Verification of Equivalence of Operator Bases: Unitarity Analysis

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第十二届新物理研讨会，青岛

The Equivalence Theorem

The *equivalence theorem* [8] [28–34] is a consequence of the re-parameterization freedom in quantum field theories. Its main application in the context of effective field theories is the following. Suppose that, to lowest order in $1/\Lambda$, two operators $\mathcal{O}$ and $\mathcal{O}'$ obey

$$a\mathcal{O} - \mathcal{O}' = \mathcal{A}\frac{\delta S_{\texttt{light}}}{\delta\phi}\,, \tag{23}$$

where a is a constant, $\mathcal{A}$ is a local operator composed of light fields and their derivatives, and $S_{\texttt{light}}$ the action of the light fields (denoted collectively by ϕ). Then the S matrix will depend on $w + aw'$ (where w, w' are the Wilson coefficients of $\mathcal{O}$, $\mathcal{O}'$, respectively), and not on these Wilson coefficients separately.

J. Wudka, 2104.01788

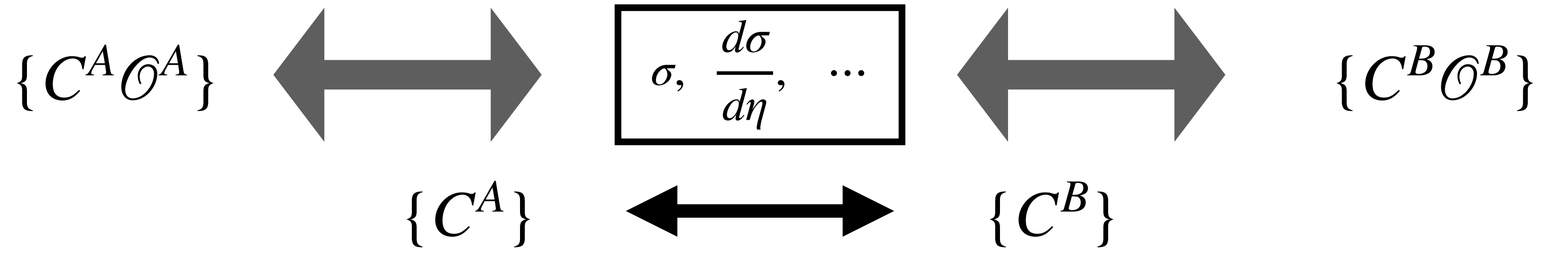
$$\{\cdots,\ \mathcal{O}'_W,\ \cdots\} \qquad\qquad \{\cdots,\ \mathcal{O}'_W,\ \mathcal{O}^{(3)}_{\varphi l}\ \cdots\}$$

$$2\mathcal{O}'_W - \frac{1}{4}g^2\mathcal{O}_{\varphi W} - \frac{1}{4}g'g\mathcal{O}_{\varphi WB} + \frac{3}{4}g^2\mathcal{O}_{\varphi\Box} = -\frac{g^2}{4}\left[\mathcal{O}^{(3)}_{\varphi l} + \mathcal{O}^{(3)}_{\varphi q}\right]$$

J. Elias-Miro, 1308.1879

How the equivalence theorem interacts with data?

- Two operators sets/bases can give rise to the equivalent description on the data?



$$W^-W^+ \to W^-W^+ : \qquad \mathcal{O}_{\varphi W} = \varphi^\dagger \varphi W^I_{\mu\nu} W^{I\mu\nu} \qquad \mathcal{O}^{(3)}_{\varphi l} = (\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$$

$$2\mathcal{O}'_W - \frac{1}{4}g^2\mathcal{O}_{\varphi W} - \frac{1}{4}g'g\mathcal{O}_{\varphi WB} + \frac{3}{4}g^2\mathcal{O}_{\varphi\Box} = -\frac{g^2}{4}\left[\mathcal{O}^{(3)}_{\varphi l} + \mathcal{O}^{(3)}_{\varphi q}\right]$$

$$\{\mathcal{O}_{\varphi W},\ \cdots\} \qquad \{\mathcal{O}^{(3)}_{\varphi l},\ \cdots\}$$

- Multiple processes with high precision measurements

Unitarity Bounds: Coupled Channel Analysis

- Unitarity bounds on dim-6 operators can be thought as “theoretical data”

$$1 = S^\dagger S = (1 + iT)^\dagger (1 + iT) \Rightarrow -i(T - T^\dagger) = T^\dagger T$$

$$\langle i \,|\, T \,|\, f \rangle \sim \sum_J (2J + 1) d^J(\theta) a^J \Rightarrow |\, Re(a^J) \,| \leq \frac{1}{2}$$

$$a^J \propto \frac{s}{\Lambda^2} \sum C_i + o(s) \Rightarrow \frac{s}{\Lambda^2} |\sum C_i| \leq \#$$

- Coupled channels analysis

$$ff \to VV, \quad VV \to VV \quad (f = u, d, e, \nu_e; V = W^\pm, Z, A, h)$$

T. Corbett, 1411.5026, 1705.09294

Operator Bases/Sets

Grzadkowski, Iskrzyński, Misiak & Rosiek, 10’

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
~~Q_G~~	~~$f^{ABC}G^{A\nu}_{\mu}G^{B\rho}_{\nu}G^{C\mu}_{\rho}$~~	~~Q_φ~~	~~$(\varphi^\dagger\varphi)^3$~~	~~$Q_{e\varphi}$~~	~~$(\varphi^\dagger\varphi)(\bar{l}_p e_r\varphi)$~~
~~$Q_{\widetilde{G}}$~~	~~$f^{ABC}\widetilde{G}^{A\nu}_{\mu}G^{B\rho}_{\nu}G^{C\mu}_{\rho}$~~	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	~~$Q_{u\varphi}$~~	~~$(\varphi^\dagger\varphi)(\bar{q}_p u_r\widetilde{\varphi})$~~
Q_W	$\varepsilon^{IJK}W^{I\nu}_{\mu}W^{J\rho}_{\nu}W^{K\mu}_{\rho}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star\left(\varphi^\dagger D_\mu\varphi\right)$	~~$Q_{d\varphi}$~~	~~$(\varphi^\dagger\varphi)(\bar{q}_p d_r\varphi)$~~
~~$Q_{\widetilde{W}}$~~	~~$\varepsilon^{IJK}\widetilde{W}^{I\nu}_{\mu}W^{J\rho}_{\nu}W^{K\mu}_{\rho}$~~				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
~~$Q_{\varphi G}$~~	~~$\varphi^\dagger\varphi G^A_{\mu\nu}G^{A\mu\nu}$~~	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}$	$Q^{(1)}_{\varphi l}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{l}_p\gamma^\mu l_r)$
~~$Q_{\varphi\widetilde{G}}$~~	~~$\varphi^\dagger\varphi\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$~~	Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q^{(3)}_{\varphi l}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi W^I_{\mu\nu}W^{I\mu\nu}$	~~Q_{uG}~~	~~$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{\varphi}G^A_{\mu\nu}$~~	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{e}_p\gamma^\mu e_r)$
~~$Q_{\varphi\widetilde{W}}$~~	~~$\varphi^\dagger\varphi\widetilde{W}^I_{\mu\nu}W^{I\mu\nu}$~~	Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{\varphi}W^I_{\mu\nu}$	$Q^{(1)}_{\varphi q}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{q}_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{\varphi}B_{\mu\nu}$	$Q^{(3)}_{\varphi q}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
~~$Q_{\varphi\widetilde{B}}$~~	~~$\varphi^\dagger\varphi\widetilde{B}_{\mu\nu}B^{\mu\nu}$~~	~~Q_{dG}~~	~~$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)\varphi G^A_{\mu\nu}$~~	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{u}_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi W^I_{\mu\nu}B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{d}_p\gamma^\mu d_r)$
~~$Q_{\varphi\widetilde{W}B}$~~	~~$\varphi^\dagger\tau^I\varphi\widetilde{W}^I_{\mu\nu}B^{\mu\nu}$~~	Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu\varphi)(\bar{u}_p\gamma^\mu d_r)$

no CP-odd operators included yet

no 4-f operators: rarely mix with other operators

$$(\mathcal{O}'_W, \mathcal{O}_B, \mathcal{O}_{2W}, \mathcal{O}_{2B}, \mathcal{O}_W^{\rm SILH}, \mathcal{O}_B^{\rm SILH})$$
$$\leftrightarrow (\mathcal{O}_{\varphi W}, \mathcal{O}_{\varphi WB}, \mathcal{O}_{\varphi\Box}, \mathcal{O}_{\varphi D}, \mathcal{O}^{(1)}_{\varphi l}, \mathcal{O}^{(3)}_{\varphi l}),$$

