Lattice QCD study of Hadron Spectroscopy

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Outline

- ✦Introduction
  - Spectroscopy on lattice
  - Scattering on lattice
- ✦Preliminary results
  - Hidden charm pentaquarks
  - H-dibaryon

Spectroscopy on lattice

✦ Write down an interpolating operator $\mathcal{O}$ with certain quantum number, e.g. pion operator $\bar{u}\gamma_5 d$

✦ Compute the correlation function

$$< 0|\mathcal{O}(t)\mathcal{O}(0)^\dagger|0 > = \sum_n \frac{< 0|\mathcal{O}|n > < n|\mathcal{O}|0 >}{2E_n} e^{-E_n t} \longrightarrow \propto e^{-E_0 t}$$

✦ At large t, fit the correlation function to an exponential.

✦ Usually only the ground state can be obtained.

Neutron-proton mass difference, Sz. Borsan et al., Science 347:1452-1455,2015

Spectroscopy on lattice

Excited states:

✦ build large basis of operators $\{\mathcal{O}_1, \mathcal{O}_2, \cdots\}$ with desired quantum numbers, construct the matrix of correlation function:

$$C_{ij} = < 0 | \mathcal{O}_i \mathcal{O}_j^\dagger | 0 > = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

✦ Solve the generalized eigenvalue problem(GEVP):

$$C_{ij} v_j^n(t) = \lambda_n(t) C_{ij}^0 v_j^n(t)$$

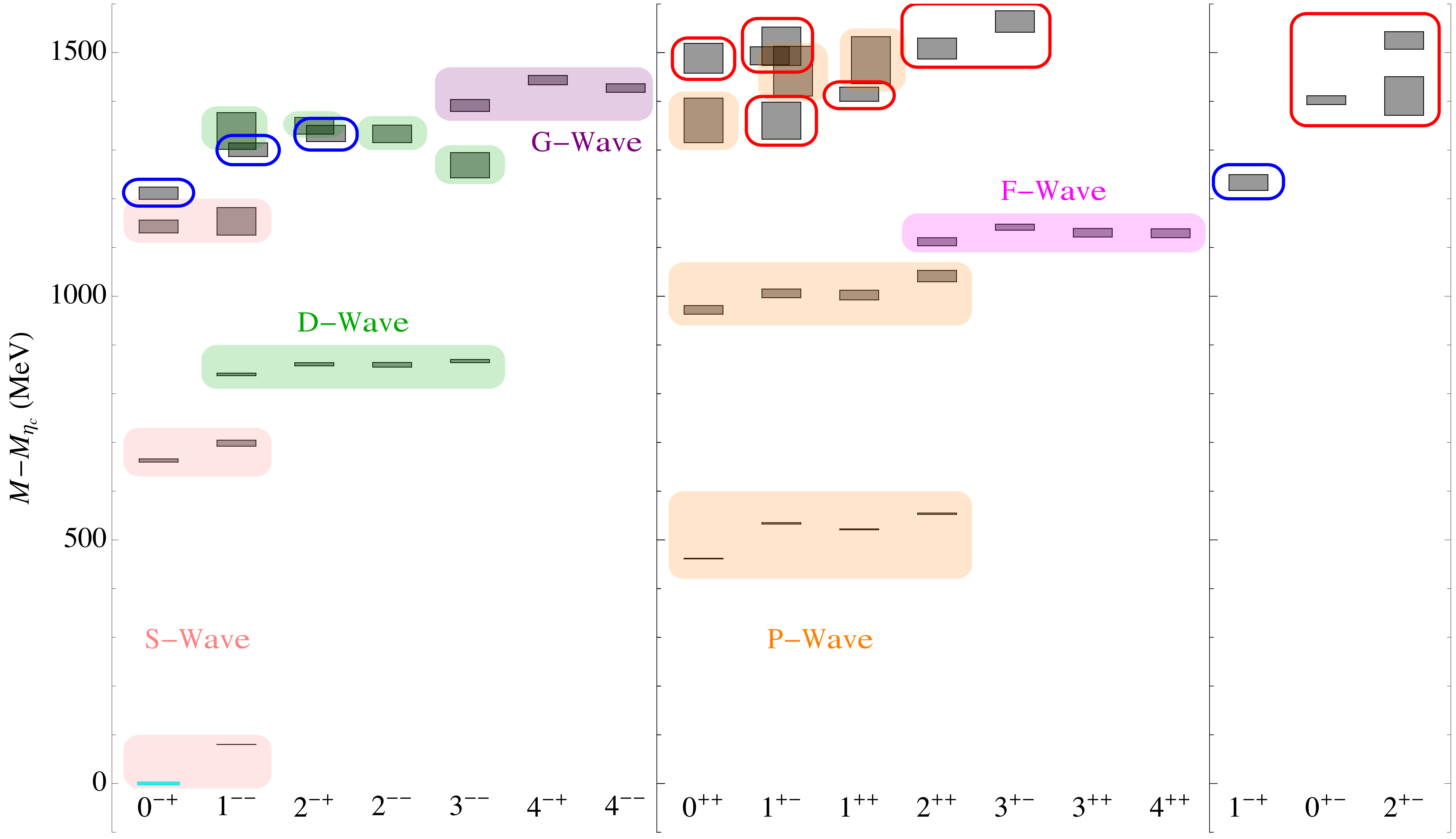
✦ Eigenvalues: $\lambda_n(t) \sim e^{-E_n t}(1 + e^{-\Delta E\, t})$

✦ Optimal linear combinations of the operators to overlap on the n'th state:

$$\Omega_n = \sum_i v_i^n \mathcal{O}_i$$

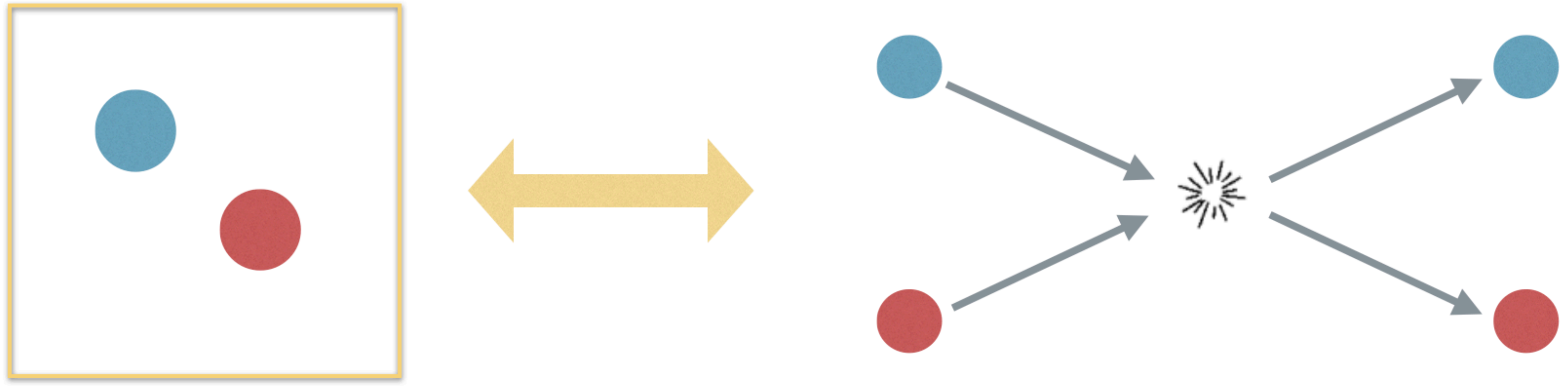
Spectroscopy on lattice

An example: Charmonium spectrum (L. Liu et al. JHEP 1207 (2012) 126)



Scattering on lattice

Lüscher's finite volume method: M. Lüscher, Nucl. Phys. B354, 531(1991)

