国科大杭州高等研究院
Hangzhou Institute for Advanced Study, UCAS

第十二届新物理研讨会

W 玻色子质量反常的标量多重态解释

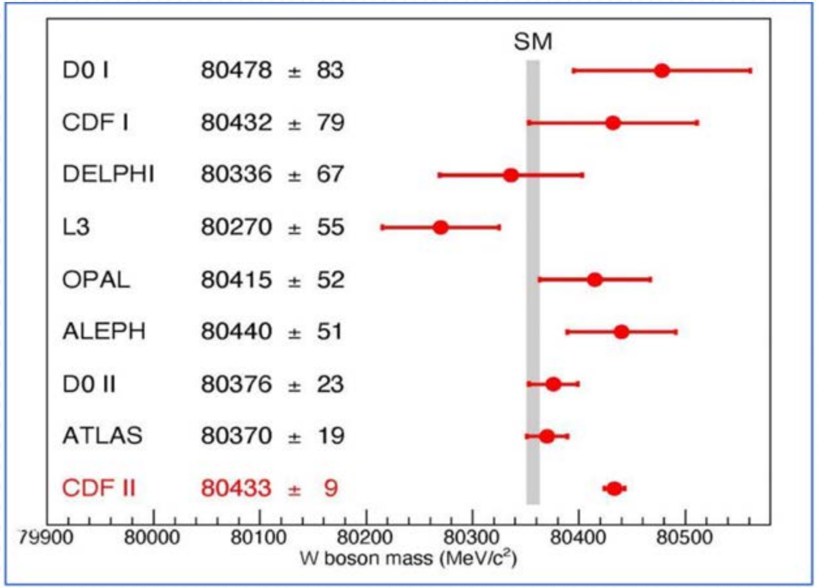
W-Boson Mass Anomaly from a General $SU(2)_L$ Scalar Multiplet

报告人：吴佳骏

青岛 · 2023年7月28日

- Jiajun Wu, Da Huang, and Chao-Qiang Geng, *Chinese Physics C* 2023, 47(6):063103.
- Jiajun Wu, Chao-Qiang Geng, and Da Huang, [arXiv:2307.12105 [hep-ph]].

W玻色子质量反常：



2022年4月

CDF-II最新结果：

明确的新物理信号

CDF合作组的结果： $M_{W,CDF} = 80.4335\pm0.0094\text{GeV}$ 与 标准模型的预测： $M_{W,SM} = 80.3570\pm0.006\ \text{GeV}$

偏离达到 7σ

- CDF Collaboration, Aaltonen T, Amerio S, et al. High-precision measurement of the W boson mass with the CDF II detector[J]. Science, 2022, 376(6589): 170-176.

◆ 诸多新物理模型被引入解决该问题，引入各类新的粒子；

◆ 其中一种是对Higgs区域的拓展，如引入额外的 $SU(2)_L$ 单态、双重态和三重态；

e.g

- K. Sakurai, F. Takahashi and W. Yin, PLB 2022.
- Y. Z. Fan, T. P. Tang, Y. L. S.Tsai and L. Wu, PRL 2022.
- H. Song, W. Su and M. Zhang, JHEP 2022.
- C. T. Lu, L. Wu, Y. Wu and B. Zhu, PRD 2022.
- K. S. Babu, S. Jana and V. P. K., PRL 2022.
- H. Song, X. Wan and J. H. Yu, [arXiv:2211.01543 [hep-ph]].
- X. K. Du, Z. Li, F. Wang and Y. K. Zhang, EPJC 2023.
- …… …… ……
- Y. Cheng, X. G. He, Z. L. Huang and M. W. Li, PLB 2022.
- N. D. Barrie, C. Han and H. Murayama, JHEP 2022.
- E. Ma, PLB 2022.
- X. F. Han, et al., CPC 2022.
- W. Abdallah, R. Gandhi and S. Roy, PLB 2022.
- J. Ellis, K. Mimasu and F. Zampedri, [arXiv:2304.06663 [hep-ph]].
- A. Addazi, et al., EPJC 2023.
- …… …… ……

本工作

- Jiajun Wu, Da Huang, and Chao-Qiang Geng, *CPC* 2023.
- Jiajun Wu, Chao-Qiang Geng, and Da Huang, [arXiv:2307.12105 [hep-ph]].

➢ 综合对比$SU(2)_L$不同表示在解决W质量问题的特点，以及向更高维情况的拓展；

➢ 分别考虑树图阶和一圈层面两种情况对W质量的修正。

模型：

标记 $SU(2)_L$ 标量多重态：
$$\Phi_{JY} = \begin{pmatrix} \cdots \\ \Phi_I^Q \\ \cdots \end{pmatrix}$$

势能项：
$$\begin{aligned} V\left(H, \Phi_{JY}\right) = & -\mu_H^2 H^\dagger H + \lambda_H \left(H^\dagger H\right)^2 + \mu_{\Phi_{JY}}^2 \Phi_{JY}^\dagger \Phi_{JY} + \lambda_1 \left(\Phi_{JY}^\dagger \Phi_{JY}\right)^2 \\ & + \lambda_2 \left(\Phi_{JY}^\dagger T_\Phi^a \Phi_{JY}\right)^2 + \lambda_3 \left(\Phi_{JY}^\dagger \Phi_{JY}\right)\left(H^\dagger H\right) \\ & + \lambda_4 \left(\Phi_{JY}^\dagger T_{\Phi_{JY}}^a \Phi_{JY}\right)\left(H^\dagger T_H^a H\right) + \lambda_5 \left(\Phi_{JY}^\dagger T_{\Phi_{JY}}^a T_{\Phi_{JY}}^b \Phi_{JY}\right)^2 \end{aligned}$$

