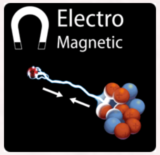
核子三维结构
Three Dimensional Imaging of the Nucleon

梁作堂 (Liang Zuo-tang)
山东大学(Shandong University)
2023年6月2日

Based on：
the plenary talk at the 21st International Symposium on Spin Physics (2014);
also a short review by K.B. Chen, S.Y. Wei and ZTL, Front. Phys. 10, 101204 (2015).

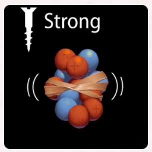
强相互作用物理



电

弱

弱电统一理论



强 ⟹ 量子色动力学 (QCD)

微扰部分 → 已被实验充分检验（1979、2004诺贝尔奖）

非微扰部分 ⟺ 尚未解决的重大难题

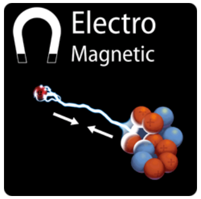
强相互作用物理

- pQCD高精度计算与应用
- 强子结构与强子产生
- 强相互作用物质形态
- ………

是当代 粒子物理 原子核物理 共同的前沿之一

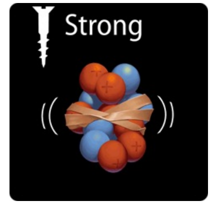
强相互作用物理：强相互作用物质形态

电磁



气体
液体
固体
晶体
超导体
超流体
等离子体
电磁波
....

强



核子/强子
(nucleon/hadron)
原子核 (nuclei)

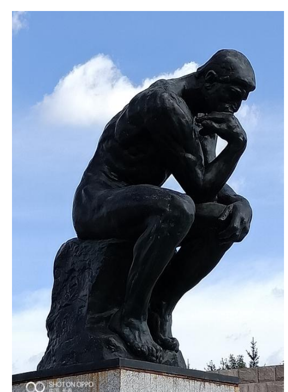
} 束缚态

色超导体？
(color super conductor)
色玻璃体？
(color glass condensate)
夸克胶子等离子体？
(quark gluon plasmas)
....

} 特殊条件下

原子分子物理
凝聚态物理
光学
等离子体物理
声学
无线电物理

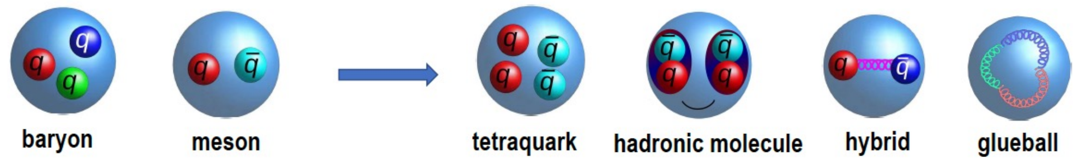
材料科学
电子科学
化学



强相互作用物理：强子结构

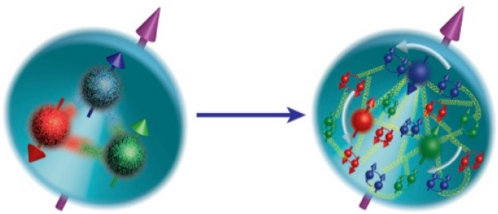
夸克模型 → 强子多重态 / 重子反常磁矩 → 强子质量谱 / 奇特强子态

（静态性质）



量子场论的基本性质: 真空涨落与激发 (vacuum excitation)

(Lamb位移 —— QED)

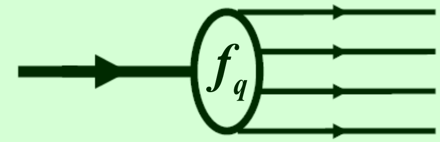


高速运动的核子的内部结构——夸克部分子模型

✶强相互作用性质研究的重要场所　　✶高能反应的初始条件

高速运动的核子的结构——直观图像

部分子模型 Parton model　　一个高速运动的质子 A fast moving proton ＝ 一束部分子 A beam of partons



其结构由部分子分布函数(parton distribution functions, PDFs)来描述

只考虑纵向运动 ⟹ One-dimensional PDF $f_q(x)$

质子内部分子q的数密度，x是q带的动量分数

考虑自旋 ⟹ spin dependent one-dimensional PDFs (totally 3):

$$f_1(x,s_q;S) = f_1(x) + \lambda_q\lambda\, g_{1L}(x) + \vec{s}_{Tq}\cdot\vec{S}_T\, h_{1T}(x)$$

helicity distribution　　transversity

考虑横向运动 ⟹ three-dimensional (or transverse momentum dependent, TMD) PDFs (totally 8):

$$f_q(x,k_\perp,S_q;p,S) = f_q(x,k_\perp) + \lambda_q\lambda\,\Delta f_q(x,k_\perp) + (\vec{S}_{\perp q}\cdot\vec{S}_T)\,\delta f_q(x,k_\perp) + \vec{S}_T\cdot(\hat{p}\times\hat{k}_\perp)\Delta^N f(x,k_\perp) + \frac{1}{M}\vec{S}_{\perp q}\cdot(\hat{p}\times\vec{k}_\perp)h_1^\perp(x,k_\perp)$$
$$+\frac{1}{M^2}(\vec{S}_{\perp q}\cdot\vec{k}_\perp)(\vec{S}_T\cdot\vec{k}_\perp)h_{1T}^\perp(x,k_\perp) + \frac{1}{M}(\vec{S}_{\perp q}\cdot\vec{k}_\perp)\,\lambda\, h_{1L}^\perp(x,k_\perp) + \lambda_q\frac{1}{M}(\vec{S}_T\cdot\vec{k}_\perp)\, g_{1T}^\perp(x,k_\perp)$$

核子三维结构：当前核子结构研究的核心前沿，美国未来唯一高能对撞机EIC以及中国EicC主要物理目标之一，内容丰富多彩，“千头万绪”！

Introduction

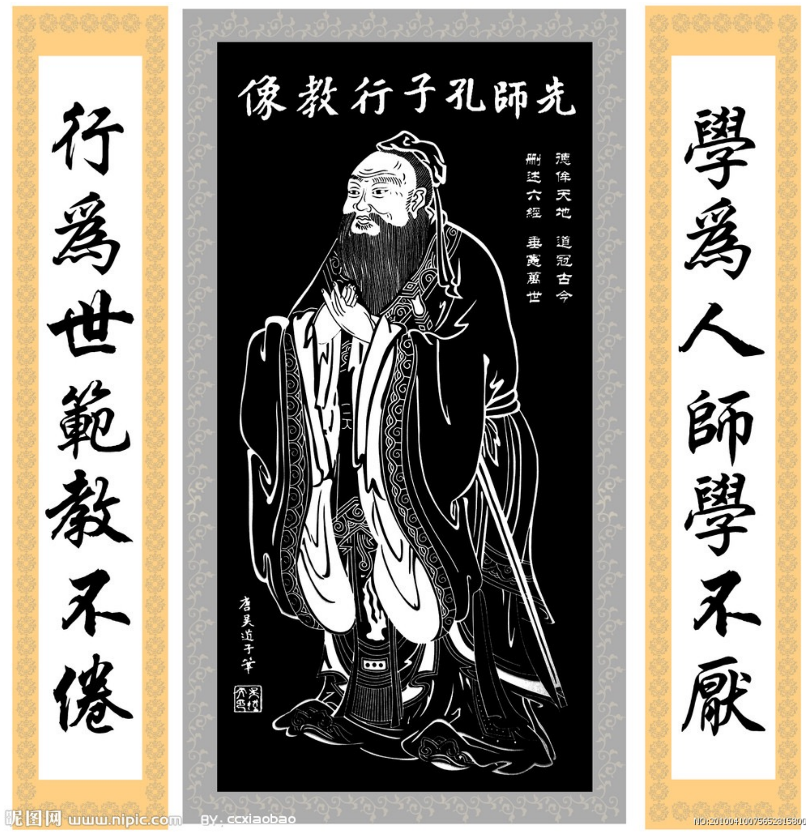
《论语》

子曰：“温故而知新，
可以为师矣”

“The Analects of Confucius”

Confucius said:

“One can be a master if he gets to know new things by reviewing the old knowledge”.



I therefore start with inclusive DIS and one-dimensional imaging of the nucleon.

Contents

I. Introduction

- Inclusive DIS and ONE dimensional PDF of the nucleon
- The need for a THREE dimensional imaging of the nucleon

温故

II. Three dimensional PDFs defined via quark-quark correlators

III. Accessing TMDs via semi-inclusive high energy reactions

- Kinematics: general forms of differential cross sections
- The theoretical framework:
  - ✪ Leading order pQCD & leading twist --- intuitive proton model
  - ✪ Leading order pQCD & higher twists --- collinear expansion
  - ✪ Leading twist & higher order pQCD --- TMD factorization
- Experiments and parameterizations
- Examples of the phenomenology

知新

IV. Summary and outlook

Parton distribution functions (PDFs)

$$f_1(x) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p | \bar{\psi}(0) \mathcal{L}(0,z^-) \frac{\gamma^+}{2} \psi(0,z^-,\vec{0}_\perp) | p \rangle$$

$$\mathcal{L}(0,z) = \mathcal{L}^\dagger(-\infty,0)\mathcal{L}(-\infty,z),$$

gauge link

$$\mathcal{L}(-\infty,z) = Pe^{ig\int_{-\infty}^{z^-} dy^- A^+(0,y^-,\vec{0}_\perp)}$$

$$= 1 + ig\int_{-\infty}^{z^-} dy^- A^+(0,y^-,\vec{0}_\perp) + \frac{1}{2}(ig)^2 \int_{-\infty}^{z^-} dy^- \int_{-\infty}^{y^-} dy'^- A^+(0,y^-,\vec{0}_\perp)A^+(0,y'^-,\vec{0}_\perp) + \ldots\ldots$$

Why? Where does it come from?

How does it look like in the three dimensional case ?

Contents

Our knowledge of parton model started from inclusive DIS

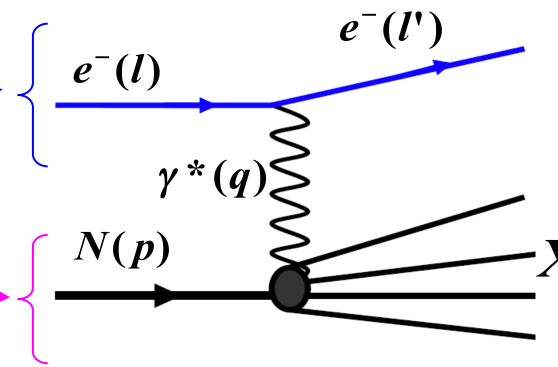
当代卢瑟福散射实验

The differential cross section

$$d\sigma = \frac{\alpha_{em}^2}{sQ^4} L^{\mu\nu}(l,\lambda_l,l',\lambda_l') W_{\mu\nu}(q,p,S) \frac{d^3 l'}{2E'}$$

leptonic tensor

hadronic tensor



$$Q^2 \equiv -q^2$$

$$x_B \equiv \frac{Q^2}{2q\cdot p}$$

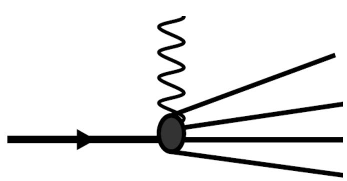
$$y \equiv \frac{q\cdot p}{l\cdot p}$$

强子张量，包含核子结构的信息

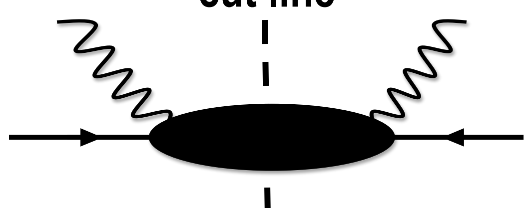
The hadronic tensor:

$$W_{\mu\nu}(q,p,S) = \sum_X \langle p,S | J_\mu(0) | X\rangle\langle X | J_\nu(0) | p,S\rangle (2\pi)^4 \delta^4(p+q-p_X)$$

$$W_{\mu\nu}(q,p,S) = \left| \quad \right|^2 =$$



$|\mathcal{M}|^2$



$\mathcal{M}\times\mathcal{M}^*$

Inclusive deep inelastic scattering (DIS) $e^- + N \to e^- + X$

Kinematic analysis: find the complete set of the "basic Lorentz tensors" and the general form of the hadronic tensor

The constraints: Gauge invariance $q^\mu W_{\mu\nu}(q,p,S)=0$ Hermiticity $W^*_{\mu\nu}(q,p,S)=W_{\nu\mu}(q,p,S)$

Parity invariance $W_{\mu\nu}(\tilde{q},\tilde{p},-\tilde{S})=W^{\mu\nu}(q,p,S)$

The unpolarized set: $\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)$, $(q+2xp)_\mu(q+2xp)_\nu$

The polarized (spin dependent) set: $\varepsilon_{\mu\nu\rho\sigma}q^\rho S^\sigma$, $\varepsilon_{\mu\nu\rho\sigma}q^\rho\left(S^\sigma-\frac{S\cdot q}{p\cdot q}p^\sigma\right)$

$\Rightarrow$ $W_{\mu\nu}(q,p,S)=W^{(S)}_{\mu\nu}(q,p)+iW^{(A)}_{\mu\nu}(q,p,S)$

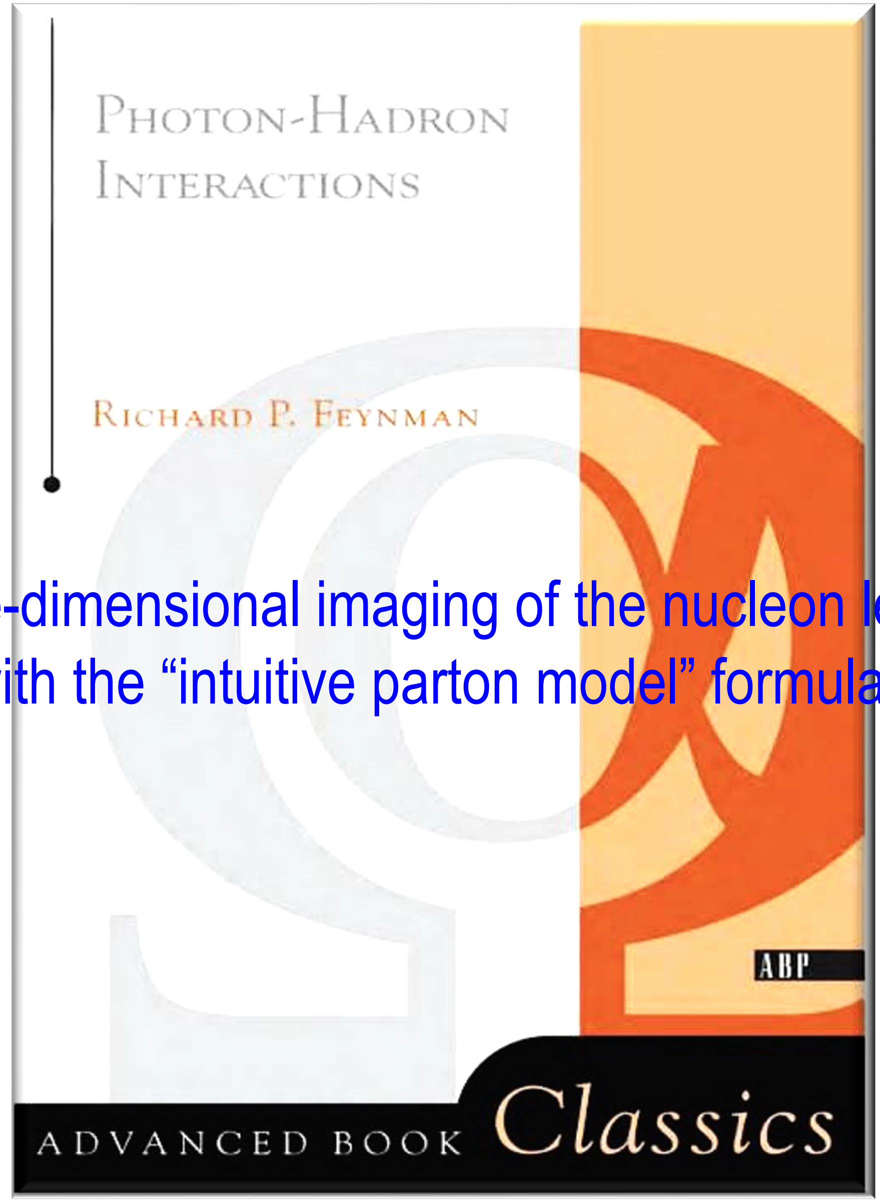
$$W^{(S)}_{\mu\nu}(q,p)=2\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)F_1(x,Q^2)+\frac{1}{xQ^2}(q+2xp)_\mu(q+2xp)_\nu F_2(x,Q^2)$$

$$W^{(A)}_{\mu\nu}(q,p,S)=\frac{2M}{p\cdot q}\varepsilon_{\mu\nu\rho\sigma}q^\rho S^\sigma g_1(x,Q^2)+\frac{2M}{p\cdot q}\varepsilon_{\mu\nu\rho\sigma}q^\rho\left(S^\sigma-\frac{S\cdot q}{p\cdot q}p^\sigma\right)g_2(x,Q^2)$$

4 independent "basic Lorentz tensors" and correspondingly 4 structure functions

包含核子结构的信息

"Original / Intuitive" Parton Model

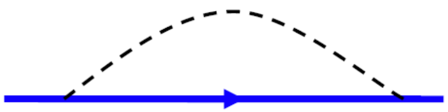


Our knowledge of one-dimensional imaging of the nucleon learned from DIS experiments started with the "intuitive parton model" formulated e.g. in this book.

"Original / Intuitive" Parton Model

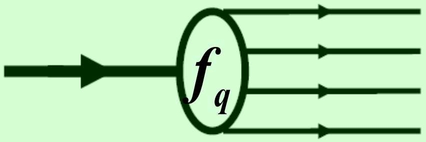
The model:

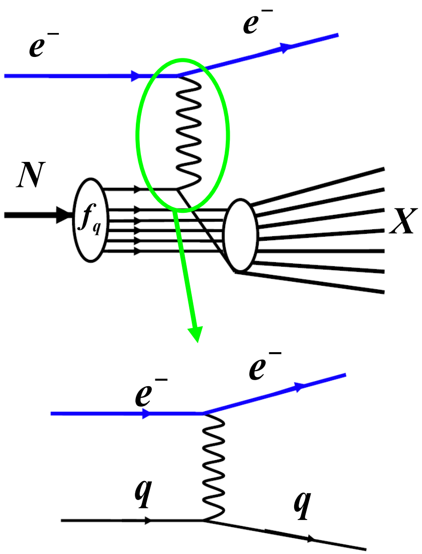
Feynman (1969);
Bjorken & Paschos (1969)

Virtual processes such as 

Because of time dilatation, in the infinite momentum frame, they exist forever.

A fast moving proton $=$ A beam of free partons





$$|\mathcal{M}(eN \to eX)|^2 = \sum_q \int dx\, f_q(x)\, |\hat{\mathcal{M}}(eq \to eq)|^2$$

scattering amplitude squared

$x = k/p$: momentum fraction carried by the parton

$f_q(x)$: parton number density, known as Parton Distribution Function (PDF)

E.g.: $F_2(x) = 2xF_1(x) = \sum_q e_q^2 f_q(x)$ $\qquad g_1(x) = \sum_q e_q^2 \Delta f_q(x)$

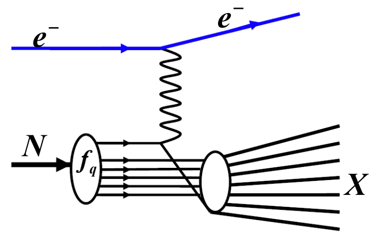
Parton Model and High Energy Reactions

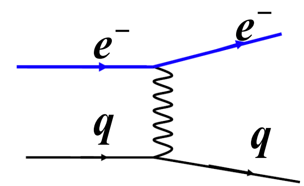
Parton model: **A fast moving proton = A beam of free partons**

The high energy scattering process | **the elementary scattering process**

Deeply inelastic lepton-nucleon scattering

$e+N\rightarrow e+X$

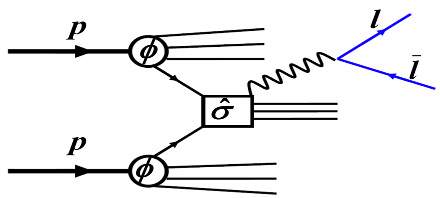


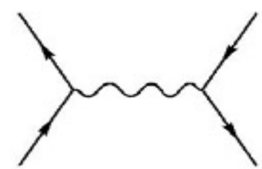


$e+q\rightarrow e+q$

Drell-Yan process:

$p+p\rightarrow l\bar{l}+X$

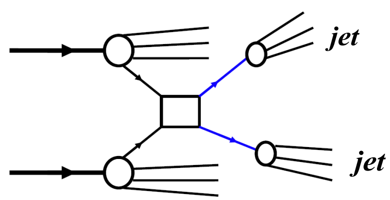


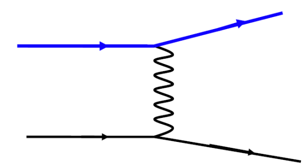


$q+\bar{q}\rightarrow \mu^{+}+\mu^{-}$

high p_t jet production:

$p+p\rightarrow jet+X$





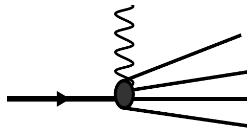
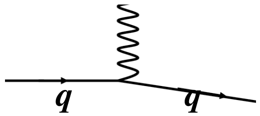
$q+q\rightarrow q+q$

$$d\sigma=\left(\text{parton distribution functions}\quad f_q(x)\right)\otimes\left(\text{cross sections for the elementary process}\quad d\hat{\sigma}\right)$$

data → parameterization → PDFLib

“Original / Intuitive” Parton Model

It is just the impulse approximation!

$W_{\mu\nu}(q,p,S) = \Big|$  $\Big|^2 = f(x) \otimes \Big|$  $\Big|^2$

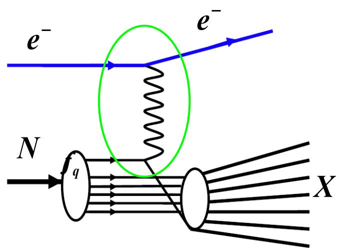
数密度 “几率”

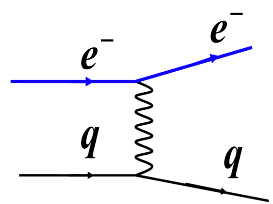
Impulse Approximation (冲量/脉冲近似):

(1) during the interaction of lepton with parton, interaction between partons is neglected;

(2) lepton interacts only with one single parton;

(3) interaction with different partons adds incoherently.





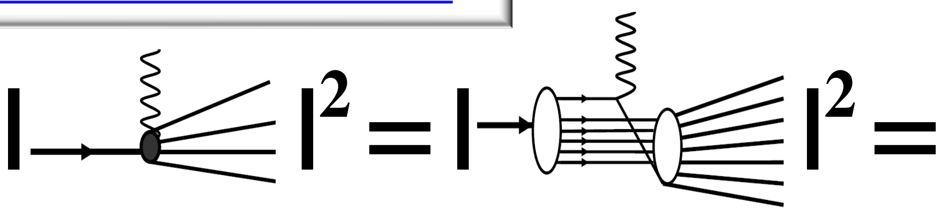
Approximation: What is neglected? Controllable?

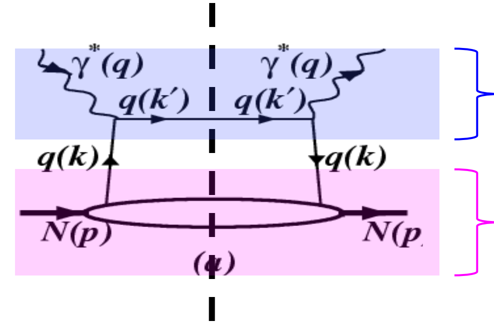
Parton distribution function (PDF): A proper (quantum field theoretical) definition?

⟹ A quantum field theoretical formulation ?

Quantum field theoretical formulation of parton model

Parton model without QCD:

$W_{\mu\nu}(q,p,S) = \Big|$  $\Big|^2 = \Big|$ ... $\Big|^2 =$



$$W_{\mu\nu}(q,p,S) = \sum_X \langle p,S \,|\, J_\mu(0) \,|\, X\rangle\langle X \,|\, J_\nu(0) \,|\, p,S\rangle (2\pi)^4 \delta^4(p+q-p_X)$$

$$= \sum_X \int d^4z \langle p,S \,|\, J_\mu(0) \,|\, X\rangle\langle X \,|\, J_\nu(z) \,|\, p,S\rangle e^{-iqz}$$

$$|X\rangle = |X'\rangle|k'\rangle$$
$$J_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$$
$$\psi(x)|X'\rangle|k'\rangle = u(k')e^{ik'\cdot x}|X'\rangle$$

$$= \int \frac{d^4k'}{(2\pi)^4}(2\pi)\delta_+(k'^2)\sum_{X'}\int d^4z e^{-iqz}\langle p,S \,|\, \bar{\psi}(0) \,|\, X'\rangle\gamma_\mu u(k')\bar{u}(k')\gamma_\nu e^{ik'z}\langle X' \,|\, \psi(z) \,|\, p,S\rangle$$

$$= \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{H}_{\mu\nu}(k,q)\hat{\phi}(k,p,S)\right]$$

the calculable hard part $\hat{H}_{\mu\nu}(k,q) = \gamma_\mu(\not k + \not q)\gamma_\nu(2\pi)\delta_+\left((k+q)^2\right)$

the quark-quark correlator $\hat{\phi}(k,p,S) = \int d^4z e^{ikz}\langle p,S \,|\, \bar{\psi}(0)\psi(z) \,|\, p,S\rangle$

4x4 matrix： $\phi_{\alpha\beta}(k,p,S) = \int d^4z\, e^{ikz}\langle p,S|\bar{\psi}_\beta(0)\psi_\alpha(z)|p,S\rangle$

no local (color) gauge invariance!

Quantum field theoretical formulation of parton model

Parton model without QCD (continued):

$x = k^+ / p^+$

$k^\pm = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$

$n = (0,1,\vec{0}_\perp)$

$\bar{n} = (1,0,\vec{0}_\perp)$

Collinear approximation(共线近似): $p \approx p^+ \bar{n}, \quad k \approx xp$

$$\hat{H}_{\mu\nu}(k,q) \approx \hat{H}_{\mu\nu}(x) \equiv \hat{H}_{\mu\nu}(k=xp,q) = \gamma_\mu \not{n} \gamma_\nu \delta(x - x_B)$$

$$W_{\mu\nu}(q,p) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}(k,q)\hat{\phi}(k,p)\right] \approx \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}(x)\hat{\phi}(k,p)\right] = \int dx \mathrm{Tr}\left[\hat{H}_{\mu\nu}(x)\hat{\phi}(x,p)\right]$$

$$\hat{\phi}(x;p) \equiv \int \frac{d^4k}{(2\pi)^4} \delta(x - k^+ / p^+)\, \hat{\phi}(k,p) = \frac{1}{2} p^+ \not{\bar{n}} f_1(x) + \ldots$$

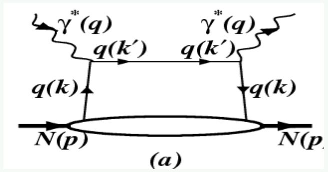
$$\Longrightarrow \quad W_{\mu\nu}(q,p) \approx \left[(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) + \frac{1}{2xq\cdot p}(q+2xp)_\mu (q+2xp)_\nu\right] f_1(x)$$

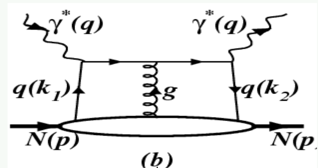
operator expression of the number density : $f_1(x) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p | \bar{\psi}(0) \frac{\gamma^+}{2} \psi(z) | p \rangle$

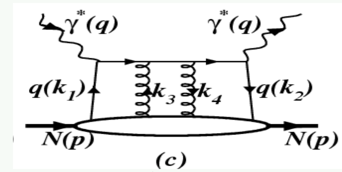
no local (color) gauge invariance!