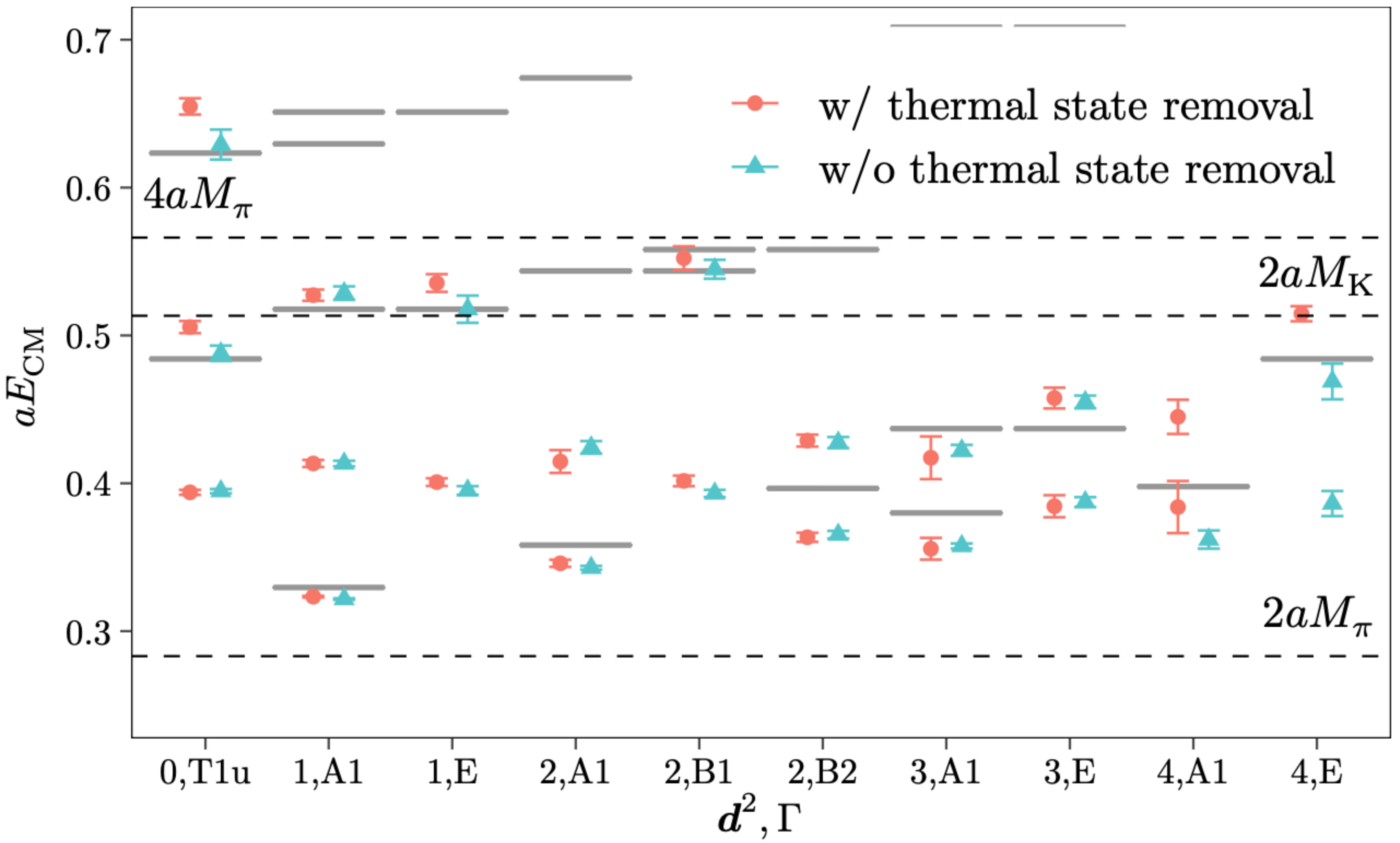


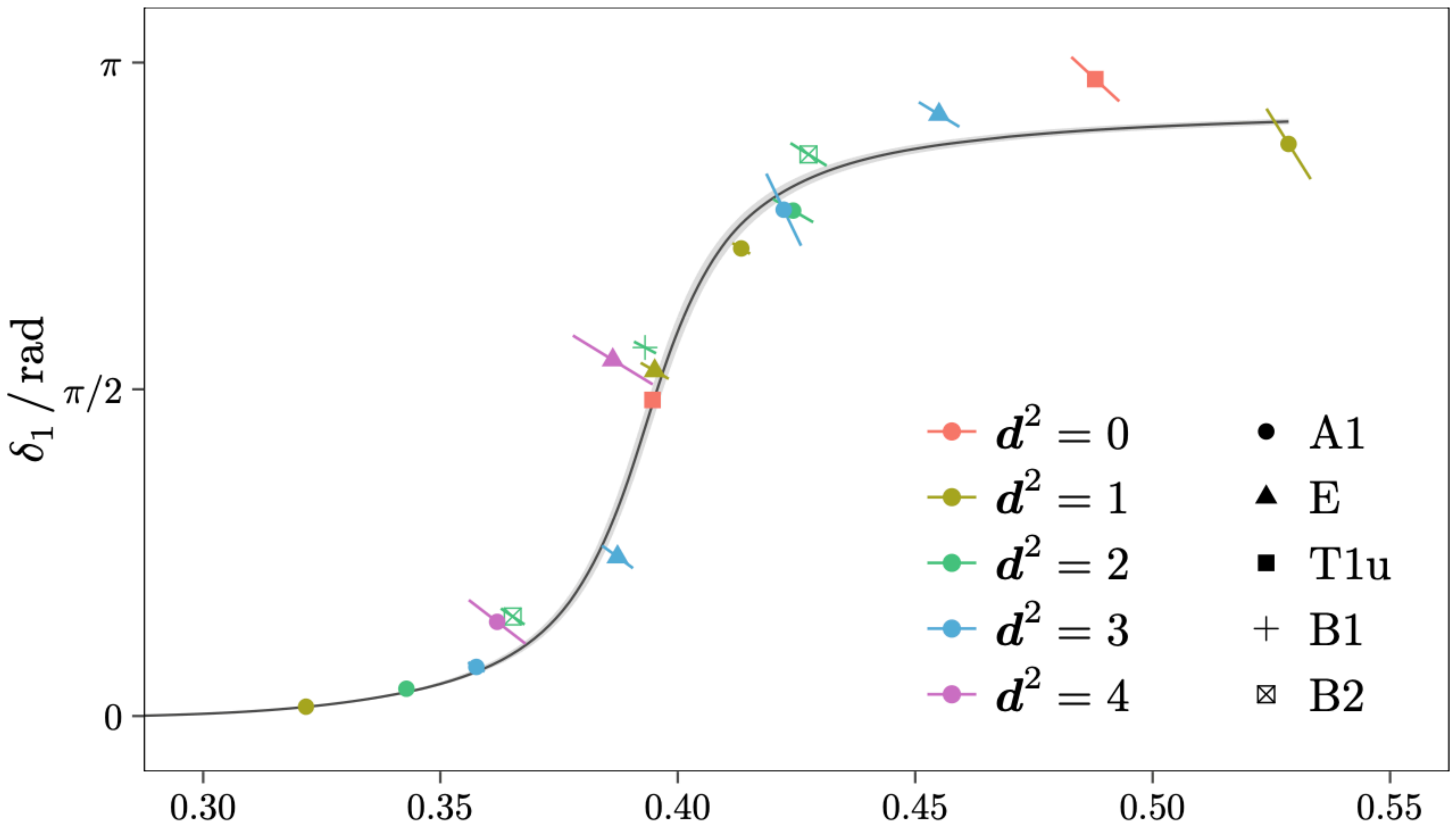
Finite volume spectrum

Infinite volume scattering parameters

Scattering on lattice

An example: ρ resonance $\rightarrow \pi\pi$ scattering





M. Werner et. al., Eur.Phys.J.A 56 (2020) 2, 61

Scattering on lattice

CLQCD

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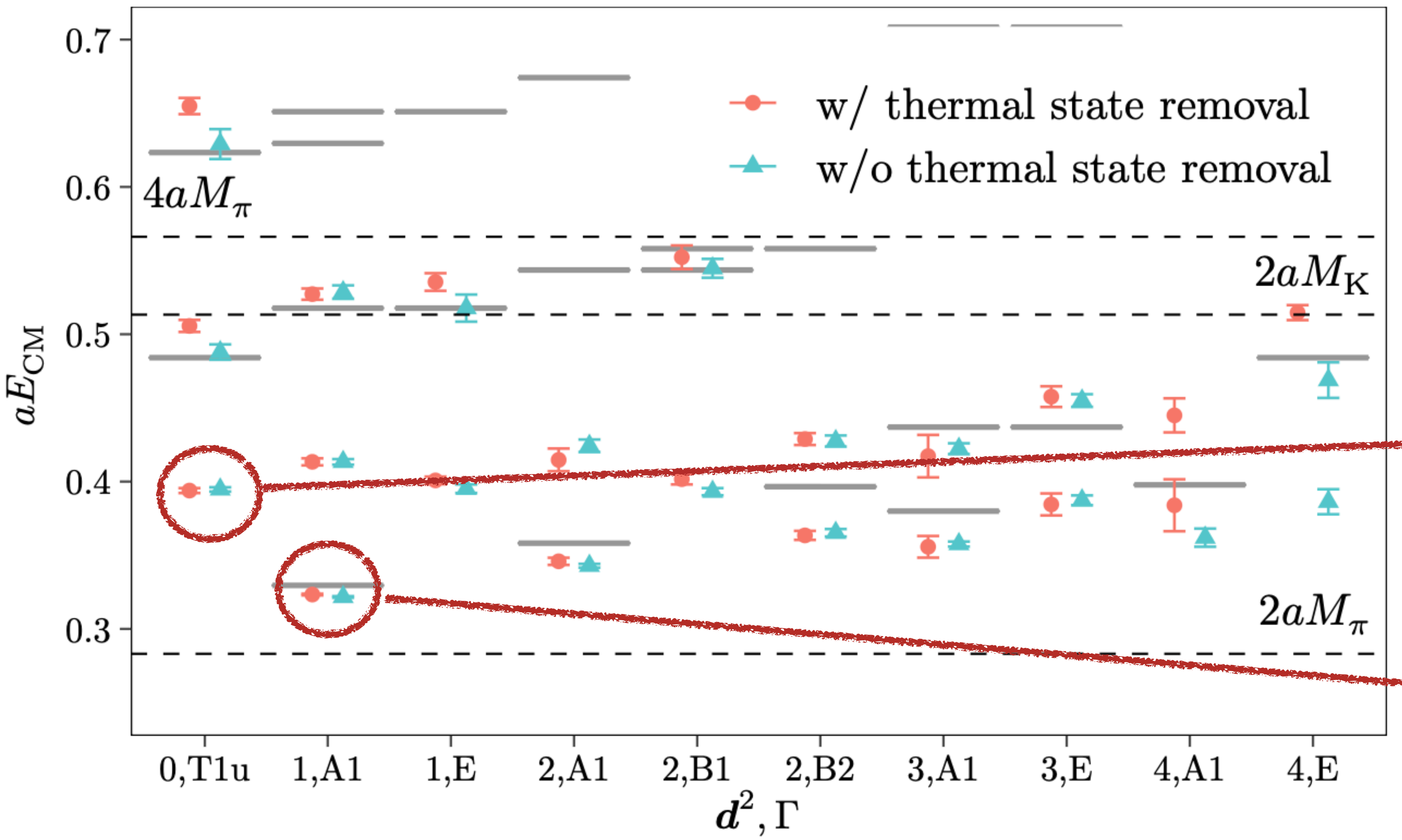