通用势能项，具有 Z_2 对称性

非独立项：
$$O_6 + O_7 \equiv \lambda_6 \left(H^\dagger \Phi_{JY}\right)\left(\Phi_{JY}^\dagger H\right) + \lambda_7 \left|\tilde{H} \Phi_{JY}\right|^2$$

仅考虑树图修正：引入额外标量场的真空期望值

Y=0的奇数维多重态（J≥1）：

$$D_\mu = \partial_\mu + ieQA_\mu + i\frac{g}{c_W}\left(T_3 - Qs_W^2\right)Z_\mu + ig\left(W_\mu^+ T_+ + W_\mu^- T_-\right)$$

Y=0时该项消失

$$D^\mu H^\dagger D_\mu H + D^\mu \Phi^\dagger D_\mu \Phi \supset -\left(\frac{1}{4}g^2 v_H^2 + \frac{1}{2}k(1+k)v_\Phi^2\right)W_\mu^+ W^{-\mu} - \frac{1}{8}(g^2+g'^2)v_H^2 Z_\mu Z^\mu$$

$$\rightarrow \quad m_W = \frac{1}{2}g\sqrt{v_H^2 + 2k(1+k)v_\Phi^2}$$

$$m_Z = \frac{v_H}{2}\sqrt{g^2+g'^2}$$

ρ 参数的限制：

$$\rho_0 \equiv \frac{m_W^2}{m_Z^2 \hat{c}_Z^2 \hat{\rho}} = 1.00038 \pm 0.00020 \quad \rightarrow \quad \frac{(\Delta v)^2}{v_H^2} \in [-0.00002, 0.00078], \quad \text{at } 2\sigma \text{ C.L.}$$

$$\frac{(\Delta v)^2}{v_H^2} \equiv \frac{2k(1+k)v_\Phi^2}{v_H^2} = \left(\frac{m_W^{\rm CDF}}{m_W^{\rm SM}}\right)^2 - 1 \sim [0.00090, 0.00201], \quad \text{at } 2\sigma \text{ C.L.}$$

仅在树图阶无法解释CDF W质量超出

仅考虑一圈修正：

Oblique Parameters

一圈图层面的定义：

$$S \equiv \frac{4s_W^2 c_W^2}{\alpha}\left[A'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{c_W s_W} A'_{Z\gamma}(0) - A'_{\gamma\gamma}(0)\right],$$

$$T \equiv \frac{1}{\alpha m_Z^2}\left[\frac{A_{WW}(0)}{c_W^2} - A_{ZZ}(0)\right],$$

$$U \equiv \frac{4s_W^2}{\alpha}\left[A'_{WW}(0) - \frac{c_W}{s_W} A'_{Z\gamma}(0) - A'_{\gamma\gamma}(0)\right] - S,$$

W 质量的一圈修正可用三个参数表示：

$$M_W = M_{W,SM}\left(1 - \frac{\alpha\left(M_Z^2\right)}{4\left(c_W^2 - s_W^2\right)}\left(S - 2c_W^2 T\right) + \frac{\alpha\left(M_Z^2\right)}{8s_W^2} U\right)$$

电弱规范场的真空极化：

$$\Pi^{\mu\nu}_{VV'}(q) = g^{\mu\nu} A_{VV'}\left(q^2\right) + q^\mu q^\nu B_{VV'}\left(q^2\right)$$

部分动能项：

$$
\begin{aligned}
\mathcal{L}_{W\Phi} = & \sum_{I=-J}^{J} \partial_\mu \left(\Phi_I^Q\right)^* \partial^\mu \Phi_I^Q \\
& + \sum_{I=-J}^{J} igW_\mu^+ \left[N_I \Phi_{I-1}^Q \partial_\mu \left(\Phi_I^Q\right)^* - N_{I+1}\left(\Phi_{I+1}^Q\right)^* \partial^\mu \Phi_I^Q\right] \\
& + \sum_{I=-J}^{J} igW_\mu^- \left[N_{I+1} \Phi_{I+1}^Q \partial_\mu \left(\Phi_I^Q\right)^* - N_I\left(\Phi_{I-1}^Q\right)^* \partial_\mu \Phi_I^Q\right] \\
& + \sum_{I=-J}^{J} g^2 N_{I+1}^2 W_\mu^+ W_\mu^- \left(\Phi_{I+1}^Q\right)^* \Phi_{I+1}^Q \\
& + \sum_{I=-J}^{J} g^2 N_I^2 W_\mu^+ W_\mu^- \left(\Phi_{I-1}^Q\right)^* \Phi_{I-1}^Q ,
\end{aligned}
$$

$$
N_I = \sqrt{(J+I)(J-I+1)/2}
$$

$$
\begin{aligned}
\mathcal{L}_{Z\Phi} = & \sum_{I=-J}^{J} \partial_\mu \left(\Phi_I^Q\right)^* \partial^\mu \Phi_I^Q \\
& + \sum_{I=-J}^{J} i\frac{g}{c_W}\left(I - Qs_W^2\right) Z_\mu \left[\partial_\mu \left(\Phi_I^Q\right)^* \Phi_I^Q - \partial^\mu \Phi_I^Q \left(\Phi_I^Q\right)^*\right] \\
& + \sum_{I=-J}^{J} \frac{g^2}{c_W^2}\left(I - Qs_W^2\right)^2 Z_\mu Z^\mu \Phi_I^Q \left(\Phi_I^Q\right)^* ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{A\Phi} = & \sum_{I=-J}^{J} \partial_\mu \left(\Phi_I^Q\right)^* \partial^\mu \Phi_I^Q \\
& + \sum_{I=-J}^{J} ieQA_\mu \left[\Phi_I^Q \partial_\mu \left(\Phi_I^Q\right)^* - \left(\Phi_I^Q\right)^* \partial_\mu \Phi_I^Q\right] .
\end{aligned}
$$

