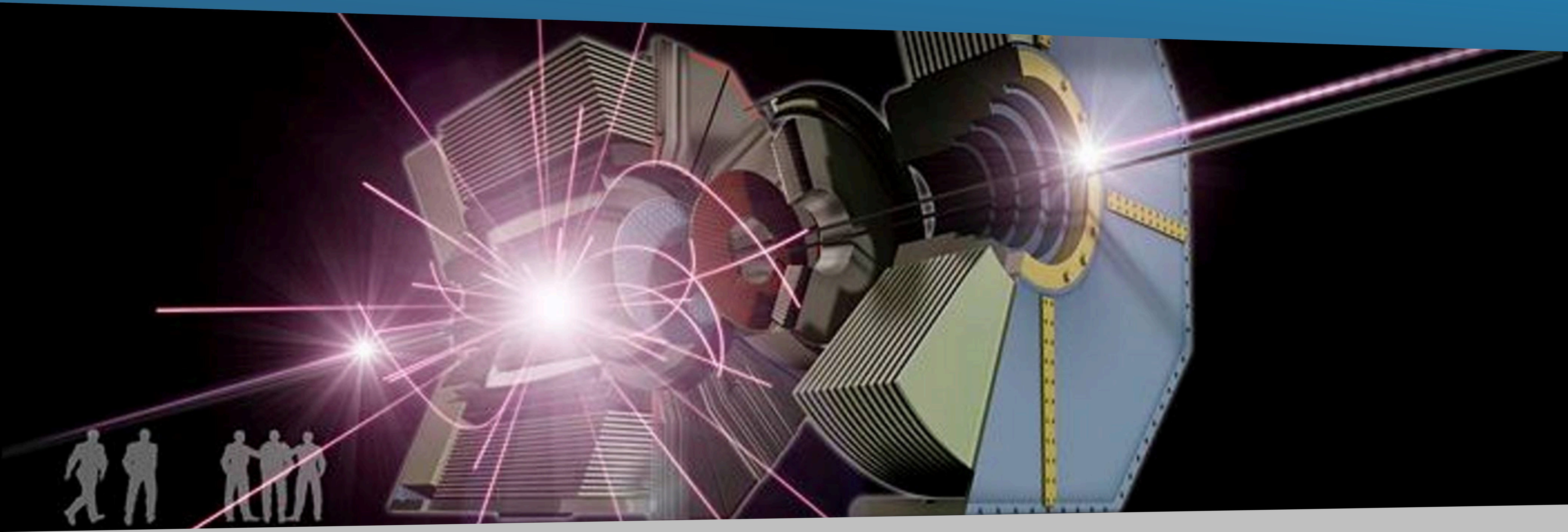
First Simultaneous Determination of Inclusive and Exclusive $|V_{ub}|$



曹 璐

(lu.cao@desy.de)

HAPOF 强子物理在线论坛 2023-04-14

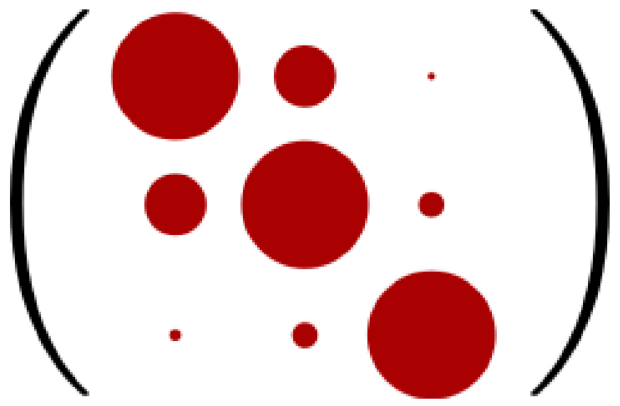
Introduction

- CKM triangle provides a potent test of the Standard Model

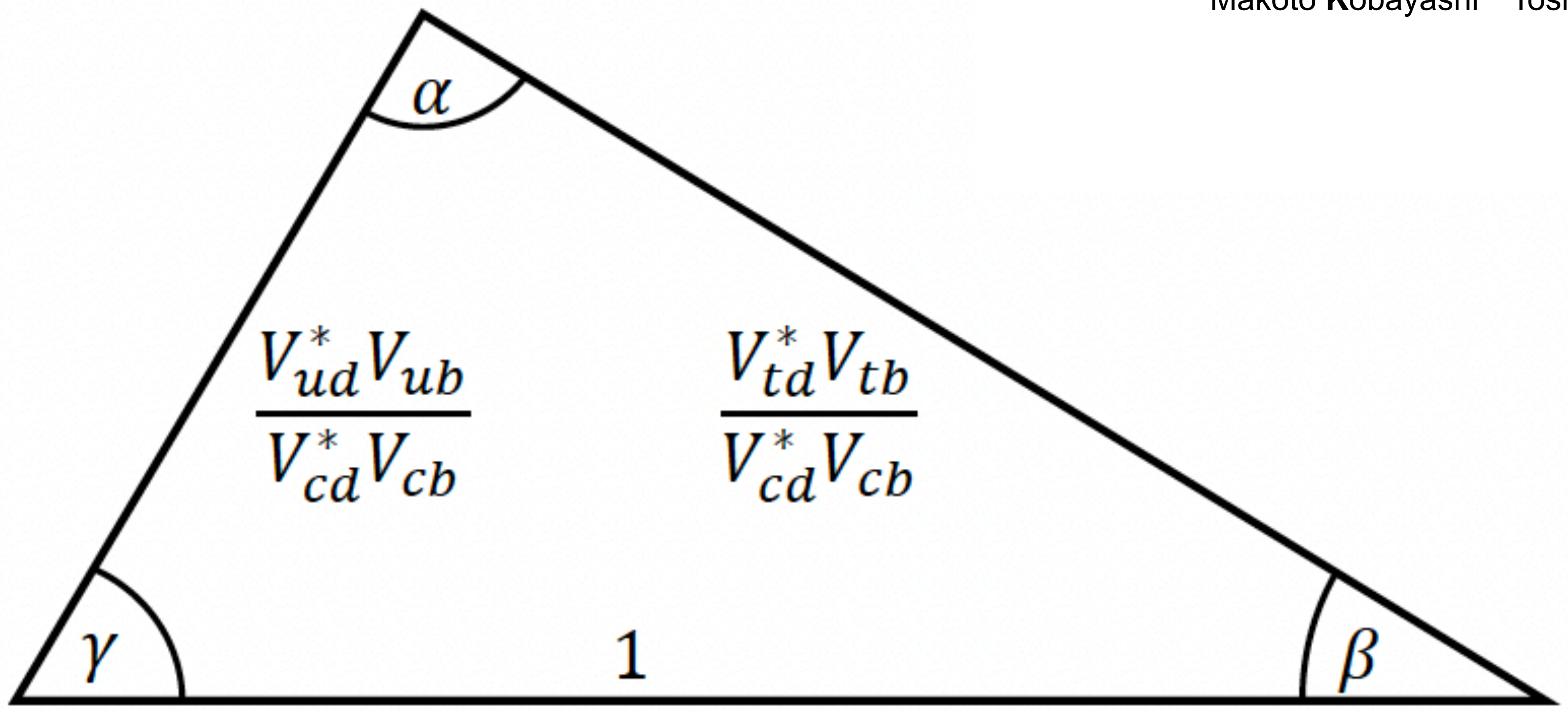
Nobel prize 2008: KM mechanism

Nicola **C**abibbo Makoto **K**obayashi Toshihide **M**asukawa

$$V_{\mathrm{CKM}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx$$



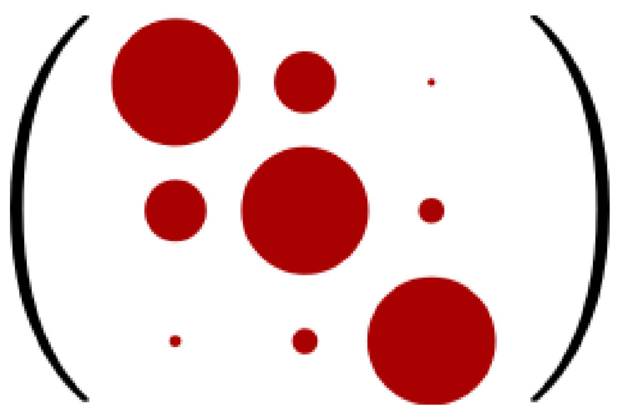
$$V_{\mathrm{CKM}} V_{\mathrm{CKM}}^{\dagger} = \mathbf{1}$$



Introduction

- CKM triangle provides a potent test of the Standard Model
- Global fit constrained by unitarity condition => 3 mixing angles + CPV phase
  - The sides and angles need to be measured to over-constrain the triangle and test that it closes.
  - If there is CP violation the triangle remains open
  - All lengths involve *b* decays

$$V_{\mathrm{CKM}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx$$



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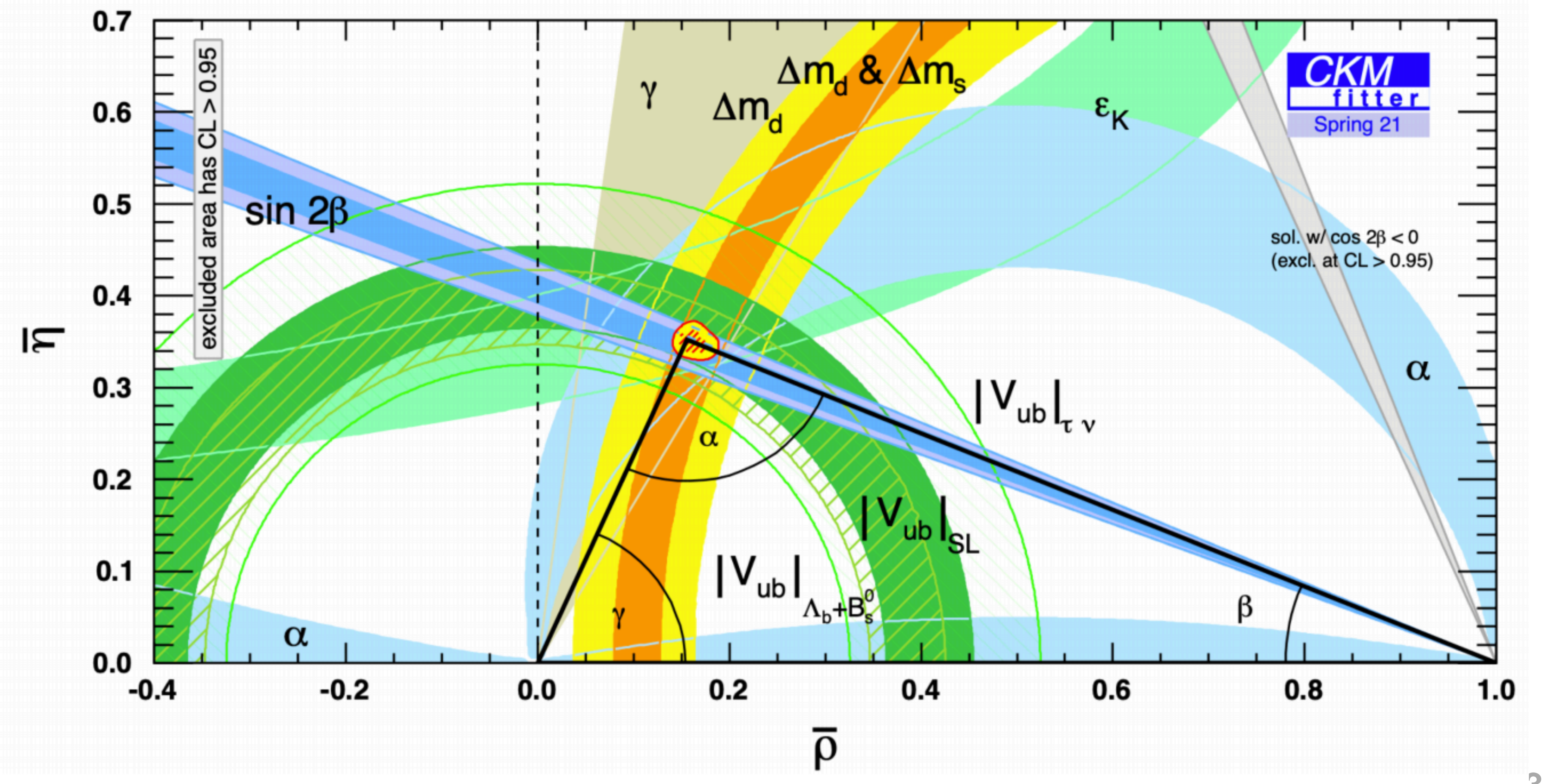

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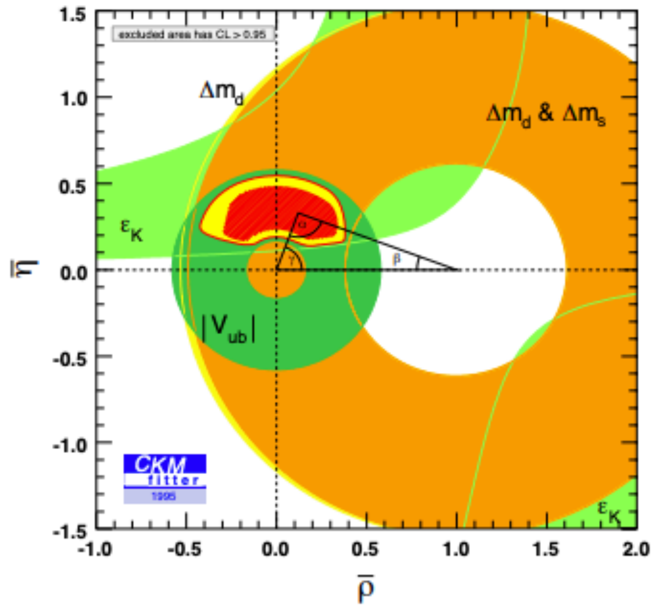
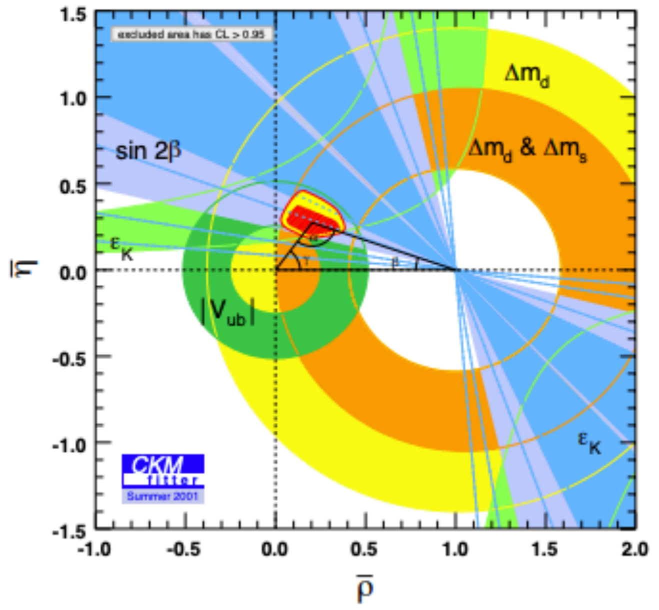
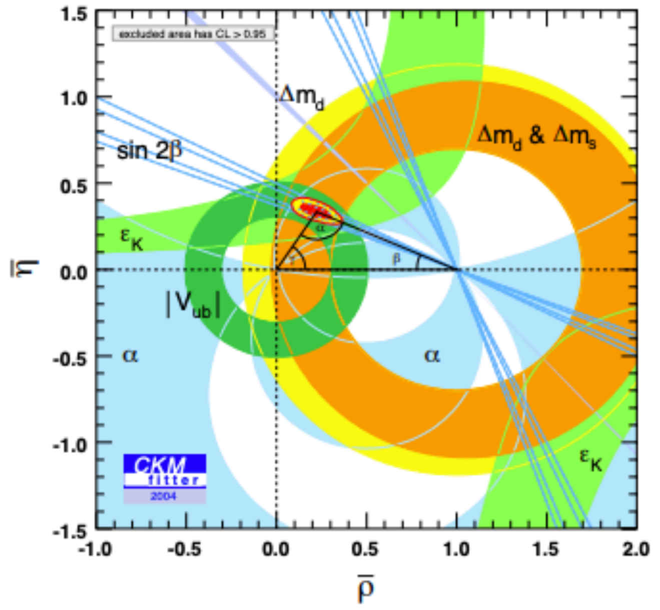


1995

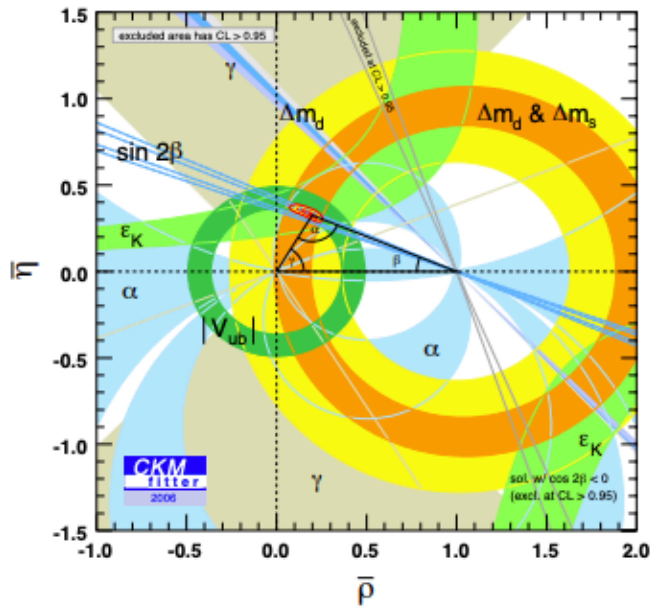


2001

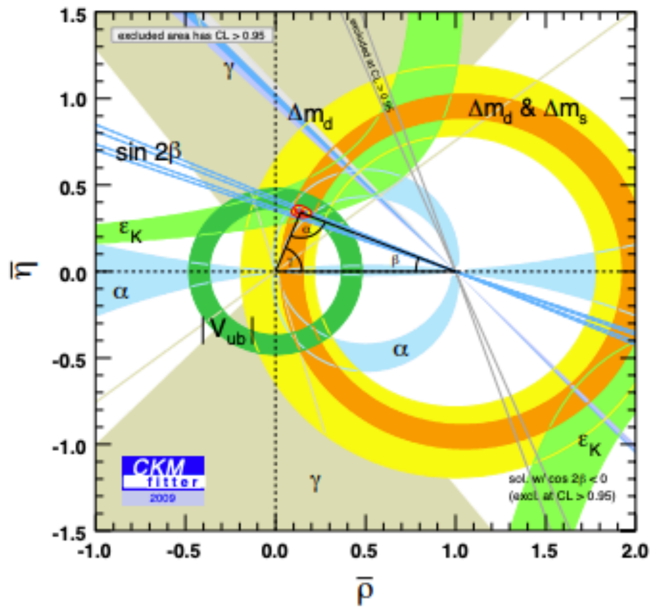


2004

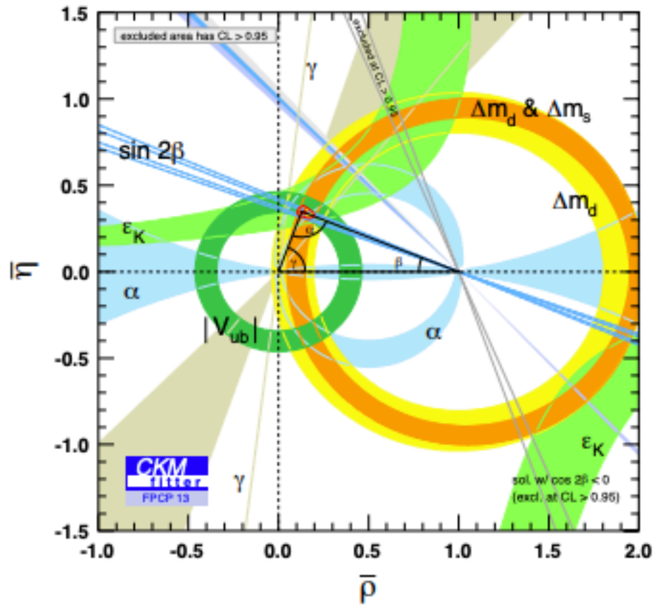
Precision of $|V_{ub}|/|V_{cb}|$ (width of the green circle) stays similar in past decades



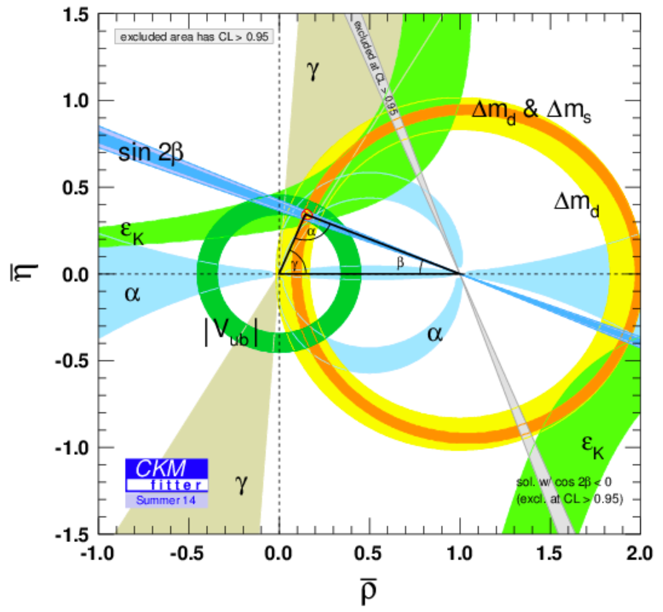
2006



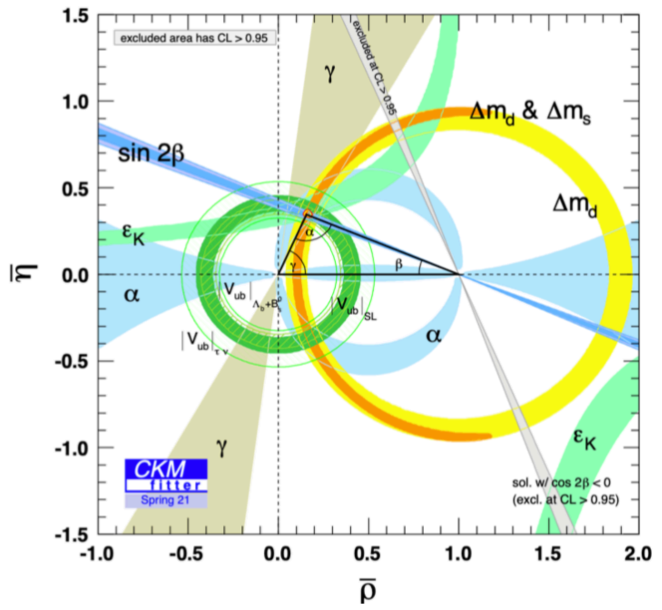
2009



2013



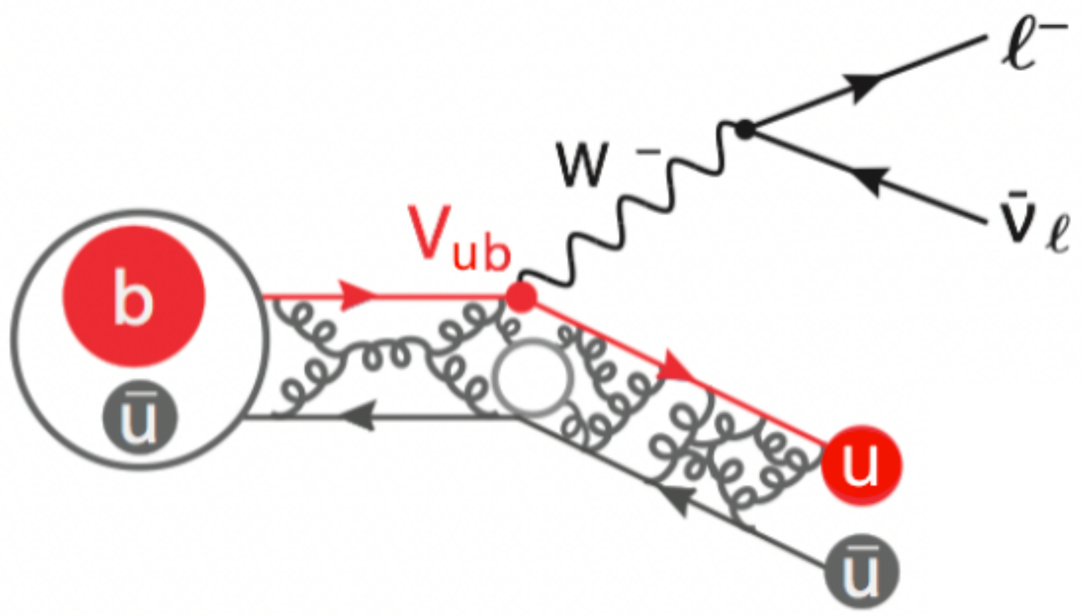
2014



2021

Introduction

- $|V_{ub}|$ can be measured in semileptonic B decays with **inclusive** or **exclusive** channels (leptonic probe refers to larger uncertainty)



Exclusive $|V_{ub}|$ 遍举

$B \to \pi \ell \nu,\ \Lambda_b \to p \ell \nu,\ B_s \to K \ell \nu$, etc.

$$\mathscr{B} \propto |V_{ub}|^2 f^2$$

Form factor f (LCSR, LQCD)

Inclusive $|V_{ub}|$ 单举

$B \to X_u \ell \nu, \quad X_u = \pi, \rho, \omega, \eta^{(\prime)}$, non-resonant contribution

$$\mathscr{B} \propto |V_{ub}|^2 \left[1 + \frac{c_5(\mu) \langle O_5 \rangle (\mu)}{m_h^2} + \frac{c_6(\mu) \langle O_6 \rangle (\mu)}{m_h^3} + O\left(m_b^4\right) \right] \qquad |V_{ub}| = \sqrt{\frac{\Delta \mathscr{B}}{\tau_B \cdot \Delta \Gamma}}$$

+ Shape Function / Fermi Motion (OPE)

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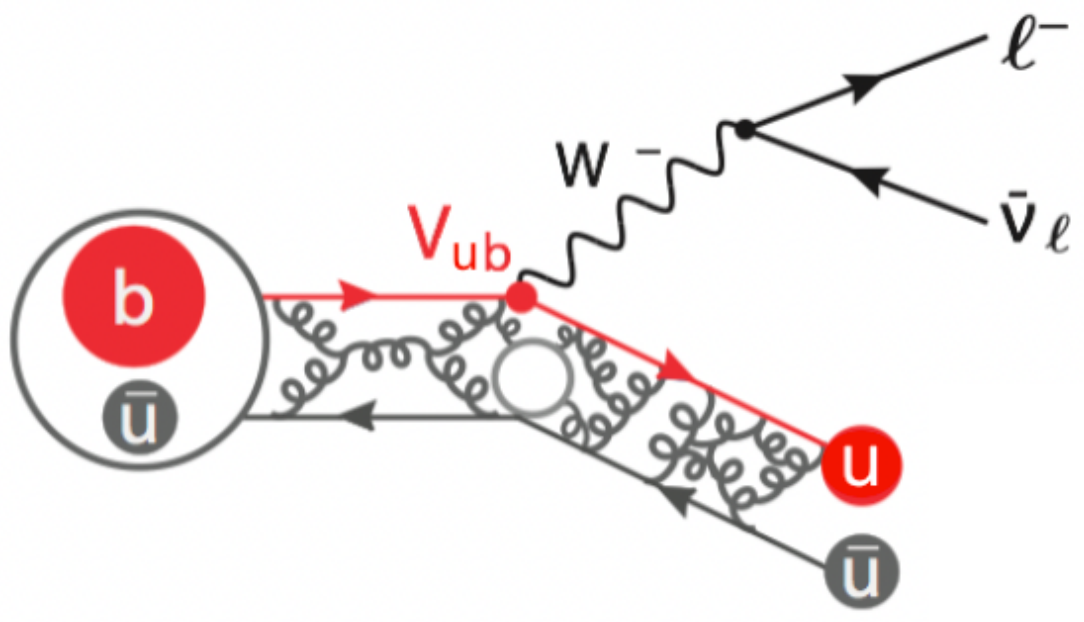

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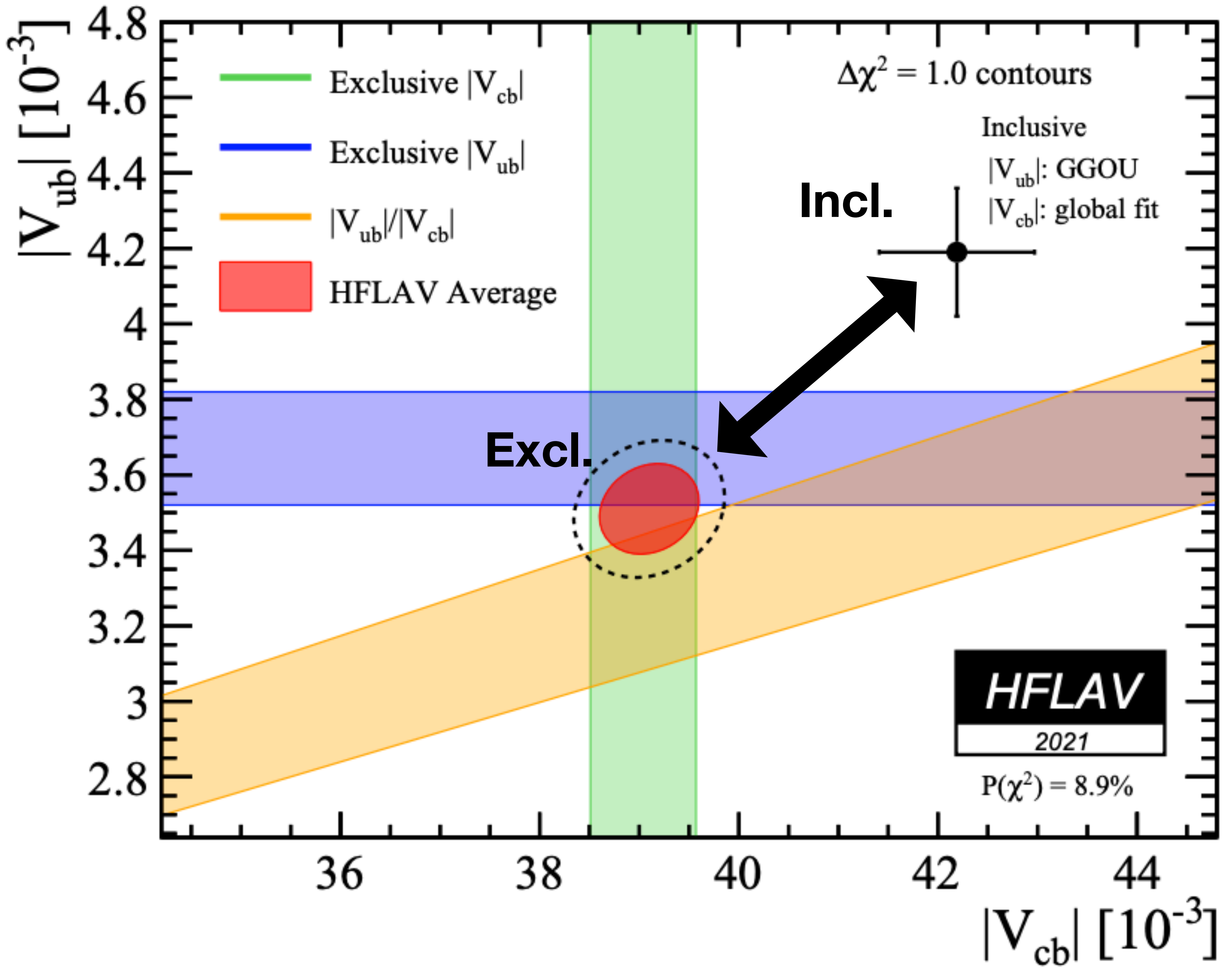

$$|V_{ub}^{\text{excl.}}| = (3.51 \pm 0.12) \times 10^{-3}$$
$$|V_{ub}^{\text{incl.}}| = (4.19 \pm 0.16) \times 10^{-3}$$

Ratio = 0.84 ± 0.04

3.7σ from unity!!