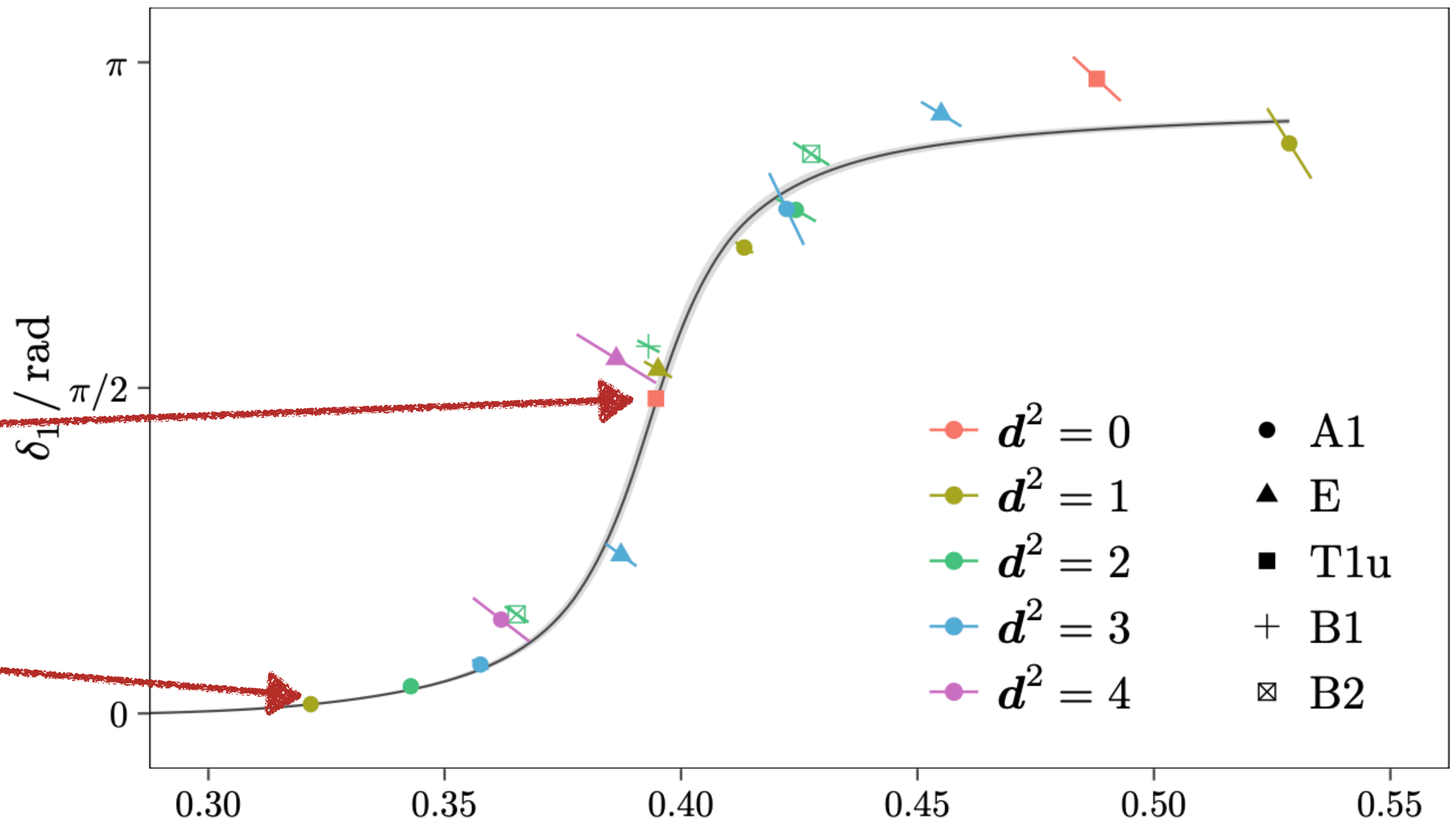

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Scattering on lattice

An example: ρ resonance $\rightarrow \pi\pi$ scattering

Breit-Wigner formula:

$$\tan \delta_1 = \frac{g^2_{\rho\pi\pi}}{6\pi} \frac{p^3}{E_{CM}(M^2_\rho - E^2_{CM})}, \qquad E_{CM} = 2\sqrt{m^2_\pi + p^2}$$

The width of ρ resonance:

$$\Gamma_\rho = \frac{2}{3} \frac{g^2_{\rho\pi\pi}}{4\pi} \frac{p^3}{M^2_\rho}$$

M. Werner et. al., Eur.Phys.J.A 56 (2020) 2, 61

Scattering on lattice

CLQCD

- General Lüscher's formula for two-body scattering:

$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (1 + i\mathbf{M})] = 0$$

Diagonal matrix of phase-space factors $\rho_{ij} = \delta_{ij}\frac{2k_i}{E_{cm}}$

Infinite-volume scattering matrix

Finite volume information $M(E_{cm}, L)$

- Resonances/bound states are formally defined as poles in scattering amplitudes.

Scattering on lattice

✦ Challenges:

- Coupled channels
- Near-threshold exotic states

→ Precise determination of the finite-volume spectra (Distillation quark smearing method)

- Multi-particle scattering

- Control systematics

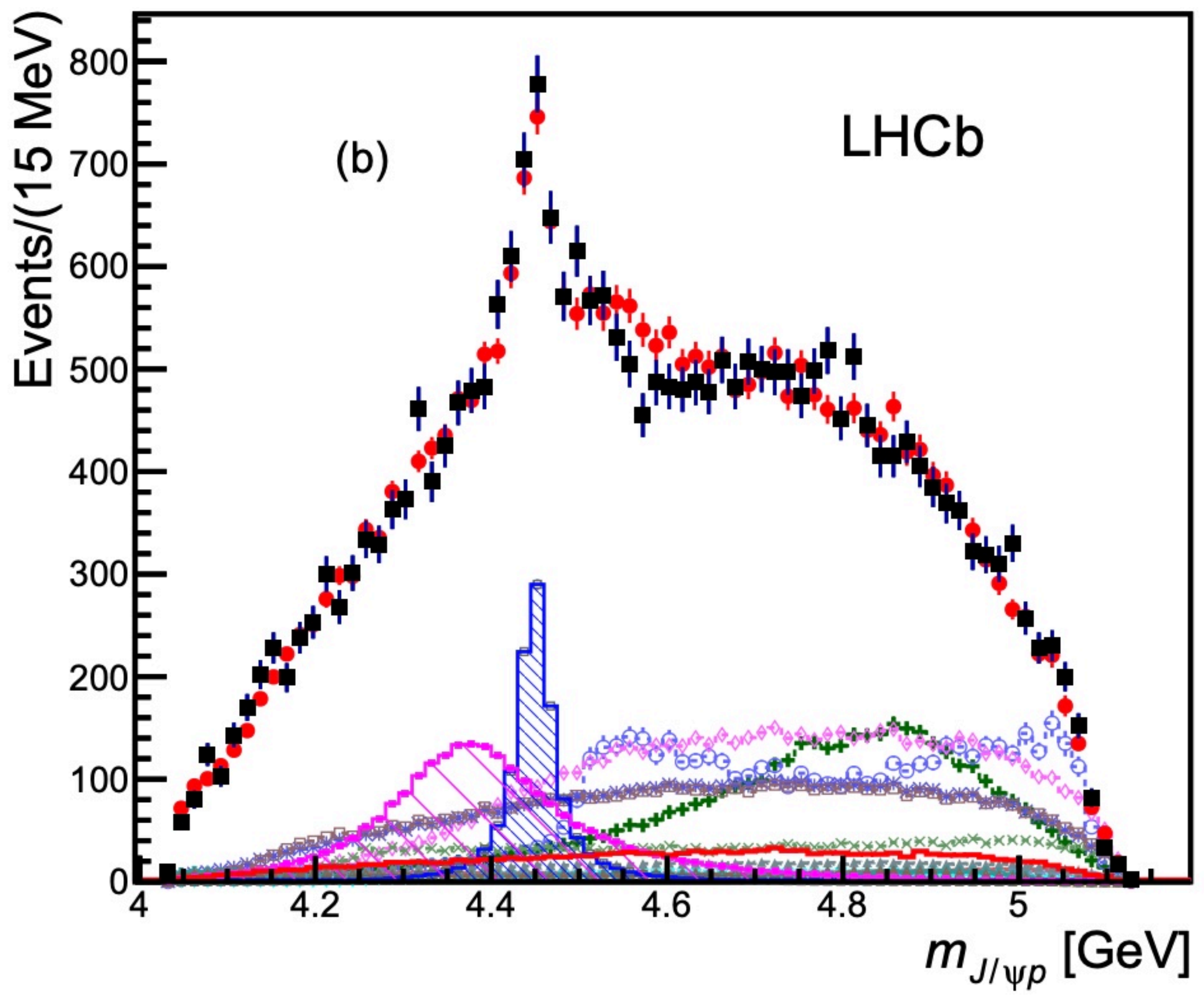
→ Large set of configurations with various parameters

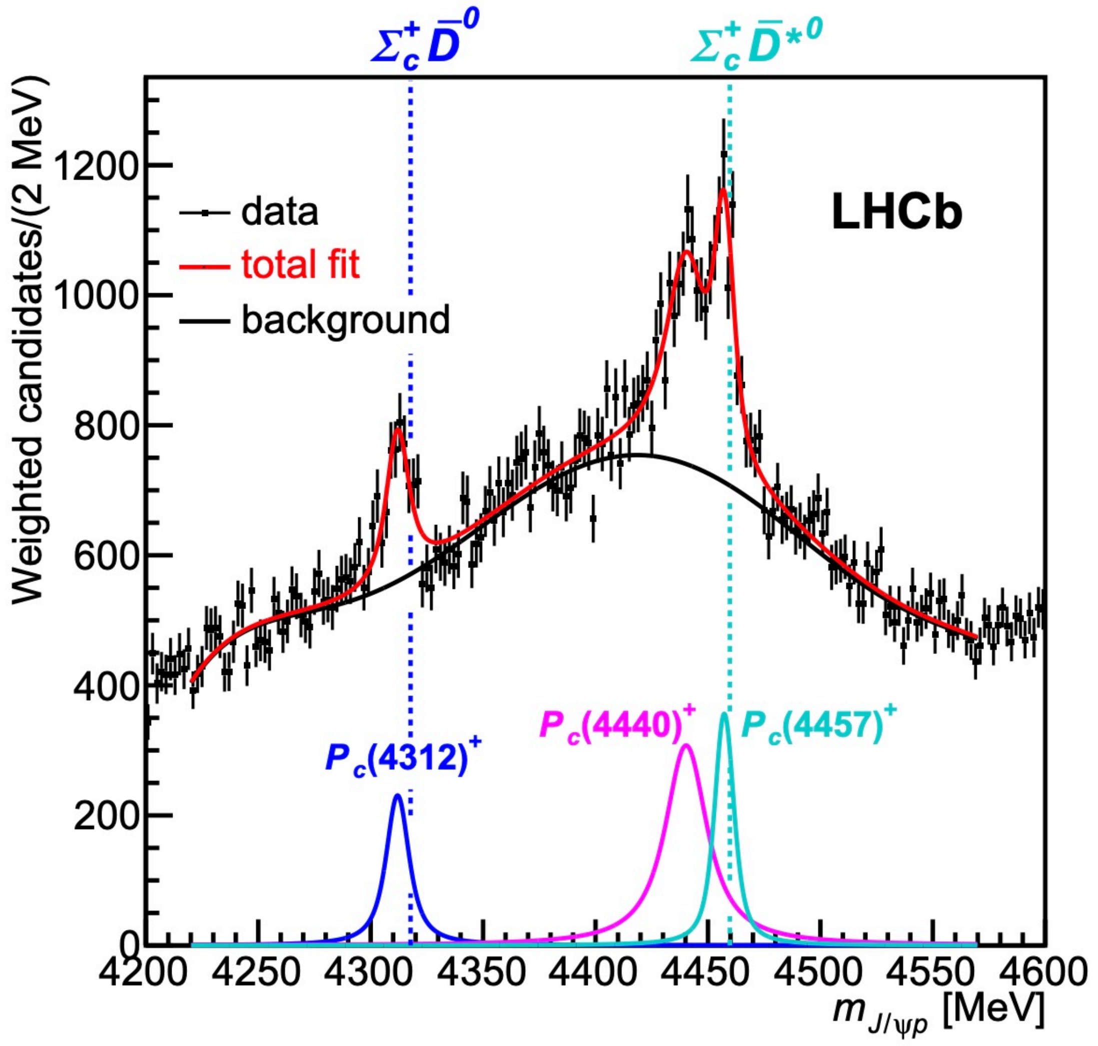
Lattice QCD configurations

Lattice spacing	Volume($L^3 \times T$)	M_π (MeV)	# of confs
~0.108fm	$24^3 \times 72$	290	1000
	$32^3 \times 64$	290	1000
	$32^3 \times 64$	220	450
	$48^3 \times 96$	220	200
	$48^3 \times 96$	140	200
~0.080fm	$32^3 \times 96$	300	480
	$48^3 \times 96$	300	200
	$32^3 \times 64$	220	460
	$48^3 \times 96$	220	200
~0.055fm	$48^3 \times 144$	300	200

