# Two-body Hadronic B Decays at NNLO in QCD Factorization

**李新强**

华中师范大学

In collaboration with G. Bell, M. Beneke, T. Huber, and S. Krankl

Based on JHEP 04 (2020), JHEP 09 (2016) 112, PLB 750 (2015) 348, NPB 832 (2010) 109

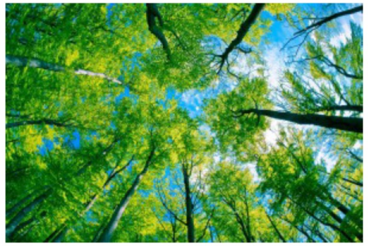
2023年度中国科学院理论物理前沿重点实验室年会，2023/06/01，北京

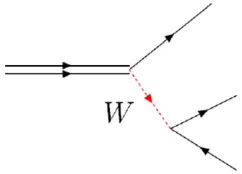
# Outline

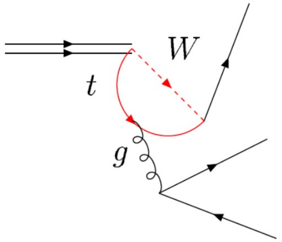
☐ **Introduction & motivation**

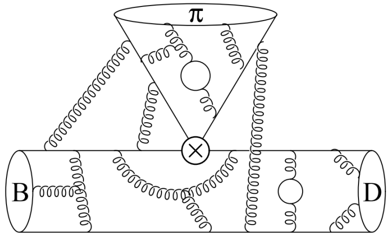
☐ **Theoretical framework & QCDF for hadronic B decays**

☐ **NNLO QCD corrections to the hadronic matrix elements**









☐ **Summary**

M. Beneke, T. Huber and Xin-Qiang Li, *"NNLO vertex corrections to non-leptonic B decays: Tree amplitudes,"* Nucl. Phys. B **832** (2010) 109 [arXiv:0911.3655 [hep-ph]].

T. Huber, S. Kränkl and Xin-Qiang Li, *"Two-body non-leptonic heavy-to-heavy decays at NNLO in QCD factorization,"* JHEP **09** (2016) 112 [arXiv:1606.02888 [hep-ph]].

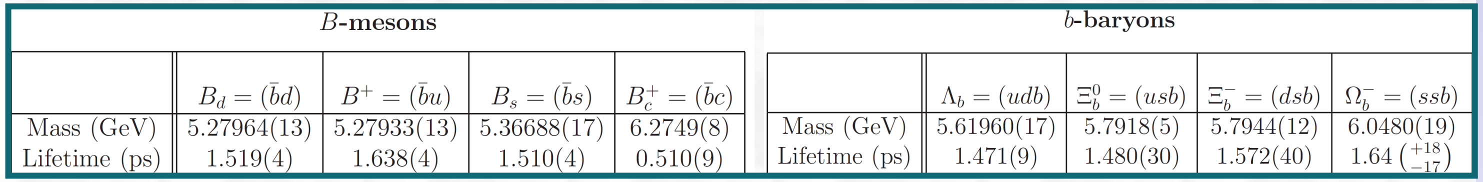
G. Bell, M. Beneke, T. Huber and Xin-Qiang Li, *"Two-loop current-current operator contribution to the non-leptonic QCD penguin amplitude,"* Phys. Lett. B **750** (2015) 348-355 [arXiv:1507.03700 [hep-ph]].

G. Bell, M. Beneke, T. Huber and Xin-Qiang Li, *"Two-loop non-leptonic penguin amplitude in QCD factorization,"* JHEP **04** (2020) 055 [arXiv:2002.03262 [hep-ph]].

# Introduction & Motivation

# B physics and B decays

- ☐ **B physics: productions & decays of various b hadrons.**



| | $B$-mesons | | | | | $b$-baryons | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $B_d = (\bar{b}d)$ | $B^+ = (\bar{b}u)$ | $B_s = (\bar{b}s)$ | $B_c^+ = (\bar{b}c)$ | | $\Lambda_b = (udb)$ | $\Xi_b^0 = (usb)$ | $\Xi_b^- = (dsb)$ | $\Omega_b^- = (ssb)$ |
| Mass (GeV) | 5.27964(13) | 5.27933(13) | 5.36688(17) | 6.2749(8) | Mass (GeV) | 5.61960(17) | 5.7918(5) | 5.7944(12) | 6.0480(19) |
| Lifetime (ps) | 1.519(4) | 1.638(4) | 1.510(4) | 0.510(9) | Lifetime (ps) | 1.471(9) | 1.480(30) | 1.572(40) | $1.64\,\left(^{+18}_{-17}\right)$ |

- ☐ **b-hadron weak decays:** at the quark level, all governed by flavor-changing charged-currents mediated by $W$-boson.

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} J^{\mu}_{\rm CC}\, W^{\dagger}_{\mu} + \text{h.c.}$$

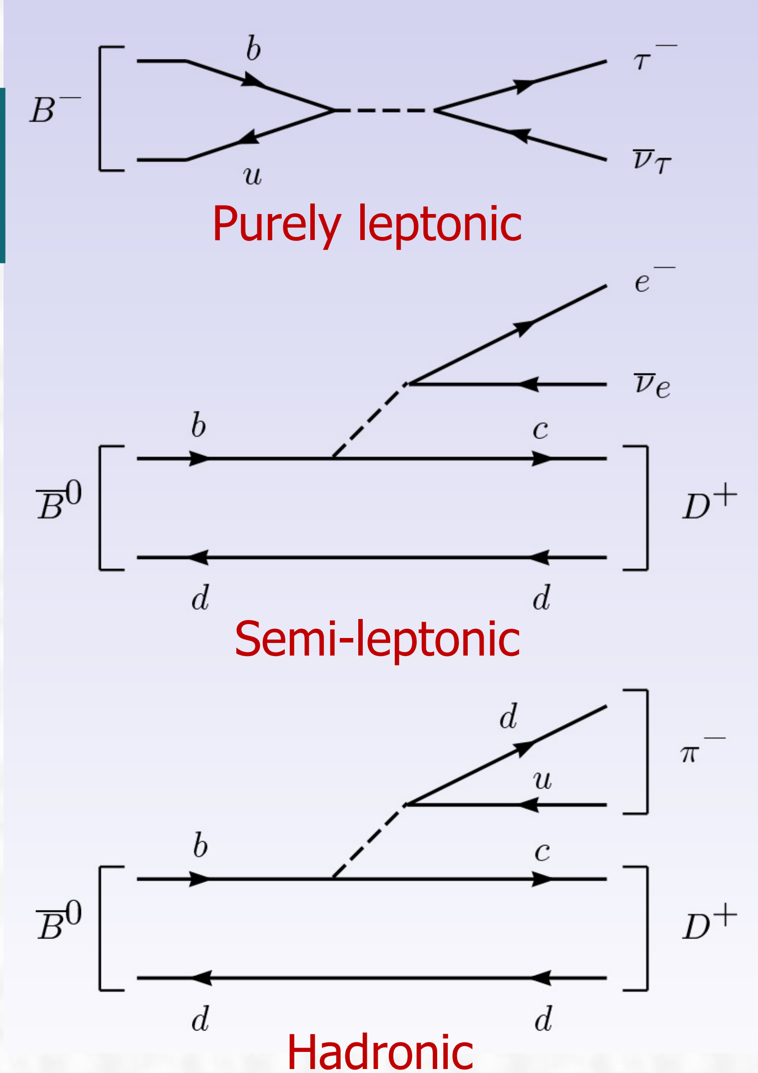
$$J^{\mu}_{\rm CC} = (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)\,\gamma^\mu \begin{pmatrix} e_{\rm L} \\ \mu_{\rm L} \\ \tau_{\rm L} \end{pmatrix}$$

$$+ (\bar{u}_{\rm L}, \bar{c}_{\rm L}, \bar{t}_{\rm L})\,\gamma^\mu\, V_{\rm CKM} \begin{pmatrix} d_{\rm L} \\ s_{\rm L} \\ b_{\rm L} \end{pmatrix}$$

$g$: $SU(2)_L$ gauge coupling

$V_{CKM}$: CKM matrix for quark mixing

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



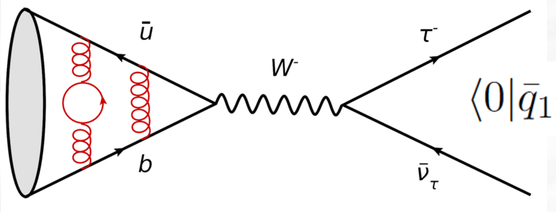
- ☐ **Classification of b-hadron weak decays:** three classes; purely leptonic, semi-leptonic, hadronic

# Interplay between weak & strong forces

- **QCD effect always matters:** in real world, quarks confined inside hadrons and no free quarks;

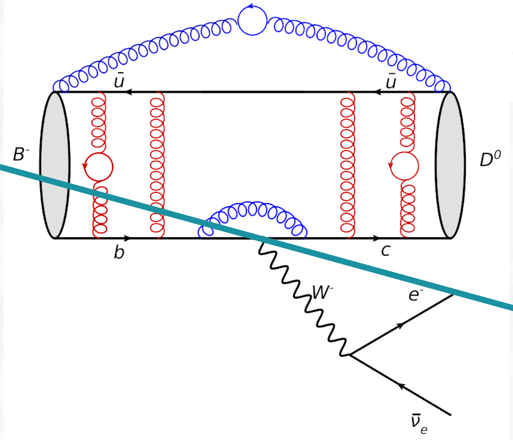
  the simplicity of weak interactions overshadowed by the complexity of strong interactions!

- **Purely leptonic decays:** decay constant



$$\langle 0|\bar{q}_1\gamma_\mu\gamma_5 q_2|P(p)\rangle = ip_\mu f_P$$

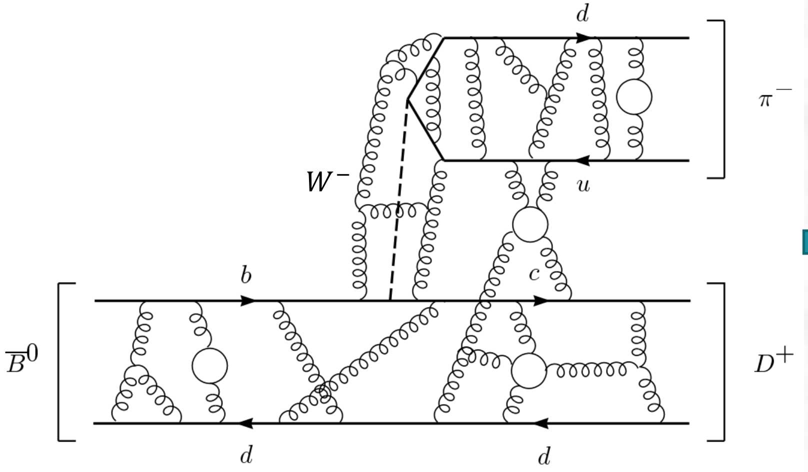
- **Semi-leptonic decays:** transition form factors



$$\langle D|\,\bar{c}\gamma^\mu b\,|\bar{B}\rangle \equiv f_+(q^2)(p_B+p_D)^\mu + [f_0(q^2)-f_+(q^2)]\,\frac{m_B^2-m_D^2}{q^2}\,q^\mu$$

***LQCD or LCSR etc.***

- **Hadronic decays:** hadronic matrix elements



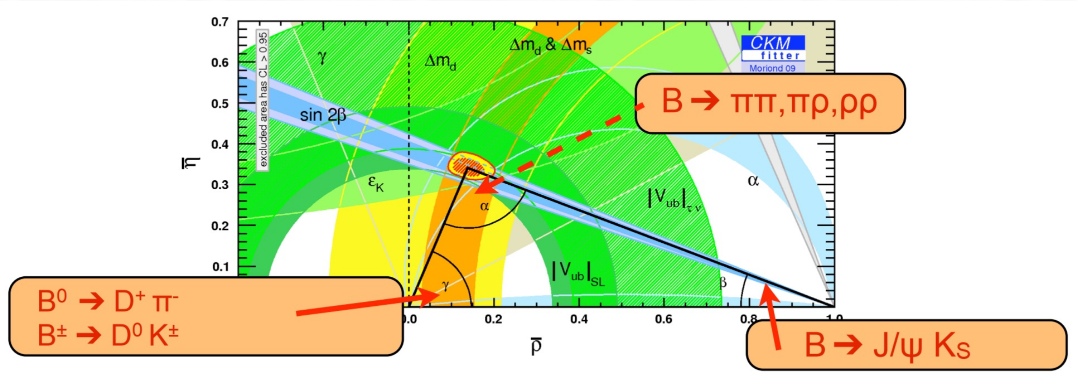
**multi-scale problem with highly hierarchical scales!**

| | | | | |
|---|---|---|---|---|
| EW interaction scale | $\gg$ | ext. mom'a in B rest frame | $\gg$ | QCD-bound state effects |
| $m_W \sim 80$ GeV<br>$m_Z \sim 91$ GeV | $\gg$ | $m_b \sim 5$ GeV | $\gg$ | $\Lambda_{\rm QCD} \sim 1$ GeV |

***the most complicated case, but very important!***

# Why hadronic B decays

- ☐ **direct access to the CKM parameters, especially to the three angles of UT.**



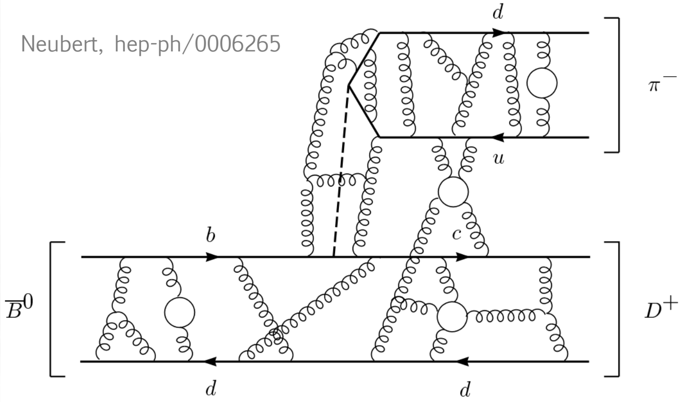
$B \to \pi\pi, \pi\rho, \rho\rho$

$B^0 \to D^+\pi^-$
$B^\pm \to D^0 K^\pm$

$B \to J/\psi K_S$

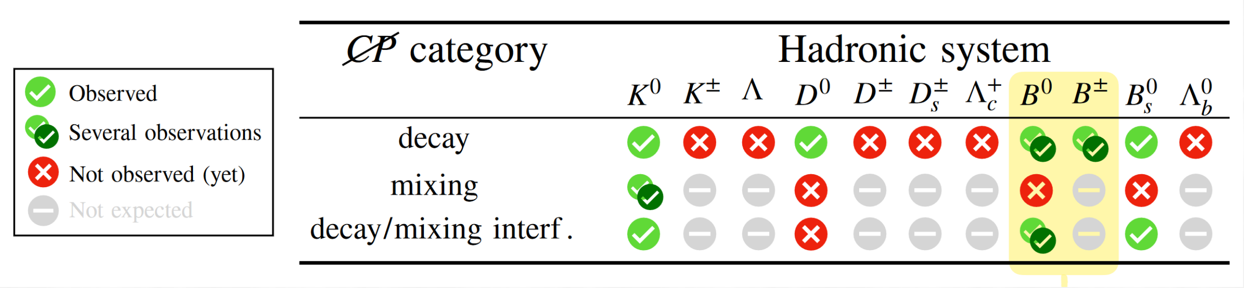
- ☐ **further insight into strong-interaction effects involved in hadronic decays.**

***factorization? strong phase origin?...***



- ☐ **deep insight into the hadron structures: especially exotic hadronic states.**

- ☐ **deepen our understanding of the origin & mechanism of CPV.**



✓ Observed
✓✓ Several observations
✗ Not observed (yet)
⊖ Not expected

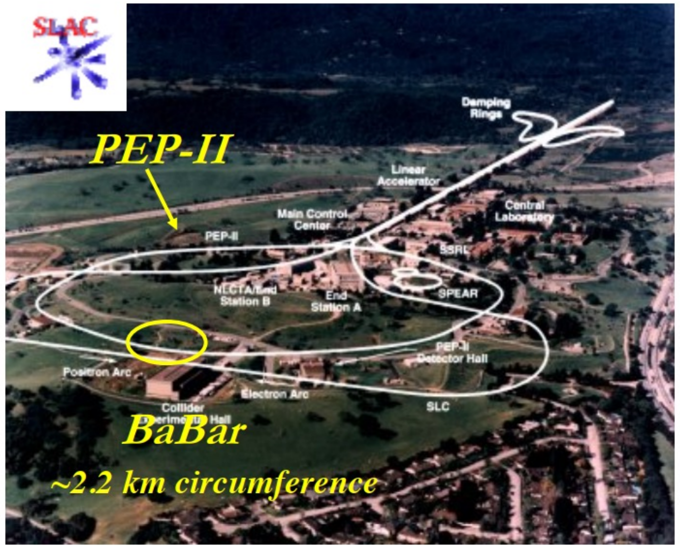
| $\not{C}\not{P}$ category | Hadronic system | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $K^0$ | $K^\pm$ | $\Lambda$ | $D^0$ | $D^\pm$ | $D_s^\pm$ | $\Lambda_c^+$ | $B^0$ | $B^\pm$ | $B_s^0$ | $\Lambda_b^0$ |
| decay | ✓ | ✗ | ✗ | ✓ | ✗ | ✗ | ✗ | ✓✓ | ✓✓ | ✓ | ✗ |
| mixing | ✓✓ | ⊖ | ⊖ | ✗ | ⊖ | ⊖ | ⊖ | ✗ | ⊖ | ✗ | ⊖ |
| decay/mixing interf. | ✓ | ⊖ | ⊖ | ✗ | ⊖ | ⊖ | ⊖ | ✓✓ | ⊖ | ✓ | ⊖ |

➡ ***although very complicated but necessary both theoretically and experimentally!***

# Exp. status of B physics

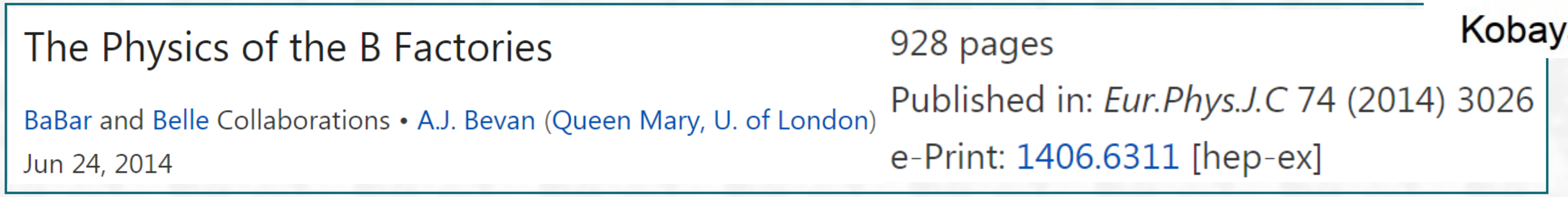
- ☐ **B-factories ($e^+e^-$): Belle and BaBar**

3.5 GeV $e^+$ 8 GeV $e^-$



3.1 GeV $e^+$ 9 GeV $e^-$

**BaBar & Belle confirmed the KM mechanism of CPV in the SM!**



The Physics of the B Factories

BaBar and Belle Collaborations • A.J. Bevan (Queen Mary, U. of London)