费曼规则

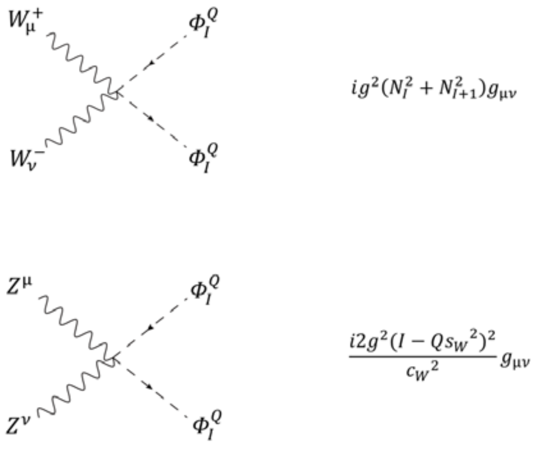


图 A.2　标量粒子的四点电弱规范耦合。

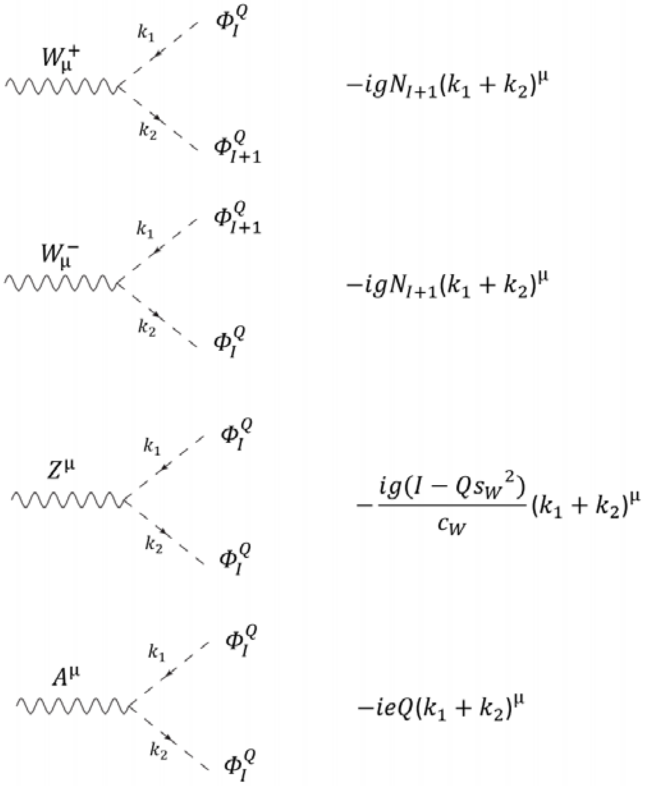
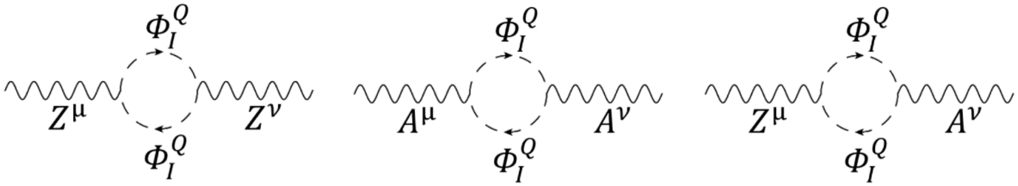
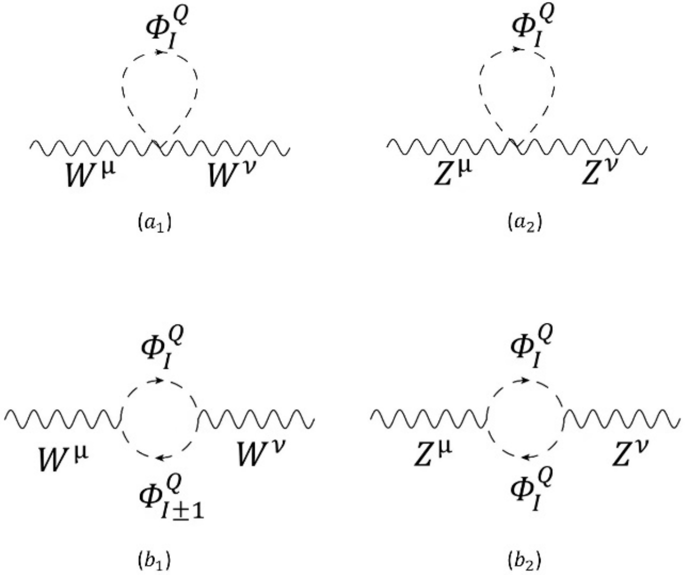