$$\mathcal{O}'_W \equiv \left(D_\mu\varphi\right)^\dagger\left(ig\frac{\tau^I}{2}W^{I;\mu\nu}\right)\left(D_\nu\varphi\right),$$

$$\mathcal{O}_B \equiv \left(D_\mu\varphi\right)^\dagger\left(i\frac{g'}{2}B^{\mu\nu}\right)\left(D_\nu\varphi\right),$$

$$\mathcal{O}_{2W} \equiv -\frac{1}{2}(D^\mu W^I_{\mu\nu})^2 \sim \mathcal{O}_{DW},$$

$$\mathcal{O}_{2B} \equiv -\frac{1}{2}(\partial^\mu B_{\mu\nu})^2 \sim \mathcal{O}_{DB}$$

$$\mathcal{O}_W^{\rm SILH} \equiv \frac{ig}{2}(\varphi^\dagger\tau^I\overleftrightarrow{D^\mu}\varphi)(D^\nu W^I_{\mu\nu}),$$

$$\mathcal{O}_B^{\rm SILH} \equiv \frac{ig'}{2}(\varphi^\dagger\overleftrightarrow{D^\mu}\varphi)(\partial^\nu B_{\mu\nu})\,.$$

$$(\mathcal{O}'_W, \mathcal{O}_B, \mathcal{O}_{2W}, \mathcal{O}_{2B}, \mathcal{O}_W^{\rm SILH}, \mathcal{O}_B^{\rm SILH})$$
$$\leftrightarrow (\mathcal{O}_{\varphi W}, \mathcal{O}_{\varphi WB}, \mathcal{O}_{\varphi\Box}, \mathcal{O}_{\varphi D}, \mathcal{O}_{\varphi l}^{(1)}, \mathcal{O}_{\varphi l}^{(3)}),$$

$$2\mathcal{O}'_W - \frac{1}{4}g^2\mathcal{O}_{\varphi W} - \frac{1}{4}g'g\mathcal{O}_{\varphi WB} + \frac{3}{4}g^2\mathcal{O}_{\varphi\Box} = -\frac{g^2}{4}\left[\mathcal{O}_{\varphi l}^{(3)} + \mathcal{O}_{\varphi q}^{(3)}\right]$$

$$2\mathcal{O}_B - \frac{1}{4}g'^2\mathcal{O}_{\varphi B} - \frac{1}{4}g'g\mathcal{O}_{\varphi WB} + g'^2\left[\mathcal{O}_{\varphi D} + \frac{1}{4}\mathcal{O}_{\varphi\Box}\right] = -\frac{g'^2}{2}\left[-\frac{1}{2}\mathcal{O}_{\varphi l}^{(1)} + \frac{1}{6}\mathcal{O}_{\varphi q}^{(1)} - \mathcal{O}_{\varphi e} + \frac{2}{3}\mathcal{O}_{\varphi u} - \frac{1}{3}\mathcal{O}_{\varphi d}\right]$$

$$\mathcal{O}_{2W} + \frac{3g^2}{8}\mathcal{O}_{\varphi\Box} = -\frac{g^2}{4}\left[\mathcal{O}_{\varphi l}^{(3)} + \mathcal{O}_{\varphi q}^{(3)}\right]$$

$$\mathcal{O}_{2B} + \frac{g'^2}{2}\left[\frac{1}{4}\mathcal{O}_{\varphi\Box} + \mathcal{O}_{\varphi D}\right] = -\frac{g'^2}{2}\left[-\frac{1}{2}\mathcal{O}_{\varphi l}^{(1)} + \frac{1}{6}\mathcal{O}_{\varphi q}^{(1)} - \mathcal{O}_{\varphi e} + \frac{2}{3}\mathcal{O}_{\varphi u} - \frac{1}{3}\mathcal{O}_{\varphi d}\right]$$

$$\mathcal{O}_W^{\rm SILH} + \frac{3g^2}{4}\mathcal{O}_{\varphi\Box} = -\frac{g^2}{4}\left[\mathcal{O}_{\varphi l}^{(3)} + \mathcal{O}_{\varphi q}^{(3)}\right]$$

$$\mathcal{O}_B^{\rm SILH} + g'^2\left[\frac{1}{4}\mathcal{O}_{\varphi\Box} + \mathcal{O}_{\varphi D}\right] = -\frac{g'^2}{2}\left[-\frac{1}{2}\mathcal{O}_{\varphi l}^{(1)} + \frac{1}{6}\mathcal{O}_{\varphi q}^{(1)} - \mathcal{O}_{\varphi e} + \frac{2}{3}\mathcal{O}_{\varphi u} - \frac{1}{3}\mathcal{O}_{\varphi d}\right]$$

Transformation Rules

$$\{\mathcal{O}_1^A, \mathcal{O}_j, \mathcal{O}_j^{\rm EoM}\} \qquad \mathcal{O}_1^A \leftrightarrow \mathcal{O}_1^B \qquad \{\mathcal{O}_1^B, \mathcal{O}_j, \mathcal{O}_j^{\rm EoM}\}$$

$$a_1\mathcal{O}_1^A + b_1\mathcal{O}_1^B + \sum_j c_j\mathcal{O}_j^{\rm EoM} = 0$$

$M_j \equiv \langle f \,|\, O_j \,|\, i\rangle$

$$M = C_1^A M_1^A + \sum_j C_j M_j + \sum_j C_j^{\rm EoM} M_j^{\rm EoM} \qquad M = C_1^B M_1^B + \sum_j C_j M_j + \sum_j C_j^{\rm EoM} M_j^{\rm EoM}$$

$$a_1 M_1^A + b_1 M_1^B + \sum_j c_j M_j^{\rm EoM} = 0$$

$$M = C_1^A M_1^A + \sum_j C_j M_j + \sum_j C_j^{\rm EoM} M_j^{\rm EoM} \qquad M = -\frac{a_1}{b_1} C_1^B M_1^A + \sum_j C_j M_j + \sum_j (C_j^{\rm EoM} - \frac{c_j}{b_1} C_1^B) M_j^{\rm EoM}$$

$$C_1^A \to -\frac{a_1}{b_1} C_1^B,$$

$$C_j^{\rm EoM} \to C_j^{\rm EoM} - \frac{c_j}{b_1} C_1^B$$

Transformation Rules

$$\{\mathcal{O}_1^A, \mathcal{O}_j, \mathcal{O}_j^{\mathrm{EoM}}\} \qquad \mathcal{O}_1^A \leftrightarrow \mathcal{O}_1^B \qquad \{\mathcal{O}_1^B, \mathcal{O}_j, \mathcal{O}_j^{\mathrm{EoM}}\}$$

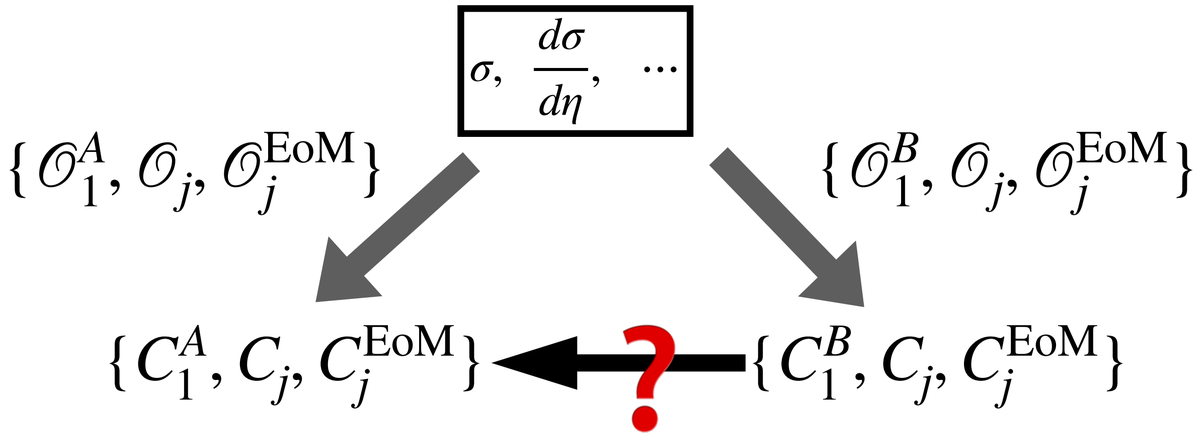
$$C_1^A \to -\frac{a_1}{b_1} C_1^B,$$

$$C_j^{\mathrm{EoM}} \to C_j^{\mathrm{EoM}} - \frac{c_j}{b_1} C_1^B$$

$$C_1^A \leftarrow -\frac{a_1}{b_1} C_1^B,$$

$$C_j^{\mathrm{EoM}} \leftarrow C_j^{\mathrm{EoM}} - \frac{c_j}{b_1} C_1^B$$

$$M_{i\to f}(\{\mathcal{O}_1^A, \mathcal{O}_j, \mathcal{O}_j^{\mathrm{EoM}}\}) \Rightarrow M_{i\to f}(\{\mathcal{O}_1^B, \mathcal{O}_j, \mathcal{O}_j^{\mathrm{EoM}}\})$$