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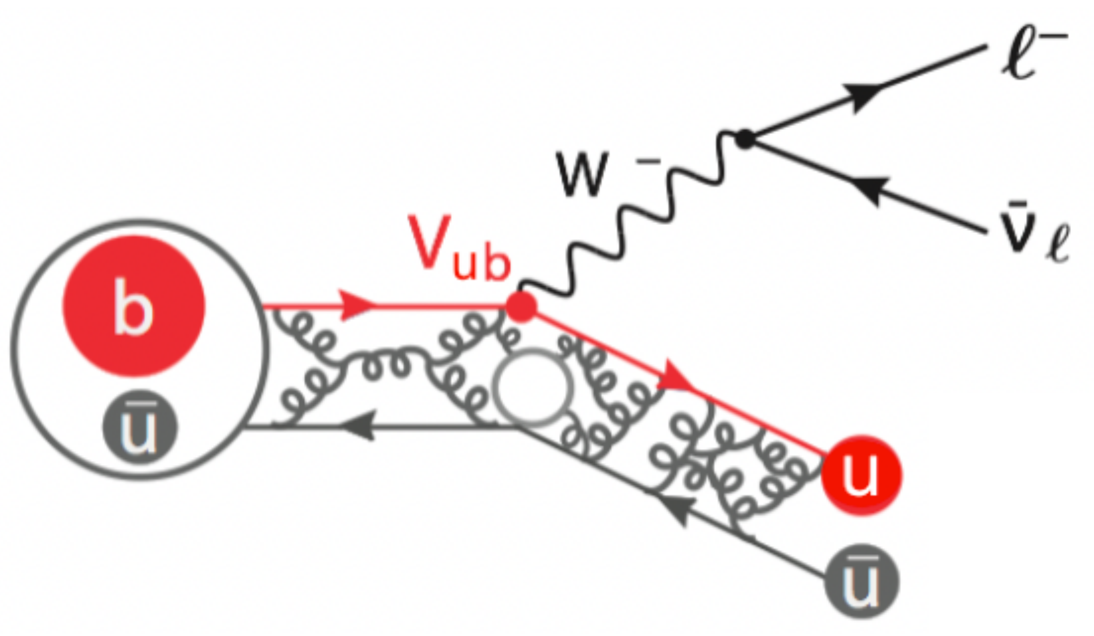

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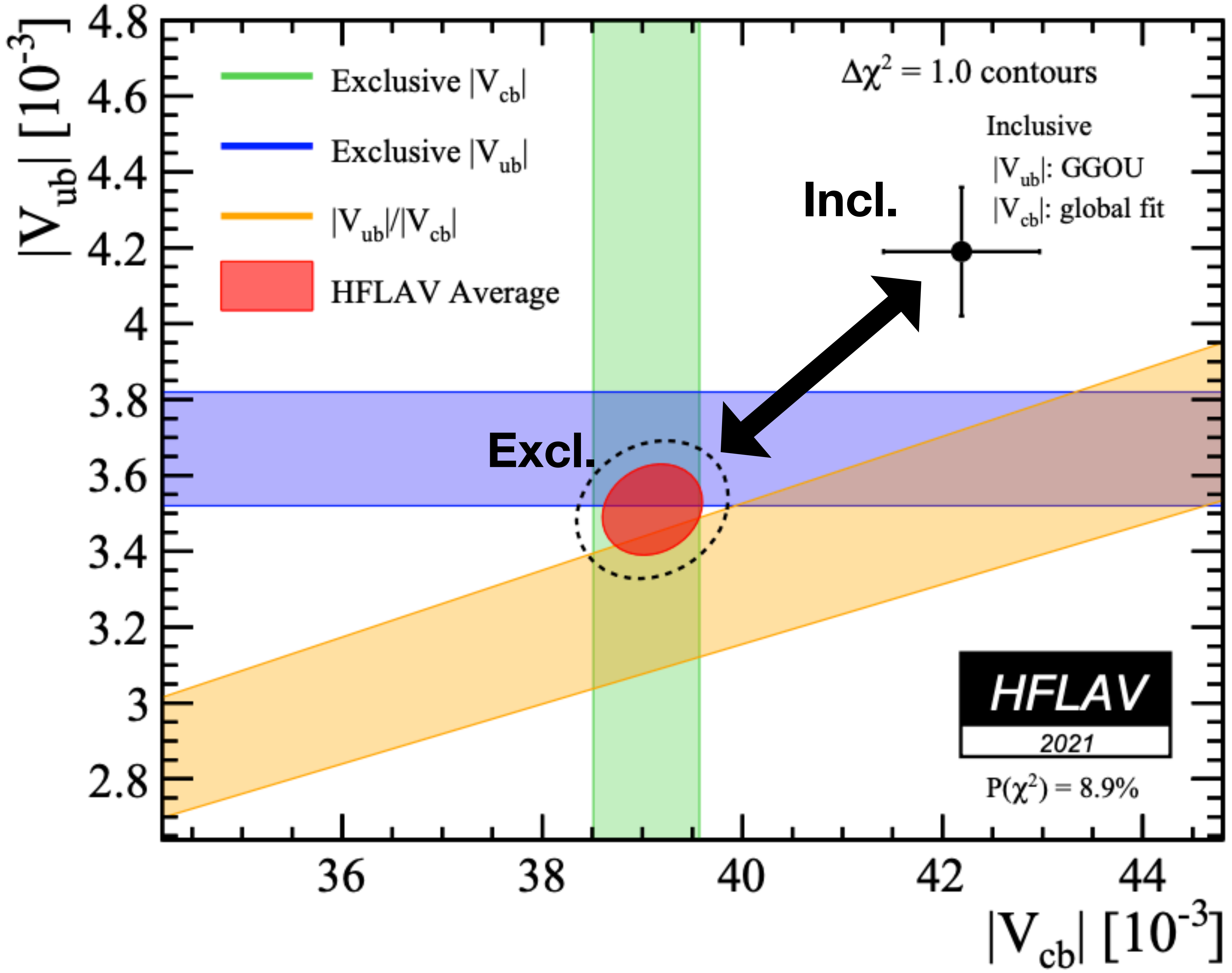

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BELLE Preliminary

Belle Preprint 2023-04, KEK Preprint 2022-53

First Simultaneous Determination of Inclusive and Exclusive $|V_{ub}|$

(The Belle Collaboration)

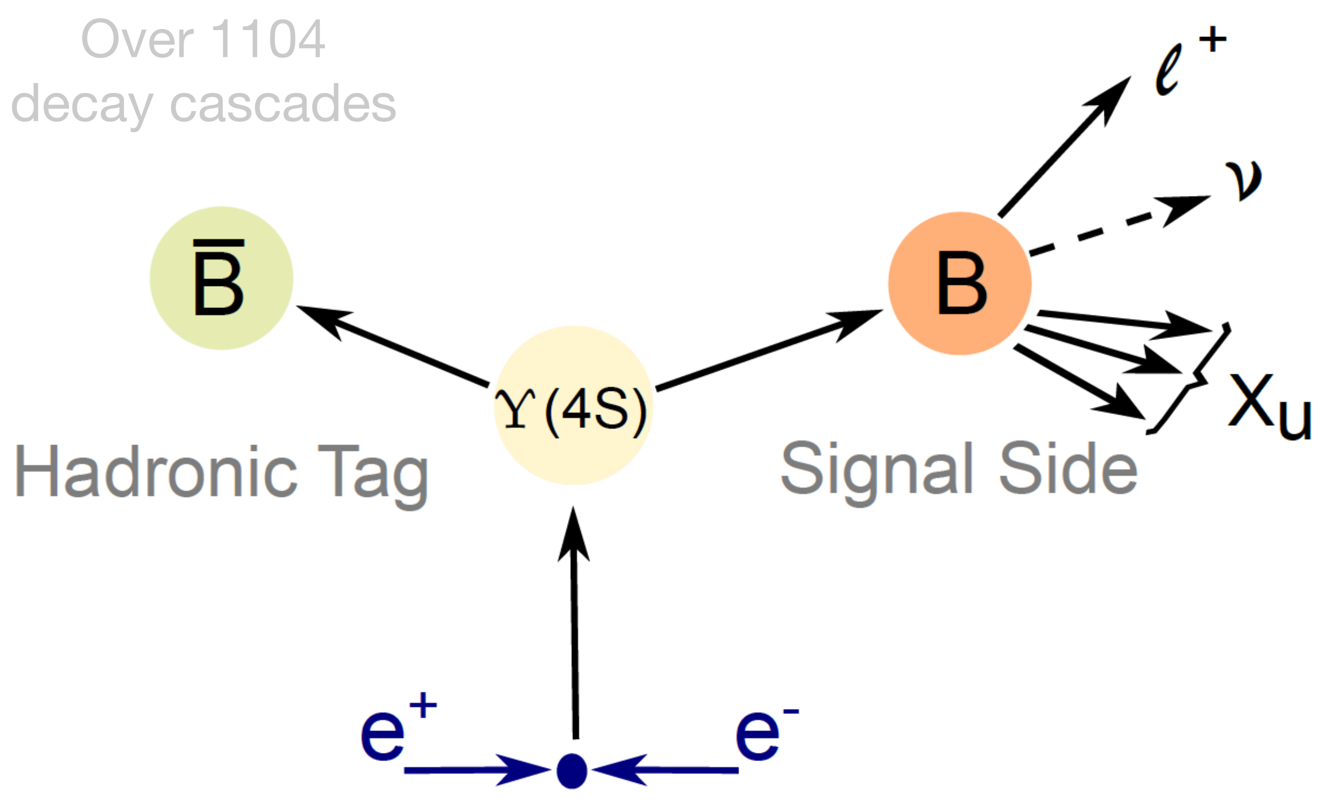
The first simultaneous determination of the absolute value of the Cabibbo-Kobayashi-Maskawa matrix element V_{ub} using inclusive and exclusive decays is performed with the full Belle data set at the $\Upsilon(4S)$ resonance, corresponding to an integrated luminosity of 711 fb^{-1}. We analyze collision events in which one B meson is fully reconstructed in hadronic modes. This allows for the reconstruction of the hadronic X_u system of the semileptonic $b \to u\ell\bar{\nu}_\ell$ decay. We separate exclusive $B \to \pi\,\ell\,\bar{\nu}_\ell$ decays from other inclusive $B \to X_u\,\ell\,\bar{\nu}_\ell$ and backgrounds with a two-dimensional fit, that utilizes the number of charged pions in the X_u system and the four-momentum transfer q^2 between the B and X_u system. Combining our measurement with information from lattice QCD and QCD calculations of the inclusive partial rate as well as external experimental information on the shape of the $B \to \pi\,\ell\,\bar{\nu}_\ell$ form factor, we determine $|V_{ub}^{\mathrm{excl.}}| = (3.78 \pm 0.23 \pm 0.16 \pm 0.14) \times 10^{-3}$ and $|V_{ub}^{\mathrm{incl.}}| = (3.90 \pm 0.20 \pm 0.32 \pm 0.09) \times 10^{-3}$, respectively, with the uncertainties being the statistical error, systematic errors, and theory errors. The ratio of $|V_{ub}^{\mathrm{excl.}}| / |V_{ub}^{\mathrm{incl.}}| = 0.97 \pm 0.12$ is compatible with unity.

arXiv: 2303.17309

submitted to PRL

Reconstruction of $B \to X_u \ell \nu$ Decays

- Using **full Belle** dataset of 711 fb^{-1}
- **Hadronic tagging** with Neutral Networks (0.2%-0.3% efficiency)
- Reconstruction strategy inherited from recent Belle's $B \to X_u \ell \nu$ measurements (phase space region $E_\ell^B > 1$ GeV)
  - $\Delta\mathcal{B}, |V_{ub}|$ @ PRD 104 , 012008 (2021)
  - Differential spectra @ PRL 127, 261801 (2021)



Can fully assign each final state particle to either the tag side or signal side

=> Allows to reconstruct X_u

Reconstructed kinematic variables

- Hadronic system X:

$$p_X = \underbrace{\textstyle\sum_i \left(\sqrt{m_\pi^2 + |\mathbf{p_i}|^2}, \mathbf{p_i}\right)}_{\text{tracks}} + \underbrace{\textstyle\sum_i (E_i, \mathbf{k_i})}_{\text{neutrals}}$$

- Missing mass squared:

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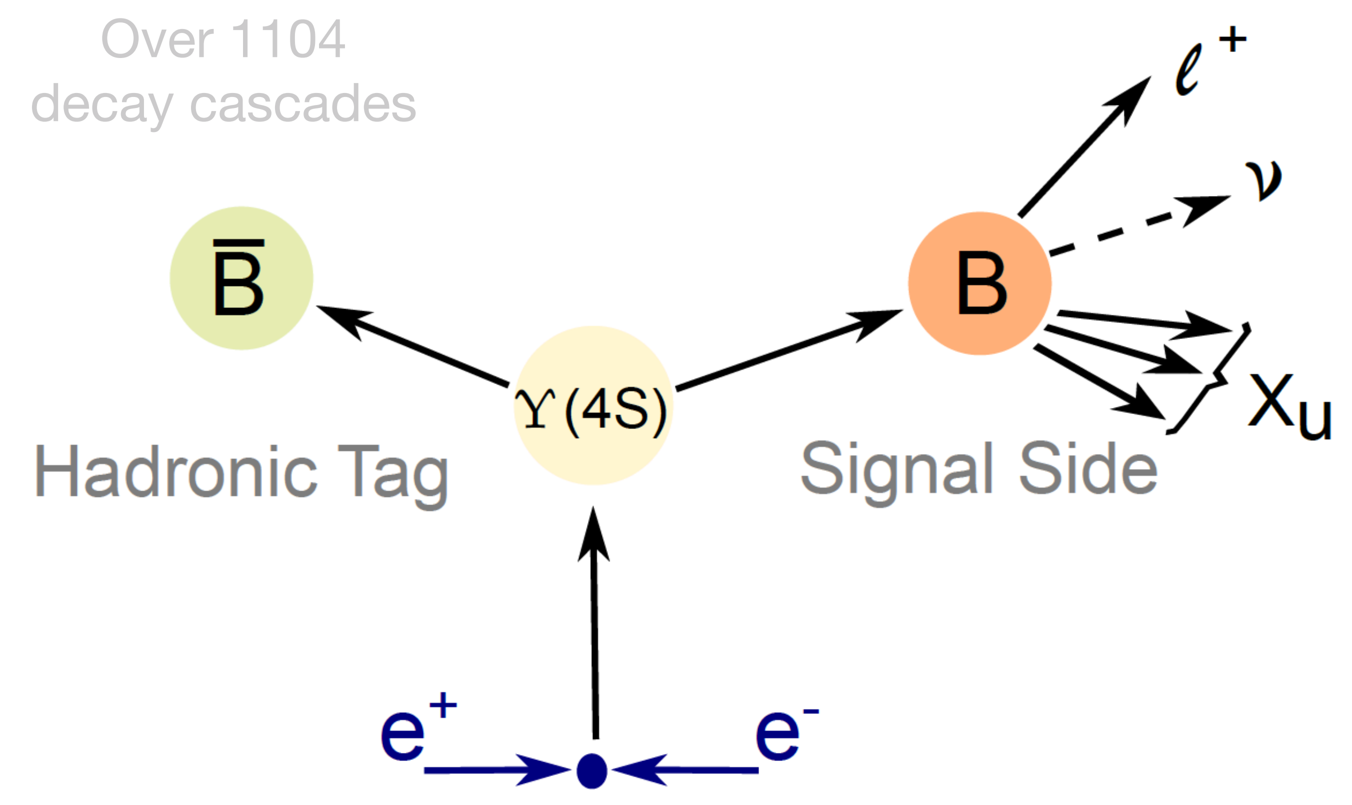

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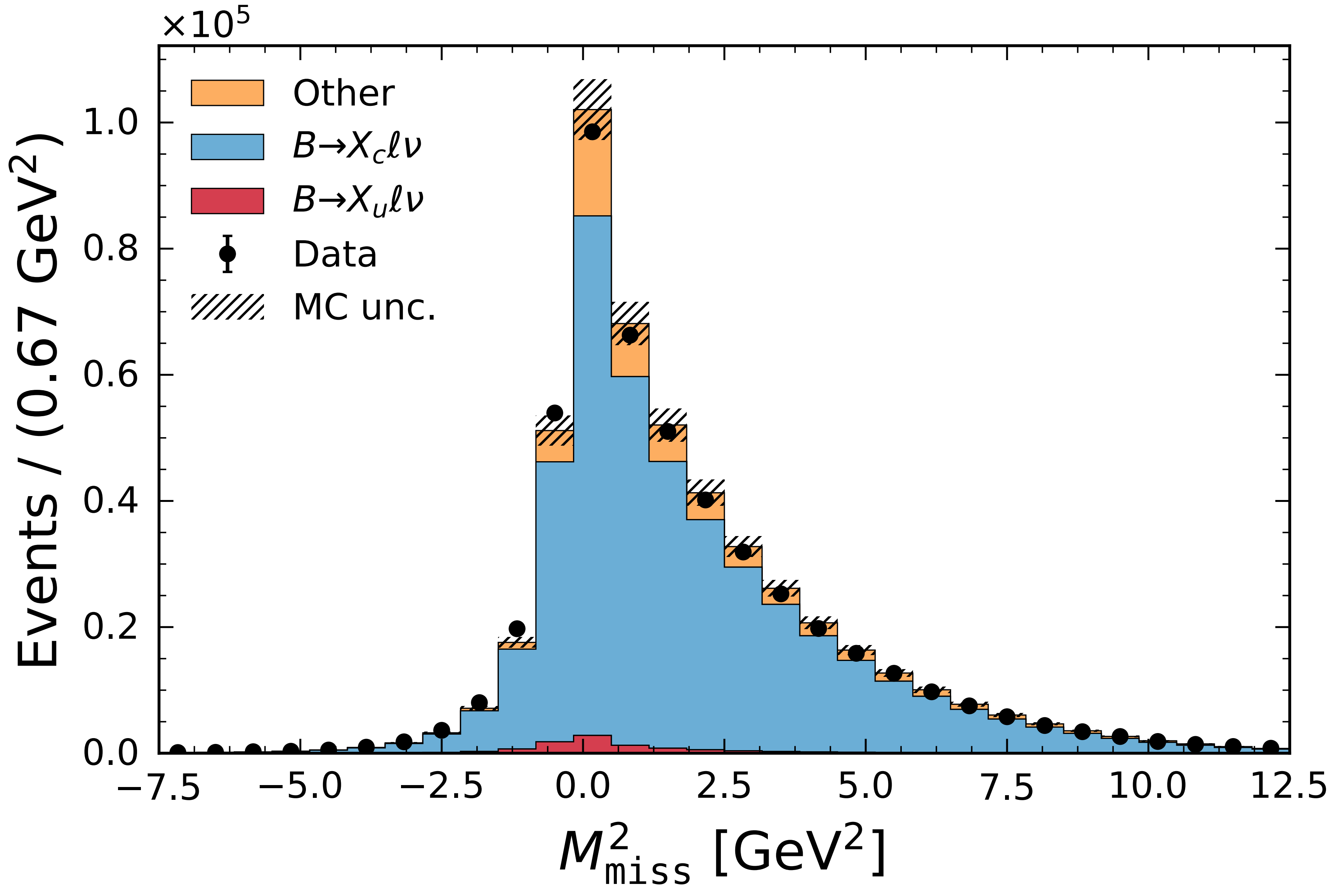

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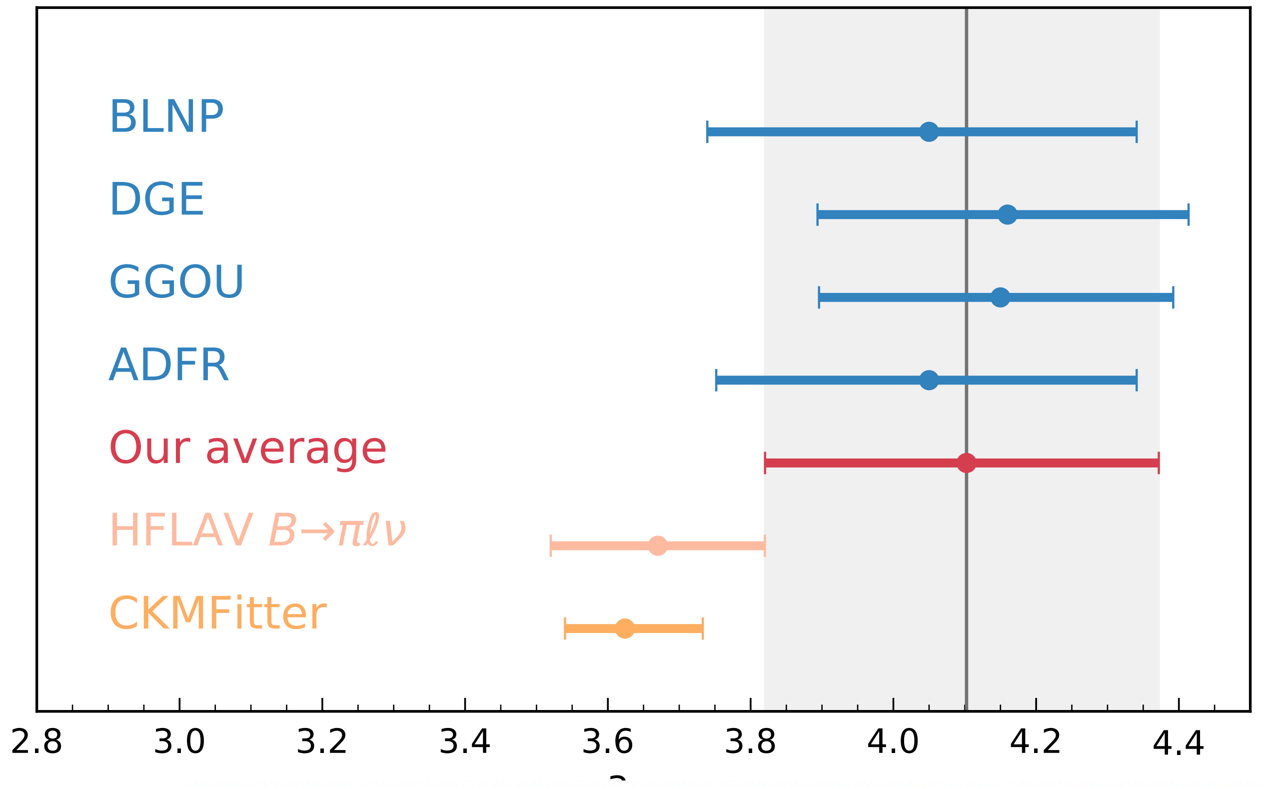
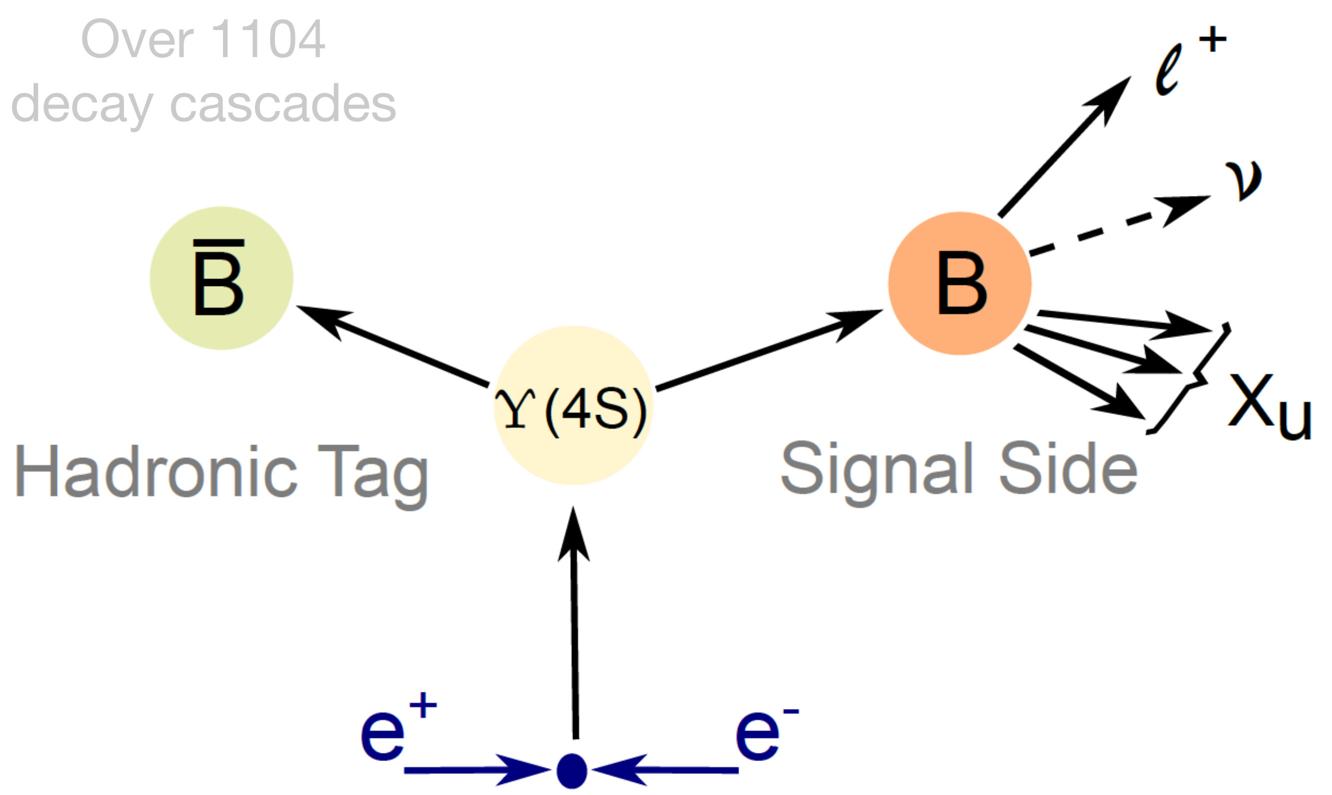