图 A.1　标量粒子的三点电弱规范耦合。



贡献到S的一圈图



贡献到T的一圈图

S, *T* 为主导，多数情况 *U* 被压低

推导出通用表达式：

$$S_{\Phi_{JY}} = -\frac{Y}{3\pi}\sum_{I=-J}^{J} I \ln m^2_{\Phi_I^Q}$$

$$T_{\Phi_{JY}} = \frac{1}{4\pi s_w^2 m_W^2}\sum_{I=-J}^{J-1} N_{I+1}^2 F\left(m^2_{\Phi_I^Q}, m^2_{\Phi_{I+1}^Q}\right)$$

$$F(A,B) \equiv \begin{cases} \frac{A+B}{2} - \frac{AB}{A-B}\ln\frac{A}{B} & A \neq B, \\ 0 & A = B. \end{cases}$$

组分间质量差是关键！

势能项：

$$\begin{aligned} V(H, \Phi_{JY}) = & -\mu_H^2 H^\dagger H + \lambda_H \left(H^\dagger H\right)^2 + \mu^2_{\Phi_{JY}} \Phi^\dagger_{JY}\Phi_{JY} + \lambda_1 \left(\Phi^\dagger_{JY}\Phi_{JY}\right)^2 \\ & + \lambda_2 \left(\Phi^\dagger_{JY} T^a_\Phi \Phi_{JY}\right)^2 + \lambda_3 \left(\Phi^\dagger_{JY}\Phi_{JY}\right)\left(H^\dagger H\right) \\ & + \lambda_4 \left(\Phi^\dagger_{JY} T^a_{\Phi_{JY}} \Phi_{JY}\right)\left(H^\dagger T^a_H H\right) + \lambda_5 \left(\Phi^\dagger_{JY} T^a_{\Phi_{JY}} T^b_{\Phi_{JY}} \Phi_{JY}\right)^2 \end{aligned}$$

唯象学研究

1. SU(2)实表示： Y=0

$$
\begin{aligned}
O_4 =&\lambda_4\left(\Phi^\dagger_{JY}T^a_{\Phi_{JY}}\Phi_{JY}\right)\left(H^\dagger T^a_H H\right)\\
=&\lambda_4\left(\Phi^\dagger_{JY}T^+_{\Phi_{JY}}\Phi_{JY}\right)\left(H^\dagger T^-_H H\right)+\lambda_4\left(\Phi^\dagger_{JY}T^-_{\Phi_{JY}}\Phi_{JY}\right)\left(H^\dagger T^+_H H\right)\\
&+\lambda_4\left(\Phi^\dagger_{JY}T^3_{\Phi_{JY}}\Phi_{JY}\right)\left(H^\dagger T^3_H H\right)\\
=&-\frac{\lambda_4}{4}(h+v)^2\sum_{I=-J}^{J} I\Phi^Q_I\left(\Phi^Q_I\right)^*\\
\supset&-\frac{\lambda_4}{4}v^2\sum_{I=-J}^{J} I\Phi^Q_I\left(\Phi^Q_I\right)^*.
\end{aligned}
$$

$\longrightarrow$

$$
\begin{cases}
-\frac{\lambda_4}{4}v^2\left[I\Phi^Q_I\left(\Phi^Q_I\right)^*-I\Phi^Q_{-I}\left(\Phi^Q_{-I}\right)^*\right]=0 & I>0,\\
-\frac{\lambda_4}{4}v^2 I\Phi^Q_I\left(\Phi^Q_I\right)^*=0 & I=0.
\end{cases}
$$

无法产生质量差，因此无法解决W质量问题

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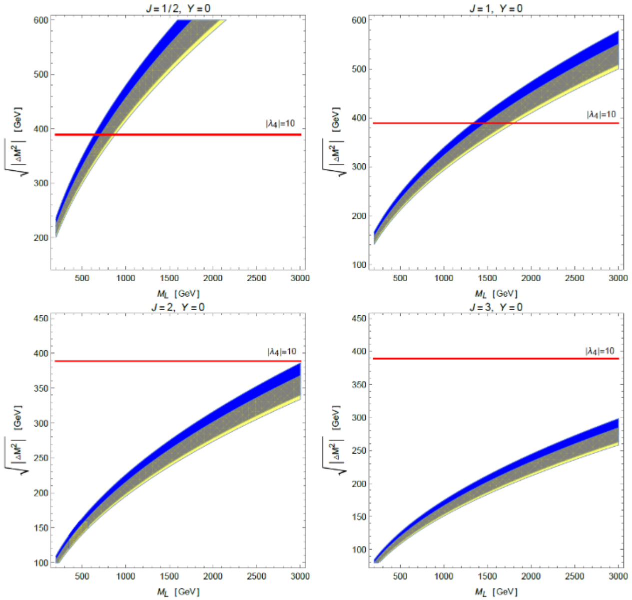
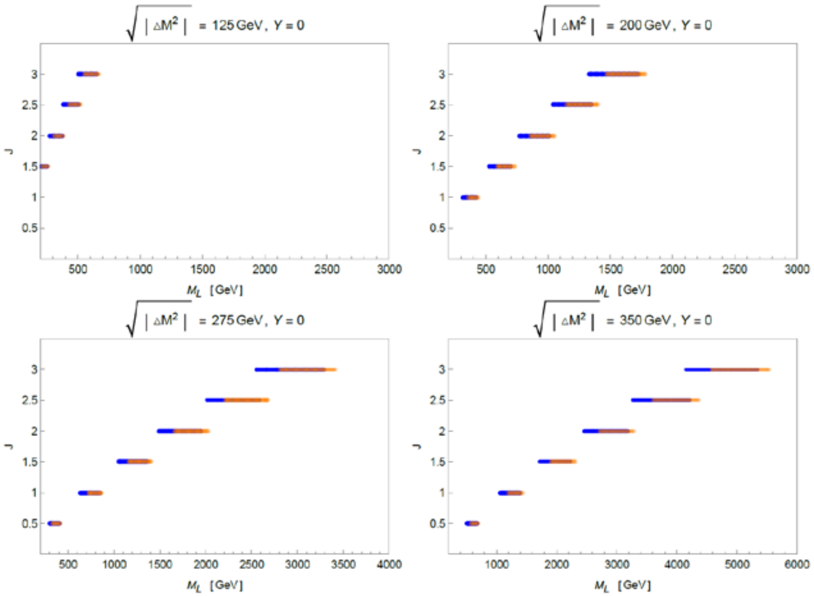
2. SU(2)复表示：

