = \frac{1}{2\pi}\int d\phi exp(-it\cos\phi)$$

$$a_l \to a(b,s) \qquad a(b,s) = \frac{1}{4\pi k}\int d^2bexp(iq\cdot b)f_{c.m.}(s,t)$$

Traditionally, σ_{tot} and the slope parameter B are given as, using the approximation of the impact- parameter representation, pure imaginary $a(b,s)$ are always adopted for the forward high-energy scattering, there are several profiles such as disk, parabolic form, Gaussian shape and Chou-Yang model etc

$$\sigma_{tot} = \frac{4\pi}{k}Imf_{cm}(s,0) = 4\int d^2bIma(b,s) \qquad B = \frac{\int d^2bb^2a(b,s)}{2\int d^2ba(b,s)}$$

Self-interaction puffy DM

Chou-Yang model

Chou-Yang model consider that the attenuation of two hadron going through each other is denoted by the evaluating the opaqueness at the impact parameter b. The density of opaqueness can be seen as the charge distribution inside the hadron.

$$a(b,s) = \frac{1}{2i}(e^{2i\delta)-1} = \frac{i}{2}(1 - e^{-\Omega(b)})$$

$$\Omega(b) = A\frac{1}{8}x^3K_3(x) \qquad x = b/r_0$$

From Rev. Mod. Phys. 57. 563

$\sigma_A(A, r_0)$ are rewritten as

$$\boxed{\sigma_A = \sigma_0 + \sigma_a}$$

$$\sigma_a = r_0^2\sigma_{tot}(\frac{1}{r_0})^2 = 4 \times 2\pi \times r_0^2 \int dx(xa)$$

$\sigma_0 = 4\pi(m_{\rm DM}\alpha\lambda^2)^2$ The reference cross section

SIDM in Astrophysical halos

When the DM-DM scatting is proceeded in the Astrophysical halos, the transfer cross section

$$\sigma_T = \int d\Omega(1 - cos\theta)\frac{d\sigma}{d\Omega}$$

The total transfer cross section is

$$\begin{aligned}\sigma_T &= \int d\Omega(1-|\cos\theta|)\frac{d\sigma}{d\Omega} \\ &= \int_{-1}^{1} -2\pi d\cos\theta(1-|\cos\theta|)(\frac{d\sigma_c}{d\Omega}+\frac{d\sigma_{int}}{d\Omega}+\frac{d\sigma_n}{d\Omega}),\end{aligned} \tag{7}$$

where

$$\frac{d\sigma_y}{d\Omega} = \frac{m^2}{4}\frac{\alpha^2 G^4(t)}{((mv)^2\frac{1-\cos\theta}{2}+m_\phi^2)^2} \tag{8}$$

$$\frac{d\sigma_{int}}{d\Omega} = -\frac{m^2 v}{4\pi}\frac{\alpha\sigma_{tot}}{2((mv)^2\frac{1-\cos\theta}{2}+m_\phi^2)}G^2(t)e^{-\frac{B\times|t|}{2}}(\rho\cos(\alpha\phi(t))+\sin(\alpha\phi(t))) \tag{9}$$

$$\frac{d\sigma_n}{d\Omega} = \frac{(mv)^2}{4\pi}\frac{\sigma_{tot}^2(1+\rho^2)e^{-B\times|t|}}{16\pi}. \tag{10}$$

SIDM in Astrophysical halos

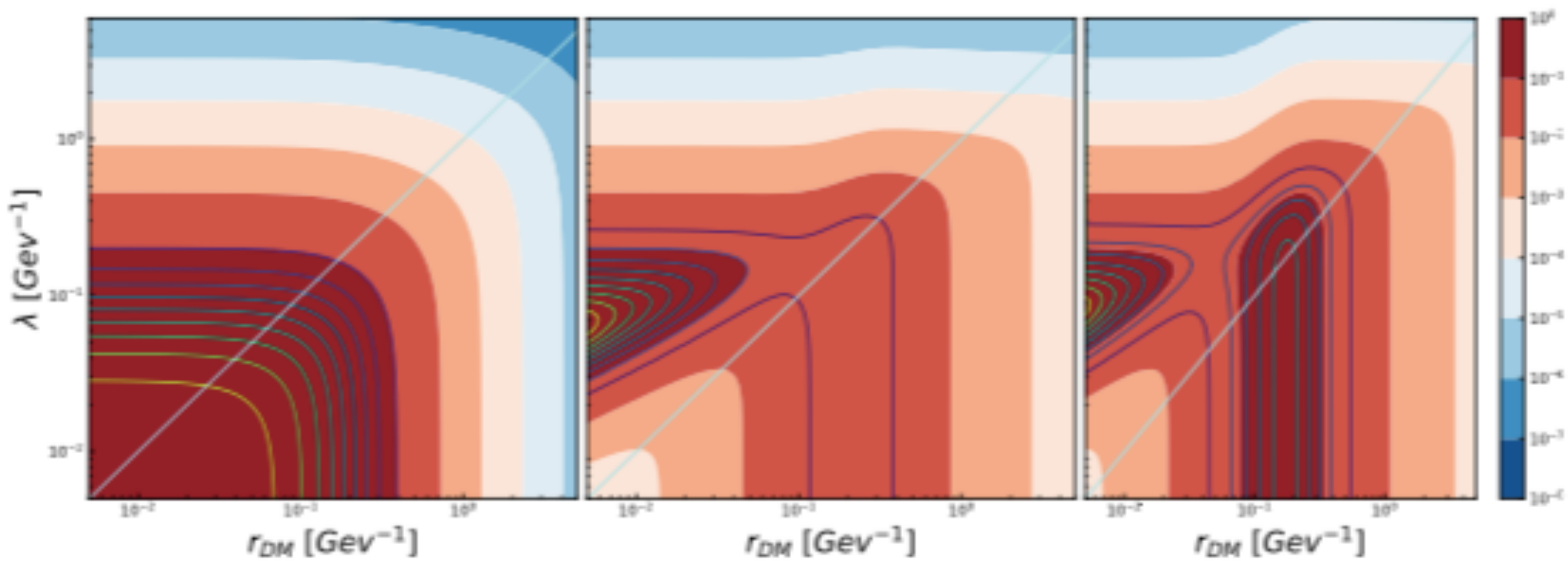


FIG. 1: Ratio σ_T/σ_A in $r_{\rm DM}$ and λ space with different absorption parameter $A = 0, 1, 10$, respectively. The other parameters are the same which is $m_{\rm DM} = 100{\rm GeV}$, $\alpha = 0.01$ and $v/c = 0.1$.