Inclusive DIS with "multiple gluon scattering"

To get the gauge invariance, we need to take the "multiple gluon scattering" into account

$W_{\mu\nu}(q,p,S) =$ 

$+$ 

$+$ 

$+ \cdots$

$$W_{\mu\nu}(q,p,S) = W^{(0)}_{\mu\nu}(q,p,S) + W^{(1)}_{\mu\nu}(q,p,S) + W^{(2)}_{\mu\nu}(q,p,S) + \ldots$$

$$W^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}^{(0)}_{\mu\nu}(k,q)\hat{\phi}^{(0)}(k,p,S)\right]$$

$$W^{(1)}_{\mu\nu}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q)\hat{\phi}^{(1)}_{\rho}(k_1,k_2,p,S)\right]$$

$$\hat{H}^{(1)\rho}_{\mu\nu} = \hat{H}^{(1,L)\rho}_{\mu\nu} + \hat{H}^{(1,R)\rho}_{\mu\nu}$$

the calculable hard part: $\hat{H}^{(0)}_{\mu\nu}(k,q) = \gamma_\mu(\not{k}+\not{q})\gamma_\nu(2\pi)\delta_+\left((k+q)^2\right)$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(k_1,k_2,q) = \gamma_\mu \frac{(\not{k}_2+\not{q})\gamma^\rho(\not{k}_1+\not{q})}{(k_2+q)^2 - i\varepsilon}\gamma_\nu(2\pi)\delta_+\left((k_1+q)^2\right)$$

the quark-quark correlator: $\hat{\phi}^{(0)}(k;p,S) = \int d^4z\, e^{ikz}\langle p,S|\bar{\psi}(0)\psi(z)|p,S\rangle$

the quark-gluon-quark correlator: $\hat{\phi}^{(1)}(k_1,k_2;p,S) = \int d^4y\, d^4z e^{ik_1z+ik_2(y-z)}\langle p,S|\bar{\psi}(0)A_\rho(y)\psi(z)|p,S\rangle$

no (local) gauge invariance!

Inclusive DIS: LO pQCD, leading twist

Collinear approximation (共线近似):

✪ Approximating the hard part as equal to that at $k = xp$:

$$\hat{H}^{(0)}_{\mu\nu}(k,q) \approx \hat{H}^{(0)}_{\mu\nu}(x)$$

$$\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q) \approx \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)$$

$$\hat{H}^{(0)}_{\mu\nu}(x) \equiv \hat{H}^{(0)}_{\mu\nu}(k = xp, q)$$

$$\hat{H}^{(1)}_{\mu\nu}(x_1,x_2) \equiv \hat{H}^{(1)}_{\mu\nu}(k_1 = x_1 p, k_2 = x_2 p, q)$$

$$x = k^+ / p^+$$

$$k^{\pm} = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$$

$$n = (0,1,\vec{0}_\perp)$$

$$\bar{n} = (1,0,\vec{0}_\perp)$$

✪ Keep only the longitudinal component of the gluon field:

$$A_\rho(y) \approx n \cdot A(y)\frac{p_\rho}{n \cdot p} = A^+(y)\frac{p_\rho}{p^+}$$

✪ Using the Ward identities such as,

$$p_\rho \hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2 - x_1 - i\varepsilon}$$

to replace hard parts for diagrams with multiple gluon scatterings by $\hat{H}^{(0)}_{\mu\nu}(x)$.

✪ Adding all terms together ⟹

Inclusive DIS: LO pQCD, leading twist

$$\Rightarrow \quad W_{\mu\nu}(q,p,S) \approx \tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Phi}^{(0)}(k;p,S)\hat{H}^{(0)}_{\mu\nu}(x)\right]$$

LO & leading twist

$$\hat{\Phi}^{(0)}(k;p,S) = \int d^4z e^{ikz} \langle p,S | \bar{\psi}(0)\mathcal{L}(0,z)\psi(z) | p,S \rangle$$

The gauge invariant un-integrated quark-quark correlator: contain QCD interaction!

gauge link

$$\mathcal{L}(0,z) = \mathcal{L}^{\dagger}(-\infty,0)\mathcal{L}(-\infty,z),$$

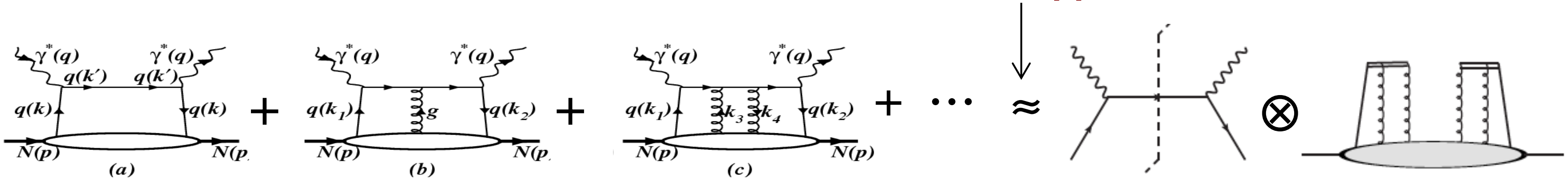
$$\mathcal{L}(-\infty,z) = Pe^{ig\int_{-\infty}^{z^-} dy^- A^+(0,y^-,\vec{0}_\perp)}$$

$$= 1 + ig\int_{-\infty}^{z^-} dy^- A^+(0,y^-,\vec{0}_\perp) + \frac{1}{2}(ig)^2 \int_{-\infty}^{z^-} dy^- \int_{-\infty}^{y^-} dy'^- A^+(0,y^-,\vec{0}_\perp) A^+(0,y'^-,\vec{0}_\perp) + \ldots\ldots$$

Gauge link comes from the multiple gluon scattering.

Graphically:

collinear approximation



Inclusive DIS: LO pQCD, leading & higher twists

Collinear expansion (共线展开):

Ellis, Furmanski, Petronzio (1982,1983); Qiu, Sterman (1990,1991)

✪ Expanding the **hard part** at $k = xp$:

$$\hat{H}^{(0)}_{\mu\nu}(x) \equiv \hat{H}^{(0)}_{\mu\nu}(k = xp, q)$$

$$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^{\rho}} \equiv \left.\frac{\partial \hat{H}^{(0)}_{\mu\nu}(k,q)}{\partial k^{\rho}}\right|_{k=xp}$$

$$\hat{H}^{(0)}_{\mu\nu}(k,q) = \hat{H}^{(0)}_{\mu\nu}(x) + \frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^{\rho}} \omega_{\rho}{}^{\rho'} k_{\rho'} + \ldots\ldots$$

$$\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q) = \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2) + \frac{\partial \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)}{\partial k_1^{\sigma}} \omega_{\sigma}{}^{\sigma'} k_{1\sigma'} + \ldots\ldots$$

✪ Decomposition of the gluon field:

$$A_{\rho}(y) = n\cdot A(y)\frac{p_{\rho}}{n\cdot p} + \omega_{\rho}{}^{\rho'} A_{\rho'}(y)$$

$$x = k^+ / p^+$$

$$\omega_{\rho}{}^{\rho'} \equiv g_{\rho}{}^{\rho'} - \bar{n}_{\rho} n^{\rho'}$$

$$\omega_{\rho}{}^{\rho'} k_{\rho'} = (k - xp)_{\rho}$$

$$k^{\pm} = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$$

$$n = (0,1,\vec{0}_{\perp})$$

$$\bar{n} = (1,0,\vec{0}_{\perp})$$

✪ Using the Ward identities such as,

$$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^{\rho}} = -\hat{H}^{(1)\rho}_{\mu\nu}(x,x), \qquad p_{\rho}\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2 - x_1 - i\varepsilon}$$

to replace the derivatives etc.

✪ Adding all terms with the same hard part together ⟹

Inclusive DIS: LO pQCD, leading & higher twists

$$W_{\mu\nu}(q,p,S)=\tilde{W}^{(0)}_{\mu\nu}(q,p,S)+\tilde{W}^{(1)}_{\mu\nu}(q,p,S)+\tilde{W}^{(2)}_{\mu\nu}(q,p,S)+\ldots$$

twist-2, 3 and 4 contributions

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S)=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Phi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(x)\right]$$

$$\hat{\Phi}^{(0)}(k;p,S)=\int d^4z\,e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

gauge invariant quark-quark correlator

twist-3, 4 and 5 contributions

$$\tilde{W}^{(1)}_{\mu\nu}(q,p,S)=\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Phi}^{(1)}_{\rho'}(k_1,k_2,p,S)\hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho{}^{\rho'}\right]$$

$$\hat{\Phi}^{(1)}(k_1,k_2;p,S)=\int d^4y\,d^4z\,e^{ik_1z+ik_2(y-z)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,y)D_\rho(y)\mathcal{L}(y,z)\psi(z)|p,S\rangle$$

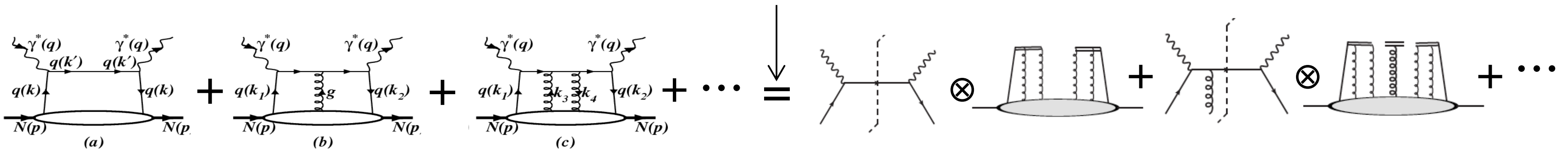
$$D_\rho(y)=-i\partial_\rho+gA_\rho(y)$$

gauge invariant quark-gluon-quark correlator

⇨ A consistent framework for inclusive DIS $e^-N\to e^-X$ including leading & higher twists

Graphically

collinear expansion



Inclusive DIS: LO pQCD, leading & higher twists

Simplified expressions for hadronic tensors

The "collinearly expanded hard parts" take the simple forms such as:

$$\hat{H}^{(0)}_{\mu\nu}(x) = \hat{h}^{(0)}_{\mu\nu}\delta(x-x_B), \qquad \hat{h}^{(0)}_{\mu\nu} = \gamma_\mu \not{n} \gamma_\nu$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'} = \frac{\pi}{2q\cdot p}\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho^{\ \rho'}\delta(x_1-x_B), \quad \hat{h}^{(1)\rho}_{\mu\nu} = \gamma_\mu \bar{\not{n}} \gamma^\rho \not{n} \gamma_\nu$$

depends only on **ONE** variable!

twist-2, 3 and 4 contributions

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int dx \mathrm{Tr}\left[\hat{\Phi}^{(0)}(x;p,S)\, h^{(0)}_{\mu\nu}\right]\delta(x-x_B)$$

$$\hat{\Phi}^{(0)}(x;p,S) \equiv \int \frac{d^4k}{(2\pi)^4}\delta(x-\frac{k^+}{p^+})\hat{\Phi}^{(0)}(k;p,S) = \int \frac{p^+dz^-}{2\pi}e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z^-)\psi(z^-)|p,S\rangle$$

one-dimensional gauge invariant quark-quark correlator

twist-3, 4 and 5 contributions

$$\tilde{W}^{(1)}_{\mu\nu}(q,p,S) = \frac{\pi}{2q\cdot p}\mathrm{Re}\int dx \mathrm{Tr}\left[\hat{\varphi}^{(1)}_{\rho'}(x;p,S)\, h^{(1)\rho}_{\mu\nu}\omega_\rho^{\ \rho'}\right]\delta(x-x_B)$$

$$\hat{\varphi}^{(1)}_{\rho}(x;p,S) \equiv \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\delta(x-\frac{k_1^+}{p^+})\hat{\Phi}^{(1)}_\rho(k_1,k_2;p,S) = \int \frac{p^+dz^-}{2\pi}e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)D_\rho(0)\mathcal{L}(0,z^-)\psi(z^-)|p,S\rangle$$

the involved one-dimensional gauge invariant quark-gluon-quark correlator

⟹ **Only ONE dimensional imaging of the nucleon is involved in inclusive DIS.**

PDFs defined via quark-quark correlator

- **Expand the quark-quark correlator in terms of the Γ-matrices:**

$$\hat{\Phi}^{(0)}(x;p,S)=\frac{1}{2}\left[\Phi^{(0)}(x;p,S)+i\gamma_5\tilde{\Phi}^{(0)}(x;p,S)+\gamma^\alpha\Phi^{(0)}_\alpha(x;p,S)+\gamma_5\gamma^\alpha\tilde{\Phi}^{(0)}_\alpha(x;p,S)+i\gamma_5{}^\alpha\sigma^{\alpha\beta}\Phi^{(0)}_{\alpha\beta}(x;p,S)\right]$$

(scalar) (pseudo-scalar) (vector) (axial vector) (tensor)

- **Make Lorentz decompositions**

$$p=p^+\bar{n}+\frac{M^2}{2p^+}n,\quad S=\lambda\frac{p^+}{M}\bar{n}+S_T-\lambda\frac{M^2}{2p^+}n$$

$$\Phi^{(0)}(x;p,S)=Me(x)$$

$$\tilde{\Phi}^{(0)}(x;p,S)=\lambda Me_L(x)$$

$$\Phi^{(0)}_\alpha(x;p,S)=p^+\bar{n}_\alpha f_1(x)+M\varepsilon_{\perp\alpha\rho}S_T^\rho f_T(x)+\frac{M^2}{p^+}n_\alpha f_3(x)$$

$$\tilde{\Phi}^{(0)}_\alpha(x;p,S)=\lambda p^+\bar{n}_\alpha g_{1L}(x)+MS_{T\alpha}g_T(x)+\lambda\frac{M^2}{p^+}n_\alpha g_{3L}(x)$$

$$\Phi^{(0)}_{\rho\alpha}(x;p,S)=p^+\bar{n}_{[\rho}S_{T\alpha]}h_{1T}(x)-M\varepsilon_{T\rho\alpha}h_T(x)+\lambda M\bar{n}_{[\rho}n_{\alpha]}h_L(x)+\frac{M^2}{p^+}n_{[\rho}S_{T\alpha]}h_{3T}(x)$$

3+6+3

blue: twist-2

black: twist-3, M/Q suppressed

brown: twist-4, $(M/Q)^2$ suppressed

$$A_{[\alpha}B_{\beta]}\equiv A_\alpha B_\beta-A_\beta B_\alpha$$

$$\varepsilon_{\perp\alpha\beta}\equiv\varepsilon_{\rho\sigma\alpha\beta}\bar{n}^\rho n^\sigma$$

the scalar functions are the one-dimensional PDFs, e.g.,

$$f_1(x)=\frac{1}{p^+}n^\alpha\Phi^{(0)}_\alpha(x;p,S)=\int\frac{dz^-}{2\pi}e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z^-)\frac{\gamma^+}{2}\psi(z^-)|p,S\rangle$$

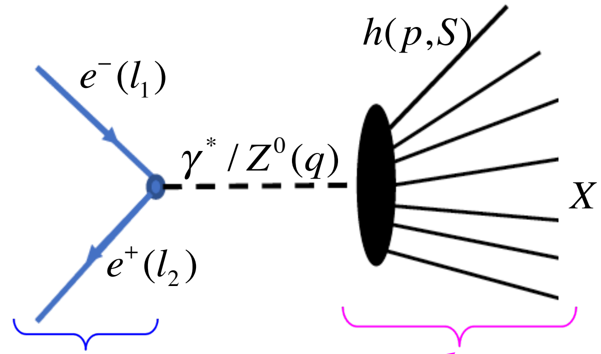
Inclusive hadron production in e⁺e⁻-annihilation $e^- + e^+ \to h + X$

The differential cross section ⟹ **definition of the fragmentation function**

$$d\sigma = \chi \frac{\alpha_{em}^2}{sQ^4} L^{\mu\nu}(l_1,\lambda_1,l_2,\lambda_2) W_{\mu\nu}(q,p,S) \frac{d^3 p}{2E}$$

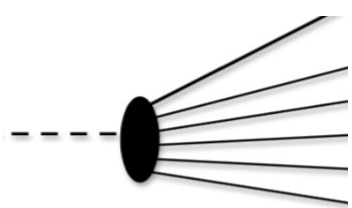
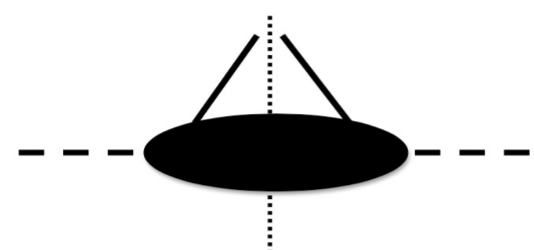
leptonic tensor

hadronic tensor



$-q^2 = Q^2$ $\qquad z_B = \frac{2q\cdot p}{Q^2}$

The hadronic tensor: $W_{\mu\nu}(q,p,S) = \sum_X \langle p,S;X | J_\mu(0) | 0 \rangle \langle 0 | J_\nu(0) | p,S;X \rangle (2\pi)^4 \delta^4(q-p-p_X)$

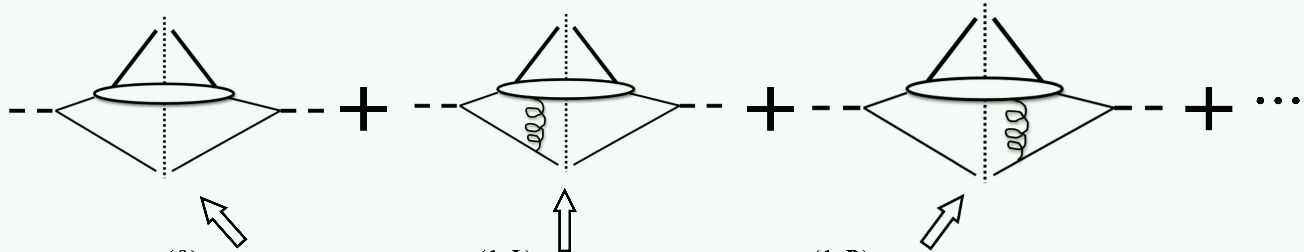
$W_{\mu\nu}(q,p,S) = \Big|$  $\Big|^2$ = 

$|\mathcal{M}|^2$ $\qquad \mathcal{M}\times\mathcal{M}^*$

Inclusive e^+e^--annihilation with "multiple gluon scattering"

To get the gauge invariance, we need to take the "multiple gluon scattering" into account

$W_{\mu\nu}(q,p,S) =$



$$W_{\mu\nu}(q,p,S) = W^{(0)}_{\mu\nu}(q,p,S) + W^{(1,L)}_{\mu\nu}(q,p,S) + W^{(1,R)}_{\mu\nu}(q,p,S) + \ldots$$

$$W^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}^{(0)}_{\mu\nu}(k,q)\Pi^{(0)}(k,p,S)\right]$$

$$W^{(1,L)}_{\mu\nu}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}^{(1,L)\rho}_{\mu\nu}(k_1,k_2,q)\Pi^{(1,L)}_{\rho}(k_1,k_2,p,S)\right]$$

the quark-quark correlator: $\hat{\Pi}^{(0)}(k;p,S) = \sum_X \int d^4z e^{-ikz}\langle 0|\psi(z)|hX\rangle\langle hX|\bar{\psi}(0)|0\rangle$

c.f.: $\hat{\phi}(k,p,S) = \sum_X \int d^4z\, e^{ikz}\langle p,S|\,\bar{\psi}(0)|X\rangle\langle X|\,\psi(z)|p,S\rangle$

the quark-gluon-quark correlator:

$$\hat{\Pi}^{(1,L)}_{\rho}(k_1,k_2;p,S) = \sum_X g\int d^4\xi d^4\eta e^{-ik_1\xi}e^{-i(k_2-k_1)\eta}\langle 0|A_\rho(\eta)\psi(0)|hX\rangle\langle hX|\bar{\psi}(0)|0\rangle$$

no (local) gauge invariance!

Inclusive e^+e^-: LO pQCD, leading & higher twists

Collinear expansion:

S.Y. Wei, Y.K. Song and ZTL, PRD89, 014024 (2014).

✪ Expanding the hard part at $k = p/z$:

$$\hat{H}^{(0)}_{\mu\nu}(z) \equiv \hat{H}^{(0)}_{\mu\nu}(k = p/z, q)$$

$$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(z)}{\partial k^\rho} \equiv \left.\frac{\partial \hat{H}^{(0)}_{\mu\nu}(k,q)}{\partial k^\rho}\right|_{k=p/z}$$

$$z = p^+ / k^+$$

$$\hat{H}^{(0)}_{\mu\nu}(k,q) = \hat{H}^{(0)}_{\mu\nu}(z) + \frac{\partial \hat{H}^{(0)}_{\mu\nu}(z)}{\partial k^\rho} \omega_\rho{}^{\rho'} k_{\rho'} + \ldots\ldots$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(k_1,k_2,q) = \hat{H}^{(1,L)\rho}_{\mu\nu}(z_1,z_2) + \frac{\partial \hat{H}^{(1)\rho}_{\mu\nu}(z_1,z_2)}{\partial k_1^\sigma} \omega_\sigma{}^{\sigma'} k_{1\sigma'} + \ldots\ldots$$

✪ Decomposition of the gluon field:

$$A_\rho(y) = n\cdot A(y)\frac{p_\rho}{n\cdot p} + \omega_\rho{}^{\rho'} A_{\rho'}(y)$$

✪ Using the Ward identities such as,

$$p_\rho \hat{H}^{(1,L)\rho}_{\mu\nu}(z_1,z_2) = -\frac{z_1 z_2}{z_2 - z_1 - i\varepsilon}\hat{H}^{(0)}_{\mu\nu}(z_1) \qquad p_\rho \hat{H}^{(1,R)\rho}_{\mu\nu}(z_1,z_2) = -\frac{z_1 z_2}{z_2 - z_1 + i\varepsilon}\hat{H}^{(0)}_{\mu\nu}(z_2)$$

to replace the derivatives etc.

✪ Adding all terms with the same hard part together ⟹

Inclusive e⁺e⁻: LO pQCD, leading & higher twists

$$W_{\mu\nu}(q,p,S) = \tilde{W}^{(0)}_{\mu\nu}(q,p,S) + \tilde{W}^{(1,L)}_{\mu\nu}(q,p,S) + \tilde{W}^{(1,R)}_{\mu\nu}(q,p,S) + \ldots$$

twist-2, 3 and 4 contributions

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Xi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(z)\right]$$

$$\hat{\Xi}^{(0)}(k;p,S) = \sum_X \int d^4\xi e^{ik\xi} \langle hX|\bar{\psi}(0)\mathcal{L}(0,\infty)|0\rangle\langle 0|\mathcal{L}^\dagger(\xi,\infty)\psi(\xi)|hX\rangle$$

gauge invariant quark-quark correlator

twist-3, 4 and 5 contributions

$$\tilde{W}^{(1,L)}_{\mu\nu}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Xi}^{(1,L)}_\rho(k_1k_2;p,S)\omega_{\rho'}{}^{\rho}\hat{H}^{(1,L)\rho'}_{\mu\nu}(z_1,z_2)\right]$$

$$\hat{\Xi}^{(1,L)}_\rho(k_1,k_2;p,S) = \sum_X \int d^4\xi d^4\eta e^{-ik_1\xi} e^{-i(k_2-k_1)\eta} \langle 0|\mathcal{L}^\dagger(\eta,\infty)D_\rho(\eta)\mathcal{L}^\dagger(0,\eta)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle$$

$$D_\rho(\eta) = -i\partial_\rho + gA_\rho(\eta)$$

gauge invariant quark-gluon-quark correlator

⇨ **A consistent framework for $e^+e^- \to hX$ including leading & higher twists**

Description of polarization of particles with different spins

Spin 1/2 hadrons:

The spin density matrix is 2x2: $\rho = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix} = \frac{1}{2}(1+\vec{S}\cdot\vec{\sigma})$

Vector polarization: $S^\mu = (0, \vec{S}_T, \lambda)$

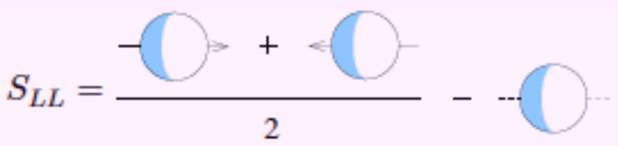
Spin 1 hadrons:

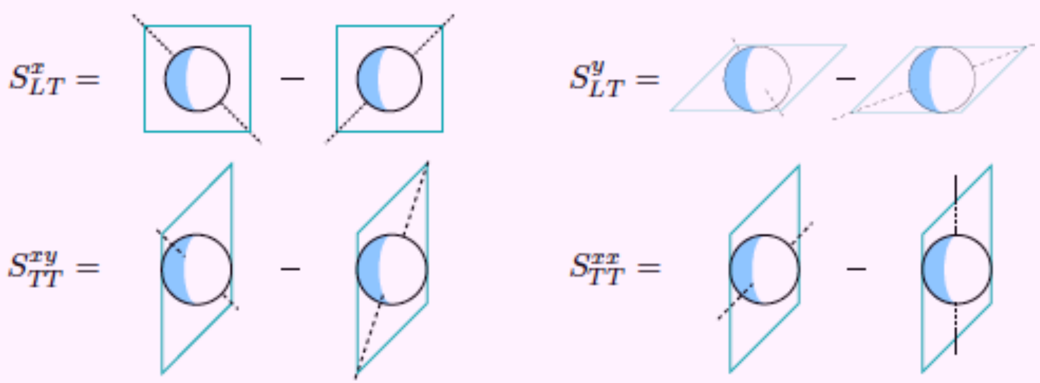
The spin density matrix is 3x3: $\rho = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{01} & \rho_{00} & \rho_{0-1} \\ \rho_{-11} & \rho_{-10} & \rho_{-1-1} \end{pmatrix} = \frac{1}{3}(1+\frac{3}{2}\vec{S}\cdot\vec{\Sigma}+3T^{ij}\Sigma^{ij})$

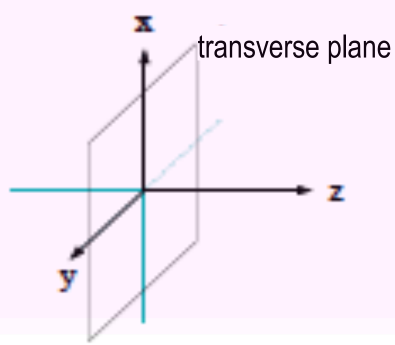
Vector polarization: $S^\mu = (0, \vec{S}_T, \lambda)$ — 3

Tensor polarization: S_{LL}, $S_{LT}^\mu = (0, S_{LT}^x, S_{LT}^y, 0)$, $S_{TT}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & S_{TT}^{xx} & S_{TT}^{xy} & 0 \\ 0 & S_{TT}^{xy} & -S_{TT}^{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ — 5

3 + 5 = 8 independent components.







