Dark Matter Detection via Migdal Effect

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Main structure

- The aim:
  This presentation introduces two papers related to the Migdal Effect: Victor Flambaum, Liangliang Su and Lei Wu (Science China), Jiwei Li and Lei Wu (JCAP)
- The not so-ancient tale:
  Explain why and what is Migdal effect
  Orthogonality problem
  Spin-dependent Migdal scattering

Are there something new in our work?

Yes, I hope to make it clear after this talk

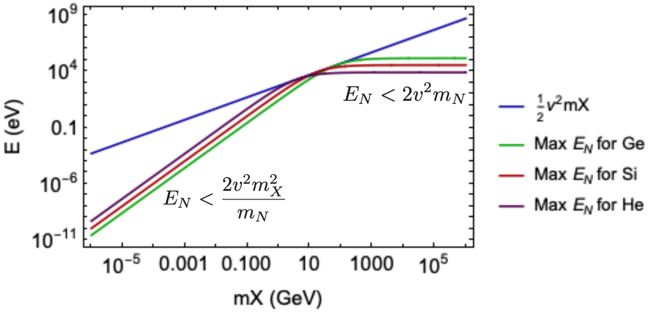
Why and What's Migdal Effect

The Kinetic No-Go Theorem for Elastic Nucleus Scattering

- Energy-momentum conservation law entails

$$E_N < \frac{(2v\mu_{XN})^2}{2m_N}$$

- When $m_X < m_N$, a significant amount of the kinetic energy of the dark matter cannot be accessed



Ways out: Inelastic Recoils

The kinetic energy is too low to excite observed nuclear recoils, Hence, we consider electromagnetic excitations

- Photon (Brehmstrallung):
  C. Kouvaris, J. Pradler: arXiv 1607.01789
- Electron (Migdal effect):
  - In atoms (Xe, Ar, He, etc)
    M. Ibe, W. Nakano, Y. Shoji and K. Suzuki: 1707.07258
    Mai Qiao, Chen Xia, and Yu-Feng Zhou: 2307.12820
  - In crystals (Si, Ge, GaAs, etc)
    Z.-L. Liang, C. Mo, F. Zheng and P. Zhang: 2011.13352
    S. Knapen, J. Kozaczuk, T. Lin: 2011.09496

Why not photon excitations?

The Migdal effect is larger than Brehmstrahlung

What is the Migdal effect?

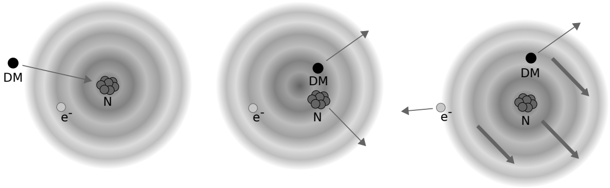


Figure from 1711.09906

The Common/Typical Definition

When the nucleus is abruptly kicked and quickly moves away, some electron wave functions may not have sufficient time to adjust, causing one or more electrons to remain behind.

What is not the Migdal effect?

- The Migdal effect pertains to the first ionization that take place in the hard scattering between the nucleus and dark matter
- It is not the secondary ionization from the recoiled nucleus, quantified by quenching factor

More Microscopic/Fundamental Definition

Altered Coulomb field transfers energy from dark matter to electrons, causing ionization

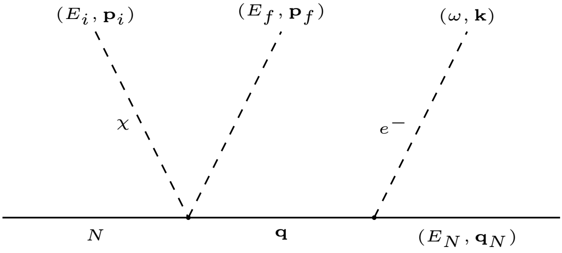


Figure from 2011.09496

Computational Framework

Migdal's Original Trick

The electron cloud cannot adjust itself to on the time scale of the DM-nucleus impact