Jun 24, 2014

928 pages

Published in: *Eur.Phys.J.C* 74 (2014) 3026

e-Print: 1406.6311 [hep-ex]

- ☐ **Hadron colliders ($p\bar{p}$): CDF & D0 @ Tevatron**

https://www-d0.fnal;
https://www-cdf.fnal.gov/gov/

Observation of $B_s$ mixing

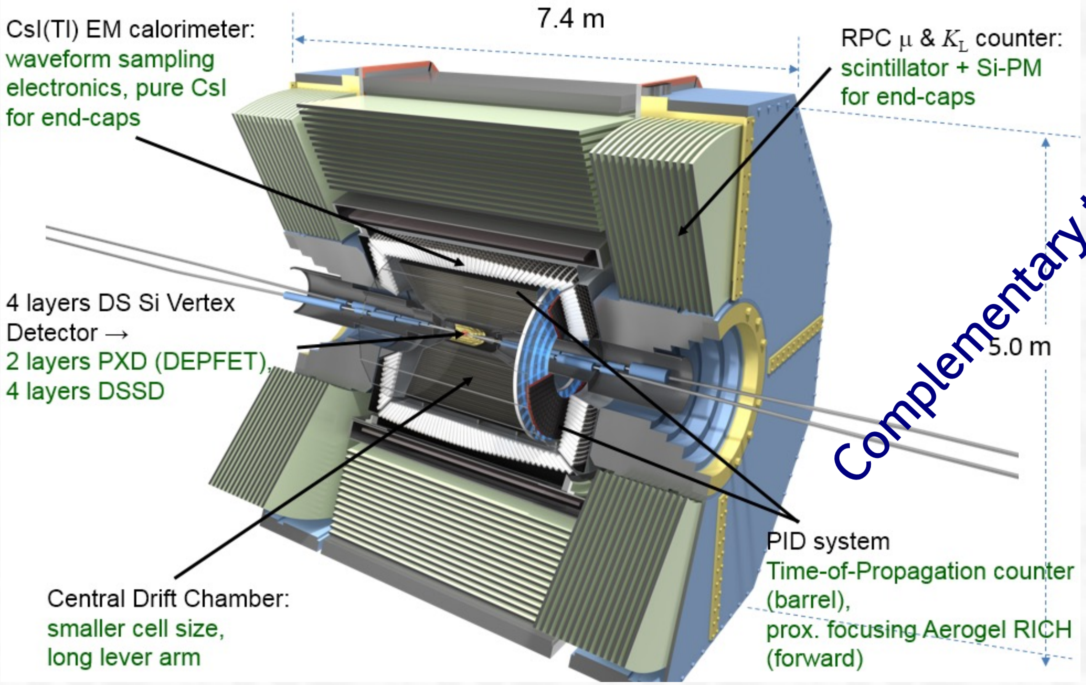
Nobel Prize 2008 for

Makoto Kobayashi

Toshihide Maskawa

# Exp. status of B physics

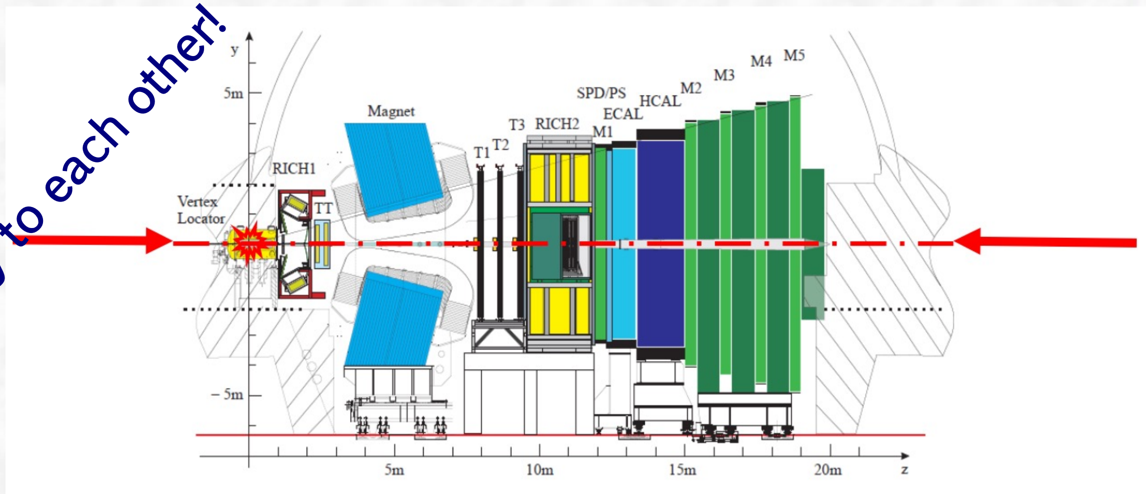
□ **Super B-factories ($e^+e^-$): Belle II**



[E. Kou *et al.* [Belle II], PTEP 2019 (2019) 123C01]

□ **Hadron colliders ($pp$): LHCb @LHC**

Complementary to each other!



[R. Aaij *et al.* [LHCb Collaboration], arXiv:1808.08865]

**LHCb & Belle II : the two currently running experiments aimed at heavy flavor physics!**

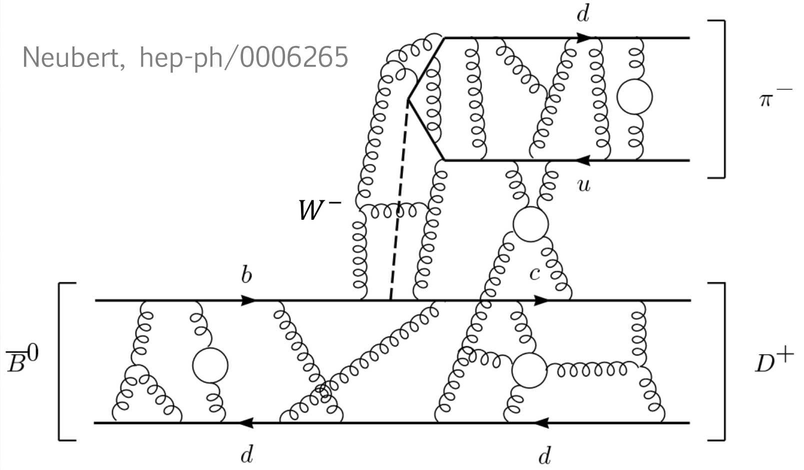
**Two main goals among others:**

- **Check if there are any extra new CP-violation mechanisms beyond the KM?**
- **Check if there are new particles/interactions that are sensitive to flavor structures?**

# Theoretical framework & QCDF for hadronic B decays

# Effective Hamiltonian for hadronic B decays

- ☐ **For hadronic B decays: typical multi-scale problem;** → **EFT formalism more suitable!**



Neubert, hep-ph/0006265

→

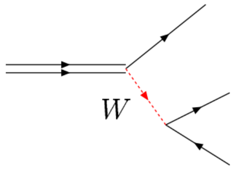
**multi-scale problem with highly hierarchical scales!**

| | | | | |
|---|---|---|---|---|
| EW interaction scale | ≫ | ext. mom'a in B rest frame | ≫ | QCD-bound state effects |
| $m_W \sim 80$ GeV<br>$m_Z \sim 91$ GeV | ≫ | $m_b \sim 5$ GeV | ≫ | $\Lambda_{\mathrm{QCD}} \sim 1$ GeV |

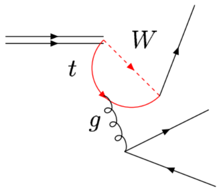
$$\mathcal{L}_{\mathrm{eff}} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pD}^{*} \left( C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2 + \sum_{i=\mathrm{pen}} C_i \mathcal{O}_{i,\mathrm{pen}} \right)$$

- ☐ **Starting point $\mathcal{H}_{\mathrm{eff}} = -\mathcal{L}_{\mathrm{eff}}$: obtained after integrating out heavy d.o.f. ($m_{W,Z,t} \gg m_b$);**

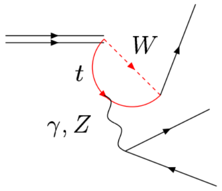
  [Buras, Buchalla, Lautenbacher '96; Chetyrkin, Misiak, Munz '98]



tree



QCD penguin



EW penguin

- ☐ **Wilson coefficients $C_i$: all physics above $m_b$; perturbatively calculable & NNLL program now complete;** [Gorbahn, Haisch '04; Misiak, Steinhauser '04]

# Hadronic matrix elements

- ☐ **For a typical two-body decay $\bar{B} \to M_1 M_2$:**

$$\boxed{\mathcal{A}(\bar{B} \to M_1 M_2) = \sum_i [\lambda_{\text{CKM}} \times C_i \times \langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle]}$$

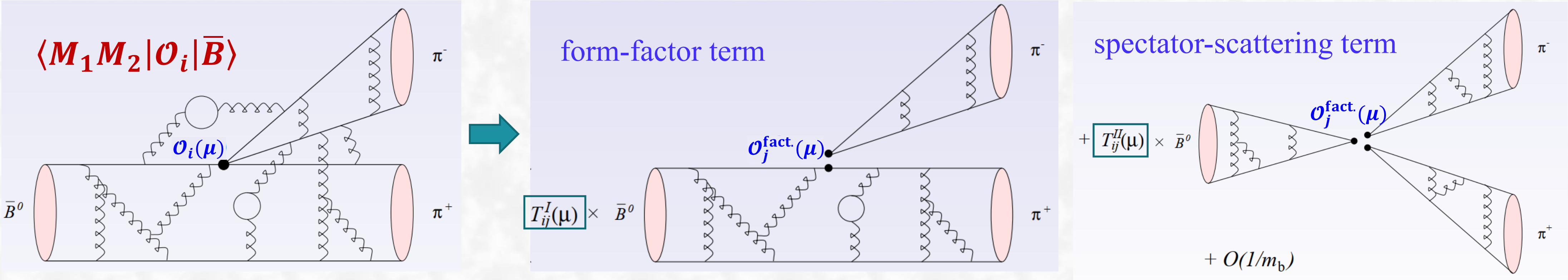
- ☐ $\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle$: depending on spin & parity of $M_{1,2}$; final-state rescattering introduces strong phases, and hence non-zero direct CPV; ⟹ *A quite difficult, multi-scale, strong-interaction problem!*

- ☐ **Different methods proposed for dealing with $\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle$:**

  - Dynamical approaches based on factorization theorems: PQCD, QCDF, SCET, · · ·
    [Keum, Li, Sanda, Lü, Yang '00;
    Beneke, Buchalla, Neubert, Sachrajda, '00;
    Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02]

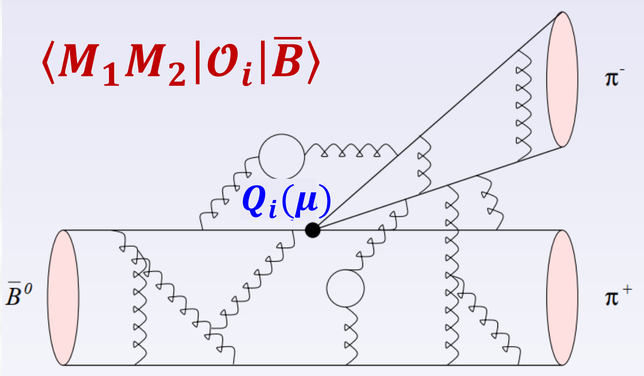
  - Symmetries of QCD: Isospin, U-Spin, V-Spin, and flavour SU(3) symmetries, · · ·
    [ Zeppenfeld, '81;
    London, Gronau, Rosner, He, Chiang, Cheng *et al.*]

- ☐ **QCDF:** systematic framework to all orders in $\alpha_s$, but limited by $\Lambda_{\text{QCD}}/m_b$ corrections. [BBNS '99-'03]



# QCDF formula

## ☐ QCDF formula: [BBNS '99-'03]



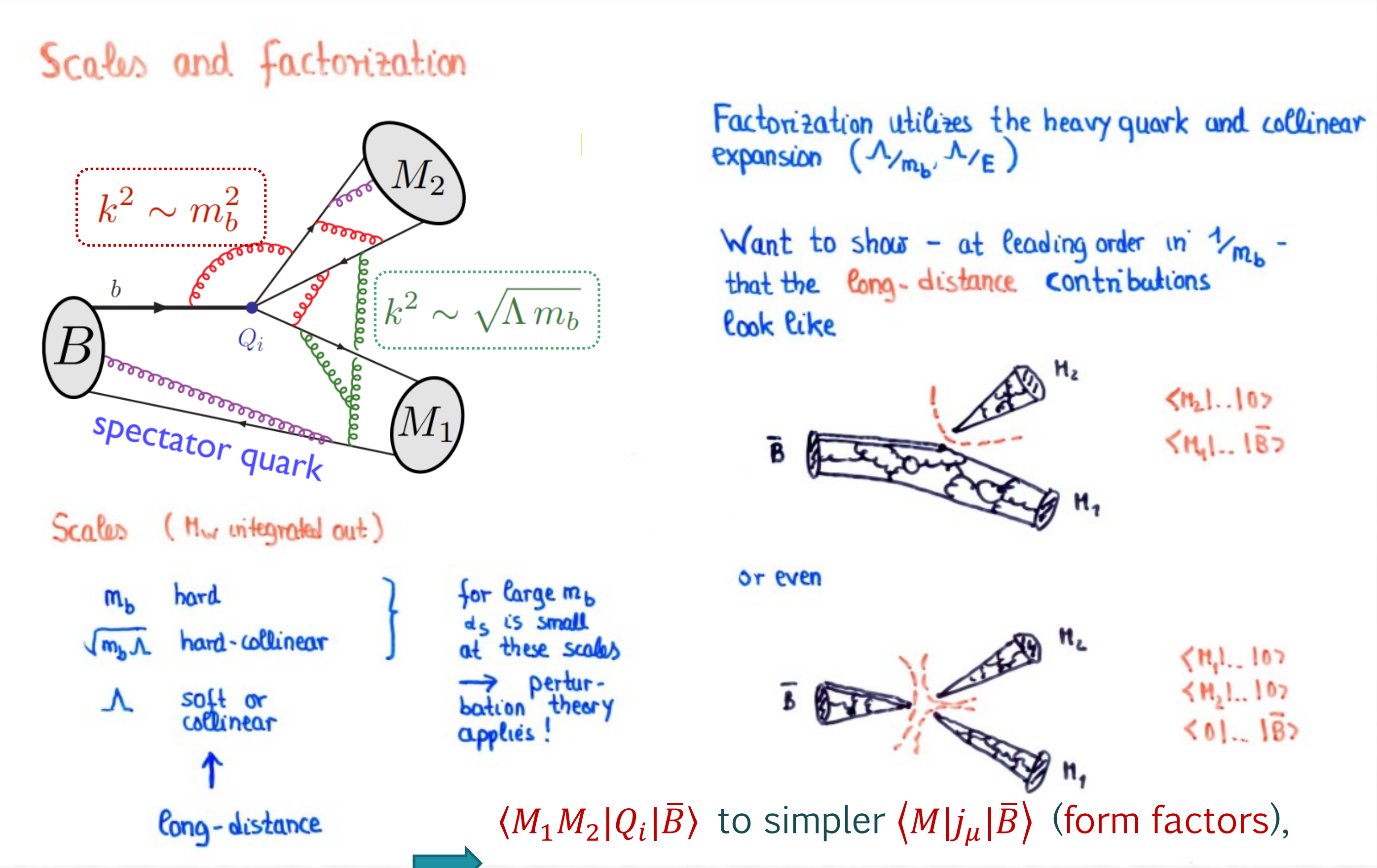
$$\langle M_1 M_2|Q_i|\bar{B}\rangle \sim F^{B\to M_1}(q^2=0)\int_0^1 dx\,\mathbf{T}_i^{\mathrm{I}}(x)\,\phi_{M_2}(x) \quad \text{form-factor term}$$

$$+\int_0^\infty \frac{d\omega}{\omega}\int_0^1 dx\,dy\;\mathbf{T}_i^{\mathrm{II}}(x,y,\omega)\,\phi_{M_1}(y)\,\phi_{M_2}(x)\,\phi_B^+(\omega)$$

spectator-scattering term

## ☐ How to proof the QCDF:

- based on diagrammatic factorization; [BBNS '99-'03]
- method of regions; [Beneke, Smirnov '97]
- heavy-quark & collinear expansion for hard processes [Lepage, Brodsky '80]



Scales and factorization

Factorization utilizes the heavy quark and collinear expansion $(\Lambda/m_b, \Lambda/E)$

Want to show – at leading order in $1/m_b$ – that the long-distance contributions look like

$\langle M_2|..|0\rangle$
$\langle M_1|..|\bar{B}\rangle$

or even

$\langle M_1|..|0\rangle$
$\langle M_2|..|0\rangle$
$\langle 0|..|\bar{B}\rangle$

Scales ($M_W$ integrated out)

| | |
|---|---|
| $m_b$ | hard |
| $\sqrt{m_b\Lambda}$ | hard-collinear |
| $\Lambda$ | soft or collinear |

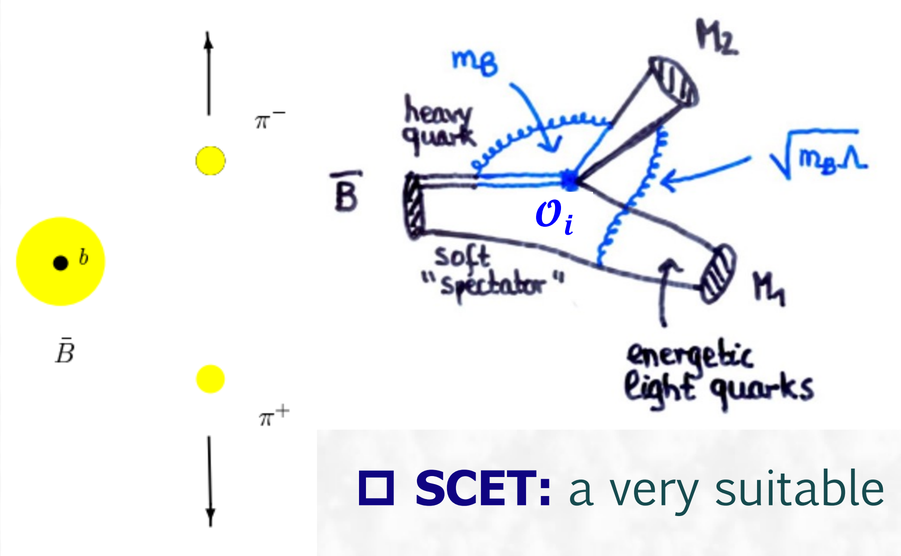
↑
long-distance

for large $m_b$ $\alpha_s$ is small at these scales → perturbation theory applies!