Transformation Rules: WARSAW and SILH

$$\mathcal{C}_{\varphi W} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{g^2}{8}\mathcal{C}'_W,$$

$$\mathcal{C}_{\varphi B} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi B} + \frac{g'^2}{8}\mathcal{C}_B,$$

$$\mathcal{C}_{\varphi WB} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{gg'}{8}(\mathcal{C}'_W + \mathcal{C}_B),$$

$$\mathcal{C}_{\varphi D} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} -g'^2\mathcal{C}_B^{\text{SILH}} - \frac{g'^2}{2}(\mathcal{C}_B + \mathcal{C}_{2B}),$$

$$\mathcal{C}^{(1)}_{\varphi l} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{g'^2}{8}\mathcal{C}_B + \frac{g'^2}{4}(\mathcal{C}_B^{\text{SILH}} + \mathcal{C}_{2B}),$$

$$\mathcal{C}^{(3)}_{\varphi l} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} -\frac{g^2}{8}\mathcal{C}'_W - \frac{g^2}{4}(\mathcal{C}_W^{\text{SILH}} + \mathcal{C}_{2W}),$$

$$\mathcal{C}^{(1)}_{\varphi q} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}^{(1)}_{\varphi q} - \frac{g'^2}{24}\mathcal{C}_B - \frac{g'^2}{12}(\mathcal{C}_B^{\text{SILH}} + \mathcal{C}_{2B}),$$

$$\mathcal{C}^{(3)}_{\varphi q} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}^{(3)}_{\varphi q} - \frac{g^2}{8}\mathcal{C}'_W - \frac{g^2}{4}(\mathcal{C}_W^{\text{SILH}} + \mathcal{C}_{2W}),$$

$$\mathcal{C}_{\varphi\Box} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} -\frac{3g^2}{4}\mathcal{C}_W^{\text{SILH}} - \frac{3g^2}{8}(\mathcal{C}'_W + \mathcal{C}_{2W}) - \frac{g'^2}{4}\mathcal{C}_B^{\text{SILH}} - \frac{g'^2}{8}(\mathcal{C}_B + \mathcal{C}_{2B}),$$

$$\mathcal{C}_{\varphi e} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi e} + \frac{g'^2}{4}(\mathcal{C}_B + 2\mathcal{C}_{2B} + 2\mathcal{C}_B^{\text{SILH}}),$$

$$\mathcal{C}_{\varphi d} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi d} + \frac{g'^2}{12}(\mathcal{C}_B + 2\mathcal{C}_{2B} + 2\mathcal{C}_B^{\text{SILH}}),$$

$$\mathcal{C}_{\varphi u} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi u} - \frac{g'^2}{6}(\mathcal{C}_B + 2\mathcal{C}_{2B} + 2\mathcal{C}_B^{\text{SILH}}).$$

$$\mathcal{C}'_W \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{8}{g^2}\mathcal{C}_{\varphi W},$$

$$\mathcal{C}_{\varphi e} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi e} - 2\mathcal{C}^{(1)}_{\varphi l},$$

$$\mathcal{C}_{\varphi d} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi d} - \frac{2}{3}\mathcal{C}^{(1)}_{\varphi l},$$

$$\mathcal{C}_{\varphi u} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi u} + \frac{4}{3}\mathcal{C}^{(1)}_{\varphi l},$$

$$\mathcal{C}^{(1)}_{\varphi q} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}^{(1)}_{\varphi q} + \frac{1}{3}\mathcal{C}^{(1)}_{\varphi l},$$

$$\mathcal{C}^{(3)}_{\varphi q} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}^{(3)}_{\varphi q} - \mathcal{C}^{(3)}_{\varphi l},$$

$$\mathcal{C}_B \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{8}{gg'}\mathcal{C}_{\varphi WB} - \frac{8}{g^2}\mathcal{C}_{\varphi W}$$

$$\mathcal{C}_{2B} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{8}{g'^2}\mathcal{C}^{(1)}_{\varphi l} + \frac{2}{g'^2}\mathcal{C}_{\varphi D}$$

$$\mathcal{C}_{\varphi B} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi B} + \frac{g'^2}{g^2}\mathcal{C}_{\varphi W} - \frac{g'}{g}\mathcal{C}_{\varphi WB}$$

$$\mathcal{C}_{2W} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{8}{3g^2}\mathcal{C}_{\varphi\Box} - \frac{2}{3g^2}\mathcal{C}_{\varphi D} - \frac{8}{g^2}\mathcal{C}^{(3)}_{\varphi l}$$

$$\mathcal{C}_B^{\text{SILH}} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{4}{g^2}\mathcal{C}_{\varphi W} - \frac{4}{gg'}\mathcal{C}_{\varphi WB} - \frac{2}{g'^2}(\mathcal{C}_{\varphi D} + 2\mathcal{C}^{(1)}_{\varphi l})$$

$$\mathcal{C}_W^{\text{SILH}} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{2}{3g^2}(\mathcal{C}_{\varphi D} + 6\mathcal{C}^{(3)}_{\varphi l} - 6\mathcal{C}_{\varphi W} - 4\mathcal{C}_{\varphi\Box}).$$

Coupled Channels and Diagonalization

organized by Q (total electric charge of the process)
and J (total angular momentum)