Excited electron wave functions in the rest frame of the recoiling nucleus

=

Ground state wave function boosted to the frame of the recoiling nucleus

$$|i\rangle \to e^{im_e \mathbf{v}_N \cdot \sum_\beta \mathbf{r}_\beta} |i\rangle$$

- v_N is velocity of the recoiling nucleus
- r_β is position operator corresponding to electron labeled with β

Dipole transition

$$\mathcal{M}_{if} = \left\langle f \left| e^{im_e \mathbf{v}_N \cdot \sum_\beta \mathbf{r}_\beta} \right| i \right\rangle \approx im_e \mathbf{v}_N \cdot \left\langle f \left| \sum_\beta \mathbf{r}_\beta \right| i \right\rangle$$

Advanced Trick

The boosting feels awkward, in particular in many-body system

Use time-dependent perturbation theory for straightforward calculations in the lab frame

$$H(t) = H_0 + H_1(t)$$

$$H_0 = -\sum_{\beta} \frac{Z_N \alpha}{|\mathbf{r}_\beta|}$$

$$H_1(t) = -\sum_{\beta} \frac{Z_N \alpha}{|\mathbf{r}_\beta - \mathbf{R}_N(t)|} + \sum_{\beta} \frac{Z_N \alpha}{|\mathbf{r}_\beta|}$$

$$\approx -Z_N \alpha \sum_{\beta} \frac{\hat{\mathbf{r}}_\beta \cdot \mathbf{v}_N}{\mathbf{r}_\beta^2} t\theta(t)$$

where

$$\mathbf{R}_N(t) = \theta(t)\mathbf{v}_N t$$

Equivalent proof for isolated atoms only

$$\mathcal{M}_{if} = i \left\langle f \left| \frac{1}{\omega^2} \sum_\beta \frac{Z_N \alpha \hat{\mathbf{r}}_\beta \cdot \mathbf{v}_N}{\mathbf{r}_\beta^2} \right| i \right\rangle$$

Everything is Fine

The full expression of event rate is simple in dipole transition

$$\frac{\mathrm{d}R_{ion}}{\mathrm{d}E_e} = N_T n_{\text{halo}} \sum_{nl} \frac{[f_p Z + f_n(A-Z)]^2}{8\mu_{\mathbf{n}}^2 E_e} \int_{q_{\min}} \mathrm{d}q q$$

$$\times \sigma_{\mathbf{n}} \left|F_N(q)\right|^2 \left|f_{ion}^{nl}(p_e, q_e)\right|^2 \eta\left(E_\chi^{\min}\right)$$

$$\left|f_{ion}^{nl}(p_e, q_e)\right|^2 \sim \left|\mathcal{M}_{if}\right|^2$$

Orthogonality consistency

Dipole transition requires $\langle f|i\rangle = \delta_{fi}$

Orthogonality Crisis

The initial bound wave-function is naturally orthogonal to the final out-going wavefunction : actually two level orthogonality crisis

- Canonical orthogonality crisis: overestimated ionization factor about several orders due to the use of plane waves for outgoing electrons
- Little orthogonality crisis: problem addressed by neglecting $L = 0$ term

$$\int \mathrm{d}r r^2 j_0\left(q_e r\right) R^*_{p_e \ell'}(r) R_{n\ell}(r)$$

where We need a way to solve little orthogonality crisis

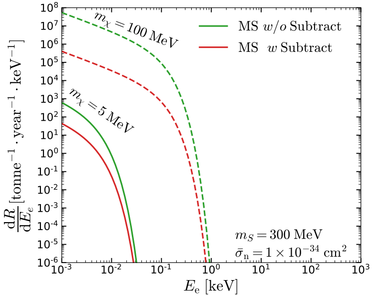
Our Proposal: Subtrction

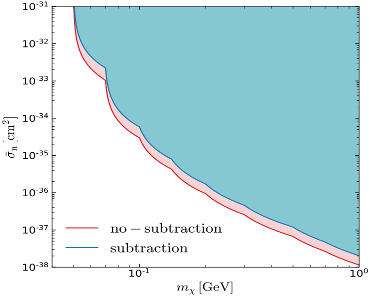
- Challenges arise from low momentum transfer

$$j_0(q_e r) \sim 1 + (q_e r)^2$$

- Radial integral modified to ensure orthogonality

$$\int \mathrm{d}r r^2 \left(j_0\left(q_e r\right) - 1\right) R^*_{p_e \ell'}(r) R_{n\ell}(r)$$