BELLE

Our average:

$|V_{ub}| = (4.10 \pm 0.09_{stat} \pm 0.22_{sys} \pm 0.15_{theo}) \times 10^{-3}$

compatible with excl. and CKM expectation within **1.3σ** and **1.6σ** respectively



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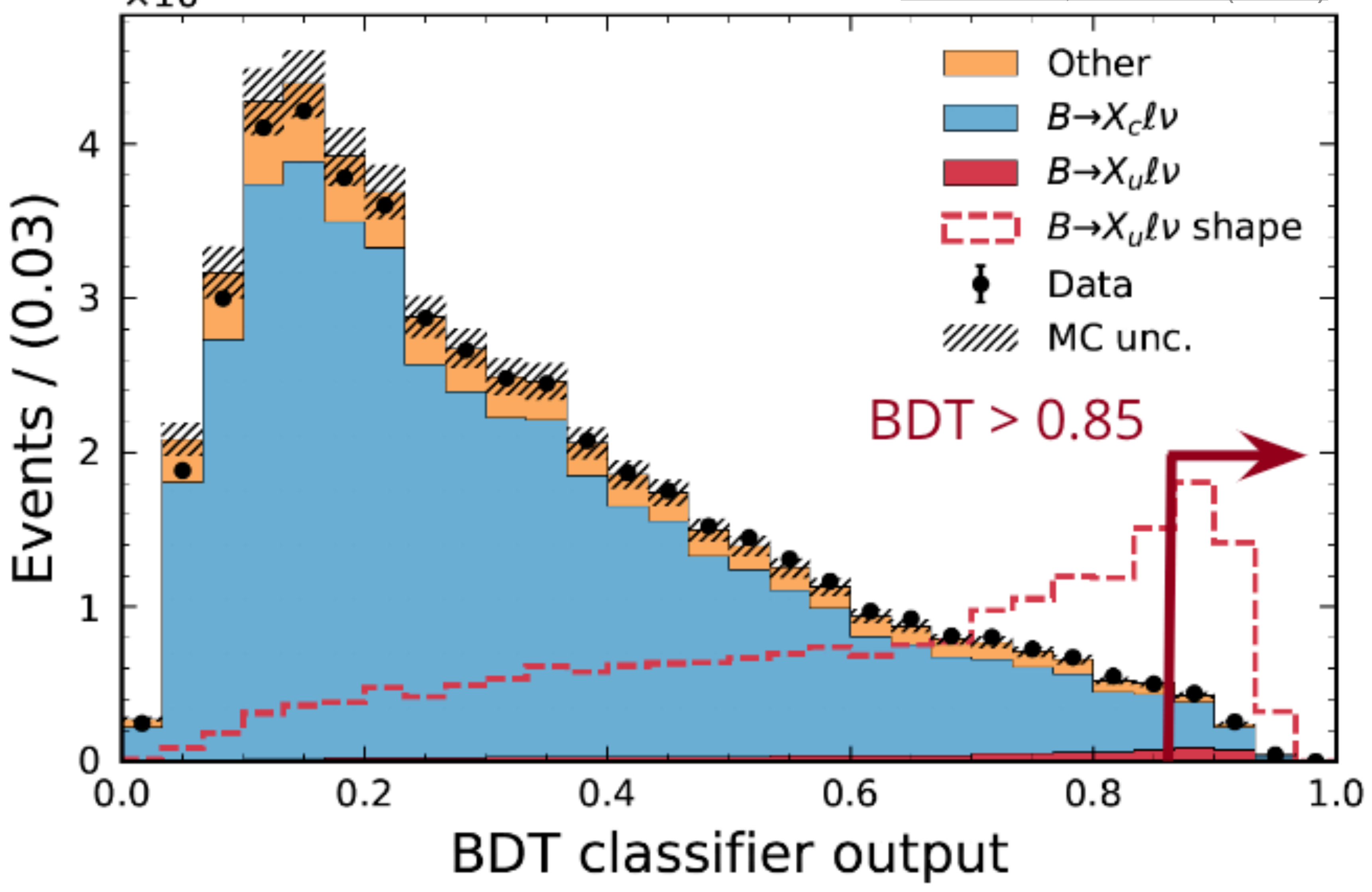
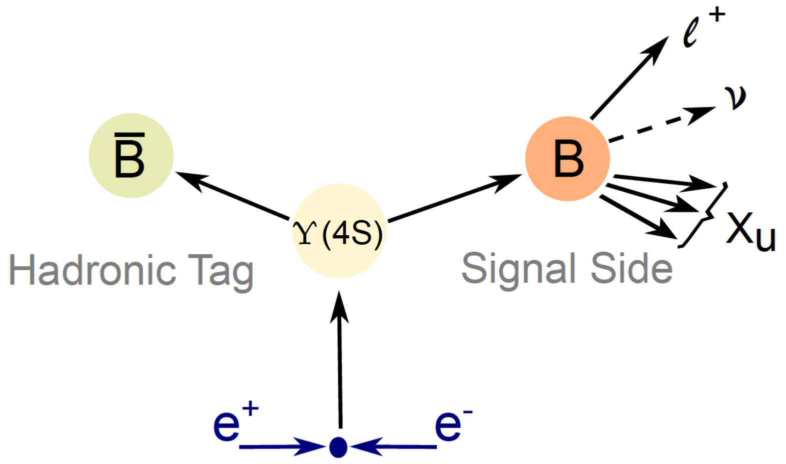
$B \to X_c \ell \nu$ Background Suppression

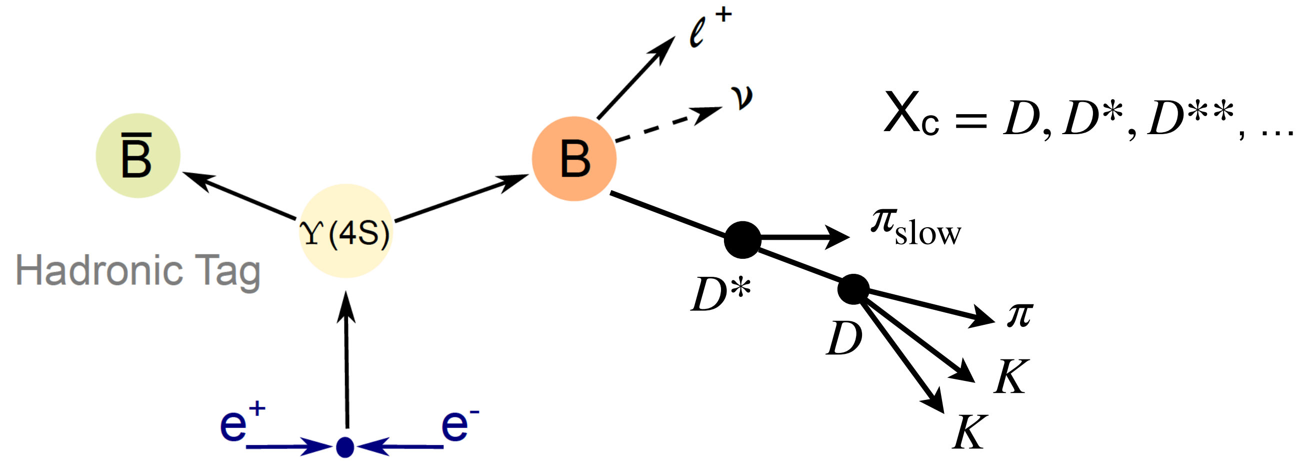
- Dominant background is from **x50 higher** $b \to c \ell \nu$

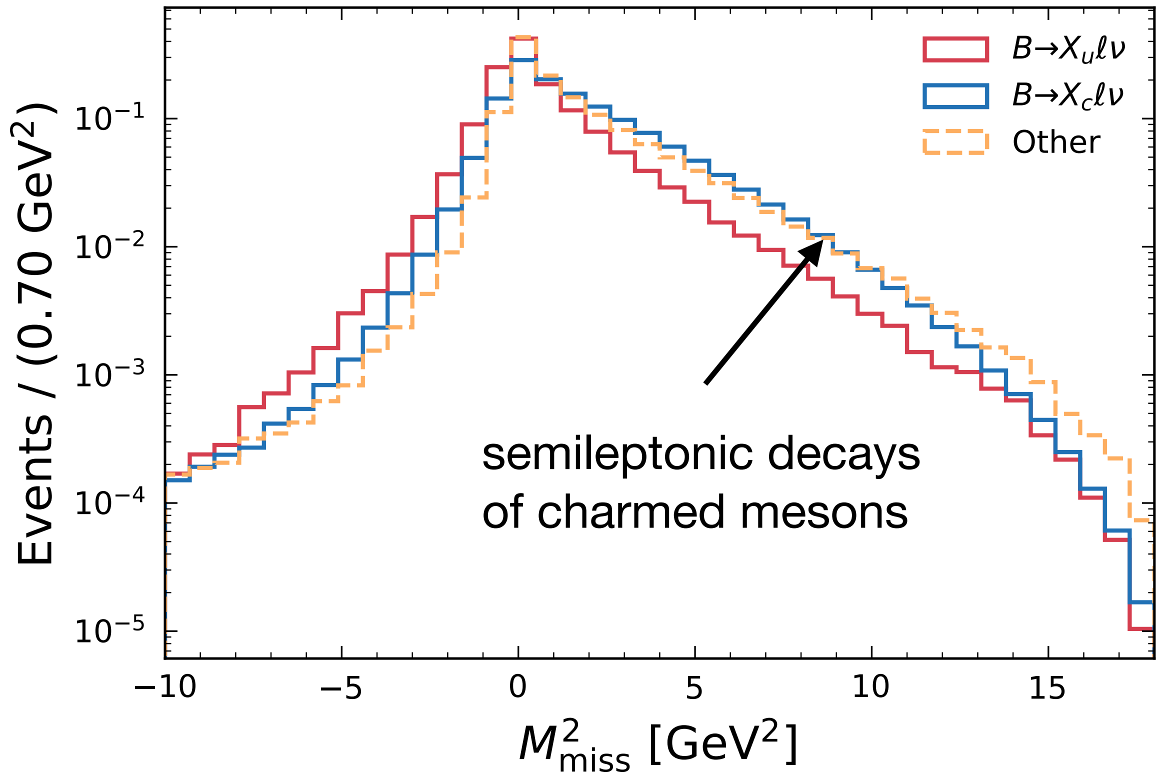
- Use machine learning (BDT) to suppress backgrounds with **11 training features**, e.g. MM^2,#K$^\pm$, #K$_S$, etc.

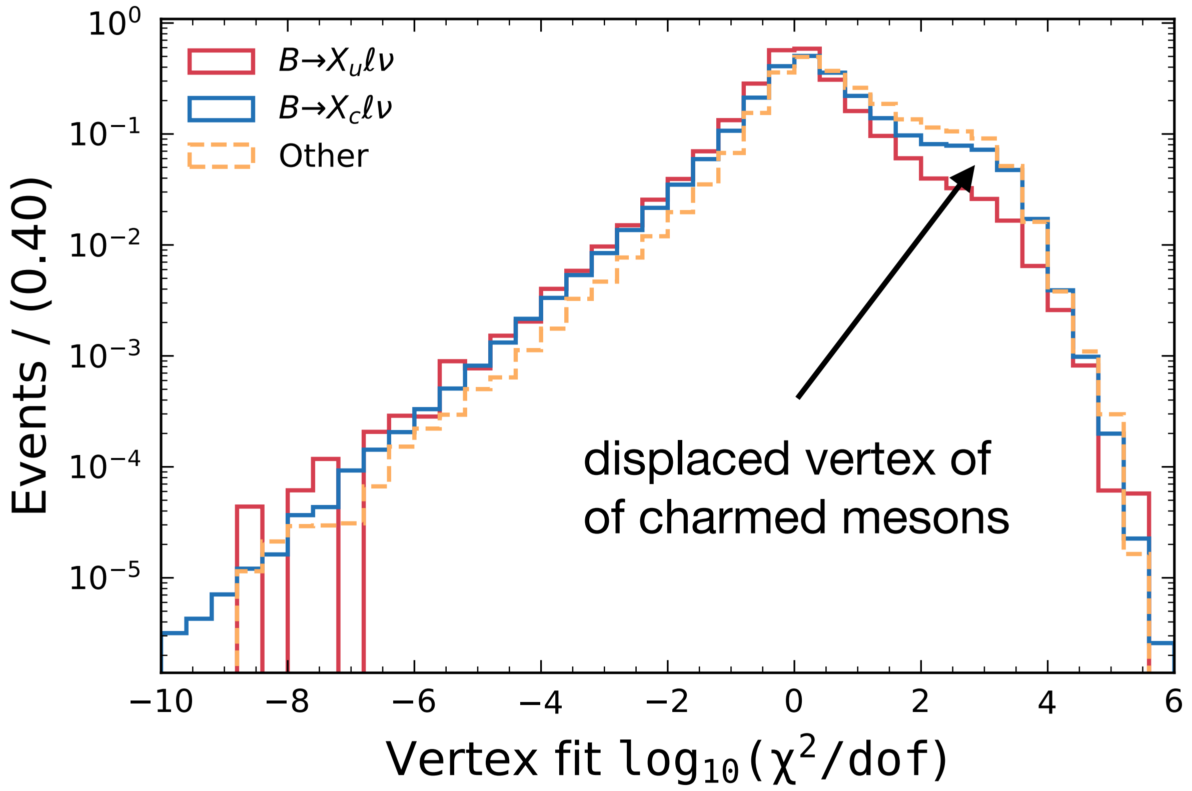
PRD 104 , 012008 (2021)

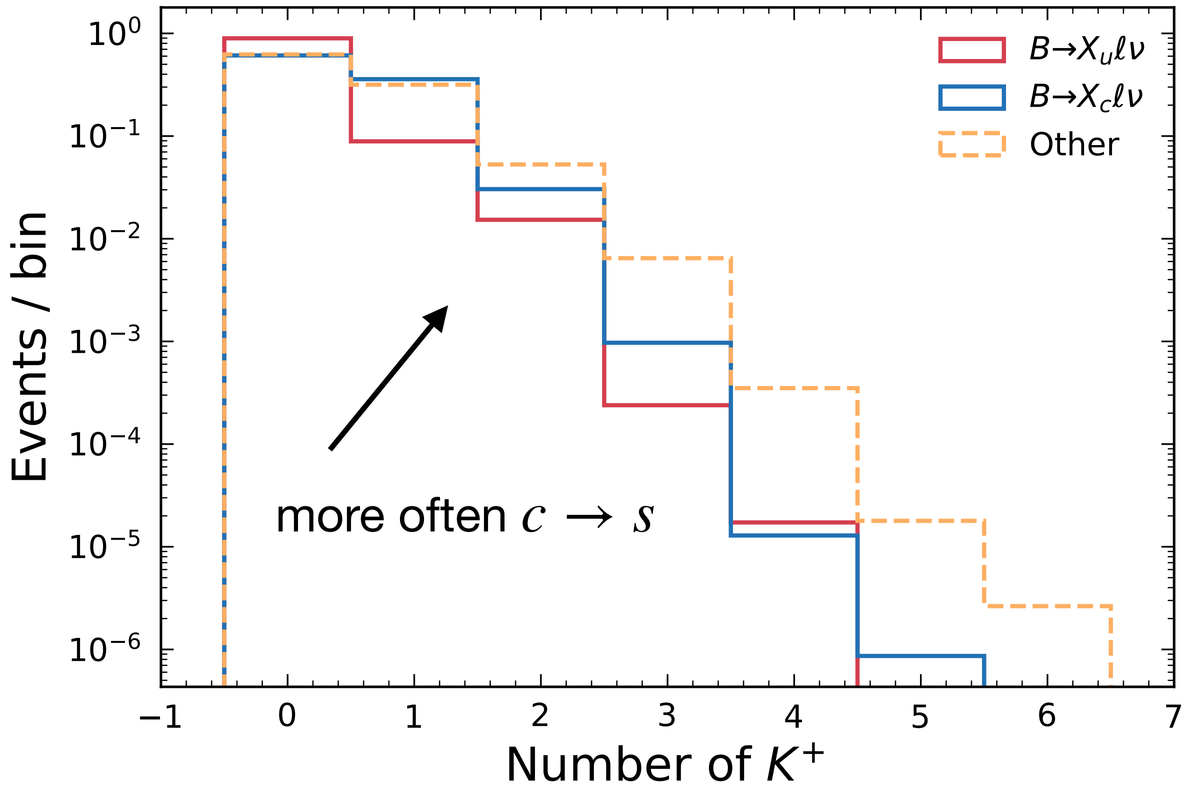


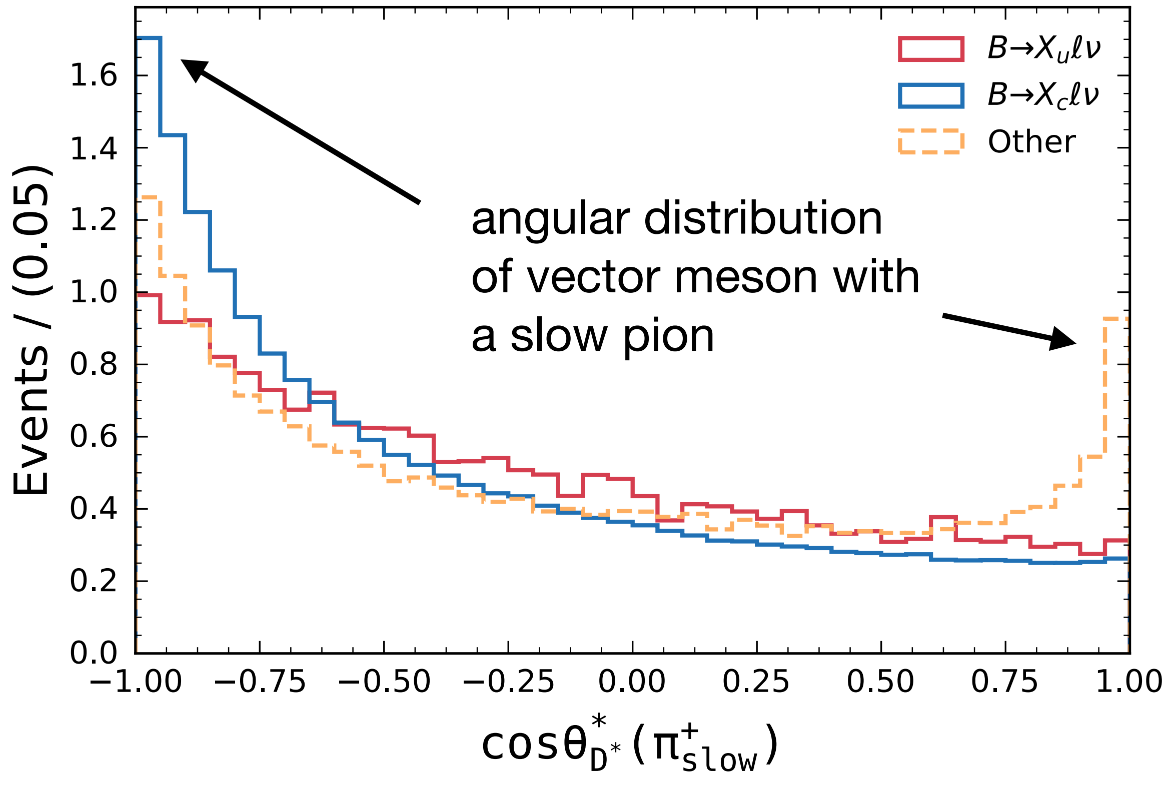












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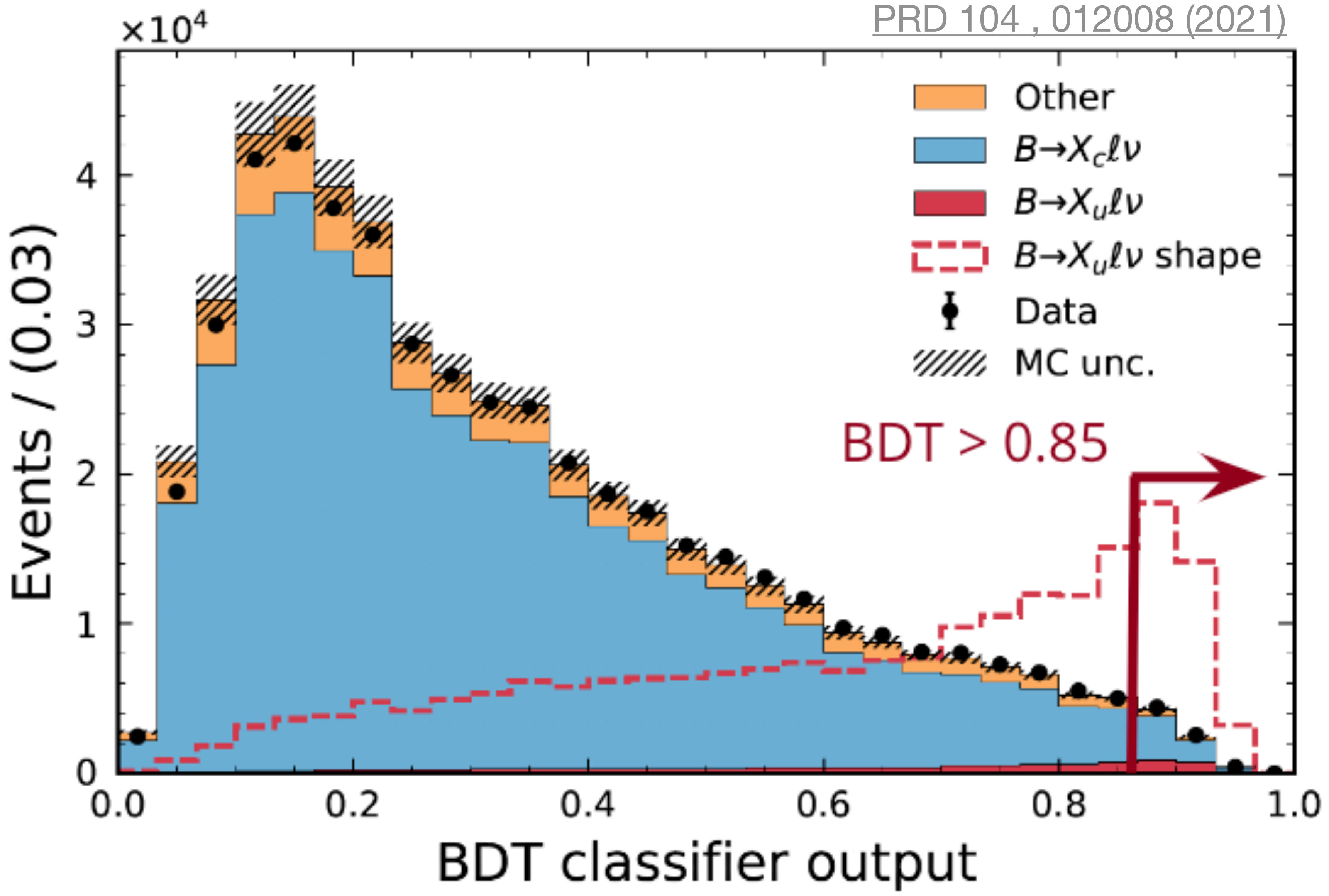
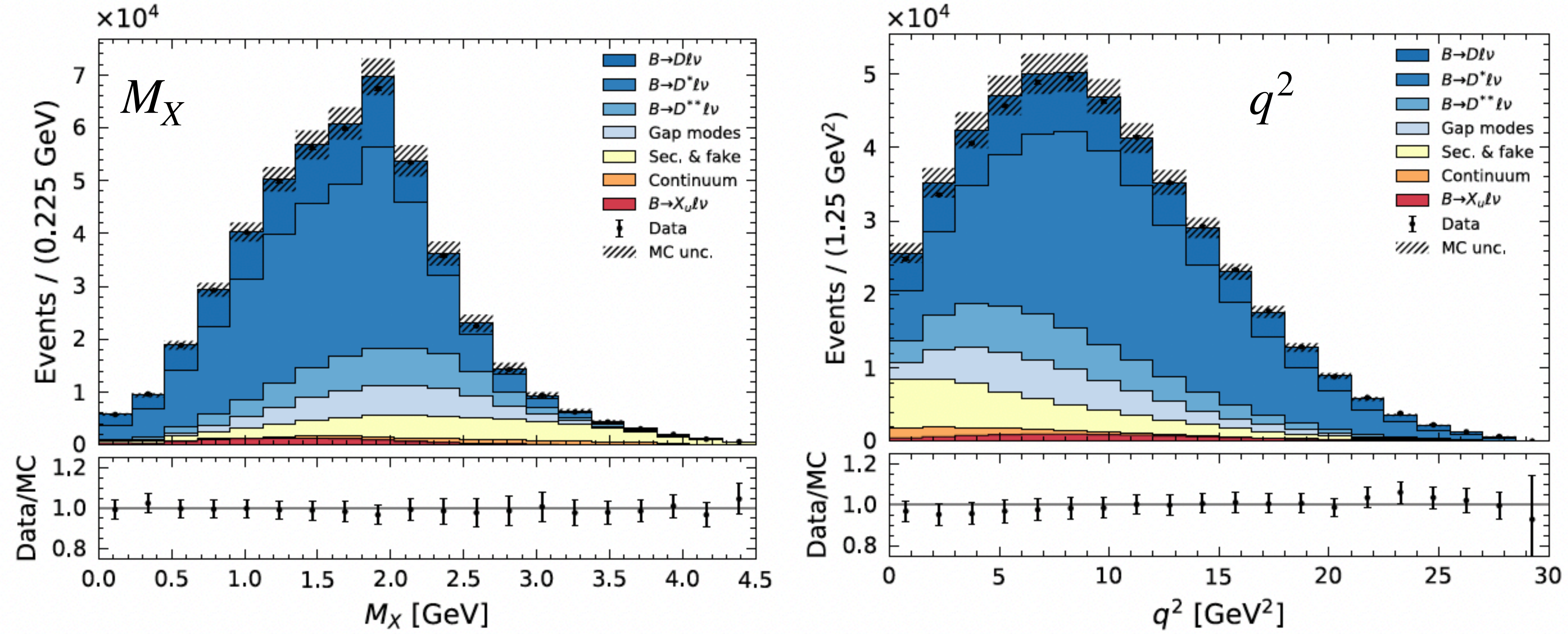
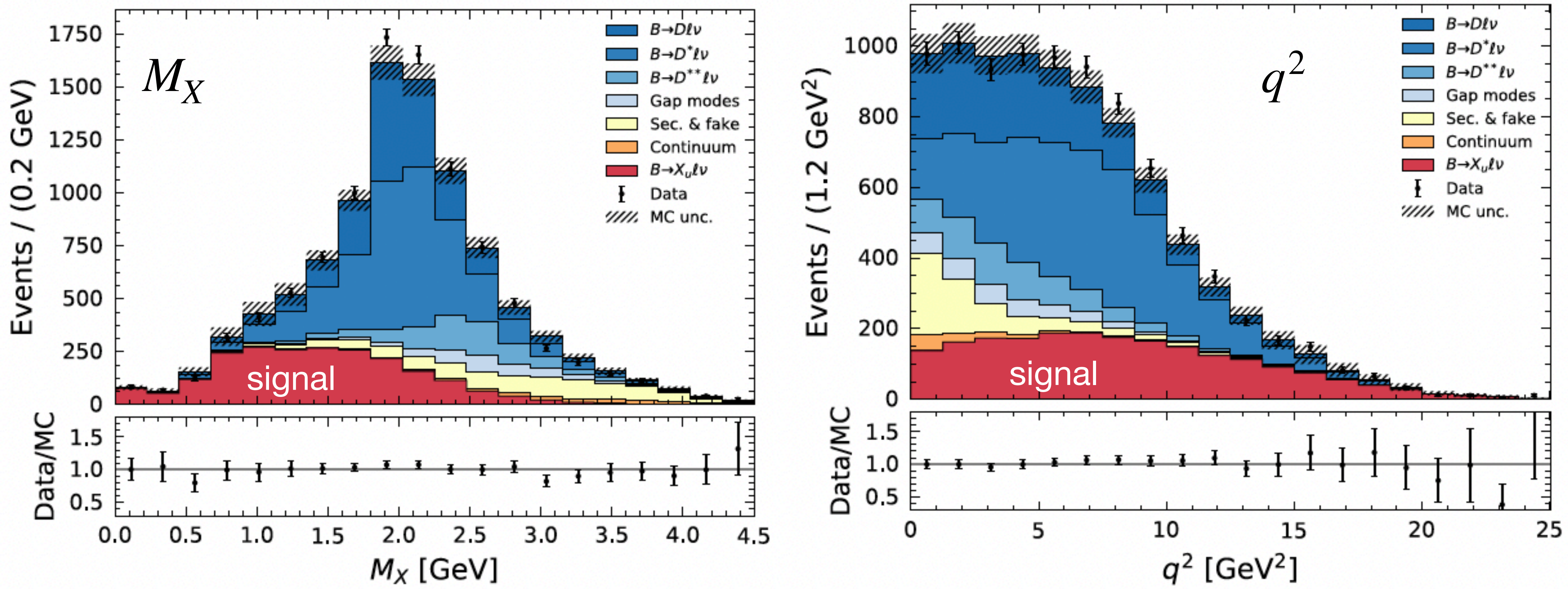