$VV \to VV \quad (Q,J) = (0,0)$

$$\begin{pmatrix}
0 & -\frac{\text{CphiB}\,\text{G1}^2+\text{GW}\,(\text{CphiW}\,\text{GW}+\text{CphiWB}\,\text{G1})}{16\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\text{CphiB}\,\text{G1}^2+\text{CphiW}\,\text{GW}^2+\text{CphiWB}\,\text{G1}\,\text{GW}}{16\pi\text{G1}^2+16\pi\text{GW}^2} & 0 & -\frac{\text{CphiB}\,\text{G1}^2+\text{GW}\,(\text{CphiW}\,\text{GW}-\text{CphiWB}\,\text{G1})}{8\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & -\frac{3\,\text{CW}\,\text{GW}^3}{4\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} \\
-\frac{\text{CphiB}\,\text{G1}^2+\text{GW}\,(\text{CphiW}\,\text{GW}+\text{CphiWB}\,\text{G1})}{16\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & -\frac{\text{CphiB}\,\text{G1}^2+\text{GW}\,(\text{CphiW}\,\text{GW}+\text{CphiWB}\,\text{G1})}{16\pi\left(\text{G1}^2+\text{GW}^2\right)} & \frac{2\,\text{G1}\,\text{GW}\,(\text{CphiB}-\text{CphiW})+\text{CphiWB}\left(\text{GW}^2-\text{G1}^2\right)}{16\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & \frac{2\,\text{G1}\,\text{GW}\,(\text{CphiB}-\text{CphiW})+\text{CphiWB}\left(\text{GW}^2-\text{G1}^2\right)}{16\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & -\frac{\text{GW}\,(\text{CphiB}\,\text{GW}-\text{CphiWB}\,\text{G1})+\text{CphiW}\,\text{G1}^2}{16\pi\left(\text{G1}^2+\text{GW}^2\right)} & -\frac{\text{GW}\,(\text{CphiB}\,\text{GW}-\text{CphiWB}\,\text{G1})+\text{CphiW}\,\text{G1}^2}{16\pi\left(\text{G1}^2+\text{GW}^2\right)} & \frac{2\,\text{CphiBox}+\text{CphiD}}{32\pi} & \frac{\text{CphiW}}{8\sqrt{2}\,\pi} & \frac{\text{CphiD}-4\,\text{CphiBox}}{32\sqrt{2}\,\pi} & \frac{\text{CphiW}}{8\sqrt{2}\,\pi} \\
0 & -\frac{\text{CphiB}\,\text{G1}^2+\text{GW}\,(\text{CphiW}\,\text{GW}+\text{CphiWB}\,\text{G1})}{16\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\text{CphiB}\,\text{G1}^2+\text{CphiW}\,\text{GW}^2+\text{CphiWB}\,\text{G1}\,\text{GW}}{16\pi\text{G1}^2+16\pi\text{GW}^2} & -\frac{3\,\text{CW}\,\text{GW}^3}{4\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & \frac{\text{CphiB}\,\text{G1}^2+\text{GW}\,(\text{CphiW}\,\text{GW}-\text{CphiWB}\,\text{G1})}{8\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 \\
0 & \frac{2\,\text{G1}\,\text{GW}\,(\text{CphiB}-\text{CphiW})+\text{CphiWB}\left(\text{GW}^2-\text{G1}^2\right)}{16\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2\,\text{G1}\,\text{GW}\,(\text{CphiW}-\text{CphiB})+\text{CphiWB}\left(\text{G1}^2-\text{GW}^2\right)}{16\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & \frac{2\,\text{G1}\,\text{GW}\,(\text{CphiB}-\text{CphiW})+\text{CphiWB}\left(\text{G1}^2-\text{GW}^2\right)}{16\pi\left(\text{G1}^2+\text{GW}^2\right)} & -\frac{3\,\text{CW}\,\text{G1}\,\text{GW}^2}{4\pi\left(\text{G1}^2+\text{GW}^2\right)} \\
0 & \frac{2\,\text{G1}\,\text{GW}\,(\text{CphiB}-\text{CphiW})+\text{CphiWB}\left(\text{GW}^2-\text{G1}^2\right)}{16\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2\,\text{G1}\,\text{GW}\,(\text{CphiW}-\text{CphiB})+\text{CphiWB}\left(\text{G1}^2-\text{GW}^2\right)}{16\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & -\frac{3\,\text{CW}\,\text{G1}\,\text{GW}^2}{4\pi\left(\text{G1}^2+\text{GW}^2\right)} & \frac{2\,\text{G1}\,\text{GW}\,(\text{CphiB}-\text{CphiW})+\text{CphiWB}\left(\text{G1}^2-\text{GW}^2\right)}{16\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{2\,\text{CphiBox}+\text{CphiD}}{32\pi} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\text{GW}\,(\text{CphiB}\,\text{GW}-\text{CphiWB}\,\text{G1})+\text{CphiW}\,\text{G1}^2}{16\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\text{CphiB}\,\text{GW}^2+\text{CphiW}\,\text{G1}^2-\text{CphiWB}\,\text{G1}\,\text{GW}}{16\pi\text{G1}^2+16\pi\text{GW}^2} & 0 & \frac{\text{GW}\,(\text{CphiB}\,\text{GW}+\text{CphiWB}\,\text{G1})+\text{CphiW}\,\text{G1}^2}{8\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & -\frac{3\,\text{CW}\,\text{G1}^2\,\text{GW}}{4\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} \\
0 & -\frac{\text{GW}\,(\text{CphiB}\,\text{GW}-\text{CphiWB}\,\text{G1})+\text{CphiW}\,\text{G1}^2}{16\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\text{CphiB}\,\text{GW}^2+\text{CphiW}\,\text{G1}^2-\text{CphiWB}\,\text{G1}\,\text{GW}}{16\pi\text{G1}^2+16\pi\text{GW}^2} & -\frac{3\,\text{CW}\,\text{G1}^2\,\text{GW}}{4\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & \frac{\text{GW}\,(\text{CphiB}\,\text{GW}+\text{CphiWB}\,\text{G1})+\text{CphiW}\,\text{G1}^2}{8\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 \\
\frac{\text{CphiB}\,\text{G1}^2+\text{CphiW}\,\text{GW}^2+\text{CphiWB}\,\text{G1}\,\text{GW}}{16\pi\text{G1}^2+16\pi\text{GW}^2} & \frac{2\,\text{CphiBox}+\text{CphiD}}{32\pi} & \frac{\text{CphiB}\,\text{G1}^2+\text{CphiW}\,\text{GW}^2+\text{CphiWB}\,\text{G1}\,\text{GW}}{16\pi\text{G1}^2+16\pi\text{GW}^2} & \frac{2\,\text{G1}\,\text{GW}\,(\text{CphiW}-\text{CphiB})+\text{CphiWB}\left(\text{G1}^2-\text{GW}^2\right)}{16\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & \frac{2\,\text{G1}\,\text{GW}\,(\text{CphiW}-\text{CphiB})+\text{CphiWB}\left(\text{G1}^2-\text{GW}^2\right)}{16\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & \frac{\text{CphiB}\,\text{GW}^2+\text{CphiW}\,\text{G1}^2-\text{CphiWB}\,\text{G1}\,\text{GW}}{16\pi\text{G1}^2+16\pi\text{GW}^2} & \frac{\text{CphiB}\,\text{GW}^2+\text{CphiW}\,\text{G1}^2-\text{CphiWB}\,\text{G1}\,\text{GW}}{16\pi\text{G1}^2+16\pi\text{GW}^2} & 0 & \frac{\text{CphiW}}{8\sqrt{2}\,\pi} & \frac{4\,\text{CphiBox}-\text{CphiD}}{32\sqrt{2}\,\pi} & \frac{\text{CphiW}}{8\sqrt{2}\,\pi} \\
0 & \frac{\text{CphiW}}{8\sqrt{2}\,\pi} & -\frac{3\,\text{CW}\,\text{GW}^3}{4\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & -\frac{3\,\text{CW}\,\text{G1}\,\text{GW}^2}{4\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & 0 & -\frac{3\,\text{CW}\,\text{G1}^2\,\text{GW}}{4\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & \frac{\text{CphiW}}{8\sqrt{2}\,\pi} & 0 & -\frac{\text{CphiW}}{8\pi} & -\frac{3\,\text{CW}\,\text{GW}}{8\pi} \\
-\frac{\text{CphiB}\,\text{G1}^2+\text{GW}\,(\text{CphiW}\,\text{GW}-\text{CphiWB}\,\text{G1})}{8\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & \frac{\text{CphiD}-4\,\text{CphiBox}}{32\sqrt{2}\,\pi} & -\frac{\text{CphiB}\,\text{G1}^2+\text{GW}\,(\text{CphiW}\,\text{GW}-\text{CphiWB}\,\text{G1})}{8\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & \frac{2\,\text{G1}\,\text{GW}\,(\text{CphiB}-\text{CphiW})+\text{CphiWB}\left(\text{G1}^2-\text{GW}^2\right)}{16\pi\left(\text{G1}^2+\text{GW}^2\right)} & \frac{2\,\text{G1}\,\text{GW}\,(\text{CphiB}-\text{CphiW})+\text{CphiWB}\left(\text{G1}^2-\text{GW}^2\right)}{16\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & -\frac{\text{GW}\,(\text{CphiB}\,\text{GW}+\text{CphiWB}\,\text{G1})+\text{CphiW}\,\text{G1}^2}{8\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & -\frac{\text{GW}\,(\text{CphiB}\,\text{GW}+\text{CphiWB}\,\text{G1})+\text{CphiW}\,\text{G1}^2}{8\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & \frac{4\,\text{CphiBox}-\text{CphiD}}{32\sqrt{2}\,\pi} & \frac{\text{CphiW}}{8\pi} & \frac{2\,\text{CphiBox}+\text{CphiD}}{32\pi} & \frac{\text{CphiW}}{8\pi} \\
\frac{3\,\text{CW}\,\text{GW}^3}{4\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & \frac{\text{CphiW}}{8\sqrt{2}\,\pi} & 0 & -\frac{3\,\text{CW}\,\text{G1}\,\text{GW}^2}{4\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & 0 & \frac{3\,\text{CW}\,\text{G1}^2\,\text{GW}}{4\sqrt{2}\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & \frac{\text{CphiW}}{8\sqrt{2}\,\pi} & -\frac{3\,\text{CW}\,\text{GW}}{8\pi} & -\frac{\text{CphiW}}{8\pi} & 0
\end{pmatrix}$$