L. Liu, M. Gong, W. Sun, P. Sun, W. Wang, Y.B. Yang

P_c Pentaquarks





$P_c(4380)$

$P_c(4450)$ → $P_c(4440)$, $P_c(4457)$

$P_c(4312)$

$P_c(4440)$

$P_c(4457)$

R. Aaij et al. (LHCb), Phys. Rev. Lett. 115, 072001 (2015)

R. Aaij et al. (LHCb), Phys. Rev. Lett. 122, 222001 (2019)

Theory interpretations:

- Molecule bound states
- Compact pentaquark states
- ……

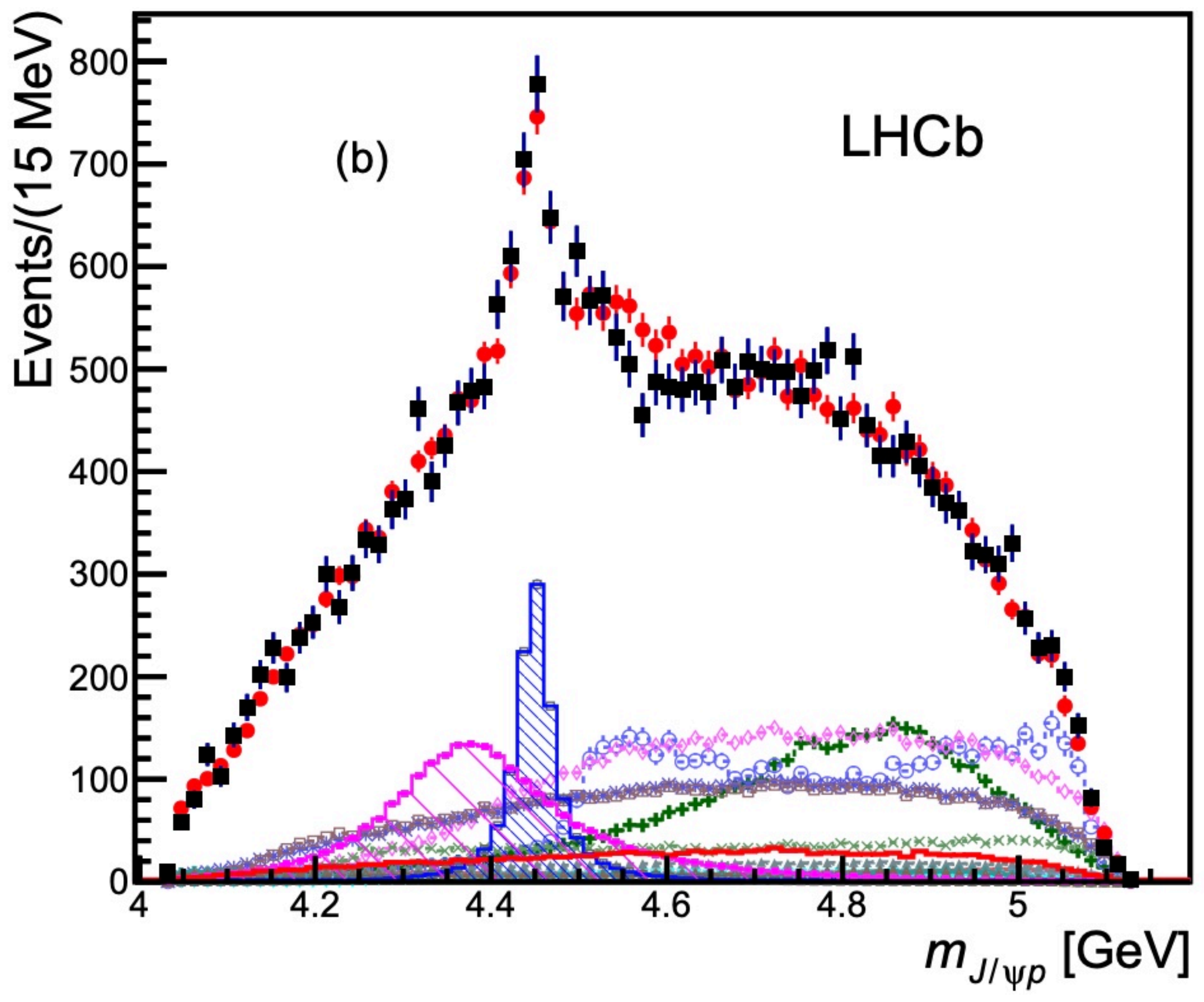
$\Sigma_c^{(*)}\bar{D}^{(*)}$ molecules:

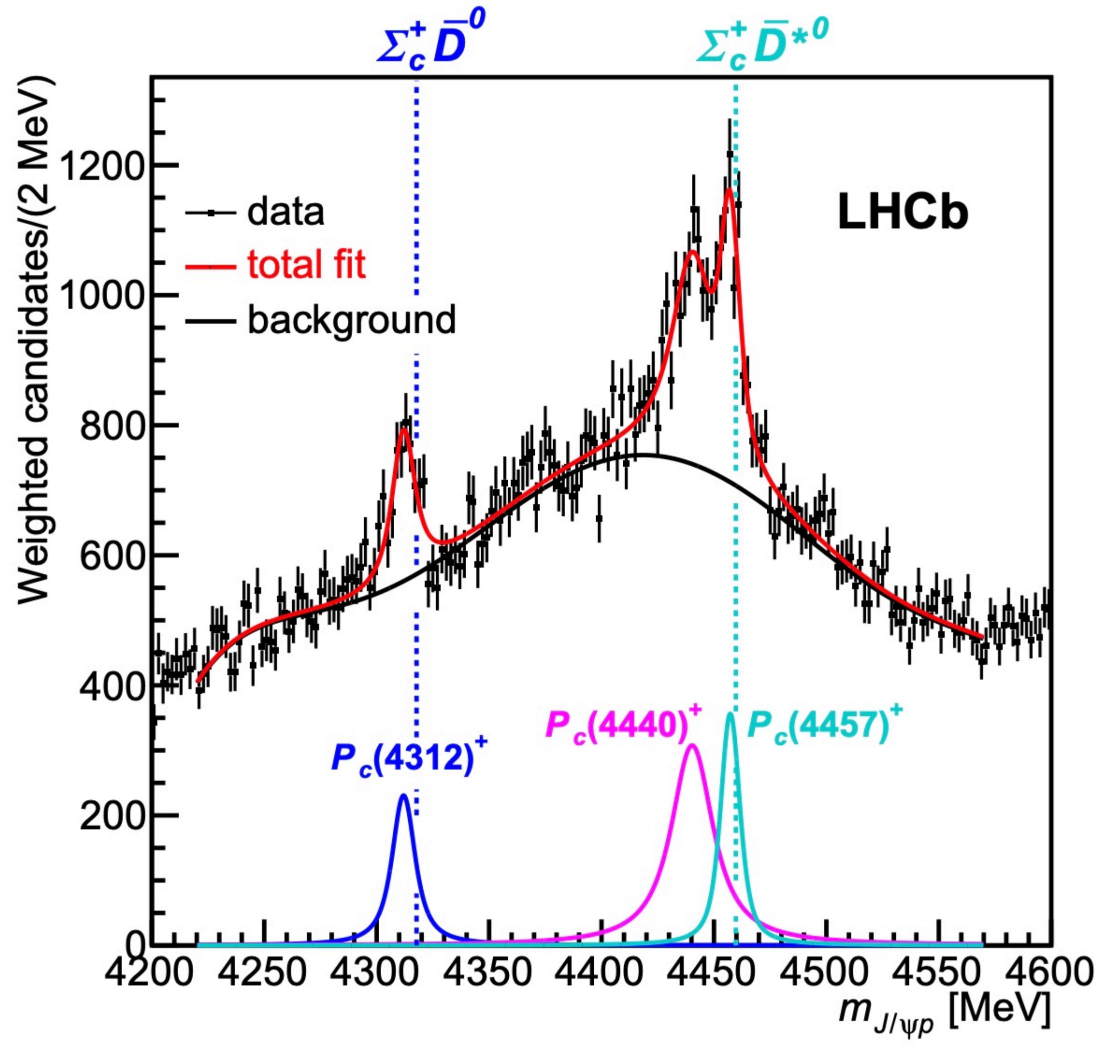
$$\Sigma_c\bar{D}, J^P = \frac{1}{2}^-, P_c(4312)$$

$$\Sigma_c\bar{D}^*, J^P = (\frac{1}{2}^-, \frac{3}{2}^-), P_c(4440)/P_c(4457)$$

$$\Sigma_c^*\bar{D}, J^P = \frac{3}{2}^-, \quad \Sigma_c^*\bar{D}^*, J^P = (\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-),$$

P_c Pentaquarks





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$\Sigma_c\bar{D}$ and $\Sigma_c\bar{D}^*$ scattering($J^P = \frac{1}{2}^-$):

- Five operators:

$$\mathcal{O}_1 = \Sigma_c(\mathbf{p})\bar{D}(-\mathbf{p}) \ (|\mathbf{p}| = 0)$$
$$\mathcal{O}_2 = \Sigma_c(\mathbf{p})\bar{D}(-\mathbf{p}) \ (|\mathbf{p}| = 1)$$
$$\mathcal{O}_3 = \Sigma_c(\mathbf{p})\bar{D}(-\mathbf{p}) \ (|\mathbf{p}| = \sqrt{2})$$
$$\mathcal{O}_4 = \Sigma_c(\mathbf{p})\bar{D}^*(-\mathbf{p}) \ (|\mathbf{p}| = 0)$$
$$\mathcal{O}_5 = \Sigma_c(\mathbf{p})\bar{D}^*(-\mathbf{p}) \ (|\mathbf{p}| = 1)$$