SIDM in Astrophysical halos

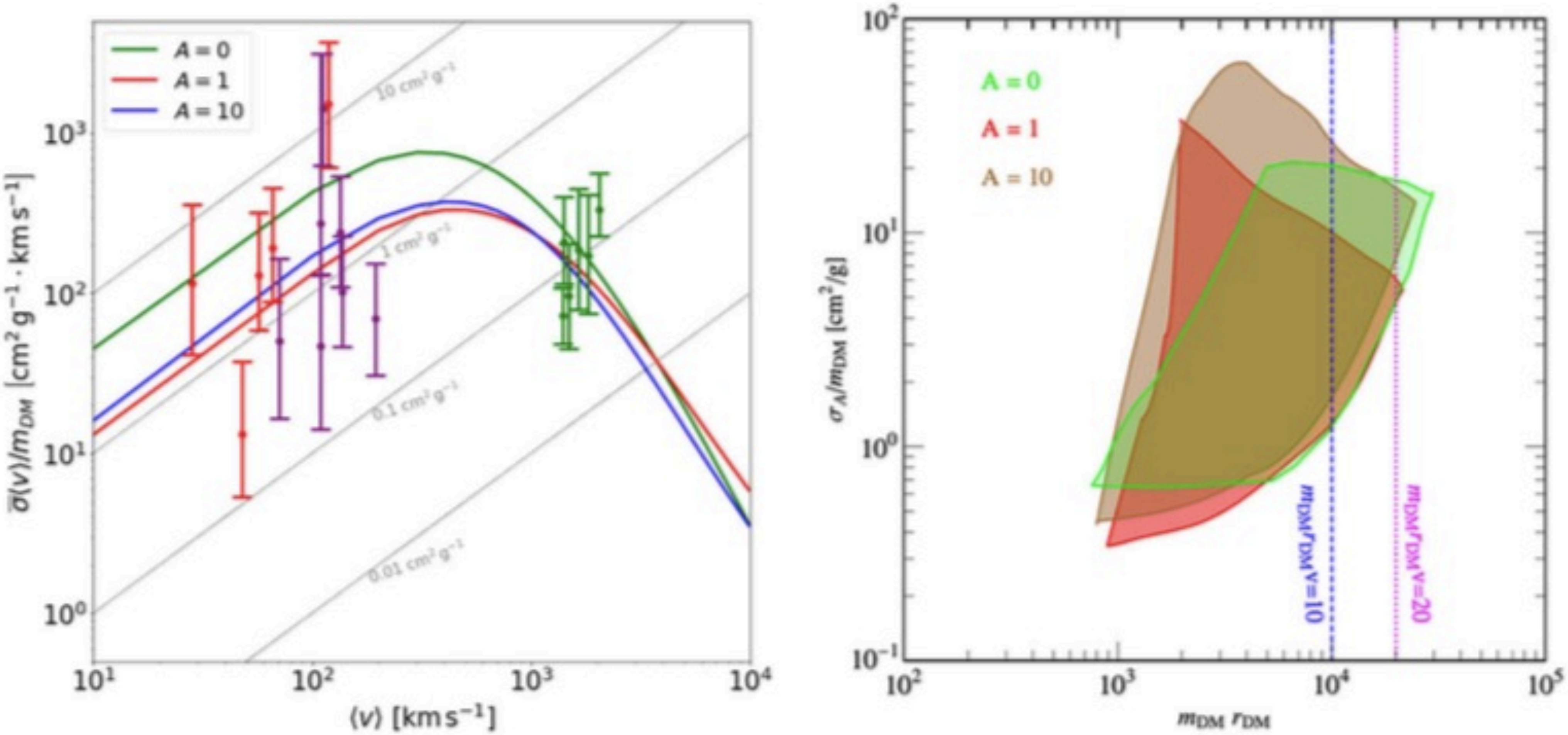


FIG. 2: Left panel: The best fit point for the velocity dependence the the puffy dark matter, Right panel: the fitted $\sigma_A/m_{\rm DM}$ versus $m_{\rm DM}r_{\rm DM}$ in the $\lambda < r_{\rm DM}$ case.