Before BDT selection



Applied BDT selection

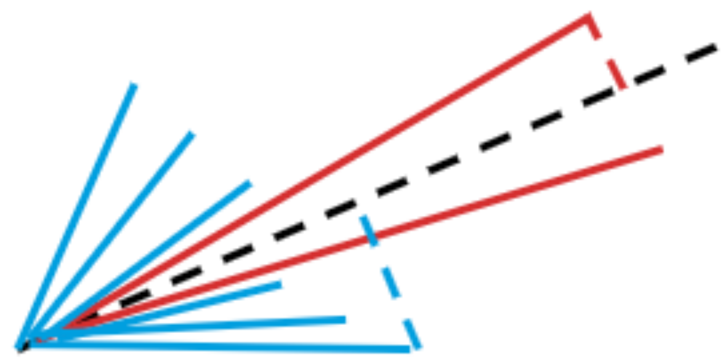
PRD 104 , 012008 (2021)

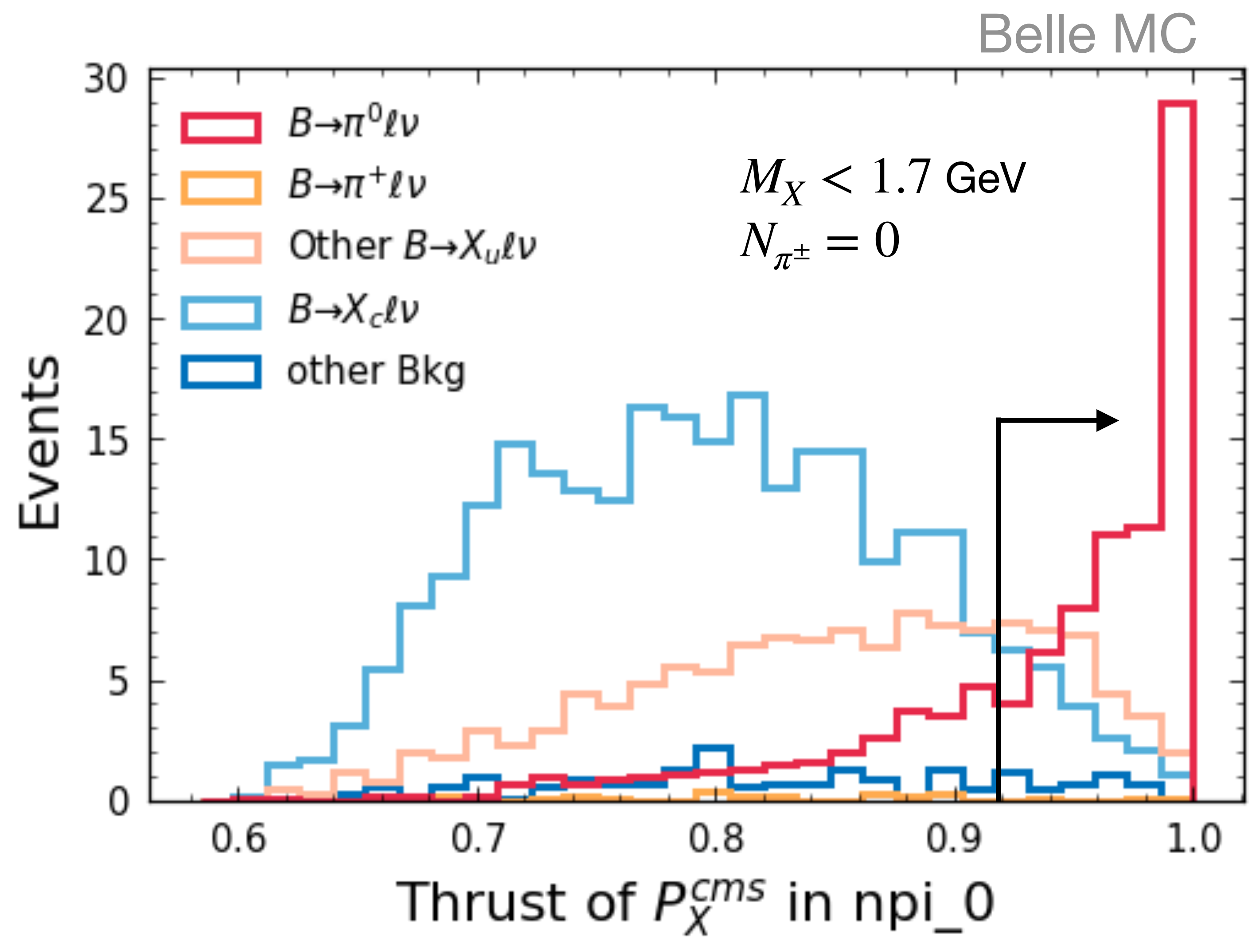


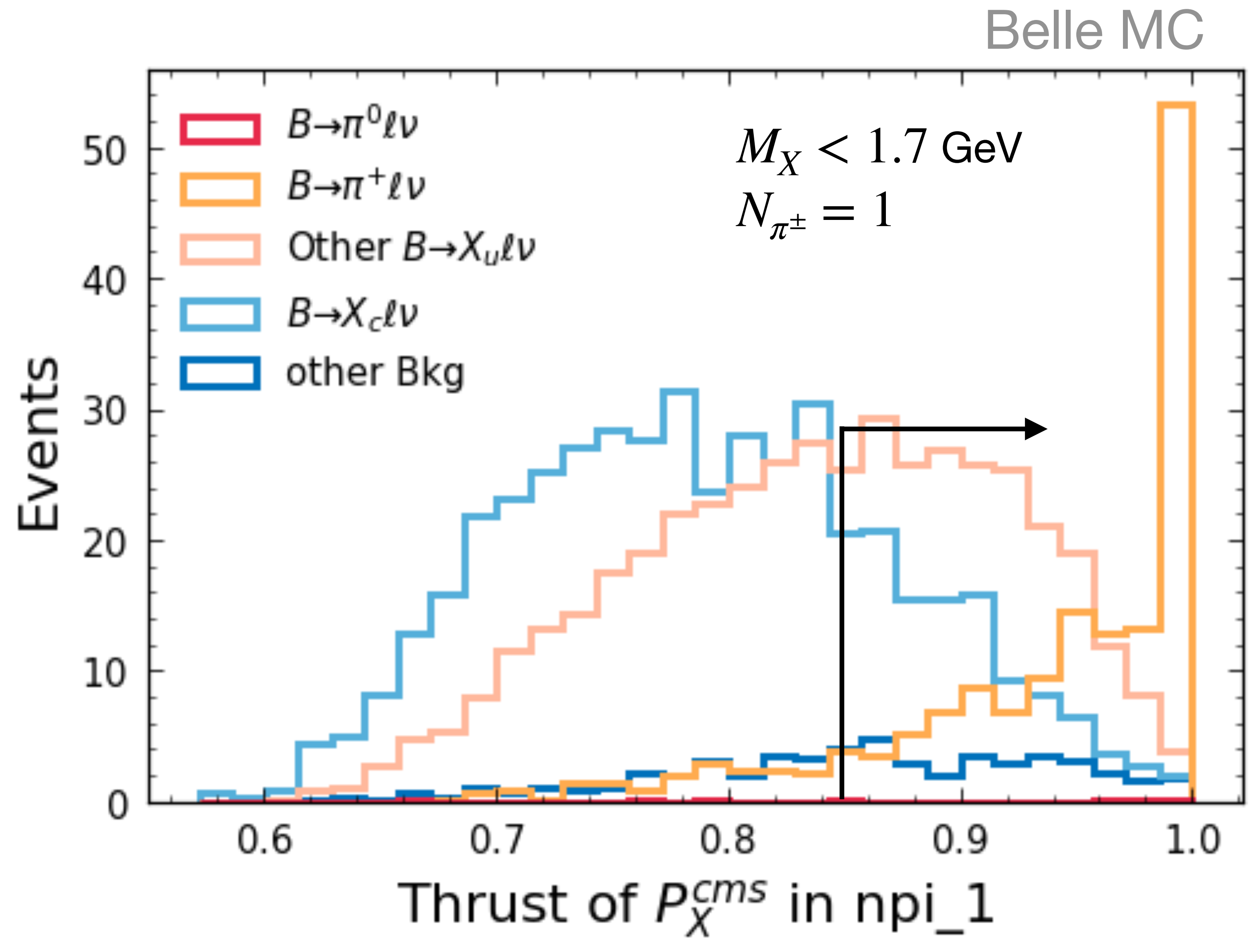
Further Criteria to Enhance $B \to \pi \ell \nu$

- Additional selections on **thrust** of X in c.m.s to **increase significance** of $B \to \pi \ell \nu$
  - For **single pion mode (π^0, $\pi^\pm$)**, a cut on thrust P_X^{CMS} to suppress high-multiplicity background
  - Selection optimized in Asimov fit to achieve high significance of single-pion modes & overall stable fit results

$$T = \max_{|n|=1} \left[\frac{\sum_i |p_i \cdot n|}{\sum_i |p_i|} \right]$$





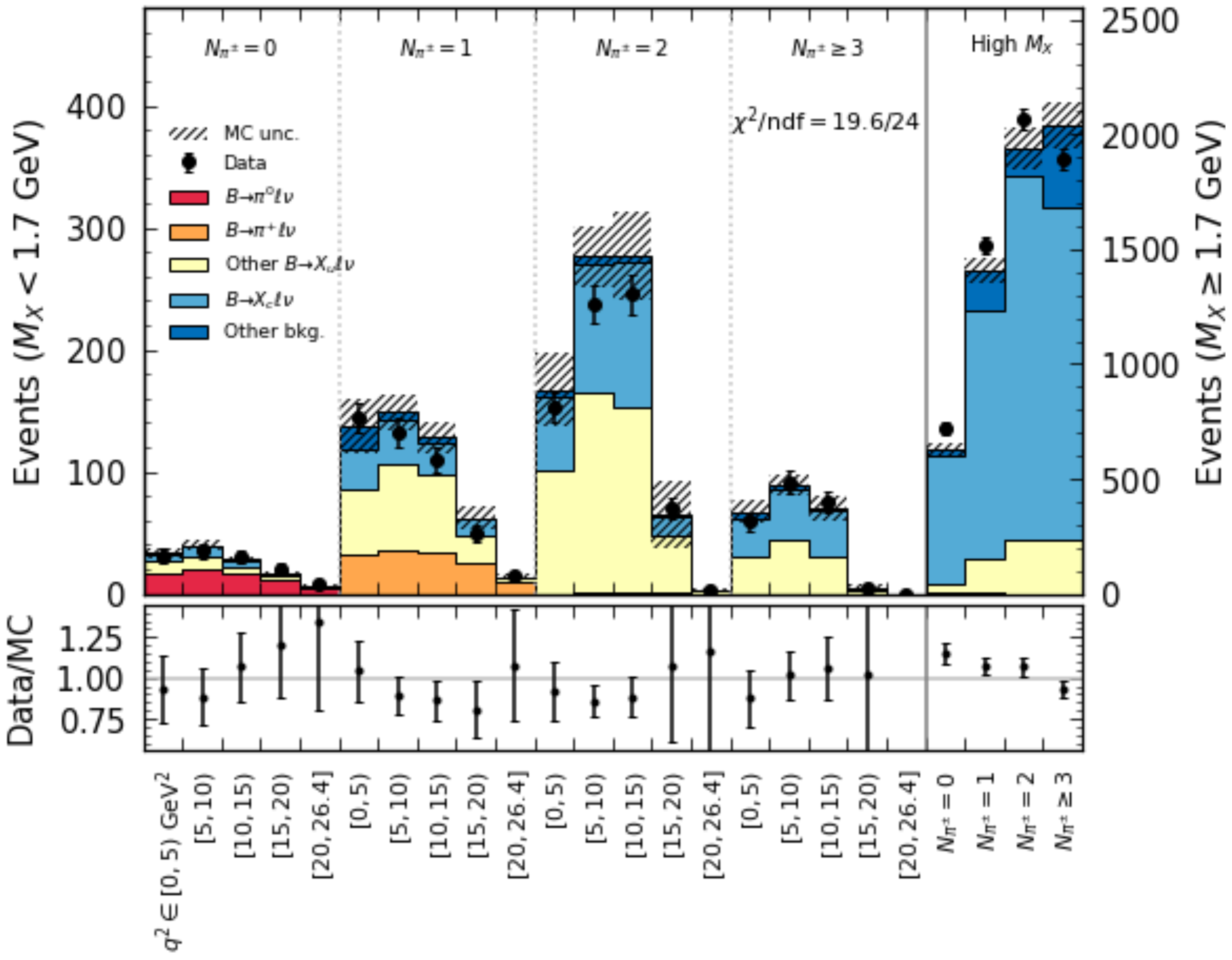


Prefit Spectrum after All Selections

arXiv: 2303.17309

- Extract signal in $\mathbf{q^2 : N_{\pi^\pm}}$ for $B \to \pi \ell \nu$ and other $B \to X_u \ell \nu$ simultaneously

Flattened 2D Spectrum (pre-fit)



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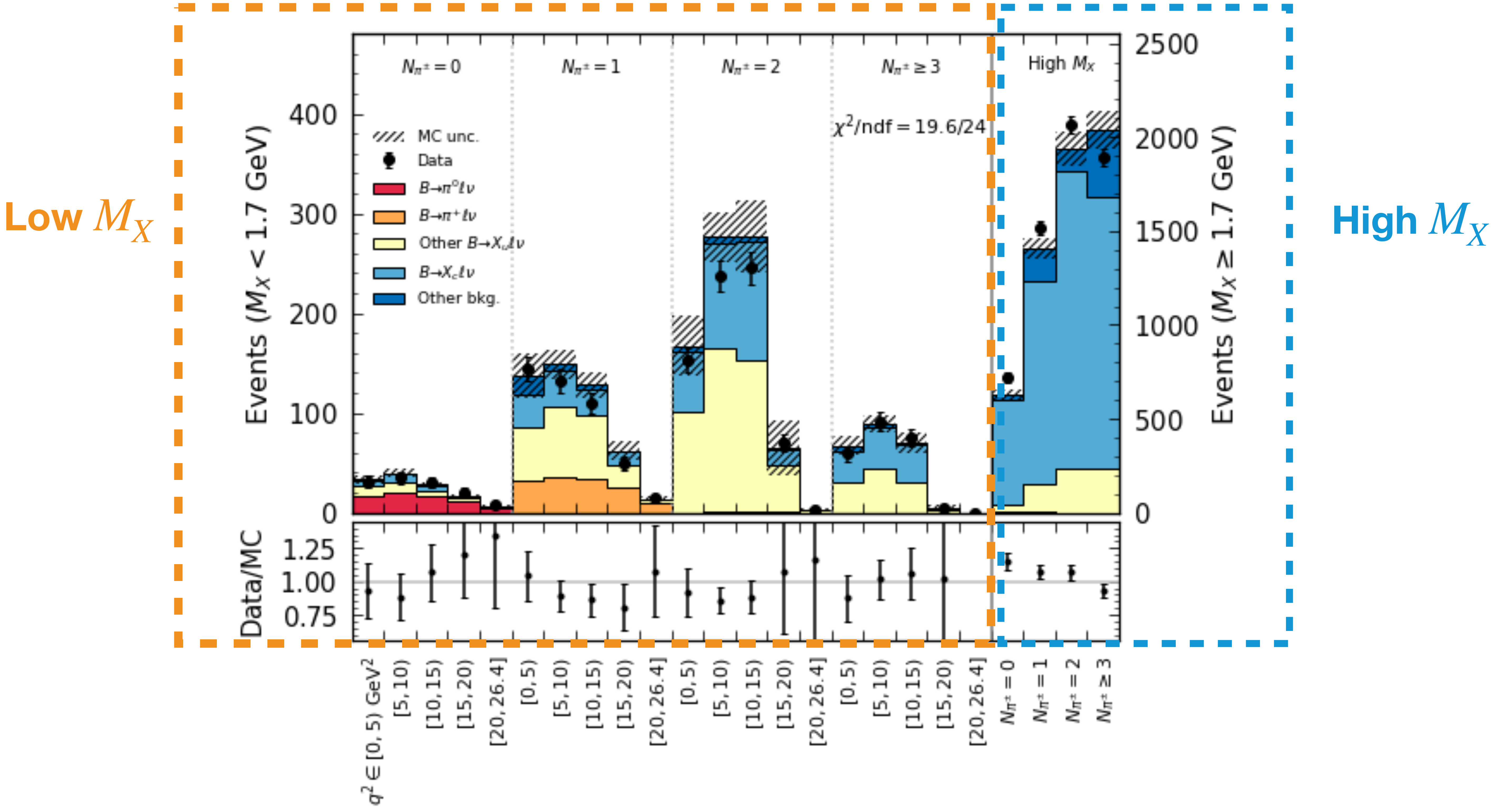

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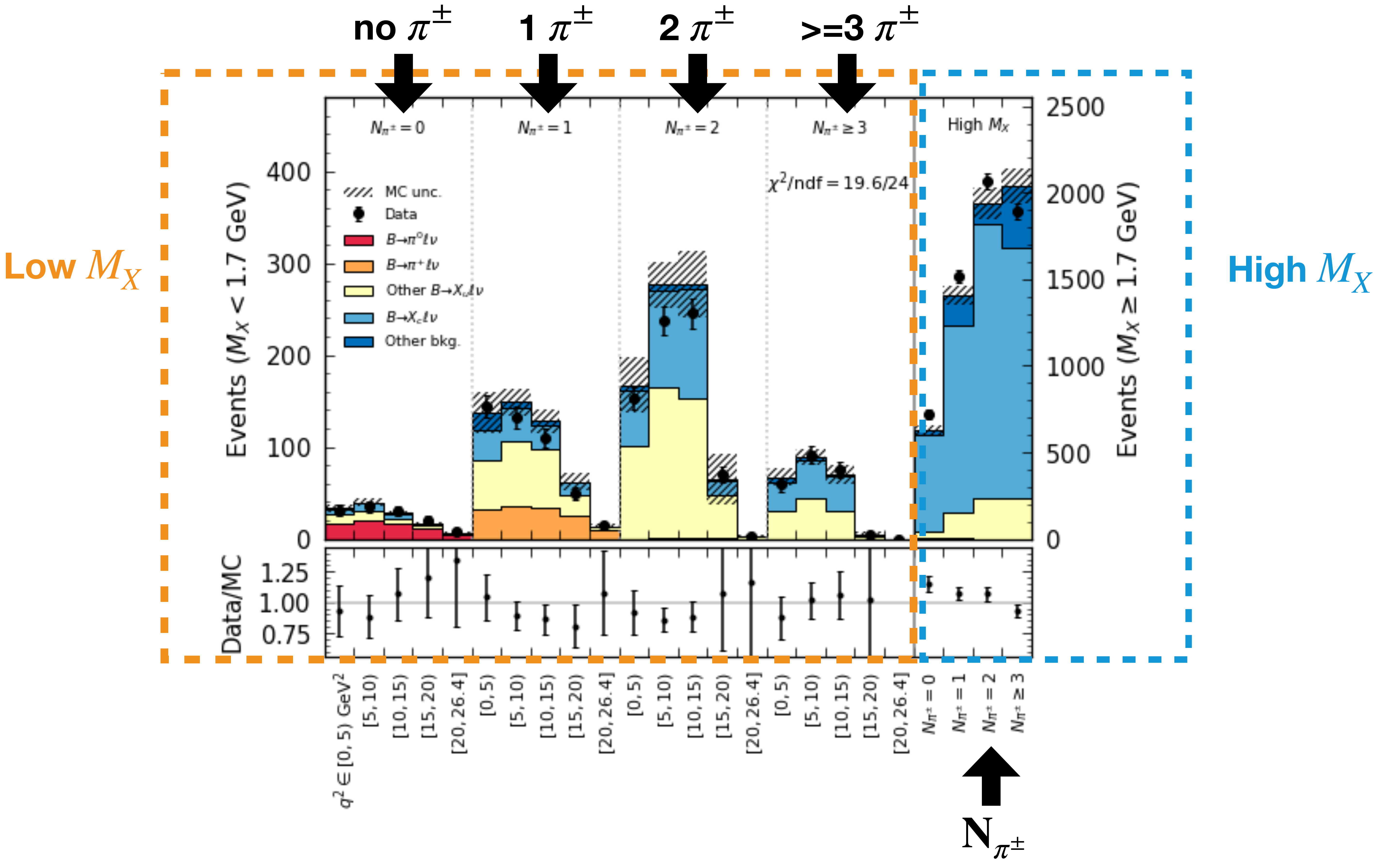

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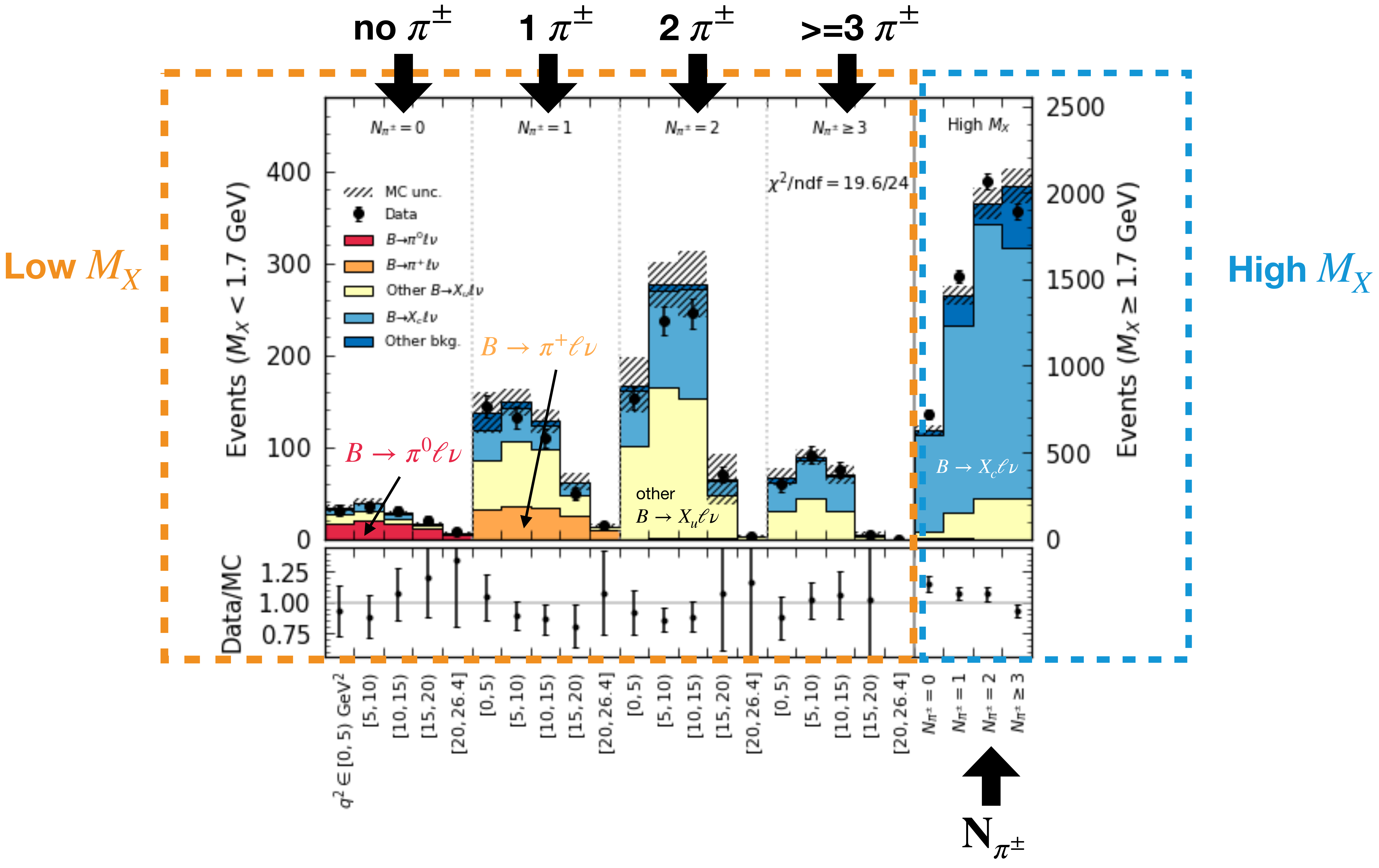

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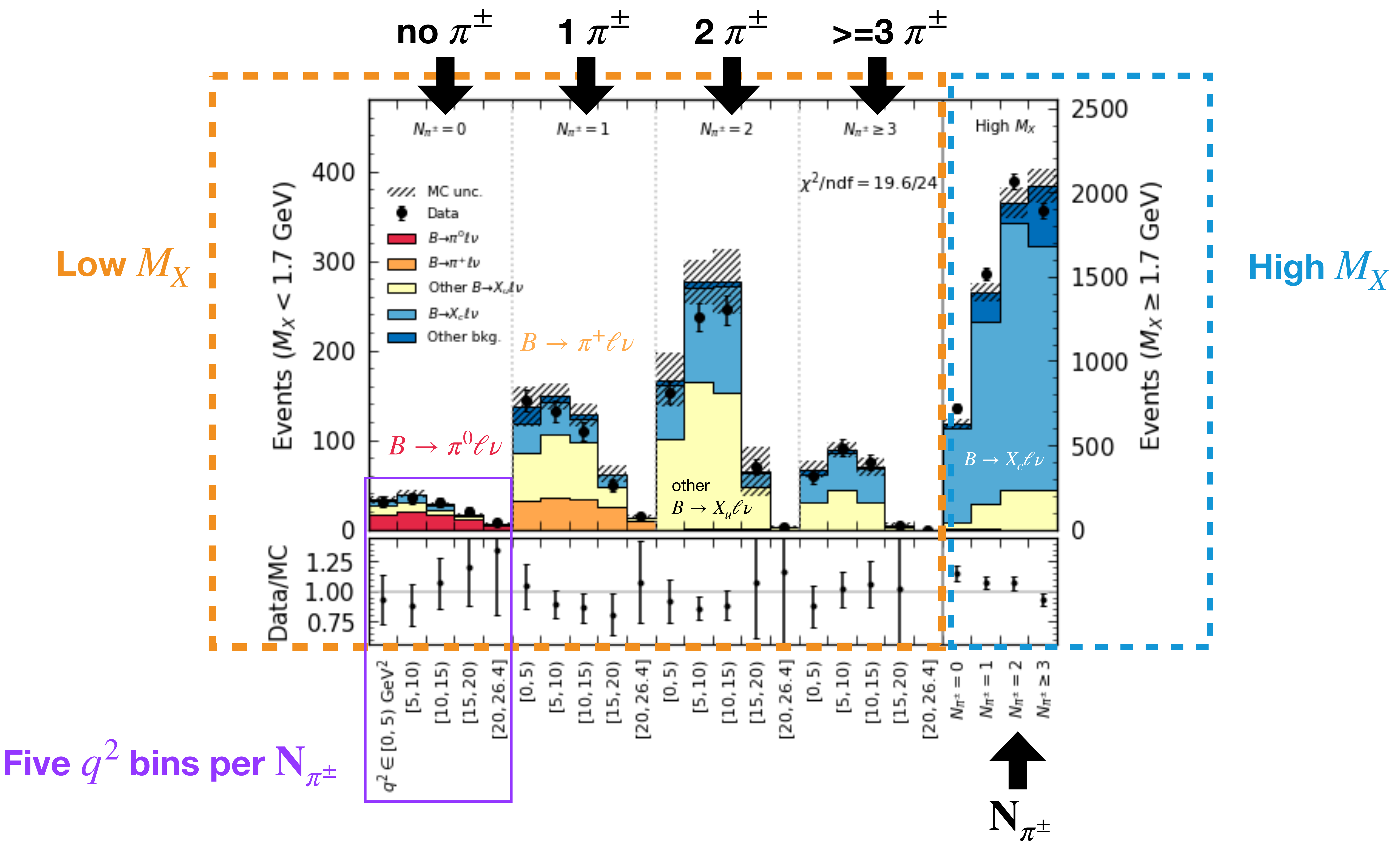
arXiv: 2303.17309

- Extract signal in $\mathbf{q^2}$: $\mathbf{N_{\pi^\pm}}$ for $B \to \pi\ell\nu$ and other $B \to X_u\ell\nu$ simultaneously