Generalize Migdal to Spin-dependent scattering

Why and What is Spin-dependent

Migdal effect currently limited to spin-independent scattering

- Some direct detection models only involve spin-dependent interactions, e.g., those with a light pseudoscalar mediator
- It may dominate in the leading order LO ! (such as real vector boson, Majorana fermion dark matter)

Which interaction is our focus?

$$\mathcal{L}_{\text{int}} \supset \bar{\chi}'\gamma^{\mu}\gamma^{5}\chi\overline{\mathcal{N}}\gamma_{\mu}\gamma^{5}\mathcal{N}. \rightarrow -4\overrightarrow{\mathbb{S}}_{\chi}\cdot\overrightarrow{\mathbb{S}}_{\mathcal{N}} = -4\boldsymbol{O}_4$$

Only SD interaction not suppressed by momentum transfer in leading order.

How to incorporate Migdal effect

soft limit: $|\mathbf{q}_N \cdot \mathbf{k}| \ll m_N \omega$ and $k \ll q_N$

Factorization at soft limit from elastic scattering helps

$$\frac{\mathrm{d}R}{\mathrm{d}E_R\ \mathrm{d}E_{\text{em}}\ \mathrm{d}v} = \frac{\mathrm{d}R_0}{\mathrm{d}E_R\ \mathrm{d}v} \times \frac{1}{2\pi} \sum_{n,l} \frac{\mathrm{d}}{\mathrm{d}E_e}\ \mathrm{d}p^c_{q_e}\left(n, l \to E_e\right)$$

Ionziation factor after subtract

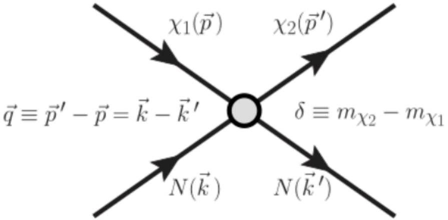
$$\sum_{n,l} \frac{\mathrm{d}}{\mathrm{d}\ln E_e}\ \mathrm{d}p^c_{q_e}\left(n, l \to E_e\right) = \frac{\pi}{2}\left|f^{nl}_{ion}\left(k_e, q_e\right)\right|^2$$

Does spin-dependent alters the dipole transition?

$S_\chi \cdot S_N$ does not affect dipole transition. Other operators might not!

Even More General: Inelastic Dark Matter

- iDM is a dark matter model designed to reconcile the DAMA spectra and CDMS experimental data
- iDM interacting with Standard Model particles exhibit a diverse phenomenology due to their kinematics
- Easily incorporated into Non-relativistic EFT



Kinematics

$\delta > 0$, Endothermic, $\delta < 0$, Exothermic

- Energy conservation

$$\frac{1}{2}\mu_N v^2 = \frac{p'^2}{2m_{\chi'}} + \frac{k'^2}{2m_N} + \Delta$$
$$= \frac{(\vec{p}+\vec{q})^2}{2m_{\chi'}} + \frac{(\vec{k}-\vec{q})^2}{2m_N} + \Delta$$

with $\Delta = \delta + E_{\rm em}$ for Migdal

- The nucleus recoil energy

$$E_R = \frac{\mu_N^2 v^2}{m_N}\left(1 - \cos\theta\sqrt{1 - \frac{2\Delta}{\mu_N v^2}}\right) - \frac{\mu_N \Delta}{m_N}$$

- Minimum DM particle velocity

$$v_{\min}(E_R) = \left|\frac{q}{2\mu_N} + \frac{\Delta}{q}\right| = \frac{1}{\mu_N\sqrt{2m_N E_R}}\left|m_N E_R + \mu_N \Delta\right|$$