The validity of Born approximation

Case of a point dark matter

Yukawa potential $V(r) = \pm \frac{\alpha_X}{r} e^{-m_\phi r}$

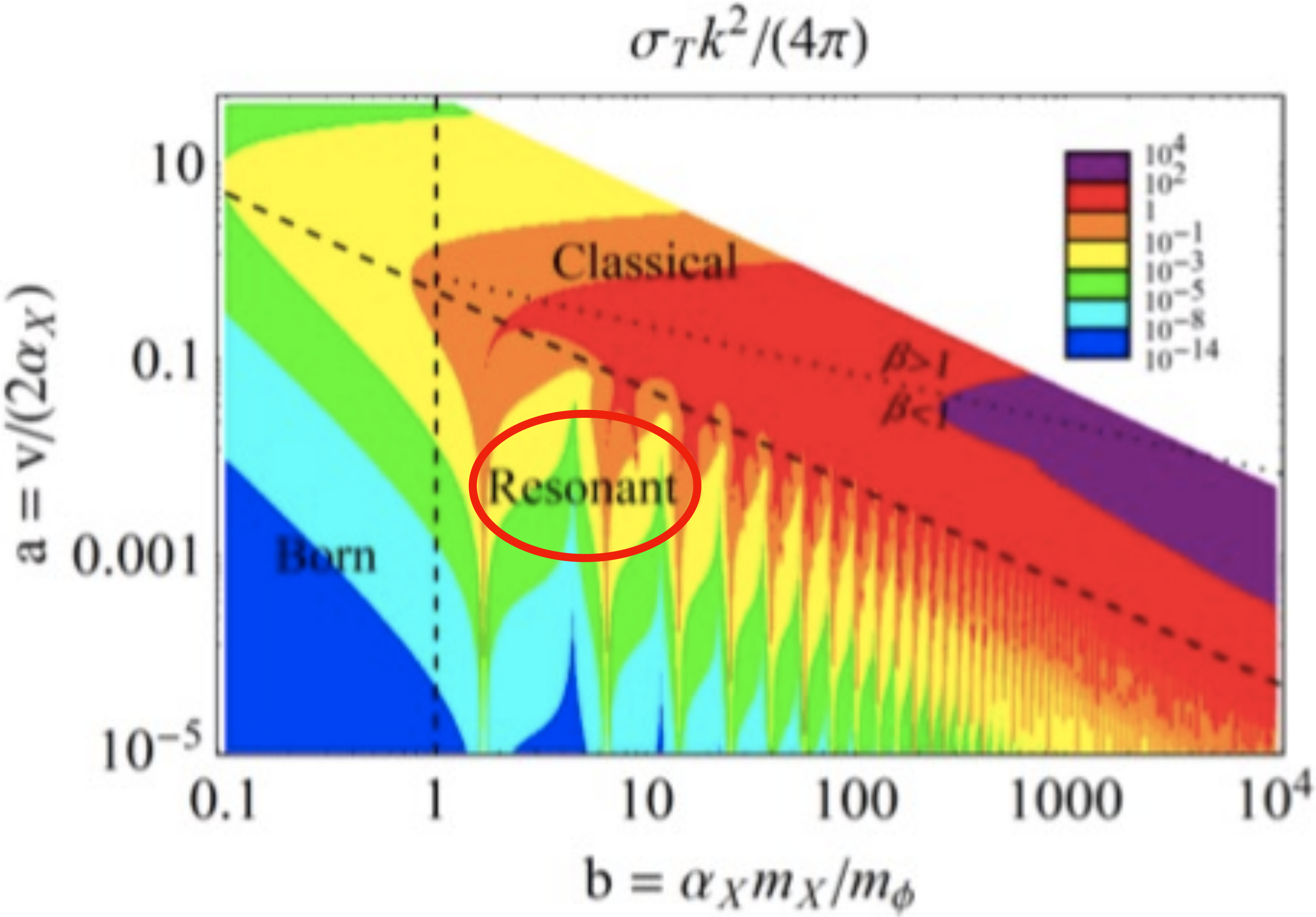
$$\frac{d\sigma}{d\Omega} = \frac{1}{k^2} \left| \sum_{\ell=0}^{\infty} (2\ell+1) e^{i\delta_\ell} P_\ell(\cos\theta) \sin\delta_\ell \right|^2$$

$$\frac{\sigma_T k^2}{4\pi} = \sum_{\ell=0}^{\infty} (\ell+1) \sin^2(\delta_{\ell+1} - \delta_\ell)$$

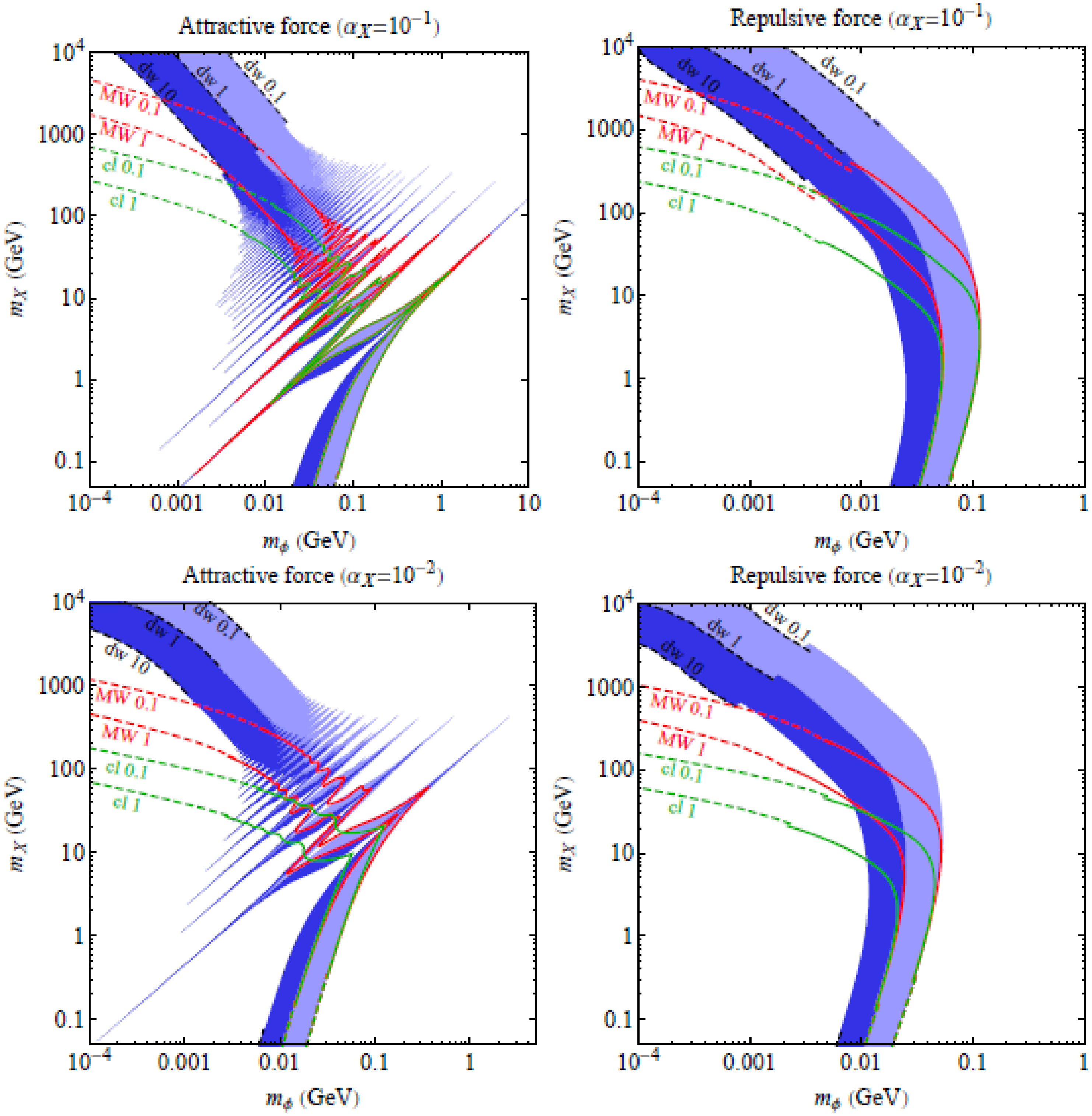
$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dR_\ell}{dr}\right) + \left(k^2 - \frac{\ell(\ell+1)}{r^2} - 2\mu V(r)\right) R_\ell = 0$$