→ $\langle M_1 M_2|Q_i|\bar{B}\rangle$ to simpler $\langle M|j_\mu|\bar{B}\rangle$ (form factors), $\langle M|j_\mu|0\rangle$, $\langle 0|j_\mu|\bar{B}\rangle$ (decay constants & LCDAs)

# Soft-collinear factorization from SCET

☐ **For a two-body decay:** simple kinematics, but complicated dynamics with several typical modes;



- low-virtuality modes:
  - ⋆ HQET fields: $p - m_b v \sim$ $\mathcal{O}(\Lambda)$
  - ⋆ soft spectators in $B$ meson:
    $p_s^\mu \sim \Lambda \ll m_b, \quad p_s^2 \sim$ $\mathcal{O}(\Lambda^2)$
  - ⋆ collinear quarks and gluons in pion:
    $E_c \sim m_b, \quad p_c^2 \sim$ $\mathcal{O}(\Lambda^2)$
- high-virtuality modes:
  - ⋆ hard modes:
    $(\text{heavy quark} + \text{collinear})^2 \sim$ $\mathcal{O}(m_b^2)$
  - ⋆ hard-collinear modes:
    $(\text{soft} + \text{collinear})^2 \sim$ $\mathcal{O}(m_b \Lambda)$

☐ **SCET:** a very suitable framework for studying factorization and re-summation for processes involving energetic & light particles/jets; [Bauer *et al.* '00; Beneke *et al.* '02]

☐ **From SCET point of view:** introduce different fields/modes for different momentum regions, and SCET diagrams must reproduce precisely QCD diagrams in collinear & soft momentum region;

↳ achieve *soft-collinear factorization* & hence QCDF formula via QFT machinery [Beneke, 1501.07374]

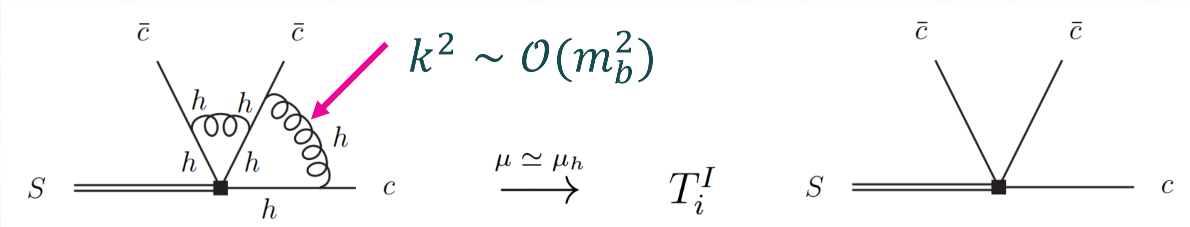
# Soft-collinear factorization from SCET

- ☐ **QCDF formula from SCET:** hard kernels $T^{I,II}$ = matching coefficients from QCD to SCET.

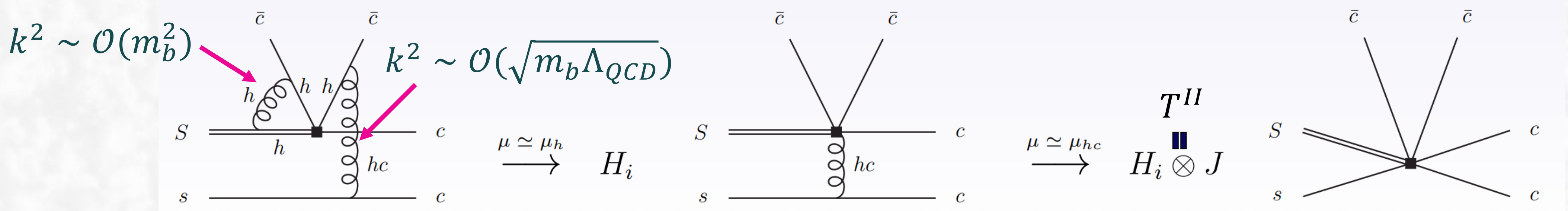
$$\langle M_1M_2|Q_i|\bar{B}\rangle = F^{BM_1}(0)\int_0^1 du\, \boxed{T_i^I(u)}\,\Phi_{M_2}(u) + \int_0^\infty d\omega \int_0^1 dudv\, \boxed{T_i^{II}(\omega,u,v)}\,\Phi_B(\omega)\Phi_{M_1}(v)\Phi_{M_2}(u)$$

➡ **QCD - SCET = $T^{I,II}$**

- ☐ **For $T^I$:** only hard scale involved, one-step matching from QCD → $\mathrm{SCET_I(hc, c, s)}$!



- ☐ **For $T^{II}$:** two scales involved, two-step matching from QCD → $\mathrm{SCET_I(hc, c, s)}$ → $\mathrm{SCET_{II}(c, s)}$!



- ☐ **SCET formalism reproduces exact QCDF result, but more apparent & efficient;** [Beneke, 1501.07374]

$$\langle M_1M_2|Q_i|\bar{B}\rangle = T^I(\mu_h) * \phi_{M_2}(\mu_h) f_+^{BM_1}(0) + H_i(\mu_h) * U_\parallel(\mu_h,\mu_{hc}) * J(\mu_{hc}) * \phi_{M_2}(\mu_h) * \phi_{M_1}(\mu_{hc}) * \phi_B(\mu_{hc})$$

# NNLO QCD corrections to hadronic matrix elements

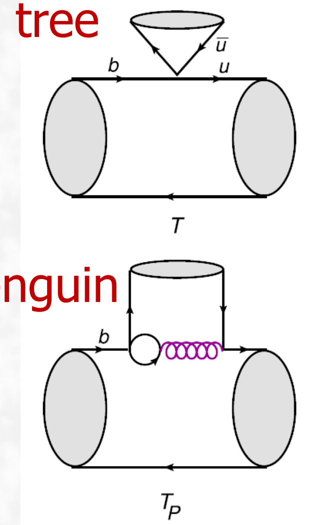
# Status of NNLO calculation of $T^I$ & $T^{II}$

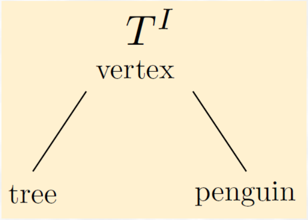
☐ **For each $Q_i$ insertion, both tree & penguin topologies relevant for charmless decays.**

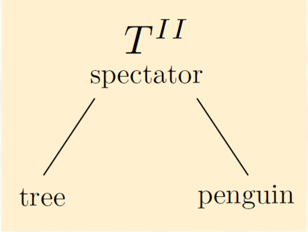
$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2}$$
$$+ T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

tree

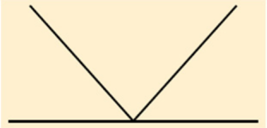
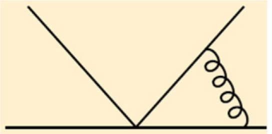
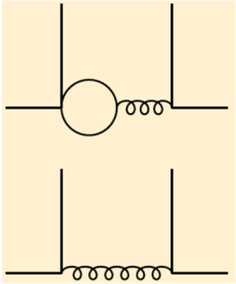
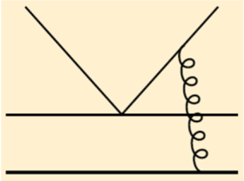
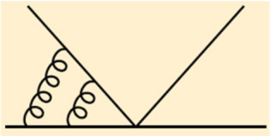
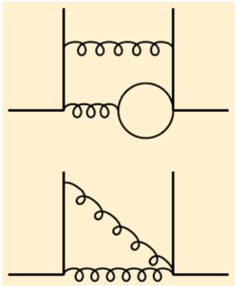
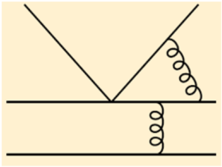
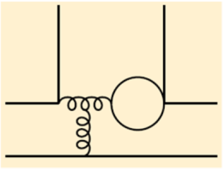
penguin





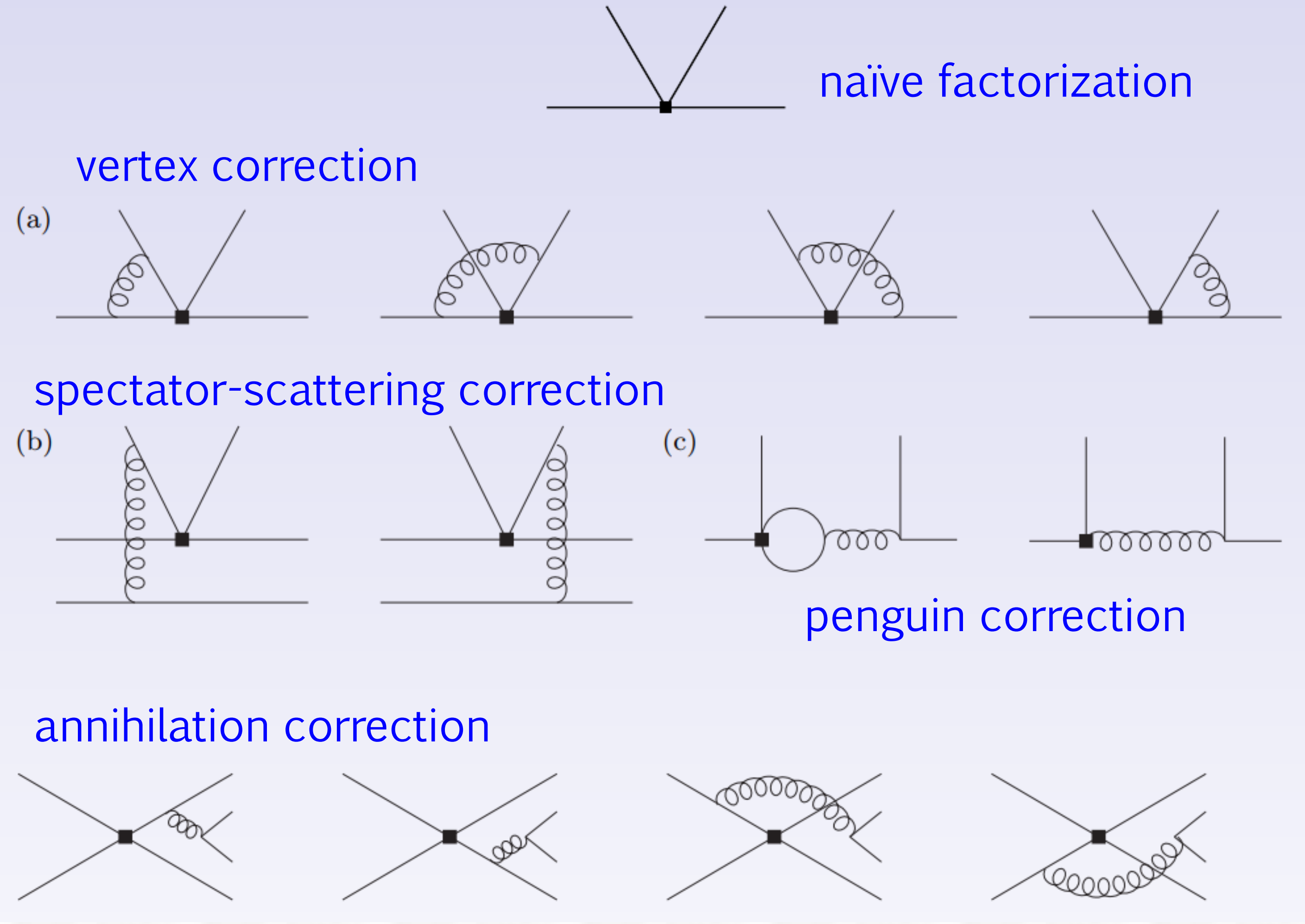


☐ **For tree & penguin topologies, both contribute to $T^I$ & $T^{II}$.**

| | $T_i^I$, tree | $T_i^I$, penguin | $T_i^{II}$, tree | $T_i^{II}$, penguin |
|---|---|---|---|---|
| LO: $\mathcal{O}(1)$ |  | $T^I = 1 + \mathcal{O}(\alpha_s) + \cdots$ | | |
| NLO: $\mathcal{O}(\alpha_s)$<br>BBNS '99-'03 |  |  |  | $T^{II} = \mathcal{O}(\alpha_s) + \cdots$ |
| NNLO: $\mathcal{O}(\alpha_s^2)$ | <br>Bell '07, '09<br>Beneke, Huber, Li '09<br>Huber, Krankl, Li '16 | <br>Kim, Yoon '11<br>Bell, Beneke, Huber, Li '15, '20 | <br>Beneke, Jager '05<br>Kivel '06, Pilipp '07 | <br>Beneke, Jager '06<br>Jain, Rothstein, Stewart '07 |

# Phenomenological analyses based on NLO

☐ **Various analyses based on NLO hard kernels.**



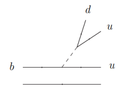
☐ **complete sets of final states:**

- $B \to PP, PV$: [Beneke, Neubert, hep-ph/0308039; Cheng, Chua, 0909.5229, 0910.5237;]
- $B \to VV$: [Beneke, Rohrer, Yang, hep-ph/0612290; Cheng, Yang, 0805.0329; Cheng, Chua, 0909.5229, 0910.5237;]
- $B \to AP, AV, AA$: [Cheng, Yang, 0709.0137, 0805.0329;]
- $B \to SP, SV$: [Cheng, Chua, Yang, hep-ph/0508104, 0705.3079; Cheng, Chua, Yang, Zhang, 1303.4403;]
- $B \to TP, TV$: [Cheng, Yang, 1010.3309;]

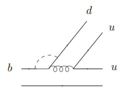
very successful but also with some problems phenomenologically. ！

# Phenomenological successes based on NLO

## ☐ Successes at NLO:



colour-allowed tree $\alpha_1$



QCD penguins $\alpha_4$

- For color-allowed tree- & penguin-dominated decay modes, branching ratios usually quantitatively OK;
- Dynamical explanation of intricate patterns of penguin interference seen in PP, PV, VP and VV modes;

$$PP \sim a_4 + r_\chi a_6, \quad PV \sim a_4 \approx \frac{PP}{3}$$
$$VP \sim a_4 - r_\chi a_6 \sim -PV$$
$$VV \sim a_4 \sim PV$$

$$\boxed{r_\chi = \frac{2m_L^2}{m_b\,(m_q + m_s)}}$$

$\Rightarrow$ $\mathrm{Br}\left(B^{\pm,0} \to \eta^{(\prime)} K^{(*)\pm,0}\right)$

- Qualitative explanation of polarization puzzle in $B \to VV$ decays, due to the large weak annihilation;
- Strong phases start at $\mathcal{O}(\alpha_s)$, dynamical explanation of smallness of direct CP asymmetries;

## ☐ Some problems encountered at NLO:

- Factorization of power correction generally broken, due to endpoint divergence;
- Could not account for some data, such as large $\mathrm{Br}(B^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K)$;
- How important the higher-order pert. corr.? Fact. theorem is still established for them?
- As strong phases start at $\mathcal{O}(\alpha_s)$, NNLO is only NLO to them; quite relevant for $A_{CP}$?

**we need go beyond the LO in pert. and power corrections!**

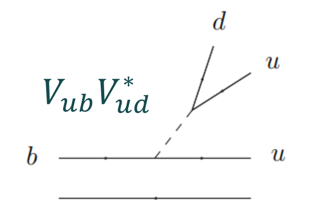
# Tree-dominated B decays

- $B \to \pi\pi$ **decay amplitudes in QCDF:**

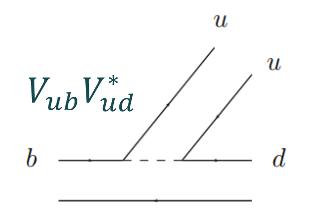
$$\sqrt{2}\,\langle \pi^-\pi^0 | \mathcal{H}_{eff} | B^- \rangle = \lambda_u \big[\alpha_1(\pi\pi) + \alpha_2(\pi\pi)\big]\; A_{\pi\pi}$$

$$\langle \pi^+\pi^- | \mathcal{H}_{eff} | \bar{B}^0 \rangle = \big\{\lambda_u \big[\alpha_1(\pi\pi) + \alpha_4^u(\pi\pi)\big] + \lambda_c\, \alpha_4^c(\pi\pi)\big\}\; A_{\pi\pi}$$

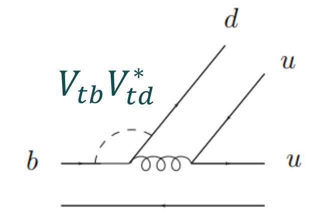
$$-\,\langle \pi^0\pi^0 | \mathcal{H}_{eff} | \bar{B}^0 \rangle = \big\{\lambda_u \big[\alpha_2(\pi\pi) - \alpha_4^u(\pi\pi)\big] - \lambda_c\, \alpha_4^c(\pi\pi)\big\}\; A_{\pi\pi}$$



colour-allowed tree $\alpha_1$



colour-suppressed tree $\alpha_2$



QCD penguins $\alpha_4$

**$b \to u\bar{u}d$:** $\lambda_u = V_{ub}V_{ud}^* \sim \mathcal{O}(\lambda^3) \sim \lambda_c = V_{cb}V_{cd}^* \sim \mathcal{O}(\lambda^3)$ → $\alpha_4$ loop-suppressed vs $\alpha_{1,2}$ → Tree-dominated!