2.1 Y=0：

S=U=0

$$S_{\Phi_{JY}} = -\frac{Y}{3\pi}\sum_{I=-J}^{J} I \ln m^2_{\Phi_I^Q}$$





无法与暗物质模型兼容

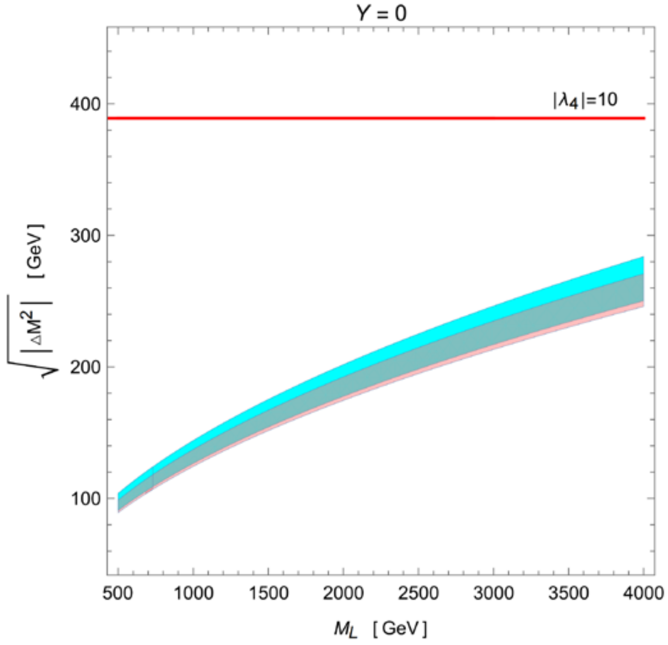
EW Global Fits: Y. Cheng, X. G. He, F. Huang, J. Sun and Z. P. Xing, [arXiv:2208.06760 [hep-ph]].

Jiajun Wu, Chao-Qiang Geng, and Da Huang, [arXiv:2307.12105 [hep-ph]].

2. SU(2)复表示：

2.1 J=4, Y=0： S=U=0

$$S_{\Phi_{JY}} = -\frac{Y}{3\pi}\sum_{I=-J}^{J} I \ln m^2_{\Phi_I^Q}$$



EW Global Fits: Y. Cheng, X. G. He, F. Huang, J. Sun and Z. P. Xing, [arXiv:2208.06760 [hep-ph]].

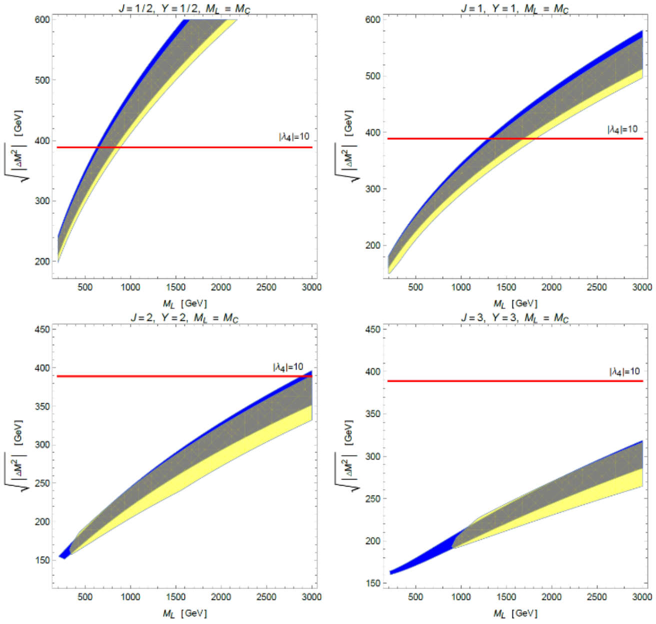
Jiajun Wu, Da Huang, and Chao-Qiang Geng, *Chinese Physics C* 2023, 47(6):063103.

2. SU(2)复表示：

2.2 Y=J： U=0

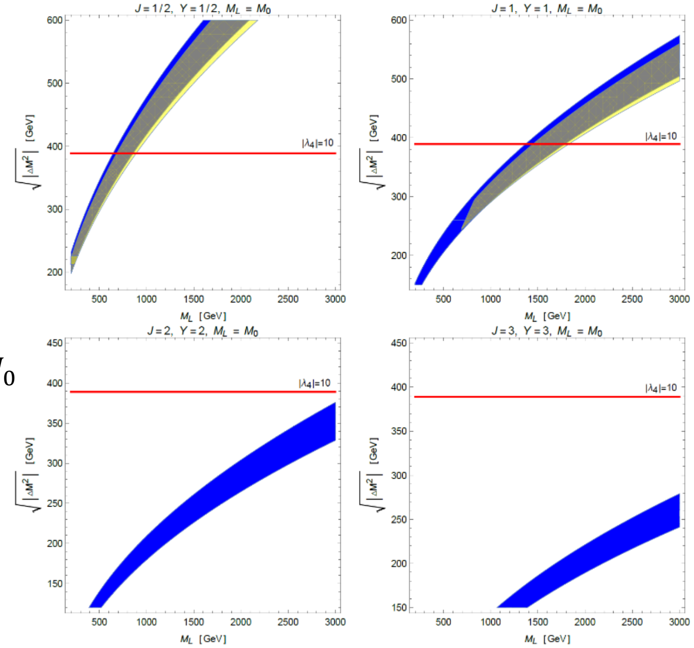
Type-A:

