Novel high-frequency gravitational waves detection with split cavity

Yiming Liu

School of Physics, Beijing Institute of Technology

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Based on 2305.00877 with Chu-Tian Gao, Yu Gao, Sichun Sun

Motivation

1, Gravitational waves can generate electromagnetic effects inside a strong electric or magnetic field within the Standard Model and general relativity.

2, We propose a new way to detect high-frequency gravitational waves from kHz to GHz, a design with a split cavity as a capacitor and readout LC circuit.

Theoretical framework

We start with the Lagrangian for the Inverse Gertsenshtein effect, which is present in the Standard model

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}\right)$$

Then we can expand the metric to the leading order in h, as below

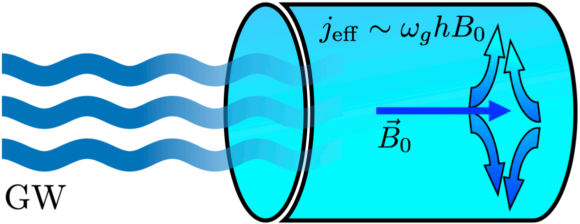
$$S \supset -\frac{1}{2}\int d^4x j^{\mu}_{\mathrm{eff}}A_{\mu}$$

where

$$j^{\mu}_{\mathrm{eff}} \equiv \partial_{\nu}\left(\frac{1}{2}hF^{\mu\nu} + h^{\nu}_{\alpha}F^{\alpha\mu} - h^{\mu}_{\alpha}F^{\alpha\nu}\right)$$

[A. Ejlli, D. Ejlli, A. M. Cruise, G. Pisano, and H. Grote]

Inverse Gertsenshtein effect



$$\begin{aligned} \nabla \cdot \mathbf{E} &= \rho_{\text{eff}} + \rho \\ \nabla \times \mathbf{B} - \partial_t \mathbf{E} &= \boldsymbol{j}_{\text{eff}} + \boldsymbol{j} \end{aligned} \tag{1}$$

[See Detecting High-Frequency Gravitational Waves with Microwave Cavities]

[M. E. Gertsenshtein, Soviet Physics JETP]

Theoretical framework

$$\partial_\nu F^{\mu\nu} = j^\mu_{\text{eff}} = (-\nabla\cdot\mathbf{P}, \nabla\times\mathbf{M} + \partial_t\mathbf{P})\,.$$

$$P_i = -h_{ij}E_j + \frac{1}{2}hE_i + h_{00}E_i - \epsilon_{ijk}h_{0j}B_k,$$

$$M_i = -h_{ij}B_j - \frac{1}{2}hB_i + h_{jj}B_i + \epsilon_{ijk}h_{0j}E_k,$$

where $h = h^\mu{}_\mu$. **From TT gauge to PD gauge:**

$$\begin{aligned}
h_{00} &= -R_{0i0j}x^i x^j \times 2\left[-\frac{i}{\omega_g z} + \frac{1-e^{-i\omega_g z}}{(\omega_g z)^2}\right]\\
h_{ij} &= -\frac{1}{3}R_{ikjl}x^k x^l \times 6\left[-\frac{1+e^{-i\omega_g z}}{(\omega_g z)^2} - 2i\frac{1-e^{-i\omega_g z}}{(\omega_g z)^3}\right]\\
h_{0i} &= -\frac{2}{3}R_{0jik}x^j x^k \times 3\left[-\frac{i}{2\omega_g z} - \frac{e^{-i\omega_g z}}{(\omega_g z)^2} - i\frac{1-e^{-i\omega_g z}}{(\omega_g z)^3}\right]
\end{aligned} \tag{2}$$

[See Maggiore M (2007) Gravitational Waves. Vol. 1: Theory and Experiments. Oxford University Press.]