$$\chi_\ell \equiv r R_\ell \,, \quad x \equiv \alpha_X m_X r \,, \quad a \equiv \frac{v}{2\alpha_X} \,, \quad b \equiv \frac{\alpha_X m_X}{m_\phi}$$

$$\left(\frac{d^2}{dx^2} + a^2 - \frac{\ell(\ell+1)}{x^2} \pm \frac{1}{x} e^{-x/b} \right) \chi_\ell(x) = 0$$



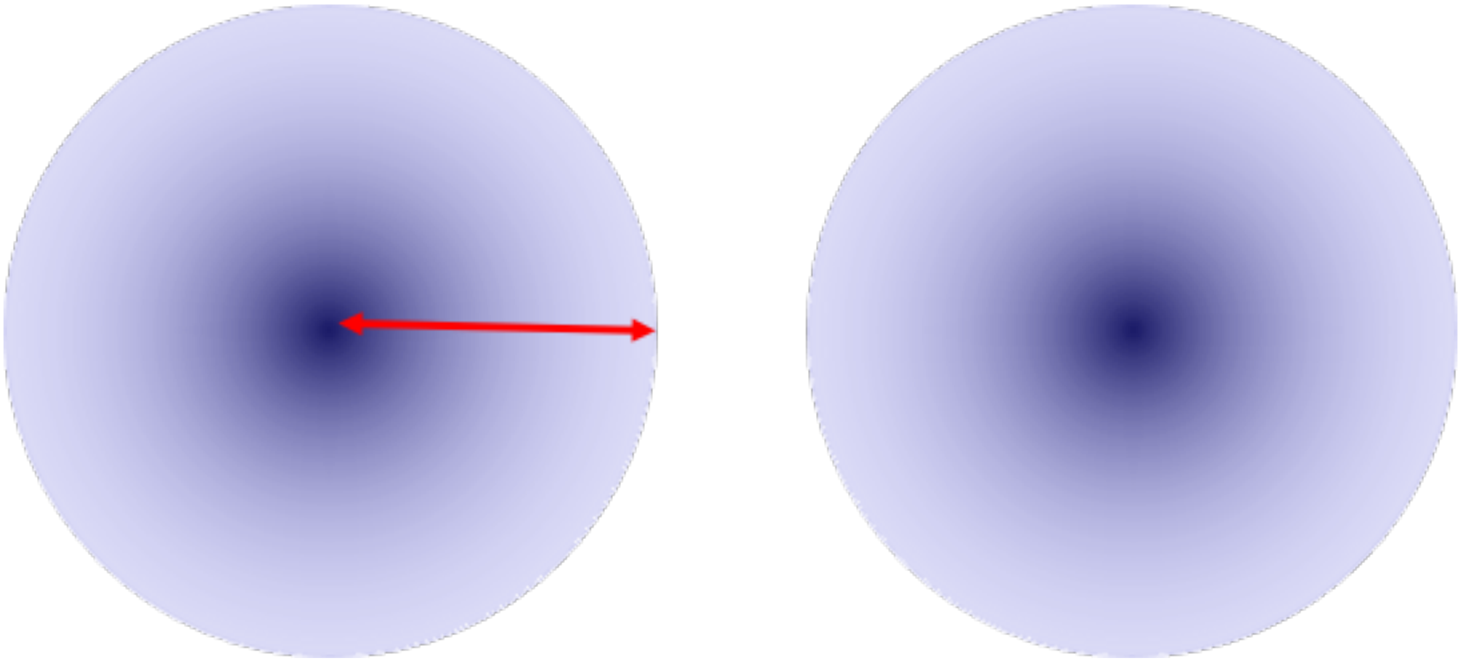
$\sigma_T k^2/(4\pi)$
Classical
Resonant
Born
$\beta>1$
$\beta<1$
$a = v/(2\alpha_X)$
$b = \alpha_X m_X/m_\phi$
10
0.1
0.001
10^{-5}
0.1
1
10
100
1000
10^4
10^4
10^2
1
10^{-1}
10^{-3}
10^{-5}
10^{-8}
10^{-14}



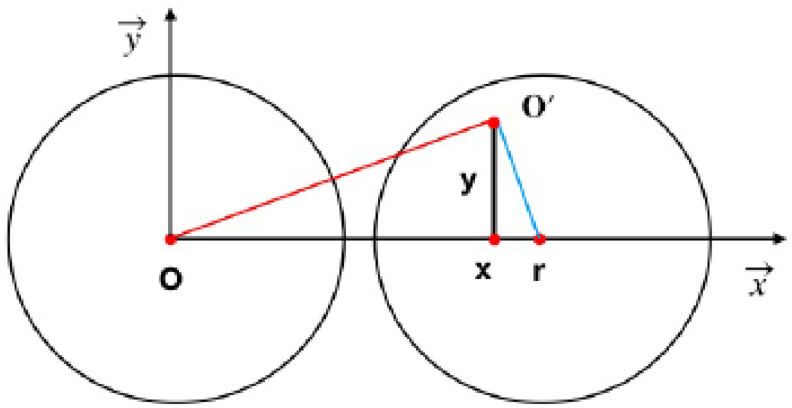
Dominant parameter space is resonant, thus the Born approximation are skeptical.