$M_L = M_C$



Type-B:

$M_L = M_0$



EW Global Fits: P. Asadi, C. Cesarotti, K. Fraser, S. Homiller and A. Parikh, [arXiv:2204.05283 [hep-ph]].

Jiajun Wu, Chao-Qiang Geng, and Da Huang, [arXiv:2307.12105 [hep-ph]].

2. SU(2)复表示：

2.2 Y=J：

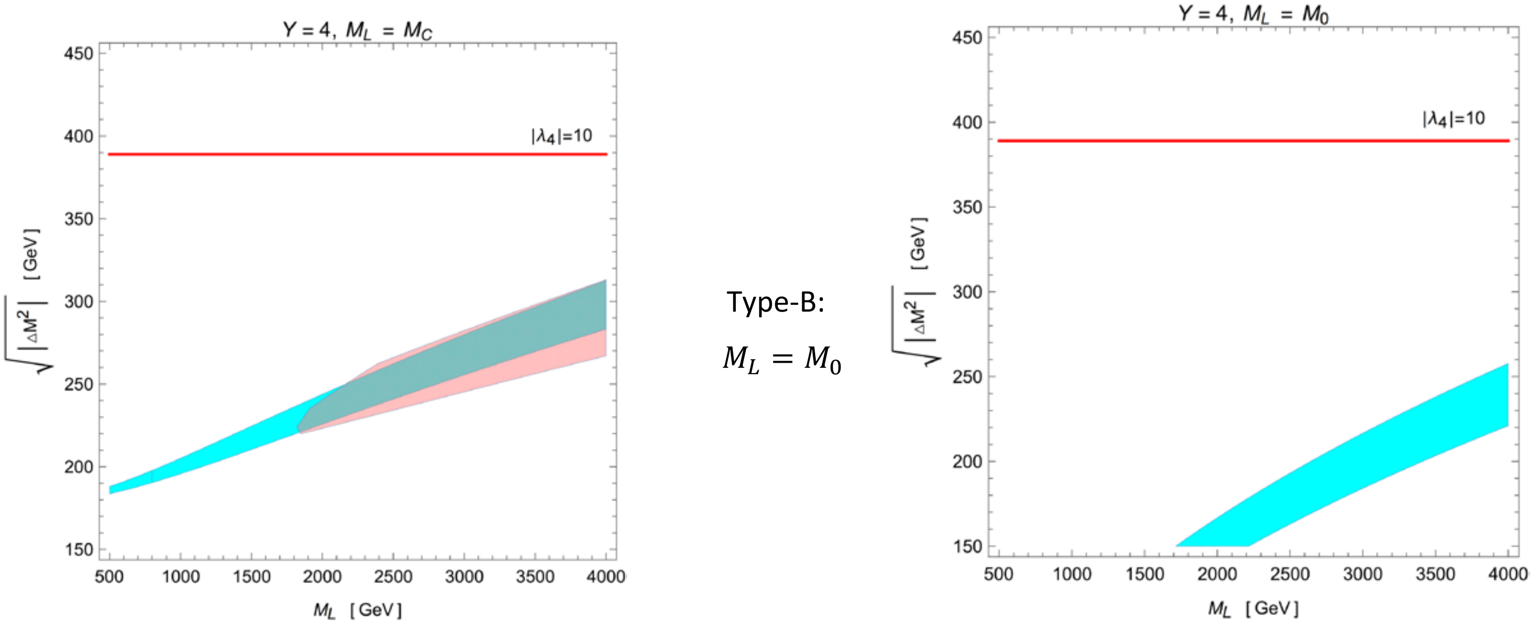
U=0

Type-A:

$M_L = M_C$

Type-B:

$M_L = M_0$



EW Global Fits: P. Asadi, C. Cesarotti, K. Fraser, S. Homiller and A. Parikh, [arXiv:2204.05283 [hep-ph]].