$VV \to VV \quad (Q,J) = (1,0)$

$$\begin{pmatrix}
0 & -\frac{\text{CphiWB}\,\text{G1}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} & \frac{3\,\text{CW}\,\text{GW}^3}{8\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & \frac{3\,\text{CW}\,\text{G1}\,\text{GW}^2}{8\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & \frac{\text{CphiWB}\,\text{G1}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} \\
-\frac{\text{CphiWB}\,\text{G1}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} & -\frac{\text{CphiD}-4\,\text{CphiBox}}{64\,\pi} & -\frac{\text{CphiWB}\,\text{G1}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} & \frac{\text{CphiWB}\,\text{GW}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} & \frac{\text{CphiWB}\,\text{GW}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} & \frac{3\,\text{CphiD}}{64\,\pi} \\
\frac{3\,\text{CW}\,\text{GW}^3}{8\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & -\frac{\text{CphiWB}\,\text{G1}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} & 0 & \frac{3\,\text{CW}\,\text{G1}\,\text{GW}^2}{8\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & \frac{\text{CphiWB}\,\text{G1}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} \\
0 & \frac{\text{CphiWB}\,\text{GW}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} & \frac{3\,\text{CW}\,\text{G1}\,\text{GW}^2}{8\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & \frac{3\,\text{CW}\,\text{G1}^2\,\text{GW}}{8\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & -\frac{\text{CphiWB}\,\text{GW}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} \\
\frac{3\,\text{CW}\,\text{G1}\,\text{GW}^2}{8\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & \frac{\text{CphiWB}\,\text{GW}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} & 0 & \frac{3\,\text{CW}\,\text{G1}^2\,\text{GW}}{8\,\pi\left(\text{G1}^2+\text{GW}^2\right)} & 0 & -\frac{\text{CphiWB}\,\text{GW}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} \\
\frac{\text{CphiWB}\,\text{G1}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} & \frac{3\,\text{CphiD}}{64\,\pi} & \frac{\text{CphiWB}\,\text{G1}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} & -\frac{\text{CphiWB}\,\text{GW}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} & -\frac{\text{CphiWB}\,\text{GW}}{16\,\pi\,\sqrt{\text{G1}^2+\text{GW}^2}} & -\frac{\text{CphiD}-4\,\text{CphiBox}}{64\,\pi}
\end{pmatrix}$$

Coupled Channels and Diagonalization

organized by Q (total electric charge of the process)
and J (total angular momentum)

$VV \to VV$

Q	J	Dimension
2	0	3 x 3
1	0	6 x 6
0	0	12 x 12
2	1	6 x 6
1	1	14 x 14
0	1	18 x 18

$$\left|\mathrm{Re}(a^J(V_i'(\lambda_i)V_j'(\lambda_j) \to V_i'(\lambda_i)V_j'(\lambda_j))\right| \leq 1$$

$ff \to VV$

Q	J	Dimension
1	0	4 x 6
0	0	8 x 11
1	1	8 x 14
0	1	16 x 25

$$\left|a^J_{V_j'V_k';x^i}\right| \leq \frac{1}{2}$$

$$\max\left\{C_j \mid \{C_j, C_k\} \in S\right\}$$

Unitarity Bounds

Operator	bounds on $\mathcal{C}_i$'s	
	Warsaw	SILH
$\mathcal{O}_W$	$\frac{4\pi}{3g}$	$\frac{4\pi}{3g}$
$\mathcal{O}_{eW}$	4π	4π
$\mathcal{O}_{eB}$	$4\sqrt{3}\pi$	$4\sqrt{3}\pi$
$\mathcal{O}_{uW}$	$\frac{4\pi}{\sqrt{3}}$	$\frac{4\pi}{\sqrt{3}}$
$\mathcal{O}_{uB}$	4π	4π
$\mathcal{O}_{dW}$	$\frac{4\pi}{\sqrt{3}}$	$\frac{4\pi}{\sqrt{3}}$
$\mathcal{O}_{dB}$	4π	4π
$\mathcal{O}_{\varphi ud}$	$4\sqrt{2}\pi$	$4\sqrt{2}\pi$
$\mathcal{O}_{\varphi\Box}$	$\frac{16\pi}{3}$	/
$\mathcal{O}_{\varphi D}$	$\frac{64\pi}{3}$	/
$\mathcal{O}_{\varphi W}$	$\frac{4\sqrt{6}\pi}{3}$	/
$\mathcal{O}_{\varphi B}$	$4\sqrt{2}\pi$	$4\sqrt{6}\pi\frac{\sqrt{3g^4+g'^4}}{3g^2}+8\pi g'/g$
$\mathcal{O}_{\varphi WB}$	8π	/

Operator	bounds on $\mathcal{C}_i$'s	
	Warsaw	SILH
$\mathcal{O}'_W$	/	$\frac{32\sqrt{6}\pi}{3g^2}$
$\mathcal{O}_B$	/	$\frac{64\pi}{gg'}+\frac{32\sqrt{6}\pi}{3g^2}$
$\mathcal{O}_{2W}$	/	$\frac{16(4+3\sqrt{6})\pi}{3g^2}$
$\mathcal{O}_{2B}$	/	$\frac{16(8+3\sqrt{6})\pi}{3g'^2}$
$\mathcal{O}_W^{\text{SILH}}$	/	$\frac{8(8+3\sqrt{6})}{3g^2}+\frac{64\pi}{11g^2}$
$\mathcal{O}_B^{\text{SILH}}$	/	$\frac{8(16+3\sqrt{6})}{3g'^2}+\frac{8\pi}{g^2}+\frac{24\pi g'^6}{g^4(8g^4+3g'^4)}$
$\mathcal{O}_{\varphi l}^{(1)}$	$2\sqrt{6}\pi$	/[a]
$\mathcal{O}_{\varphi l}^{(3)}$	$2\sqrt{6}\pi$	/[b]
$\mathcal{O}_{\varphi e}$	$4\sqrt{3}\pi$	12π
$\mathcal{O}_{\varphi q}^{(1)}$	$2\sqrt{2}\pi$	$4\sqrt{\frac{2}{3}}\pi$
$\mathcal{O}_{\varphi q}^{(3)}$	$2\sqrt{2}\pi$	$4\sqrt{2}\pi$
$\mathcal{O}_{\varphi u}$	4π	$4\sqrt{\frac{11}{3}}\pi$
$\mathcal{O}_{\varphi d}$	4π	$4\sqrt{\frac{5}{3}}\pi$

$$\mathcal{C}'_W \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{8}{g^2}\mathcal{C}_{\varphi W},$$

$$\mathcal{C}_{\varphi e} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi e} - 2\mathcal{C}^{(1)}_{\varphi l},$$

$$\mathcal{C}_{\varphi d} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi d} - \frac{2}{3}\mathcal{C}^{(1)}_{\varphi l},$$

$$\mathcal{C}_{\varphi u} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi u} + \frac{4}{3}\mathcal{C}^{(1)}_{\varphi l},$$

$$\mathcal{C}^{(1)}_{\varphi q} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}^{(1)}_{\varphi q} + \frac{1}{3}\mathcal{C}^{(1)}_{\varphi l},$$

$$\mathcal{C}^{(3)}_{\varphi q} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}^{(3)}_{\varphi q} - \mathcal{C}^{(3)}_{\varphi l},$$

$$\mathcal{C}_B \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{8}{gg'}\mathcal{C}_{\varphi WB} - \frac{8}{g^2}\mathcal{C}_{\varphi W}$$

$$\mathcal{C}_{2B} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{8}{g'^2}\mathcal{C}^{(1)}_{\varphi l} + \frac{2}{g'^2}\mathcal{C}_{\varphi D}$$

$$\mathcal{C}_{\varphi B} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi B} + \frac{g'^2}{g^2}\mathcal{C}_{\varphi W} - \frac{g'}{g}\mathcal{C}_{\varphi WB}$$

$$\mathcal{C}_{2W} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{8}{3g^2}\mathcal{C}_{\varphi\Box} - \frac{2}{3g^2}\mathcal{C}_{\varphi D} - \frac{8}{g^2}\mathcal{C}^{(3)}_{\varphi l}$$

$$\mathcal{C}^{\text{SILH}}_B \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{4}{g^2}\mathcal{C}_{\varphi W} - \frac{4}{gg'}\mathcal{C}_{\varphi WB} - \frac{2}{g'^2}(\mathcal{C}_{\varphi D} + 2\mathcal{C}^{(1)}_{\varphi l})$$

$$\mathcal{C}^{\text{SILH}}_W \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{2}{3g^2}(\mathcal{C}_{\varphi D} + 6\mathcal{C}^{(3)}_{\varphi l} - 6\mathcal{C}_{\varphi W} - 4\mathcal{C}_{\varphi\Box}).$$