See e.g. A. Bacchetta, & P.J. Mulders, PRD62, 114004 (2000).

One dimensional FFs defined via quark-quark correlator

- **Expand the quark-quark correlator in terms of the Γ-matrices:**

$$\hat{\Xi}^{(0)}(z;p,S)=\frac{1}{2}\left[\Xi^{(0)}(z;p,S)+i\gamma_5\tilde{\Xi}^{(0)}(z;p,S)+\gamma^{\alpha}\Xi^{(0)}_{\alpha}(z;p,S)+\gamma_5\gamma^{\alpha}\tilde{\Xi}^{(0)}_{\alpha}(z;p,S)+i\gamma_5{}^{\alpha}\sigma^{\alpha\beta}\,\Xi^{(0)}_{\alpha\beta}(z;p,S)\right]$$

(scalar) (pseudo-scalar) (vector) (tensor)

5+10+5

- **Make Lorentz decompositions**

blue: twist-2

black: twist-3, M/Q suppressed

brown: twist-4, $(M/Q)^2$ suppressed

$$z\Xi^{(0)}(z;p,S)=ME(z)+MS_{LL}E_{LL}(z)$$

$$z\tilde{\Xi}^{(0)}(z;p,S)=\lambda ME_L(z)$$

$$z\Xi^{(0)}_{\alpha}(z;p,S)=p^+\bar{n}_\alpha D_1(z)+p^+\bar{n}_\alpha S_{LL}D_{1LL}(z)-M\tilde{S}_{T\alpha}D_T(z)+MS_{LT\alpha}D_{LT}(z)+\frac{M^2}{p^+}n_\alpha D_3(z)+\frac{M^2}{p^+}n_\alpha S_{LL}D_{3LL}(z)$$

$$z\tilde{\Xi}^{(0)}_{\alpha}(z;p,S)=\lambda p^+\bar{n}_\alpha G_{1L}(z)-MS_{T\alpha}G_T(z)-M\tilde{S}_{LT\alpha}G_{LT}(z)+\lambda\frac{M^2}{p^+}n_\alpha G_{3L}(z)$$

$$z\Xi^{(0)}_{\rho\alpha}(z;p,S)=p^+\bar{n}_{[\rho}S_{T\alpha]}H_{1T}(z)-p^+\bar{n}_{[\rho}\tilde{S}_{LT\alpha]}H_{1LT}(z)-M\varepsilon_{T\rho\alpha}H_T(z)+\lambda M\bar{n}_{[\rho}n_{\alpha]}H_L(z)+MS_{LL}\varepsilon_{T\rho\alpha}H_{LL}(z)$$
$$+\frac{M^2}{p^+}n_{[\rho}S_{T\alpha]}H_{3T}(z)-\frac{M^2}{p^+}n_{[\rho}\tilde{S}_{LT\alpha]}H_{3LT}(z)$$

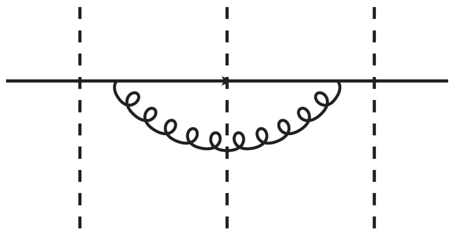
$$A_{[\alpha}B_{\beta]}\equiv A_\alpha B_\beta-A_\beta B_\alpha$$

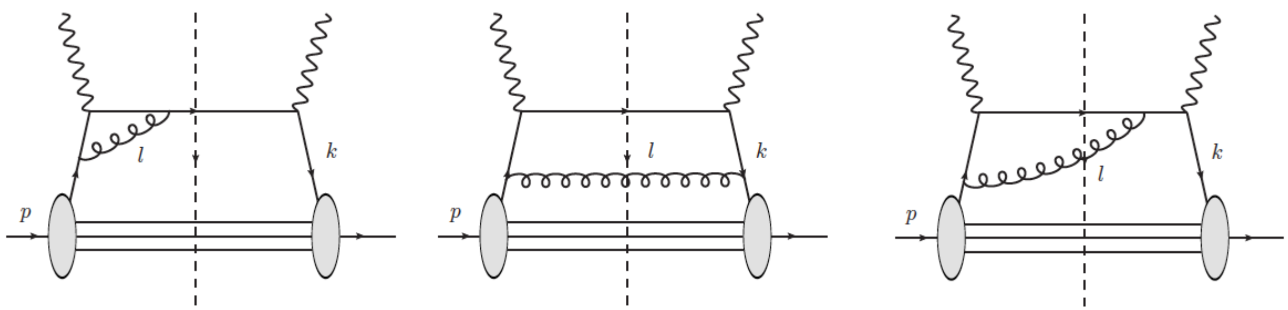
$$\varepsilon_{\perp\alpha\beta}\equiv\varepsilon_{\rho\sigma\alpha\beta}\bar{n}^\rho n^\sigma \qquad \tilde{A}_{T\alpha}\equiv\varepsilon_{\perp\alpha\beta}A_T^\beta$$

Inclusive DIS: Higher order pQCD

Factorization theorem and QCD evolution of PDFs

"Loop diagram contributions"



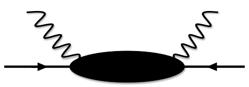


(a) (b) (c)

factorization & resummation →

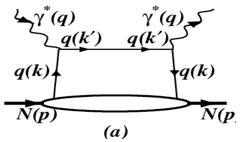
- Higher order pQCD contributions;
- Evolution of PDFs (DGLAP equation)

List of to do's --- the recipe



kinematics
(symmetries, … …)

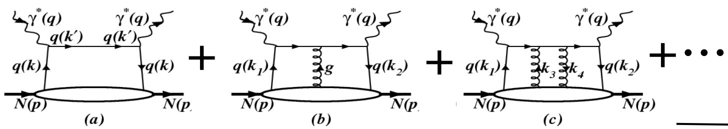
→ general form of
the cross section



parton model without
QCD interaction

→ collinear approximation →

leading order pQCD,
leading twist,
no evolution, no gauge invariance



parton model +
"multiple gluon scattering"

→ collinear expansion →

leading order pQCD,
leading & higher twist,
no evolution, but gauge invariance

parton model +
"multiple gluon scattering" +
"loop diagram contributions"

→ collinear expansion +
factorization & resummation →

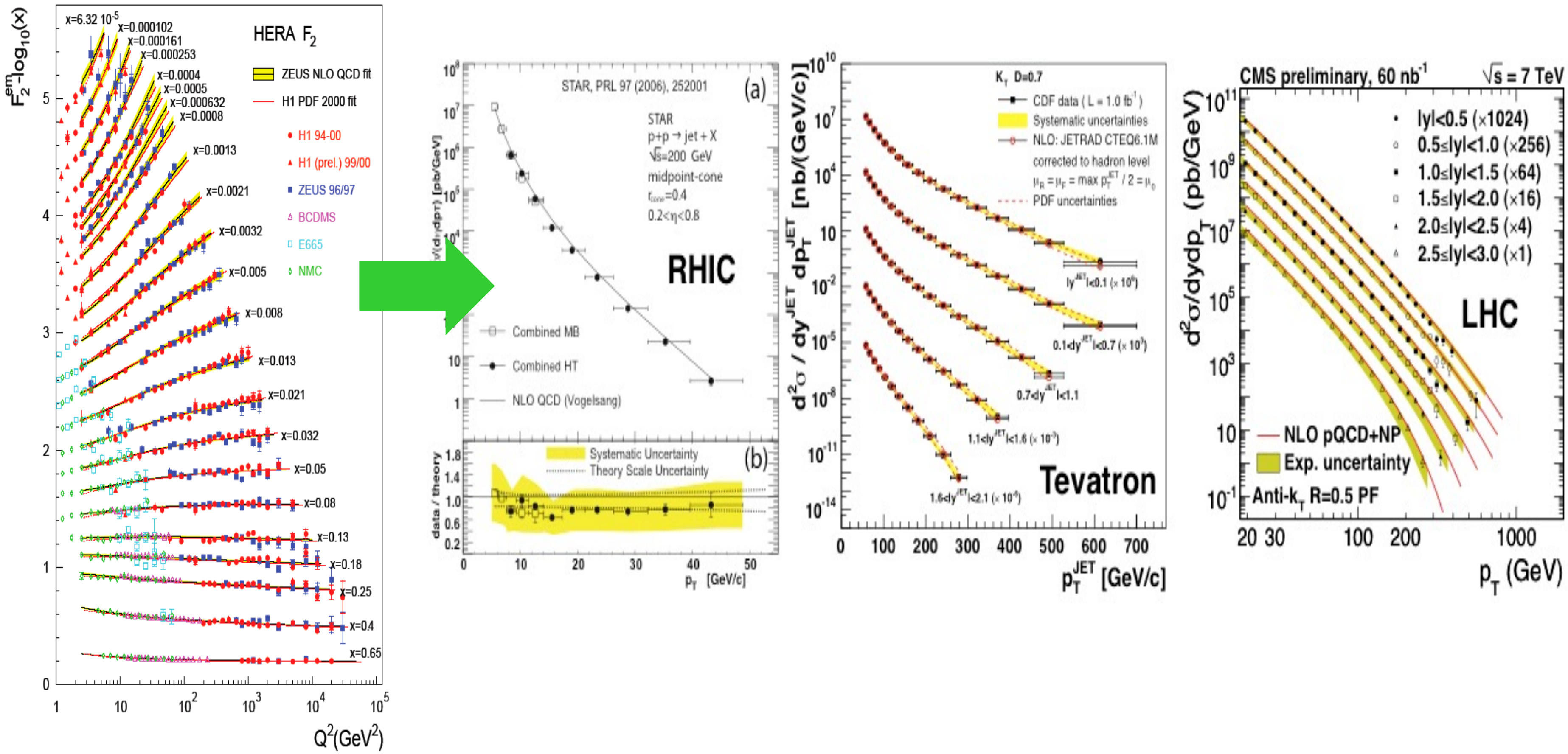
leading & higher order pQCD,
leading & higher twist,
evolution & gauge invariance

experiments
calculations: model, LQCD, etc

→ global QCD fit
整体QCD拟合 →

parameterizations (PDFLib)

Very successful!

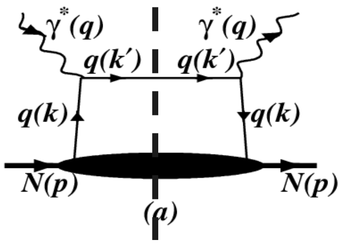


parameterize at 0.3 TeV e-p (HERA), predict p-p and p-p-bar at 0.2, 1.96, and 7 TeV.

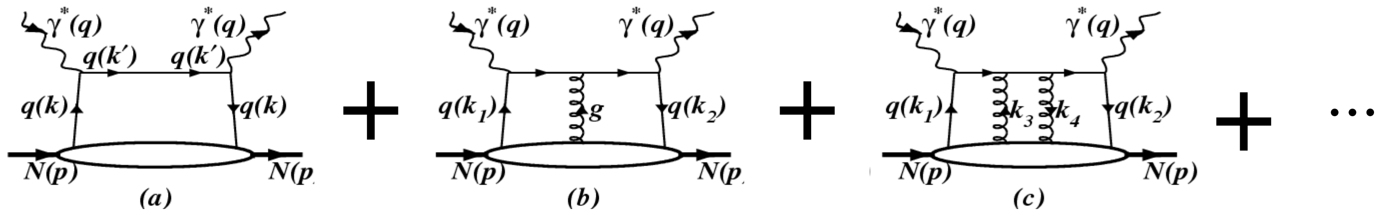
J.W. Qiu, lectures at Weihai High Energy Physics Summer School(WHEPS2015), 2015, Weihai, China.

Inclusive DIS and parton model: brief summary

- (Gauge invariant) PDF is not merely



but



i.e., it always contains "intrinsic motion" and "multiple gluon scattering".

- "Multiple gluon scattering" gives rise to the gauge link.

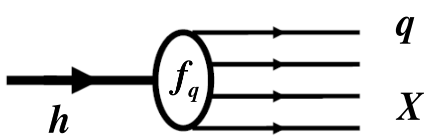
- Collinear expansion is the necessary procedure to obtain the correct formulism in terms of gauge invariant parton distribution functions (PDFs).

- Collinear expansion $\Longleftrightarrow$ power $\left(\frac{M}{Q}\right)^n$ expansion

Fragmentation Function v.s. Parton Distribution Function

Parton distribution functions (PDFs):

$h \to q + X$



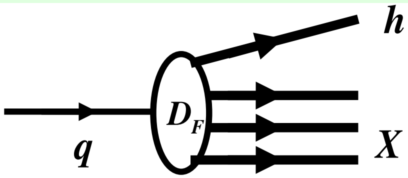
a hadron ⟶ a beam of partons

number density of parton in the beam

$$\hat{\Phi}(k;p,S) = \sum_X \int d^4 z e^{ikz} \times \langle h|\bar{\psi}(0)|X\rangle\langle X|\mathcal{L}(0,z)\psi(z)|h\rangle$$

Fragmentation functions (FFs):

$q \to h + X$



a quark ⟶ a jet of hadrons

number density of hadron in the jet

$$\hat{\Xi}(k_F;p,S) = \sum_X \int d^4 \xi e^{ik_F \xi} \times \langle 0|\mathcal{L}(0,\xi)\psi(\xi)|hX\rangle\langle hX|\bar{\psi}(0)|0\rangle$$

"conjugate" to each other

Deeply inelastic scattering (DIS)

Hadron production in e^+e^--annihilation

⇨ **FFs and PDFS should be studied simultaneously!**

The need for three-dimensional imaging of the nucleon

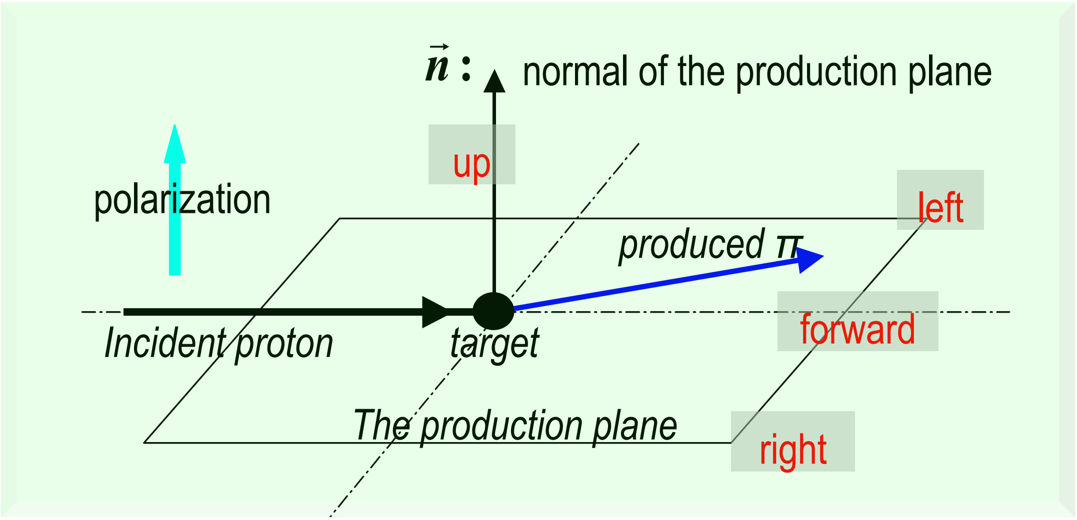
Single-spin left-right asymmetry

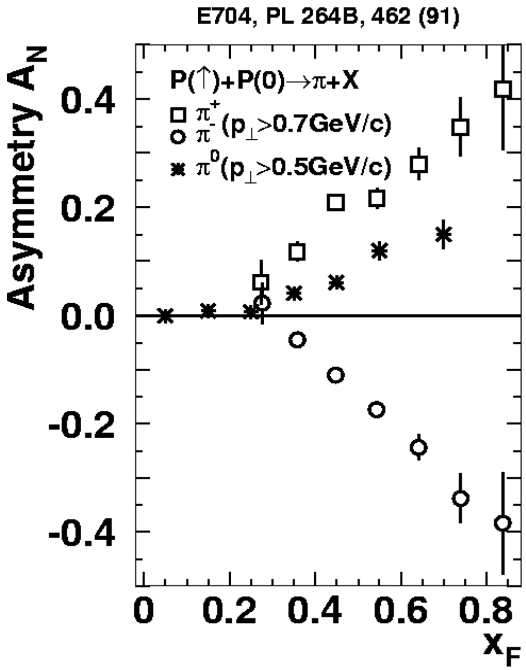
where study of TMD PDFs started.

$$p(\uparrow)+p\rightarrow\pi+X$$

$$A_N=\frac{N_L-N_R}{N_L+N_R}$$

azimuthal asymmetry $A_N=A^{\cos\phi}$





<u>Theory:</u> Kane, Pumplin, Repko (1978), pQCD leads to $a_N[q(\uparrow)q\rightarrow qq]=0$

The need for three-dimensional imaging of the nucleon

Single-spin left-right asymmetry studies

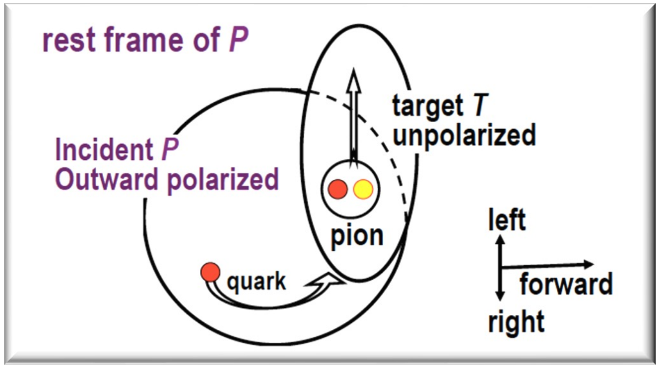
a brief history for Sivers function

1991, Sivers: asymmetric quark distribution in polarized nucleon (Sivers function)

$$f_q(x,k_\perp;S_\perp) = f_q(x,k_\perp) + (\hat{k}_\perp \times \hat{p}) \cdot \vec{S}_\perp \ \Delta^N f(x,k_\perp)$$

1993, Boros, Liang & Meng:

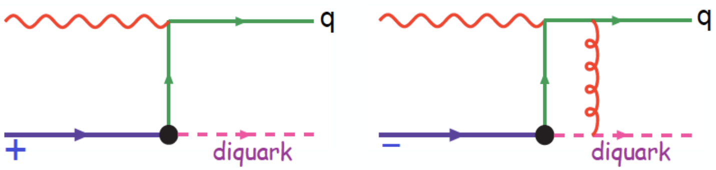
an intuitive picture: quark orbital angular momentum + "surface effect"



1993, Collins: P&T invariance $\Longrightarrow$ $\Delta^N f(x,k_\perp) = 0$ (proof of non-existence of Sivers effect).

2002, Brodsky, Hwang, Schmidt: take "final state interaction" + orbital angular momentum into account,

consider



$\Longrightarrow$ $\Delta^N f(x,k_\perp) \neq 0$

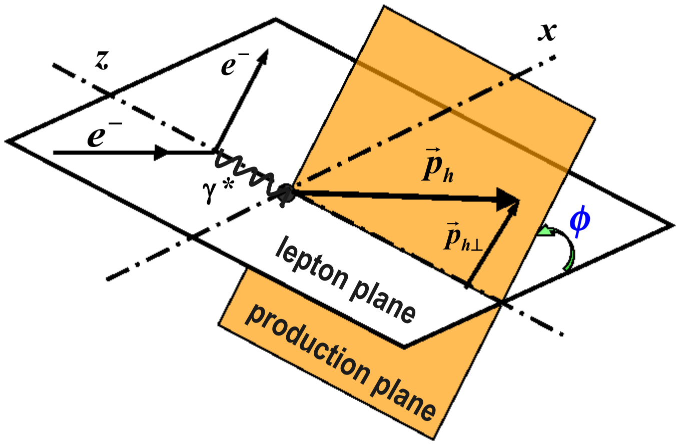
2002, Collins: "final state interaction" = "gauge link".

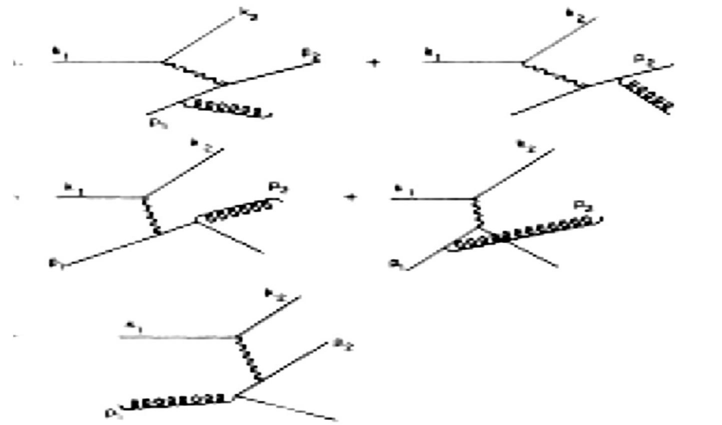
Lesson: do not forget the gauge link!

The need for three-dimensional imaging of the nucleon

Azimuthal asymmetry studies: $e^- + N \to e^- + q(jet) + X$

1977, Georgi & Politzer: gluon radiation ⇨ azimuthal asymmetry ⇨ "Clean test to pQCD"





1978, Cahn: generalize parton model to include an intrinsic $\vec{k}_\perp$:

"Cahn effect"

$$\langle \cos\varphi \rangle = -\frac{|\vec{k}_\perp|}{Q}\frac{2(2-y)\sqrt{1-y}}{1+(1-y)^2}$$ (twist 3)

$$\langle \cos 2\phi \rangle = \frac{|\vec{k}_\perp|^2}{Q^2}\frac{2(1-y)}{1+(1-y)^2}$$ (twist 4)

higher twist, nevertheless significant ! $|\vec{k}_\perp| \sim 0.3-0.7 GeV$ $|\vec{k}_\perp|/Q \sim 0.1$

⇨ **Lesson: do not forget higher twists!**

The need for three-dimensional imaging of the nucleon

A short summary:

Study of single-spin asymmetry ⇒ gauge link

Study of azimuthal asymmetry ⇒ higher twists

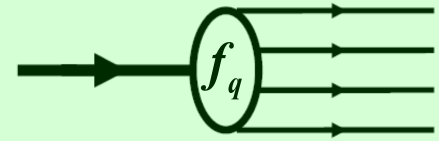
⇒ collinear expansion!

⇒ We need to use the field theoretical formulation rather than the intuitive parton model

Contents

TMD PDFs: Intuitive definitions

Parton model: A fast moving proton $=$ A beam of partons



One-dimensional PDF $f_1(x)$: the number density of partons in the proton.

Including spin ⟹ spin dependent one-dimensional PDFs (totally 3 independent):

$$f_1(x, s_q; S) = f_1(x) + \lambda_q \lambda\, g_{1L}(x) + \vec{s}_{Tq} \cdot \vec{S}_T\, h_{1T}(x)$$

helicity distribution (g_{1L}) transversity (h_{1T})

Including transverse momentum ⟹ three-dimensional (or TMD) PDFs (totally 8):

$$\begin{aligned} f_q(x, k_\perp, S_q; p, S) = {} & f_q(x, k_\perp) + \lambda_q \lambda\, \Delta f_q(x, k_\perp) + (\vec{S}_{\perp q} \cdot \vec{S}_T)\, \delta f_q(x, k_\perp) \\ & + \vec{S}_T \cdot (\hat{p} \times \hat{k}_\perp) \Delta^N f(x, k_\perp) + \frac{1}{M} \vec{S}_{\perp q} \cdot (\hat{p} \times \vec{k}_\perp) h_1^\perp(x, k_\perp) \\ & + \frac{1}{M^2} (\vec{S}_{\perp q} \cdot \vec{k}_\perp)(\vec{S}_T \cdot \vec{k}_\perp) h_{1T}^\perp(x, k_\perp) + \frac{1}{M} (\vec{S}_{\perp q} \cdot \vec{k}_\perp)\, \lambda\, h_{1L}^\perp(x, k_\perp) \\ & + \lambda_q \frac{1}{M} (\vec{S}_T \cdot \vec{k}_\perp)\, g_{1T}^\perp(x, k_\perp) \end{aligned}$$

TMD PDFs defined via quark-quark correlator

The quark-quark correlator $\hat{\Phi}^{(0)}(k;p,S)=\int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$

integrate over k^-: $\hat{\Phi}^{(0)}(x,k_\perp;p,S)=\int dz^- d^2z_\perp e^{i(xp^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$

Expansion in terms of the Γ-matrices

$$\begin{aligned}\hat{\Phi}^{(0)}(x,k_\perp;p,S)=\frac{1}{2}\Big[\ &\Phi^{(0)}(x,k_\perp;p,S) && \text{—— scalar}\\ &+i\gamma_5\,\tilde{\Phi}^{(0)}(x,k_\perp;p,S) && \text{—— pseudo-scalar}\\ &+\lambda^\alpha\,\Phi^{(0)}_\alpha(x,k_\perp;p,S) && \text{—— vector}\\ &+\gamma_5\lambda^\alpha\,\tilde{\Phi}^{(0)}_\alpha(x,k_\perp;p,S) && \text{—— axial vector}\\ &+i\gamma_5\sigma^{\alpha\beta}\,\Phi^{(0)}_{\alpha\beta}(x,k_\perp;p,S)\Big] && \text{—— tensor}\end{aligned}$$

e.g.: $\Phi^{(0)}_\alpha(x,k_\perp;p,S)=\frac{1}{2}\mathrm{Tr}\left[\gamma_\alpha\hat{\Phi}^{(0)}(x,k_\perp;p,S)\right]$

$$=\int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\frac{\gamma_\alpha}{2}\psi(z)|p,S\rangle$$

TMD PDFs defined via quark-quark correlator

The Lorentz decomposition

totally 8(twist 2)+16(twist 3)+8(twist 4) components

$$\Phi_S^{(0)}(x,k_\perp;p,S) = M\left[e(x,k_\perp) + \frac{\varepsilon_{\perp\rho\sigma}k_\perp^\rho S_T^\sigma}{M} e_T^\perp(x,k_\perp)\right]$$ ← twist-3

$$\Phi_\alpha^{(0)}(x,k_\perp;p,S) = p^+\bar{n}_\alpha\left[f_1(x,k_\perp) + \frac{\varepsilon_{\perp\rho\sigma}k_\perp^\rho S_T^\sigma}{M} f_{1T}^\perp(x,k_\perp)\right]$$ ← twist-2

$$+\ k_{\perp\alpha} f^\perp(x,k_\perp) + M\varepsilon_{\perp\alpha\sigma}S_T^\sigma f_T(x,k_\perp) + \varepsilon_{\perp\alpha\rho}k_\perp^\rho\left[\lambda f_L^\perp(x,k_\perp) + \frac{k_\perp\cdot S_T}{M} f_T^\perp(x,k_\perp)\right]$$ ← twist-3

$$+\ \frac{M^2}{p^+}n_\alpha\left[f_3(x,k_\perp) + \frac{\varepsilon_{\perp\rho\sigma}k_\perp^\rho S_T^\sigma}{M} f_{3T}^\perp(x,k_\perp)\right]$$ ← twist-4

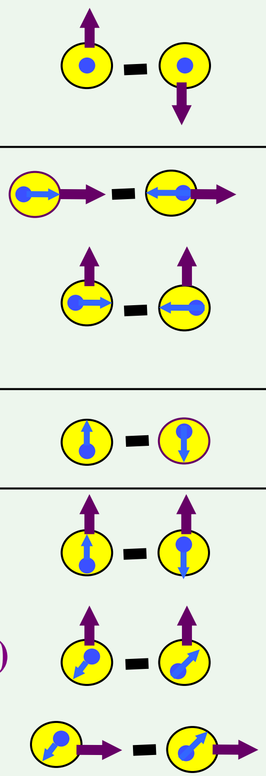
$$p = p^+\bar{n} + \frac{M^2}{2p^+}n, \qquad S = \lambda\frac{p^+}{M}\bar{n} + S_T - \lambda\frac{M^2}{2p^+}n$$

See e.g., K. Goeke, A. Metz, M. Schlegel, PLB 618, 90 (2005);
P. J. Mulders, lectures in 17th Taiwan nuclear physics summer school, August, 2014.