- **$\alpha_2$ at NLO:** large cancellation between 1-loop vertex correction & LO result; also dominated by spectator-scattering;

$$\pi) = 0.220 - [0.179 + 0.077i]_{\rm NLO} + \left[\frac{r_{\rm sp}}{0.485}\right]\left\{[0.123]_{\rm LOsp} + [0.072]_{\rm tw3}\right\}$$

$$r_{\rm sp} = \frac{9 f_{M_1} \hat{f}_B}{m_b f_+^{B\pi}(0)\lambda_B}$$

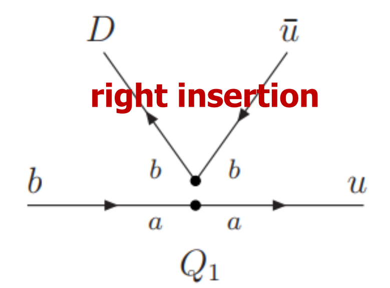
↳ **making $\alpha_2$ sensitive to NNLO corrections, and large effect possible!**

# Hard kernel $T^I$ at NNLO

## ☐ QCD → SCETI matching calculation:

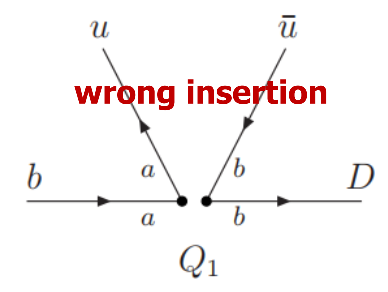
- For "right insertion":

$$\langle Q_i \rangle = T_i \, \langle O_{\rm QCD} \rangle + \sum_{a>1} H_{ia} \langle O_a \rangle$$



- For "wrong insertion":

$$\langle Q_i \rangle = \widetilde{T}_i \, \langle O_{\rm QCD} \rangle + \tilde{H}_{i1} \langle \tilde{O}_1 - O_1 \rangle + \sum_{a>1} \tilde{H}_{ia} \langle \tilde{O}_a \rangle$$



## ☐ On-shell matrix elements at NNLO: full QCD side

$$\begin{aligned}\langle Q_i \rangle = \Big\{ & A_{ia}^{(0)} + \frac{\alpha_s}{4\pi} \left[ A_{ia}^{(1)} + Z_{ext}^{(1)} A_{ia}^{(0)} + Z_{ij}^{(1)} A_{ja}^{(0)} \right] \\ & + \left( \frac{\alpha_s}{4\pi} \right)^2 \Big[ A_{ia}^{(2)} + Z_{ij}^{(1)} A_{ja}^{(1)} + Z_{ij}^{(2)} A_{ja}^{(0)} + Z_{ext}^{(1)} A_{ia}^{(1)} + Z_{ext}^{(2)} A_{ia}^{(0)} \\ & + Z_{ext}^{(1)} Z_{ij}^{(1)} A_{ja}^{(0)} + Z_{\alpha}^{(1)} A_{ia}^{(1)} + (-i)\, \delta m^{(1)} A_{ia}'^{(1)} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \langle O_a \rangle^{(0)}\end{aligned}$$

## ☐ On-shell matrix elements at NNLO: SCET side

$$\begin{aligned}\langle O_a \rangle = \Big\{ & \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi} \left[ M_{ab}^{(1)} + Y_{ext}^{(1)} \delta_{ab} + Y_{ab}^{(1)} \right] + \left( \frac{\hat{\alpha}_s}{4\pi} \right)^2 \Big[ M_{ab}^{(2)} + Y_{ac}^{(1)} M_{cb}^{(1)} \\ & + Y_{ab}^{(2)} + Y_{ext}^{(1)} M_{ab}^{(1)} + Y_{ext}^{(2)} \delta_{ab} + Y_{ext}^{(1)} Y_{ab}^{(1)} + \hat{Z}_{\alpha}^{(1)} M_{ab}^{(1)} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \langle O_b \rangle^{(0)}\end{aligned}$$

## ☐ Master formula for $T^I$: right insertion

$$\begin{aligned}T_i^{(0)} &= A_{i1}^{(0)} \,, \\ T_i^{(1)} &= A_{i1}^{(1)\rm nf} + Z_{ij}^{(1)} A_{j1}^{(0)} \,, \\ T_i^{(2)} &= A_{i1}^{(2)\rm nf} + Z_{ij}^{(1)} A_{j1}^{(1)} + Z_{ij}^{(2)} A_{j1}^{(0)} + Z_{\alpha}^{(1)} A_{i1}^{(1)\rm nf} + (-i)\, \delta m^{(1)} A_{i1}'^{(1)\rm nf} \\ & \quad - T_i^{(1)} [C_{FF}^{(1)} + Y_{11}^{(1)} - Z_{ext}^{(1)}] - \sum_{b>1} H_{ib}^{(1)} Y_{b1}^{(1)} \,.\end{aligned}$$

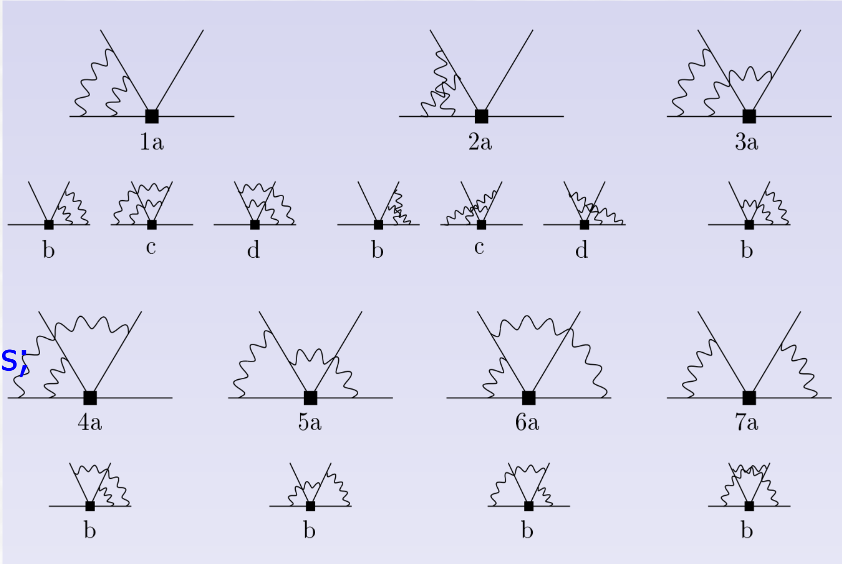
## ☐ Master formula for $T^I$: wrong insertion

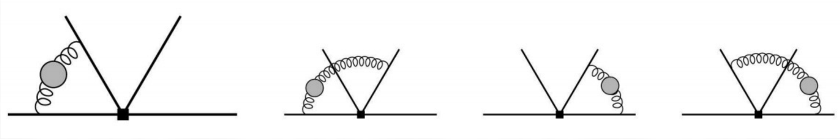
$$\begin{aligned}\widetilde{T}_i^{(0)} &= \widetilde{A}_{i1}^{(0)} \,, \\ \widetilde{T}_i^{(1)} &= \widetilde{A}_{i1}^{(1)\rm nf} + Z_{ij}^{(1)} \widetilde{A}_{j1}^{(0)} + \underbrace{\widetilde{A}_{i1}^{(1)\rm f} - A_{21}^{(1)\rm f} \widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] \widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} \,, \\ \widetilde{T}_i^{(2)} &= \widetilde{A}_{i1}^{(2)\rm nf} + Z_{ij}^{(1)} \widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)} \widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)} \widetilde{A}_{i1}^{(1)\rm nf} \\ & \quad + (-i)\, \delta m^{(1)} \widetilde{A}_{i1}'^{(1)\rm nf} + Z_{ext}^{(1)} [\widetilde{A}_{i1}^{(1)\rm nf} + Z_{ij}^{(1)} \widetilde{A}_{j1}^{(0)}] \\ & \quad - \widetilde{T}_i^{(1)} [C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}] - \sum_{b>1} \widetilde{H}_{ib}^{(1)} \widetilde{Y}_{b1}^{(1)} \\ & \quad + [\widetilde{A}_{i1}^{(2)\rm f} - A_{21}^{(2)\rm f} \widetilde{A}_{i1}^{(0)}] + (-i)\, \delta m^{(1)} [\widetilde{A}_{i1}'^{(1)\rm f} - A_{21}'^{(1)\rm f} \widetilde{A}_{i1}^{(0)}] \\ & \quad + (Z_{\alpha}^{(1)} + Z_{ext}^{(1)}) [\widetilde{A}_{i1}^{(1)\rm f} - A_{21}^{(1)\rm f} \widetilde{A}_{i1}^{(0)}] \\ & \quad - [\widetilde{M}_{11}^{(2)} - M_{11}^{(2)}] \widetilde{A}_{i1}^{(0)} \\ & \quad - (C_{FF}^{(1)} - \xi_{45}^{(1)}) [\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] \widetilde{A}_{i1}^{(0)} - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}] \widetilde{A}_{i1}^{(0)} \,.\end{aligned}$$

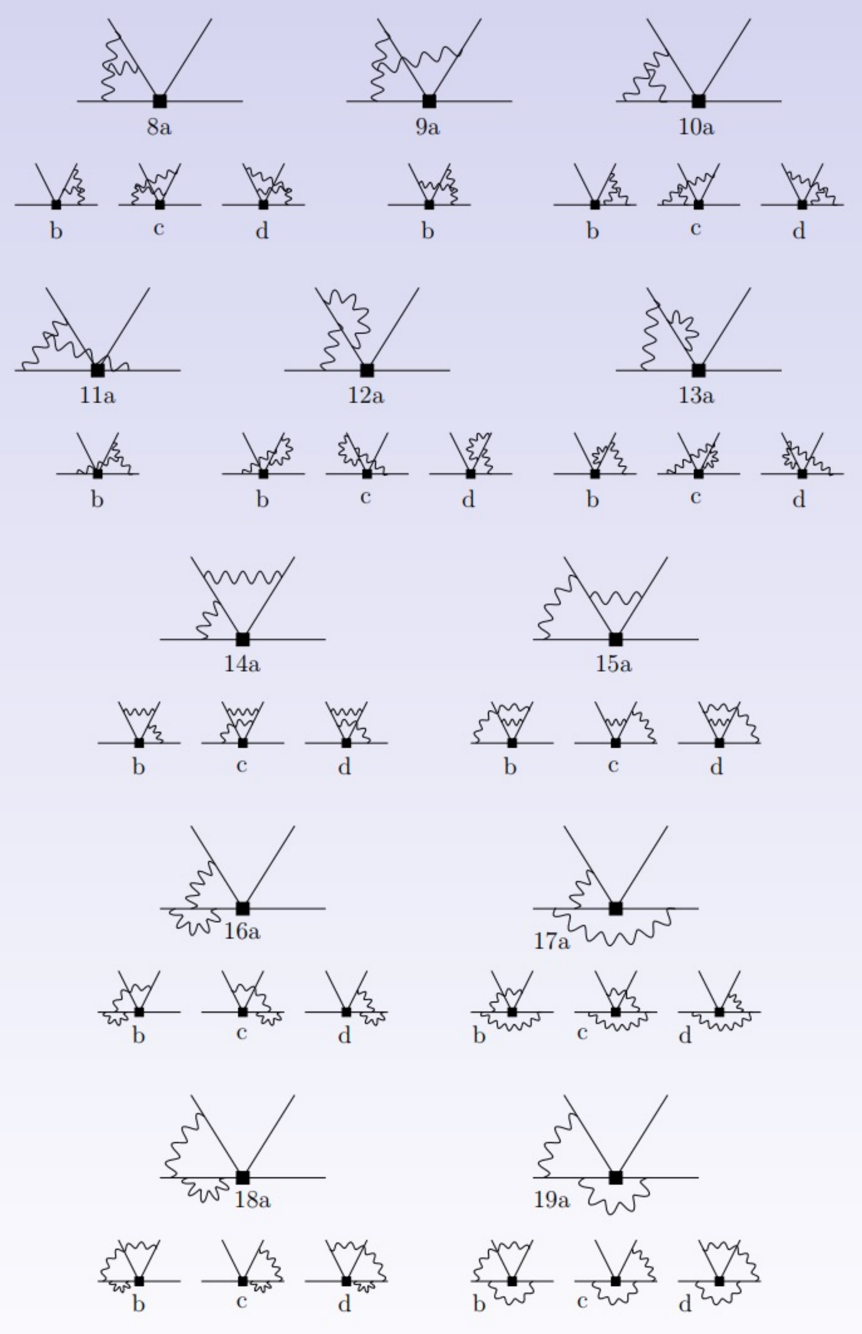
# Two-loop QCD diagrams

**☐ Relevant two-loop non-factorizable Feynman diagrams in full QCD: $\widetilde{A}_{i1}^{(2)\mathrm{nf}}$**

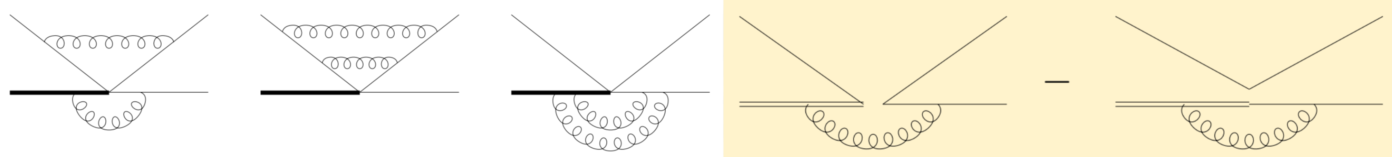
- totally ~ 70 diagrams;
- needs modern multi-loop Feynman diagrams techniques;
- IBP reduction, Mellin-Barnes representation, Differential equations, …







**☐ Complicated counter-terms from SCET operators:**



# Final results for $\alpha_{1,2}$

$$V_{1j}^{(0)} = \int_0^1 du\, T_j^{(0)} \phi_M(u), \qquad \frac{C_F}{2N_c} V_{1j}^{(l)} = \int_0^1 du\, T_j^{(l)}(u)\phi_M(u),$$

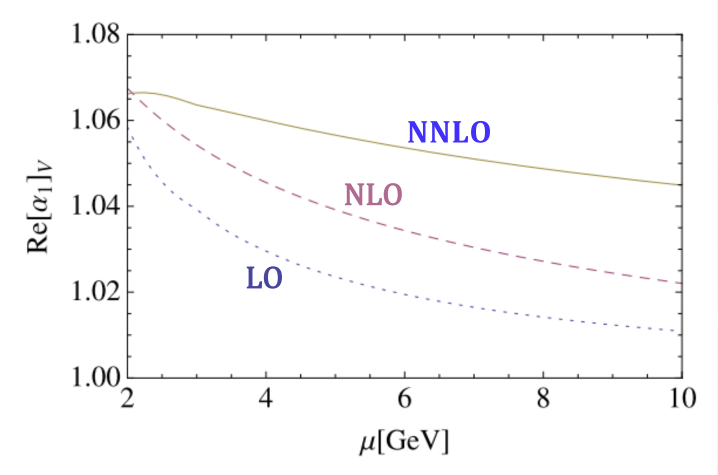
$$V_{2j}^{(0)} = \int_0^1 du\, \widetilde{T}_j^{(0)} \phi_M(u), \qquad \frac{C_F}{2N_c} V_{2j}^{(l)} = \int_0^1 du\, \widetilde{T}_j^{(l)}(u)\phi_M(u).$$

- ☐ **Tree amplitudes $\alpha_{1,2}$, after convolution with LCDAs:**

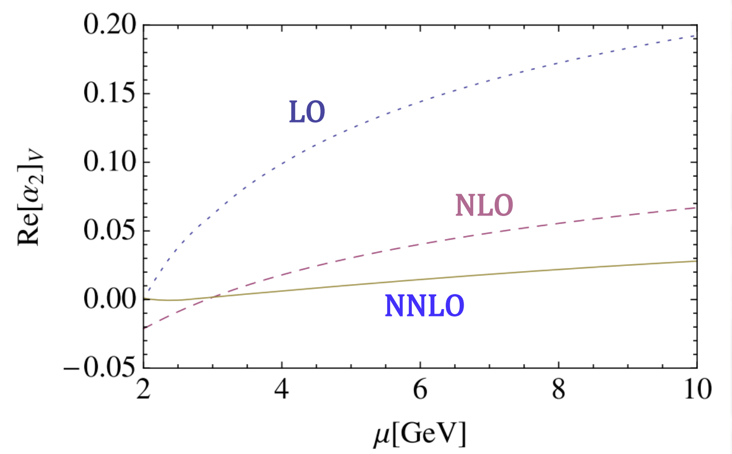
$$\alpha_i(M_1M_2) = \sum_j C_j V_{ij}^{(0)} + \sum_{l\geqslant 1}\left(\frac{\alpha_s}{4\pi}\right)^l \left[\frac{C_F}{2N_c}\sum_j C_j V_{ij}^{(l)} + P_i^{(l)}\right] + \cdots$$

- ☐ **Numerical results including the NNLO corrections:**

$$\begin{aligned}
\alpha_1(\pi\pi) &= 1.009 + [0.023 + 0.010\,i]_{\rm NLO} + [0.026 + 0.028\,i]_{\rm NNLO} \\
&\quad - \left[\frac{r_{\rm sp}}{0.445}\right]\Big\{[0.014]_{\rm LOsp} + [0.034 + 0.027i\,]_{\rm NLOsp} + [0.008]_{\rm tw3}\Big\} \\
&= 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})i \\
\alpha_2(\pi\pi) &= 0.220 - [0.179 + 0.077\,i]_{\rm NLO} - [0.031 + 0.050\,i]_{\rm NNLO} \\
&\quad + \left[\frac{r_{\rm sp}}{0.445}\right]\Big\{[0.114]_{\rm LOsp} + [0.049 + 0.051i\,]_{\rm NLOsp} + [0.067]_{\rm tw3}\Big\} \\
&= 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078})i
\end{aligned}$$



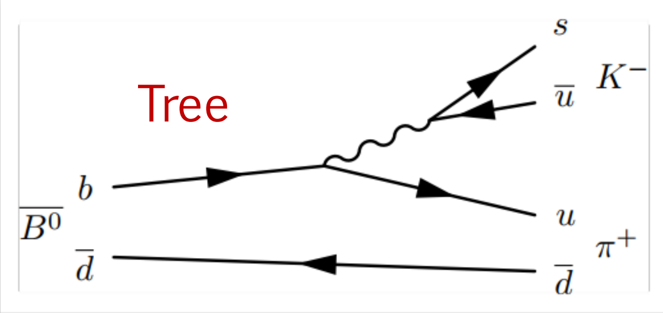
**Scale-dependence much reduced!**

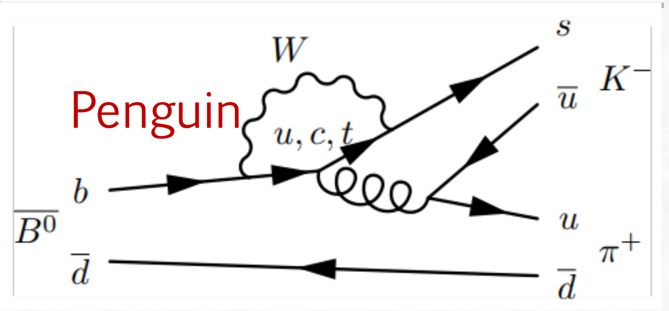


- ☐ **For tree amplitudes $\alpha_{1,2}$, cancellation between $T^I$ & $T^{II}$;**

$$\langle M_1M_2|\mathcal{O}_i|\bar{B}\rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

# Penguin-dominated B decays





- $B \to \pi K$ **decay amplitudes:** mediated by $b \to sq\bar q$ transitions;

$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} = A_{\pi\bar K}\big[\delta_{pu}\,\alpha_1 + \hat\alpha_4^p\big] + A_{\bar K\pi}\big[\delta_{pu}\alpha_2 + \delta_{pc}\tfrac{3}{2}\alpha_{3,\mathrm{EW}}^c\big],$$
$$\mathcal{A}_{\bar B^0\to\pi^+K^-} = A_{\pi\bar K}\big[\delta_{pu}\,\alpha_1 + \hat\alpha_4^p\big],$$

$\lambda_u = V_{ub}V_{us}^* \sim \mathcal{O}(\lambda^4) \ll \lambda_c = V_{cb}V_{cs}^* \sim \mathcal{O}(\lambda^2)$ ➡ Penguin-dominated!