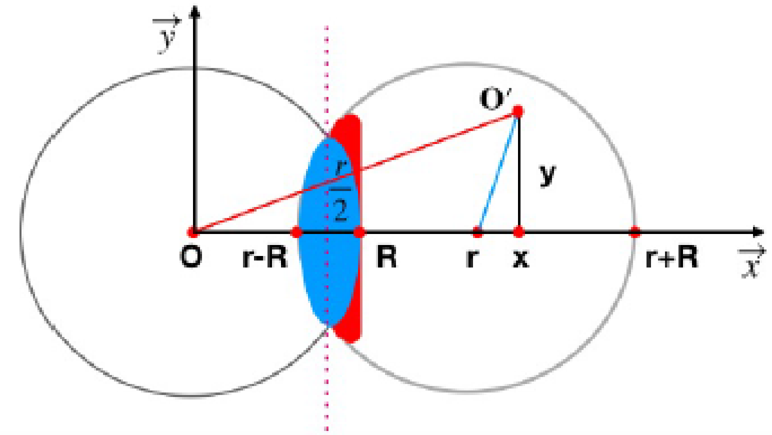
From S. Tulin et. al. Phys. Rev. D 87, 115007

Case of a puffy dark matter



$$\rho(r) = \frac{3Q}{4\pi R_\chi^3}\theta(R_\chi - r),$$



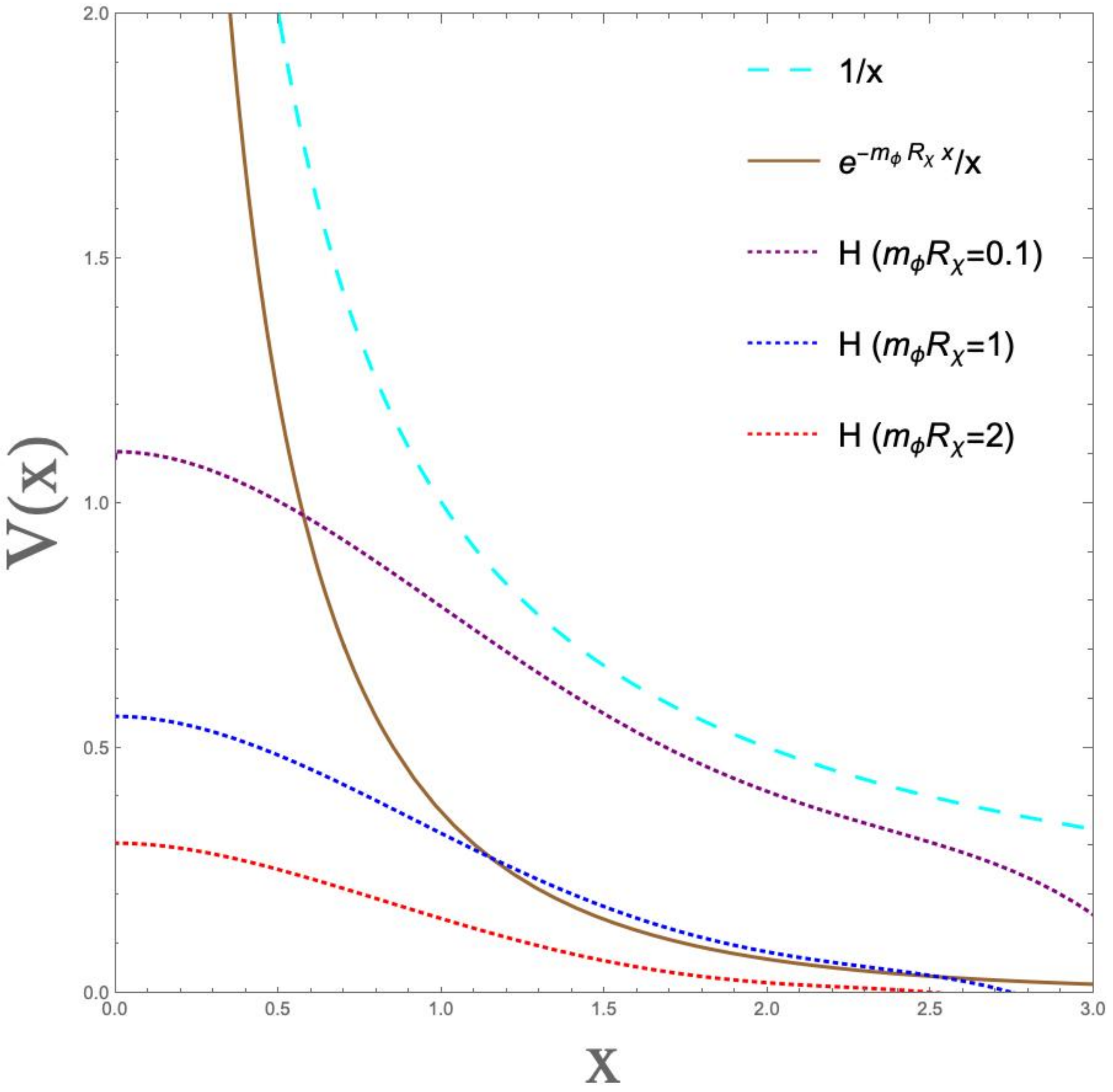


$$V(r = R_\chi x) = \begin{cases} \frac{\alpha}{R_\chi}\frac{1}{x} & \text{Coulomb potential,} \\ \frac{\alpha}{R_\chi}\frac{e^{-yx}}{x} & \text{Yukawa potential,} \\ \frac{\alpha}{R_\chi}H(x,y), & \text{Puffy potential,} \end{cases}$$