Operator	bounds on $\mathcal{C}_i$'s	
	Warsaw	SILH
$\mathcal{O}_W$	$\frac{4\pi}{3g}$	$\frac{4\pi}{3g}$
$\mathcal{O}_{eW}$	4π	4π
$\mathcal{O}_{eB}$	$4\sqrt{3}\pi$	$4\sqrt{3}\pi$
$\mathcal{O}_{uW}$	$\frac{4\pi}{\sqrt{3}}$	$\frac{4\pi}{\sqrt{3}}$
$\mathcal{O}_{uB}$	4π	4π
$\mathcal{O}_{dW}$	$\frac{4\pi}{\sqrt{3}}$	$\frac{4\pi}{\sqrt{3}}$
$\mathcal{O}_{dB}$	4π	4π
$\mathcal{O}_{\varphi ud}$	$4\sqrt{2}\pi$	$4\sqrt{2}\pi$
$\mathcal{O}_{\varphi\Box}$	$\frac{16\pi}{3}$	/
$\mathcal{O}_{\varphi D}$	$\frac{64\pi}{3}$	/
$\mathcal{O}_{\varphi W}$	$\frac{4\sqrt{6}\pi}{3}$	/
$\mathcal{O}_{\varphi B}$	$4\sqrt{2}\pi$	$4\sqrt{6}\pi\frac{\sqrt{3g^4+g'^4}}{3g^2} + 8\pi g'/g$
$\mathcal{O}_{\varphi WB}$	8π	/
$\mathcal{O}'_W$	/	$\frac{32\sqrt{6}\pi}{3g^2}$
$\mathcal{O}_B$	/	$\frac{64\pi}{gg'} + \frac{32\sqrt{6}\pi}{3g^2}$
$\mathcal{O}_{2W}$	/	$\frac{16(4+3\sqrt{6})\pi}{3g^2}$
$\mathcal{O}_{2B}$	/	$\frac{16(8+3\sqrt{6})\pi}{3g'^2}$
$\mathcal{O}^{\text{SILH}}_W$	/	$\frac{8(8+3\sqrt{6})}{3g^2} + \frac{64\pi}{11g^2}$
$\mathcal{O}^{\text{SILH}}_B$	/	$\frac{8(16+3\sqrt{6})}{3g'^2} + \frac{8\pi}{g^2} + \frac{24\pi g'^6}{g^4(8g^4+3g'^4)}$
$\mathcal{O}^{(1)}_{\varphi l}$	$2\sqrt{6}\pi$	/[a]
$\mathcal{O}^{(3)}_{\varphi l}$	$2\sqrt{6}\pi$	/[b]
$\mathcal{O}_{\varphi e}$	$4\sqrt{3}\pi$	12π
$\mathcal{O}^{(1)}_{\varphi q}$	$2\sqrt{2}\pi$	$4\sqrt{\frac{2}{3}}\pi$
$\mathcal{O}^{(3)}_{\varphi q}$	$2\sqrt{2}\pi$	$4\sqrt{2}\pi$
$\mathcal{O}_{\varphi u}$	4π	$4\sqrt{\frac{11}{3}}\pi$
$\mathcal{O}_{\varphi d}$	4π	$4\sqrt{\frac{5}{3}}\pi$

$$C_{2B} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{8}{g'^2}C_{\varphi l}^{(1)} + \frac{2}{g'^2}C_{\varphi D}$$

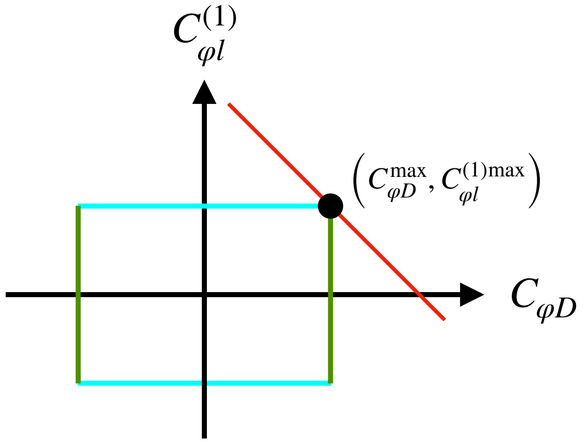
$$C_{2B} = \frac{8}{g'^2}C_{\varphi l}^{(1)} + \frac{2}{g'^2}C_{\varphi D} = \frac{16(8+3\sqrt{6})\pi}{3g'^2}$$

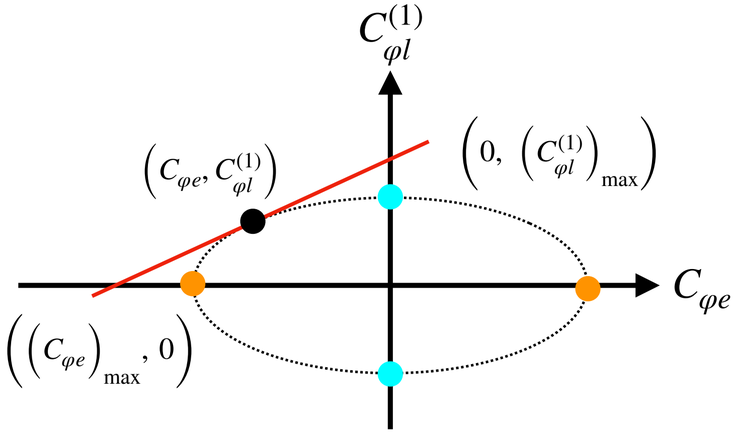
$$C_{\varphi e} \underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} C_{\varphi e} - 2C_{\varphi l}^{(1)},$$

$$C_{\varphi e} = C_{\varphi e} - 2C_{\varphi l}^{(1)} \neq 4\sqrt{3}\pi + 2 \times 2\sqrt{6}\pi$$

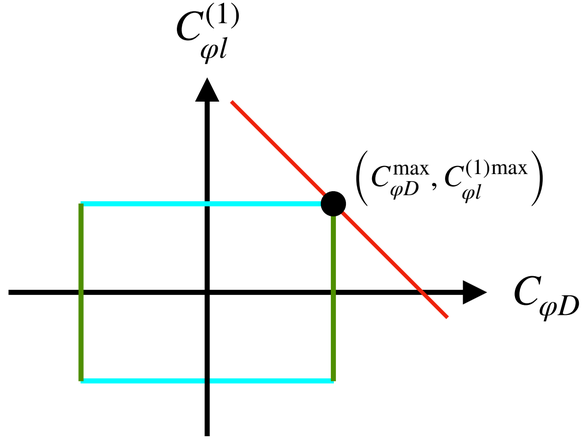
Operator	bounds on $\mathcal{C}_i$'s	
	Warsaw	SILH
$\mathcal{O}_W$	$\frac{4\pi}{3g}$	$\frac{4\pi}{3g}$
$\mathcal{O}_{eW}$	4π	4π
$\mathcal{O}_{eB}$	$4\sqrt{3}\pi$	$4\sqrt{3}\pi$
$\mathcal{O}_{uW}$	$\frac{4\pi}{\sqrt{3}}$	$\frac{4\pi}{\sqrt{3}}$
$\mathcal{O}_{uB}$	4π	4π
$\mathcal{O}_{dW}$	$\frac{4\pi}{\sqrt{3}}$	$\frac{4\pi}{\sqrt{3}}$
$\mathcal{O}_{dB}$	4π	4π
$\mathcal{O}_{\varphi ud}$	$4\sqrt{2}\pi$	$4\sqrt{2}\pi$
$\mathcal{O}_{\varphi\Box}$	$\frac{16\pi}{3}$	/
$\mathcal{O}_{\varphi D}$	$\frac{64\pi}{3}$	/
$\mathcal{O}_{\varphi W}$	$\frac{4\sqrt{6}\pi}{3}$	/
$\mathcal{O}_{\varphi B}$	$4\sqrt{2}\pi$	$4\sqrt{6}\pi\frac{\sqrt{3g^4+g'^4}}{3g^2} + 8\pi g'/g$
$\mathcal{O}_{\varphi WB}$	8π	/
$\mathcal{O}'_W$	/	$\frac{32\sqrt{6}\pi}{3g^2}$
$\mathcal{O}_B$	/	$\frac{64\pi}{gg'} + \frac{32\sqrt{6}\pi}{3g^2}$
$\mathcal{O}_{2W}$	/	$\frac{16(4+3\sqrt{6})\pi}{3g^2}$
$\mathcal{O}_{2B}$	/	$\frac{16(8+3\sqrt{6})\pi}{3g'^2}$
$\mathcal{O}_W^{\text{SILH}}$	/	$\frac{8(8+3\sqrt{6})}{3g^2} + \frac{64\pi}{11g^2}$
$\mathcal{O}_B^{\text{SILH}}$	/	$\frac{8(16+3\sqrt{6})}{3g'^2} + \frac{8\pi}{g^2} + \frac{24\pi g'^6}{g^4(8g^4+3g'^4)}$
$\mathcal{O}_{\varphi l}^{(1)}$	$2\sqrt{6}\pi$	/[a]
$\mathcal{O}_{\varphi l}^{(3)}$	$2\sqrt{6}\pi$	/[b]
$\mathcal{O}_{\varphi e}$	$4\sqrt{3}\pi$	12π
$\mathcal{O}_{\varphi q}^{(1)}$	$2\sqrt{2}\pi$	$4\sqrt{\frac{2}{3}}\pi$
$\mathcal{O}_{\varphi q}^{(3)}$	$2\sqrt{2}\pi$	$4\sqrt{2}\pi$
$\mathcal{O}_{\varphi u}$	4π	$4\sqrt{\frac{11}{3}}\pi$
$\mathcal{O}_{\varphi d}$	4π	$4\sqrt{\frac{5}{3}}\pi$