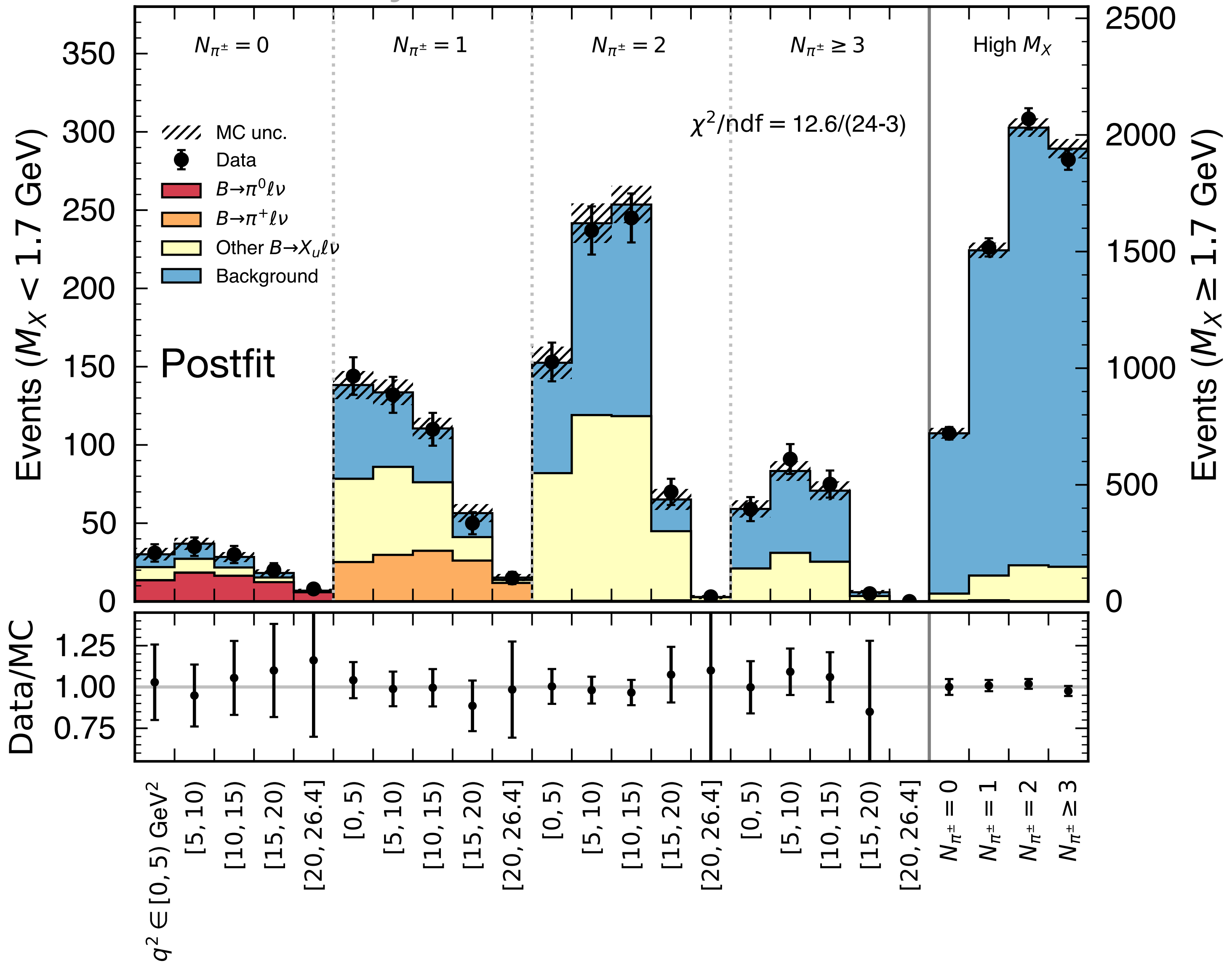


Simultaneous Extraction of $B \to \pi \ell \nu$ and other $B \to X_u \ell \nu$

- Fitter corporates experimental observation of templates' **normalisations** η and $B \to \pi \ell \nu$ **form factor (FF) parameters** $\mathbf{a}^{+,0}$

$$-2\log\mathscr{L} = -2\log\prod_i \text{Poisson}\left(\eta_{\text{obs}}, \eta_{\text{pred}}\cdot(1+\epsilon\cdot\theta)\right) + \theta\rho_\theta^{-1}\theta^T + \chi^2_{\text{FF}}$$

Preliminary



Normalizations can be linked with isospin relation, or floating separately **(nominal: linked)**

$B \to \pi^0 \ell \nu$
$B \to \pi^+ \ell \nu$
other $B \to X_u \ell \nu$
all background

Simultaneous Extraction of $B \to \pi \ell \nu$ and other $B \to X_u \ell \nu$

- Fitter corporates experimental observation of templates' **normalisations** η and $B \to \pi \ell \nu$ **form factor (FF) parameters** $\mathbf{a}^{+,0}$
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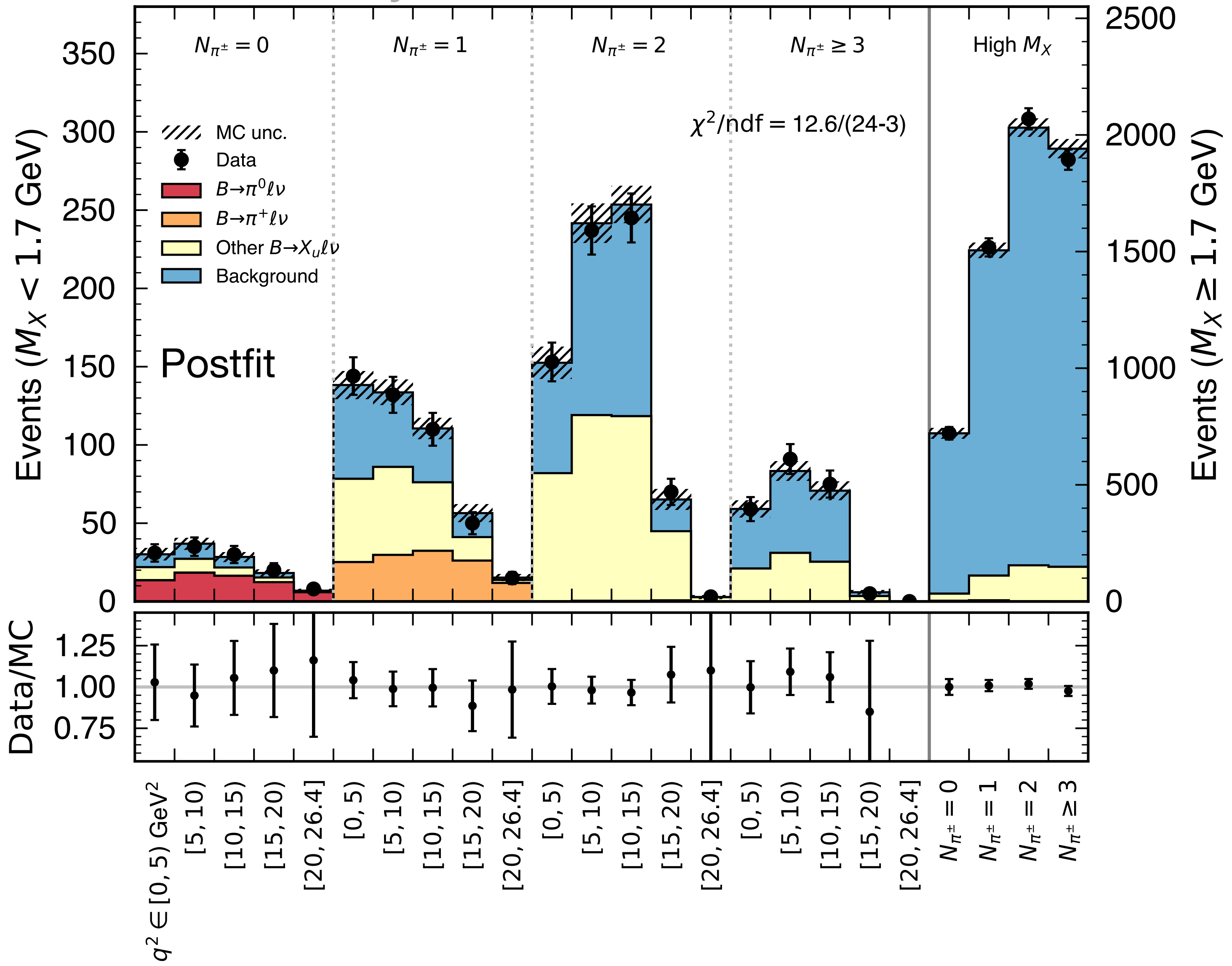
Preliminary



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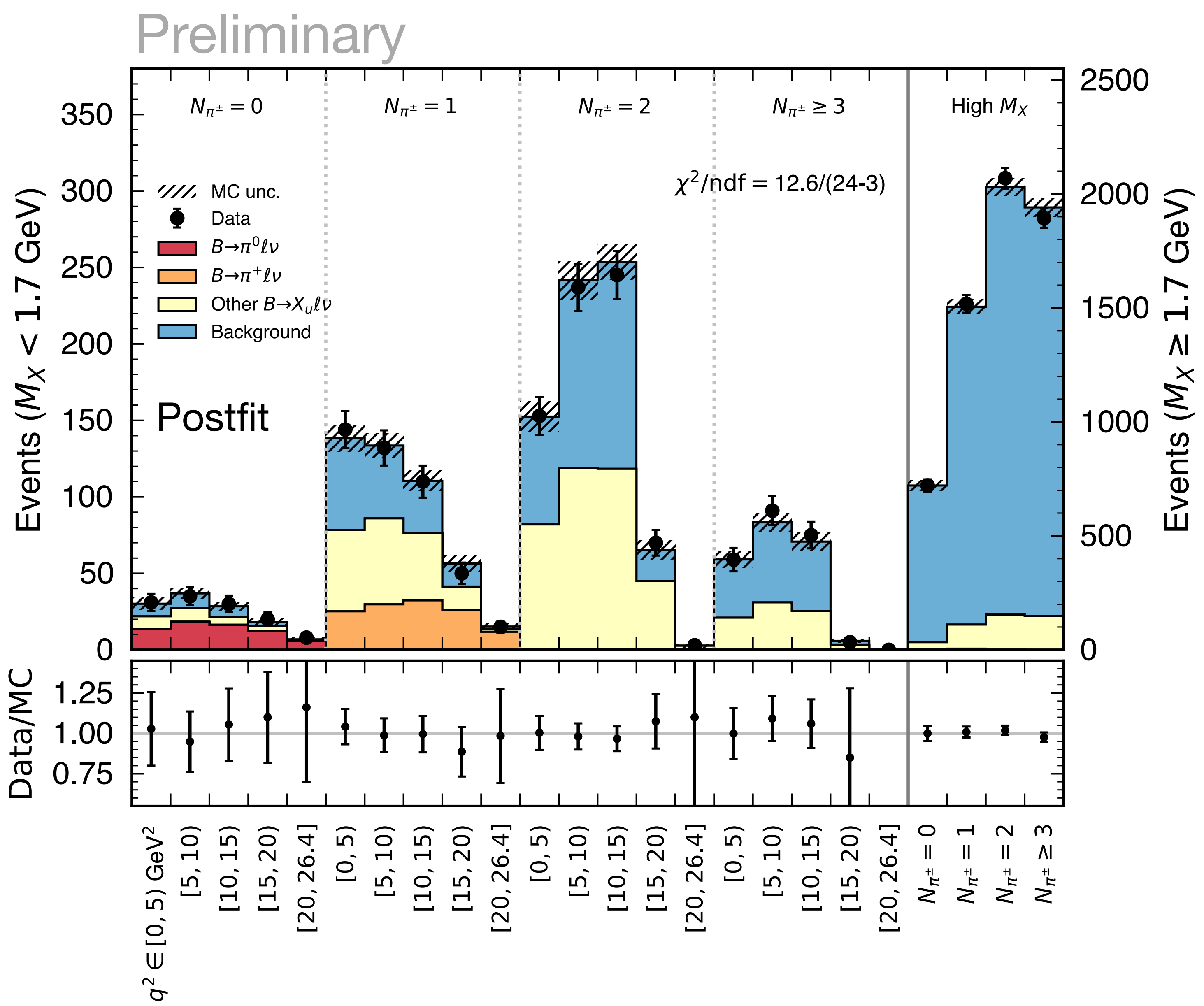

Normalizations
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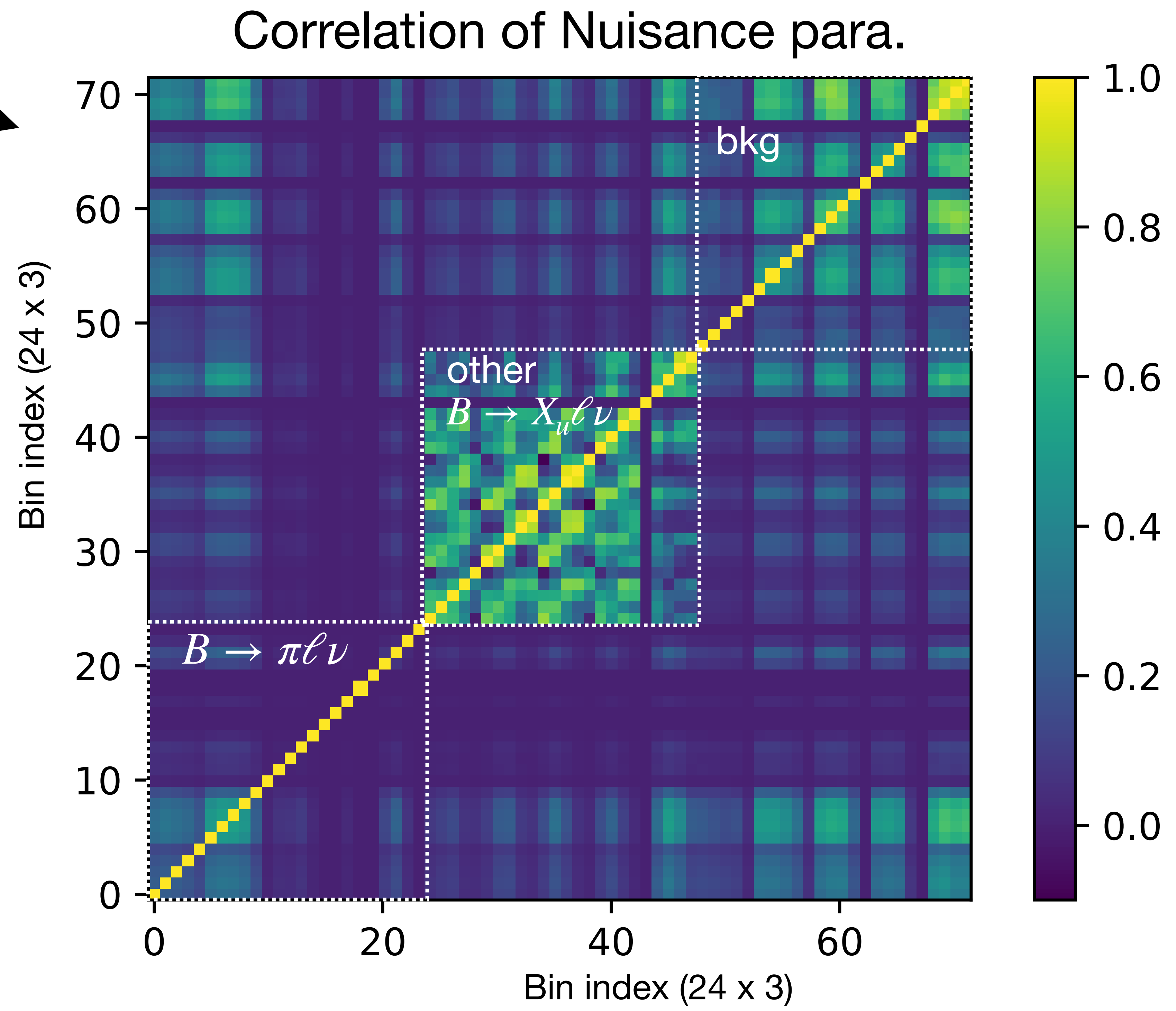


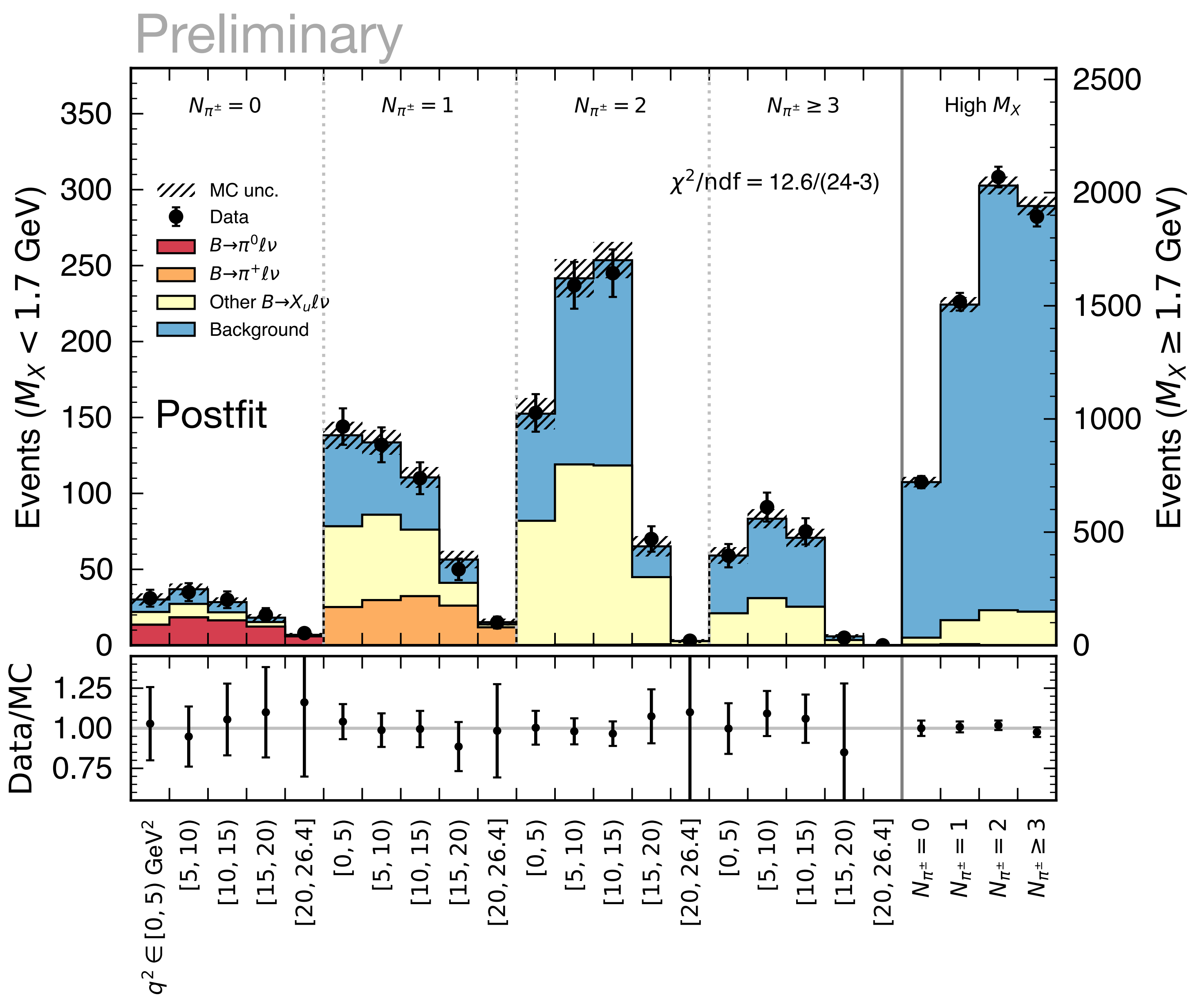
First Simultaneous Determination of Incl. & Excl. $|V_{ub}|$

- Fitter corporates experimental observation of templates' **normalisations** η and $B \to \pi \ell \nu$ **form factor (FF) parameters** $\mathbf{a}^{+,0}$
- Systematic uncertainties included via **bin-wise Nuisance parameters** θ of each template for both of additives and multiplicative impacts, (normalized to relative bin uncertainty ϵ)

$$-2\log\mathscr{L} = -2\log\prod_i \text{Poisson}\left(\eta_{\text{obs}}, \eta_{\text{pred}}\cdot(1+\epsilon\cdot\theta)\right) + \theta\rho_\theta^{-1}\theta^T + \chi^2_{\text{FF}}$$

Constraints on BCL parameters , input taken from LQCD / LQCD+exp fits in FLAG Review 2021

$$\chi^2_{\text{FF}} = (\mathbf{a}_{\text{obs}} - \mathbf{a}_{\text{pred}})\text{Cov}^{-1}_{\text{FF}}(\mathbf{a}_{\text{obs}} - \mathbf{a}_{\text{pred}})^T$$



Normalizations can be linked with isospin relation, or floating separately **(nominal: linked)**

$B \to \pi^0 \ell \nu$
$B \to \pi^+ \ell \nu$
other $B \to X_u \ell \nu$
all background

BCL parameterization of $B \to \pi \ell \nu$ decay form factors ($N^+ = N^0 = 3$):

$$f_+(q^2) = \frac{1}{1 - q^2/m^2_{B^*}} \sum_{n=0}^{N^+-1} \boxed{a_n^+} \left[z^n - (-1)^{n-N^+}\frac{n}{N^+}z^{N^+}\right]$$

$$f_0(q^2) = \sum_{n=0}^{N^0-1} \boxed{a_n^0} z^n$$

Simultaneous Extraction of $B \to \pi \ell \nu$ and other $B \to X_u \ell \nu$

- Fitter corporates experimental observation of templates' **normalisations** η and $B \to \pi \ell \nu$ **form factor (FF) parameters** $\mathbf{a}^{+,0}$
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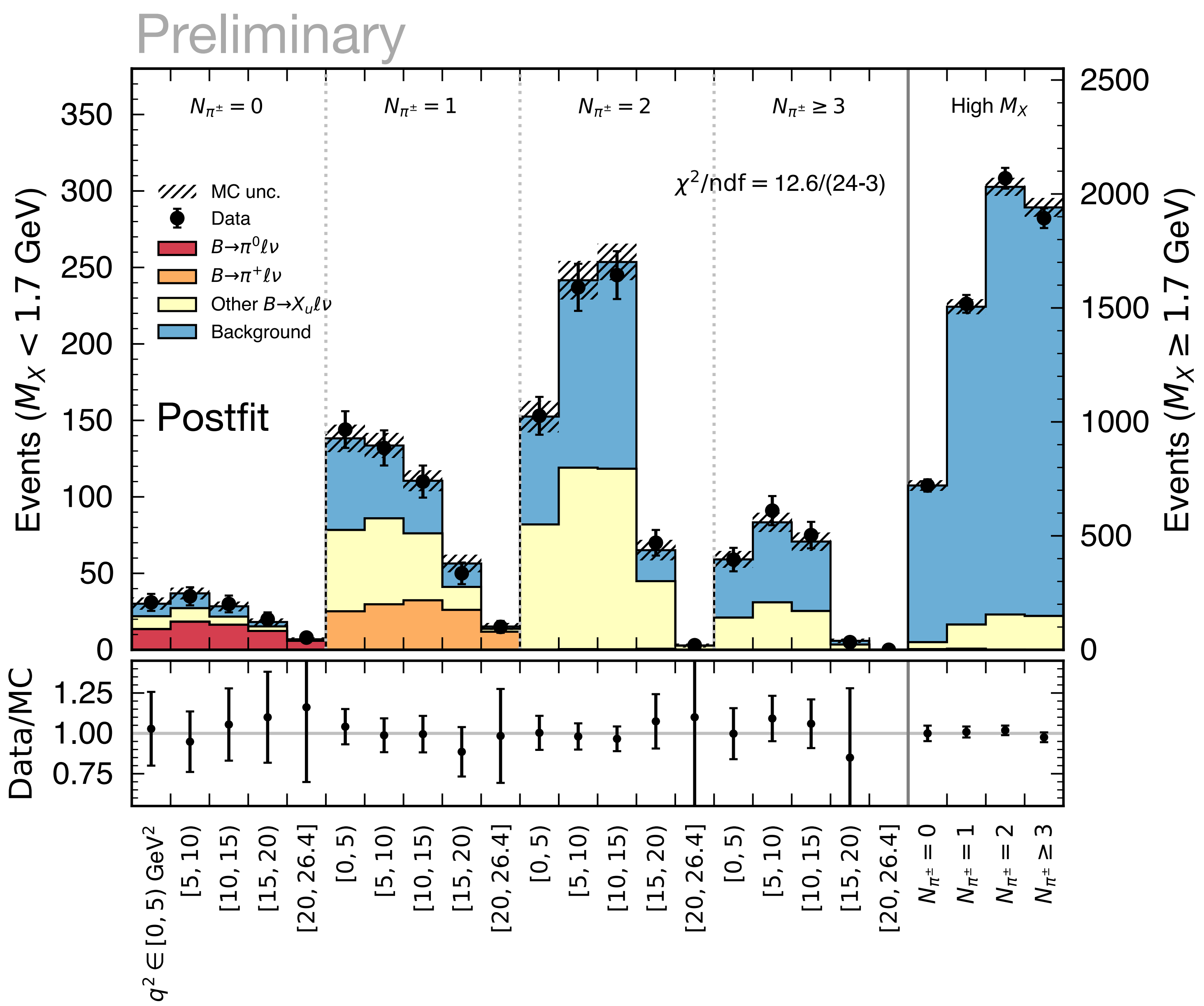
Constraints on BCL parameters , input taken from LQCD / LQCD+exp fits in FLAG Review 2021

Differential decay rates

Acceptance & reco. efficiency

Forward-folding q^2



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$B \to \pi^+ \ell \nu$
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all background

Shape described by BCL para.