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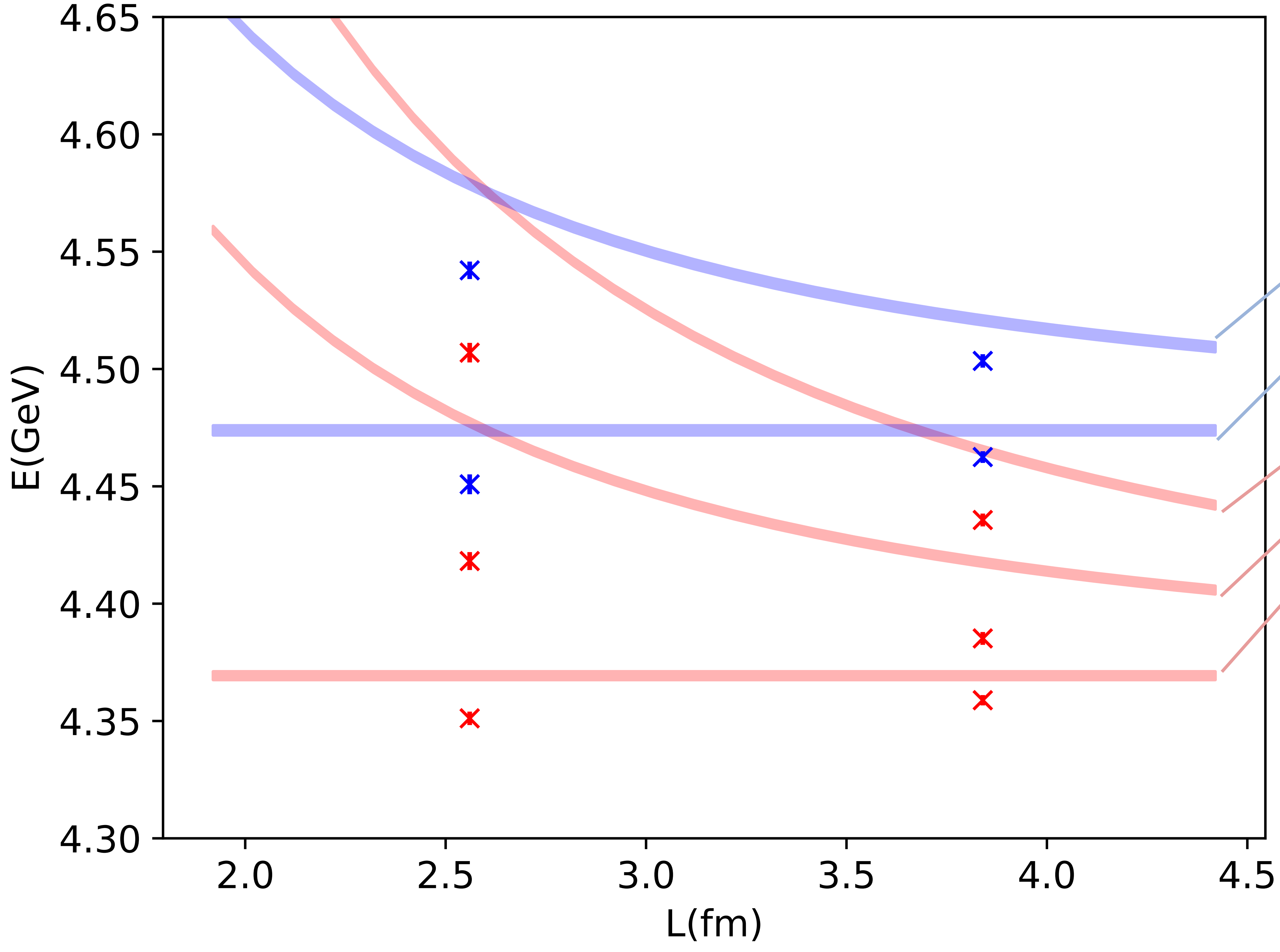
P_c Pentaquarks



$$E^{free} = \sqrt{m_{\Sigma_c}^2 + \mathbf{p}^2} + \sqrt{m_{D^*}^2 + \mathbf{p}^2}$$

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- ✦ The finite-volume energies lie below the free energies, indicating rather strong attractive interactions.
- ✦ Mixing between $\Sigma_c \bar{D}$ and $\Sigma_c \bar{D}^*$ is negligible.

P_c Pentaquarks

Scattering amplitude:

$$T \sim \frac{1}{p\cot\delta - ip}$$

Bound state pole:

$$p = i|p_B|$$

Effective range expansion:

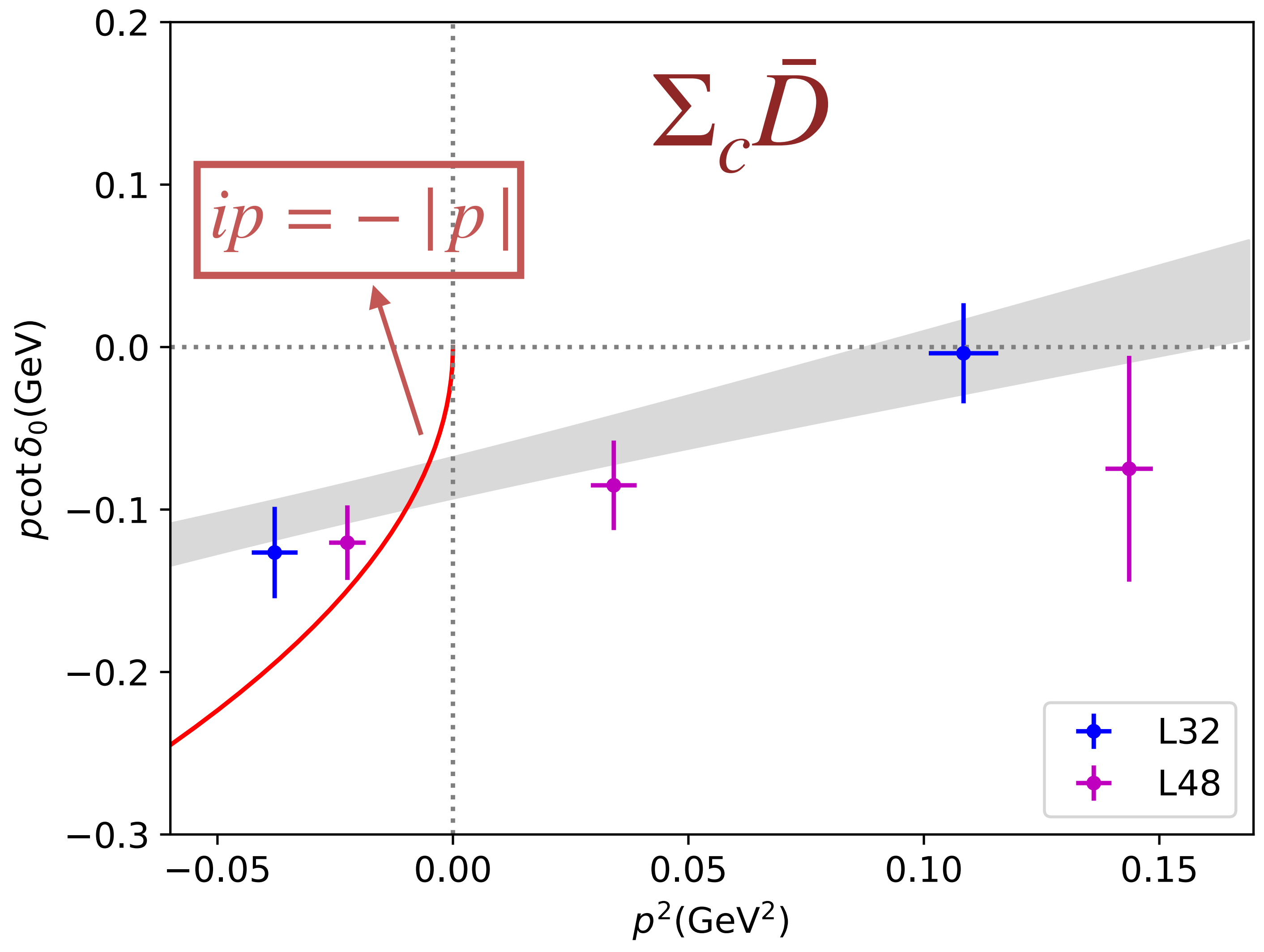
$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2}r_0 p^2 + \cdots$$

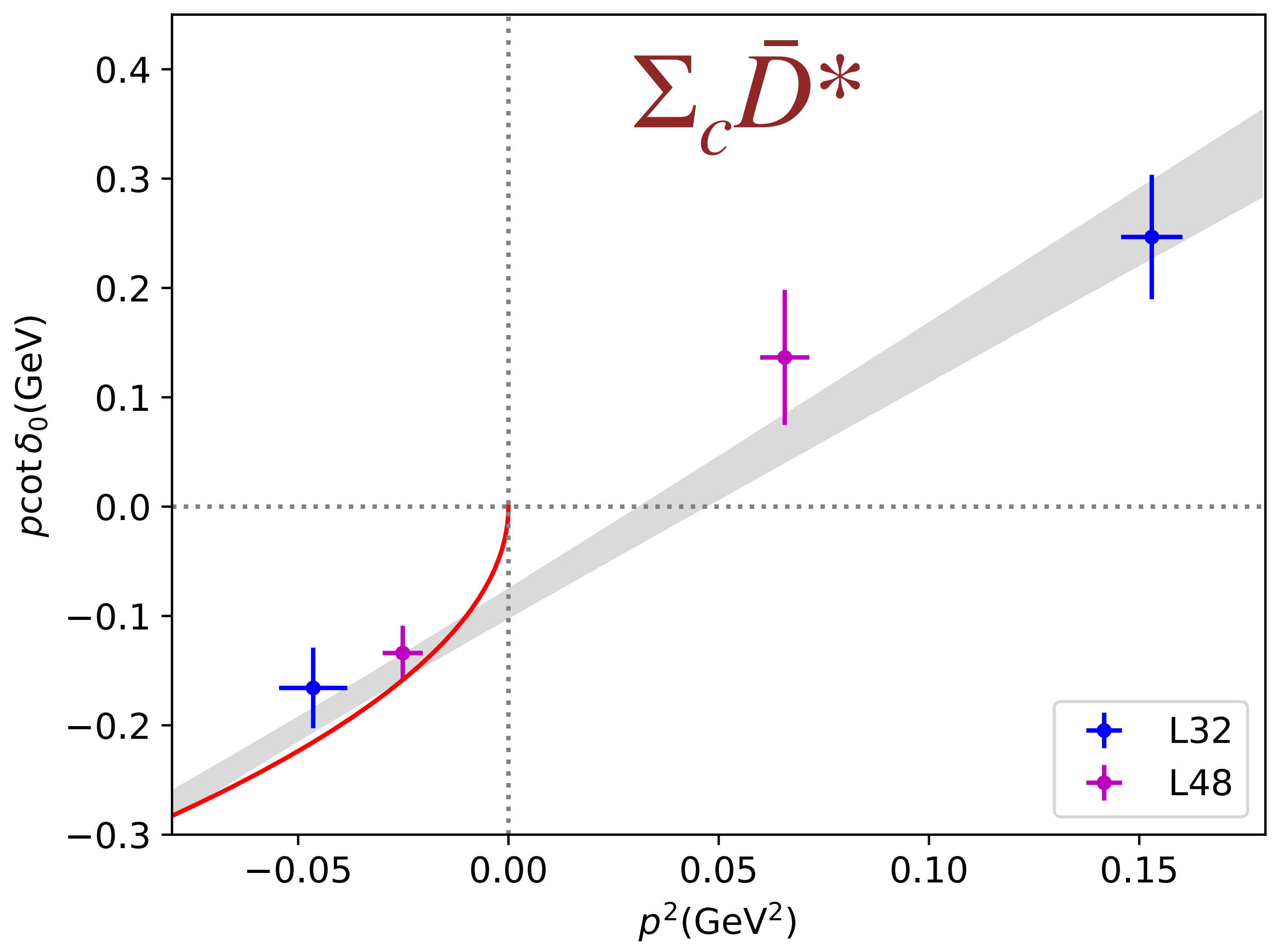
$\Sigma_c\bar{D}$: $P_c(4312)$?
$a_0 = -2.0(3)(5)\mathrm{fm}$
$E_B = 6(2)(2)\mathrm{MeV}$