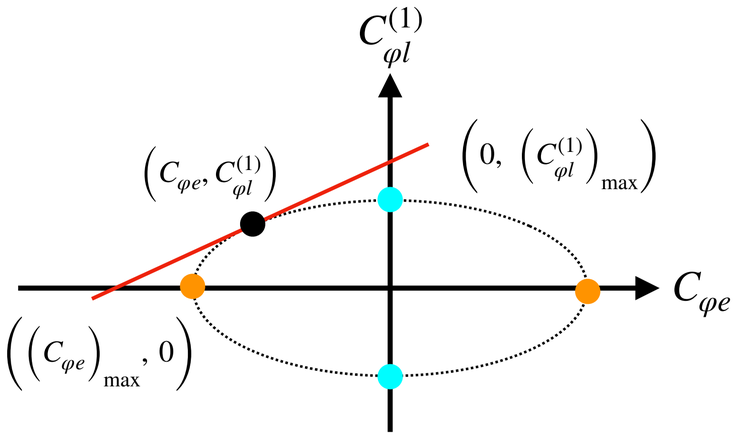
Warsaw Basis	SILH Basis
$\{\mathcal{O}_W, \mathcal{O}_{\varphi\Box}, \mathcal{O}_{\varphi W}, \mathcal{O}_{\varphi B}\}$, $\{\mathcal{O}_{\varphi D}, \mathcal{O}_{\varphi\Box}, \mathcal{O}_{\varphi WB}\}$,	$\{\mathcal{O}_W, \mathcal{O}_{\varphi B}, \mathcal{O}'_W, \mathcal{O}_B, \mathcal{O}_{2W}, \mathcal{O}_{2B}, \mathcal{O}_W^{\rm SILH}, \mathcal{O}_B^{\rm SILH}\}$ $\{\mathcal{O}'_W, \mathcal{O}_B, \mathcal{O}_{2W}, \mathcal{O}_{2B}, \mathcal{O}_W^{\rm SILH}, \mathcal{O}_B^{\rm SILH}\}$,
$\{\mathcal{O}_W, \mathcal{O}_{\varphi l}^{(3)}, \mathcal{O}_{\varphi q}^{(3)}\}$,	$\{\mathcal{O}_W, \mathcal{O}_{\varphi q}^{(3)}, \mathcal{O}'_W, \mathcal{O}_W^{\rm SILH}, \mathcal{O}_{2W}\}$,
$\{\mathcal{O}_{\varphi d}, \mathcal{O}_{\varphi u}, \mathcal{O}_{\varphi l}^{(1)}, \mathcal{O}_{\varphi q}^{(1)}, \mathcal{O}_{\varphi e}\}$,	$\{\mathcal{O}_{\varphi d}, \mathcal{O}_{\varphi u}, \mathcal{O}_{\varphi q}^{(1)}, \mathcal{O}_{\varphi e}, \mathcal{O}_B, \mathcal{O}_B^{\rm SILH}\}$,
$\{\mathcal{O}_{uW}, \mathcal{O}_{uB}, \mathcal{O}_{dW}, \mathcal{O}_{dB}, \mathcal{O}_{eW}, \mathcal{O}_{eB}\}$;	$\{\mathcal{O}_{uW}, \mathcal{O}_{uB}, \mathcal{O}_{dW}, \mathcal{O}_{dB}, \mathcal{O}_{eW}, \mathcal{O}_{eB}\}$,
$\{\mathcal{O}_{\varphi ud}\}$,	$\{\mathcal{O}_{\varphi ud}\}$,
$\{\mathcal{O}_W\}$	$\{\mathcal{O}_W\}$.





$$\left(C_{2B}^{\rm SILH}\right)_{\max} = \frac{2}{g'^2}\left(C_{\varphi D}\right)_{\max} + \frac{8}{g'^2}\left(C_{\varphi l}^{(1)}\right)_{\max} \qquad \left(C_{\varphi e}^{\rm SILH}\right)_{\max} = C_{\varphi e} - 2C_{\varphi l}^{(1)}$$

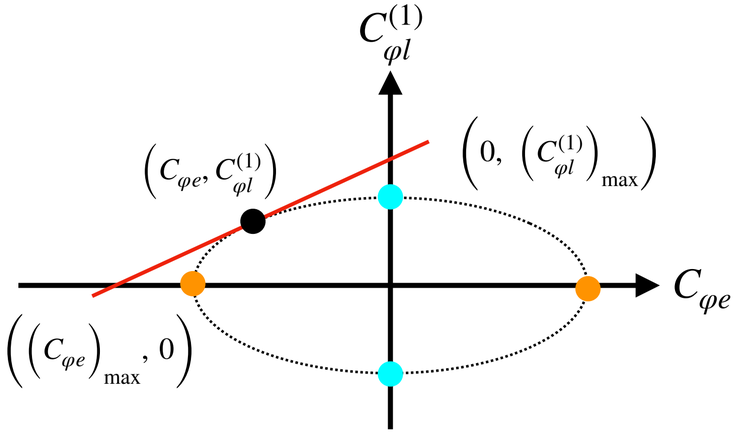




$$\left(C_{2B}^{\mathrm{SILH}}\right)_{\max} = \frac{2}{g'^2}\left(C_{\varphi D}\right)_{\max} + \frac{8}{g'^2}\left(C_{\varphi l}^{(1)}\right)_{\max} \qquad \left(C_{\varphi e}^{\mathrm{SILH}}\right)_{\max} = C_{\varphi e} - 2C_{\varphi l}^{(1)}$$

$$C_{2B} = \frac{8}{g'^2}C_{\varphi l}^{(1)} + \frac{2}{g'^2}C_{\varphi D} = \frac{16(8+3\sqrt{6})\pi}{3g'^2} \qquad C_{\varphi e} = C_{\varphi e} - 2C_{\varphi l}^{(1)} \neq 4\sqrt{3}\pi + 2\times 2\sqrt{6}\pi$$

Warsaw Basis	SILH Basis
$\{\mathcal{O}_W, \mathcal{O}_{\varphi\Box}, \mathcal{O}_{\varphi W}, \mathcal{O}_{\varphi B}\}$, $\{\mathcal{O}_{\varphi D}, \mathcal{O}_{\varphi\Box}, \mathcal{O}_{\varphi WB}\}$,	$\{\mathcal{O}_W, \mathcal{O}_{\varphi B}, \mathcal{O}'_W, \mathcal{O}_B, \mathcal{O}_{2W}, \mathcal{O}_{2B}, \mathcal{O}_W^{\rm SILH}, \mathcal{O}_B^{\rm SILH}\}$ $\{\mathcal{O}'_W, \mathcal{O}_B, \mathcal{O}_{2W}, \mathcal{O}_{2B}, \mathcal{O}_W^{\rm SILH}, \mathcal{O}_B^{\rm SILH}\}$,
$\{\mathcal{O}_W, \mathcal{O}_{\varphi l}^{(3)}, \mathcal{O}_{\varphi q}^{(3)}\}$,	$\{\mathcal{O}_W, \mathcal{O}_{\varphi q}^{(3)}, \mathcal{O}'_W, \mathcal{O}_W^{\rm SILH}, \mathcal{O}_{2W}\}$,
$\{\mathcal{O}_{\varphi d}, \mathcal{O}_{\varphi u}, \mathcal{O}_{\varphi l}^{(1)}, \mathcal{O}_{\varphi q}^{(1)}, \mathcal{O}_{\varphi e}\}$,	$\{\mathcal{O}_{\varphi d}, \mathcal{O}_{\varphi u}, \mathcal{O}_{\varphi q}^{(1)}, \mathcal{O}_{\varphi e}, \mathcal{O}_B, \mathcal{O}_B^{\rm SILH}\}$,
$\{\mathcal{O}_{uW}, \mathcal{O}_{uB}, \mathcal{O}_{dW}, \mathcal{O}_{dB}, \mathcal{O}_{eW}, \mathcal{O}_{eB}\}$,	$\{\mathcal{O}_{uW}, \mathcal{O}_{uB}, \mathcal{O}_{dW}, \mathcal{O}_{dB}, \mathcal{O}_{eW}, \mathcal{O}_{eB}\}$,
$\{\mathcal{O}_{\varphi ud}\}$,	$\{\mathcal{O}_{\varphi ud}\}$,
$\{\mathcal{O}_W\}$	$\{\mathcal{O}_W\}$.