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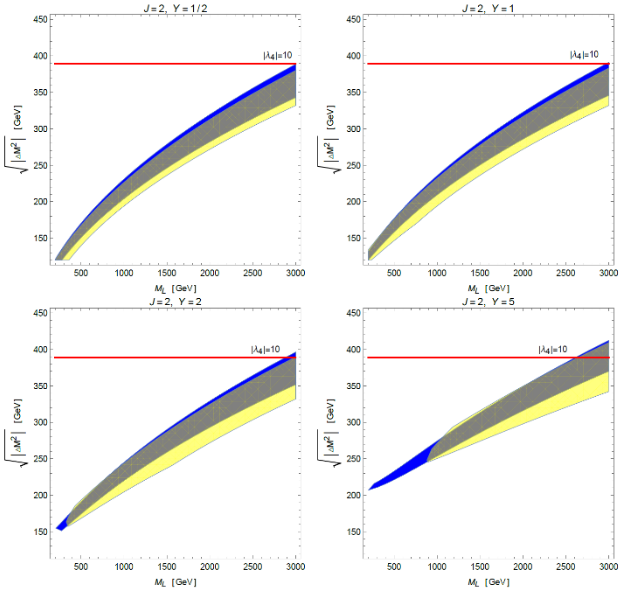
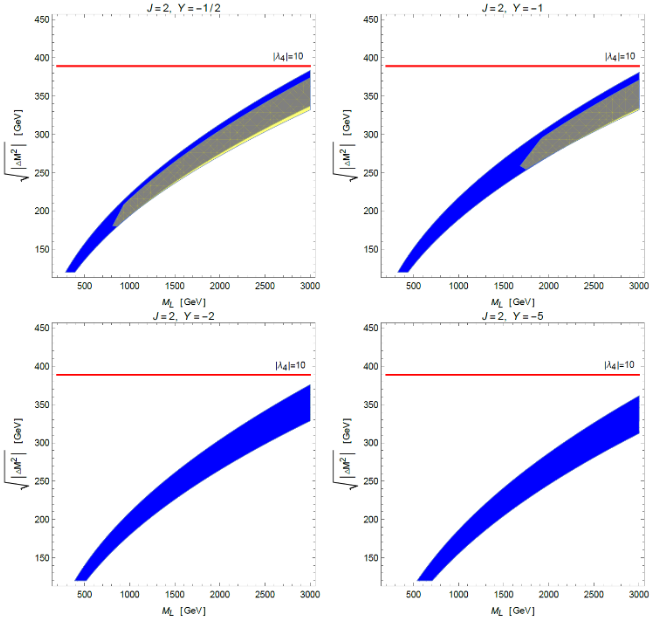
2. SU(2)复表示：

2.3 Y取任意值：

Y取正值:



Y取负值:



EW Global Fits: P. Asadi, C. Cesarotti, K. Fraser, S. Homiller and A. Parikh, [arXiv:2204.05283 [hep-ph]].

分析:

主要受S取值范围的限制　　$-0.024 \leq S \leq 0.364$

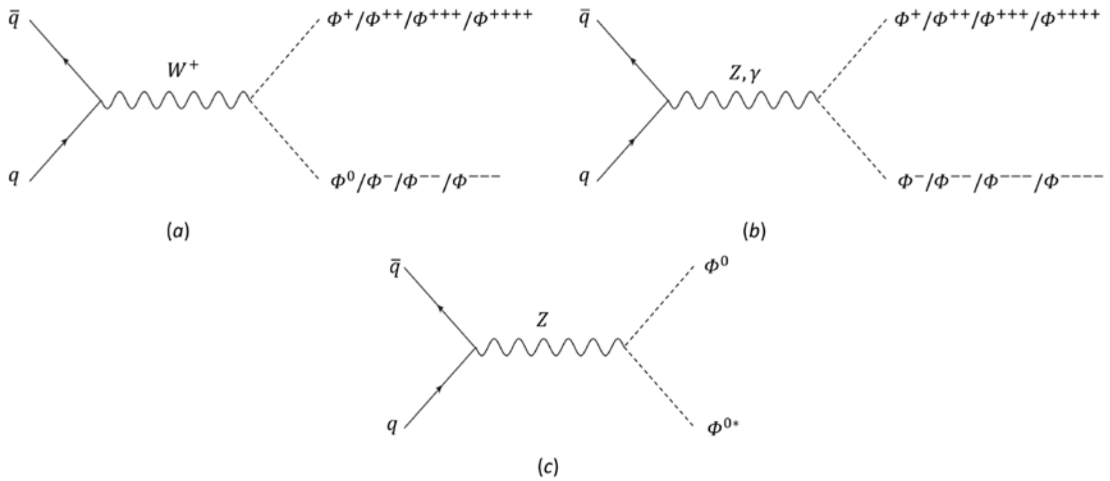
以J=2为例

$$
\begin{aligned}
S_{\Phi_{2Y}} &= -\frac{Y}{3\pi}\left(2\ln m^2_{\Phi^Q_2} + \ln m^2_{\Phi^Q_1} - \ln m^2_{\Phi^Q_{-1}} - 2\ln m^2_{\Phi^Q_{-2}}\right) \\
&= -\frac{Y}{3\pi}\left[2\ln\left(\frac{m^2_{\Phi^Q_{-2}} - \lambda_4 v^2}{m^2_{\Phi^Q_{-2}}}\right) + \ln\left(\frac{m^2_{\Phi^Q_{-1}} - \frac{\lambda_4 v^2}{2}}{m^2_{\Phi^Q_{-1}}}\right)\right] \\
&\cong \lambda_4 Y\left(\frac{2v^2}{3\pi m^2_{\Phi^Q_{-2}}} + \frac{v^2}{6\pi m^2_{\Phi^Q_{-1}}}\right).
\end{aligned}
$$