- **In QCDF, strong phases generated firstly at NLO;**

$$A_{\rm CP} = [c \times \alpha_s]_{\rm NLO} + \mathcal{O}(\alpha_s^2, \Lambda/m_b)$$

➡ ***NNLO is only NLO for $A_{CP}$, large effects still possible***

↳ **To predict accurately direct CPV, we must calculate both tree & penguin up to NNLO;**

- **Driven by the current exp. data on $B \to \pi K$;**

$$\Delta A_{CP}(\pi K) = A_{CP}(B^- \to \pi^0 K^-) - A_{CP}(\bar B^0 \to \pi^+ K^-)$$
$$= (11.5 \pm 1.4)\%$$ **differs from 0 by ~8$\sigma$**

$\Delta A_{CP}$ puzzle

➡ ***How about the situation @ NNLO?***

# Penguin topologies with various insertions

**☐ Effective Hamiltonian including penguin operators:** [BBL '96; CMM '98]

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} V_{pD}^* V_{pb} \left( C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$$

$$Q_1^p = (\bar{p}_L \gamma^\mu T^A b_L)\,(\bar{D}_L \gamma_\mu T^A p_L),$$
$$Q_2^p = (\bar{p}_L \gamma^\mu b_L)\,(\bar{D}_L \gamma_\mu p_L),$$

current-current operators

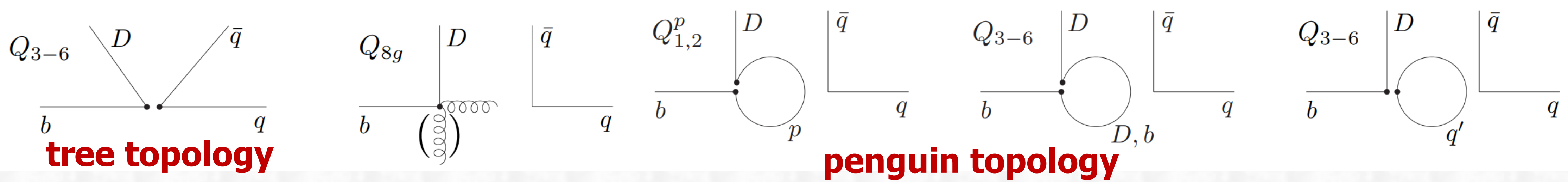
$$Q_3 = (\bar{D}_L \gamma^\mu b_L) \textstyle\sum_q (\bar{q}\gamma_\mu q),$$
$$Q_4 = (\bar{D}_L \gamma^\mu T^A b_L) \textstyle\sum_q (\bar{q}\gamma_\mu T^A q),$$
$$Q_5 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \textstyle\sum_q (\bar{q}\gamma_\mu \gamma_\nu \gamma_\rho q),$$
$$Q_6 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho T^A b_L) \textstyle\sum_q (\bar{q}\gamma_\mu \gamma_\nu \gamma_\rho T^A q)\,.$$

QCD penguin operators

$$Q_{8g} = \frac{-g_s}{32\pi^2}\, \overline{m}_b\, \bar{D}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu} b,$$

chromo-magnetic dipole operators

**☐ Various operator insertions:**



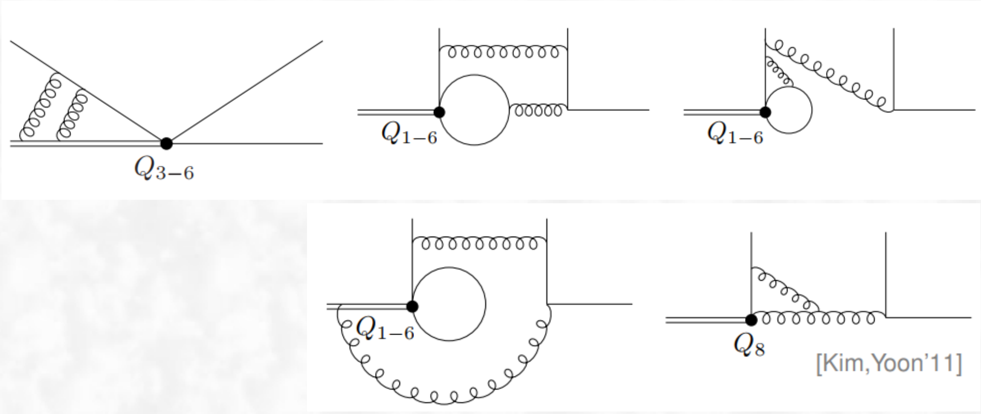
(i) Dirac structure of $Q_i$, (ii) color structure of $Q_i$, (iii) types of contraction, and (iv) quark mass in the fermion loop

# $T^I$ up to NNLO

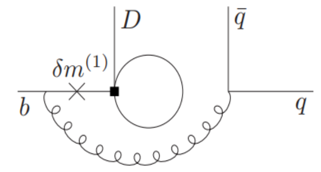
## ☐ Master formulae for $T^I$:

$$
\begin{aligned}
\frac{1}{2}\,\widetilde{T}_i^{(2)} = {} & \boxed{\widetilde{A}_{i1}^{(2)\mathrm{nf}}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\,\widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)}\,\widetilde{A}_{i1}^{(1)\mathrm{nf}} \\
& + \boxed{(-i)\,\delta m^{(1)}\,\widetilde{A}_{i1}^{\prime(1)\mathrm{nf}}} + Z_{\mathrm{ext}}^{(1)}\,[\widetilde{A}_{i1}^{(1)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)}] \\
& - \frac{1}{2}\,\widetilde{T}_i^{(1)}\,[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}] - \sum_{b>1} \widetilde{H}_{ib}^{(1)}\,\widetilde{Y}_{b1}^{(1)} \\
& + [\widetilde{A}_{i1}^{(2)\mathrm{f}} - A_{31}^{(2)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}] + (-i)\,\delta m^{(1)}\,[\widetilde{A}_{i1}^{\prime(1)\mathrm{f}} - A_{31}^{\prime(1)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)} \\
& + (Z_{\alpha}^{(1)} + Z_{\mathrm{ext}}^{(1)})\,[\widetilde{A}_{i1}^{(1)\mathrm{f}} - A_{31}^{(1)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}] \\
& - [\widetilde{M}_{11}^{(2)} - M_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)} \\
& - (C_{FF}^{(1)} - \xi_{45}^{(1)})\,[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\,\widetilde{A}_{i1}^{(0)} - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)} \\
& - \boxed{\sum_{b>1} \widetilde{A}_{ib}^{(0)}\,\widetilde{M}_{b1}^{(2)} - \sum_{b>1} \widetilde{A}_{ib}^{(0)}\,\widetilde{Y}_{b1}^{(2)}}\,.
\end{aligned}
$$

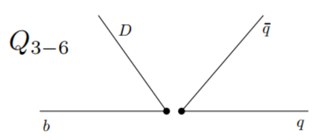
**~ 100 two-loop Feynman diagrams**



[Kim,Yoon'11]



non-vanishing fermion-tadpole contraction of four-quark operators



tree-level matching of $Q_i$ involves already evanescent SCET operators

## ☐ Complication during calculations:

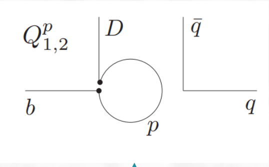
(i) fermion loop with either $m = 0, m = m_c$ or $m = m_b$.

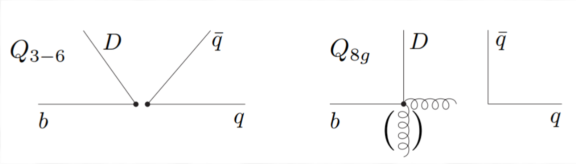
(ii) genuine 2-loop two-scale problem: $\bar{u},\ z_c = m_c^2/m_b^2$.

(iii) threshold at $\bar{u} = 4z_c$ introduces strong phase.

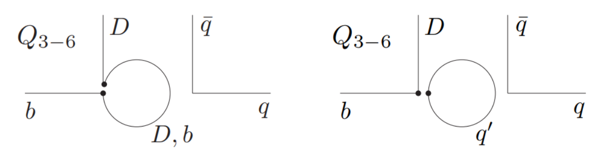
# Final results for $a_4^p$

## ☐ Final numerical results:





$$
\begin{aligned}
a_4^u(\pi\bar K)/10^{-2} &= -2.87 - [0.09+0.09i]_{\mathrm{V}_1} + [0.49-1.32i]_{\mathrm{P}_1} - [0.32+0.71i]_{\mathrm{P}_2,\,\mathrm{Q}_{1,2}} + [0.33+0.38i]_{\mathrm{P}_2,\,\mathrm{Q}_{3-6,8}} \\
&\quad + \left[\frac{r_{\mathrm{sp}}}{0.434}\right]\Big\{[0.13]_{\mathrm{LO}} + [0.14+0.12i]_{\mathrm{HV}} - [0.01-0.05i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}}\Big\} \\
&= (-2.12^{+0.48}_{-0.29}) + (-1.56^{+0.29}_{-0.15})i\,,
\end{aligned}
$$

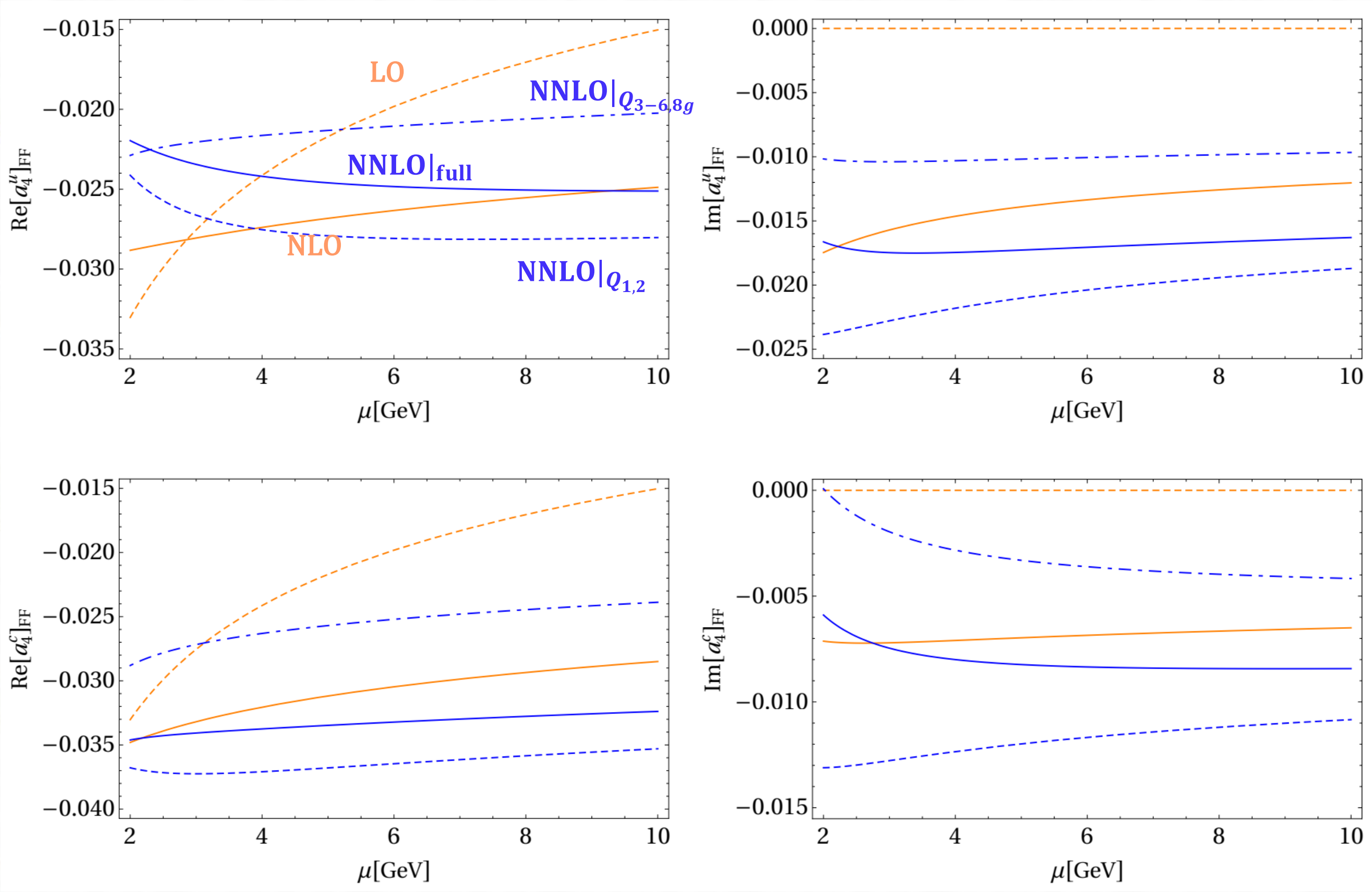


$$
\begin{aligned}
a_4^c(\pi\bar K)/10^{-2} &= -2.87 - [0.09+0.09i]_{\mathrm{V}_1} + [0.05-0.62i]_{\mathrm{P}_1} - [0.77+0.50i]_{\mathrm{P}_2,\,\mathrm{Q}_{1,2}} + [0.33+0.38i]_{\mathrm{P}_2,\,\mathrm{Q}_{3-6,8}} \\
&\quad + \left[\frac{r_{\mathrm{sp}}}{0.434}\right]\Big\{[0.13]_{\mathrm{LO}} + [0.14+0.12i]_{\mathrm{HV}} + [0.01+0.03i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}}\Big\} \\
&= (-3.00^{+0.45}_{-0.32}) + (-0.67^{+0.50}_{-0.39})i\,.
\end{aligned}
$$

- individual NNLO contributions from $Q^p_{1,2}$ and $Q_{3-6,8g}$ are significant.
- strong cancellation between NNLO corrections from $Q^p_{1,2}$ and $Q_{3-6,8g}$.

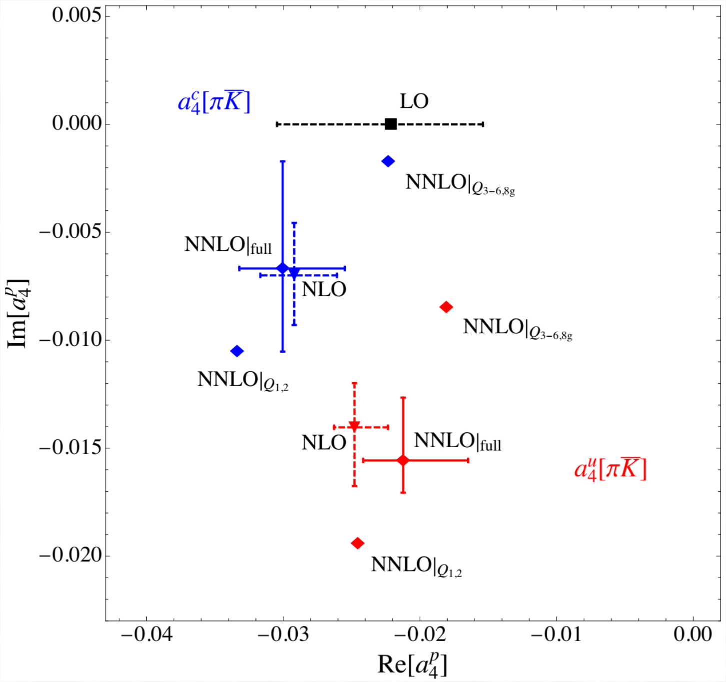
# Scale dependence of $a_4^p$

**☐ Scale dependence of $a_4^p$: only form-factor term;**



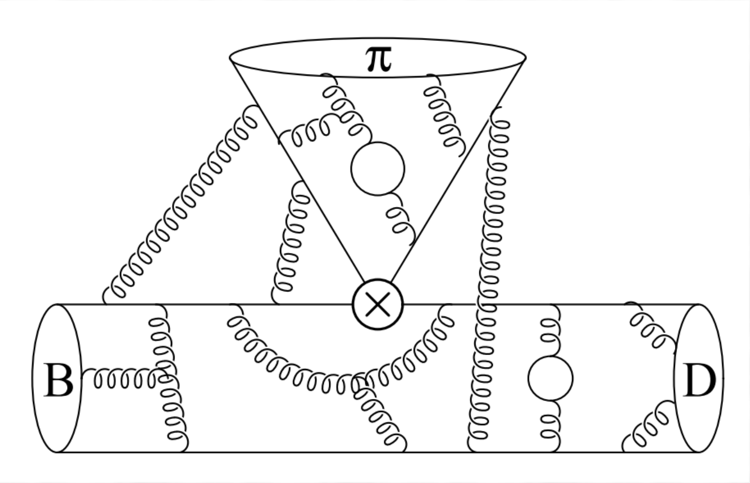
- Scale dependence negligible, especially for μ > 4 GeV.

**☐ Results at different orders:**



- Total NNLO effects small.
- Theoretical uncertainty is larger at NNLO than at NLO.

# $B_q^0 \to D_q^{(*)-} L^+$ class-I decays



- **At quark-level, mediated by $b \to c\bar{u}d(s)$;**

  all four flavors different from each other,

  no penguin operators & no penguin topologies!

- **For class-I decays:** QCDF formula much simpler;

  only the form-factor term at leading power;

  [Beneke, Buchalla, Neubert, Sachrajda '99-'03; Bauer, Pirjol, Stewart '01]

$$\mathcal{Q}_2 = \bar{d}\gamma_\mu(1-\gamma_5)u \;\; \bar{c}\gamma^\mu(1-\gamma_5)b$$
$$\mathcal{Q}_1 = \bar{d}\gamma_\mu(1-\gamma_5)T^A u \;\; \bar{c}\gamma^\mu(1-\gamma_5)T^A b$$

$$\langle D_q^{(*)+} L^- | \, \mathcal{Q}_i \, | \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du \, T_{ij}(u) \phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)$$

i) only color-allowed tree topology $a_1$;

ii) spectator & annihilation power-suppressed;

iii) annihilation absent in $B_{d(s)}^0 \to D_{d(s)}^- K(\pi)^+$ etc.;

iv) they are theoretically simpler and cleaner!

- **Hard kernel $T$:** both NLO and NNLO results known;

  [Beneke, Buchalla, Neubert, Sachrajda '01; Huber, Kränkl, Li '16]

$$T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)$$

# Calculation of $T^I$

## ☐ Matching QCD onto SCET$_{\mathrm{I}}$ : [Huber, Kränkl, Li '16]

$m_c$ also heavy, must keep $m_c/m_b$ fixed as $m_b \to \infty$,

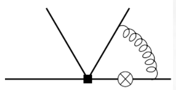
thus needing two sets of SCET operator basis.

$$\langle \mathcal{Q}_i \rangle = \hat{T}_i \langle \mathcal{Q}^{\mathrm{QCD}} \rangle + \hat{T}'_i \langle \mathcal{Q}'^{\mathrm{QCD}} \rangle + \sum_{a>1} [H_{ia} \langle \mathcal{O}_a \rangle + H'_{ia} \langle \mathcal{O}'_a \rangle]$$

## ☐ Renormalized on-shell QCD amplitudes:

$$\begin{aligned} \langle \mathcal{Q}_i \rangle = \Big\{ & A_{ia}^{(0)} + \frac{\alpha_s}{4\pi} \Big[ A_{ia}^{(1)} + Z_{ext}^{(1)} A_{ia}^{(0)} + Z_{ij}^{(1)} A_{ja}^{(0)} \Big] \\ & + \Big( \frac{\alpha_s}{4\pi} \Big)^2 \Big[ A_{ia}^{(2)} + Z_{ij}^{(1)} A_{ja}^{(1)} + Z_{ij}^{(2)} A_{ja}^{(0)} + Z_{ext}^{(1)} A_{ia}^{(1)} + Z_{ext}^{(2)} A_{ia}^{(0)} + Z_{ext}^{(1)} Z_{ij}^{(1)} A_{ja}^{(0)} \\ & + (-i)\delta m_b^{(1)} A_{ia}^{*(1)} + \boxed{(-i)\delta m_c^{(1)} A_{ia}^{**(1)}} + Z_\alpha^{(1)} A_{ia}^{(1)} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \langle \mathcal{O}_a \rangle^{(0)} \\ & + (A \leftrightarrow A') \langle \mathcal{O}'_a \rangle^{(0)} . \end{aligned}$$

on QCD side



## ☐ Renormalized on-shell SCET amplitudes:

$$\begin{aligned} \langle \mathcal{O}_a \rangle = \Big\{ & \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi} \Big[ M_{ab}^{(1)} + Y_{ext}^{(1)} \delta_{ab} + Y_{ab}^{(1)} \Big] \\ & + \Big( \frac{\hat{\alpha}_s}{4\pi} \Big)^2 \Big[ M_{ab}^{(2)} + Y_{ext}^{(1)} M_{ab}^{(1)} + Y_{ac}^{(1)} M_{cb}^{(1)} + \hat{Z}_\alpha^{(1)} M_{ab}^{(1)} + Y_{ext}^{(2)} \delta_{ab} \\ & + Y_{ext}^{(1)} Y_{ab}^{(1)} + Y_{ab}^{(2)} \Big] + \mathcal{O}(\hat{\alpha}_s^3) \Big\} \langle \mathcal{O}_b \rangle^{(0)} , \end{aligned}$$

on SCET side

physical operators and factorizes into FF*LCDA.

$$\begin{aligned} \mathcal{O}_1 =& \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \chi \;\; \bar{h}_{v'} \not{n}_+ (1-\gamma_5) h_v , \\ \mathcal{O}_2 =& \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \chi \;\; \bar{h}_{v'} \not{n}_+ (1-\gamma_5) \gamma_{\perp,\beta} \gamma_{\perp,\alpha} h_v , \\ \mathcal{O}_3 =& \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma \gamma_\perp^\delta \chi \;\; \bar{h}_{v'} \not{n}_+ (1-\gamma_5) \gamma_{\perp,\delta} \gamma_{\perp,\gamma} \gamma_{\perp,\beta} \gamma_{\perp,\alpha} h_v \end{aligned}$$

$$\begin{aligned} \mathcal{O}'_1 =& \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \chi \;\; \bar{h}_{v'} \not{n}_+ (1+\gamma_5) h_v , \\ \mathcal{O}'_2 =& \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \chi \;\; \bar{h}_{v'} \not{n}_+ (1+\gamma_5) \gamma_{\perp,\alpha} \gamma_{\perp,\beta} h_v , \\ \mathcal{O}'_3 =& \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma \gamma_\perp^\delta \chi \;\; \bar{h}_{v'} \not{n}_+ (1+\gamma_5) \gamma_{\perp,\alpha} \gamma_{\perp,\beta} \gamma_{\perp,\gamma} \gamma_{\perp,\delta} h_v \end{aligned}$$

evanescent operators and must be renormalized to zero.