Twist-2 TMD PDFs defined via quark-quark correlator

Leading twist (twist 2) f, g, h: quark un-, longitudinally, transversely polarized

polarization quark	nucleon	pictorially	TMD PDFs (8)	if no gauge link	integrated over $k_\perp$	name
U	U		$f_1(x,k_\perp)$		$q(x)$	number density
	T		$f_{1T}^\perp(x,k_\perp)$	0	×	Sivers function
L	L		$g_{1L}(x,k_\perp)$		$\Delta q(x)$	helicity distribution
	T		$g_{1T}^\perp(x,k_\perp)$		×	worm gear/trans-helicity
T	U		$h_1^\perp(x,k_\perp)$	0	×	Boer-Mulders function
	$T(//)$		$h_{1T}(x,k_\perp)$		$\delta q(x)$	transversity distribution
	$T(\perp)$		$h_{1T}^\perp(x,k_\perp)$			pretzelocity
	L		$h_{1L}^\perp(x,k_\perp)$		×	worm gear/ longi-transversity



Twist-3 TMD PDFs defined via quark-quark correlator

Next to the leading twist (twist-3)

they are NOT probability distributions but contribute in different polarization.

polarization: quark	nucleon	pictorially	TMD PDFs (16)	if no gauge link	integrated over $k_\perp$	name
U	U		$e(x,k_\perp)$, $f^\perp(x,k_\perp)$	0	$e(x)$, ×	number density
	L		$f_L^\perp(x,k_\perp)$	0	×	
	T		$e_T^\perp(x,k_\perp)$,	0	×	Sivers function
			$f_T(x,k_\perp)$, $f_T^\perp(x,k_\perp)$	0 0	$f_T(x)$	
L	U		$g^\perp(x,k_\perp)$	0	×	
	L		$e_L(x,k_\perp)$, $g_L^\perp(x,k_\perp)$	0 $\frac{g_{1L}(x,k_\perp)}{x}$	$e_L(x)$, ×	helicity distribution
	T		$e'^\perp_T(x,k_\perp)$,	0	×	
			$g_T(x,k_\perp)$, $g_T^\perp(x,k_\perp)$	0 $\frac{g_{1T}(x,k_\perp)}{x}$	$g'_T(x)$	worm gear/trans-helicity
T	U		$h(x,k_\perp)$	0	$h(x)$	Boer-Mulders function
	T(//)		$h_T^\perp(x,k_\perp)$	$\frac{h_{1T}^\perp(x,k_\perp)}{x}$	×	transversity distribution
	T(⊥)		$h_T^{\perp\prime}(x,k_\perp)$	$\frac{k_\perp^2 h_{1T}^\perp(x,k_\perp)}{M^2 x}$	×	pretzelocity
	L		$h_L(x,k_\perp)$	$\frac{k_\perp^2 h_{1L}^\perp(x,k_\perp)}{M^2 x}$	$h_L(x)$	worm gear/ longi-transversity

TMD PDFs defined via quark-quark correlator

quark polarization →

<u>Twist-2 TMD PDFs</u>

nucleon polarization ↓ / quark polarization →	U	L	T
U	$f_1(x,k_\perp)$ number density		$h_1^\perp(x,k_\perp)$ Boer-Mulders function
L		$g_{1L}(x,k_\perp)$ helicity distribution	$h_{1L}^\perp(x,k_\perp)$ Worm-gear/longi-transversity
T	$f_{1T}^\perp(x,k_\perp)$ Sivers function	$g_{1T}^\perp(x,k_\perp)$ Worm-gear/trans-helicity	$h_{1T}(x,k_\perp)$ transversity distribution; $h_{1T}^\perp(x,k_\perp)$ pretzelosity

<u>Twist-3 TMD PDFs</u>

nucleon polarization ↓ / quark polarization →	U	L	T
U	$e(x,k_\perp), f^\perp(x,k_\perp)$ number density	$g^\perp(x,k_\perp)$	$h(x,k_\perp)$ Boer-Mulders function
L	$f_L^\perp(x,k_\perp)$	$e_L(x,k_\perp), g_L^\perp(x,k_\perp)$ helicity distribution	$h_L(x,k_\perp)$ Worm gear/ longi-transversity
T	$e_T^\perp(x,k_\perp)$, $f_T^{\perp 1}(x,k_\perp), f_T^{\perp 2}(x,k_\perp)$ Sivers function	$e_T(x,k_\perp)$, $g_T(x,k_\perp), g_T^\perp(x,k_\perp)$ Worm gear/ trans-helicity	$h_T^\perp(x,k_\perp)$ transversity distribution; $h_T(x,k_\perp)$ pretzelosity

Twist-2 TMD FFs defined via quark-quark correlator

Leading twist (twist 2)

$D, G, H:$ quark un-, longitudinally, transversely polarized

quark	polarization: hadron	pictorially	TMD FFs (8)	integrated over $k_{F\perp}$	name
U	U		$D_1(z,k_{F\perp})$	$D_1(z)$	number density
	T		$D_{1T}^{\perp}(z,k_{F\perp})$	×	Sivers-type function
L	L		$G_{1L}(z,k_{F\perp})$	$G_{1L}(z)$	spin transfer (longitudinal)
	T		$G_{1T}^{\perp}(x,k_{\perp})$	×	
T	U		$H_1^{\perp}(z,k_{F\perp})$	×	Collins function
	T(//)		$H_{1T}(z,k_{F\perp})$	$H_{1T}(z)$	spin transfer (transverse)
	$T(\perp)$		$H_{1T}^{\perp}(z,k_{F\perp})$		
	L		$H_{1L}^{\perp}(z,k_{F\perp})$	×	

Twist-2 TMD FFs defined via quark-quark correlator (spin-1)

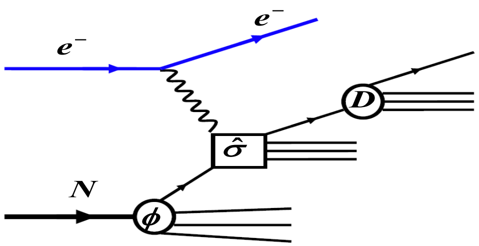
Quark pol	Hadron pol	TMD FFs (2+6+10=18)	integrated over $k_{F\perp}$	name
U	U	$D_1(z,k_{F\perp})$	$D_1(z)$	number density
	T	$D_{1T}^{\perp}(z,k_{F\perp})$	×	Sivers-type function
	LL	$D_{1LL}(z,k_{F\perp})$	$D_{1LL}(z)$	spin alignment
	LT	$D_{1LT}^{\perp}(z,k_{F\perp})$	×	
	TT	$D_{1TT}^{\perp}(z,k_{F\perp})$	×	
L	L	$G_{1L}(z,k_{F\perp})$	$G_{1L}(z)$	spin transfer (longitudinal)
	T	$G_{1T}^{\perp}(z,k_{F\perp})$	×	
	LT	$G_{1LT}^{\perp}(z,k_{F\perp})$	×	
	TT	$G_{1TT}^{\perp}(z,k_{F\perp})$	×	
T	U	$H_1^{\perp}(z,k_{F\perp})$	×	Collins function
	$T(//)$	$H_{1T}(z,k_{F\perp})$	$H_{1T}(z)$	spin transfer (transverse)
	$T(\perp)$	$H_{1T}^{\perp}(z,k_{F\perp})$		
	L	$H_{1L}^{\perp}(z,k_{F\perp})$	×	
	LL	$H_{1LL}^{\perp}(z,k_{F\perp})$	×	
	LT	$H_{1LT}(z,k_{F\perp})$, $H_{1LT}^{\perp}(z,k_{F\perp})$	$H_{1LT}(z)$	
	TT	$H_{1TT}^{\perp}(z,k_{F\perp})$, $H'^{\perp}_{1TT}(z,k_{F\perp})$	×, ×	

See e.g., K.B. Chen, S.Y. Wei, W.H. Yang, & ZTL, PRD94, 034003 (2016).

Contents

Access TMDs via semi-inclusive high energy reactions

Semi-inclusive reactions



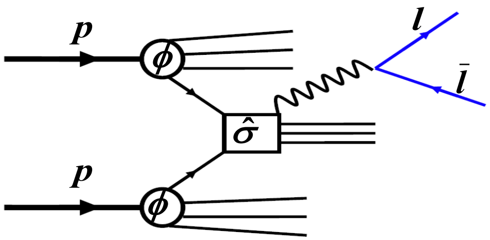
DIS: $e+N \to e+h+X$

DESY COMPASS Jefferson Lab EIC

TMD PDFs:

$f_1, f_{1T}^{\perp}, g_{1L}, g_{1T}, h_1, h_1^{\perp}, h_{1L}^{\perp}, h_{1T}^{\perp} \ldots$

TMD FFs: $D_1, H_1^{\perp}, \ldots$

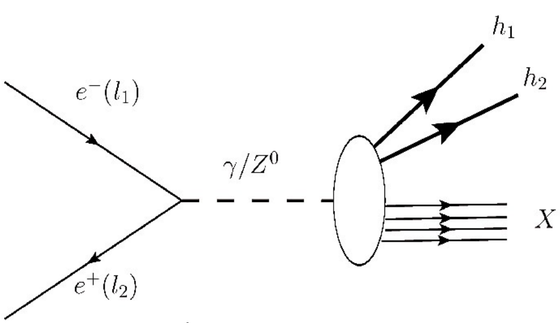


Drell-Yan: $p+p \to l+\bar{l}+X$

FNAL BNL JPARC GSI

TMD PDFs:

$f_1, f_{1T}^{\perp}, g_{1L}, g_{1T}, h_1, h_1^{\perp}, h_{1L}^{\perp}, h_{1T}^{\perp} \ldots$

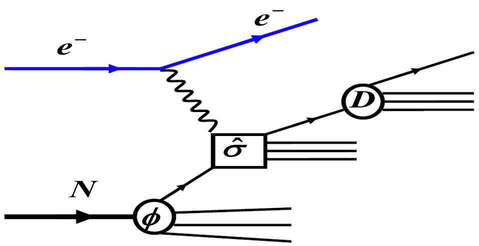


$e^- + e^+ \to h_1 + h_2 + X$

BESIII BELLE CERN BABAR

TMD FFs: $D_1, H_1^{\perp}, \ldots$

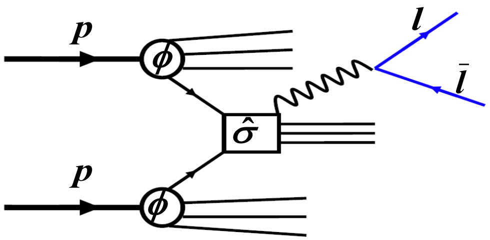
Semi-inclusive reactions: general form of the hadronic tensors and cross sections



DIS: $e + N \to e + h + X$

Gourdin, NPB 49, 501 (1972);
Kotzinian, NPB 441, 234 (1995);
Diehl, Sapeta, EPJ C41, 515 (2005);
Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP 02, 093 (2007);
……

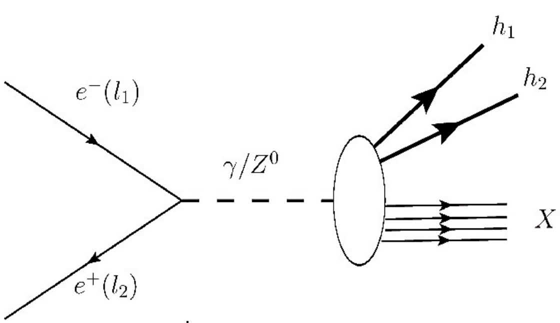
→ 18 independent "structure functions" for spinless hadron *h*



Drell-Yan: $p + p \to l + \bar{l} + X$

Arnold, Metz, Schlegel, Phys. Rev. D79 ,034005 (2009).

→ 48 independent "structure functions"



$e^- + e^+ \to h_1 + h_2 + X$

Pitonyak, Schlegel, Metz, PRD89, 054032 (2014).

→ 36 independent "structure functions" for spin-1/2 hadrons

K.B. Chen, W.H. Yang, S.Y. Wei, & ZTL, PRD94, 034003 (2016).

→ 81 independent "structure functions" for spin-1 hadrons

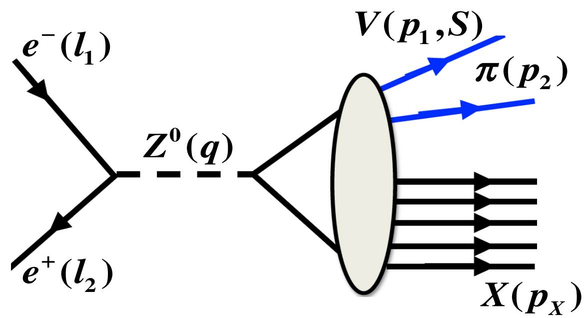
Kinematic analysis for $e^+e^- \to Z \to V\pi X$

$e^-e^+ \to Z \to V(p_1,S)\pi(p_2)X$: the best place to study tensor polarization dependent FFs

The differential cross section:

$$\frac{2E_1E_2}{d^3p_1d^3p_2} = \frac{\alpha^2}{sQ^4}\chi L_{\mu\nu}(l_1,l_2)W^{\mu\nu}(q,p_1,S,p_2)$$

$$L_{\mu\nu}(l_1,l_2) = c_1^e\left[l_{1\mu}l_{2\nu} + l_{1\nu}l_{2\mu} - (l_1\cdot l_2)g_{\mu\nu}\right] + ic_3^e\varepsilon_{\mu\nu\rho\sigma}l_1^\rho l_2^\sigma$$



The hadronic tensor:

$W_{\mu\nu}(q,p_1,S,p_2) = W^{S\mu\nu}$ (the Symmetric part) $+iW^{A\mu\nu}$ (the Anti-symmetric part)

$$= \sum_{\sigma,i} W_{\sigma i}^S\, h_{\sigma i}^{S\mu\nu} + \sum_{\sigma,j} \tilde{W}_{\sigma j}^S\, \tilde{h}_{\sigma j}^{S\mu\nu} + i\sum_{\sigma,i} W_{\sigma i}^A\, h_{\sigma i}^{A\mu\nu} + i\sum_{\sigma,j} \tilde{W}_{\sigma j}^A\, \tilde{h}_{\sigma j}^{A\mu\nu}$$

$\sigma = U, V, S_{LL}, S_{LT}, S_{TT}$
polarization

the basic Lorentz tensors: $h_{\sigma i}^{S\mu\nu} = h_{\sigma i}^{S\nu\mu}$, $h_{\sigma i}^{A\mu\nu} = -h_{\sigma i}^{A\nu\mu}$ space reflection P-even: $\hat{\mathcal{P}}h^{\mu\nu} = h_{\mu\nu}$

$\tilde{h}_{\sigma i}^{S\mu\nu} = \tilde{h}_{\sigma i}^{S\nu\mu}$, $\tilde{h}_{\sigma i}^{A\mu\nu} = -\tilde{h}_{\sigma i}^{A\nu\mu}$ space reflection P-odd: $\hat{\mathcal{P}}\tilde{h}^{\mu\nu} = -\tilde{h}_{\mu\nu}$

Constraints: $W^{\mu\nu*} = W^{\nu\mu}$ (hermiticity), $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$ (current conservation)

K.B. Chen, W.H. Yang, S.Y. Wei, & ZTL, PRD94, 034003 (2016) (spin-1).

Kinematic analysis for $e^+e^- \to Z \to V\pi X$

The basic Lorentz tensor sets for the hadronic tensor

unpolarized: 5+4=9

$$a^{[\alpha}b^{\beta]} \equiv a^\alpha b^\beta - a^\beta b^\alpha$$
$$a^{\{\alpha}b^{\beta\}} \equiv a^\alpha b^\beta + a^\beta b^\alpha$$
$$\varepsilon^{\mu\nu\alpha p} \equiv \varepsilon^{\mu\nu\alpha\beta} p_\beta, \quad \varepsilon_\perp^{\mu\nu} \equiv \varepsilon^{\mu\nu\alpha\beta} \bar{n}_\alpha n_\beta$$
$$p_q \equiv p - \frac{p\cdot q}{q^2} q \quad (p_q \cdot q = 0)$$

$$h_{Ui}^{S\mu\nu} = \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2},\ p_{1q}^\mu p_{1q}^\nu,\ p_{2q}^\mu p_{2q}^\nu,\ p_{1q}^{\{\mu} p_{2q}^{\nu\}} \right\}$$
symmetric (S), P-even

$$\tilde{h}_{Ui}^{S\mu\nu} = \left\{ \varepsilon^{\{\mu q p_1 p_2}(p_{1q}^{\nu\}},\ p_{2q}^{\nu\}}) \right\}$$
symmetric (S), P-odd

$$h_U^{A\mu\nu} = p_{1q}^{[\mu} p_{2q}^{\nu]}$$
anti-symmetric (A), P-even

$$\tilde{h}_{Ui}^{A\mu\nu} = \left\{ \varepsilon^{\mu\nu q p_1},\ \varepsilon^{\mu\nu q p_2} \right\}$$
anti-symmetric (A), P-odd

A regularity: (spin dependent Lorentz tensor set) = (spin dependent Lorentz (pseudo)scalar) × (the unpolarized set)

Unpolarized 5+4=9

Vector polarization S-dependent: 13+14=27

longitudinal polarization $\lambda \sim \vec{p}_1 \cdot \vec{S}$

transverse polarization

$$\begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix} \qquad \begin{pmatrix} h_{Li}^{S\mu\nu} \\ \tilde{h}_{Li}^{S\mu\nu} \\ h_{Li}^{A\mu\nu} \\ \tilde{h}_{Li}^{A\mu\nu} \end{pmatrix} = \lambda \begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_{U}^{A\mu\nu} \end{pmatrix} \qquad \begin{pmatrix} h_{Ti}^{S\mu\nu} \\ \tilde{h}_{Ti}^{S\mu\nu} \\ h_{Ti}^{A\mu\nu} \\ \tilde{h}_{Ti}^{A\mu\nu} \end{pmatrix} = \left\{ (p_2\cdot S) \begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_{U}^{A\mu\nu} \end{pmatrix},\ \varepsilon^{Sqp_1p_2} \begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_{U}^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix} \right\}$$

Kinematic analysis for $e^+e^- \to Z \to V\pi X$

The basic Lorentz tensor sets for the hadronic tensor (continued)

$$S_{TT}^{p\beta} \equiv p_\alpha S_{TT}^{\alpha\beta}$$

$$\varepsilon^{abcd} \equiv \varepsilon^{\alpha\beta\gamma\delta} a_\alpha b_\beta c_\gamma d_\delta$$

S_{LL}-dependent part: 5+4=9

$$S_{LL}^{\mathcal{P}} = S_{LL}$$

$$\begin{pmatrix} h_{LLi}^{S\mu\nu} \\ \tilde{h}_{LLi}^{S\mu\nu} \\ h_{LLi}^{A\mu\nu} \\ \tilde{h}_{LLi}^{A\mu\nu} \end{pmatrix} = S_{LL} \begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_{U}^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix}$$

S_{LT}-dependent part: 9+9=18

$$S_{LT} = (0, S_{LT}^x, S_{LT}^y, 0)$$

$$p_1 \cdot S_{LT} = 0, \quad q \cdot S_{LT} = 0 \qquad S_{LT\mu}^{\mathcal{P}} = S_{LT}^{\mu}$$

$$\begin{pmatrix} h_{LTi}^{S\mu\nu} \\ \tilde{h}_{LTi}^{S\mu\nu} \\ h_{LTi}^{A\mu\nu} \\ \tilde{h}_{LTi}^{A\mu\nu} \end{pmatrix} = \left\{ (p_2 \cdot S_{LT}) \begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_{U}^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix}, \quad \varepsilon^{S_{LT} q p_1 p_2} \begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_{U}^{A\mu\nu} \end{pmatrix} \right\}$$

S_{TT}-dependent part: 9+9=18

$$S_{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & S_{TT}^{xx} & S_{TT}^{xy} & 0 \\ 0 & S_{TT}^{xy} & -S_{TT}^{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$S_{TT\mu\nu}^{\mathcal{P}} = S_{TT}^{\mu\nu}$$

$$S_{TT}^{p_1\beta} = S_{TT}^{\alpha p_1} = 0$$

$$S_{TT}^{q\beta} = S_{TT}^{\alpha q} = 0$$

$$\begin{pmatrix} h_{TTi}^{S\mu\nu} \\ \tilde{h}_{TTi}^{S\mu\nu} \\ h_{TTi}^{A\mu\nu} \\ \tilde{h}_{TTi}^{A\mu\nu} \end{pmatrix} = \left\{ S_{TT}^{p_2 p_2} \begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_{U}^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix}, \quad \varepsilon^{S_{TT}^{p_2} q p_1 p_2} \begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_{U}^{A\mu\nu} \end{pmatrix} \right\}$$

K.B. Chen, W.H. Yang, S.Y. Wei, & ZTL, PRD94, 034003 (2016).

Kinematic analysis for $e^+e^- \to Z \to V\pi X$

The cross section in Helicity-GJ-frame: unpolarized and longitudinally polarized parts

The structure functions: $F_{jxx}^{yy} = F_{jxx}^{yy}(s,\xi_1,\xi_2,p_{2T})$, $\tilde{F}_{jxx}^{yy} = \tilde{F}_{jxx}^{yy}(s,\xi_1,\xi_2,p_{2T})$

$$\frac{2E_1E_2 d\sigma^U}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi(\mathcal{F}_U + \tilde{\mathcal{F}}_U)$$

$$\mathcal{F}_U = (1+\cos^2\theta)F_{1U} + \sin^2\theta F_{2U} + \cos\theta F_{3U} + \cos\varphi[\sin\theta F_{1U}^{\cos\varphi} + \sin 2\theta F_{2U}^{\cos\varphi}] + \cos 2\varphi \sin^2\theta F_U^{\cos 2\varphi}$$

$$\tilde{\mathcal{F}}_U = \sin\varphi[\sin\theta \tilde{F}_{1U}^{\sin\varphi} + \sin 2\theta \tilde{F}_{2U}^{\sin\varphi}] + \sin 2\varphi \sin^2\theta \tilde{F}_U^{\sin 2\varphi}$$

1, $\cos\varphi$, $\cos 2\varphi$ — $\sin\varphi$, $\sin 2\varphi$

$$\frac{2E_1E_2 d\sigma^L}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi\lambda(\mathcal{F}_L + \tilde{\mathcal{F}}_L)$$

$\mathcal{F}_L \leftrightarrow \tilde{\mathcal{F}}_U$, $\tilde{\mathcal{F}}_L \leftrightarrow \mathcal{F}_U$

$F_{jL}^{xxx} \leftrightarrow \tilde{F}_{jU}^{xxx}$, $\tilde{F}_{jL}^{xxx} \leftrightarrow F_{jU}^{xxx}$

$$\mathcal{F}_L = \sin\varphi[\sin\theta F_{1L}^{\sin\varphi} + \sin 2\theta F_{2L}^{\sin\varphi}] + \sin 2\varphi \sin^2\theta F_L^{\sin 2\varphi}$$

$$\tilde{\mathcal{F}}_L = (1+\cos^2\theta)\tilde{F}_{1L} + \sin^2\theta \tilde{F}_{2L} + \cos\theta \tilde{F}_{3L} + \cos\varphi[\sin\theta \tilde{F}_{1L}^{\cos\varphi} + \sin 2\theta \tilde{F}_{2L}^{\cos\varphi}] + \cos 2\varphi \sin^2\theta \tilde{F}_L^{\cos 2\varphi}$$

$$\frac{2E_1E_2 d\sigma^{LL}}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi S_{LL}(\mathcal{F}_{LL} + \tilde{\mathcal{F}}_{LL})$$

$\mathcal{F}_{LL} \leftrightarrow \mathcal{F}_U$, $\tilde{\mathcal{F}}_{LL} \leftrightarrow \tilde{\mathcal{F}}_U$

$F_{jLL}^{xxx} \leftrightarrow F_{jU}^{xxx}$, $\tilde{F}_{jLL}^{xxx} \leftrightarrow \tilde{F}_{jU}^{xxx}$

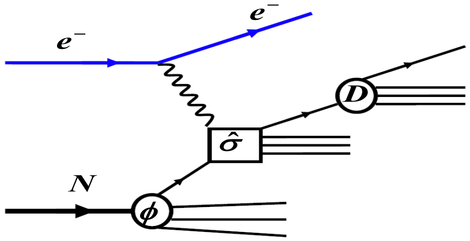
$$\mathcal{F}_{LL} = (1+\cos^2\theta)F_{1LL} + \sin^2\theta F_{2LL} + \cos\theta F_{3LL} + \cos\varphi[\sin\theta F_{1LL}^{\cos\varphi} + \sin 2\theta F_{2LL}^{\cos\varphi}] + \cos 2\varphi \sin^2\theta F_{LL}^{\cos 2\varphi}$$

$$\tilde{\mathcal{F}}_{LL} = \sin\varphi[\sin\theta \tilde{F}_{1LL}^{\sin\varphi} + \sin 2\theta \tilde{F}_{2LL}^{\sin\varphi}] + \sin 2\varphi \sin^2\theta \tilde{F}_{LL}^{\sin 2\varphi}$$

Semi-inclusive DIS $e^-(\lambda_l) + N(\lambda, S_T) \rightarrow e^- + h + X$: Kinematics