Luscher's formula:

$$p\cot\delta(p) = \frac{2Z_{00}(1;(\frac{pL}{2\pi})^2)}{L\sqrt{\pi}}$$

$\Sigma_c\bar{D}^*$: $P_c(4440)/P_c(4457)$?
$a_0 = -2.3(5)(1)\mathrm{fm}$
$E_B = 7(3)(1)\mathrm{MeV}$





P_c Pentaquarks

Coupled channels: $\eta_c N, J/\psi N, \Lambda_c \bar{D}, \Lambda_c \bar{D}^*, \Sigma_c \bar{D}, \Sigma_c \bar{D}^*$

- 15 operators:

$$\mathcal{O}_{1,2,3} = N(\mathbf{p})\eta_c(-\mathbf{p}) \; (|\mathbf{p}| = 0,1,\sqrt{2})$$

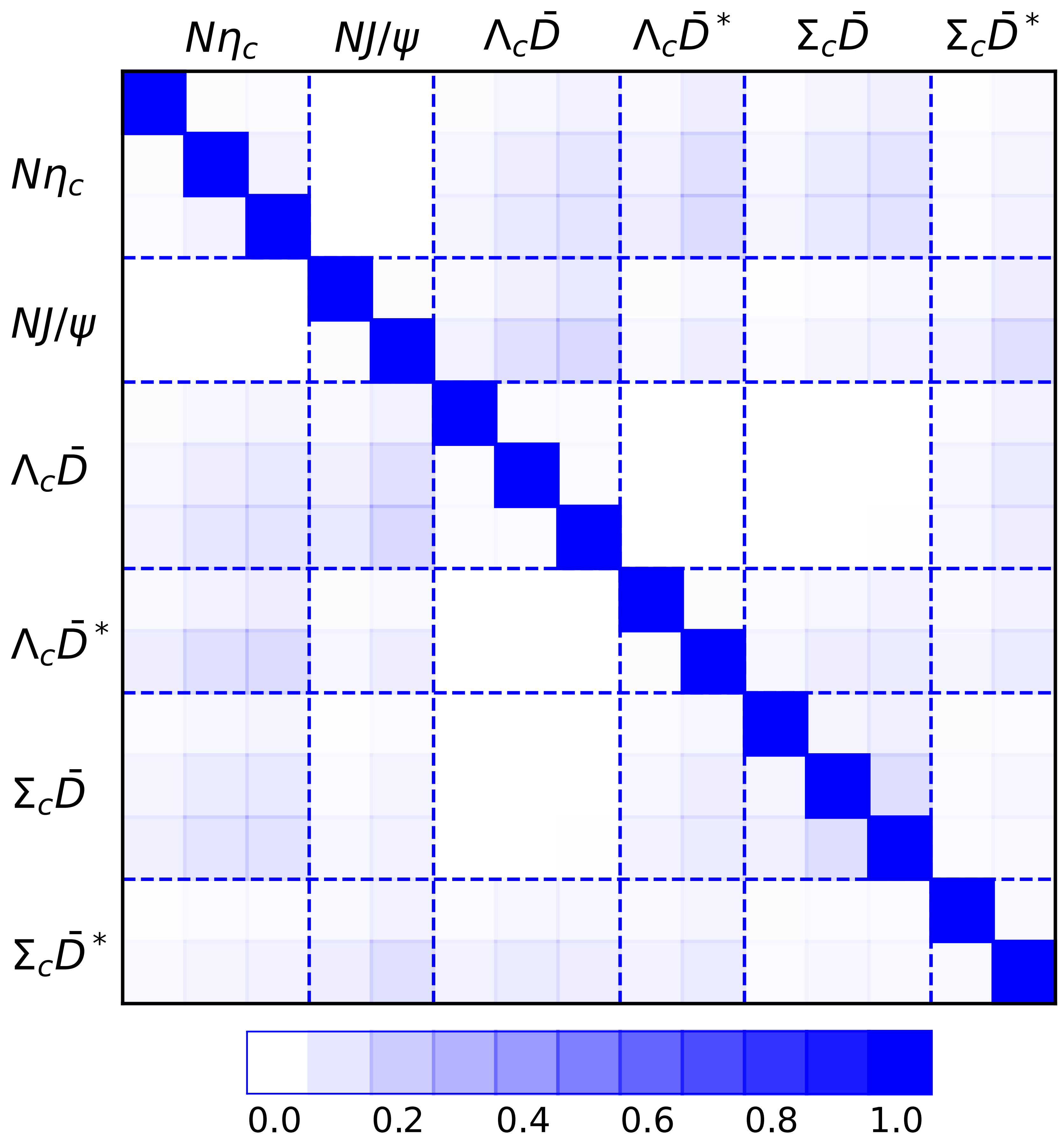
$$\mathcal{O}_{4,5} = N(\mathbf{p})J/\psi(-\mathbf{p}) \; (|\mathbf{p}| = 0,1)$$

$$\mathcal{O}_{6,7,8} = \Lambda_c(\mathbf{p})\bar{D}(-\mathbf{p}) \; (|\mathbf{p}| = 0,1,\sqrt{2})$$

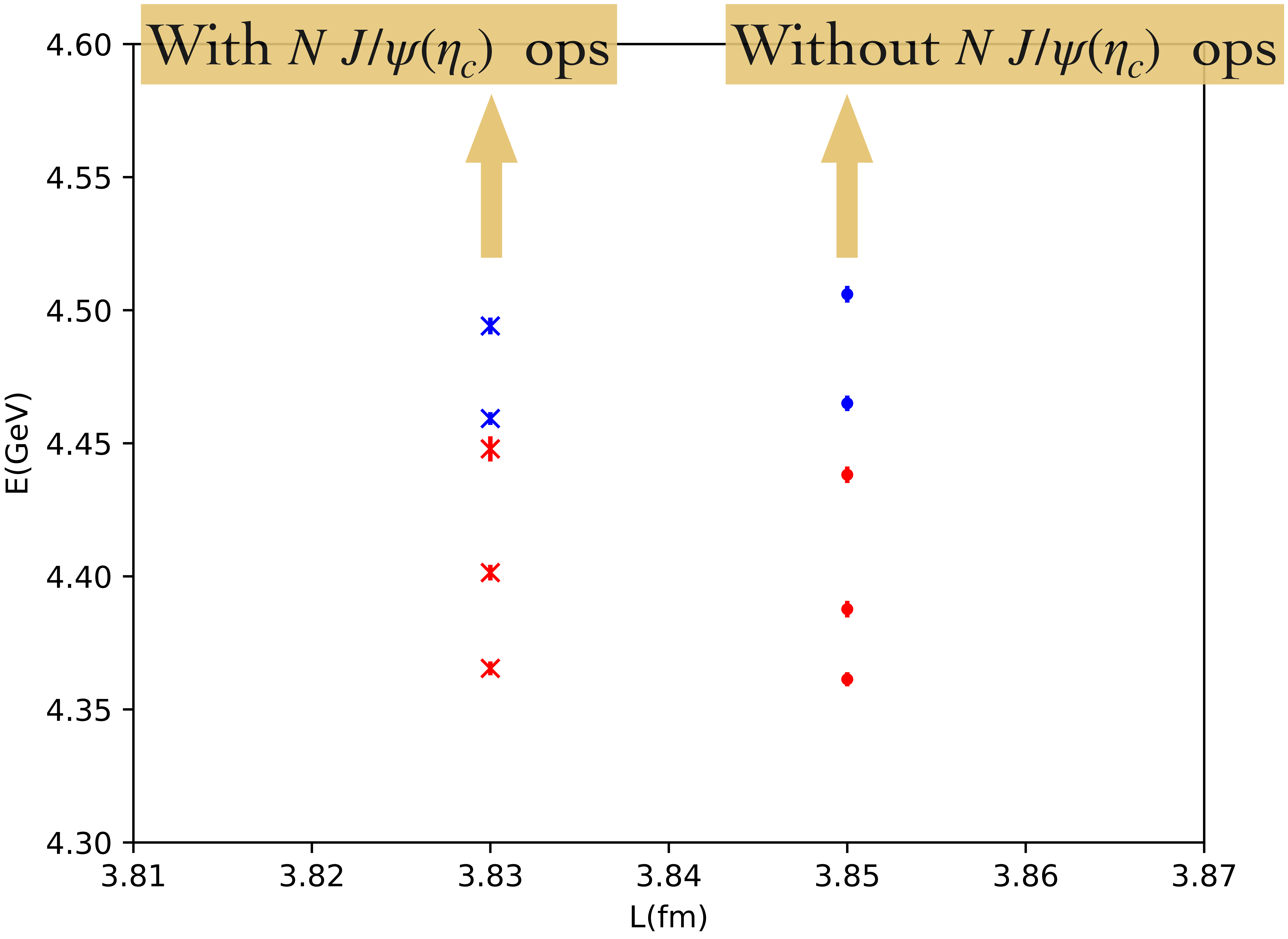
$$\mathcal{O}_{9,10} = \Lambda_c(\mathbf{p})\bar{D}^*(-\mathbf{p}) \; (|\mathbf{p}| = 0,1)$$

$$\mathcal{O}_{11,12,13} = \Sigma_c(\mathbf{p})\bar{D}(-\mathbf{p}) \; (|\mathbf{p}| = 0,1,\sqrt{2})$$

$$\mathcal{O}_{14,15} = \Sigma_c(\mathbf{p})\bar{D}^*(-\mathbf{p}) \; (|\mathbf{p}| = 0,1)$$