Simultaneous Extraction of $B \to \pi \ell \nu$ and other $B \to X_u \ell \nu$

- Fitter corporates experimental observation of templates' **normalisations** η and $B \to \pi \ell \nu$ **form factor (FF) parameters a$^{+,0}$**
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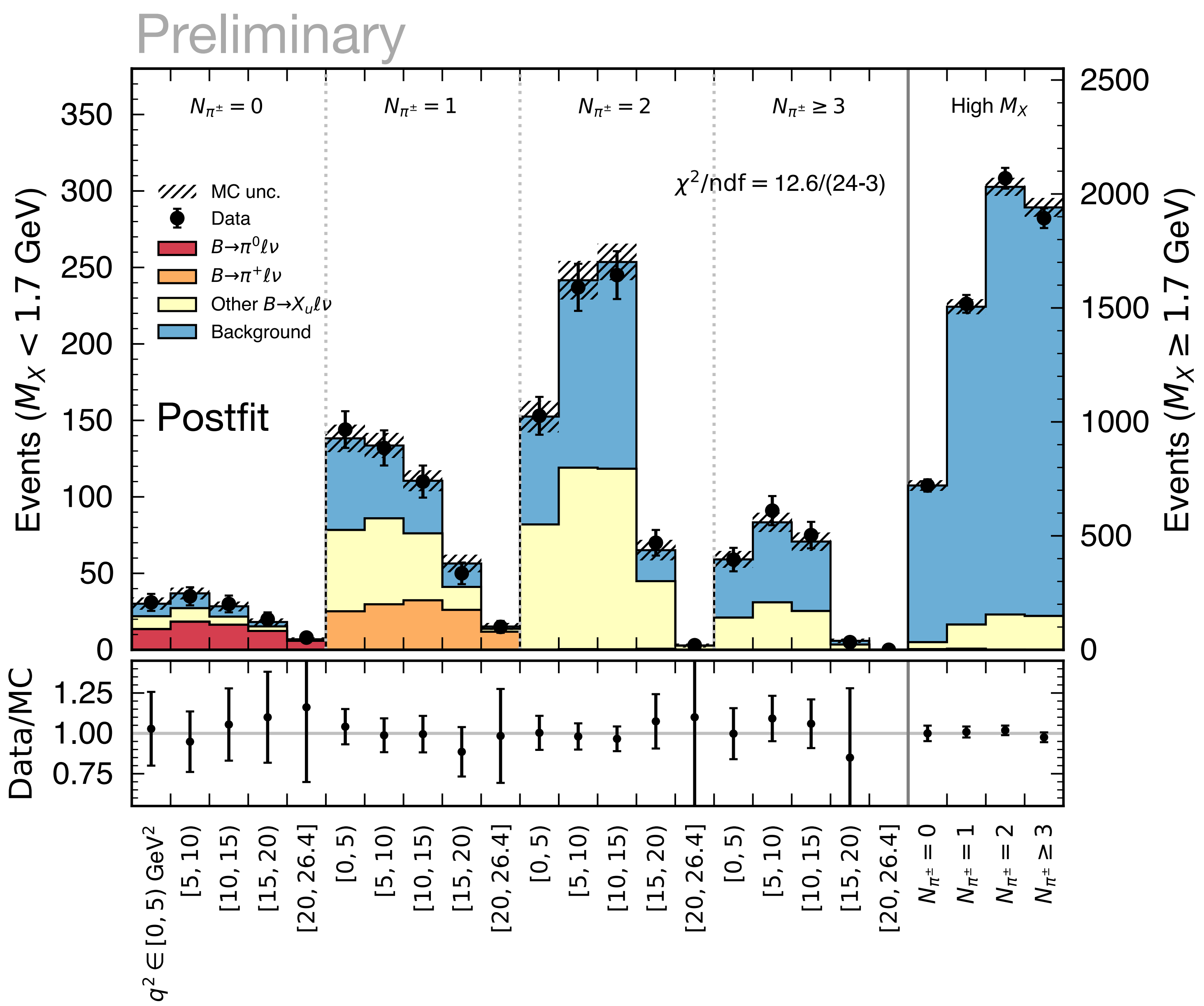

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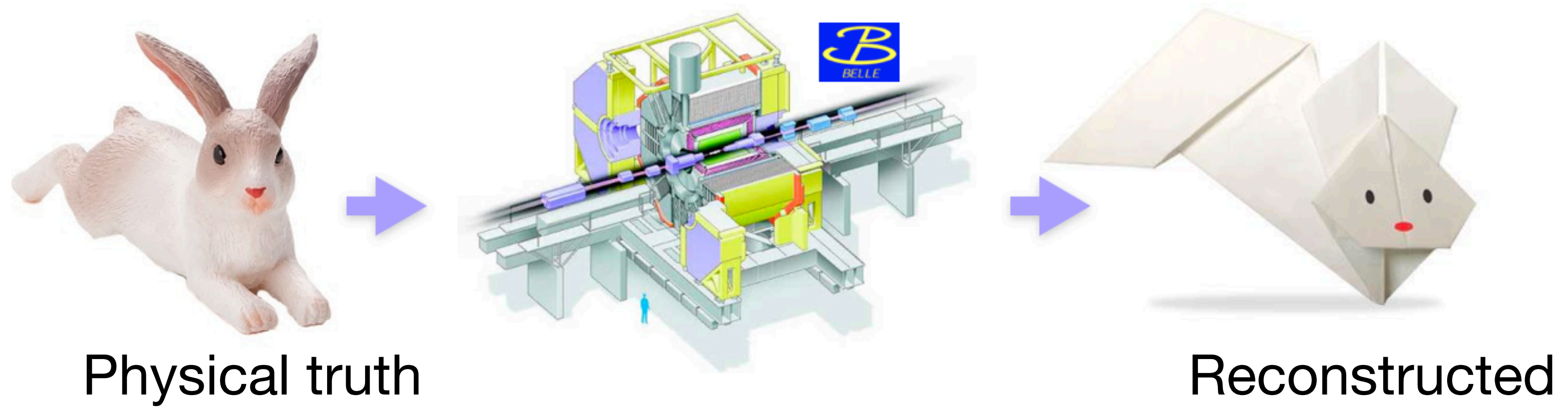

Simultaneous Extraction of $B \to \pi\ell\nu$ and other $B \to X_u\ell\nu$

- Fitter corporates experimental observation of templates' **normalisations** η and $B \to \pi\ell\nu$ **form factor (FF) parameters a$^{+,0}$**
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Constraints on BCL parameters , input taken from LQCD / LQCD+exp fits in FLAG Review 2021

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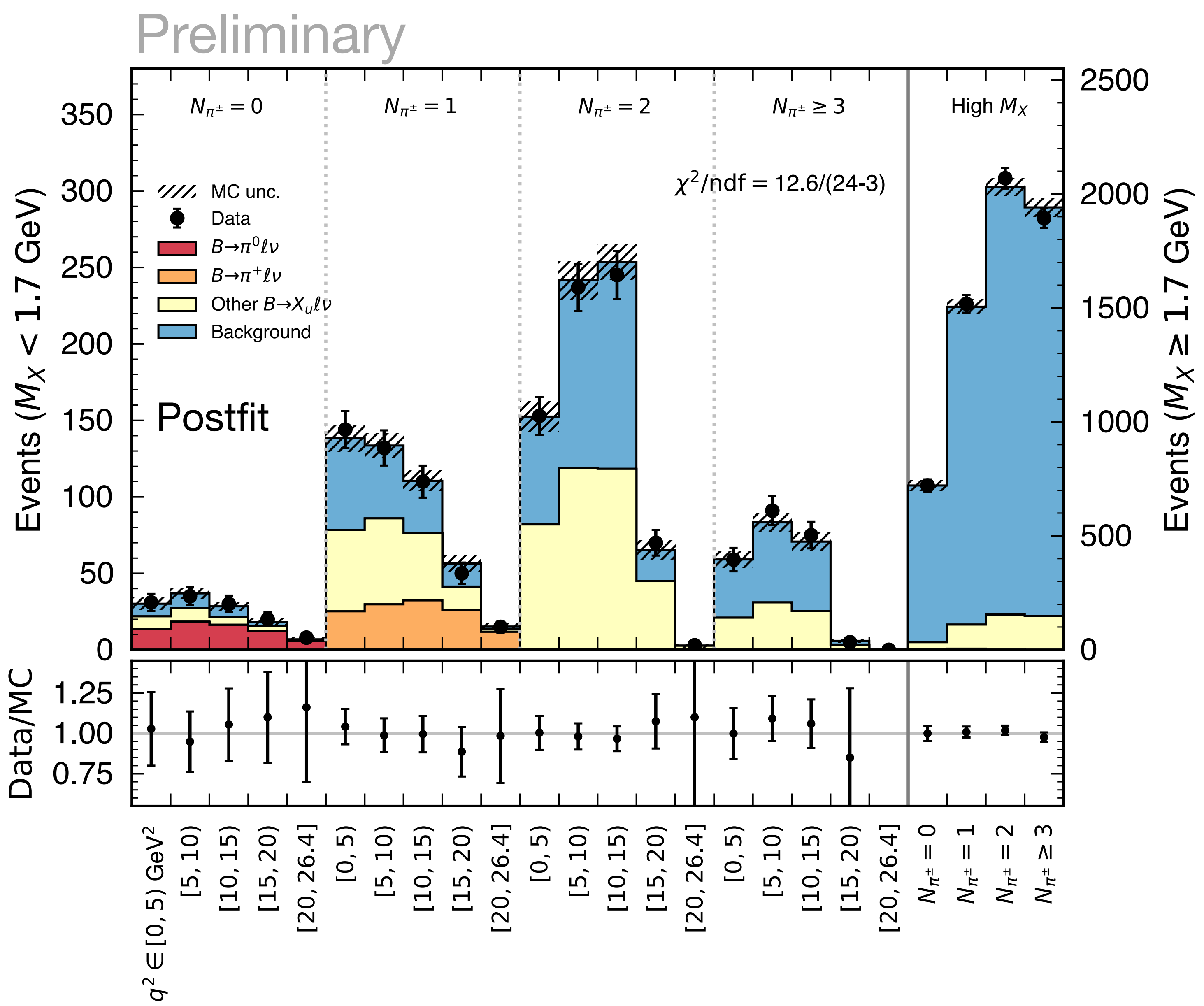

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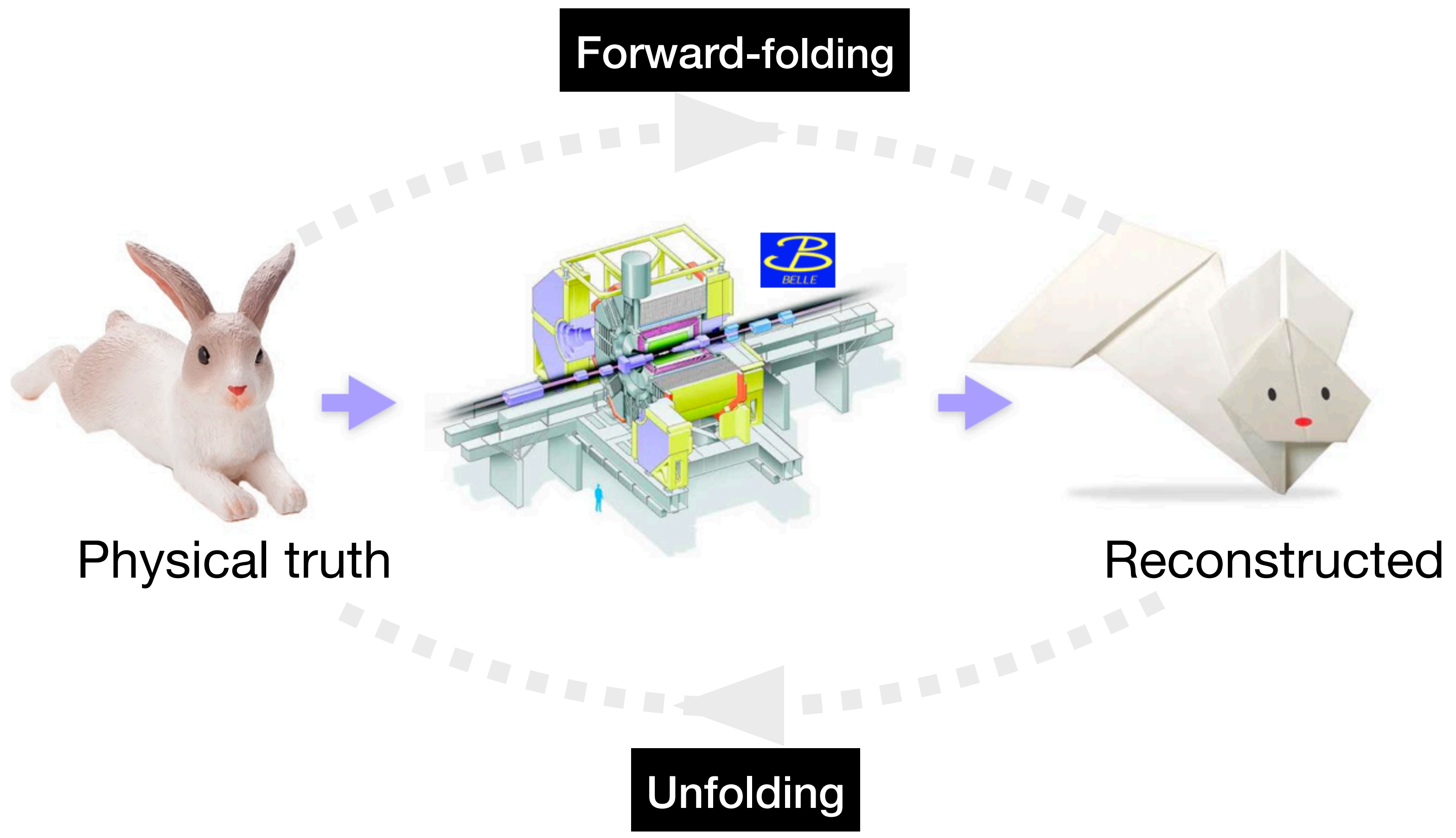

Simultaneous Extraction of $B \to \pi\ell\nu$ and other $B \to X_u\ell\nu$

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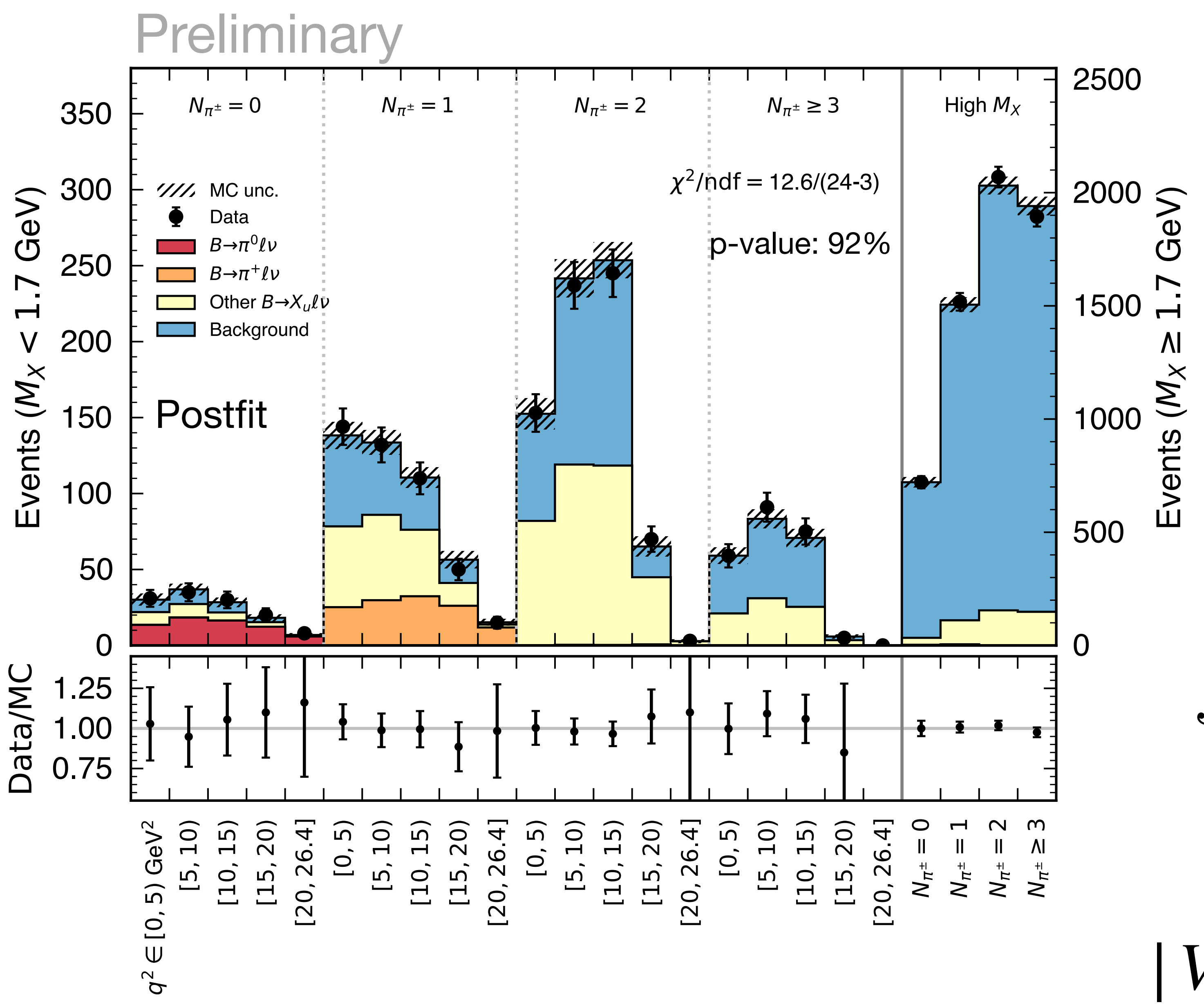
Constraints on BCL parameters , input taken from LQCD / LQCD+exp fits in FLAG Review 2021

Differential decay rates

Acceptance & reco. efficiency

Forward-folding q^2



Normalizations can be linked with isospin relation, or floating separately **(nominal: linked)**

$B \to \pi^0\ell\nu$

$B \to \pi^+\ell\nu$

other $B \to X_u\ell\nu$

all background

Shape described by BCL para.

$$\mathscr{B}(B\to X_u\ell\nu) = \mathscr{B}(B\to\pi^0\ell\nu) + \mathscr{B}(B\to\pi^+\ell\nu) + \mathscr{B}(B\to X_u^{\text{other}}\ell\nu)$$

Fit results provide all $\mathscr{B}$ and $B \to \pi\ell\nu$ FF (decay rate) => sufficient to derive exclusive and inclusive $|V_{ub}|$

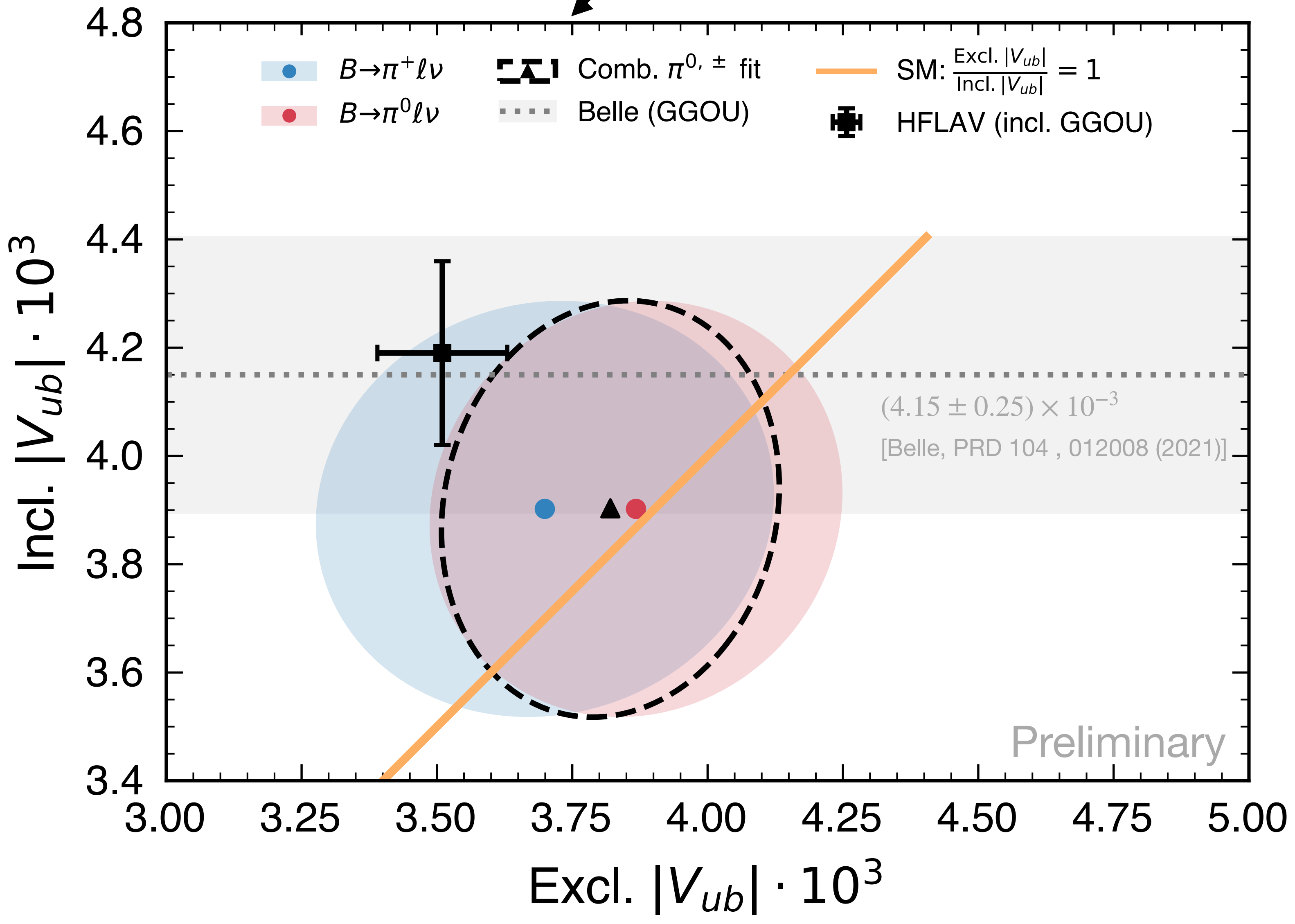
$$|V_{ub}^{\text{incl.}}| = \sqrt{\frac{\Delta\mathscr{B}(B\to X_u\ell\nu)}{\tau_B\cdot\Delta\Gamma_{\text{GGOU}}}} \qquad |V_{ub}^{\text{excl.}}| = \sqrt{\frac{\mathscr{B}(B\to\pi\ell\nu)}{\tau_B\cdot\Gamma_{\text{FF}}}}$$

$\Delta\mathscr{B}(B\to X_u\ell\nu) = \mathscr{B}(B\to X_u\ell\nu)\cdot\epsilon_{\Delta\text{PS}:E_\ell^B>1\text{GeV}}$

Exclusive and Inclusive $|V_{ub}|$

arXiv: 2303.17309

- Various fit scenarios applied:
  - **Linked** or separate $B \to \pi^+ \ell \nu$, $B \to \pi^0 \ell \nu$
  - Input BCL constraint: **LQCD + exp.** or **only LQCD**



$|V_{ub}|$ in **combined** scenario with **LQCD+exp** const.:

Excl. $(3.78 \pm 0.23_{\text{stat}} \pm 0.16_{\text{syst}} \pm 0.14_{\text{theo}}) \times 10^{-3}$

Incl. $(3.90 \pm 0.20_{\text{stat}} \pm 0.32_{\text{syst}} \pm 0.09_{\text{theo}}) \times 10^{-3}$

Ratio = 0.97 ± 0.12 Correlation: 0.10

Weighted average of excl. & incl.

$(3.85 \pm 0.26) \times 10^{-3}$

CKM global fit (w/o $|V_{ub}|$)**:** $(3.64 \pm 0.07) \times 10^{-3}$**,**
compatible within 0.8σ

Exclusive and Inclusive $|V_{ub}|$

arXiv: 2303.17309

- Various fit scenarios applied:
  - **Linked** or separate $B \to \pi^+ \ell \nu$, $B \to \pi^0 \ell \nu$
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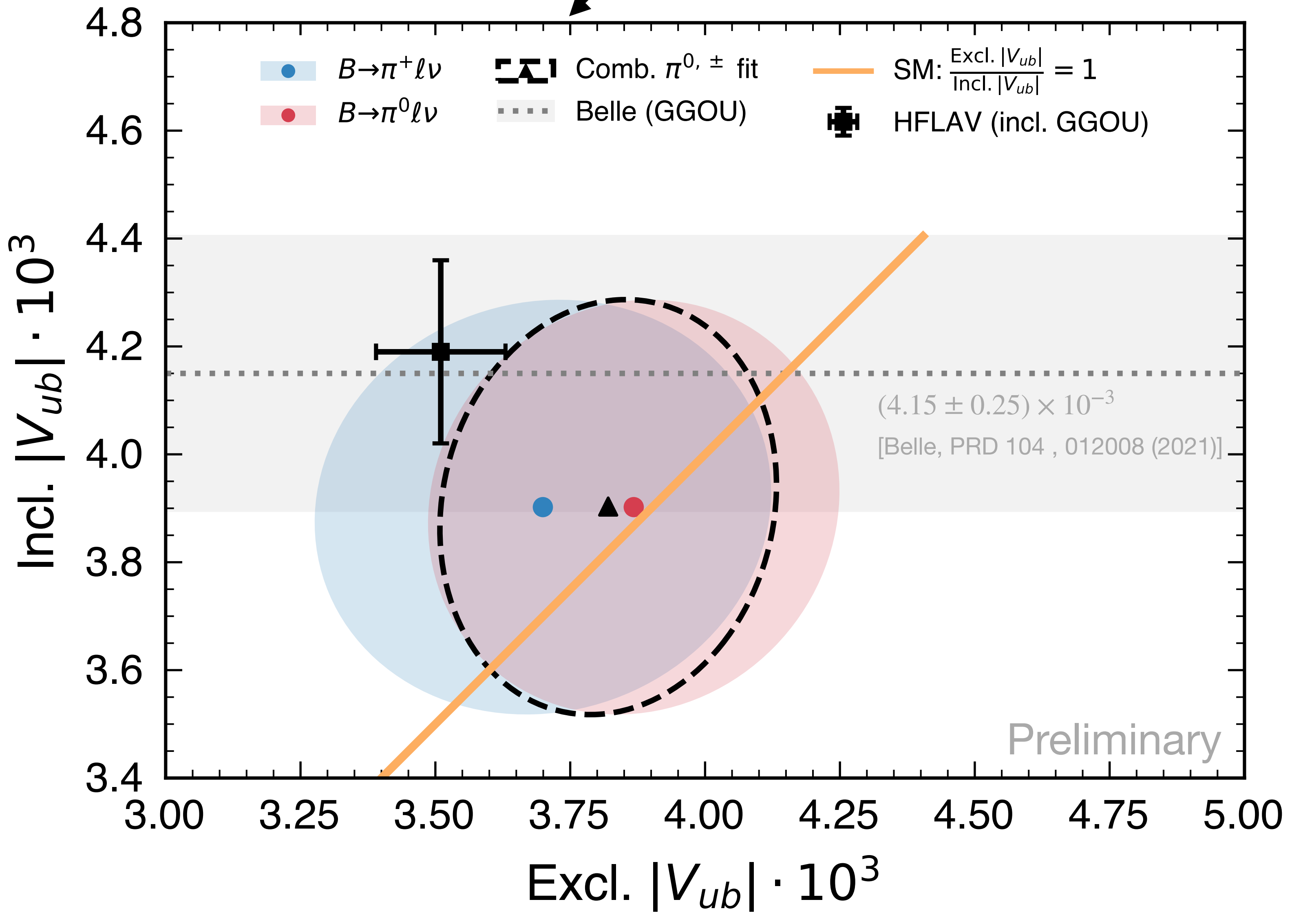

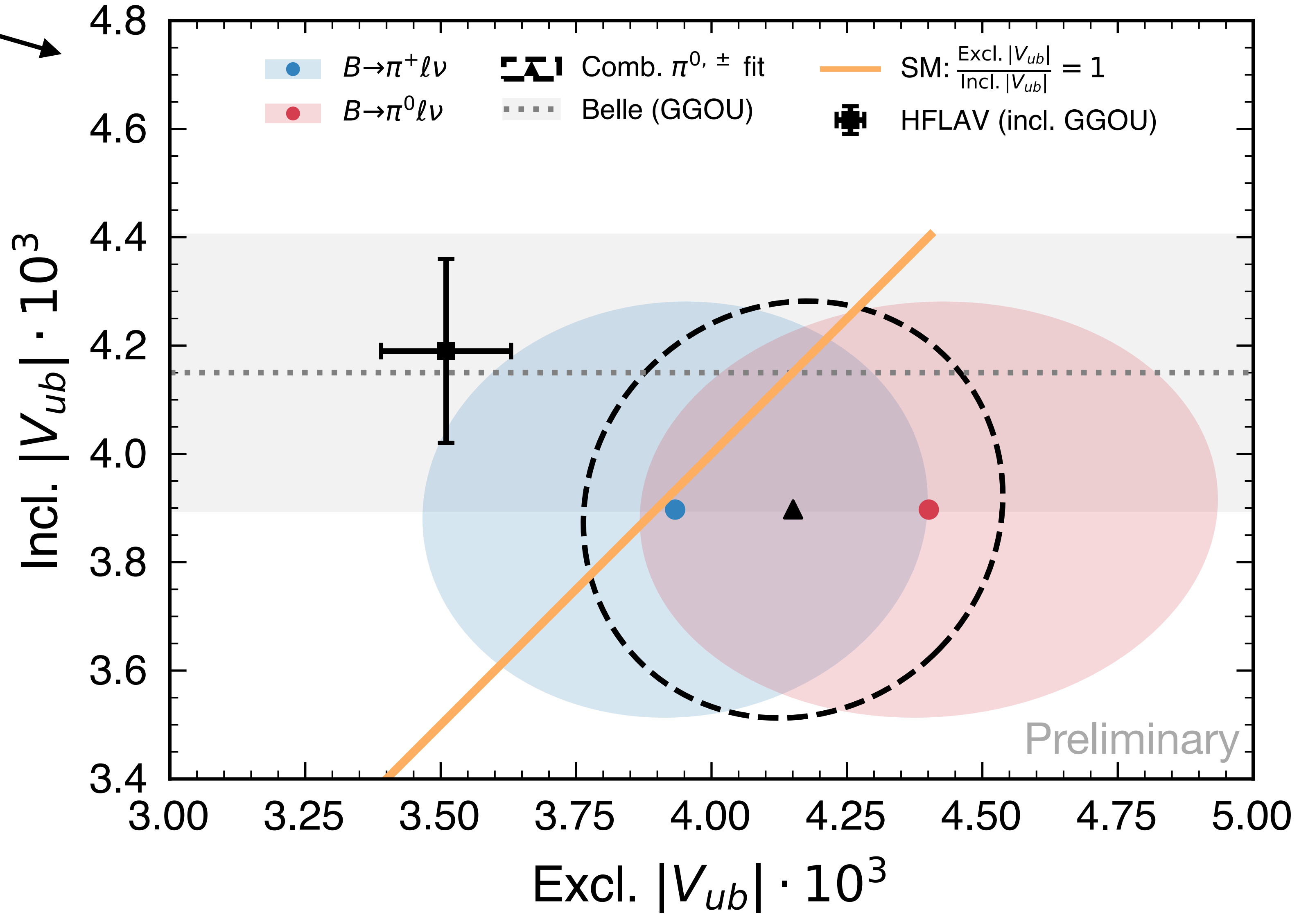

$|V_{ub}|$ in **combined** scenario with **LQCD+exp** const.:

Excl. $(3.78 \pm 0.23_{\text{stat}} \pm 0.16_{\text{syst}} \pm 0.14_{\text{theo}}) \times 10^{-3}$

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$(3.85 \pm 0.26) \times 10^{-3}$