## ☐ Master formulas for hard kernels:

$$T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)$$

$$\boxed{\begin{aligned} \hat{T}_i^{(0)} =& A_{i1}^{(0)} \\ \hat{T}_i^{(1)} =& A_{i1}^{(1)nf} + Z_{ij}^{(1)} A_{j1}^{(0)} \\ \hat{T}_i^{(2)} =& A_{i1}^{(2)nf} + Z_{ij}^{(1)} A_{j1}^{(1)} + Z_{ij}^{(2)} A_{j1}^{(0)} + Z_\alpha^{(1)} A_{i1}^{(1)nf} - \hat{T}_i^{(1)} \Big[ C_{FF}^{\mathrm{D}(1)} + Y_{11}^{(1)} - Z_{\mathrm{ext}}^{(1)} \Big] \\ & - C_{FF}^{\mathrm{ND}(1)} \hat{T}_i'^{(1)} + (-i)\delta m_b^{(1)} A_{i1}^{*(1)nf} + (-i)\delta m_c^{(1)} A_{i1}^{**(1)nf} - \sum_{b\neq 1} H_{ib}^{(1)} Y_{b1}^{(1)} . \end{aligned}}$$

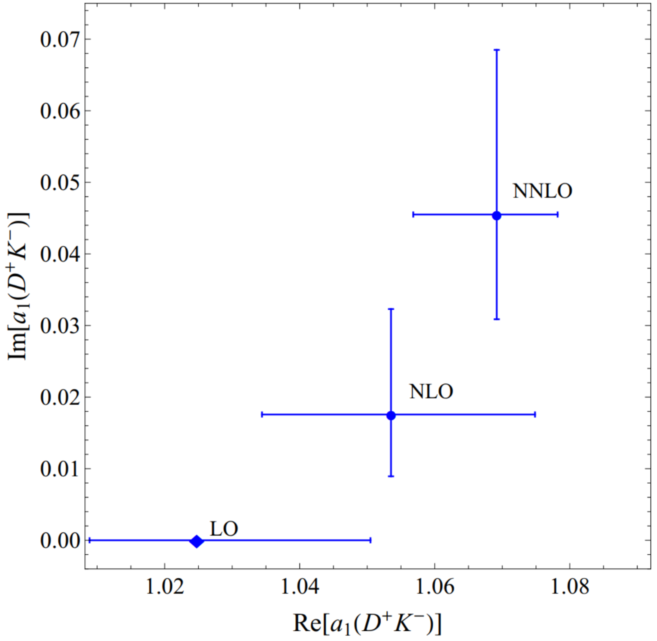
# Decay amplitudes for $B_q^0 \to D_q^- L^+$

☐ **Color-allowed tree amplitude $a_1$:**

$$a_1(D^+L^-) = \sum_{i=1}^{2} C_i(\mu) \int_0^1 du \left[\hat{T}_i(u,\mu) + \hat{T}_i'(u,\mu)\right] \Phi_L(u,\mu)\,,$$
$$a_1(D^{*+}L^-) = \sum_{i=1}^{2} C_i(\mu) \int_0^1 du \left[\hat{T}_i(u,\mu) - \hat{T}_i'(u,\mu)\right] \Phi_L(u,\mu)\,,$$

☐ **Numerical result:**

$$\begin{aligned} a_1(D^+K^-) &= 1.025 + [0.029 + 0.018i]_{\rm NLO} + [0.016 + 0.028i]_{\rm NNLO} \\ &= (1.069^{+0.009}_{-0.012}) + (0.046^{+0.023}_{-0.015})i\,, \end{aligned}$$



- both NLO and NNLO add always constructively to LO result!
- NNLO corrections quite small in real (2%), but rather large in imaginary part (60%).

$$\begin{aligned} a_1(D^+K^-) &= (1.069^{+0.009}_{-0.012}) + (0.046^{+0.023}_{-0.015})i\,, \\ a_1(D^+\pi^-) &= (1.072^{+0.011}_{-0.013}) + (0.043^{+0.022}_{-0.014})i\,, \\ a_1(D^{*+}K^-) &= (1.068^{+0.010}_{-0.012}) + (0.034^{+0.017}_{-0.011})i\,, \\ a_1(D^{*+}\pi^-) &= (1.071^{+0.012}_{-0.013}) + (0.032^{+0.016}_{-0.010})i\,. \end{aligned}$$

☐ **For different decay modes:** *quasi-universal*, with small process dependence from *non-factorizable correction*.

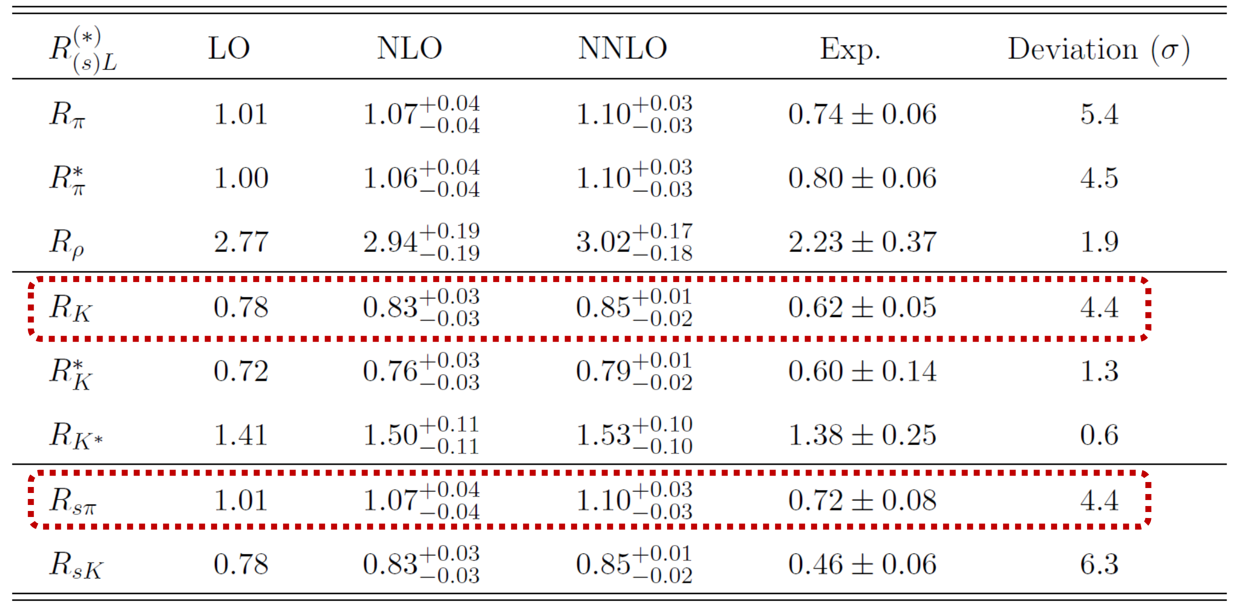
# Non-leptonic/semi-leptonic ratios

## ☐ Non-leptonic/semi-leptonic ratios : [Bjorken '89; Neubert, Stech '97; Beneke, Buchalla, Neubert, Sachrajda '01]

$$R^{(*)}_{(s)L} \equiv \frac{\Gamma(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} L^-)}{d\Gamma(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} \ell^- \bar{\nu}_\ell)/dq^2\,|_{q^2=m_L^2}} = 6\pi^2\,|V_{uq}|^2\, f_L^2\,|a_1(D^{(*)+}_{(s)} L^-)|^2\, X_L^{(*)}$$

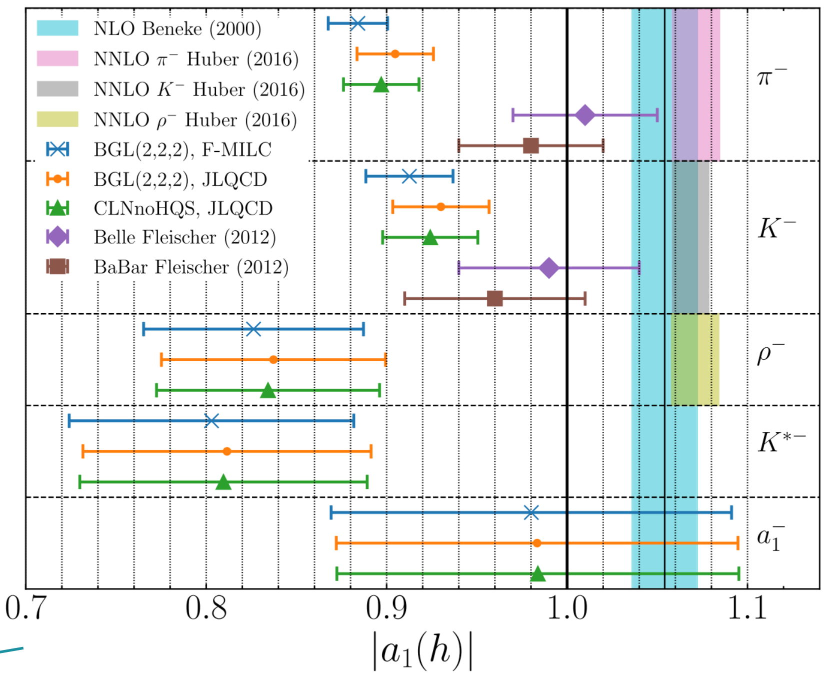
free from uncertainties from $V_{cb}$ & $B_{d,s} \to D^{(*)}_{d,s}$ form factors.

## ☐ Updated predictions vs data: [Huber, Kränkl, Li '16; Cai, Deng, Li, Yang '21]



| $R^{(*)}_{(s)L}$ | LO | NLO | NNLO | Exp. | Deviation $(\sigma)$ |
|---|---|---|---|---|---|
| $R_\pi$ | 1.01 | $1.07^{+0.04}_{-0.04}$ | $1.10^{+0.03}_{-0.03}$ | $0.74 \pm 0.06$ | 5.4 |
| $R^*_\pi$ | 1.00 | $1.06^{+0.04}_{-0.04}$ | $1.10^{+0.03}_{-0.03}$ | $0.80 \pm 0.06$ | 4.5 |
| $R_\rho$ | 2.77 | $2.94^{+0.19}_{-0.19}$ | $3.02^{+0.17}_{-0.18}$ | $2.23 \pm 0.37$ | 1.9 |
| $R_K$ | 0.78 | $0.83^{+0.03}_{-0.03}$ | $0.85^{+0.01}_{-0.02}$ | $0.62 \pm 0.05$ | 4.4 |
| $R^*_K$ | 0.72 | $0.76^{+0.03}_{-0.03}$ | $0.79^{+0.01}_{-0.02}$ | $0.60 \pm 0.14$ | 1.3 |
| $R_{K^*}$ | 1.41 | $1.50^{+0.11}_{-0.11}$ | $1.53^{+0.10}_{-0.10}$ | $1.38 \pm 0.25$ | 0.6 |
| $R_{s\pi}$ | 1.01 | $1.07^{+0.04}_{-0.04}$ | $1.10^{+0.03}_{-0.03}$ | $0.72 \pm 0.08$ | 4.4 |
| $R_{sK}$ | 0.78 | $0.83^{+0.03}_{-0.03}$ | $0.85^{+0.01}_{-0.02}$ | $0.46 \pm 0.06$ | 6.3 |

## ☐ Latest Belle data: 2207.00134



$|a_1(\bar{B} \to D^{*+}\pi^-)| = 0.884 \pm 0.004 \pm 0.003 \pm 0.016$ $[1.071^{+0.020}_{-0.016}]$;

15% lower than SM

$|a_1(\bar{B} \to D^{*+}K^-)| = 0.913 \pm 0.019 \pm 0.008 \pm 0.013$ $[1.069^{+0.020}_{-0.016}]$;

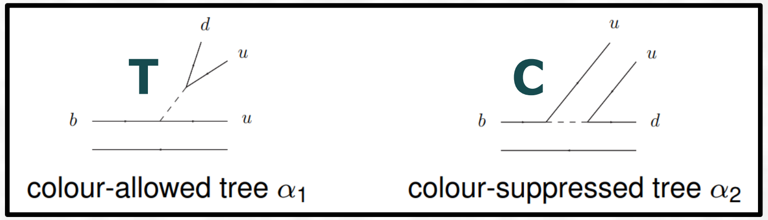
# Status of NNLO calculation of $T^I$ & $T^{II}$

☐ **Complete NNLO calculation for $T^I$ & $T^{II}$ at leading power in QCDF/SCET now complete;**

☐ **Soft-collinear factorization at 2-loop level established via explicit calculations;**

☐ **For tree amplitudes, cancellation between $T^I$ & $T^{II}$;**

$$\langle M_1 M_2|\mathcal{O}_i|\bar{B}\rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$



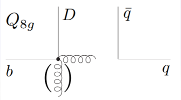
colour-allowed tree $\alpha_1$

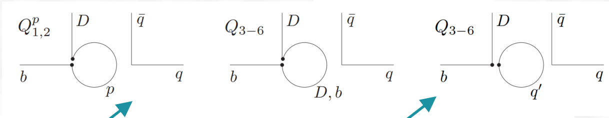
colour-suppressed tree $\alpha_2$

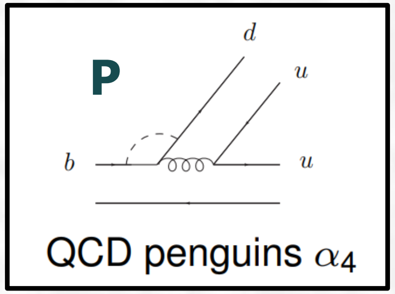
$$\begin{aligned}\alpha_1(\pi\pi) &= 1.009 + [0.023 + 0.010\,i]_{\rm NLO} + [0.026 + 0.028\,i]_{\rm NNLO} \\ &\quad - \left[\frac{r_{\rm sp}}{0.445}\right]\left\{[0.014]_{\rm LOsp} + [0.034 + 0.027i\,]_{\rm NLOsp} + [0.008]_{\rm tw3}\right\} \\ &= 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})i\end{aligned}$$

$$\begin{aligned}\alpha_2(\pi\pi) &= 0.220 - [0.179 + 0.077\,i]_{\rm NLO} - [0.031 + 0.050\,i]_{\rm NNLO} \\ &\quad + \left[\frac{r_{\rm sp}}{0.445}\right]\left\{[0.114]_{\rm LOsp} + [0.049 + 0.051i\,]_{\rm NLOsp} + [0.067]_{\rm tw3}\right\} \\ &= 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078})i\end{aligned}$$

☐ **For QCD penguin amplitude, cancellation between $Q^p_{1,2}$ & $Q_{3-6,8g}$;**







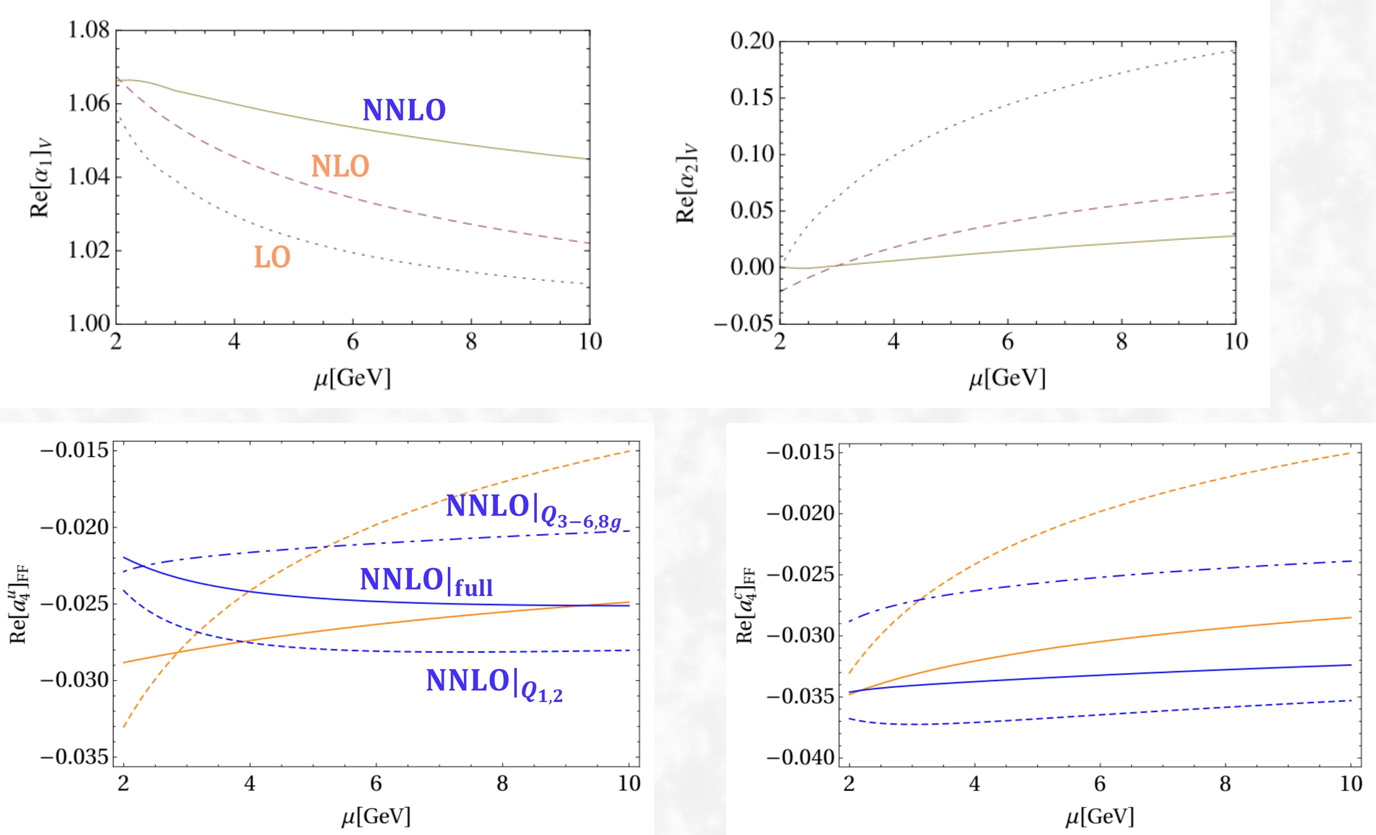
QCD penguins $\alpha_4$

$$\begin{aligned}a_4^u(\pi\bar{K})/10^{-2} &= -2.87 - [0.09 + 0.09i]_{\rm V_1} + [0.49 - 1.32i]_{\rm P_1} - [0.32 + 0.71i]_{\rm P_2,\,Q_{1,2}} + [0.33 + 0.38i]_{\rm P_2,\,Q_{3-6,8}} \\ &\quad + \left[\frac{r_{\rm sp}}{0.434}\right]\left\{[0.13]_{\rm LO} + [0.14 + 0.12i]_{\rm HV} - [0.01 - 0.05i]_{\rm HP} + [0.07]_{\rm tw3}\right\} \\ &= (-2.12^{+0.48}_{-0.29}) + (-1.56^{+0.29}_{-0.15})i\,,\end{aligned}$$

# Scale dependence of $a_{1,2}$ and $a_4^p$

- ☐ **Phen., NNLO corrections have no much effects compared to the NLO predictions;** [w.i.p]

- ☐ **The scale dependence much reduced for $a_{1,2}$ & $a_4^p$ : only form−factor term**
  - ➢ **scale dependence negligible, especially for $\mu > 4$ GeV.**



- ☐ **More precise than NLO results, and hence welcome for precision data @ LHCb & Belle II;**

Factorization also valid? New sources of strong phases?