$$j^{\mu}_{\rm eff} = -\frac{B_0 \omega_g^2 r}{6\sqrt{2}} e^{i\omega_g t} \left(0, ie^{-2i\phi} h_{+2} - ie^{2i\phi} h_{-2}, e^{-2i\phi} h_{+2} + e^{2i\phi} h_{-2}, 0\right) \times f(\omega_g z)\,,$$

where $f(x) \equiv -3 - 6ix^{-1} - 12e^{-ix}x^{-2} - 12i\left(1 - e^{-ix}\right) x^{-3}$ is a dimensionless function with $\lim_{x\to 0} f(x) = 1$ and $\lim_{x\to\infty} f(x) = -3$. GW helicity components are defined as $h_{\pm 2} \equiv (h_{+} \pm ih_{\times}) / \sqrt{2}$. In the SI unit, we can replace B_0 with $B_0 / \left(\mu_0 c^2\right)$ above in $j^{\mu}_{\rm eff}$. We illustrate $j_{\rm eff}$ in the x-y plane in Fig.1.

Theoretical framework

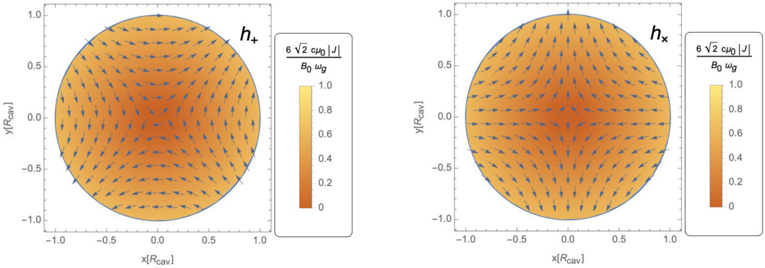


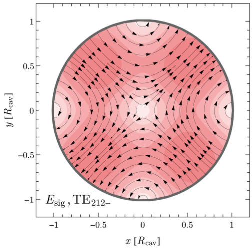
FIG. 1: We plot the $j_{\rm eff}$ with only cross/plus mode at z=0 in x-y plane in Eq.2. The arrows denote the direction of j^μ and the different color scales represent the magnitude.

Theoretical framework

Taking the lowest E mode numbers, the base mode that the gravitational waves along the z direction excite is TE211$\pm$ and its spatial components are given as

$$
\begin{aligned}
E_r^{\pm} &= \begin{bmatrix} A_+ \cos 2\phi \\ -A_- \sin 2\phi \end{bmatrix} \sin\left(\frac{\pi z}{L_{\mathrm{cav}}}\right) \frac{i\omega_{211}}{\omega_{211}^2 - \left(\frac{2\pi}{L_{\mathrm{cav}}}\right)^2} \frac{2}{r} J_2\left(r\gamma_{21}\right), \\
E_\phi^{\pm} &= -\begin{bmatrix} A_+ \sin 2\phi \\ A_- \cos 2\phi \end{bmatrix} \frac{i\omega_{211}}{\omega_{211}^2 - \left(\frac{2\pi}{L_{\mathrm{cav}}}\right)^2} \sin\left(\frac{\pi z}{L_{\mathrm{cav}}}\right) J_2'\left(\gamma_{21} r\right)\gamma_{21}, \\
E_z^{\pm} &= 0 \\
\omega_{211} &= \sqrt{k_z^2 + \gamma_{mn}^2} = \sqrt{\left(\frac{\pi}{L_{\mathrm{cav}}}\right)^2 + \left(\frac{x_{21}'}{R_{\mathrm{cav}}}\right)^2}
\end{aligned}
\tag{3}
$$

A quarterly split cavity



[See Detecting High-Frequency Gravitational Waves with Microwave Cavities]

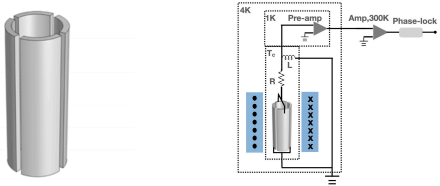


FIG. 2: Split cavity: the top, bottom, and quarterly split periphery of the cylinder cavity are separated by insulation to form induction signal currents through the connection wiring. The slit angle is 3° in this case. The connecting wires and amplification LC circuit are also shown.