假设 $\lambda_4 v^2 \ll m^2_\Phi$

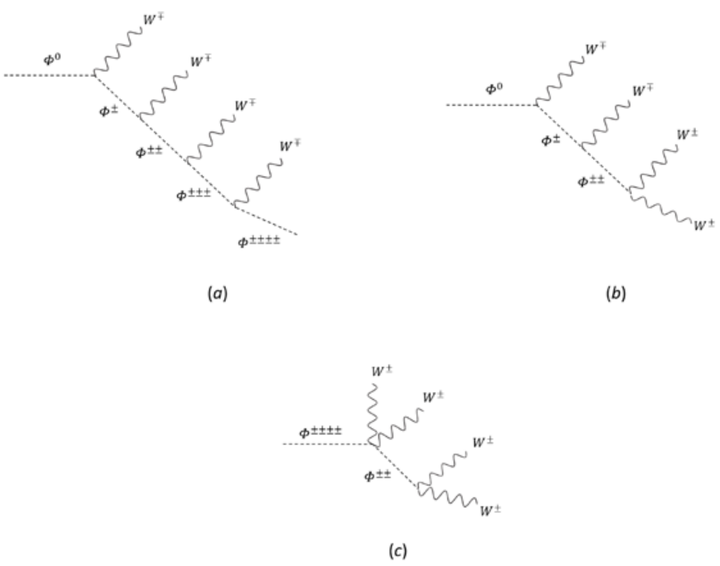
对撞机过程分析，以 J=Y=2 的情况为例：

产生过程：

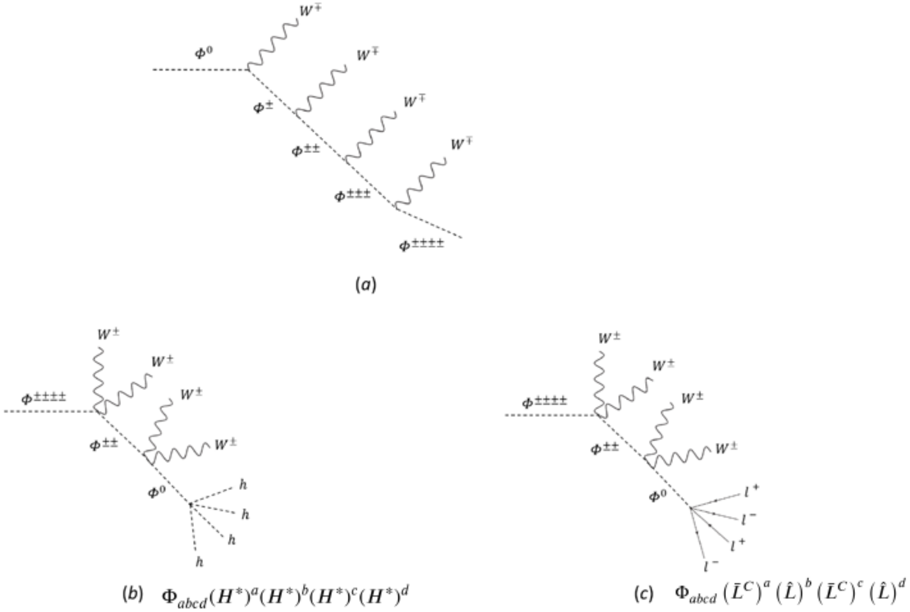


Drell-Yan 过程

对撞机信号： multi-W or multi-higgs or multi-leptons



引入一个极小的真空期望值



引入高量纲算符

总结：

- ***W* 玻色子质量反常问题具有重要的物理意义，如果后续实验证实了CDF合作组结果的准确性，那么这将成为一个显著的新物理信号。**

  - 在树图阶：受 ρ 参数限制，无法仅靠树图修正解释CDF测得的W质量超出。
  - 在一圈层面上：
    - SU(2)实表示组分间无法产生质量差，故无法解决W质量问题。
    - SU(2)复表示各组分间可以产生质量差，存在解决W质量问题的参数空间；对于Y=0的情况，参数空间比较大，但是无法同时兼顾W质量问题和暗物质；对于Y=J的情况，随着维数增大参数空间减小，尤其是$M_L = M_0$的情况。
    - Y的取值会产生影响，随着Y绝对值增大存活的参数空间会缩小，尤其是当Y取负值的时候。

THANKS!

非独立势能项

$$
\begin{aligned}
O_6 &= \lambda_6 \left(H^\dagger \Phi_{JY}\right)\left(\Phi_{JY}^\dagger H\right) \\
&= \lambda_6 \sum_{I=-J}^{J} C_{2J}^{J-I} \left[\left(H^*\right)^a \left(\Phi_I\right)_{abc\ldots m}\right]\left[\left(\Phi_I^*\right)^{abc\ldots m} H_a\right] \\
&= \lambda_6 \sum_{I=-J}^{J} C_{2J-1}^{J-I-1} \left[\left(H^*\right)^0 \left(\Phi_I\right)_0 + \left(H^*\right)^1 \left(\Phi_I\right)_1\right]_{bc\ldots m} \\
&\quad \times \left[\left(\Phi_I^*\right)^0 H_0 + \left(\Phi_I^*\right)^1 H_1\right]^{bc\ldots m} \\
&= \lambda_6 \frac{(v+h)^2}{2} \sum_{I=-J}^{J} C_{2J-1}^{J-I-1} \left(\Phi_I\right)_{0bc\ldots m} \left(\Phi_I^*\right)^{0bc\ldots m} \\
&= \lambda_6 \frac{(v+h)^2}{2} \frac{C_{2J-1}^{J-I-1}}{C_{2J}^{J-I}} \Phi_I^Q \left(\Phi_I^Q\right)^* \\
&= \lambda_6 \frac{(v+h)^2}{4} \sum_{I=-J}^{J} \Phi_I^Q \left(\Phi_I^Q\right)^* - \frac{\lambda_6}{J} \frac{(v+h)^2}{4} \sum_{I=-J}^{J} I \Phi_I^Q \left(\Phi_I^Q\right)^*
\end{aligned}
$$

$$
\left(\Phi_I\right)_{abc\ldots m} = \frac{1}{\sqrt{C_{2J}^{J-I}}} \Phi_I^Q
$$