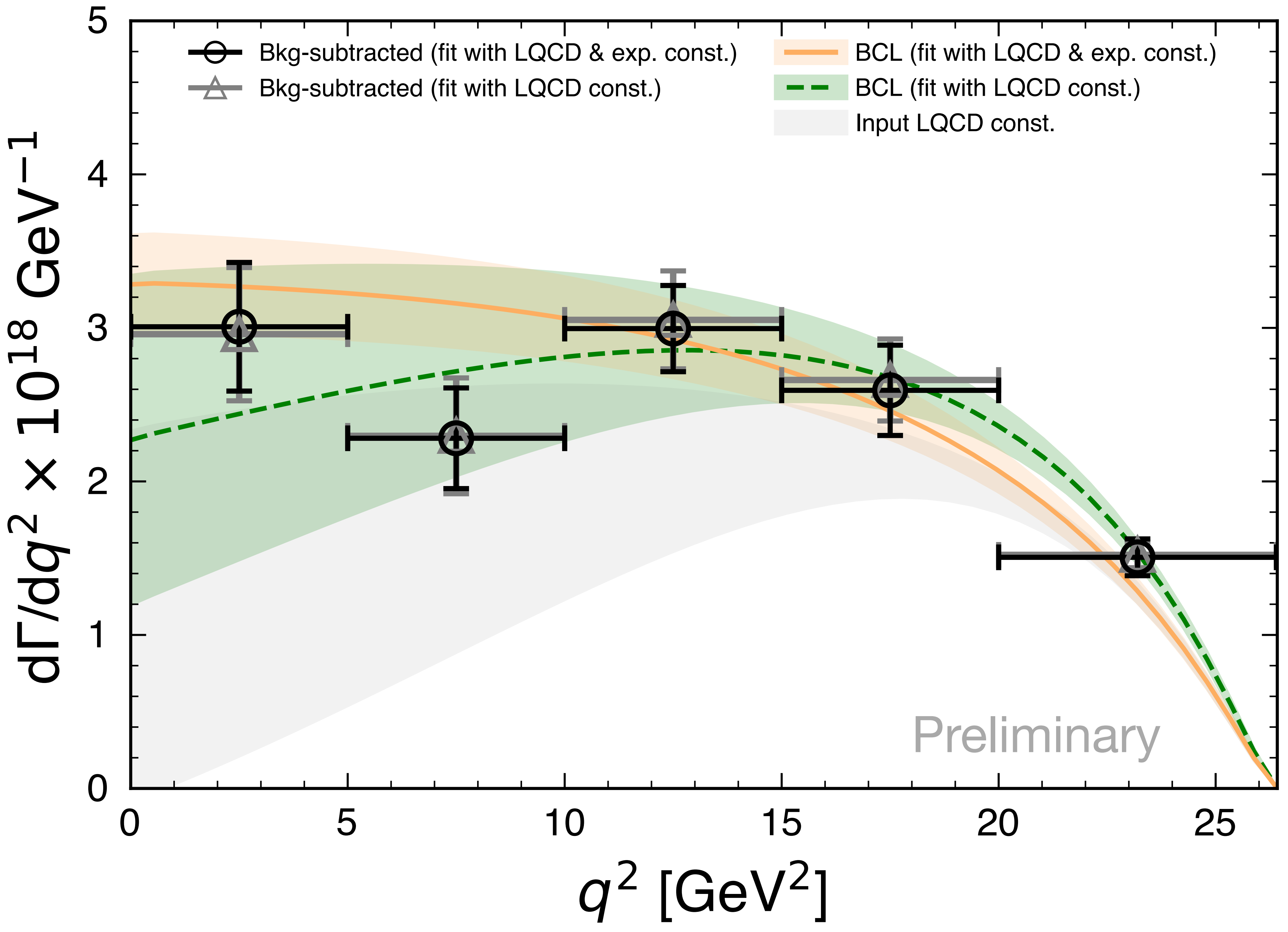
CKM global fit (w/o $|V_{ub}|$)**:** $(3.64 \pm 0.07) \times 10^{-3}$**,**
compatible within 0.8σ

Fitted $B \to \pi \ell \nu$ Decay Form Factor

- Various fit scenarios applied:
  - **Linked** or separate $B \to \pi^+ \ell \nu$, $B \to \pi^0 \ell \nu$
  - Input BCL constraint: **LQCD + exp.** or **only LQCD**

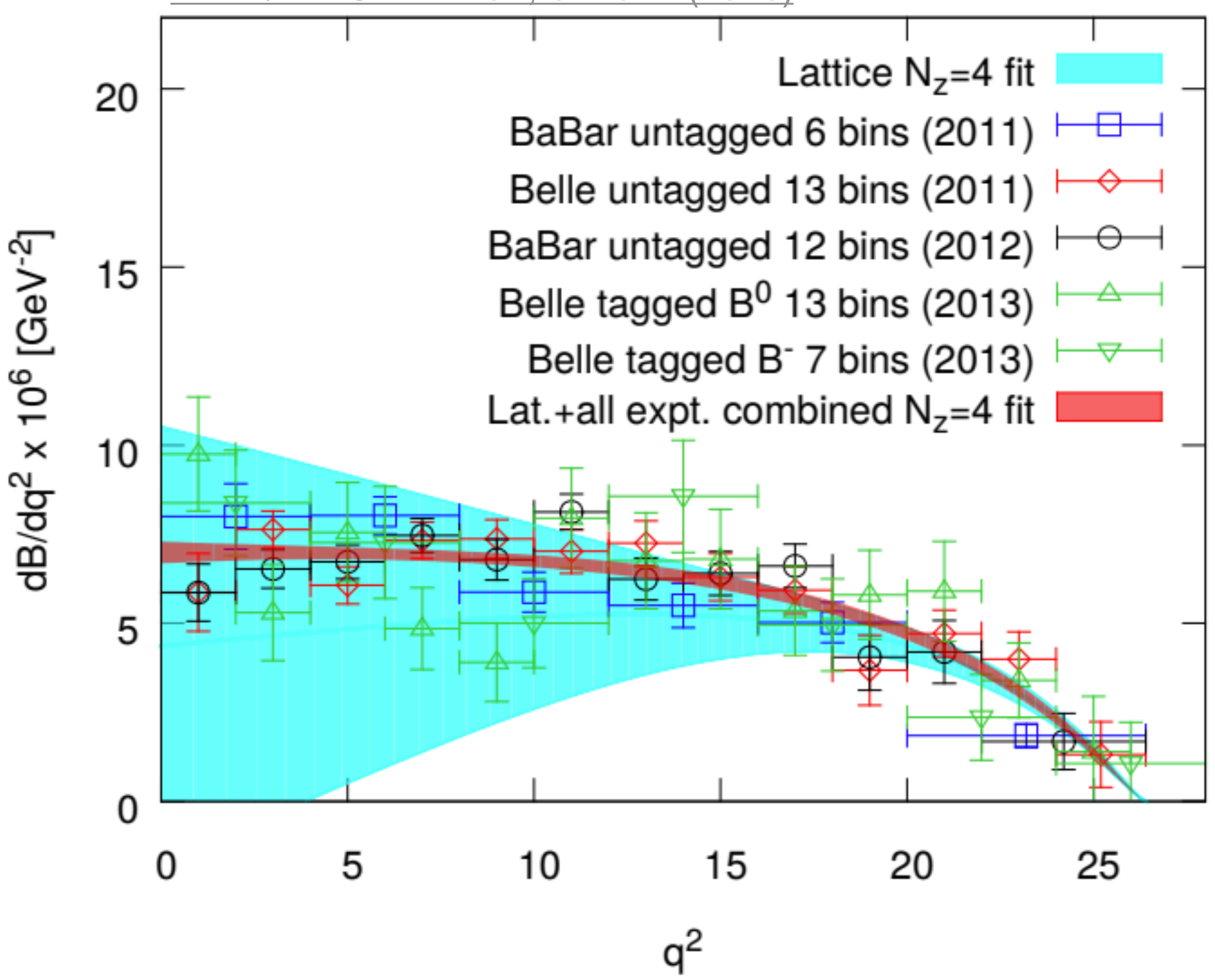
arXiv: 2303.17309

q^2 spectra with **fitted BCL para.** (linked $\pi^{0,\pm}$)

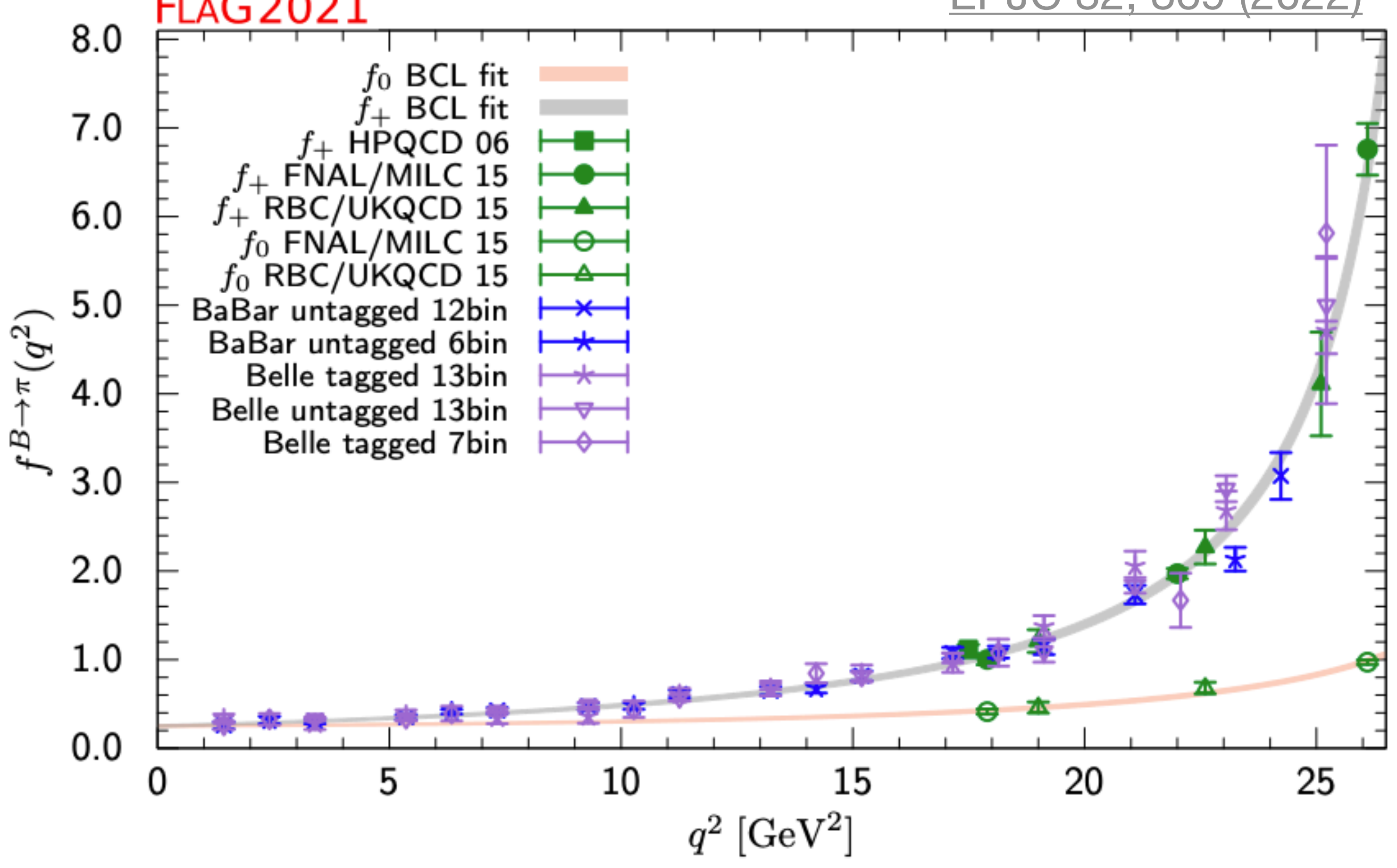


Points: subtract fitted other $B \to X_u \ell \nu$ and background yields from data, and apply unfolding + eff. correction

FNAL/MILC: PRD 92, 014024 (2015)



EPJC 82, 869 (2022)

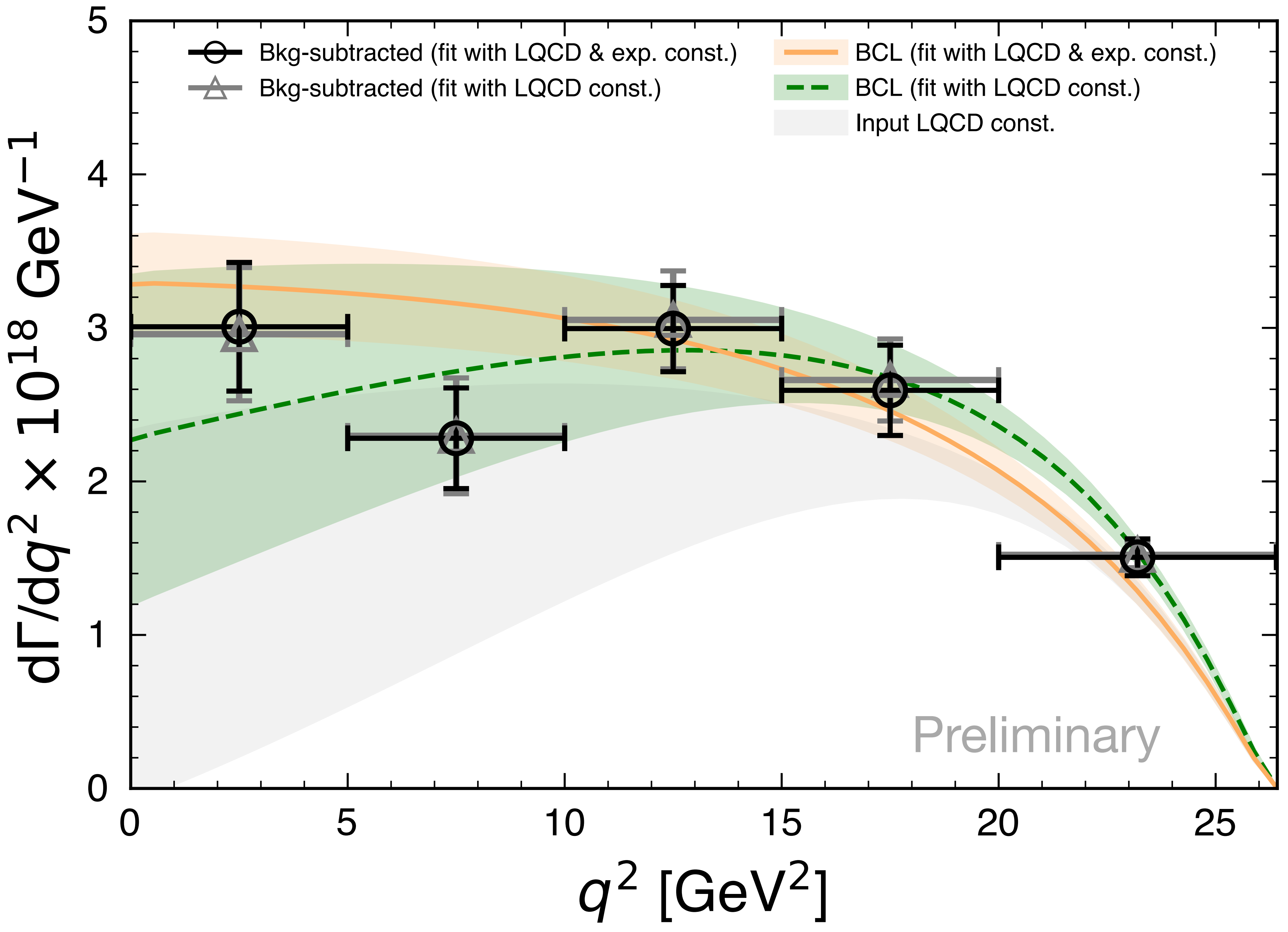


Fitted $B \to \pi \ell \nu$ Decay Form Factor

arXiv: 2303.17309

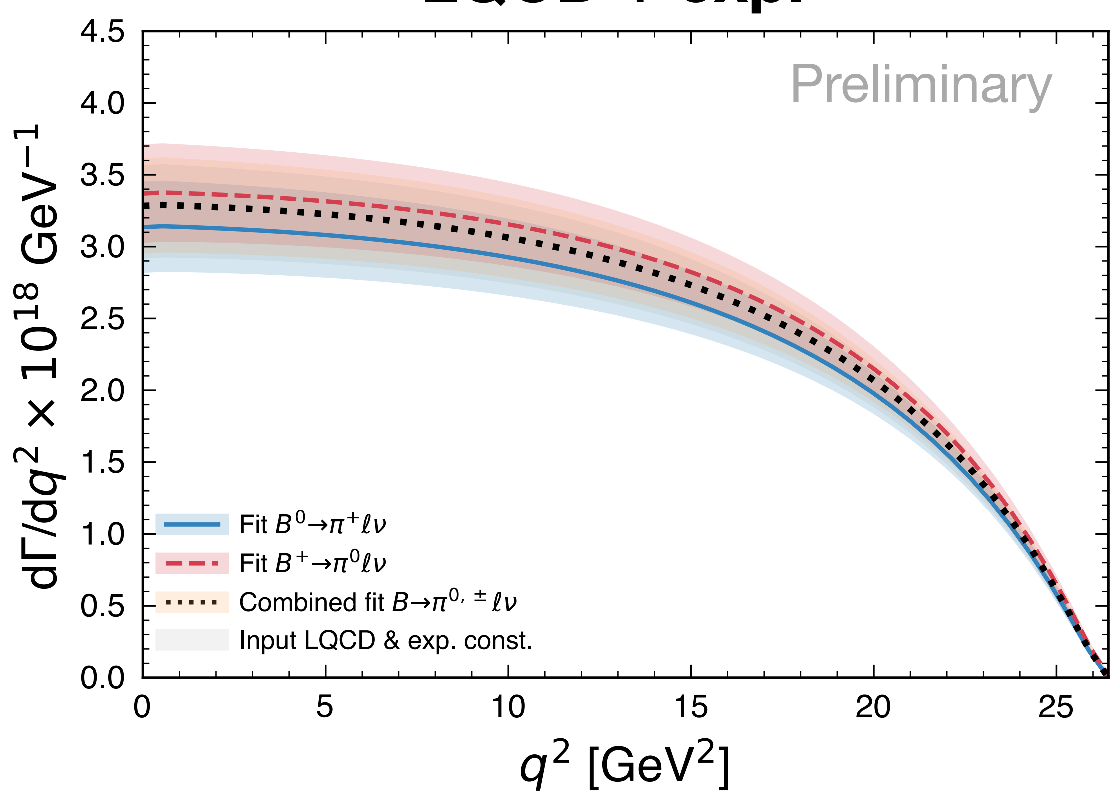
- Various fit scenarios applied:
  - **Linked** or separate $B \to \pi^+ \ell \nu$, $B \to \pi^0 \ell \nu$
  - Input BCL constraint: **LQCD + exp.** or **only LQCD**

q^2 spectra with **fitted BCL para.** (linked $\pi^{0,\pm}$)

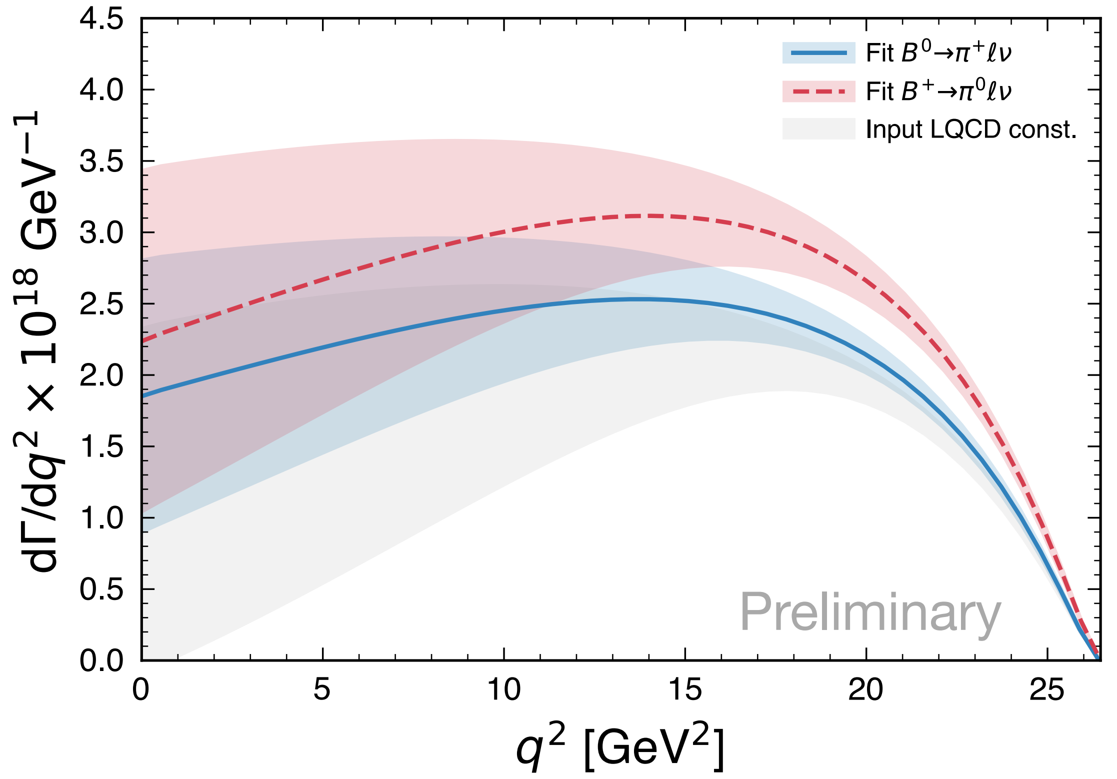


Points: subtract other $B \to X_u \ell \nu$ and background in data, and apply unfolding + eff. correction

LQCD + exp.



Only LQCD



LQCD + exp. (linked $\pi^{0,\pm}$)

	$\lvert V_{ub}\rvert \times 10^3$	a_0^+	a_1^+	a_2^+	a_0^0	a_1^0
Central	3.783	0.414	-0.494	-0.297	0.500	-1.426
Uncertainty	0.308	0.014	0.053	0.180	0.023	0.054
$\lvert V_{ub}\rvert$	1.000	-0.453	-0.171	0.232	-0.109	-0.106
a_0^+		1.000	0.152	-0.451	0.259	0.142
a_1^+			1.000	-0.798	-0.096	0.215
a_2^+				1.000	0.012	-0.097
a_0^0					1.000	-0.451
a_1^0						1.000

Only LQCD (linked $\pi^{0,\pm}$)

	$\lvert V_{ub}\rvert \times 10^3$	a_0^+	a_1^+	a_2^+	a_0^0	a_1^0
Central	4.122	0.406	-0.615	-0.550	0.494	-1.527
Uncertainty	0.384	0.012	0.082	0.401	0.020	0.098
$\lvert V_{ub}\rvert$	1.000	-0.419	-0.486	-0.328	-0.191	-0.478
a_0^+		1.000	0.274	-0.177	0.255	0.174
a_1^+			1.000	0.371	0.109	0.731
a_2^+				1.000	0.200	0.712
a_0^0					1.000	-0.027
a_1^0						1.000

Summary

- **Novel** approach allows to measure inclusive and exclusive $|V_{ub}|$ **simultaneously**
- Obtained $|V_{ub}^{\text{excl.}}|/|V_{ub}^{\text{incl.}}|$ ratio is **compatible with SM** expectation but **1.6σ higher than current WA**
- Future Belle II larger dataset will be helpful for improving the precision (e.g. single-pion mode)

$$|V_{ub}^{\text{excl.}}| = (3.78 \pm 0.23_{\text{stat}} \pm 0.16_{\text{syst}} \pm 0.14_{\text{theo}}) \times 10^{-3} \quad \text{(LQCD + exp.)}$$
$$|V_{ub}^{\text{incl.}}| = (3.90 \pm 0.20_{\text{stat}} \pm 0.32_{\text{syst}} \pm 0.09_{\text{theo}}) \times 10^{-3} \quad \text{(LQCD + exp.)}$$

Ratio = 0.97 ± 0.12

$$|V_{ub}^{\text{excl.}}| = (4.12 \pm 0.30_{\text{stat}} \pm 0.18_{\text{syst}} \pm 0.16_{\text{theo}}) \times 10^{-3} \quad \text{(LQCD)}$$
$$|V_{ub}^{\text{incl.}}| = (3.90 \pm 0.20_{\text{stat}} \pm 0.32_{\text{syst}} \pm 0.09_{\text{theo}}) \times 10^{-3} \quad \text{(LQCD)}$$

Ratio = 1.06 ± 0.14

Weighted average of excl. & incl. :

$$|V_{ub}| = (3.85 \pm 0.26) \times 10^{-3} \quad \text{(LQCD + exp.)}$$
$$|V_{ub}| = (4.01 \pm 0.27) \times 10^{-3} \quad \text{(LQCD)}$$

CKM global fit (w/o $|V_{ub}|$)**:** $(3.64 \pm 0.07) \times 10^{-3}$**,**
compatible within 0.8σ and 1.4σ, respectively