P_c Pentaquarks



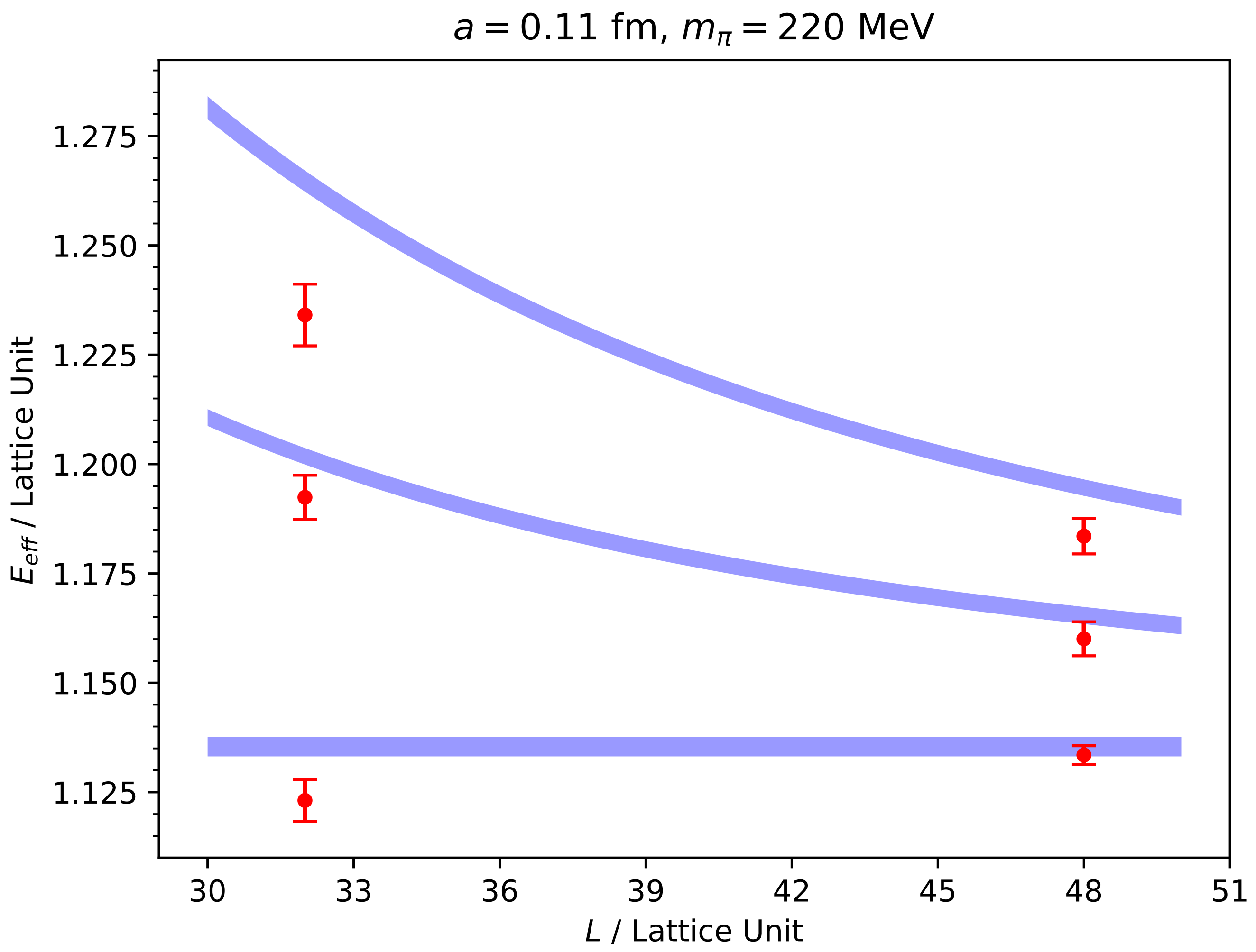
H-dibaryon

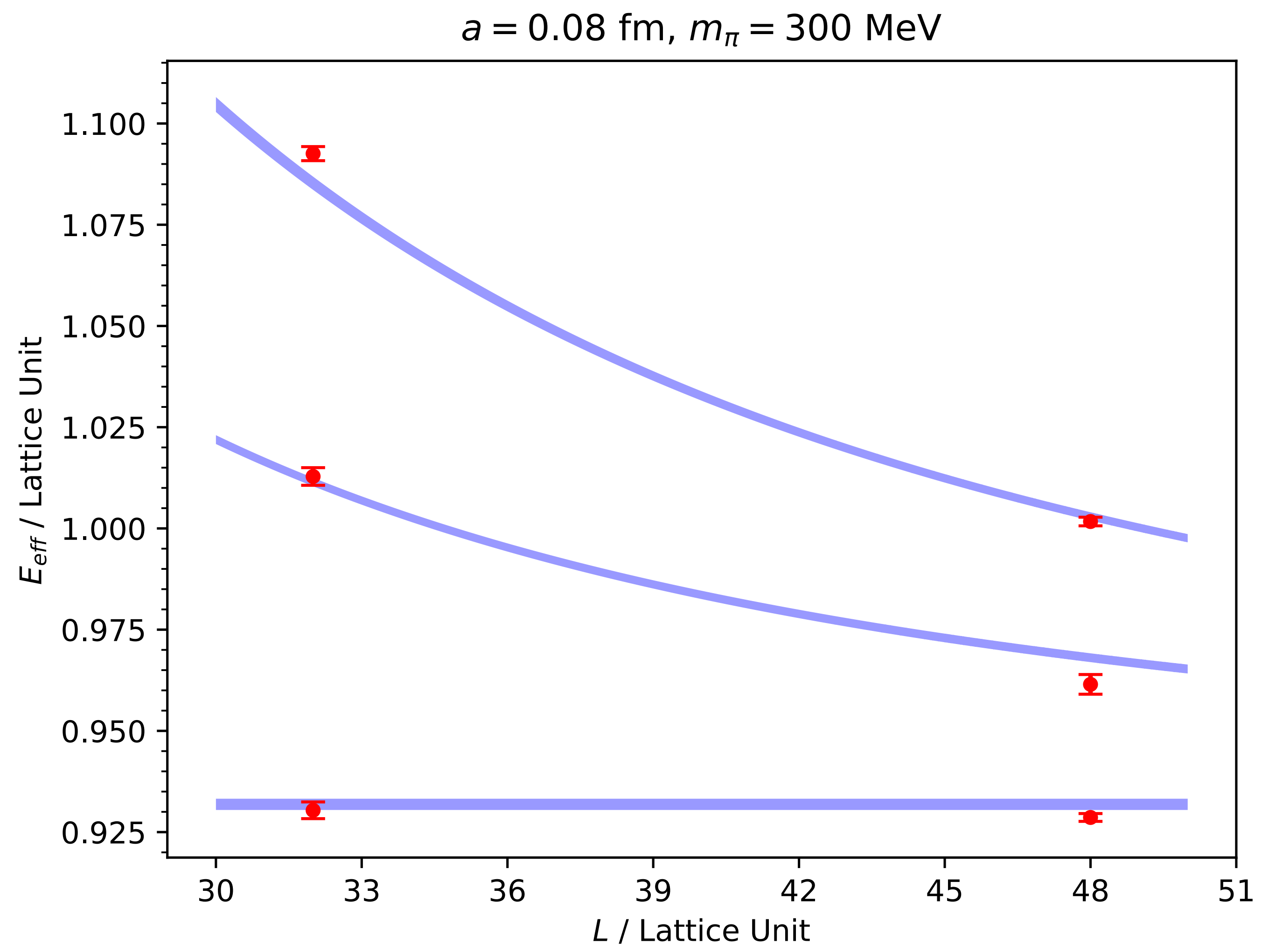
✦Theoretical prediction of a deeply bound di-baryon with quark content uuddss.
✦No solid experimental evidence.
✦Controvertial lattice results.