Dynamics for NREFT

A. L. Fitzpatrick, W. Haxton, E. Katz, N. Lubbers, and Y. Xu, JCAP 02, 004 (2013)

- Define a Hermitian invariant

$$\vec{v}^{\perp} \equiv \vec{v} + \frac{\vec{q}}{2\mu_N} = \frac{1}{2}\left(\vec{v}_{\chi,\text{ in}} + \vec{v}_{\chi,\text{ out}} - \vec{v}_{N,\text{ in}} - \vec{v}_{N,\text{ out}}\right)$$

  such that $\vec{v}^{\perp} \cdot \vec{q} = 0$

- Four Galilean invariants

$$i\frac{\vec{q}}{m_N}, \quad \vec{v}^{\perp}, \quad \vec{\mathbb{S}}_N, \quad \vec{\mathbb{S}}_\chi$$

Generalize to iDM

$\vec{v}^{\perp}_{\text{el}} \to \vec{v}^{\perp}_{\text{inel}} \equiv \vec{v} + \frac{\vec{q}}{2\mu_N} + \frac{\Delta}{|\vec{q}|^2}\vec{q}$

NREFT

15 operators in total

$$\mathcal{H}_{\text{eff}} = \sum_{\alpha=n,p} \sum_{i=1}^{15} c_i^{\alpha} \boldsymbol{O}_i^{\alpha} = \sum_{\tau=0,1} \sum_{i=1}^{15} c_{\tau}^{i} \boldsymbol{O}_i t^{\tau}$$

- Multipole expansions in spherical harmonics

$$\mathcal{M} = \sum_{\tau=0,1} \left\langle j'_{\chi}, M'_{\chi}; j'_N, M'_N \left| \boldsymbol{O}_{JM;\tau}(q) \right| j_{\chi}, M_{\chi}; j_N, M_N \right\rangle$$

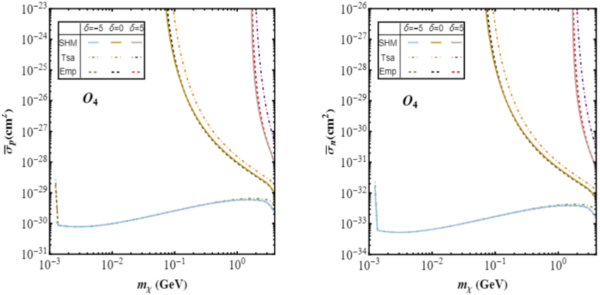
- Differential cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_R} = \frac{2m_T}{4\pi v^2} \left[\frac{1}{2j_{\chi}+1} \frac{1}{2j_N+1} \sum_{\text{spins}} |\mathcal{M}|^2 \right].$$

Results

$$\bar{\sigma}_{\chi n}(q=0) \equiv \frac{c_i^{n2}\mu_{\chi n}^2}{\pi}$$

Data from XENON1T S2-Only, Set 90% C.L. :



Back up

Equivalence between boost picture and time-dependent perturbation theory

$$\begin{aligned}
\mathcal{M}_{if}^{(\text{Migdal})} &= i m_e \mathbf{v}_N \cdot \left\langle f \left| \sum_\beta \mathbf{r}_\beta \right| i \right\rangle \\
&= -i \frac{m_e}{\omega} \mathbf{v}_N \cdot \left\langle f \left| \sum_\beta [\mathbf{r}_\beta, H_0] \right| i \right\rangle \\
&= \frac{1}{\omega} \mathbf{v}_N \cdot \left\langle f \left| \sum_\beta \mathbf{p}_\beta \right| i \right\rangle \\
&= -\frac{1}{\omega^2} \mathbf{v}_N \cdot \left\langle f \left| \sum_\beta [\mathbf{p}_\beta, H_0] \right| i \right\rangle \\
&= i \frac{1}{\omega^2} \mathbf{v}_N \cdot \left\langle f \left| \sum_\beta \frac{dV}{d\mathbf{r}_\beta} \right| i \right\rangle
\end{aligned}$$