The differential cross section:

$$d\sigma = \frac{\alpha^2}{sQ^4} L_{\mu\nu}(l,\lambda_e,l')W^{\mu\nu}(q,p,S,p')\frac{d^3 l'}{2E_l'(2\pi)3}\frac{d^3 p'}{2E_h'(2\pi)3}$$



$$W_{\mu\nu}(q,p,S,p') = \sum_{\sigma,i} W_{\sigma i}^{S} h_{\sigma i}^{S\mu\nu} + \sum_{\sigma,j} \tilde{W}_{\sigma j}^{S} \tilde{h}_{\sigma j}^{S\mu\nu} + i\sum_{\sigma,i} W_{\sigma i}^{A} h_{\sigma i}^{A\mu\nu} + i\sum_{\sigma,j} \tilde{W}_{\sigma j}^{A} \tilde{h}_{\sigma j}^{A\mu\nu}$$

$\sigma = U, V:$ polarization

basic Lorentz tensors

The basic Lorentz sets

unpolarized part: 5+4=9

$$h_{Ui}^{S\mu\nu} = \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, \tilde{p}^\mu \tilde{p}^\nu, \tilde{p}^{\{\mu}\tilde{p}'^{\nu\}}, \tilde{p}'^\mu \tilde{p}'^\nu \right\}$$

$$\tilde{h}_{Ui}^{S\mu\nu} = \left\{ \varepsilon^{\{\mu qpp'}(\tilde{p}^{\nu\}}, \tilde{p}'^{\nu\}}) \right\}$$

$$h_U^{A\mu\nu} = \tilde{p}^{[\mu}\tilde{p}'^{\nu]}$$

$$\tilde{p} = p - \frac{p\cdot q}{q^2} q$$

$$\tilde{h}_{Ui}^{A\mu\nu} = \left\{ \varepsilon^{\mu\nu qp}, \varepsilon^{\mu\nu qp'} \right\}$$

spin dependent part: 13+5=18

$$h_{Vi}^{S\mu\nu} = \left\{ \left[(q\cdot S), (p'\cdot S) \right] \tilde{h}_{Ui}^{S\mu\nu}, \varepsilon^{Sqpp'} h_{Uj}^{S\mu\nu} \right\}$$

$$\tilde{h}_{Vi}^{S\mu\nu} = \left\{ \left[(q\cdot S), (p'\cdot S) \right] h_{Ui}^{S\mu\nu}, \varepsilon^{Sqpp'} \tilde{h}_{Uj}^{S\mu\nu} \right\}$$

$$h_{Vi}^{A\mu\nu} = \left\{ \left[(q\cdot S), (p'\cdot S) \right] \tilde{h}_{Ui}^{A\mu\nu}, \varepsilon^{Sqpp'} h_{U}^{A\mu\nu} \right\}$$

$$\tilde{h}_{Vi}^{A\mu\nu} = \left\{ \left[(q\cdot S), (p'\cdot S) \right] h_{U}^{A\mu\nu}, \varepsilon^{Sqpp'} h_{U}^{A\mu\nu} \right\}$$

Semi-inclusive DIS $e^-(\lambda_l) + N(\lambda, S_T) \to e^- + h + X$: Kinematics

The cross section in the γ*p c.m. frame (only parity conserved part)

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,d^2p_{hT}} = \frac{\alpha^2}{xyQ^2}\mathcal{K}(\mathcal{F}_{UU} + \lambda_l \mathcal{F}_{LU} + \lambda_N \mathcal{F}_{UL} + \lambda_l\lambda_N \mathcal{F}_{LL} + |\vec{S}_T|\mathcal{F}_{UT} + \lambda_l|\vec{S}_T|\mathcal{F}_{LT})$$

$h_{Ui}^{S\mu\nu}$ (4)
$h_{U}^{A\mu\nu}$ (1) } (5)

$h_{Li}^{S\mu\nu}$ (2)
$h_{Li}^{A\mu\nu}$ (2) } (4)

$h_{Ti}^{S\mu\nu}$ (6)
$h_{Ti}^{A\mu\nu}$ (3) } (9)

totally 18

nucleon
electron

$$\mathcal{F}_{UU} = F_{UU,T} + \varepsilon F_{UU,L} + \cos\phi\sqrt{2\varepsilon(1+\varepsilon)}F_{UU}^{\cos\phi} + \cos 2\phi\,\varepsilon F_{UU}^{\cos 2\phi}$$

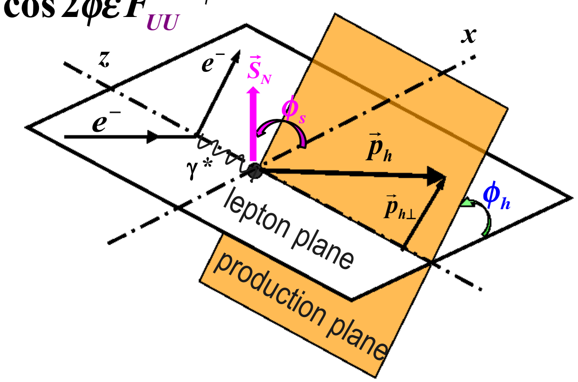
$$\mathcal{F}_{LU} = \sin\phi\sqrt{2\varepsilon(1-\varepsilon)}F_{LU}^{\sin\phi}$$

$$\mathcal{F}_{UL} = \sin\phi\sqrt{2\varepsilon(1+\varepsilon)}F_{UL}^{\sin\phi} + \sin 2\phi\,\varepsilon F_{UL}^{\sin 2\phi}$$

$$\mathcal{F}_{LL} = \sqrt{1-\varepsilon^2}F_{LL} + \cos\phi\sqrt{2\varepsilon(1-\varepsilon)}F_{LL}^{\cos\phi}$$

$$\mathcal{F}_{UT} = \sin\phi_S\sqrt{2\varepsilon(1+\varepsilon)}F_{UT}^{\sin\phi_S} + \sin(\phi-\phi_S)(F_{UT,T}^{\sin(\phi-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi-\phi_S)})$$
$$+ \sin(\phi+\phi_S)\varepsilon F_{UT}^{\sin(\phi+\phi_S)} + \sin(2\phi-\phi_S)\sqrt{2\varepsilon(1+\varepsilon)}F_{UT}^{\sin(2\phi-\phi_S)} + \sin(3\phi-\phi_S)\varepsilon F_{UT}^{\sin(3\phi-\phi_S)}$$

$$\mathcal{F}_{LT} = \cos\phi_S\sqrt{2\varepsilon(1-\varepsilon)}F_{LT}^{\cos\phi_S} + \cos(\phi-\phi_S)\sqrt{1-\varepsilon^2}F_{LT}^{\cos(\phi-\phi_S)}$$
$$+ \cos(2\phi-\phi_S)\sqrt{2\varepsilon(1-\varepsilon)}F_{LT}^{\cos(2\phi-\phi_S)}$$



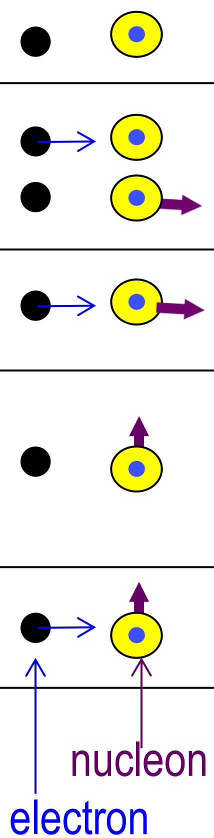
The structure functions: $F_{jxx}^{yy} = F_{jxx}^{yy}(x, \xi, p_{hT}^2, Q)$

$$\varepsilon = (1 - y - \frac{1}{4}\gamma^2 y^2)/(1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2) \qquad \mathcal{K} = (1 + \gamma^2/2x)y^2/(1-\varepsilon), \quad \gamma = 2Mx/Q$$

Semi-inclusive DIS: LO & Leading twist parton model results

for the structure functions (8 non-zero F's)

$$e(\lambda_l)+N(\lambda,S_T)\rightarrow e+h+X$$



$$F_{UU,T}=\mathcal{C}\left[f_1 D_1\right] \qquad F_{UU,L}=0 \qquad F_{UU}^{\cos\phi_h}=0 \qquad F_{UU}^{\cos 2\phi_h}=\mathcal{C}\left[w_1 h_1^{\perp} H_1^{\perp}\right]$$

$$F_{LU}^{\sin\phi_h}=0 \qquad F_{UL}^{\sin\phi_h}=0 \qquad F_{UL}^{\sin 2\phi_h}=\mathcal{C}\left[w_1 h_{1L}^{\perp} H_1^{\perp}\right]$$

$$F_{LL}=\mathcal{C}\left[g_{1L} D_1\right] \qquad F_{LL}^{\cos\phi_h}=0$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)}=-2\mathcal{C}\left[w_2 f_{1T}^{\perp} D_1\right] \qquad F_{UT,L}^{\sin(\phi_h-\phi_S)}=0 \qquad F_{UT}^{\sin(\phi_h+\phi_S)}=-2\mathcal{C}\left[w_3 h_{1T} H_1^{\perp}\right]$$

$$F_{UT}^{\sin\phi_S}=0 \qquad F_{UT}^{\sin(2\phi_h-\phi_S)}=0 \qquad F_{UT}^{\sin(3\phi_h-\phi_S)}=\mathcal{C}\left[w_4 h_{1T}^{\perp} H_1^{\perp}\right]$$

$$F_{LT}^{\cos(\phi_h-\phi_S)}=\mathcal{C}\left[w_2 g_{1T} D_1\right] \qquad F_{LT}^{\cos\phi_S}=0 \qquad F_{LT}^{\cos(2\phi_h-\phi_S)}=0$$

$$\mathcal{C}\left[w_i f\, D\right]\equiv x\sum_q e_q^2\int d^2k_\perp d^2k_{F\perp}\delta^{(2)}(\vec{k}_\perp-\vec{k}_{F\perp}-\vec{p}_{h\perp}/z)\; w_i\, f_q(x,k_\perp) D_q(z,k_{F\perp})$$

$$w_1=-\left[2(\hat{\vec{p}}_{h\perp}\cdot\vec{k}_{F\perp})(\hat{\vec{p}}_{h\perp}\cdot\vec{k}_\perp)-\vec{k}_\perp\cdot\vec{k}_{F\perp}\right]/MM_h, \quad w_2=\hat{\vec{p}}_{h\perp}\cdot\vec{k}_\perp/M, \quad w_3=\hat{\vec{p}}_{h\perp}\cdot\vec{k}_{F\perp}/M_h$$

See e.g., Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP 0702, 093 (2007); ....

Semi-inclusive DIS: LO & Leading twist parton model results

for the cross section

$$e(\lambda_l) + N(\lambda, S_T) \rightarrow e + h + X$$

$$\frac{d\sigma}{dxdydzd^2p_{h\perp}} = \frac{\alpha^2}{xyQ^2} \times$$

$$\Big\{ (1 - y + \frac{1}{2}y^2)\mathcal{C}\left[f_1 D_1\right] + (1-y)\cos 2\phi_h \mathcal{C}\left[w_1 h_1^\perp H_1^\perp\right]$$

Boer-Mulders ⊗ Collins

$$+ \lambda_l \lambda y (1 - \frac{1}{2}y)\mathcal{C}\left[g_{1L} D_1\right]$$

longi-transversity ⊗ Collins

$$+ \lambda (1-y) \sin 2\phi_h \mathcal{C}\left[w_1 h_{1L}^\perp H_1^\perp\right]$$

Sivers ⊗ unpolarized FF

$$+ |\vec{S}_T| \Big((1 - y + \frac{1}{2}y^2)\sin(\phi_h - \phi_S)\mathcal{C}\left[w_2 f_{1T}^\perp D_1\right]$$

$$+ 2(1-y)\sin(\phi_h + \phi_S)\mathcal{C}\left[w_3 h_{1T} H_1^\perp\right] + 2(1-y)\sin(3\phi_h - \phi_S)\mathcal{C}\left[w_4 h_{1T}^\perp H_1^\perp\right] \Big)$$

$$+ \lambda_l |\vec{S}_T| y (1 - \frac{1}{2}y)\cos(\phi_h - \phi_S)\mathcal{C}\left[w_2 g_{1T} D_1\right] \Big\}$$

transversity ⊗ Collins

pretzelosity ⊗ Collins

trans-helicity ⊗ unpolarized FF

nucleon

electron

Semi-inclusive DIS: LO & Leading twist parton model results

for the azimuthal asymmetries (6 leading twist asymmetries)

$$A_{UU}^{\cos 2\phi_h} = \langle \cos 2\phi_h \rangle_{UU} = \frac{(1-y)}{A(y)} \frac{\mathcal{C}\left[w_1 h_1^{\perp} H_1^{\perp}\right]}{\mathcal{C}\left[f_1 D_1\right]}$$

Boer-Mulders ⊗ Collins

$$A_{UL}^{\sin 2\phi_h} = \langle \sin 2\phi_h \rangle_{UL} = \frac{(1-y)}{A(y)} \frac{\mathcal{C}\left[w_1 h_{1L}^{\perp} H_1^{\perp}\right]}{\mathcal{C}\left[f_1 D_1\right]}$$

longi-transversity ⊗ Collins

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \langle \sin(\phi_h-\phi_S) \rangle_{UT} = \frac{1}{2} \frac{\mathcal{C}\left[w_2 f_{1T}^{\perp} D_1\right]}{\mathcal{C}\left[f_1 D_1\right]} \equiv A_{Sivers}$$

Sivers ⊗ unpolarized FF

$$A_{UT}^{\sin(\phi_h+\phi_S)} = \langle \sin(\phi_h+\phi_S) \rangle_{UT} = \frac{(1-y)}{A(y)} \frac{\mathcal{C}\left[w_3 h_{1T} H_1^{\perp}\right]}{\mathcal{C}\left[f_1 D_1\right]} \equiv A_{Collins}$$

transversity ⊗ Collins

$$A_{UT}^{\sin(3\phi_h-\phi_S)} = \langle \sin(3\phi_h-\phi_S) \rangle_{UT} = \frac{(1-y)}{A(y)} \frac{\mathcal{C}\left[w_4 h_{1T}^{\perp} H_1^{\perp}\right]}{\mathcal{C}\left[f_1 D_1\right]}$$

pretzelosity ⊗ Collins

$$A_{LT}^{\cos(\phi_h-\phi_S)} = \langle \cos(\phi_h-\phi_S) \rangle_{LT} = \frac{y(2-y)}{2A(y)} \frac{\mathcal{C}\left[-w_2 g_{1T} D_1\right]}{\mathcal{C}\left[f_1 D_1\right]}$$

trans-helicity ⊗ unpolarized FF

nucleon

electron

$$A(y) \equiv 1 + (1-y)^2$$

Other semi-inclusive reactions

Similar for $p + p \to l + \bar{l} + X$ and $e^- + e^+ \to h_1 + h_2 + X$

We have: (1) General form of

the hadronic tensor and cross sections

in terms of "structure functions";

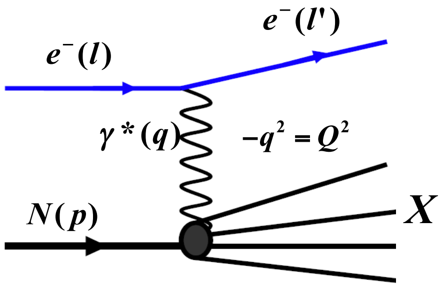
(2) Leading order pQCD & leading twist (intuitive) parton model results

in terms of leading twist TMD PDFs and FFs.

Going beyond
LO pQCD and/or leading twist → collinear expansion,
factorization … …

Collinear expansion in high energy reactions

Inclusive DIS $e^- N \to e^- X$



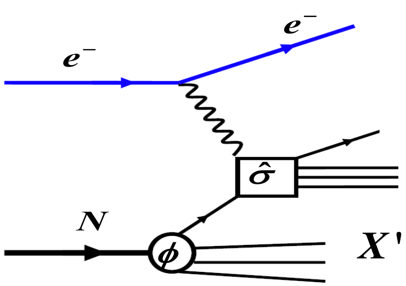
Yes!

where collinear expansion was first formulated.

R. K. Ellis, W. Furmanski and R. Petronzio,
Nucl. Phys. B207,1 (1982);   B212, 29 (1983).

Semi-Inclusive DIS

$$e + N \to e + q(jet) + X$$

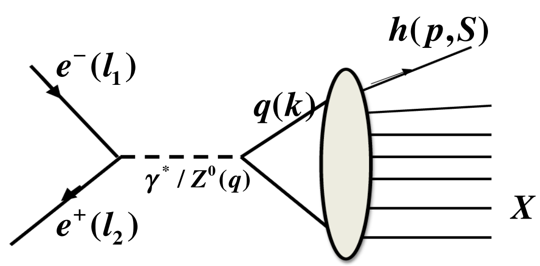


Yes!

ZTL & X.N. Wang,
PRD (2007);

Inclusive

$$e^- + e^+ \to h + X$$

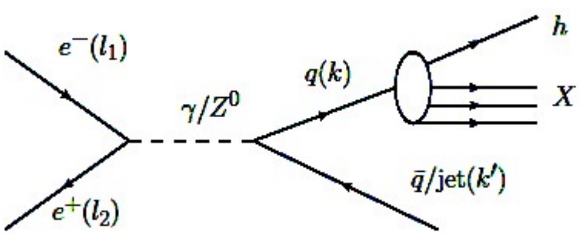


Yes!

S.Y. Wei, Y.K. Song, ZTL,
PRD (2014);

Semi-Inclusive

$$e^- + e^+ \to h + \bar{q}(jet) + X$$



Yes!

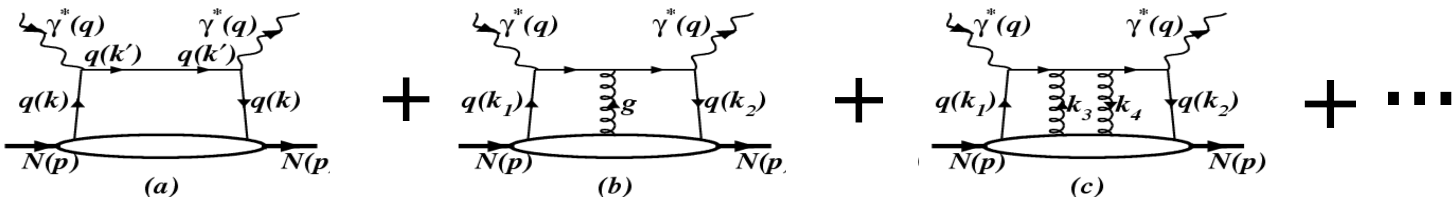
S.Y. Wei, K.B. Chen, Y.K. Song,
ZTL, PRD (2015).

Successfully to all processes where only ONE hadron is explicitly involved.

Semi-Inclusive DIS $e^- + N \rightarrow e^- + q(jet) + X$ with QCD interaction:

$$W_{\mu\nu}^{(si)}(q,p,S,k') = \sum_X \langle p,S | J_\mu(0) | k',X \rangle \langle k',X | J_\nu(0) | p,S \rangle (2\pi)^4 \delta^4(p+q-k'-p_X)$$

$$= W_{\mu\nu}^{(0,si)}(q,p,S,k') + W_{\mu\nu}^{(1,si)}(q,p,S,k') + W_{\mu\nu}^{(2,si)}(q,p,S,k') + \ldots$$



$$W_{\mu\nu}^{(0,si)}(q,p,S,k') = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(0,si)}(k,k',q)\ \hat{\phi}^{(0)}(k,p,S) \right]$$

$$\hat{H}_{\mu\nu}^{(0,si)}(k,k',q) = \gamma_\mu(\not{k} + \not{q})\gamma_\nu (2\pi)^4 \delta^4\left(k' - k - q\right)$$

c.f.: $$W_{\mu\nu}^{(0)}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(0)}(k,q)\ \hat{\phi}^{(0)}(k,p,S) \right]$$

$$\hat{H}_{\mu\nu}^{(0)}(k,q) = \gamma_\mu(\not{k} + \not{q})\gamma_\nu (2\pi)\delta_+\left((k+q)^2\right)$$

$$W_{\mu\nu}^{(1,si,L)}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\widehat{H}_{\mu\nu}^{(1,si,L)\rho}(k_1,k_2,k',q) \widehat{\phi}_\rho^1(k_1,k_2,p,S) \right]$$

$$\widehat{H}_{\mu\nu}^{(1,si,L)\rho}(k_1,k_2,k',q) = \gamma_\mu \frac{(\not{k}_2 + \not{q})\gamma^\rho(\not{k}_1 + \not{q})}{(k_2+q)^2 - i\epsilon} \gamma_\nu (2\pi)^4 \delta^4(k' - k_1 - q)$$

Collinear expansion in semi-Inclusive DIS $e^- + N \to e^- + q(jet) + X$

An identity: $$(2\pi)^4\delta^4(k'-k-q) = (2\pi)\delta_+\left((k-q)^2\right)(2\pi)^3(2E_{k'})\delta^3\left(\vec{k}'-\vec{k}-\vec{q}\right)$$

We obtain: $$\hat{H}^{(0,si)}_{\mu\nu}(k,k',q) = \hat{H}^{(0)}_{\mu\nu}(k,q)(2\pi)^3(2E_{k'})\delta^3\left(\vec{k}'-\vec{k}-\vec{q}\right)$$

$$\hat{H}^{(1,c,si)\rho}_{\mu\nu}(k_1,k_2,k',q) = \hat{H}^{(1,c)\rho}_{\mu\nu}(k_1,k_2,q)(2\pi)^3(2E_{k'})\delta^3\left(\vec{k}'-\vec{k}_c-\vec{q}\right)$$

Hence:

$$W^{(0,si)}_{\mu\nu}(q,p,S,k') = \underbrace{\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{H}^{(0)}_{\mu\nu}(k,q)\hat{\phi}^{(0)}(k,p,S)\right]}_{W^{(0)}_{\mu\nu}(q,p,S)}(2\pi)^3(2E_{k'})\delta^3\left(\vec{k}'-\vec{k}-\vec{q}\right)$$

a common factor !

$$W^{(1,si)}_{\mu\nu}(q,p,S,k') = \underbrace{\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\sum_{c=L,R}\mathrm{Tr}\left[\hat{H}^{(1,c)\rho}_{\mu\nu}(k_1,k_2,q)\hat{\phi}^{(1)}_{\rho}(k_1,k_2,p,S)\right]}_{W^{(1)}_{\mu\nu}(q,p,S)}(2\pi)^3(2E_{k'})\delta^3\left(\vec{k}'-\vec{k}_c-\vec{q}\right)$$

Semi-Inclusive DIS $e^- + N \to e^- + q(jet) + X$

$$W_{\mu\nu}^{(si)}(q,p,S,k') = \tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(2,si)}(q,p,S,k') + \ldots$$

twist-2, 3 and 4 contributions

$$\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\hat{\Phi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(x)]\ (2E_{k'})(2\pi)^3\delta^3(\vec{k}' - \vec{k} - \vec{q})$$

$$\hat{\Phi}^{(0)}(k,p,S) = \int d^4z e^{ikz} \langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

depends on x only!

twist-3, 4 and 5 contributions

$$\tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \sum_{c=L,R} \mathrm{Tr}[\hat{\Phi}_{\rho'}^{(1)}(k_1,k_2,p,S)\ \hat{H}_{\mu\nu}^{(1,c)\rho}(x_1,x_2)\ \omega_\rho^{\ \rho'}]\ (2E_{k'})(2\pi)^3\delta^3(\vec{k}' - \vec{k}_c - \vec{q})$$

$$\hat{\Phi}_\rho^{(1)}(k_1,k_2,p,S) = \int d^4z d^4y e^{ik_1y + ik_2(z-y)} \langle p,S|\bar{\psi}(0)\mathcal{L}(0,y)D_\rho(y)\mathcal{L}(y,z)\psi(z)|p,S\rangle$$

⇨ **A consistent framework for $e^- N \to e^- + q(jet) + X$ at LO pQCD including higher twists**

ZTL & X.N. Wang, PRD (2007); Y.K. Song, J.H. Gao, ZTL & X.N. Wang, PRD (2011) & PRD (2014).

Simplified expressions for hadronic tensors

The "collinearly expanded hard parts" take the simple forms such as:

$$\hat{H}^{(0)}_{\mu\nu}(x) = \hat{h}^{(0)}_{\mu\nu}\delta(x - x_B), \qquad \hat{h}^{(0)}_{\mu\nu} = \gamma_\mu \not{n} \gamma_\nu$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'} = \frac{\pi}{2q\cdot p}\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho^{\ \rho'}\delta(x_1 - x_B),$$

where $\hat{h}^{(1)\rho}_{\mu\nu} = \gamma_\mu \bar{\not{n}} \gamma^\rho \not{n} \gamma_\nu$, depends only on x_1 !

twist-2, 3 and 4

$$\tilde{W}^{(0,si)}_{\mu\nu}(q,p,S;k_\perp) = \mathrm{Tr}\left[\hat{\Phi}^{(0)}(x_B,k_\perp)\, h^{(0)}_{\mu\nu}\right]$$

$$\hat{\Phi}^{(0)}(x,k_\perp) = \int \frac{p^+ dz^-}{2\pi} d^2 z_\perp e^{ixp^+z^- - ik_\perp\cdot z_\perp}\langle N|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|N\rangle$$

three-dimensional gauge invariant quark-quark correlator

twist-3, 4 and 5

$$\tilde{W}^{(1,si)}_{\mu\nu}(q,p,S;k_\perp) = \frac{\pi}{2q\cdot p}\mathrm{Tr}\left[\hat{\varphi}^{(1)}_{\rho'}(x_B,k_\perp)\, h^{(1)\rho}_{\mu\nu}\omega_\rho^{\ \rho'}\right]$$

$$\hat{\varphi}^{(1)}_\rho(x,k_\perp) \equiv \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\delta(x - \frac{k_1^+}{p^+})\delta^2(k_{1\perp} - k_\perp)\hat{\Phi}^{(1)}_\rho(k_1,k_2)$$

$$= \int \frac{p^+ dz^-}{2\pi} d^2 z_\perp e^{ixp^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp}\langle N|\bar{\psi}(0)D_\rho(0)\mathcal{L}(0,z)\psi(z)|N\rangle$$

the involved three-dimensional gauge invariant quark-gluon-quark correlator

THREE dimensional, depend only on ONE parton momentum!