- ☐ **Main issue in QCDF/SCET: sub-leading power-corrections** $\sim \Lambda_{QCD}/m_b \simeq 0.2$ **unknown!**

# Summary

- **With exp. and theor. progress, we are now entering a precision era for flavour physics!**

- **Within QCDF/SCET framework, NNLO QCD corrections to color-allowed, color-suppressed tree & leading-power penguin amplitudes complete, factorization at 2-loop established.**

- **Due to delicate cancellation, NNLO corrections small; some puzzles still remain:**
  - long-standing $\mathrm{Br}(\bar{B}^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K) = A_{CP}(B^- \to \pi^0 K^-) - A_{CP}(\bar{B}^0 \to \pi^+ K^-)$;
  - for class-I $B_q^0 \to D_q^{(*)-} L^+$ decays, $\mathcal{O}(4\text{–}5\sigma)$ discrepancies observed in branching ratios;

➡ **sub-leading power corrections in QCDF/SCET need to be considered!**

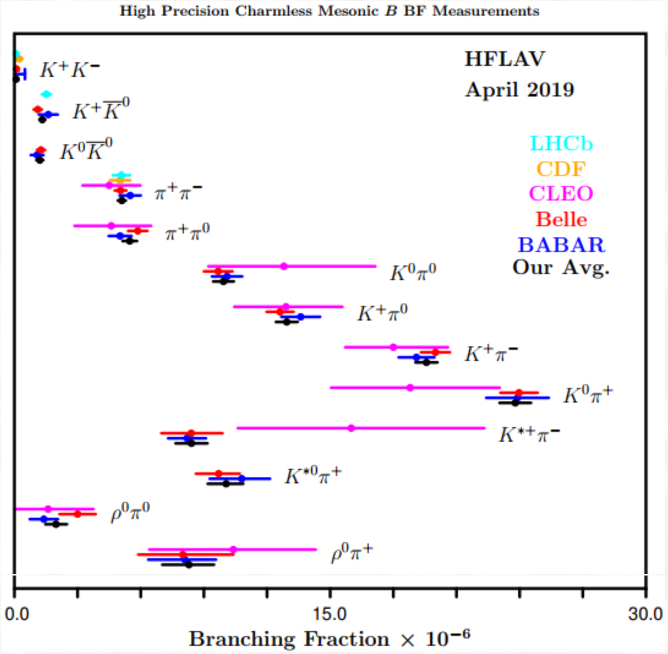
  - Sub-leading color-octet matrix elements $\langle M_1 M_2 | [\bar{u}_c T^A h_v]_{\Gamma_1} [\bar{s}_{\bar{c}} T^A u_{\bar{c}}]_{\Gamma_2}(u) | \bar{B} \rangle$ ; [w.i.p]
  - improved treatments of annihilation amplitudes: SU(3)-breaking effects & flavor-dependence of the building blocks $A_{1,2}^i$; [w.i.p]

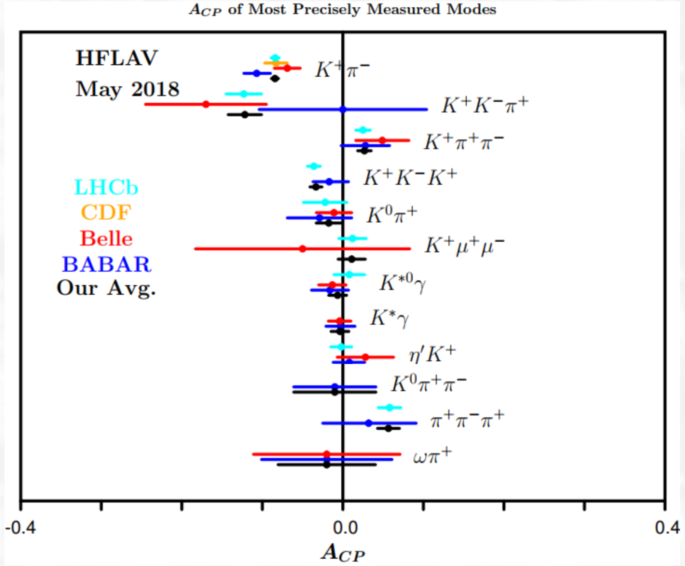
Thank You for your attention!

# Backup

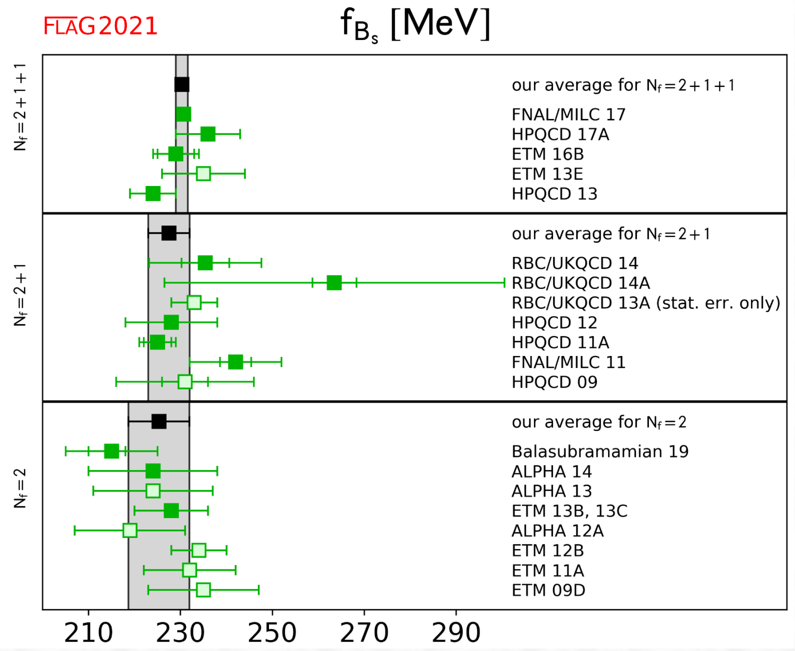
# Precision era of B physics

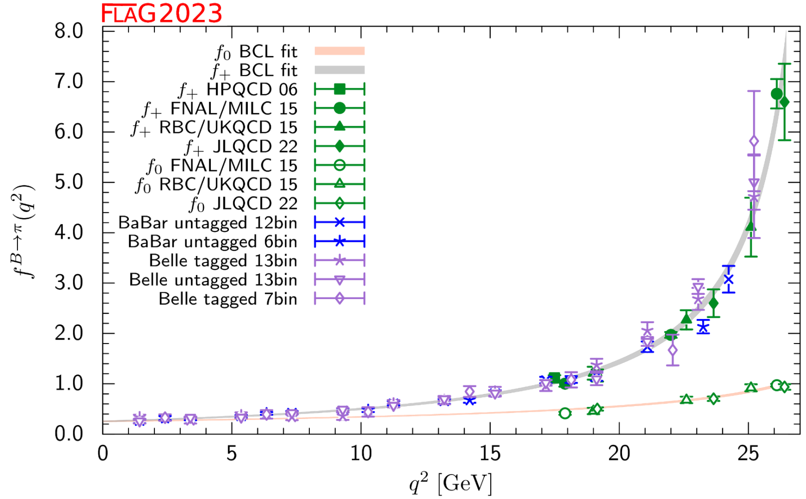
- ☐ **More precise data from these B-dedicated experiments!**





https://hflav.web.cern.ch/





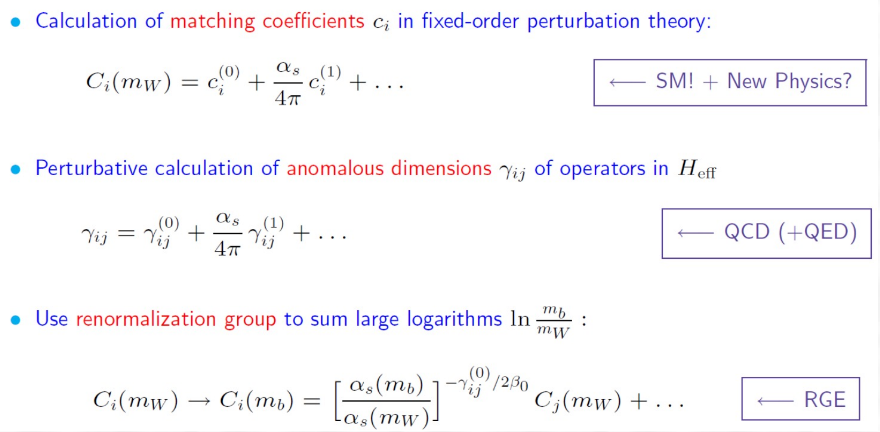
http://flag.unibe.ch/2021/

- ☐ **Lattice QCD & LCSR etc. also provide more precise results for the non-pert. hadronic parameters!**

↳ **we are entering an *era of precision flavor physics!***

# Local operators for hadronic B decays

## □ Three steps for Wilson coefficients:



- Calculation of matching coefficients $c_i$ in fixed-order perturbation theory:

$$C_i(m_W) = c_i^{(0)} + \frac{\alpha_s}{4\pi} c_i^{(1)} + \ldots$$

⟵ SM! + New Physics?

- Perturbative calculation of anomalous dimensions $\gamma_{ij}$ of operators in $H_{\rm eff}$

$$\gamma_{ij} = \gamma_{ij}^{(0)} + \frac{\alpha_s}{4\pi} \gamma_{ij}^{(1)} + \ldots$$

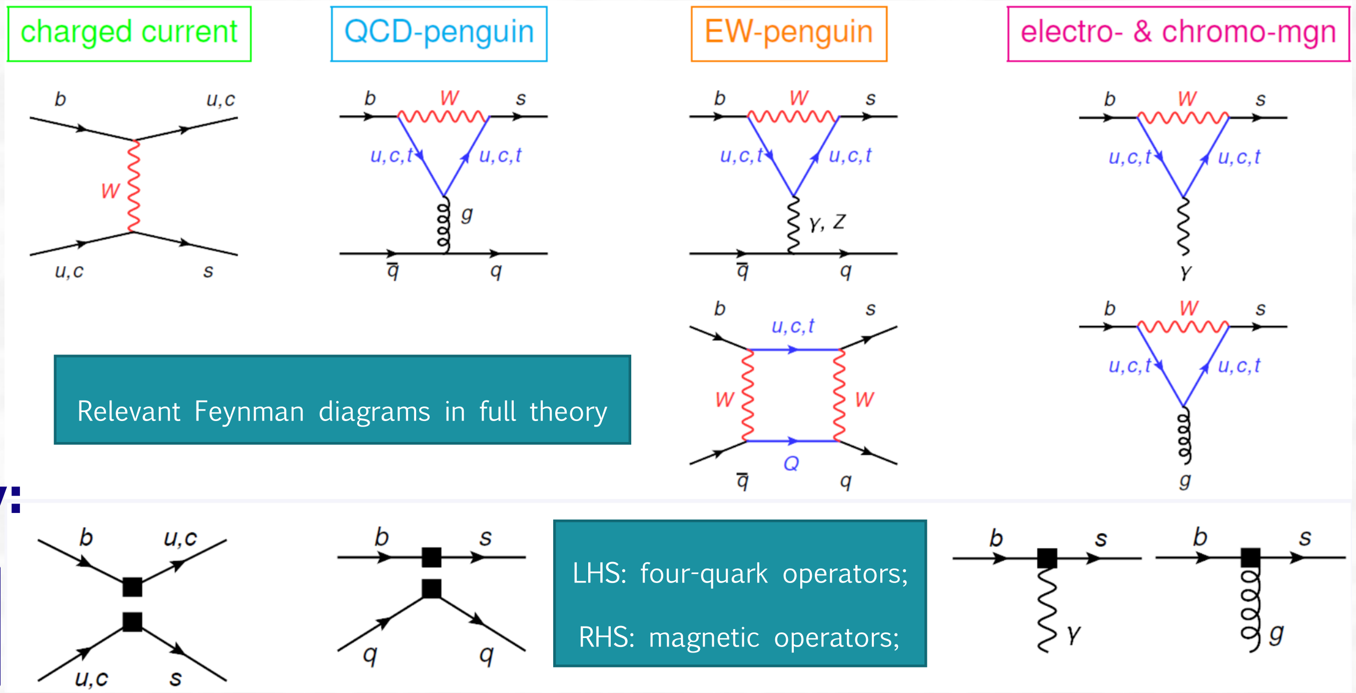
⟵ QCD (+QED)

- Use renormalization group to sum large logarithms $\ln \frac{m_b}{m_W}$ :

$$C_i(m_W) \to C_i(m_b) = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_W)}\right]^{-\gamma_{ij}^{(0)}/2\beta_0} C_j(m_W) + \ldots$$

⟵ RGE

## □ Local operators $\mathcal{O}_i$:



Relevant Feynman diagrams in full theory

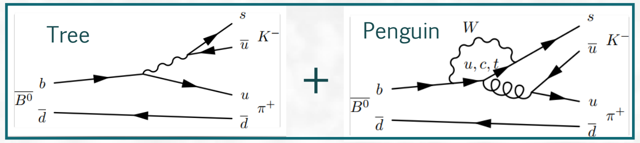
LHS: four-quark operators;

RHS: magnetic operators;

## □ Decay amplitude for a given decay:

$$\boxed{\mathcal{A}(\bar{B} \to f) = \sum_i \left[\, \lambda_{\rm CKM} \times C \times \langle f|\mathcal{O}|\bar{B}\rangle_{\rm QCD+QED} \,\right]_i}$$

hadronic matrix elements at $m_b$



## □ CKM factors $\lambda_p^{(D)} \equiv V_{pb}V_{pD}^*$:

- for $b \to d$, $\lambda_u^{(d)} \sim \lambda_c^{(d)} \sim \lambda^3$, tree-dominated, like $\bar{B}^0 \to \pi^+ \pi^-$

-for $b \to s$, $\lambda_u^{(s)} \sim \lambda^4 \ll \lambda_c^{(s)} \sim \lambda^2$, penguin-dominated, like $\bar{B}^0 \to \pi^+ K^-$ ➡ *interference induces CPV!*

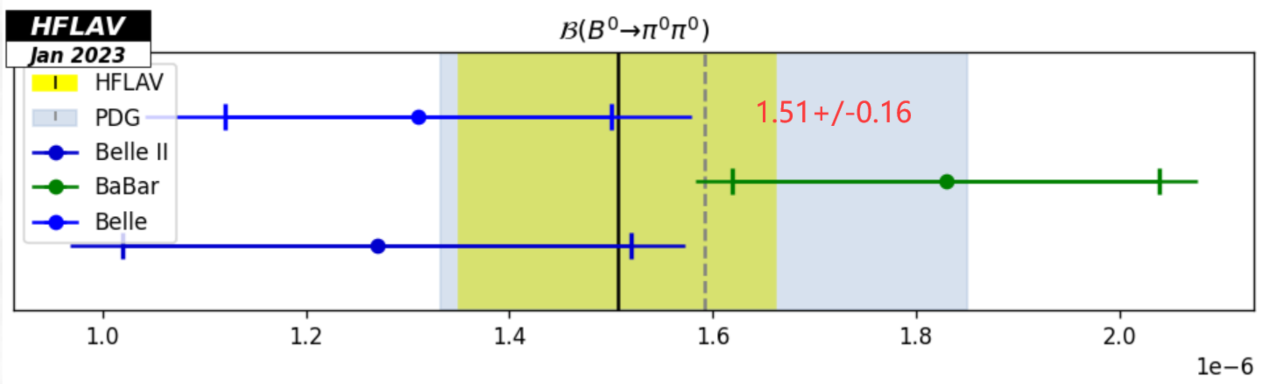
# Charmless two-body hadronic B decays

- ☐ **Long-standing puzzles in $\mathrm{Br}(\bar B^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K) = A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-)$:** [HFLAV '23]

$$\mathrm{Br}(B^0 \to \pi^0\pi^0) = (0.3 - 0.9)\times 10^{-6}$$

$$\Delta A_{CP}(\pi K) = (11.5 \pm 1.4)\%$$

**differs from 0 by ~8$\sigma$**



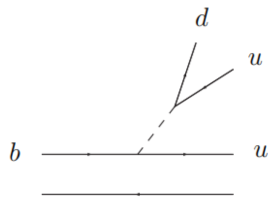
- ☐ **Decay amplitudes in QCDF:**

$$-\mathcal{A}_{\bar B^0\to\pi^0\pi^0} = A_{\pi\pi}\big[\delta_{pu}(\alpha_2 - \beta_1) - \hat\alpha_4^p - 2\beta_4^p\big]$$

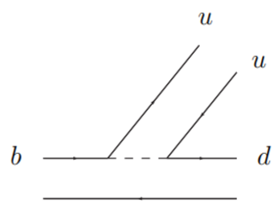
$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} = A_{\pi\bar K}\big[\delta_{pu}\,\alpha_1 + \hat\alpha_4^p\big] + A_{\bar K\pi}\big[\delta_{pu}\alpha_2 + \delta_{pc}\tfrac{3}{2}\alpha^c_{3,\mathrm{EW}}\big],$$
$$\mathcal{A}_{\bar B^0\to\pi^+K^-} = A_{\pi\bar K}\big[\delta_{pu}\,\alpha_1 + \hat\alpha_4^p\big],$$

$$A_{\mathrm{CP}}(\pi^0K^\pm) - A_{\mathrm{CP}}(\pi^\mp K^\pm) = -2\sin\gamma\Big(\mathrm{Im}(r_C) - \mathrm{Im}(r_T\, r_{\mathrm{EW}})\Big) + \ldots$$

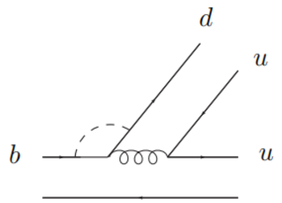
- ☐ **Dominant topologies: LP NNLO known**



colour-allowed tree $\alpha_1$



colour-suppressed tree $\alpha_2$



QCD penguins $\alpha_4$

$\alpha_2$ always plays a key role here!