$$\left(C_{\varphi e}^{\rm SILH}\right)_{\max} = C_{\varphi e} - 2C_{\varphi l}^{(1)}$$

$$C_{\varphi e}^2 + 2C_{\varphi l}^{(1)^2} \leq 48\pi^2$$

$$C_{\varphi e} = C_{\varphi e} - 2C_{\varphi l}^{(1)} = C_{\varphi e} + 2\sqrt{24\pi^2 - C_{\varphi e}^2/2} \leq 12\pi$$

$$(C_{\varphi e},\ C_{\varphi l}^{(1)}) = (-4\pi,\ 4\pi)$$

$\mathcal{O}_{\varphi l}^{(1)}$	$2\sqrt{6}\pi$	/[a]
$\mathcal{O}_{\varphi l}^{(3)}$	$2\sqrt{6}\pi$	/[b]
$\mathcal{O}_{\varphi e}$	$4\sqrt{3}\pi$	12π

Summary

Equivalence of operator bases at the dim-6 is verified by unitarity analysis

$$
\begin{aligned}
\mathcal{C}_{\varphi W} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{g^2}{8}\mathcal{C}'_W,\\
\mathcal{C}_{\varphi B} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi B}+\frac{g'^2}{8}\mathcal{C}_B,\\
\mathcal{C}_{\varphi WB} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{gg'}{8}(\mathcal{C}'_W+\mathcal{C}_B),\\
\mathcal{C}_{\varphi D} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} -g'^2\mathcal{C}_B^{\text{SILH}}-\frac{g'^2}{2}(\mathcal{C}_B+\mathcal{C}_{2B}),\\
\mathcal{C}^{(1)}_{\varphi l} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{g'^2}{8}\mathcal{C}_B+\frac{g'^2}{4}(\mathcal{C}_B^{\text{SILH}}+\mathcal{C}_{2B}),\\
\mathcal{C}^{(3)}_{\varphi l} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} -\frac{g^2}{8}\mathcal{C}'_W-\frac{g^2}{4}(\mathcal{C}_W^{\text{SILH}}+\mathcal{C}_{2W}),\\
\mathcal{C}^{(1)}_{\varphi q} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}^{(1)}_{\varphi q}-\frac{g'^2}{24}\mathcal{C}_B-\frac{g'^2}{12}(\mathcal{C}_B^{\text{SILH}}+\mathcal{C}_{2B}),\\
\mathcal{C}^{(3)}_{\varphi q} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}^{(3)}_{\varphi q}-\frac{g^2}{8}\mathcal{C}'_W-\frac{g^2}{4}(\mathcal{C}_W^{\text{SILH}}+\mathcal{C}_{2W}),\\
\mathcal{C}_{\varphi\Box} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} -\frac{3g^2}{4}\mathcal{C}_W^{\text{SILH}}-\frac{3g^2}{8}(\mathcal{C}'_W+\mathcal{C}_{2W})\\
&\quad -\frac{g'^2}{4}\mathcal{C}_B^{\text{SILH}}-\frac{g'^2}{8}(\mathcal{C}_B+\mathcal{C}_{2B}),\\
\mathcal{C}_{\varphi e} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi e}+\frac{g'^2}{4}(\mathcal{C}_B+2\mathcal{C}_{2B}+2\mathcal{C}_B^{\text{SILH}}),\\
\mathcal{C}_{\varphi d} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi d}+\frac{g'^2}{12}(\mathcal{C}_B+2\mathcal{C}_{2B}+2\mathcal{C}_B^{\text{SILH}}),\\
\mathcal{C}_{\varphi u} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi u}-\frac{g'^2}{6}(\mathcal{C}_B+2\mathcal{C}_{2B}+2\mathcal{C}_B^{\text{SILH}}).
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{C}'_W &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{8}{g^2}\mathcal{C}_{\varphi W},\\
\mathcal{C}_{\varphi e} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi e}-2\mathcal{C}^{(1)}_{\varphi l},\\
\mathcal{C}_{\varphi d} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi d}-\frac{2}{3}\mathcal{C}^{(1)}_{\varphi l},\\
\mathcal{C}_{\varphi u} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi u}+\frac{4}{3}\mathcal{C}^{(1)}_{\varphi l},\\
\mathcal{C}^{(1)}_{\varphi q} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}^{(1)}_{\varphi q}+\frac{1}{3}\mathcal{C}^{(1)}_{\varphi l},\\
\mathcal{C}^{(3)}_{\varphi q} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}^{(3)}_{\varphi q}-\mathcal{C}^{(3)}_{\varphi l},\\
\mathcal{C}_B &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{8}{gg'}\mathcal{C}_{\varphi WB}-\frac{8}{g^2}\mathcal{C}_{\varphi W}\\
\mathcal{C}_{2B} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{8}{g'^2}\mathcal{C}^{(1)}_{\varphi l}+\frac{2}{g'^2}\mathcal{C}_{\varphi D}\\
\mathcal{C}_{\varphi B} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \mathcal{C}_{\varphi B}+\frac{g'^2}{g^2}\mathcal{C}_{\varphi W}-\frac{g'}{g}\mathcal{C}_{\varphi WB}\\
\mathcal{C}_{2W} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{8}{3g^2}\mathcal{C}_{\varphi\Box}-\frac{2}{3g^2}\mathcal{C}_{\varphi D}-\frac{8}{g^2}\mathcal{C}^{(3)}_{\varphi l}\\
\mathcal{C}_B^{\text{SILH}} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{4}{g^2}\mathcal{C}_{\varphi W}-\frac{4}{gg'}\mathcal{C}_{\varphi WB}-\frac{2}{g'^2}(\mathcal{C}_{\varphi D}+2\mathcal{C}^{(1)}_{\varphi l})\\
\mathcal{C}_W^{\text{SILH}} &\underset{\text{Constraint}}{\overset{\text{Amplitude}}{\rightleftharpoons}} \frac{2}{3g^2}(\mathcal{C}_{\varphi D}+6\mathcal{C}^{(3)}_{\varphi l}-6\mathcal{C}_{\varphi W}-4\mathcal{C}_{\varphi\Box}).
\end{aligned}
$$

$$
\begin{aligned}
&\{\mathcal{O}_W,\mathcal{O}_{\varphi\Box},\mathcal{O}_{\varphi W},\mathcal{O}_{\varphi B}\},\\
&\{\mathcal{O}_{\varphi D},\mathcal{O}_{\varphi\Box},\mathcal{O}_{\varphi WB}\},\\
&\{\mathcal{O}_W,\mathcal{O}^{(3)}_{\varphi l},\mathcal{O}^{(3)}_{\varphi q}\},\\
&\{\mathcal{O}_{\varphi d},\mathcal{O}_{\varphi u},\mathcal{O}^{(1)}_{\varphi l},\mathcal{O}^{(1)}_{\varphi q},\mathcal{O}_{\varphi e}\},\\
&\{\mathcal{O}_{uW},\mathcal{O}_{uB},\mathcal{O}_{dW},\mathcal{O}_{dB},\mathcal{O}_{eW},\mathcal{O}_{eB}\},\\
&\{\mathcal{O}_{\varphi ud}\},\\
&\{\mathcal{O}_W\}
\end{aligned}
$$

$$
\begin{aligned}
&\{\mathcal{O}_W,\mathcal{O}_{\varphi B},\mathcal{O}'_W,\mathcal{O}_B,\mathcal{O}_{2W},\mathcal{O}_{2B},\mathcal{O}_W^{\text{SILH}},\mathcal{O}_B^{\text{SILH}}\}\\
&\{\mathcal{O}'_W,\mathcal{O}_B,\mathcal{O}_{2W},\mathcal{O}_{2B},\mathcal{O}_W^{\text{SILH}},\mathcal{O}_B^{\text{SILH}}\},\\
&\{\mathcal{O}_W,\mathcal{O}^{(3)}_{\varphi q},\mathcal{O}'_W,\mathcal{O}_W^{\text{SILH}},\mathcal{O}_{2W}\},\\
&\{\mathcal{O}_{\varphi d},\mathcal{O}_{\varphi u},\mathcal{O}^{(1)}_{\varphi q},\mathcal{O}_{\varphi e},\mathcal{O}_B,\mathcal{O}_B^{\text{SILH}}\},\\
&\{\mathcal{O}_{uW},\mathcal{O}_{uB},\mathcal{O}_{dW},\mathcal{O}_{dB},\mathcal{O}_{eW},\mathcal{O}_{eB}\},\\
&\{\mathcal{O}_{\varphi ud}\},\\
&\{\mathcal{O}_W\}.
\end{aligned}
$$

Thanks!