Semi-Inclusive DIS: $e^- + N \to e^- + q(jet) + X$

Complete results for structure functions up to twist-4

$\kappa_M \equiv \frac{M}{Q}, \quad \bar{k}_\perp \equiv \frac{|\vec{k}_\perp|}{M}$

$$W_{UU,T} = xf_1 + 4x^2\kappa_M^2 f_{+3dd}, \quad W_{UU,L} = 8x^3\kappa_M^2 f_3$$

$$W_{UU}^{\cos 2\phi} = -2x^2\kappa_M^2 \bar{k}_\perp^2 f_{-3d}^\perp$$

$$W_{UL}^{\sin 2\phi} = 2x^2\kappa_M^2 \bar{k}_\perp^2 f_{+3dL}^\perp$$

$$W_{LL} = xg_{1L} + 4x^2\kappa_M^2 f_{+3ddL}$$

$$W_{UT,T}^{\sin(\phi-\phi_S)} = \bar{k}_\perp(xf_{1T}^\perp + 4x^2\kappa_M^2 f_{+3ddT}), \quad W_{UT,L}^{\sin(\phi-\phi_S)} = 8x^3\kappa_M^2 \bar{k}_\perp f_{3T}^\perp$$

$$W_{UT}^{\sin(\phi+\phi_S)} = -x^2\kappa_M^2 \bar{k}_\perp^3 (f_{+3dT}^{\perp 4} + f_{-3dT}^{\perp 2})$$

$$W_{UT}^{\sin(3\phi-\phi_S)} = -x^2\kappa_M^2 \bar{k}_\perp^3 (f_{+3dT}^{\perp 4} - f_{-3dT}^{\perp 2})$$

$$W_{LT}^{\cos(\phi-\phi_S)} = \bar{k}_\perp(xg_{1T}^\perp + 4x^2\kappa_M^2 f_{+3ddT}^{\perp 3})$$

$$W_{UU}^{\cos\phi} = -2x^2\kappa_M \bar{k}_\perp f^\perp$$

$$W_{UL}^{\sin\phi} = -2x^2\kappa_M \bar{k}_\perp f_L^\perp$$

$$W_{LU}^{\sin\phi} = 2x^2\kappa_M \bar{k}_\perp g^\perp$$

$$W_{LL}^{\cos\phi} = -2x^2\kappa_M \bar{k}_\perp g_L^\perp$$

$$W_{UT}^{\sin\phi_S} = -2x^2\kappa_M f_T$$

$$W_{UT}^{\sin(2\phi-\phi_S)} = -x^2\kappa_M \bar{k}_\perp^2 f_T^\perp$$

$$W_{LT}^{\cos\phi_S} = -2x^2\kappa_M g_T$$

$$W_{LT}^{\cos(2\phi-\phi_S)} = -x^2\kappa_M \bar{k}_\perp^2 g_T^\perp$$

(1) twist 2 and 4 ⟺ even number of ϕ and ϕ_S ; twist-3 ⟺ odd number of ϕ and ϕ_S

(2) Wherever there is twist-2 contribution, there is a twist-4 addendum to it.

S.Y. Wei, Y.K. Song, K.B. Chen, & ZTL, PRD95, 074017 (2017).

Semi-inclusive high energy reactions: TMD factorization

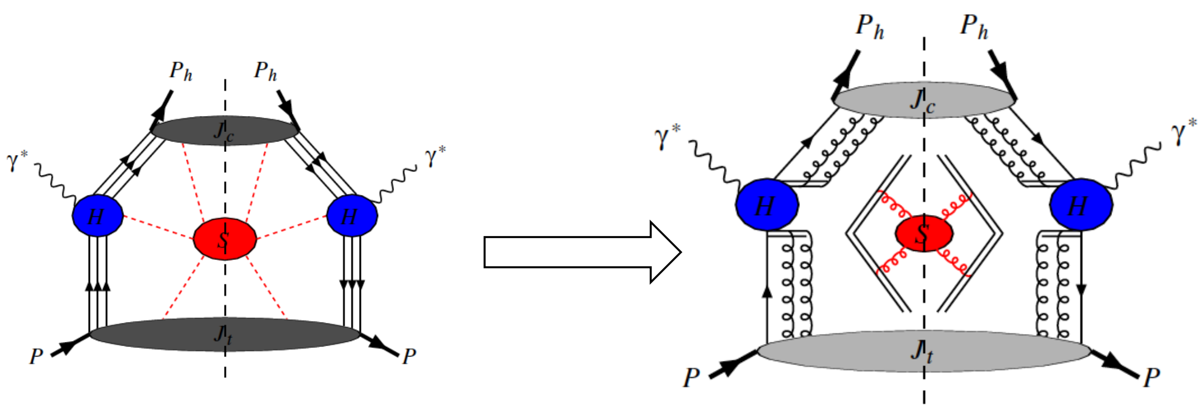
TMD factorization theorem at the leading twist has been proven both for

Semi-inclusive e^+e^--annihilation: $e^+e^- \to h_1 + h_2 + X$ and
semi-inclusive DIS: $e^- + N \to e^- + h + X$

Collins, Soper, NPB (1981,1982); Collins, Sterman, Soper, NPB (1985);

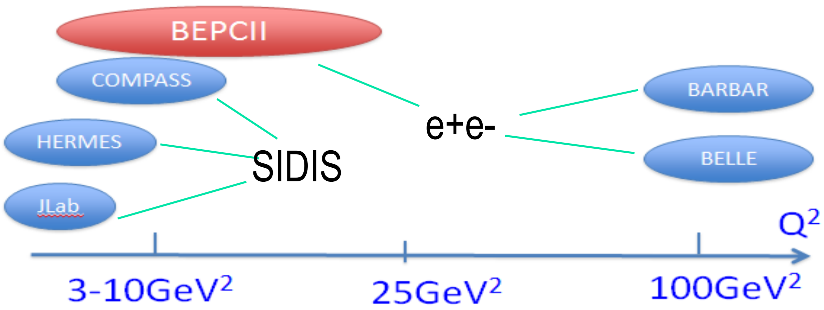
Collins, Oxford Press 2011;

Ibildi, Ji, Ma, Yuan, PRD (2004); Ji, Ma, Yuan, PLB (2004), PRD (2005); ....

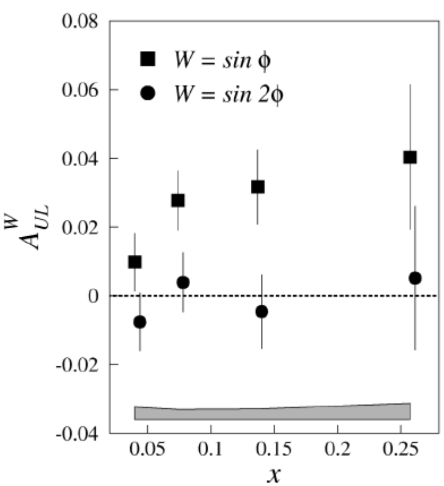


Measurements & Parameterizations

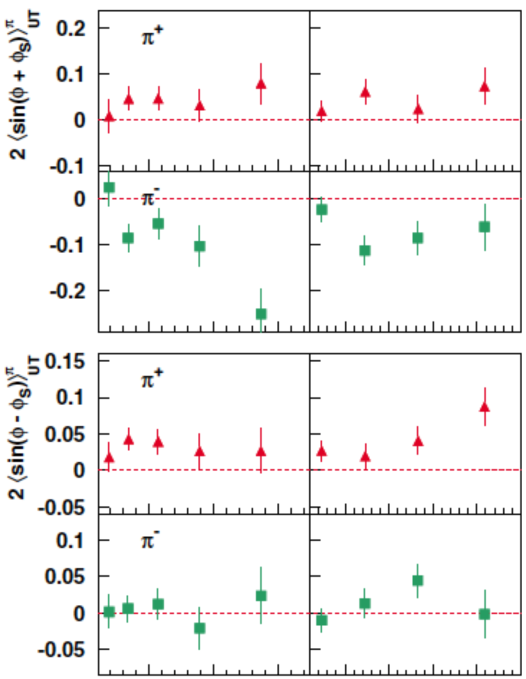
Measurements



HERMES, PRL(2000)



HERMES, PRL(2005)



SIDIS experiments --- Summary

collaboration	reaction	asymmetries
HERMES	$e^+N \to e^+\pi^\pm X$	$A_{UL}^{\sin\phi_h}$, $A_{UL}^{\sin 2\phi_h}$
	$e^+N \to e^+\pi^\pm X$	A_{Siv}, A_{Coll}
	$e^+N \to e^+\pi^{\pm,0}(K^\pm)X$	A_{Siv}
	$e^+N \to e^+\pi^{\pm,0}(K^\pm)X$	A_{Coll}
	$e^+N \to e^+\pi^\pm(K^\pm)X$	$A_{UU}^{\cos\phi_h}, A_{UU}^{\cos 2\phi_h}$
COMPASS	$\mu^-\ {}^6LiD \to \mu^- h^\pm X$	A_{Siv}, A_{Coll}
	$\mu^-\ {}^6LiD \to \mu^-\pi^\pm(K^{\pm,0})X$	A_{Siv}, A_{Coll}
	$\mu^- NH_3 \to \mu^- h^\pm X$	A_{Siv}, A_{Coll}
	$\mu^- NH_3 \to \mu^- h^\pm X$	A_{Coll}
	$\mu^- NH_3 \to \mu^- h^\pm X$	A_{Siv}
	$\mu^- NH_3 \to \mu^-\pi^\pm(K^{\pm,0})X$	A_{Siv}, A_{Coll}
	$\mu^-\ {}^6LiD \to \mu^- h^\pm X$	$A_{UU}^{\cos\phi_h}, A_{UU}^{\cos 2\phi_h}$
CLAS	$e^- p \to e^-\pi^{\pm,0}X$	$A_{UL}^{\sin 2\phi_h}$
	$e^- p \to e^-\pi^0 X$	$A_{LU}^{\sin\phi_h}$
JLab Hall A	$e^-\ {}^3He \to e^-\pi^\pm X$	A_{Siv}, A_{Coll}
	$e^-\ {}^3He \to e^-\pi^\pm X$	$A_{LT}^{\cos(\phi_h-\phi_S)}$
	$e^-\ {}^3He \to e^-\pi^\pm X$	$A_{UT}^{\sin(3\phi_h-\phi_S)}$
	$e^-\ {}^3He \to e^- K^\pm X$	A_{Siv}, A_{Coll}

TMD parameterizations: the first phase

(1) transverse momentum dependence

Gaussian ansatz:

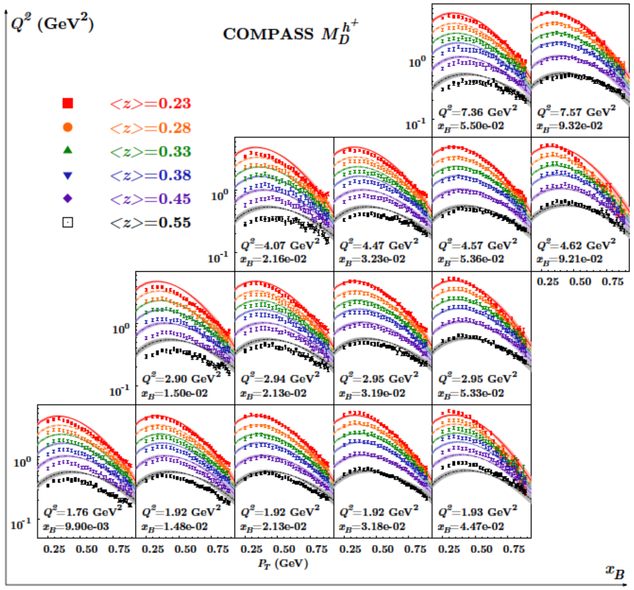
$$f_1(x,k_\perp) = f_1(x)\frac{1}{\pi\langle\vec{k}_\perp^2\rangle}e^{-\vec{k}_\perp^2/\langle\vec{k}_\perp^2\rangle}$$

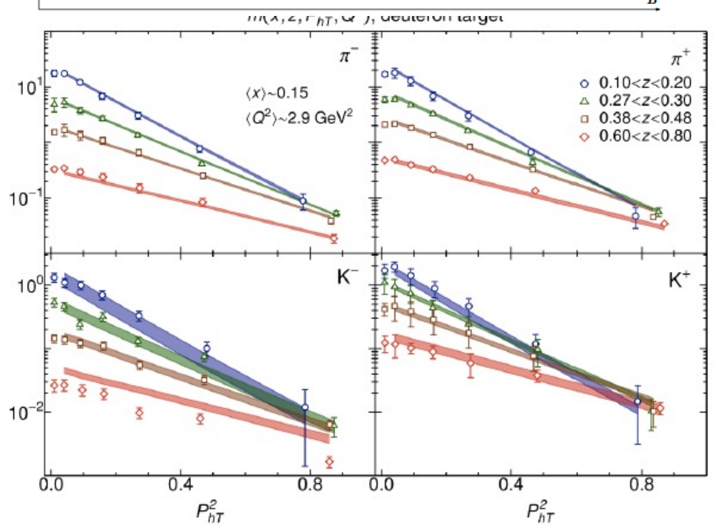
$$D_1(z,k_{F\perp}) = D_1(z)\frac{1}{\pi\langle\vec{k}_{F\perp}^2\rangle}e^{-\vec{k}_{F\perp}^2/\langle\vec{k}_{F\perp}^2\rangle}$$

- the width is fitted
- the form is tested
- flavor dependence

See, e.g.,
Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Prokudin, PRD 71, 074006 (2005);
Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Prokudin, Tuerk, PRD 75, 054032 (2007);
Schweitzer,Teckentrup, Metz, PRD 81, 094019 (2010);
Signori, Bacchetta, Radicic, Schnell, JHEP 11,194 (2013);
Anselmino, Boglione, Gonzalez, Melis, Prokudin, JHEP 04, 005 (2014);
……………..

first phase: without evolution





TMD parameterizations: the first phase

(2) Sivers function

Efremov, Goeke, Menzel, Metz, Schweitzer, PLB 612, 233 (2005);
Collins, Efremov, Goeke, Menzel, Metz, Schweitzer, PRD 73, 014021 (2006);
Arnold, Efremov, Goeke, Schlegel, Schweitzer, 0805.2137[hep-ph] (2008);

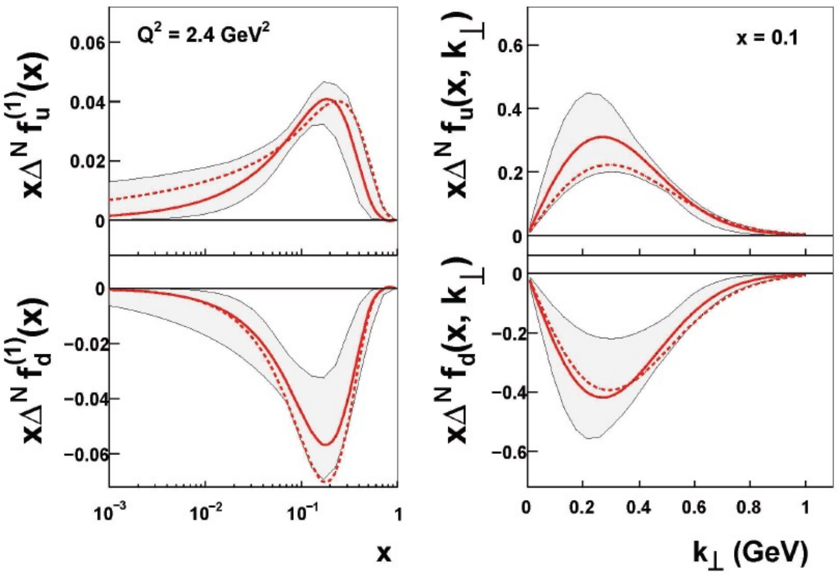
Bochum fits

Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Prokudin, PRD 71, 074006 (2005);
Anselmino, Boglione, D'Alesio, Kotzinian, Melis, Murgia, Prokudin, Tuerk, EPJA 39, 89 (2009);

Torino fits

Vogelsang, Yuan, PRD 72, 054028 (2005);
Bacchetta, Radici, PRL 107, 212001 (2011);

Vogelsang-Yuan fits



$$\Delta^N f_{q/p^\uparrow}(x,k_\perp) = 2\mathcal{N}_q(x)\,h(k_\perp)\,f_{q/p}(x,k_\perp)$$

$$\mathcal{N}_q(x) = \mathcal{N}_q x^{\alpha_q}(1-x)^{\beta_q}\frac{(\alpha_q+\beta_q)^{\alpha_q+\beta_q}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

$$h(k_\perp) = \sqrt{2e}\,\frac{|\vec{k}_\perp|}{M_1}\,e^{-\vec{k}_\perp^2/M_1^2}$$

Already different sets of parameterizations, though not very much different from each other.

TMD parameterizations: the first phase

(3) Transversity & Collins function

Simultaneous extraction of transversity and Collins function

Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Prokudin, PRD 75, 054032 (2007);
Anselmino, Boglione, D'Alesio, Melis, Murgia, Prokudin, PRD 87, 094019 (2013);
………………..

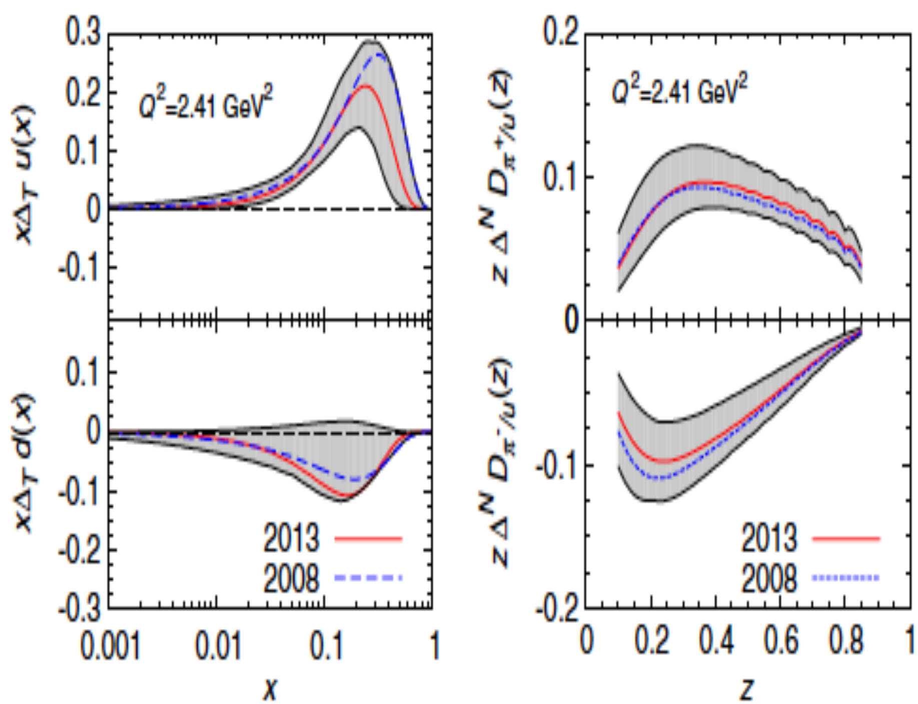
$$\Delta_T q(x,k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)\left[f_{q/p}(x)+\Delta q(x)\right]\frac{e^{-\vec{k}_\perp^2/\langle \vec{k}_\perp^2\rangle_T}}{\pi\langle \vec{k}_\perp^2\rangle_T}$$

$$\Delta^N D_{h/q^\uparrow}(z,k_{F\perp}) = 2\mathcal{N}_q^C(z)D_{h/q}(z)h(k_{F\perp})\frac{e^{-\vec{k}_{F\perp}^2/\langle \vec{k}_{F\perp}^2\rangle}}{\pi\langle \vec{k}_{F\perp}^2\rangle}$$

$$\mathcal{N}_q^T(x) = \mathcal{N}_q^T x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{\alpha+\beta}}{\alpha^\alpha \beta^\beta}$$

$$\mathcal{N}_q^C(z) = \mathcal{N}_q^C z^\gamma (1-z)^\delta \frac{(\gamma+\delta)^{\gamma+\delta}}{\gamma^\gamma \delta^\delta}$$

$$h(k_{F\perp}) = \sqrt{2e}\,\frac{|\vec{k}_{F\perp}|}{M_F}e^{-\vec{k}_{F\perp}^2/M_F^2}$$



TMD parameterizations: the first phase

(4) Boer-Mulders, pretzelosity, ……

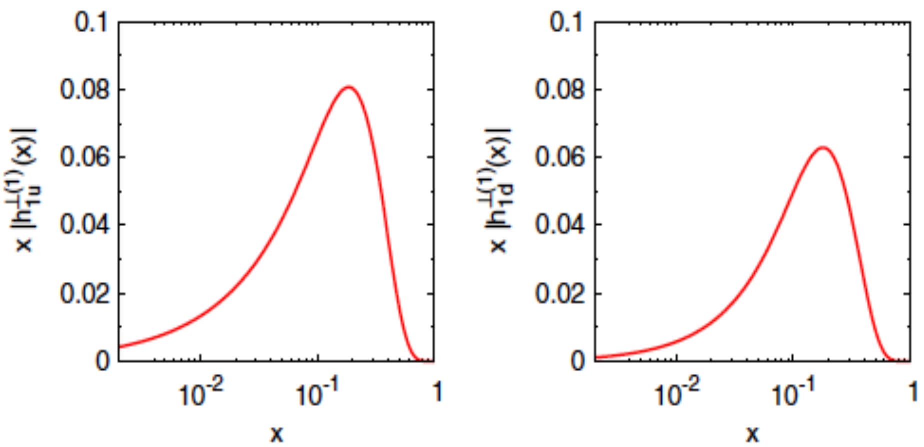
Zhang, Lu, Ma, Schmidt, PRD 77, 054011 (2008); D78, 034035 (2008);
Barone, Prokudin, Ma, PRD 78, 045022 (2008);
Barone, Melis, Prokudin, PRD 81, 114026 (2010);
Lu, Schmidt, PRD 81, 034023 (2010);
……………..

$$h_1^{\perp q}(x,k_\perp) = \lambda_q f_{1T}^{\perp q}(x,k_\perp)$$

$$f_{1T}^{\perp}(x,k_\perp) = 2\mathcal{N}_q(x) h(k_\perp) f_{1q}(x,k_\perp)$$

$$\mathcal{N}_q(x) = \mathcal{N}_q x^{\alpha_q}(1-x)^{\beta_q}\frac{(\alpha_q+\beta_q)^{\alpha_q+\beta_q}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

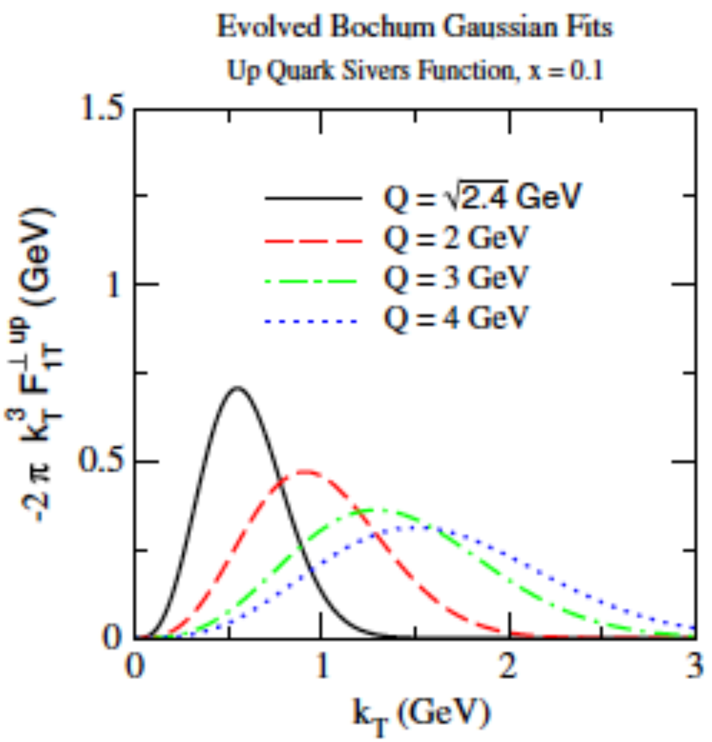
$$h(k_\perp) = \sqrt{2e}\,\frac{M_p}{M_1}e^{-\vec{k}_\perp^2/M_1^2}$$

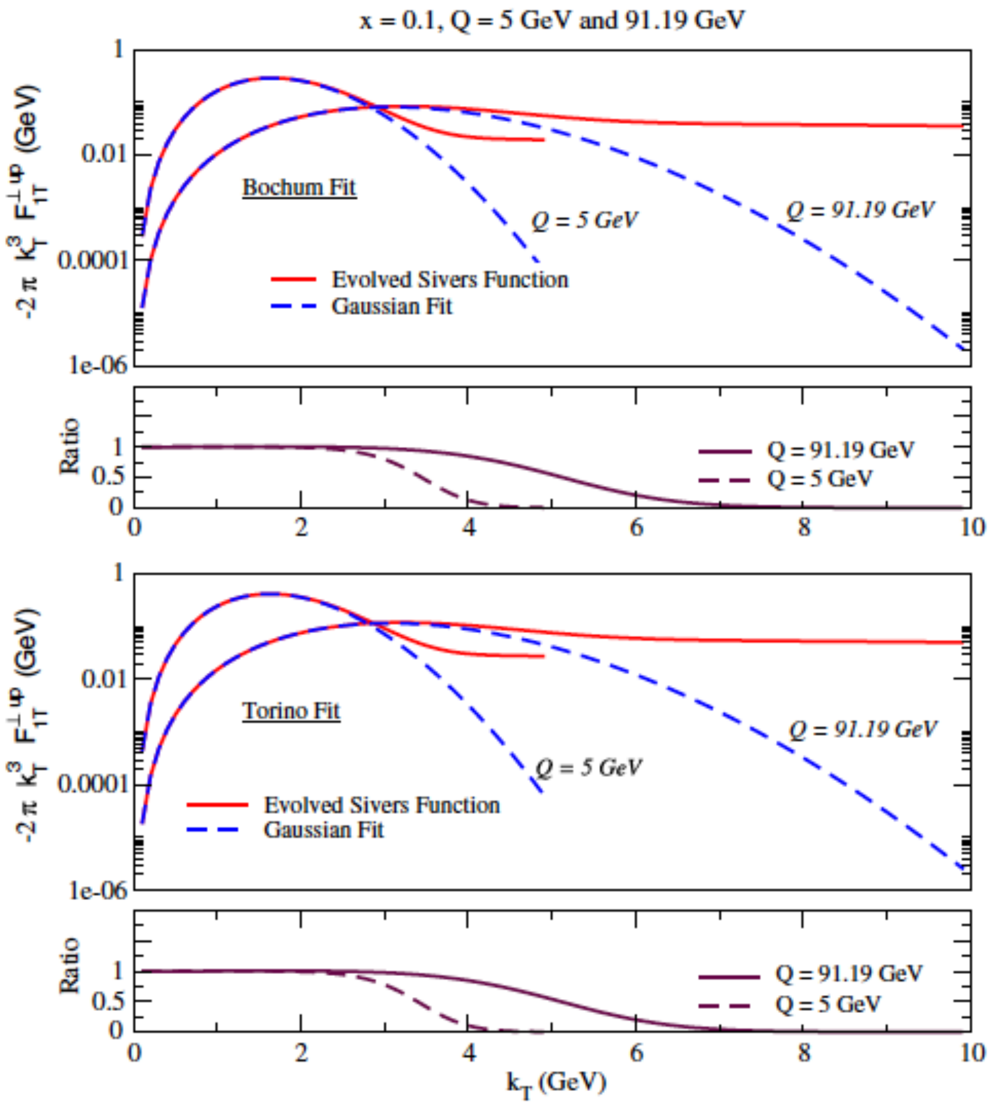


nonzero Boer-Mulders function from SIDIS data on $\langle \cos 2\phi \rangle$

TMD parameterizations: QCD evolution

TMD evolution:





Aybat, Collins, Qiu, Rogers,
PRD 85, 034043 (2012)

TMD parameterizations: TMDlib & TMDplotter

Published in Eur. Phys. J. C74 (2014) 3220

DESY 14-059
NIKHEF 2014-024
YITP-SB-14-24
August 2014

First version is there!

arXiv:1408.3015v1 [hep-ph] 13 Aug 2014

TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions Version 1.0.0

F. Hautmann[1,2], H. Jung[3,4], M. Krämer[3],
P. J. Mulders[5,6], E. R. Nocera[7], T. C. Rogers[8], A. Signori[5,6]

[1] Rutherford Appleton Laboratory, UK
[2] Dept. of Theoretical Physics, University of Oxford, UK
[3] DESY, Hamburg, FRG
[4] University of Antwerp, Belgium
[5] Department of Physics and Astronomy, VU University Amsterdam, the Netherlands
[6] Nikhef, the Netherlands
[7] Università degli Studi di Milano and INFN Milano, Italy
[8] C.N. Yang Institute for Theoretical Physics, Stony Brook University, USA

Abstract

Transverse-momentum-dependent distributions (TMDs) are central in high-energy physics from both theoretical and phenomenological points of view. In this manual we introduce the library, TMDlib, of fits and parameterisations for transverse-momentum-dependent parton distribution functions (TMD PDFs) and fragmentation functions (TMD FFs) together with an online plotting tool, TMDplotter. We provide a description of the program components and of the different physical frameworks the user can access via the available parameterisations.