Results (EM numerical simulation of the split cylinder with COMSOL package)

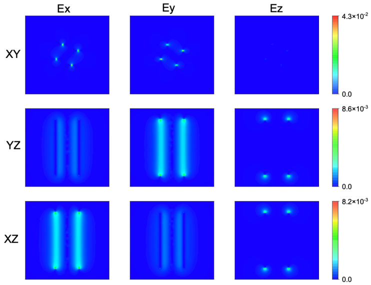


FIG. 3: EM response simulation of the incoming gravitational waves $h = 10^{-10}$ in the z-direction with a quarterly split cylinder $R = 1$cm and $L = 5$cm. The unit is V/m.

Results (The EM simulation results for the plates' charge accumulation and effective capacitance)

R:L = 1cm:5cm			R:L = 1m:1m		
f (Hz)	C (pF)	q (C)	f (Hz)	C (pF)	q (C)
10^7	1.53	2.39×10^{-19}	10^5	31.05	2.04×10^{-16}
10^8	1.53	2.39×10^{-18}	10^6	31.05	2.04×10^{-15}
10^9	1.52	2.29×10^{-17}	10^7	31.05	2.03×10^{-14}

TABLE I: Selected values of the cavity's effective capacitance C and the accumulated charge q from EM simulations. The GW is assumed to propagate along the cavity axis with an amplitude $h_0 = 10^{-10}$. The left and right columns show two experimental scales: a multi-center-sized cavity with $f_{211} = 1.5 \times 10^{10}$ Hz, and a meter-sized cavity with $f_{211} = 2.1 \times 10^8$ Hz. Notice here our working frequencies are away from the closed cavity's resonance frequencies.

SENSITIVITY PROSPECTS(Narrow-band scheme or $\Delta f \sim \omega / (2\pi Q_c)$)

A narrow band measurement scheme involves frequency filtering at this favored frequency range, and there are two popular ways to achieve this

1. a high geometric quality factor of the cavity Q_c
2. electronic filtering with an LCR circuit with a quality factor

When the signal's and the pickup's quality factors match each other, the high pickup quality factor is capable of enhancing the output power from a monochromatic perturbation source by a factor of the order Q_c, Assuming this condition could be met, the enhanced signal current is $I_a = Q_c \cdot q_0 \omega \cos(\omega t)$

[See Erratum: Phys.Rev.Lett. 52, 695 (1984)]

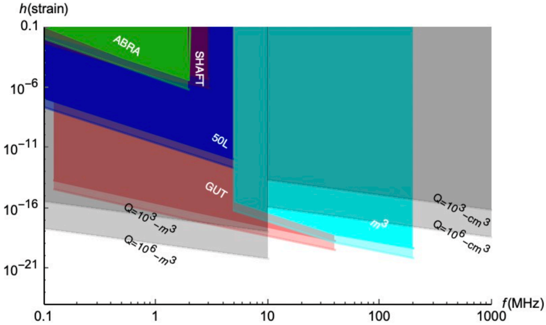
[See Proposal for Axion Dark Matter Detection Using an LC Circuit]

SENSITIVITY PROSPECTS(Narrow-band scheme or $\Delta f \sim \omega / (2\pi Q_c)$)

$$Q_c = (\omega C R_s)^{-1}$$
$$q_0 = \int \vec{E} \cdot \mathrm{d}\vec{A}$$
$$P_{\text{sig.}} = \langle I_a^2 \rangle \cdot R_s = \frac{(Q_c \omega q_0)^2}{2 Q_c \omega C} = Q_c \cdot \langle q_0^2 \rangle \omega C^{-1}$$
$$\mathrm{SNR} = \frac{P_{\text{sig.}}}{k_{\mathrm{B}} T_N} \sqrt{\frac{\Delta t}{\Delta f}}$$
$$\Delta f \sim \omega / (2\pi Q_c)$$
$$T_N = 0.1 \mathrm{~K}$$
$$\Delta t = 60 s$$

Where C, R_s denote the LCR's capacitance and resistance at the resonant point, k_{B} is the Boltzmann constant, and we can require SNR= 3 for a 3 sensitivity criterion.