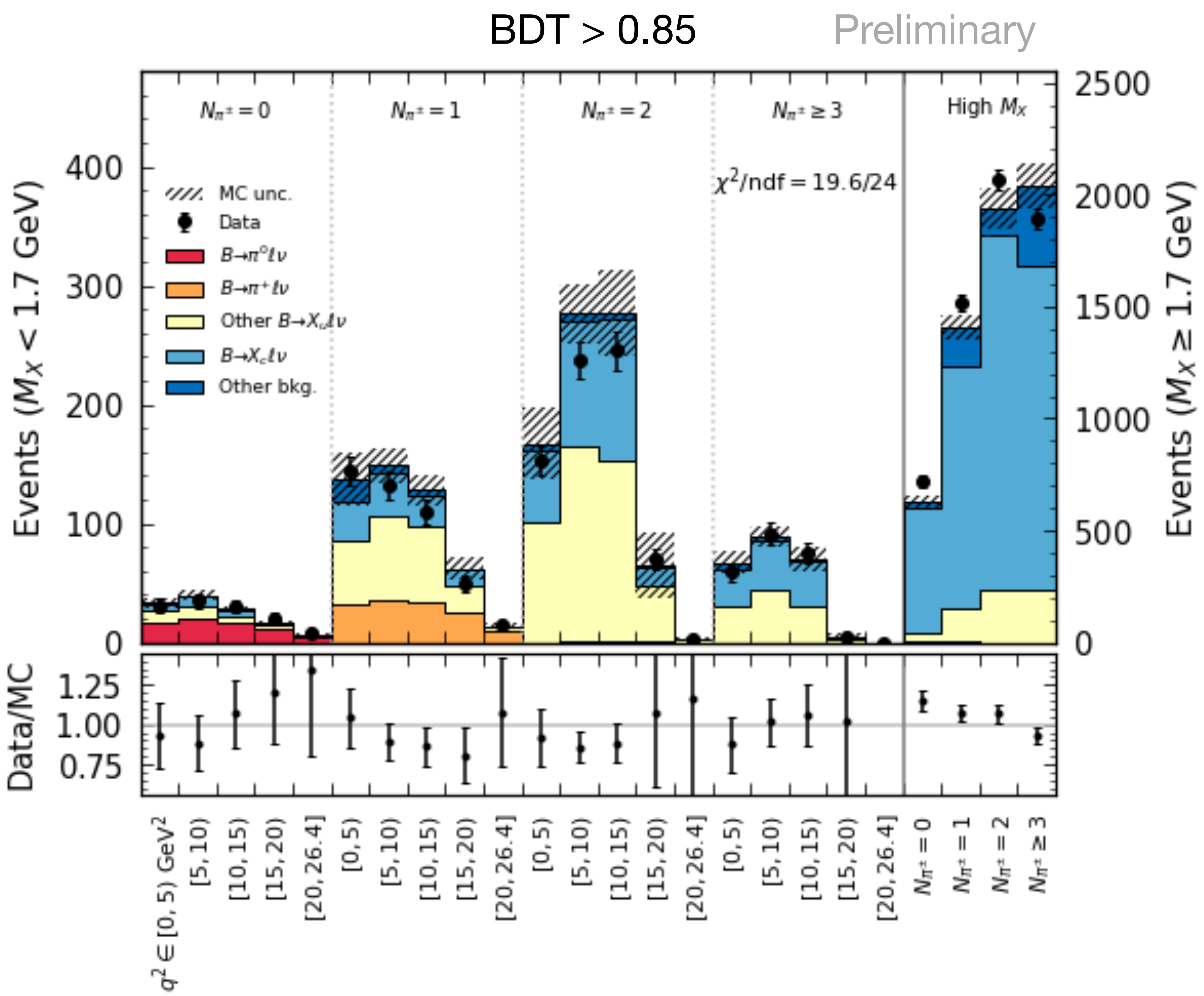
THANK YOU

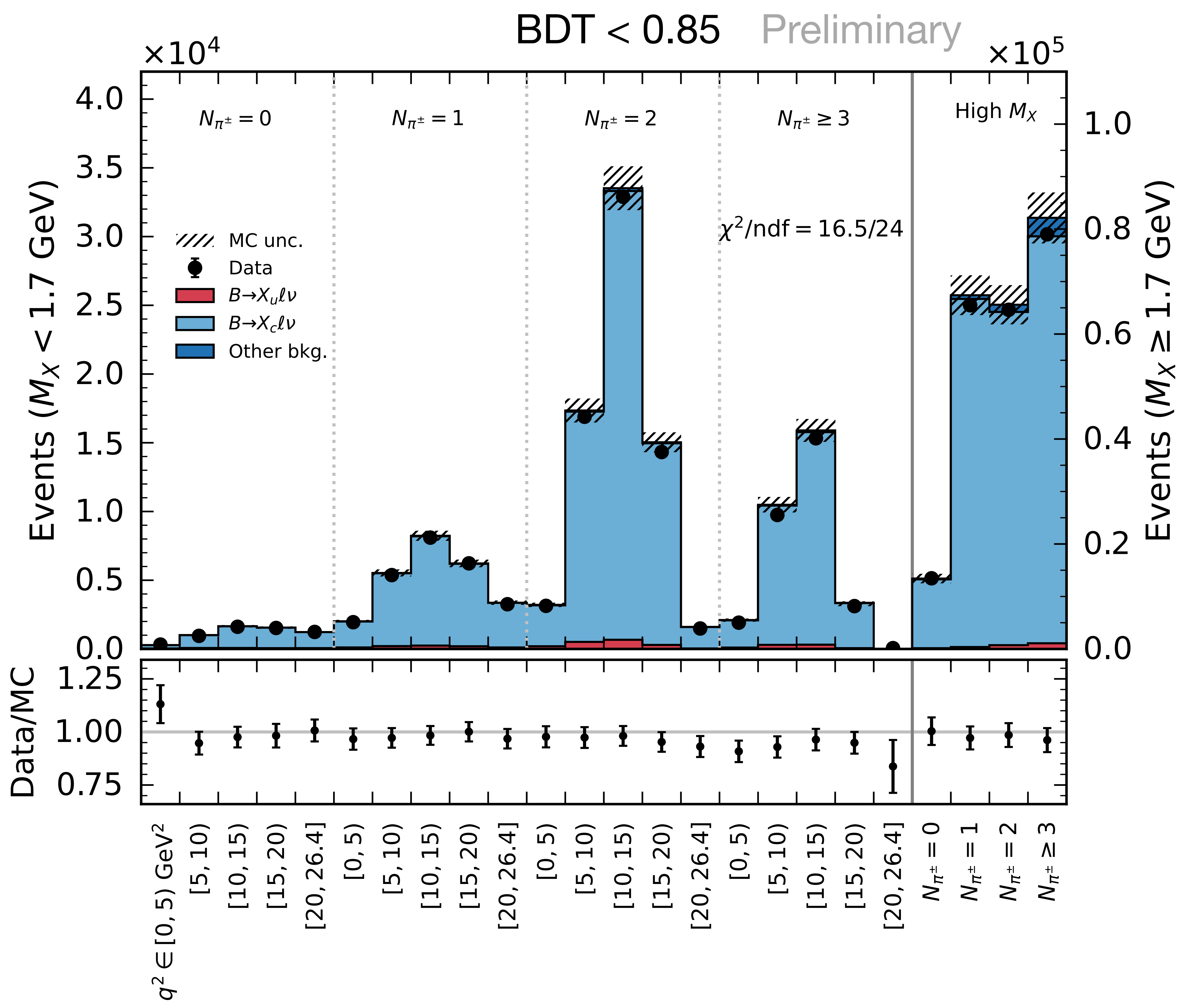
THANK YOU

Backup

Preliminary

- Prefit distributions





Various Inclusive Decay Rates

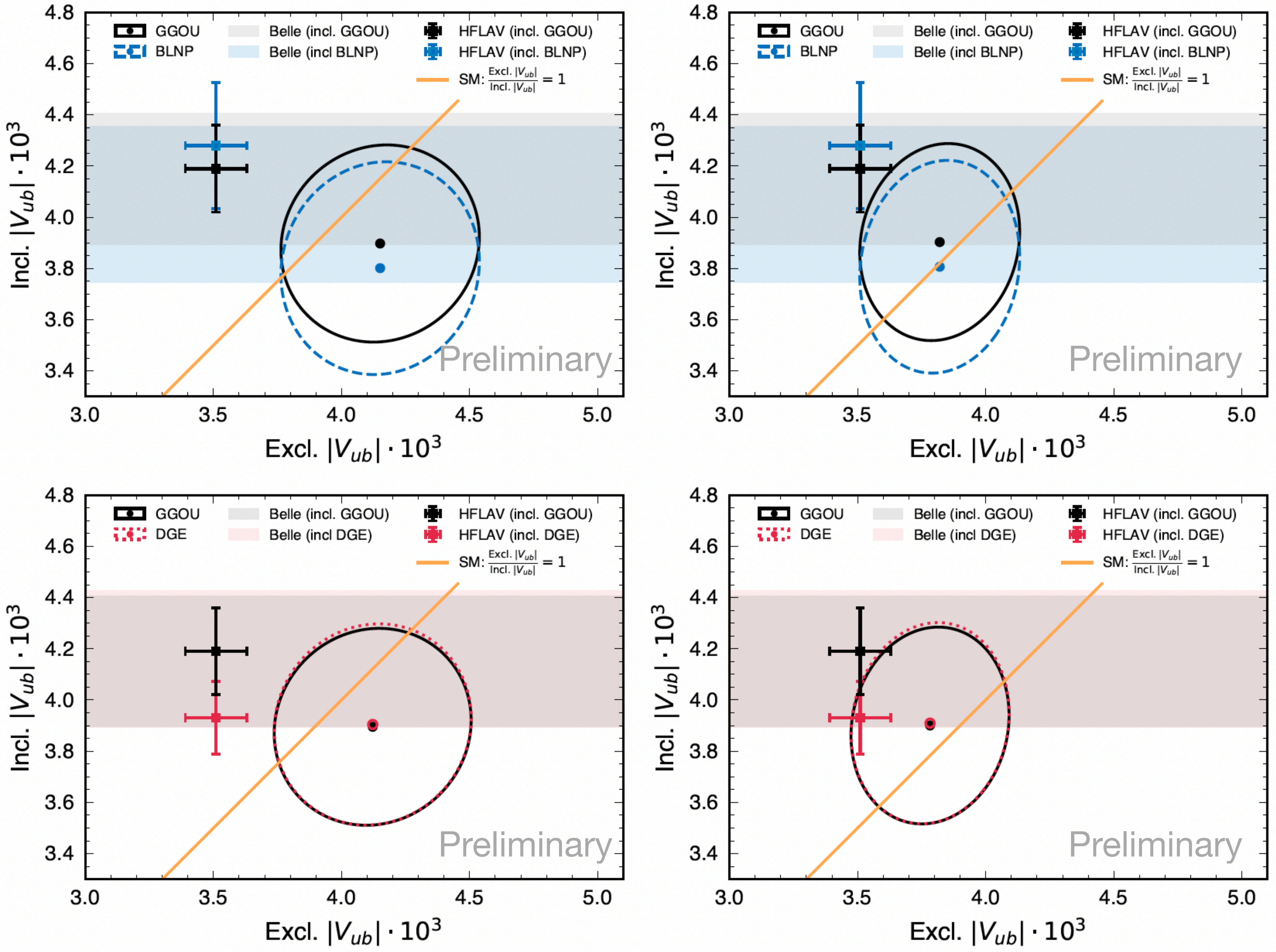


FIG. 4. The $|V_{ub}|$ values obtained using the different theoretical inclusive decay rates are compared: GGOU versus BLNP (up) and GGOU versus DGE (low). The left column shows the fit with only LQCD constraints and the results from combined LQCD-experimental constraints are in the right column.

More Details on Fit Results

Preliminary

- Setup 1-a: fit $q^2 : N_{\pi^\pm}$ spectra with LQCD and external experimental constraint on the BCL form factor and shared $B \to \pi \, \ell \, \bar{\nu}_\ell$ normalization based on the isospin relation.
- Setup 1-b: same as 1-a, but with only LQCD constraint for the form factor.
- Setup 2-a: separate normalizations of the $B^0 \to \pi^- \, \ell^+ \, \nu_\ell$ and $B^+ \to \pi^0 \, \ell^+ \, \nu_\ell$ decays and with LQCD-experimental constraint.
- Setup 2-b: same as 2-a, but with only LQCD constraint.

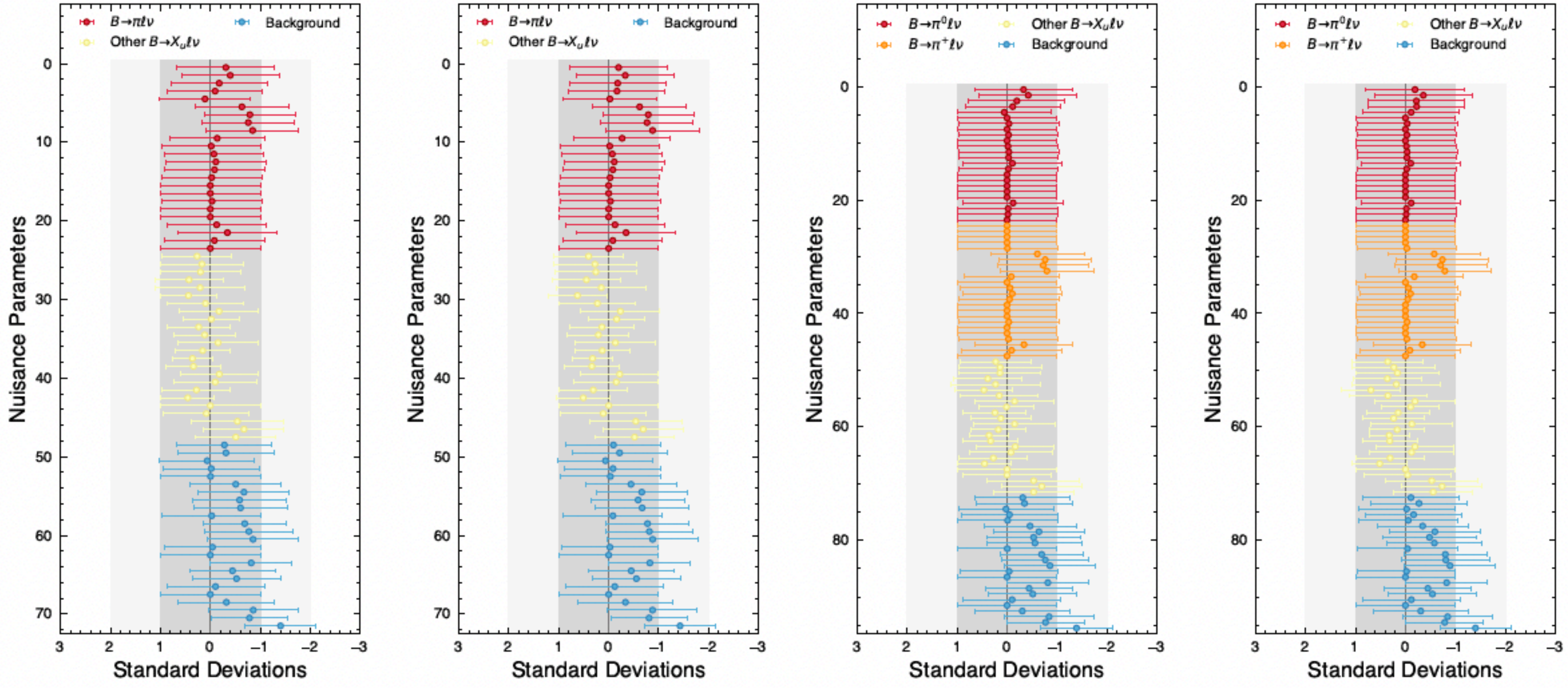


FIG. 7. The pulls of Nuisance parameters. From left to right, the results are shown for the setup 1-a, 1-b, 2-a and 2-b. The uncertainty of each pull shows the post-fit error normalized to the pre-fit constraint.

More Details on Fit Results

Preliminary

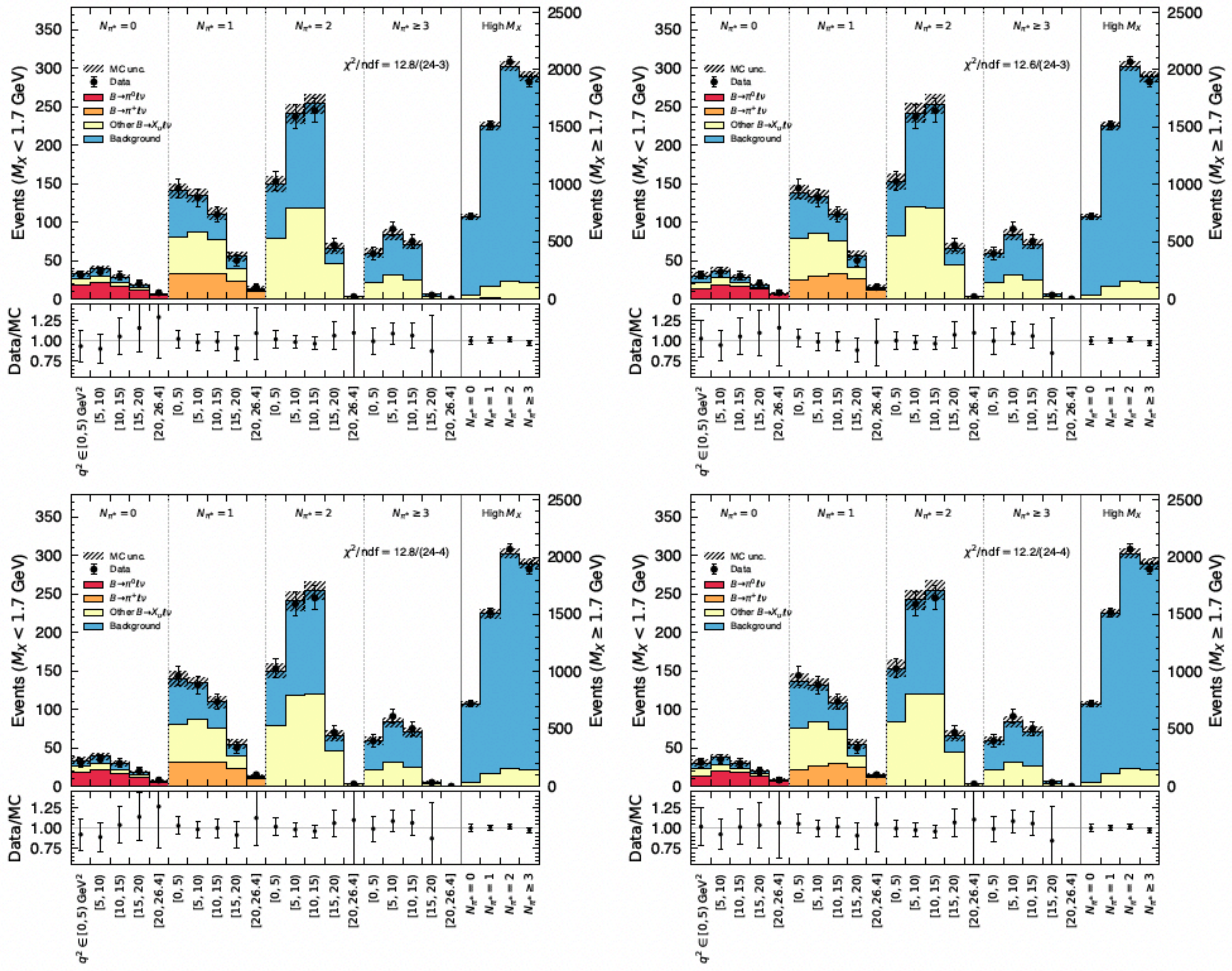
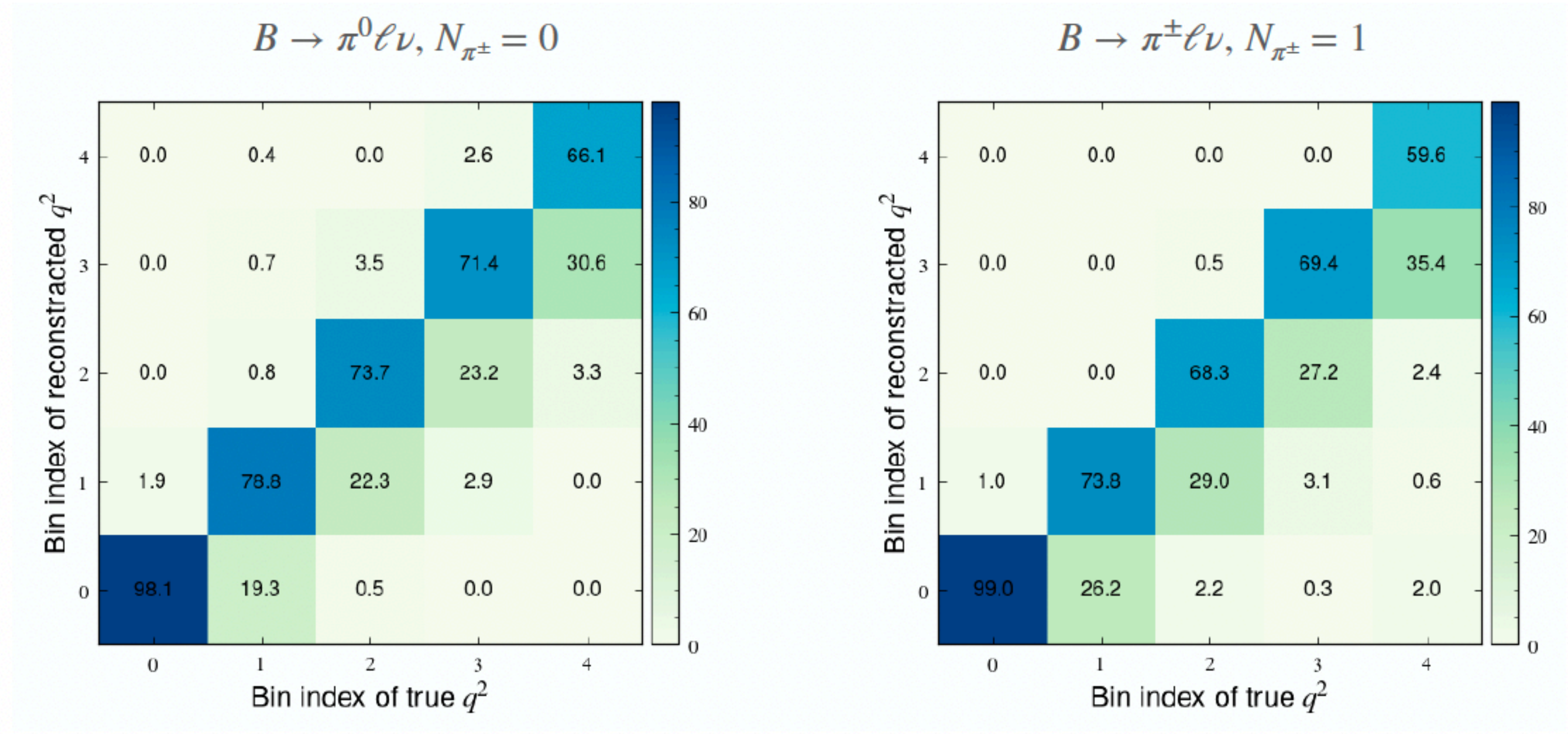
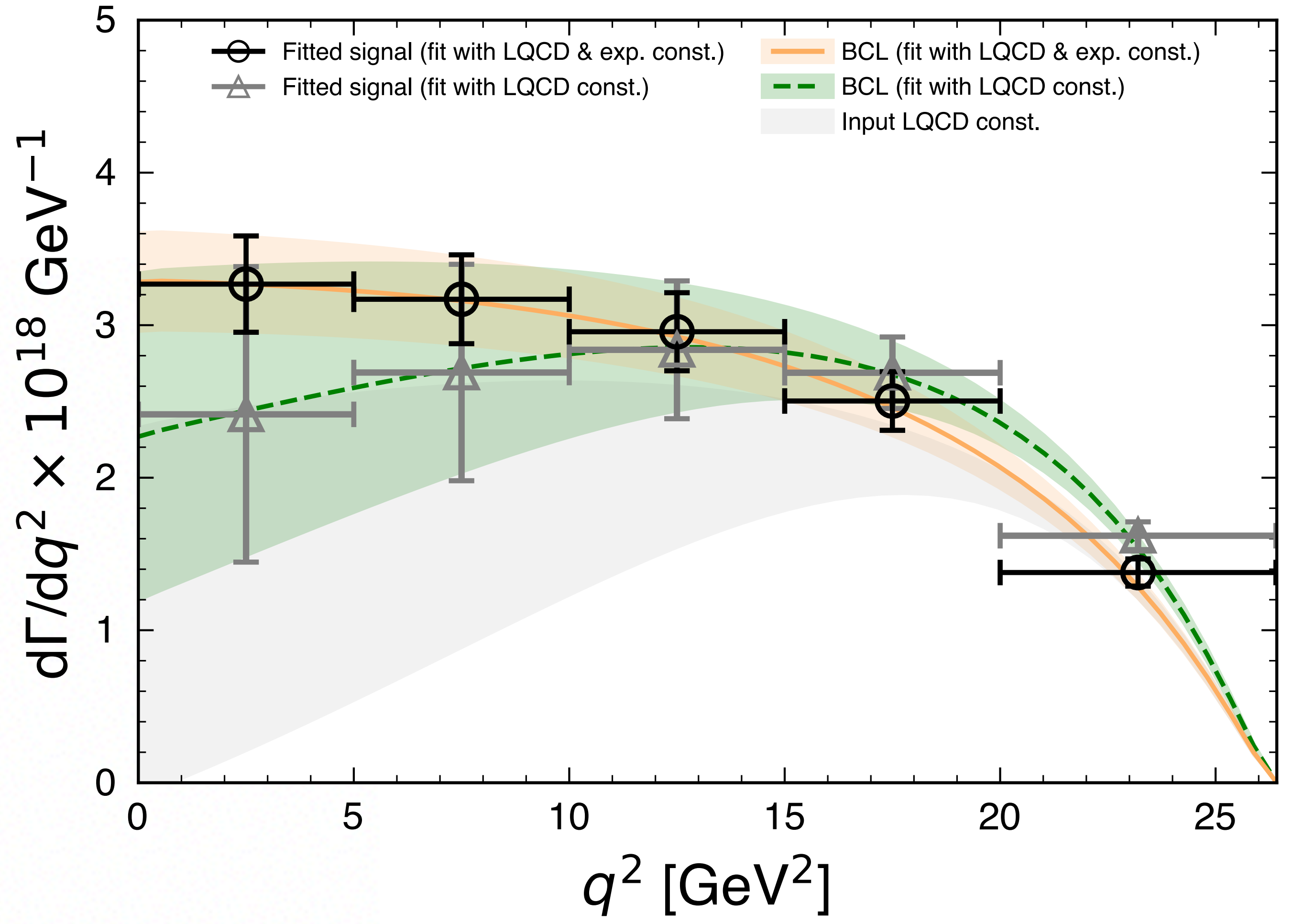


FIG. 6. The postfit $q^2 : N_{\pi^\pm}$ spectra with various setups.From top left to bottom right, the results are shown for the setup 1-a, 1-b, 2-a and 2-b. The uncertainties incorporate all post-fit uncertainties discussed in the main text.

Migration and Efficiency

Preliminary





Setup	$B^+ \to \pi^0 \ell^+ \nu_\ell$	$B^0 \to \pi^- \ell^+ \nu_\ell$	Other $B \to X_u \ell \bar{\nu}_\ell$	Bkg.
1-a	78 ± 11	147 ± 21	1087 ± 241	7209 ± 631
1-b	72 ± 11	143 ± 23	1095 ± 241	7226 ± 631
2-a	80 ± 14	141 ± 31	1089 ± 241	7200 ± 631
2-b	77 ± 14	125 ± 32	1104 ± 241	7205 ± 632
$10^3 \cdot \epsilon_{\mathrm{sig}}$	0.30	0.32	0.26	-

TABLE I. The fitted yields for $B^+ \to \pi^0 \ell^+ \nu_\ell$, $B^0 \to \pi^- \ell^+ \nu_\ell$, other $B \to X_u \ell \bar{\nu}_\ell$ decays and backgrounds with various fitter setups. The uncertainties assigned to the fitted yields include the statistical and systematic impacts in the fitting procedure. The signal efficiencies ϵ_{sig} are also listed.

Details on Systematic Uncertainties

Main systematic uncertainties for the measured branching fractions are listed below

Observable	Sources	Relative Error (%)
$\mathcal{B}(B \to \pi \ell \nu)$	Tagging efficiency	**4.0**
	$B \to X_u \ell \nu$ modelling	**3.5**
	$B \to X_c \ell \nu$ background	**1.2**
$\mathcal{B}(B \to X_u \ell \nu)$	$B \to X_u \ell \nu$ modelling	**12.1**
	X_u fragmentation	**5.3**
	Tagging efficiency	**4.0**
	$B \to X_c \ell \nu$ background	**2.8**

Hybrid Model of $B \to X_u \ell \nu$

Hybrid MC is a **combination** of **resonances** (exclusive decays) and **non-resonant** contribution in the inclusive $B \to X_u \ell \nu$ decays

- EvtGen simulation:

(1) exclusive modes $B \to (\pi, \rho, \omega, \eta^{(\prime)})\ell\nu$ with latest WA form factors & branching fractions

(2) fully inclusive $B \to X_u \ell \nu$ (only non-resonant shapes, e.g. BLNP, GGOU)

- Calculate hybrid weights to mix resonance & non-res. in **3D binning of** (q^2, E_ℓ^B, M_X) to recover total $\mathscr{B}(B \to X_u \ell \nu)$ in each bin

$$H_i = R_i + \omega_i N_i$$

- Systematic uncertainties include the impact from exclusive FFs & BRs, total $\mathscr{B}(B \to X_u \ell \nu)$, inclusive models

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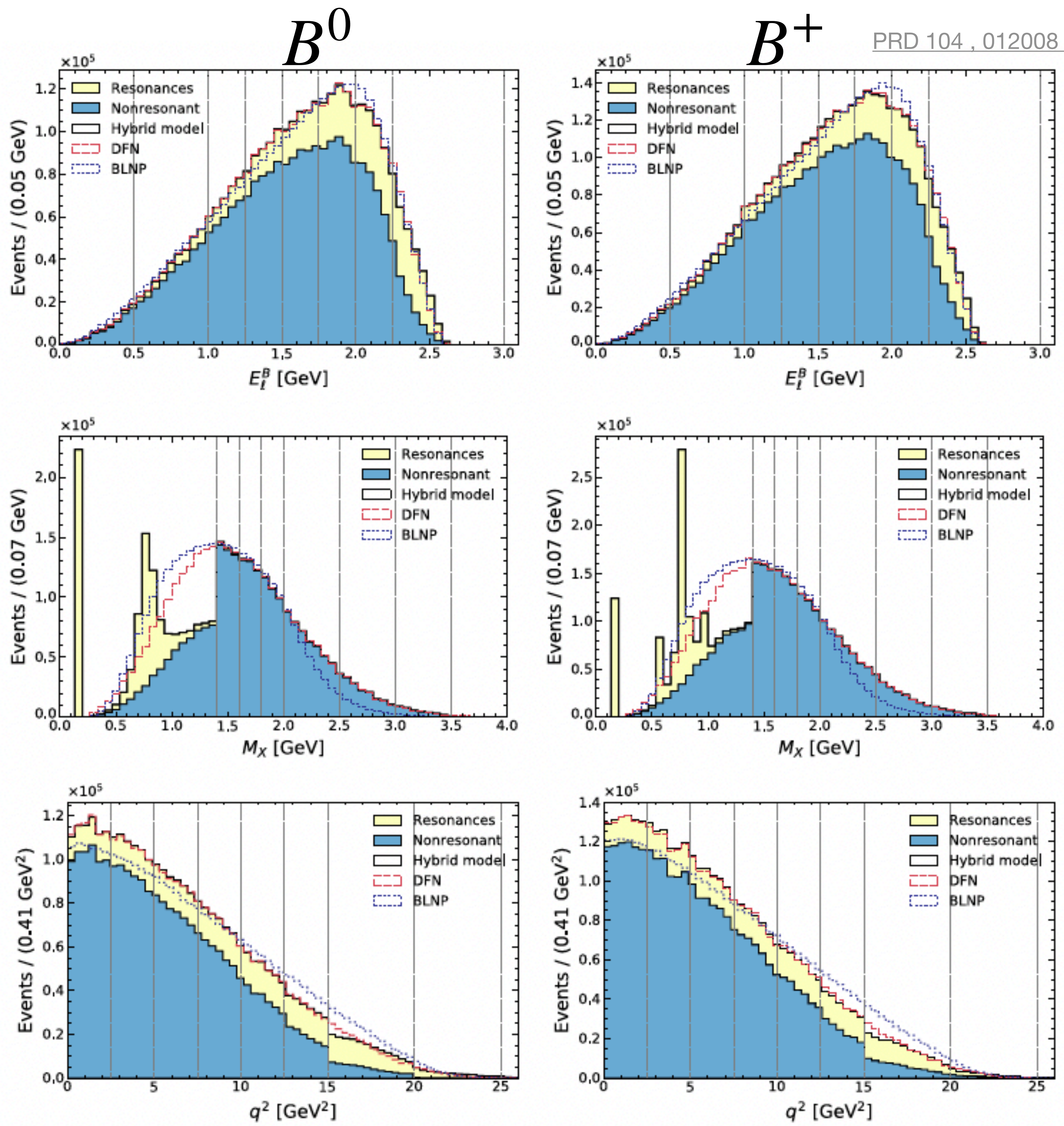

EvtGen truth