Contents

Vector spin alignment (自旋排列)

Spin 1 hadrons:

$$\hat{\rho} = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{01} & \rho_{00} & \rho_{0-1} \\ \rho_{-11} & \rho_{-10} & \rho_{-1-1} \end{pmatrix}$$

Spin 1/2 hadrons:

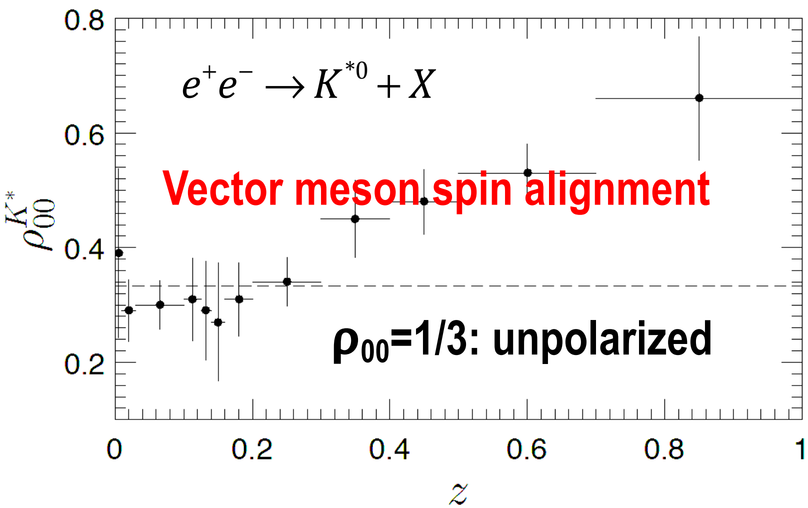
$$\rho = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix} = \frac{1}{2}(1+\vec{S}\cdot\vec{\sigma})$$

For $V \to 1+2$ (pseudoscalar mesons):

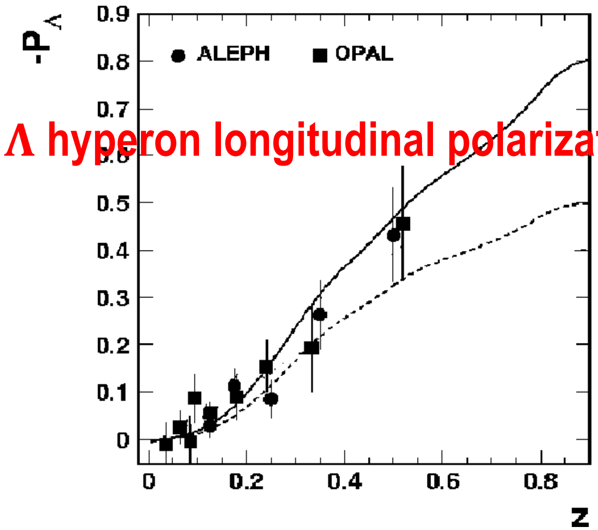
$$\frac{dN}{d\cos\theta} = \frac{3}{4}[(1-\rho_{00}) + (3\rho_{00}-1)\cos^2\theta]$$

For $H \to N + \pi$ (weak decay):

$$\frac{dN}{d\cos\theta} = \frac{1}{2}(1+\alpha P\cos\theta))$$



OPAL, PLB412, 210 (1997).



At the Z-pole, $e^+e^- \to Z \to \bar{q}q \to h + X$, quark/anti-quark is longitudinally polarized.

Is the vector meson spin alignment induced by the polarization of the fragmenting quark?

Vector spin alignment (自旋排列)

Spin 1 hadrons:

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{01} & \rho_{00} & \rho_{0-1} \\ \rho_{-11} & \rho_{-10} & \rho_{-1-1} \end{pmatrix} = \frac{1}{3}(1 + \frac{3}{2}\vec{S}\cdot\vec{\Sigma} + 3T^{ij}\Sigma^{ij})$$

Vector polarization: $S^\mu = (0, \vec{S}) = (0, \vec{S}_T, S_L)$

Tensor polarization: S_{LL}, $S_{LT}^\mu = (0, S_{LT}^x, S_{LT}^y, 0)$, $S_{TT}^{x\mu} = (0, S_{TT}^{xx}, S_{TT}^{xy}, 0)$

标量　矢量　张量

$$\rho = \begin{pmatrix} \frac{1+S_{LL}}{3} + \frac{S_L}{2} & \frac{(S_{LT}^x - iS_{LT}^y) + (S_T^x - iS_T^y)}{2\sqrt{2}} & \frac{S_{TT}^{xx} - iS_{TT}^{xy}}{2} \\ \frac{(S_{LT}^x + iS_{LT}^y) + (S_T^x + iS_T^y)}{2\sqrt{2}} & \frac{1-2S_{LL}}{3} & \frac{-(S_{LT}^x - iS_{LT}^y) + (S_T^x - iS_T^y)}{2\sqrt{2}} \\ \frac{S_{TT}^{xx} + iS_{TT}^{xy}}{2} & \frac{-(S_{LT}^x + iS_{LT}^y) + (S_T^x + iS_T^y)}{2\sqrt{2}} & \frac{1+S_{LL}}{3} - \frac{S_L}{2} \end{pmatrix}$$

One dimensional FFs defined via quark-quark correlator

The Lorentz decomposition

vector meson spin alignment

$$z\Xi_\alpha(z;p,S) = p^+\bar{n}_\alpha[D_1(z) + S_{LL}D_{1LL}(z)] + \cdots$$

$$p = p^+\bar{n} + \frac{M^2}{2p^+}n,$$

$$D_1(z) + S_{LL}D_{1LL}(z) = \frac{1}{p^+}n^\alpha\, z\Xi_\alpha(z;p,S)$$

$$= \frac{1}{8\pi}\sum_X \int zd\xi^-\, e^{-ip^+\xi^-/z}\langle hX|\bar{\psi}(\xi)n\cdot\gamma\,|0\rangle\langle 0|\psi\,(0)|hX\rangle$$

$$= \frac{1}{8\pi}\sum_X \int zd\xi^-\, e^{-ip^+\xi^-/z}\sum_{\lambda_q=L,R}\left\langle hX\middle|\bar{\psi}_{\lambda_q}(\xi)\gamma^+\,\middle|0\right\rangle\left\langle 0\middle|\psi_{\lambda_q}\,(0)\middle|hX\right\rangle$$

$$\psi_{L/R} \equiv \frac{1}{2}(1\pm\gamma_5)\psi$$

space reflection invariance

$$\Rightarrow \quad \langle hX|\bar{\psi}_L(\xi)n\cdot\gamma\,|0\rangle\langle 0|\psi_L\,(0)|hX\rangle = \langle hX|\bar{\psi}_R(\xi)\gamma^+|0\rangle\langle 0|\psi_R\,(0)|hX\rangle$$

independent of the spin λ_q of the fragmenting quark!

One dimensional FFs defined via quark-quark correlator

The Lorentz decomposition

longitudinal spin transfer

$$z\widetilde{\Xi}_\alpha(z;p,S) = p^+\bar{n}_\alpha S_L G_{1L}(z) + \cdots$$

S_L is the longitudinal component of $S = (0, \vec{S}_T, S_L)$

$$S_L G_{1L}(z) = \frac{1}{p^+} n^\alpha \, z\widetilde{\Xi}_\alpha(z;p,S)$$

$$= \frac{1}{8\pi} \sum_X \int z d\xi^- \, e^{-ip^+\xi^-/z} \langle hX|\bar{\psi}(\xi)\gamma_5\gamma^+ |0\rangle\langle 0|\psi(0)|hX\rangle$$

$$= \frac{1}{8\pi} \sum_X \int z d\xi^- \, e^{-ip^+\xi^-/z} \sum_{\lambda_q} \left\langle hX\middle|\bar{\psi}_{\lambda_q}(\xi)\gamma^+\middle|0\right\rangle \lambda_q \left\langle 0\middle|\psi_{\lambda_q}(0)\middle|hX\right\rangle$$

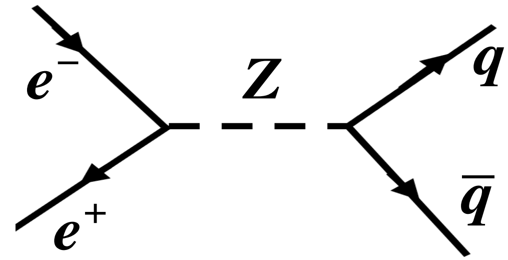
$$= \frac{1}{8\pi} \sum_X \int z d\xi^- \, e^{-ip^+\xi^-/z} [\langle hX|\bar{\psi}_L(\xi)\gamma^+|0\rangle\langle 0|\psi_L(0)|hX\rangle - \langle hX|\bar{\psi}_R(\xi)\gamma^+|0\rangle\langle 0|\psi_R(0)|hX\rangle]$$

dependent of the spin λ_q of the fragmenting quark!

Quark polarization in $e^+e^- \to q\bar{q}$

At the Z-pole: $e^+e^- \to Z \to q\bar{q}$

The cross section: $\frac{d\hat{\sigma}^{ZZ}}{d\Omega} = \frac{\alpha^2}{4s}\chi\left[c_1^e c_1^q(1+\cos^2\theta) + 2c_3^e c_3^q \cos\theta\right]$

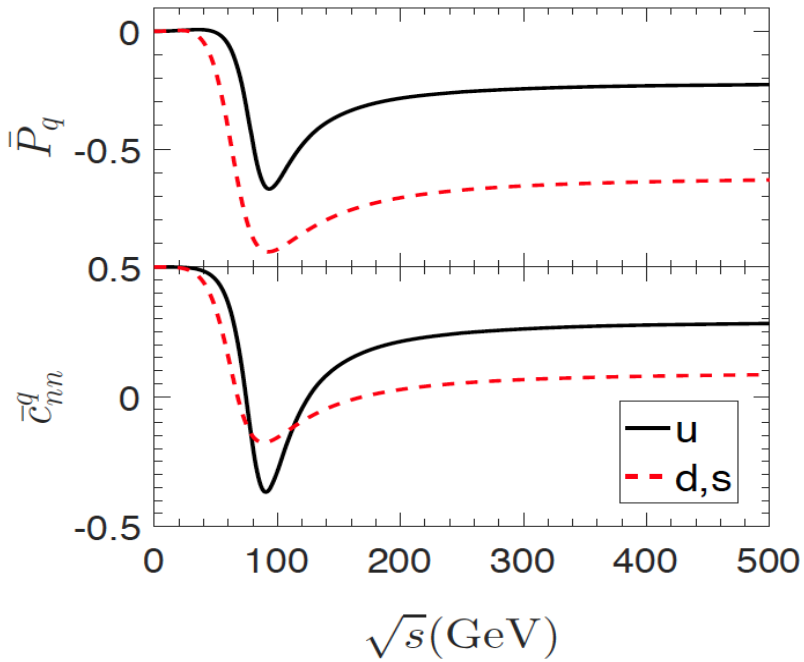


Longitudinal polarization of q or $\bar{q}$: $\bar{P}_q^{ZZ} = -\frac{c_3^q}{c_1^q}$

At any energy: $e^+e^- \to \gamma^*/Z \to q\bar{q}$

$$\frac{d\hat{\sigma}}{d\Omega} = \frac{d\hat{\sigma}^{ZZ}}{d\Omega} + \frac{d\hat{\sigma}^{Z\gamma}}{d\Omega} + \frac{d\hat{\sigma}^{\gamma\gamma}}{d\Omega}$$

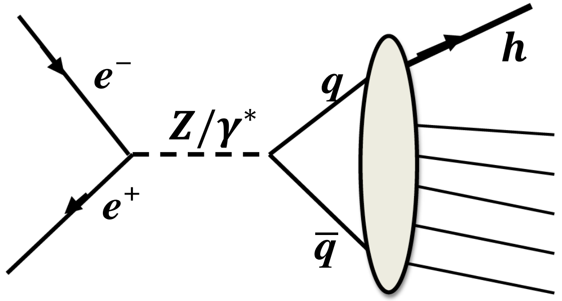
$$\bar{P}_q = -\frac{\chi c_1^e c_3^q + \chi_{\text{int}}^q c_V^e c_A^q}{e_q^2 + \chi c_1^e c_1^q + \chi_{\text{int}}^q c_V^e c_V^q},$$



Hadron polarization in $e^+e^- \to hX$

Hyperon polarization:

$$P_{L\Lambda}(z,Q) = \frac{\sum_q P_q(Q) W_q(Q) G_{1Lq}(z,Q)}{\sum_q W_q(Q) D_{1q}(z,Q)}$$



$$W_q(Q) = \frac{2}{3}\left(e_q^2 + \chi c_1^e c_1^q + \chi_{int}^q c_V^e c_V^q\right)$$

$$\chi = s^2 / \left[(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2\right] \sin^4 2\theta_W$$

$$\chi_{int}^q = -2 e_q s (s - M_Z^2) / \left[(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2\right] \sin^2 2\theta_W$$

Vector meson spin alignment:

$$\langle S_{LL}\rangle(z,Q) = \frac{1}{2}\frac{\sum_q W_q(Q)\, D_{1LLq}(z,Q)}{\sum_q W_q(Q) D_{1q}(z,Q)}$$

K.B. Chen, W.H. Yang, S.Y. Wei and ZTL, PRD94, 034003 (2016);
K.B. Chen, W.H. Yang, Y.J. Zhou and ZTL, PRD95, 034009 (2017).

Hyperon polarization in $e^+e^- \to H + X$

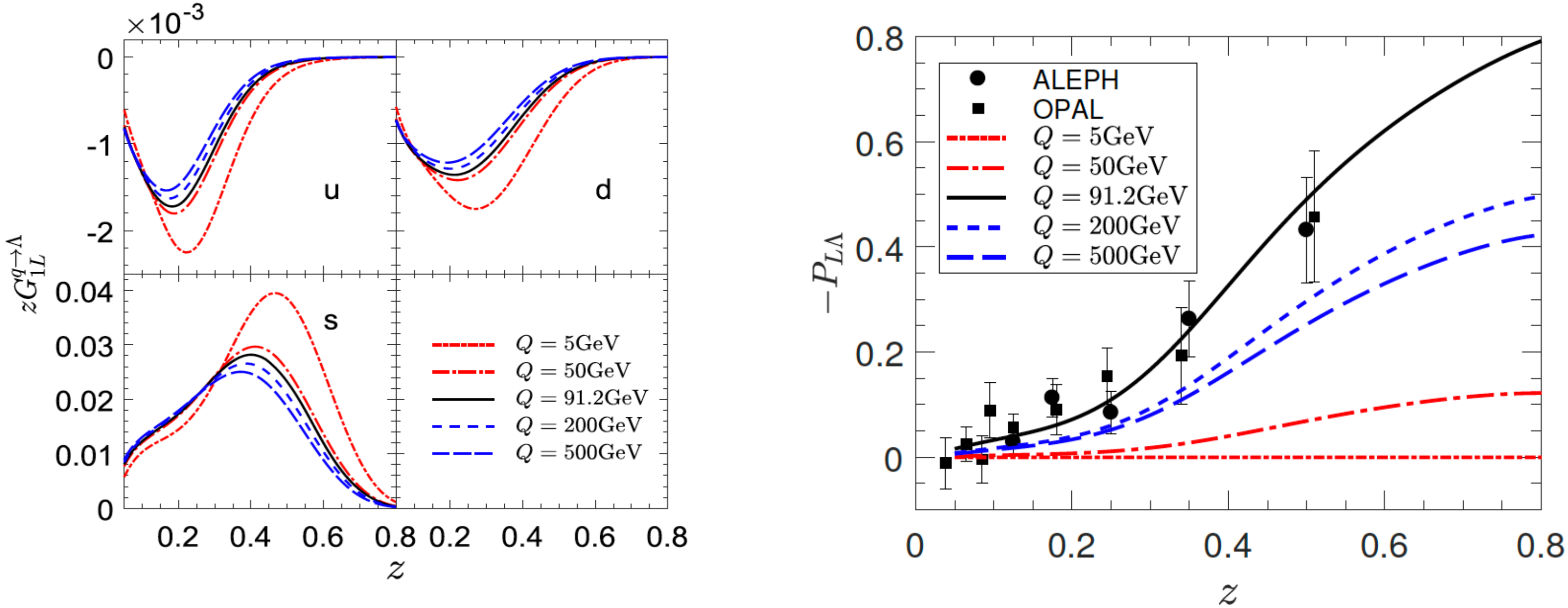
Parameterization at a initial scale:

$$G_{1L}^{s\to\Lambda}(z,\mu_0) = z^a D_1^{s\to\Lambda}(z,\mu_0)$$

$$G_{1L}^{u/d\to\Lambda}(z,\mu_0) = N z^a D_1^{u/d\to\Lambda}(z,\mu_0)$$

QCD Evolution:
(DGLAP equation)

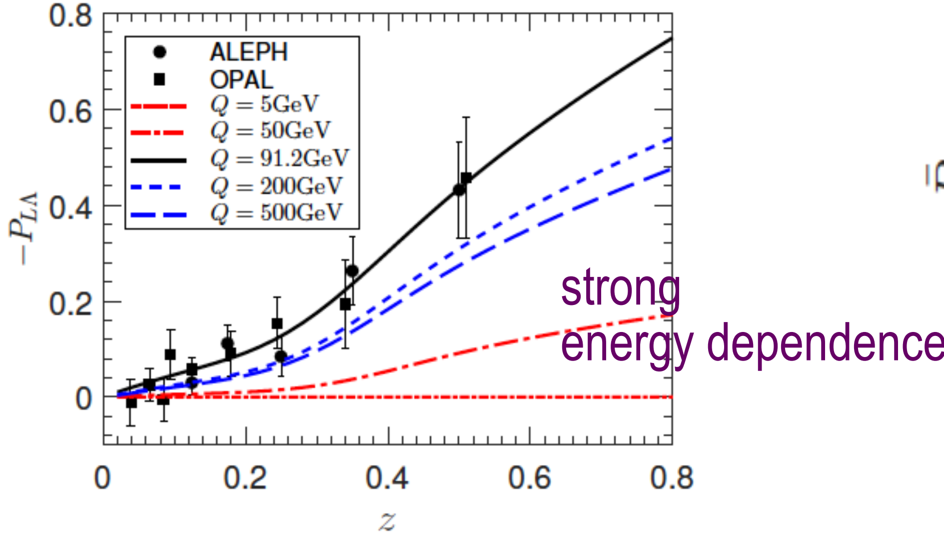
$$\frac{\partial}{\partial \ln Q^2} G_{1L}^{i\to h}(z,Q^2) = \frac{\alpha_s}{2\pi}\sum_j \int_z^1 \frac{dy}{y} G_{1L}^{j\to h}\left(\frac{z}{y},Q^2\right)\Delta P_{ij}(y,\alpha_s)$$

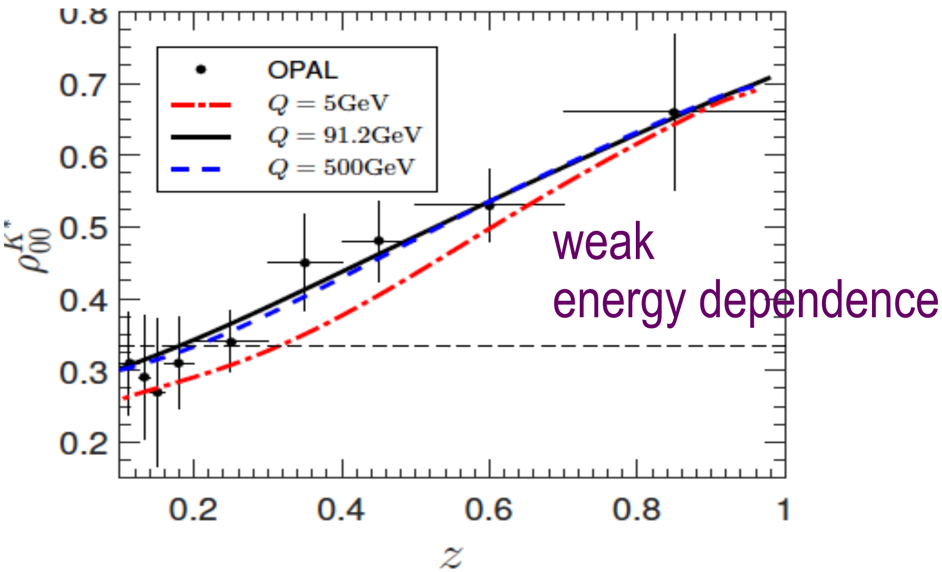


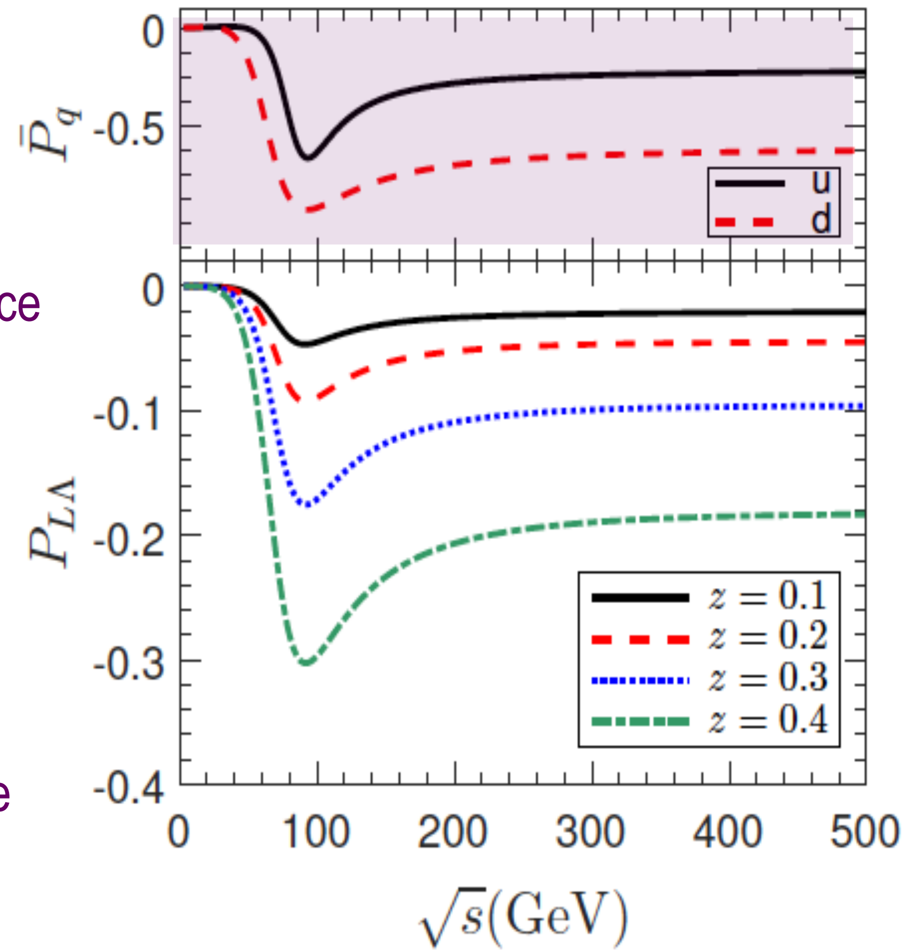
K.B. Chen, W.H. Yang, Y.J. Zhou and ZTL, PRD95, 034009 (2017).