- ☐ **Find some mechanism to enhance $\alpha_2$, may we explain both puzzles!**

➢ **necessary & urgent to consider sub-leading power corrections!**

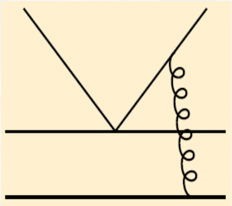
# Power-suppressed colour-octet contribution

- ☐ **Sub-leading power corrections to $a_2$: spectator scattering or final-state interactions**
- ☐ **Every four-quark operator in $H_{\mathrm{eff}}$ has a colour-octet piece in QCD:**

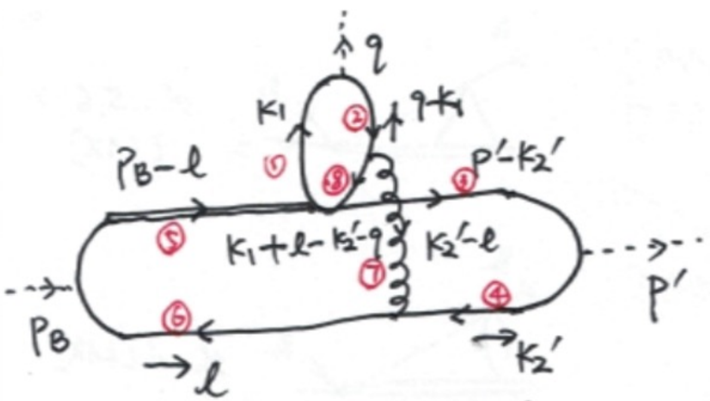
$$t^a_{ik} t^a_{jl} = \frac{1}{2}\delta_{il}\delta_{jk} - \frac{1}{2N_c}\delta_{ik}\delta_{jl},$$

$$Q_1 = (\bar{u}_i b_i)_{V-A} \otimes (\bar{s}_j u_j)_{V-A} = \frac{1}{N_c}(\bar{s}_i b_i)_{V-A} \otimes (\bar{u}_j u_j)_{V-A} + 2(\bar{s} T^A b)_{V-A} \otimes (\bar{u} T^A u)_{V-A}$$

$$Q_2 = (\bar{u}_i b_j)_{V-A} \otimes (\bar{s}_j u_i)_{V-A} = \frac{1}{N_c}(\bar{u}_i b_i)_{V-A} \otimes (\bar{s}_j u_j)_{V-A} + 2(\bar{u} T^A b)_{V-A} \otimes (\bar{s} T^A u)_{V-A}$$



- ☐ **Three-loop correlators with colour-octet operator insertion:**



BBNS_NPB591_313.

- ➢ The gluon propagator can be in the hard-collinear region;
  - ➡ hard-spectator scattering contribution;
- ➢ Can also be in the soft region; expected to be $\mathcal{O}(1/m_b)$;
  - ➡ can be non-zero at sub-leading power;
- ➢ Other four regions suppressed by more powers of $1/m_b$;

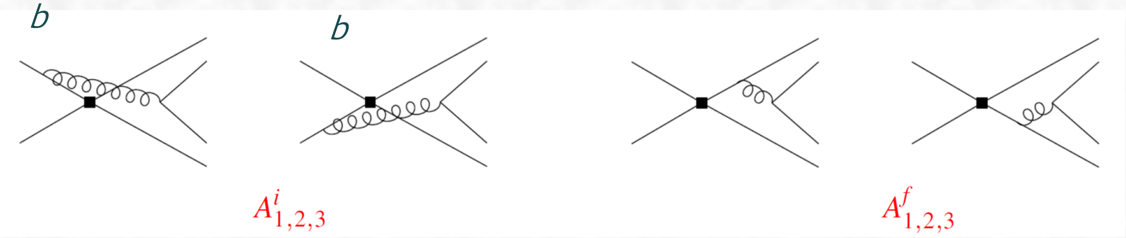
# Pure annihilation B decays

- ☐ **Two typical pure annihilation decay modes:** $\bar{B}_s^0 \to \pi^+\pi^-$ vs $\bar{B}^0 \to K^+K^-$

$$\mathcal{A}(\bar{B}_s \to \pi^+\pi^-) = B_{\pi\pi}\left[\delta_{pu} b_1 + 2b_4^p + \frac{1}{2} b_{4,\mathrm{EW}}^p\right]$$

$$\mathcal{A}(\bar{B}_d \to K^+K^-) = A_{\bar{K}K}\left[\delta_{pu}\beta_1 + \beta_4^p + b_{4,\mathrm{EW}}^p\right] + B_{K\bar{K}}\left[b_4^p - \frac{1}{2} b_{4,\mathrm{EW}}^p\right]$$

$$= A_{\bar{K}K}\left[\delta_{pu}\beta_1 + \beta_4^p\right] + B_{K\bar{K}}\left[b_4^p\right]$$



- ☐ **Both involve the building blocks $b_1 = \frac{C_F}{N_c^2} C_1 A_1^i$ & $b_4^p = \frac{C_F}{N_c^2}[C_4 A_1^i + C_6 A_2^i]$:**

$A_1^i$: $(\mathrm{V}-\mathrm{A}) \otimes (V-A)$

$A_2^i$: $(\mathrm{V}-\mathrm{A}) \otimes (V+A)$

$$A_1^i(M_1M_2) = \pi\alpha_s \int_0^1 dxdy \left\{\Phi_{M_2}(x)\,\Phi_{M_1}(y)\left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y}\right] + r_\chi^{M_1} r_\chi^{M_2}\,\Phi_{m_2}(x)\,\Phi_{m_1}(y)\,\frac{2}{\bar{x}y}\right\},$$

$$A_2^i(M_1M_2) = \pi\alpha_s \int_0^1 dxdy \left\{\Phi_{M_2}(x)\,\Phi_{M_1}(y)\left[\frac{1}{\bar{x}(1-x\bar{y})} + \frac{1}{\bar{x}y^2}\right] + r_\chi^{M_1} r_\chi^{M_2}\,\Phi_{m_2}(x)\,\Phi_{m_1}(y)\,\frac{2}{\bar{x}y}\right\},$$

- ☐ **With the asymptotic LCDAs, we have $A_1^i = A_2^i$ :** [BBNS '99-'03]

$$A_1^i(M_1M_2) = \pi\alpha_s \left\{18X_A - 18 - 6(9-\pi^2) + r_\chi^{M_1} r_\chi^{M_2}\left(2X_A^2\right)\right\},$$

$$A_2^i(M_1M_2) = \pi\alpha_s \left\{18X_A - 18 - 6(9-\pi^2) + r_\chi^{M_1} r_\chi^{M_2}\left(2X_A^2\right)\right\},$$

$$X_A = \left(1 + \circlearrowright_A e^{i\varphi_A}\right)\ln\left(m_B / \Lambda_h\right),$$

$\Lambda_h = 0.5\mathrm{GeV}$, $\circlearrowright_A \leq 1$ and an arbitrary phase $\varphi_A$

# Ways to improve the modelling of annihilations

## ☐ With universal $X_A$ and different scenarios, we have:

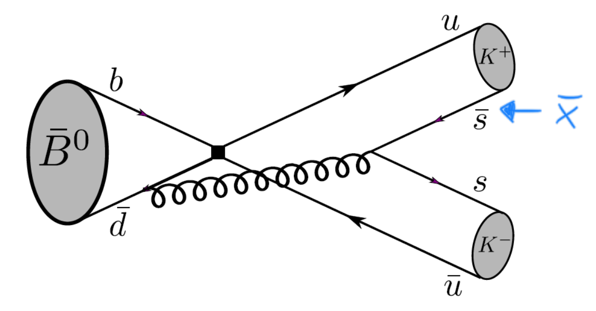
[BBNS '03]

| Mode | Theory | S1 (large $\gamma$) | S2 (large $a_2$) | S3 ($\varphi_A = -45^{\circ}$) | S4 ($\varphi_A = -55^{\circ}$) | Exp. |
|---|---|---|---|---|---|---|
| $\bar{B}_s^0 \to \pi^+\pi^-$ | $0.024^{+0.003+0.025+0.000+0.163}_{-0.003-0.012-0.000-0.021}$ | 0.027 | 0.032 | 0.149 | 0.155 | $0.671 \pm 0.083$ |
| $\bar{B}^0 \to K^-K^+$ | $0.013^{+0.005+0.008+0.000+0.087}_{-0.005-0.005-0.000-0.011}$ | 0.007 | 0.014 | 0.079 | 0.070 | $0.0803 \pm 0.0147$ |

## ☐ Large SU(3)-flavor symmetry breaking or flavor-dependent $A^i_{1,2}$ ?

[Wang, Zhu '03; Bobeth *et al.* '14; Chang, Sun *et al.* '14-15]

## ☐ How to improve the situation:



- including higher Gegenbauer moments to include SU(3)-breaking effects;

$$\Phi_M(x,\mu) = 6x\bar{x}\left[1 + \sum_{n=1}^{\infty} a_n^M(\mu)\, C_n^{(3/2)}(2x-1)\right]$$

due to G-parity, $a^{\pi}_{odd} = 0$, but $a^{K}_{odd} \neq 0$

- Making the parameter $X_A$ to be flavor-dependent & depending on its origins;

  mediated by a soft strange quark ($X_A^s$) or a soft up or down quark ($X_A^{ud}$)?

- including the difference between the chirality factors to include SU(3)-breaking effects;

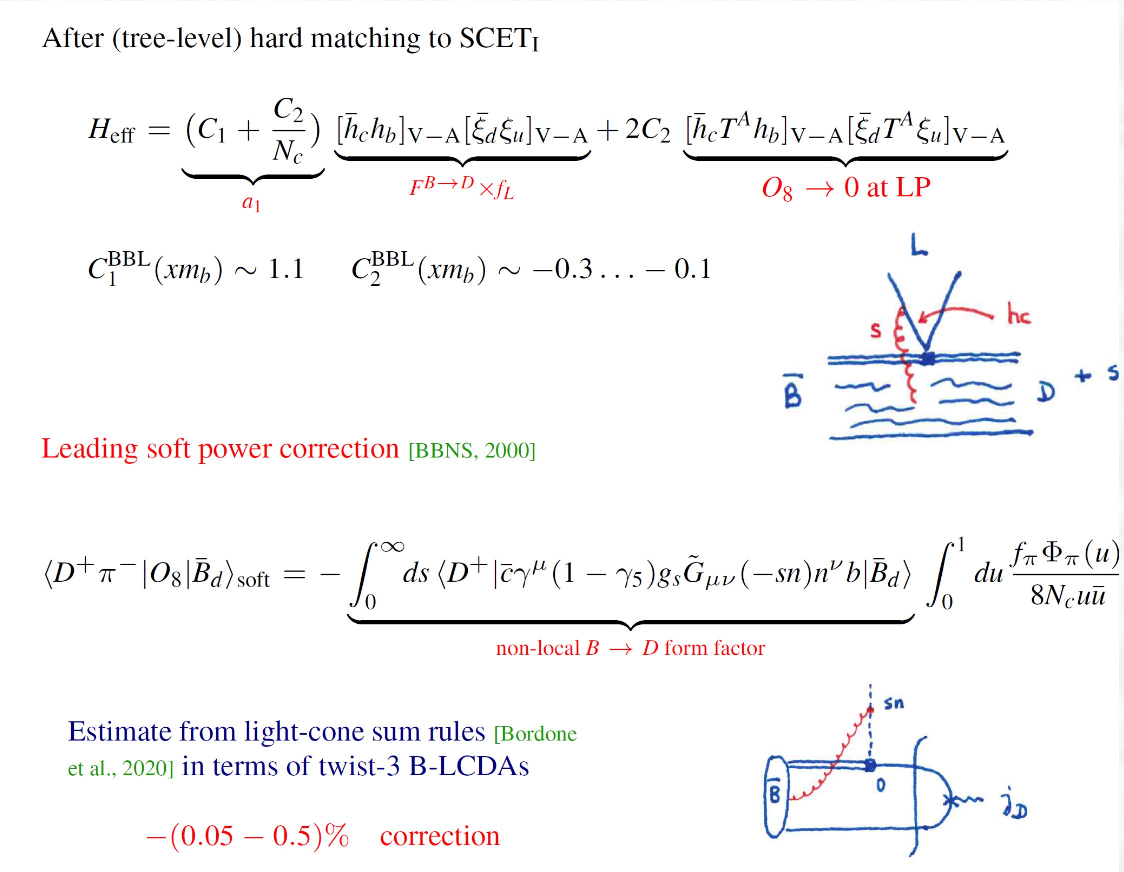
$$r_\chi^\pi(1.5\text{GeV}) = \frac{2m_\pi^2}{m_b(\mu)\big(m_u(\mu) + m_d(\mu)\big)} \simeq 0.86, \qquad r_\chi^K(1.5\text{GeV}) = \frac{2m_K^2}{m_b(\mu)\big(m_u(\mu) + m_s(\mu)\big)} \simeq 0.91$$

$$\boxed{A_1^i(M_1M_2) \neq A_2^i(M_1M_2)}$$

# Power corrections

Beneke, talk @“Status and Prospects of Non-Leptonic B Decays”,
Siegen, May 31 - June 02, 2022

## ☐ Leading soft power corrections:



After (tree-level) hard matching to SCET$_\text{I}$

$$H_\text{eff} = \underbrace{\left(C_1 + \frac{C_2}{N_c}\right)}_{a_1} \underbrace{[\bar{h}_c h_b]_{\text{V}-\text{A}}[\bar{\xi}_d \xi_u]_{\text{V}-\text{A}}}_{F^{B\to D}\times f_L} + 2C_2 \underbrace{[\bar{h}_c T^A h_b]_{\text{V}-\text{A}}[\bar{\xi}_d T^A \xi_u]_{\text{V}-\text{A}}}_{O_8 \,\to\, 0 \text{ at LP}}$$

$$C_1^\text{BBL}(xm_b) \sim 1.1 \qquad C_2^\text{BBL}(xm_b) \sim -0.3 \ldots -0.1$$

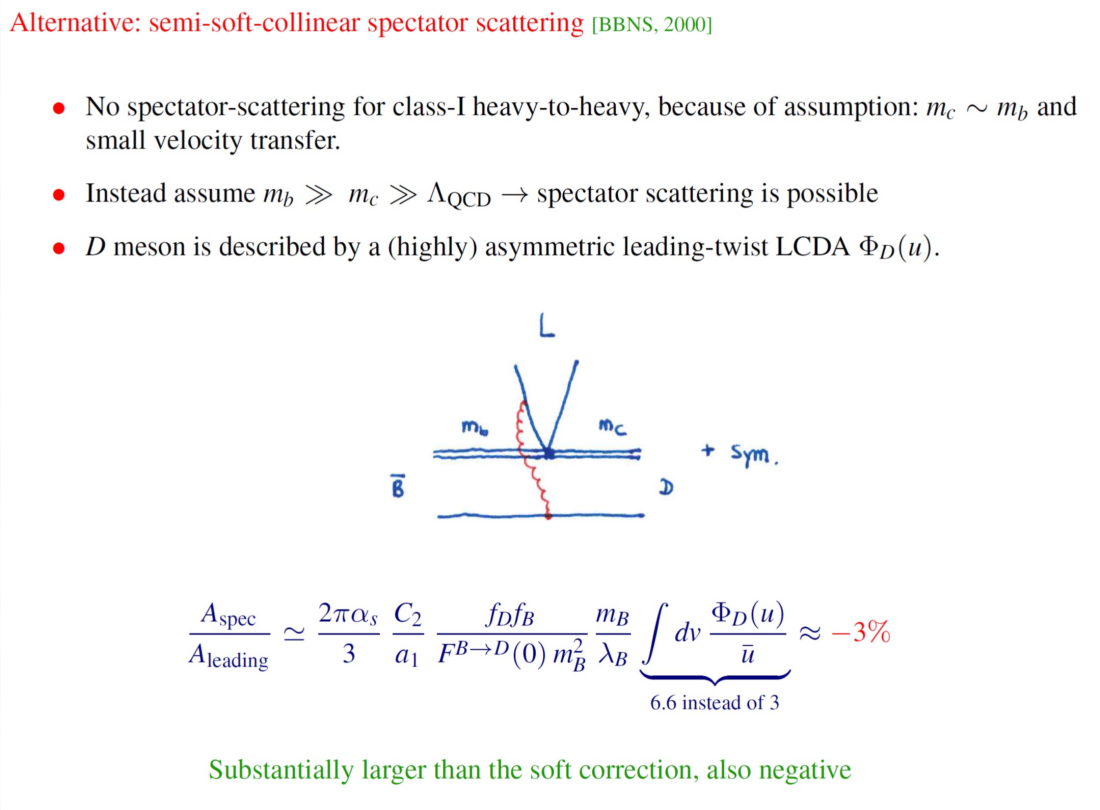
Leading soft power correction [BBNS, 2000]

$$\langle D^+\pi^-|O_8|\bar{B}_d\rangle_\text{soft} = -\underbrace{\int_0^\infty ds\, \langle D^+|\bar{c}\gamma^\mu(1-\gamma_5)g_s\tilde{G}_{\mu\nu}(-sn)n^\nu b|\bar{B}_d\rangle}_{\text{non-local } B\,\to\, D \text{ form factor}} \int_0^1 du\, \frac{f_\pi \Phi_\pi(u)}{8N_c u\bar{u}}$$

Estimate from light-cone sum rules [Bordone et al., 2020] in terms of twist-3 B-LCDAs

$-(0.05 - 0.5)\%$ correction

## ☐ Semi-soft-collinear spectator scattering:



Alternative: semi-soft-collinear spectator scattering [BBNS, 2000]

- No spectator-scattering for class-I heavy-to-heavy, because of assumption: $m_c \sim m_b$ and small velocity transfer.
- Instead assume $m_b \gg m_c \gg \Lambda_\text{QCD} \to$ spectator scattering is possible
- $D$ meson is described by a (highly) asymmetric leading-twist LCDA $\Phi_D(u)$.

$$\frac{A_\text{spec}}{A_\text{leading}} \simeq \frac{2\pi\alpha_s}{3}\,\frac{C_2}{a_1}\,\frac{f_D f_B}{F^{B\to D}(0)\, m_B^2}\,\frac{m_B}{\lambda_B}\underbrace{\int dv\, \frac{\Phi_D(u)}{\bar{u}}}_{\text{6.6 instead of 3}} \approx -3\%$$

Substantially larger than the soft correction, also negative

➢ Must take seriously into account these power corrections in QCDF/SCET !