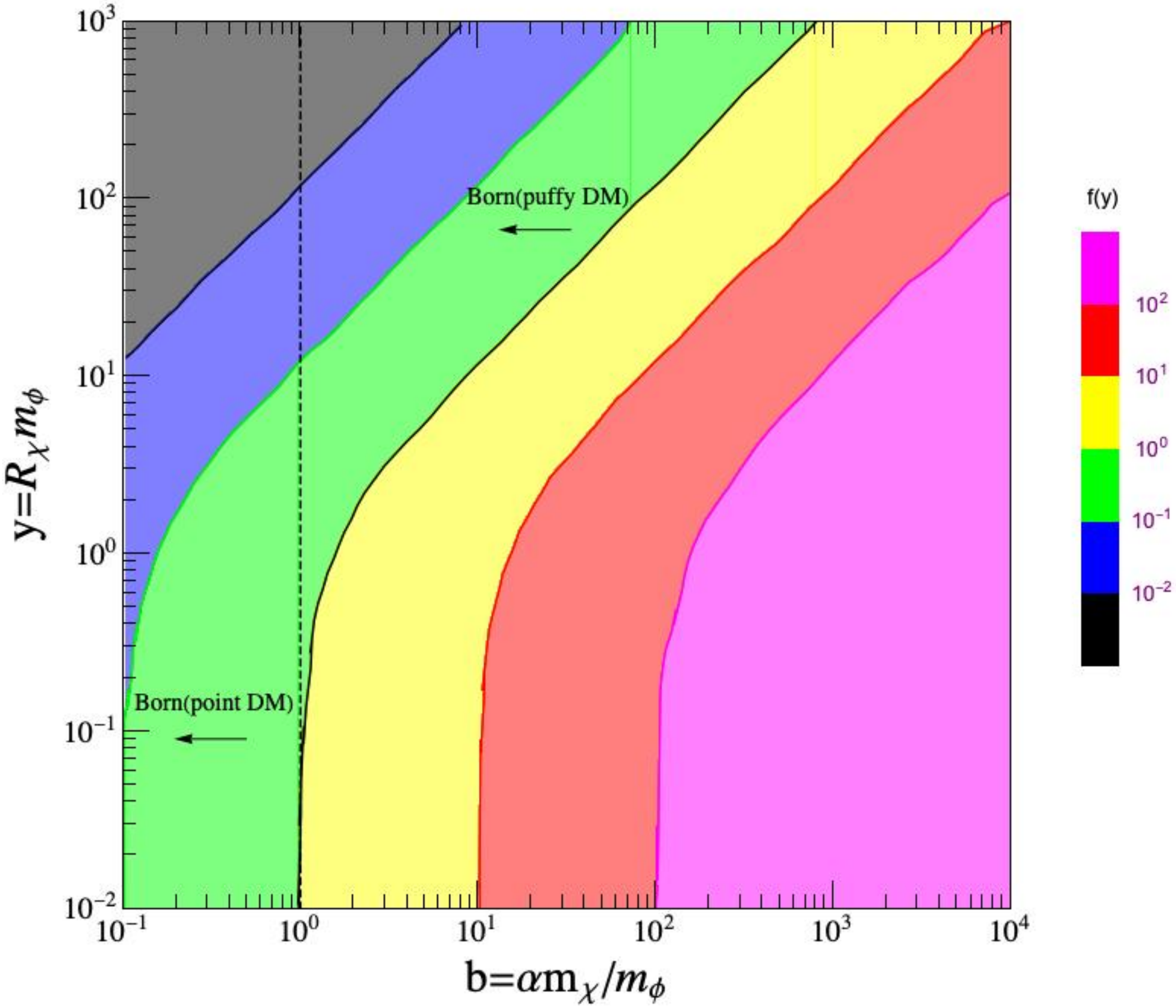
$$\begin{aligned} H(x,y) =& 3(y^4(-2+x)^3 x(4+x) - 6e^{-y(2+x)}(1+y+e^{2y}(-1+y)) \\ & \times (-2(1+y) + e^{yx}(2+y(2+(-2+y(-2+x))x))) \\ & + 6e^{-y}(1+y)(2(2+(y)^2(-2+x)x)\cosh(y) - 4\cosh(y(-1+x) \\ & + 4y(-1+x)\sinh(y) - \sinh(y(-1+x)))))/(16y^6 x). \end{aligned}$$

The validity of Born approximation

1. The Yukawa potential are approaching the Coulomb potential in force range with a pole
2. The puffy potential removes the pole and the strength decrease along with the increase of the radius.