SENSITIVITY PROSPECTS(Narrow-band scheme or $\Delta f \sim \omega/(2\pi Q_c)$)



Projected sensitivity to the stress of a GW signal, with coherent factor $Q = 10^6$ and $Q = 10^3$, corresponding to narrowband searches.

[Challenges and Opportunities of Gravitational Wave Searches at MHz to GHz Frequencies]

[High-Frequency Gravitational Wave Detection via Optical Frequency Modulation]

[Potential of Radio Telescopes as High-Frequency Gravitational Wave Detectors]

[Prospects for detection of ultra high frequency gravitational waves from compact binary coalescenses with resonant cavities]

[Prototype superfluid gravitational wave detector]

SENSITIVITY PROSPECTS(Non-resonant scheme or $\Delta f \sim f$)

For occasional signals, the SNR can be estimated by comparing the signal and noise power during the characteristic time-duration Δt of the signal

$\mathrm{SNR}=\frac{\left\langle P_{\text{sig. }}\right\rangle}{P_{\text{noise}}}=\frac{R \cdot \sum_{n=-\infty}^{+\infty}\left|c_n\right|^2}{k_B T_N \Delta f}$,

(We use Parseval's Theorem: $\int\left|I_a(t)\right|^2 \mathrm{~d} t=\Delta t \cdot \sum_n\left|c_n\right|^2$, and

$c_n \equiv \frac{1}{\Delta t} \int_0^{\Delta t} I_a(t) e^{i \frac{2 \pi}{\Delta t} n t} \mathrm{~d} t, \quad n=1,2, \ldots$

$\lim_{\Delta t \rightarrow \infty} \frac{2 \pi}{\Delta t} \sum\left|I_a\left(\omega_n\right)\right|^2 \rightarrow \int\left|I_a(\omega)\right|^2 \mathrm{~d} \omega$)

$\mathrm{SNR}=\frac{\left\langle P_{\text{sig. }}\right\rangle}{P_{\text{noise}}}=\frac{R}{2 \pi \Delta t} \frac{2 \int_{\Delta \omega}\left|\tilde{I}_a(\omega)\right|^2 \mathrm{~d} \omega}{k_B T_N \Delta f}$.

Where $\Delta t=1 s$, $T_N=0.1 K$, $\Delta f \sim f$.

SENSITIVITY PROSPECTS(Non-resonant scheme or $\Delta f \sim f$)

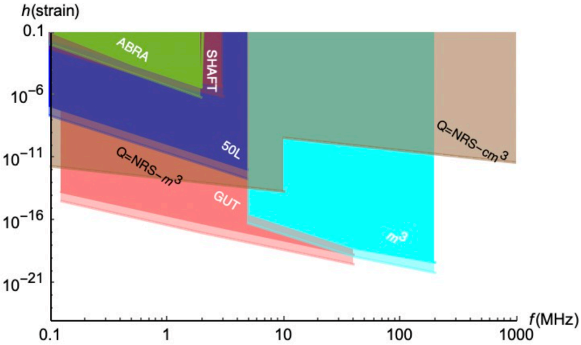


FIG. 5: SNR=3 sensitivity for transient events, like PBH merger, and BH superradiance collapse. We use a bandwidth of $\Delta f = f$ for sensitivity estimate.

Conclusions

1 We consider a cryogenic detection scheme of the electric signals induced by gravitational waves via the inverse Gertsenshtein effect, inside a strong static magnetic field.

2 In the case of a narrow-frequency gravitational wave, the signal can be further enhanced by a large quality factor Q that matches the bandwidth of the gravitational wave, and the corresponding strain sensitivity is projected to around $h \sim 10^{20}(10^6/Q)$ for a meter-sized split cavity

3 For signals more extended in the frequency domain, we consider a broad-band signal with a characteristic frequency range wider than the cavity's geometric bandwidth and make an estimate of our design's sensitivity in broadband, non-resonant $(\Delta f \sim f)$ mode. The broadband sensitivity is found to be $h \sim 10^{13}$ for a meter-scale cavity.