Numerical estimations for inclusive processes

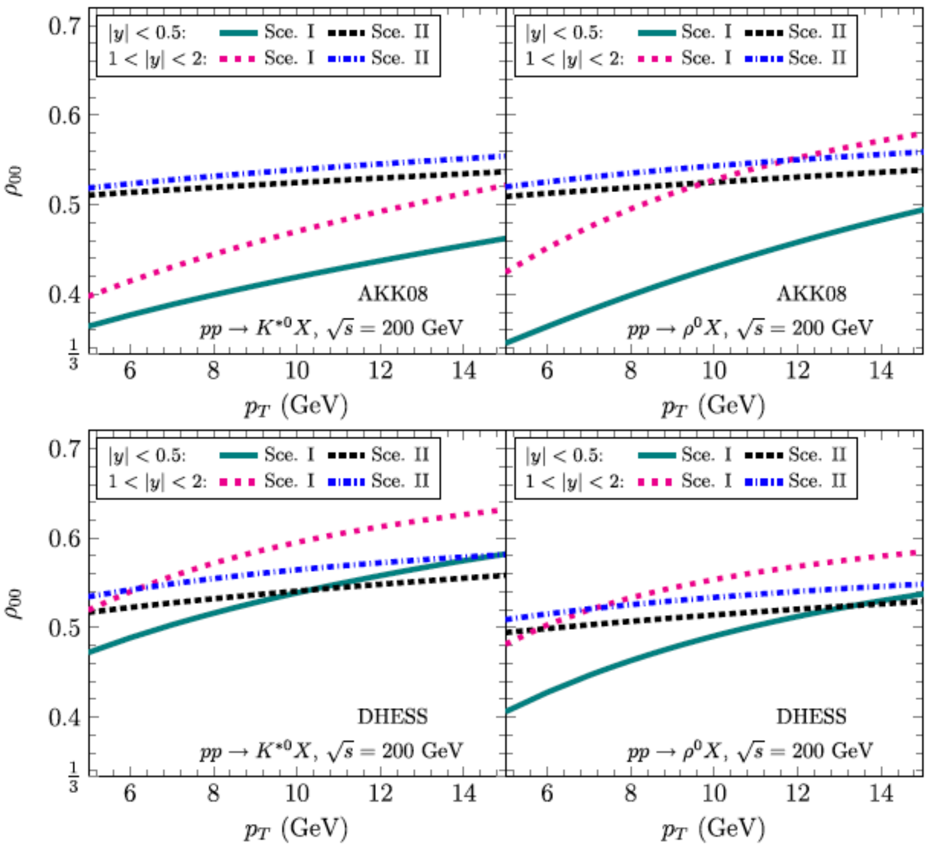
Leading twist and leading order pQCD evolution





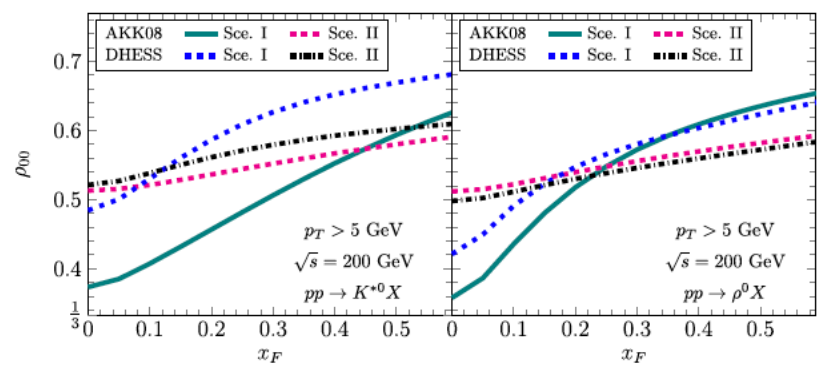


Spin alignment in $pp \to VX$



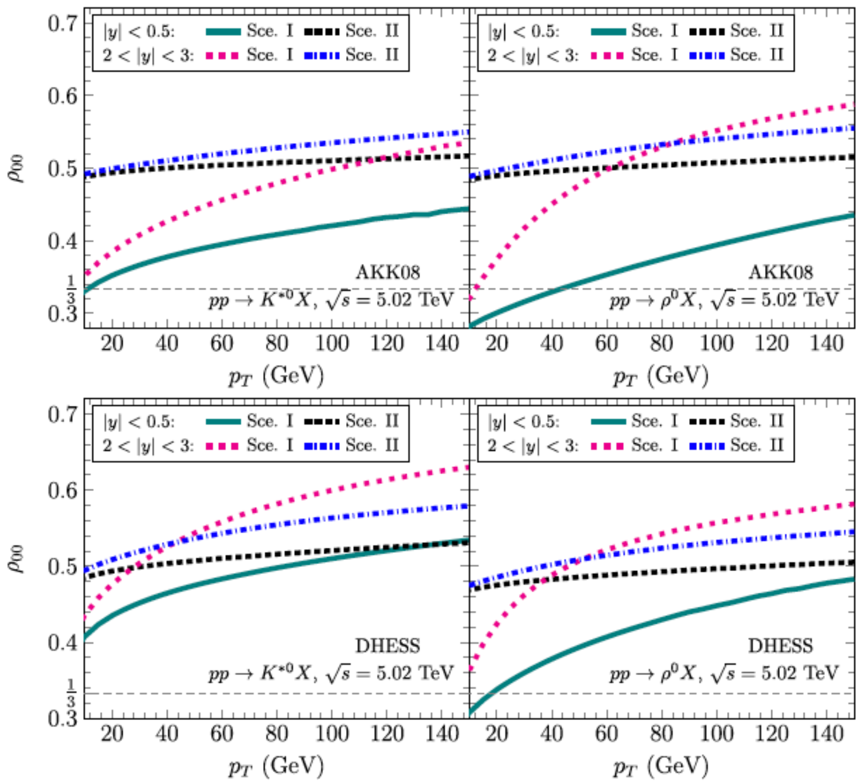
$\sqrt{s} = 200$GeV

$\rho_{00} > 1/3$ and
increase with increasing p_T or x_F



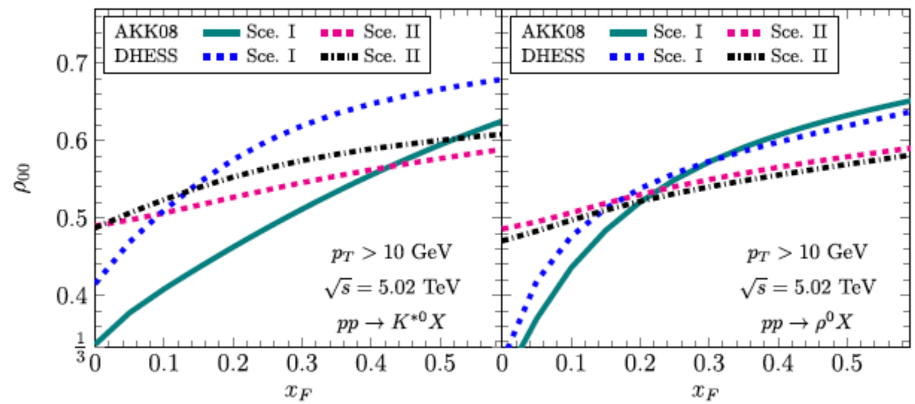
K.B. Chen, ZTL, Y.K. Song and S.Y. Wei, PRD102, 034001 (2020).

Spin alignment in $pp \to VX$



$\sqrt{s} = 5.02$TeV

$\rho_{00} > 1/3$ and
increase with increasing p_T or x_F



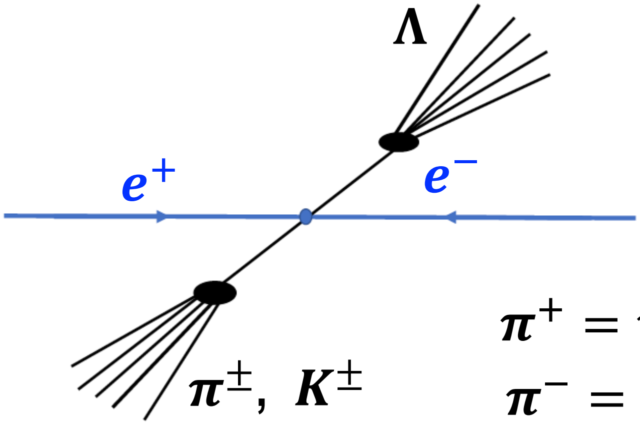
K.B. Chen, ZTL, Y.K. Song and S.Y. Wei, PRD102, 034001 (2020).

Isospin symmetry of Fragmentation Functions

Belle Collaboration, PRL 122, 042001 (2019).

Transverse polarization of Λ in the fragmentation of unpolarized quarks

Significant differences have been observed



$\pi^+ = u\bar{d}$

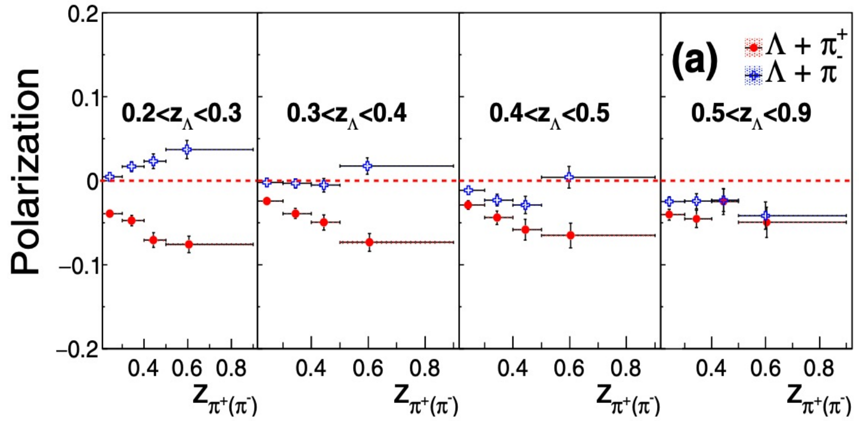
$\pi^- = d\bar{u}$

$\pi^+: \bar{d} \to \pi^+ + X,\ d \to \Lambda + X$

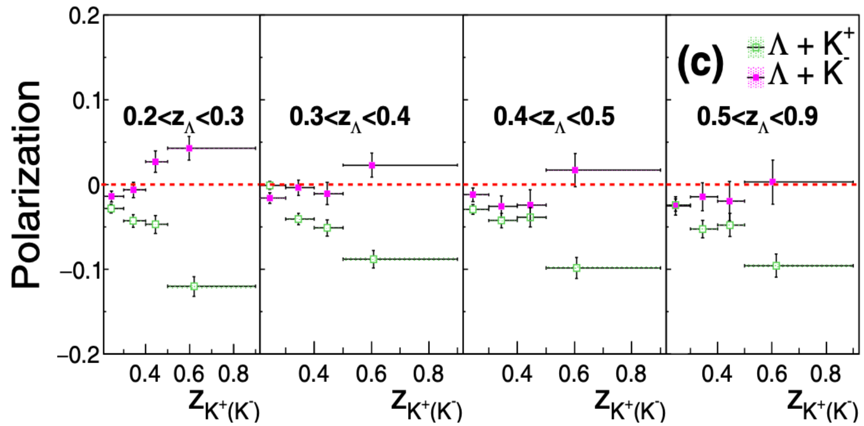
$\pi^-: \bar{u} \to \pi^- + X,\ u \to \Lambda + X$

$P_{T\Lambda} \leftrightarrow D_{1T}^{\perp\Lambda}(z, p_T)$

$e^+e^- \to \Lambda + \pi^\pm + X$

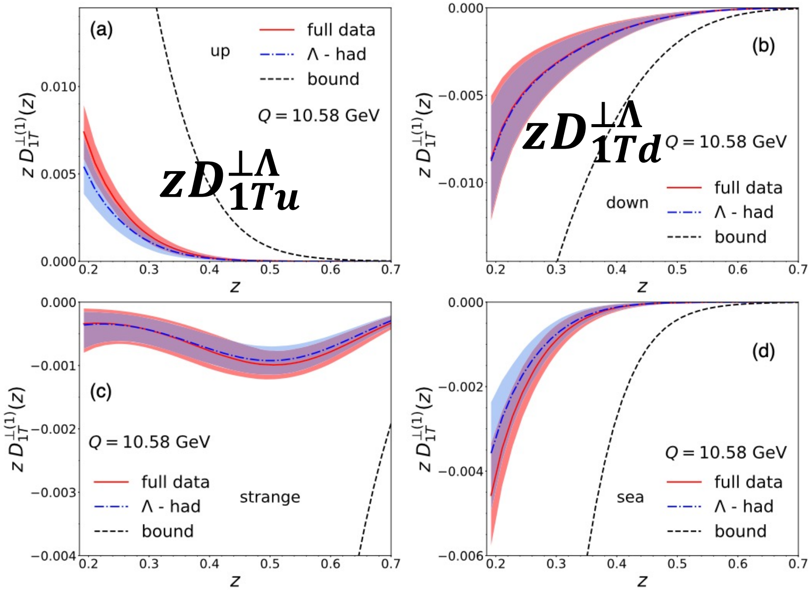


$e^+e^- \to \Lambda + K^\pm + X$

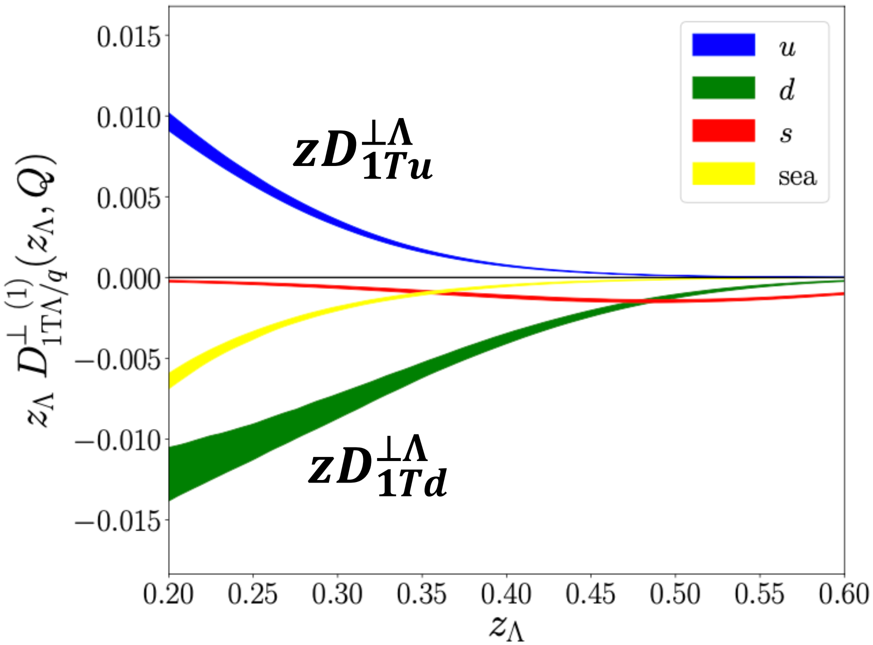


Isospin symmetry of Fragmentation Functions

U. D'Alesio, F. Murgia, and M. Zaccheddu, PRD 102, 054001 (2020)



D. Callos, Z.B. Kang, and J. Terry, PRD 102, 096007 (2020)

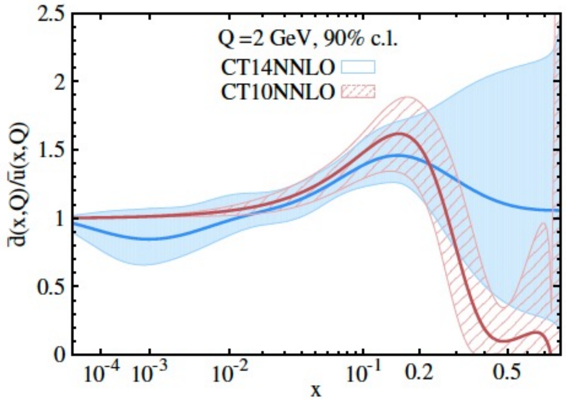


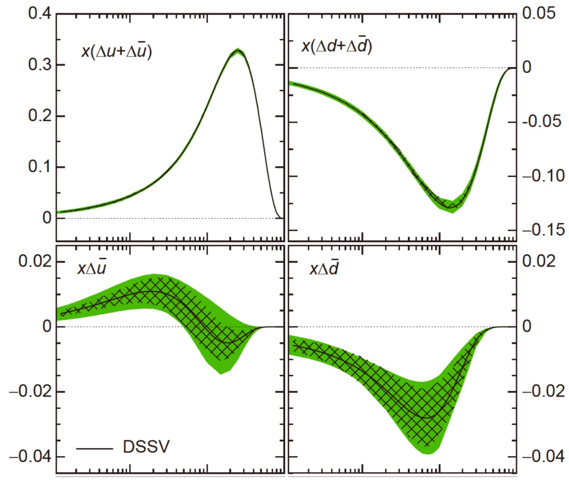
$$D_{1Tu}^{\perp\Lambda} \neq D_{1Td}^{\perp\Lambda}$$

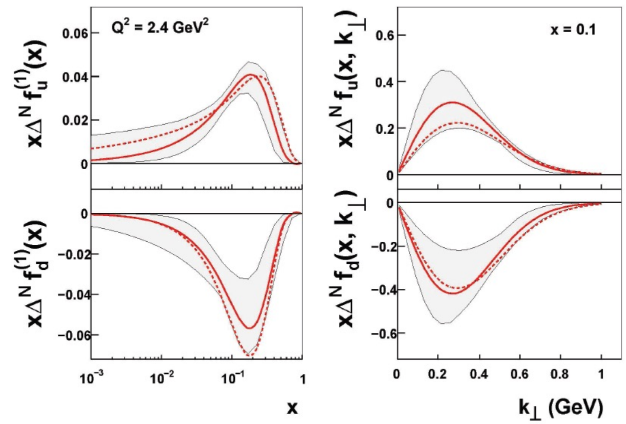
Very strong isospin symmetry violation!

Isospin symmetry of Fragmentation Functions

"Isospin symmetry violation" in parton distribution functions (PDFs)







However, this is NOT real isospin symmetry violation!

Isospin symmetry demands:

$$\bar{u}^p(x) = \bar{d}^n(x) \quad \Delta\bar{u}^p(x) = \Delta\bar{d}^n(x) \quad \Delta^N f_u^p(x) = \Delta^N f_d^n(x)$$

It does NOT demands:

$$\bar{u}^p(x) = \bar{d}^p(x) \quad \Delta\bar{u}^p(x) = \Delta\bar{d}^p(x) \quad \Delta^N f_u^p(x) = \Delta^N f_d^p(x)$$

Isospin symmetry of Fragmentation Functions

For fragmentation functions (FFs), isospin symmetry demands,

$$D_{1u}^{\Lambda}(z, p_T) = D_{1d}^{\Lambda}(z, p_T) \qquad \Delta D_{1u}^{\Lambda}(z, p_T) = \Delta D_{1d}^{\Lambda}(z, p_T)$$

$$D_{1Tu}^{\perp\Lambda}(z, p_T) = D_{1Td}^{\perp\Lambda}(z, p_T)$$

If we assume fragmentation is determined by strong interaction and isospin symmetry is hold in strong interaction, we have

$$D_{1u}^{\Lambda,\mathrm{dir}}(z, p_T) = D_{1d}^{\Lambda,\mathrm{dir}}(z, p_T) \qquad \Delta D_{1u}^{\Lambda,\mathrm{dir}}(z, p_T) = \Delta D_{1d}^{\Lambda,\mathrm{dir}}(z, p_T)$$

$$D_{1Tu}^{\perp\Lambda,\mathrm{dir}}(z, p_T) = D_{1Td}^{\perp\Lambda,dir}(z, p_T)$$

For the final hadrons, $D_{1u}^{\Lambda}(z, p_T) = D_{1d}^{\Lambda,\mathrm{dir}}(z, p_T) + D_{1d}^{\Lambda,dec}(z, p_T)$

There could be isospin symmetry from the electroweak decay contributions!

⇨ A systematical study of decay contributions to isospin violation in FFs.

Isospin symmetry of Fragmentation Functions

Electroweak decay contributions to FFs of the Λ-hyperon:

$$D_{1q}^{\Lambda,dec}(z) = D_{1q}^{\Lambda,\Sigma^0}(z) + D_{1q}^{\Lambda,\Xi}(z) + D_{1q}^{\Lambda,\Omega}(z)$$

$$D_{1q}^{\Lambda,\Xi}(z) \equiv D_{1q}^{\Lambda,\Xi^0}(z) + D_{1d}^{\Lambda,\Xi^-}(z)$$

$$D_{1q}^{\Xi}(z) \equiv D_{1q}^{\Xi^0}(z) + D_{1d}^{\Xi^-}(z)$$

We see clear that

$$D_{1u}^{\Lambda,\Sigma^0}(z) = D_{1d}^{\Lambda,\Sigma^0}(z) \quad \text{if } D_{1u}^{\Sigma^0}(z) = D_{1d}^{\Sigma^0}(z)$$

$$D_{1u}^{\Lambda,\Xi}(z) = D_{1d}^{\Lambda,\Xi}(z) \quad \text{if } D_{1u}^{\Xi}(z) = D_{1d}^{\Xi}(z)$$

$$D_{1u}^{\Lambda,\Omega^-}(z) = D_{1d}^{\Lambda,\Omega^-}(z) \quad \text{if } D_{1u}^{\Omega^-}(z) = D_{1d}^{\Omega^-}(z)$$

They lead to no isospin violation in Λ production!

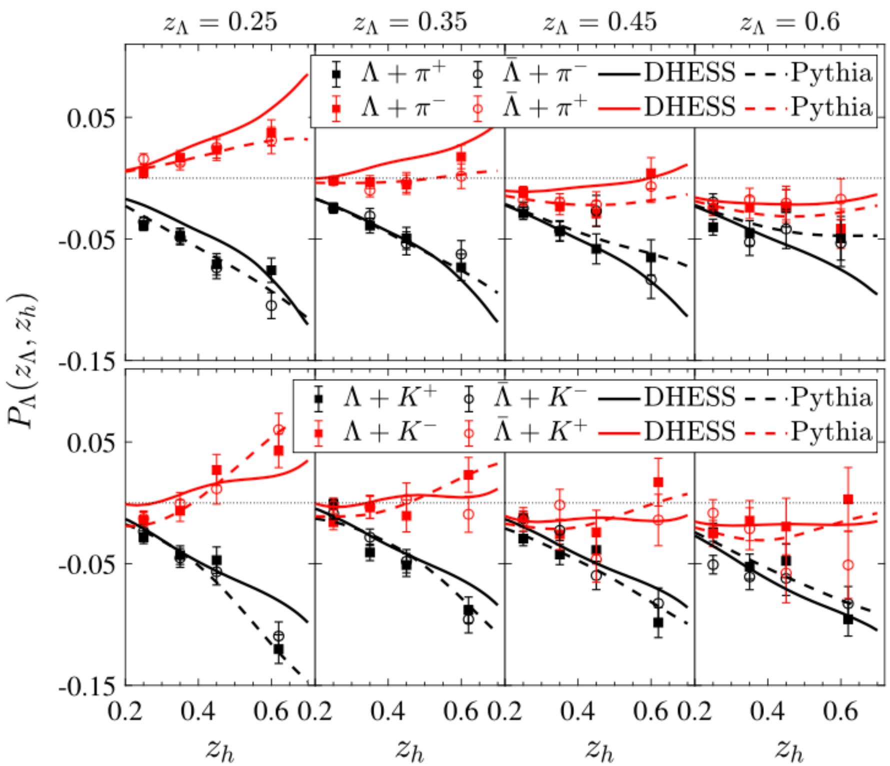
Electroweak decays of $J^P = \left(\frac{1}{2}\right)^+$ octet and of $J^P = \left(\frac{3}{2}\right)^+$ decuplet baryons

hyperon	decay mode	branch ratio
Ω^-	ΛK^- $\Xi^0\pi^-$ $\Xi^-\pi^0$	67.8 ± 0.7 23.6 ± 0.7 8.6 ± 0.4
Λ	$p\pi^-$ $n\pi^0$	63.9 ± 0.5 35.8 ± 0.5
Σ^0	$\Lambda\gamma$	100
Σ^+	$p\pi^0$ $n\pi^+$	51.57 ± 0.30 48.31 ± 0.30
Σ^-	$n\pi^-$	100
Ξ^0	$\Lambda\pi^0$	100
Ξ^-	$\Lambda\pi^-$	100

Isospin symmetry should be hold for FFs of Λ hyperon!

Isospin symmetry of Fragmentation Functions

Fit to Belle data if we demand isospin symmetry for $D_{1Tq}^{\perp\Lambda}(z, p_T)$,



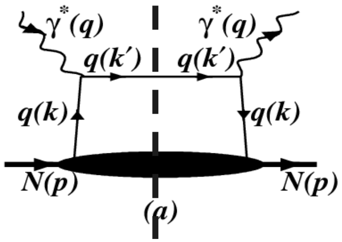
K.b. Chen, ZTL, Y.l. Pan, Y.k. Song, S.y. Wei, PLB 816, 136217 (2021).

Predictions for EIC/EicC, that can be used to test isospin symmetry of $D_{1Tq}^{\perp\Lambda}(z, p_T)$, see, K.b. Chen, ZTL, Y.k. Song, S.y. Wei, PRD 105, 034027 (2022).

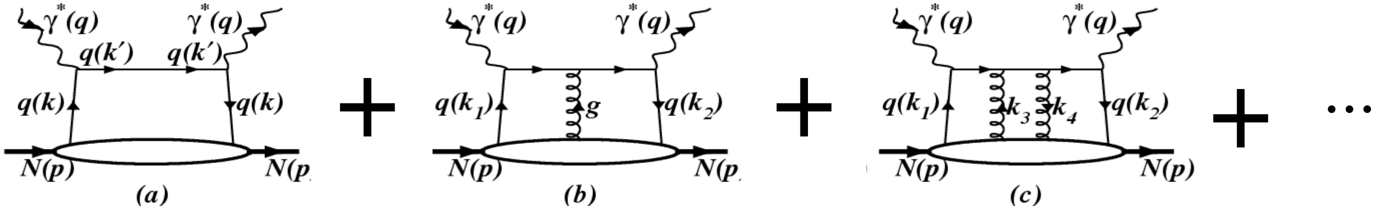
Contents

Summary

- (Gauge invariant) PDF is not merely



but



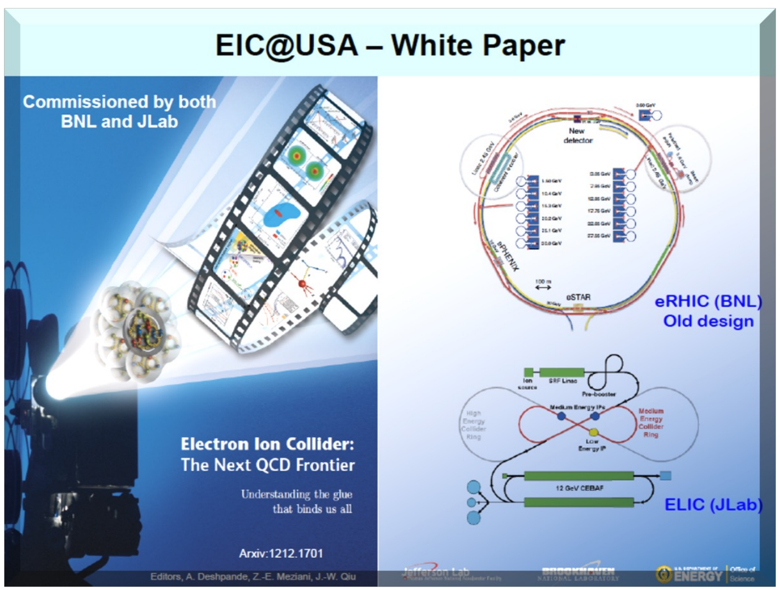
i.e., it always contains "intrinsic motion" and "multiple gluon scattering".

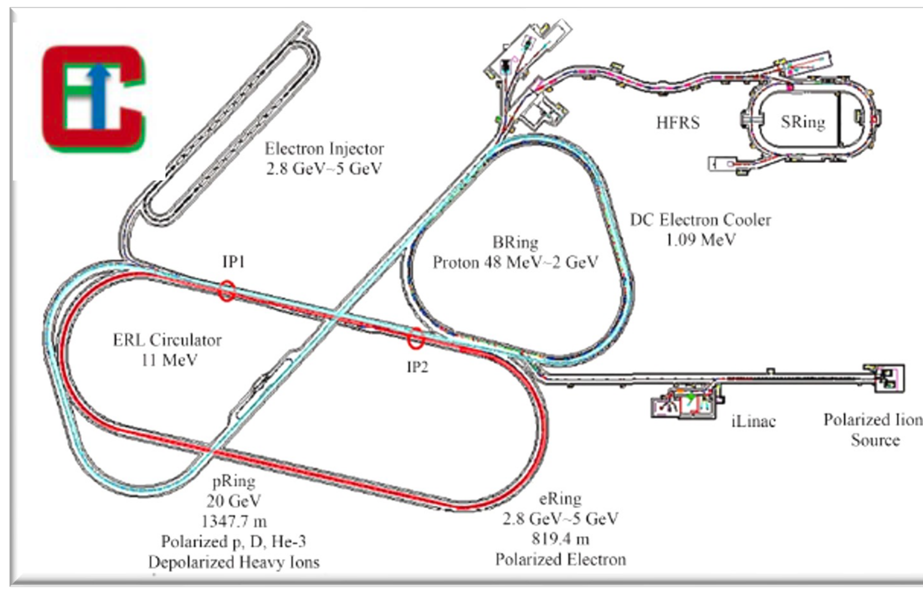
- "Multiple gluon scattering" gives rise to the gauge link.
- Collinear expansion is the necessary procedure to obtain the correct formulism in terms of gauge invariant parton distribution functions (PDFs).
- Collinear expansion has been proven to be applicable to all processes where one hadron is explicitly involved.

Summary & Outlook

- Rapid developing
- Much progress
- 未来实验(EIC, EicC)
  重要核心物理目标

本报告注重了“程序”与“基础”以及个人熟悉的方面，不完整，不系统，特别是没有试图对当前研究热点系统总结，谨慎参考！





Thank you for your attention!