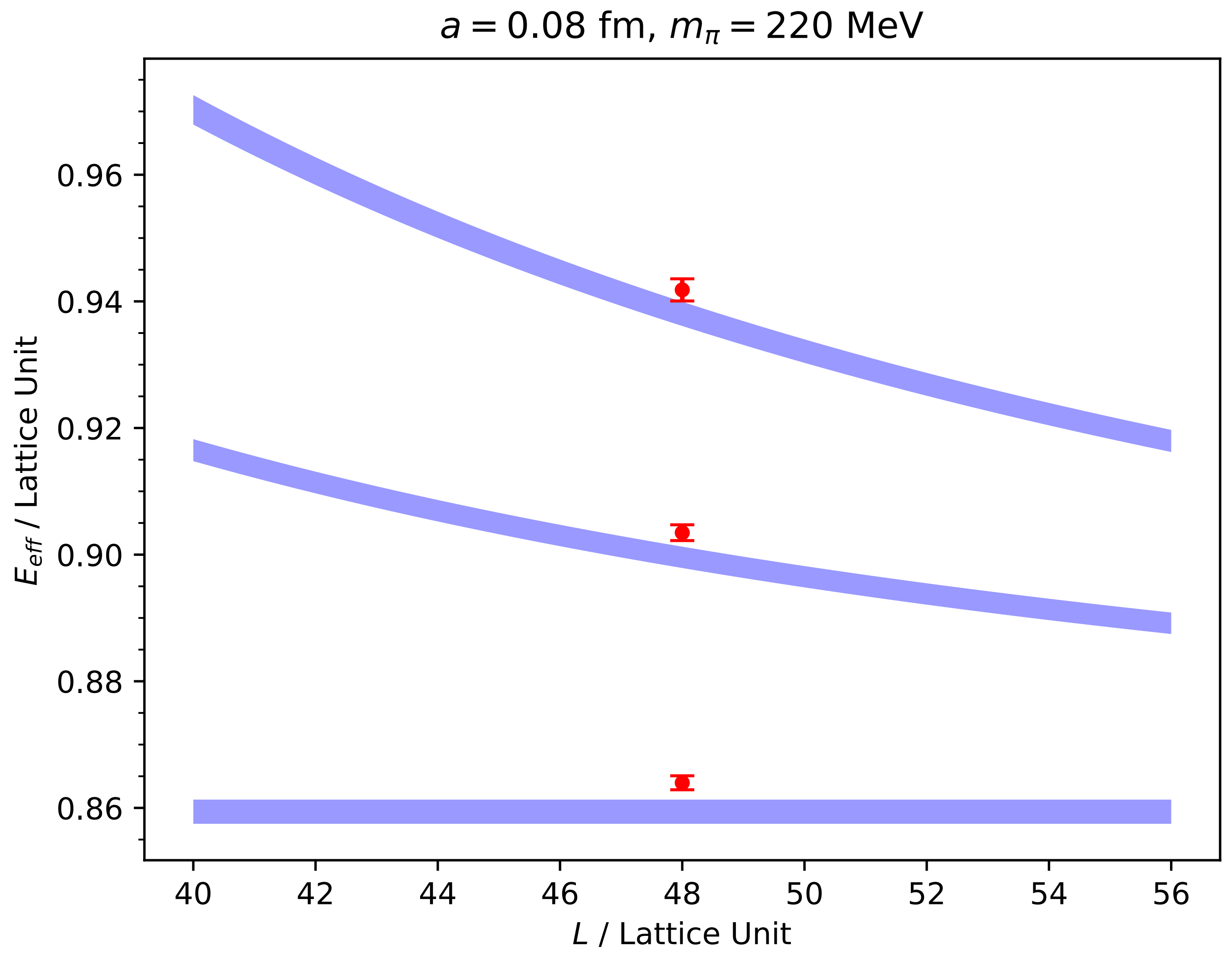
- arXiv: 2108.09644(HALQCD), weekly attractive without a bound state.
- arXiv: 2103.01054, weakly bound, binding energy 4.56(1.13)(0.63)MeV.
- arXiv: 1912.08630(HALQCD), virtual state.
- arXiv:1805.03966, bound state with binding energy 19(10)MeV
- arXiv:1109.2889(NPLQCD), bound state with binding energy 13.2(1.8)(4.0)MeV
- arXiv:1012.5968(HALQCD), bound state with binding energy 30-40MeV
- ……

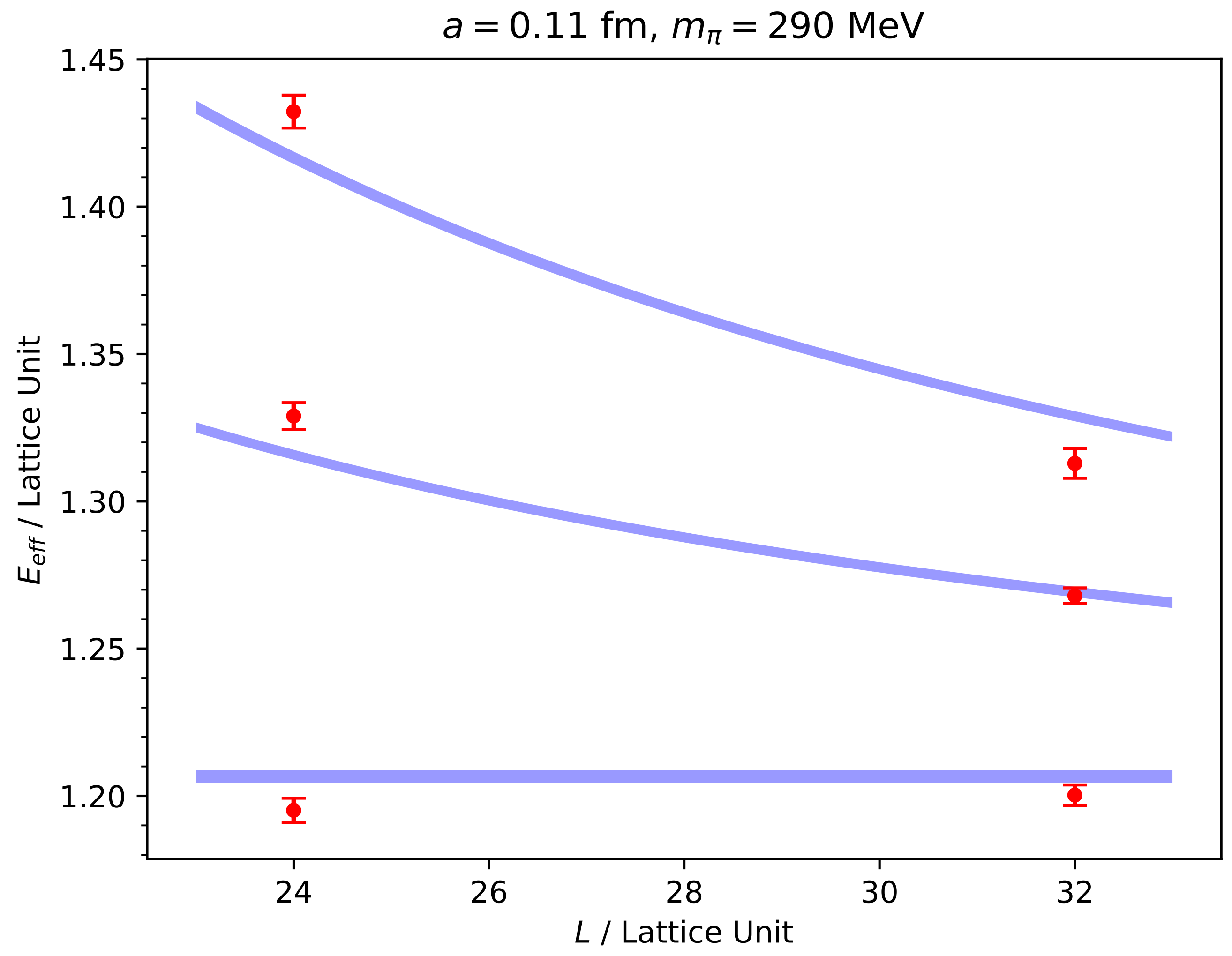
H-dibaryon

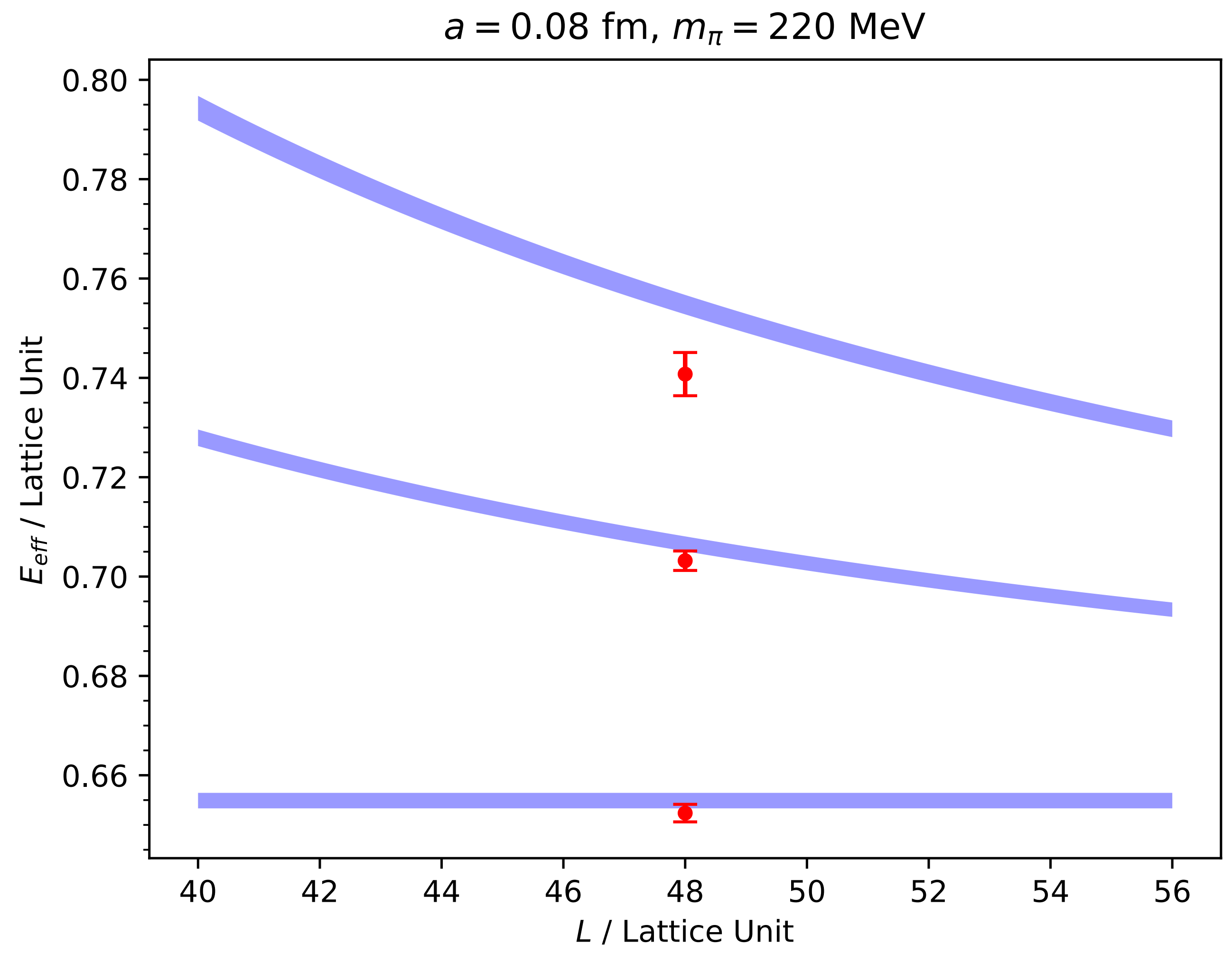
$\Lambda\Lambda$ operators: $\mathcal{O} = \Lambda(\mathbf{p})\Lambda(-\mathbf{p})\ (|\mathbf{p}| = 0, 1, \sqrt{2})$











Summary

- Lattice QCD study of hadron spectroscopy has entered the era of precise determination of the properties of resonances and exotic states.
- We have setup the framework and methodology to systematically study hadron spectroscopy. Coupled channels, multi-particle scattering and nucleon related scattering remain to be challenging.
- Preliminary results on pentaquark and di-Lambda have been obtained. Other ongoing projects: T_{cc}, doubly-charmed baryon, XYZ states, ρ resonance…