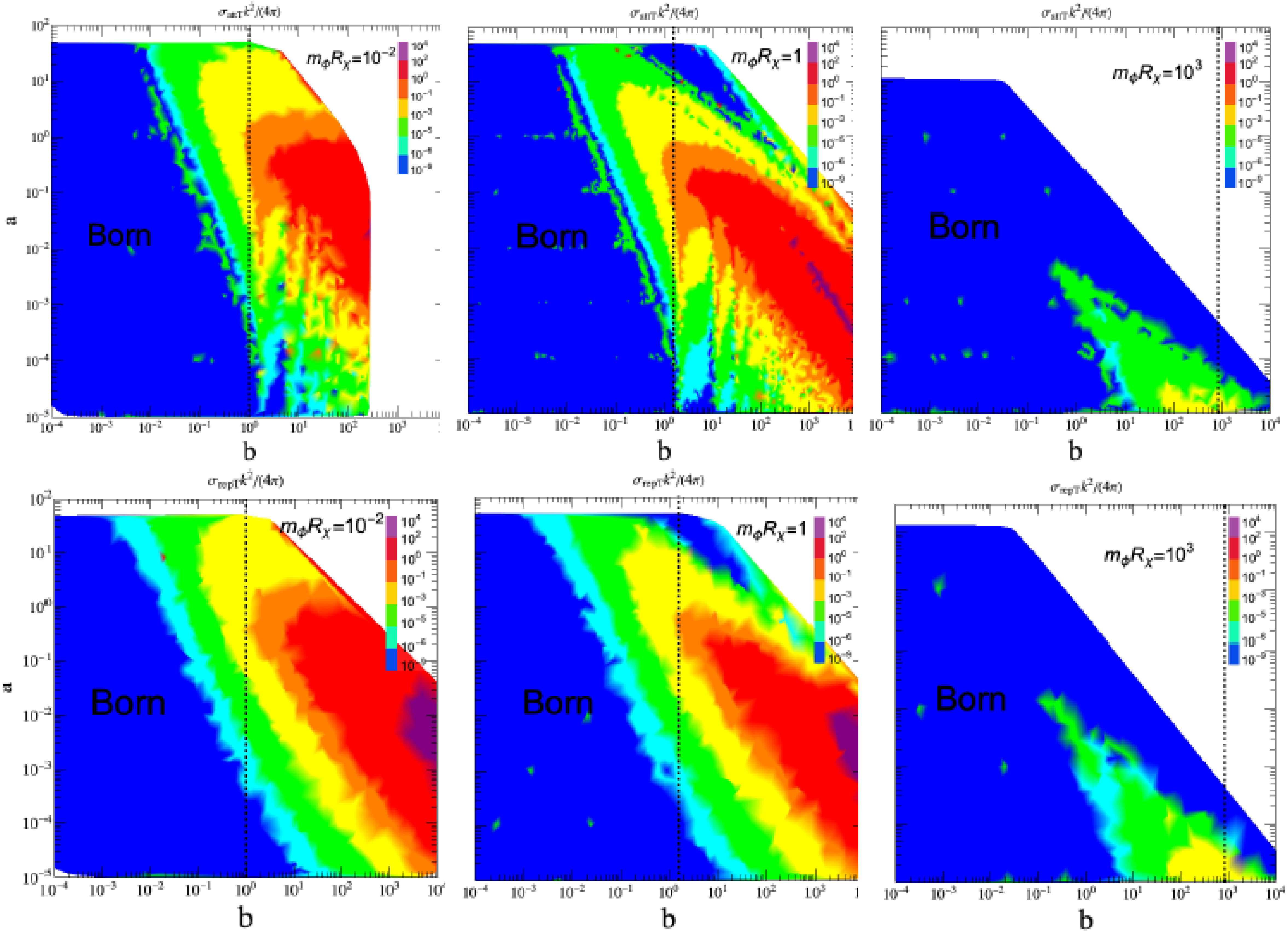


The validity of Born approximation

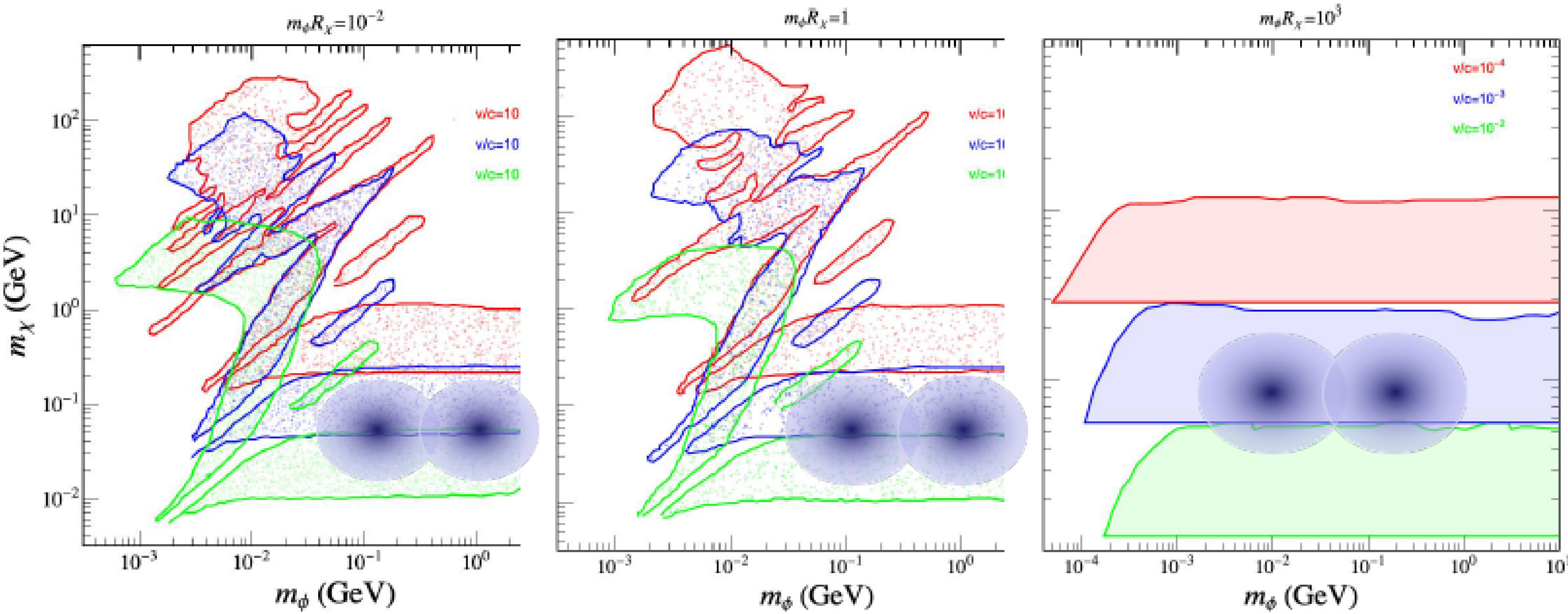


$$m_\chi \left| \int_0^\infty rV(r)dr \right| = \frac{\alpha m_\chi}{m_\phi} \frac{3\left(-15 + y^2(15 - 10y + 4y^3) + 15(1 + y^2)e^{-2y}\right)}{10y^6}$$

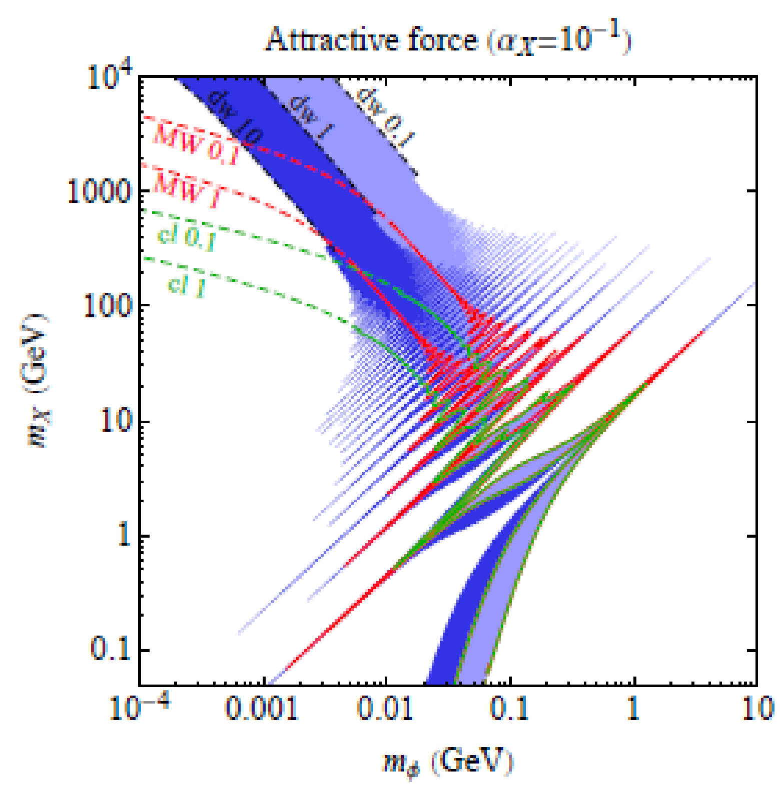
$$= f(y) \ll 1 \,.$$

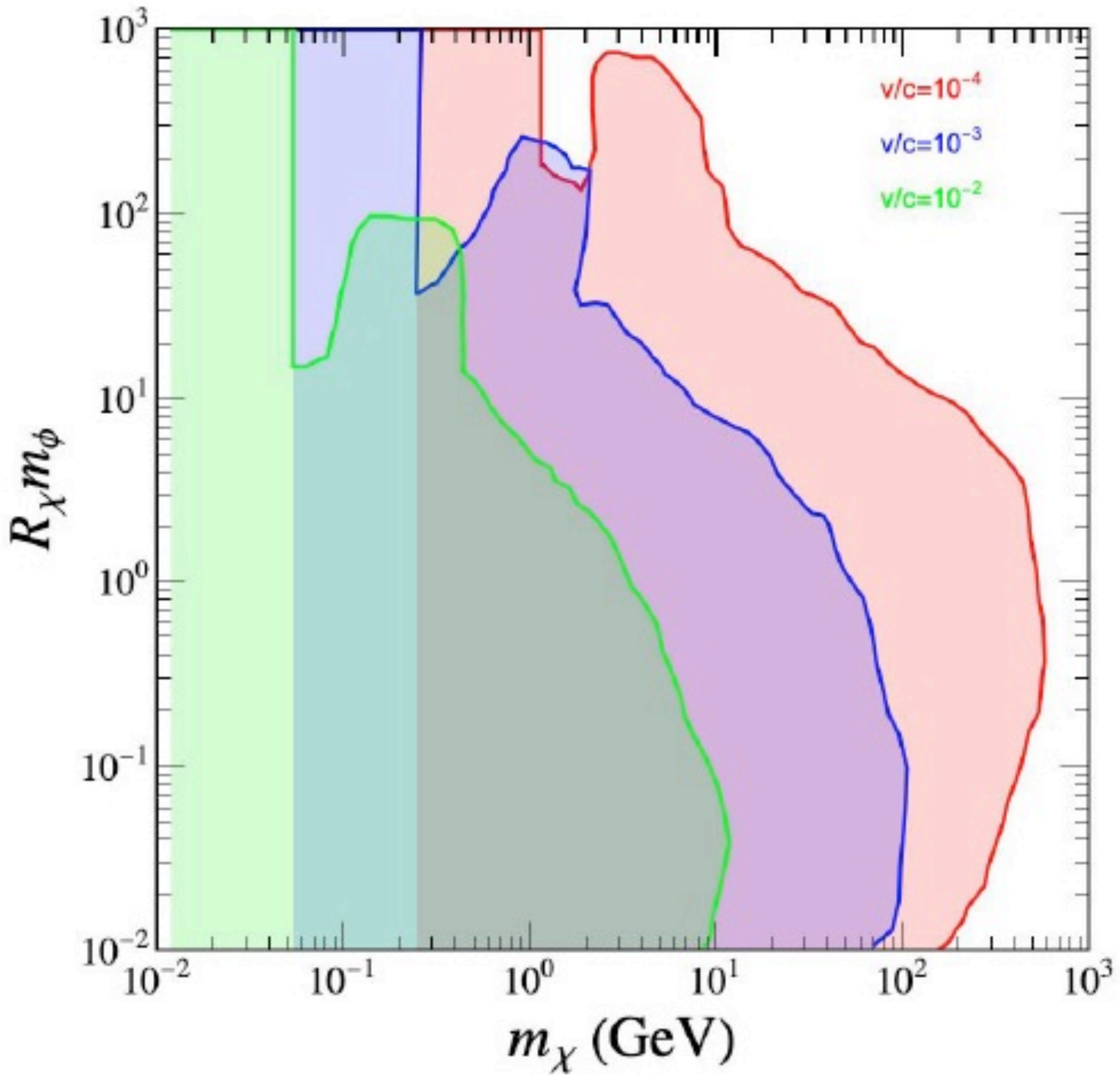


The transfer scattering cross section in puffy dark matter parameter



1.The parameter space for puffy dark matter scattering for the attractive force case. The resonant cross section may be produced for a small fixed radius-force range ratio.
2.Additional interacting parameter space are caused by radius effect (tough each other). However, the resonant effect disappears.





The puffy SIDM self-scattering can explain the dwarf galaxies in the Born and resonant regimes, and can also explain the cluster and Milky Way galaxy in the non-Born regime. **So the cross sections merely in the Born approximation can not solve the small-scale anomalies**

CONCLUSION

- Chou-Yang model are very beautiful and predictive.
- Simplicity and analyticity is our struggle!
- Self-interacting of dark matter needs deep studies.
- The cross sections merely in the Born approximation can not solve the small-scale anomalies.