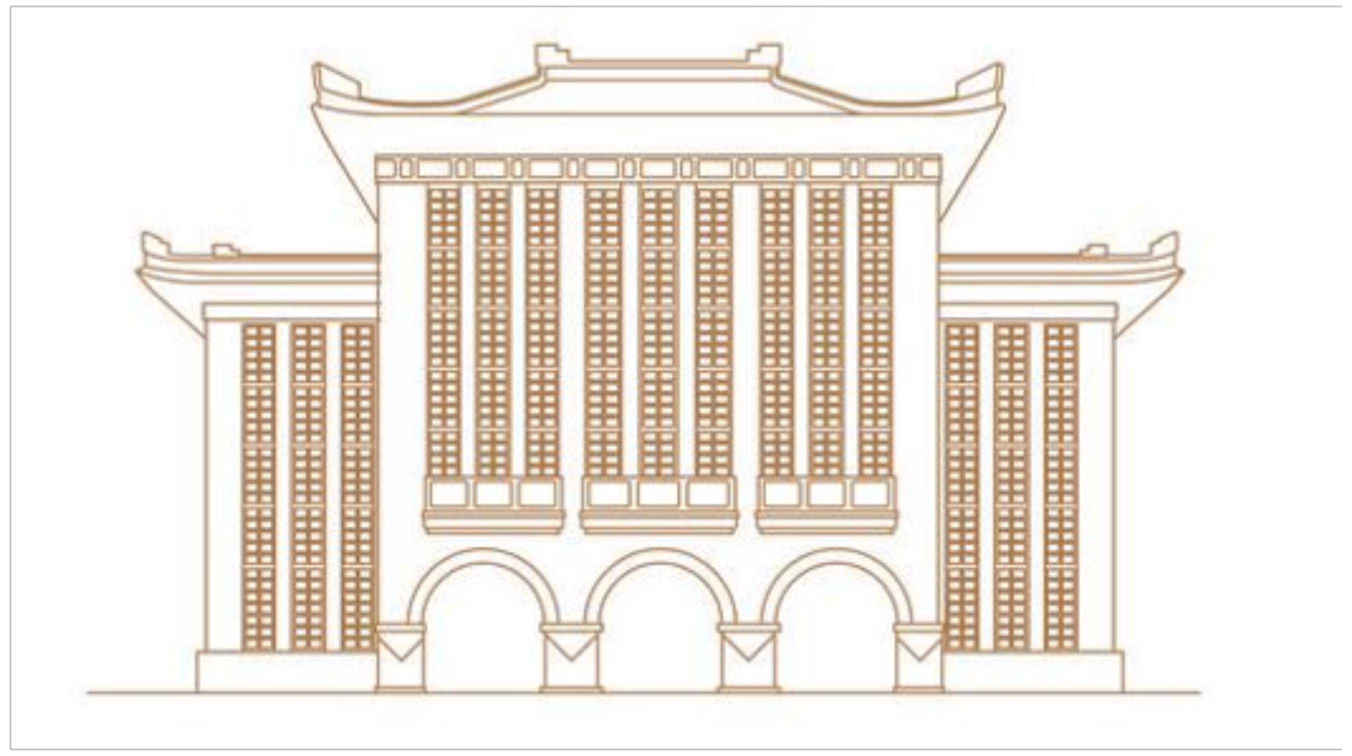


第十二届“新物理研讨会”

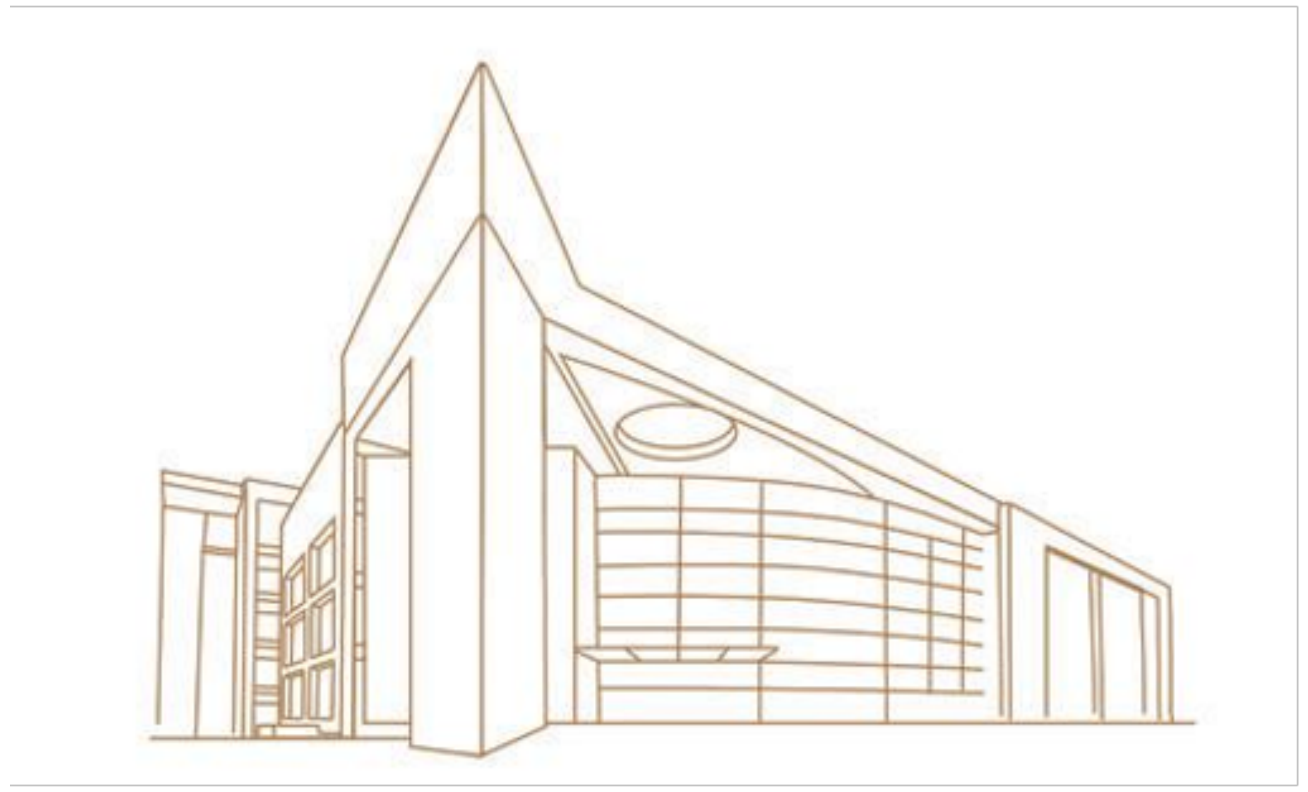
Progress and prospect in baryon chiral perturbation theory

De-Liang Yao

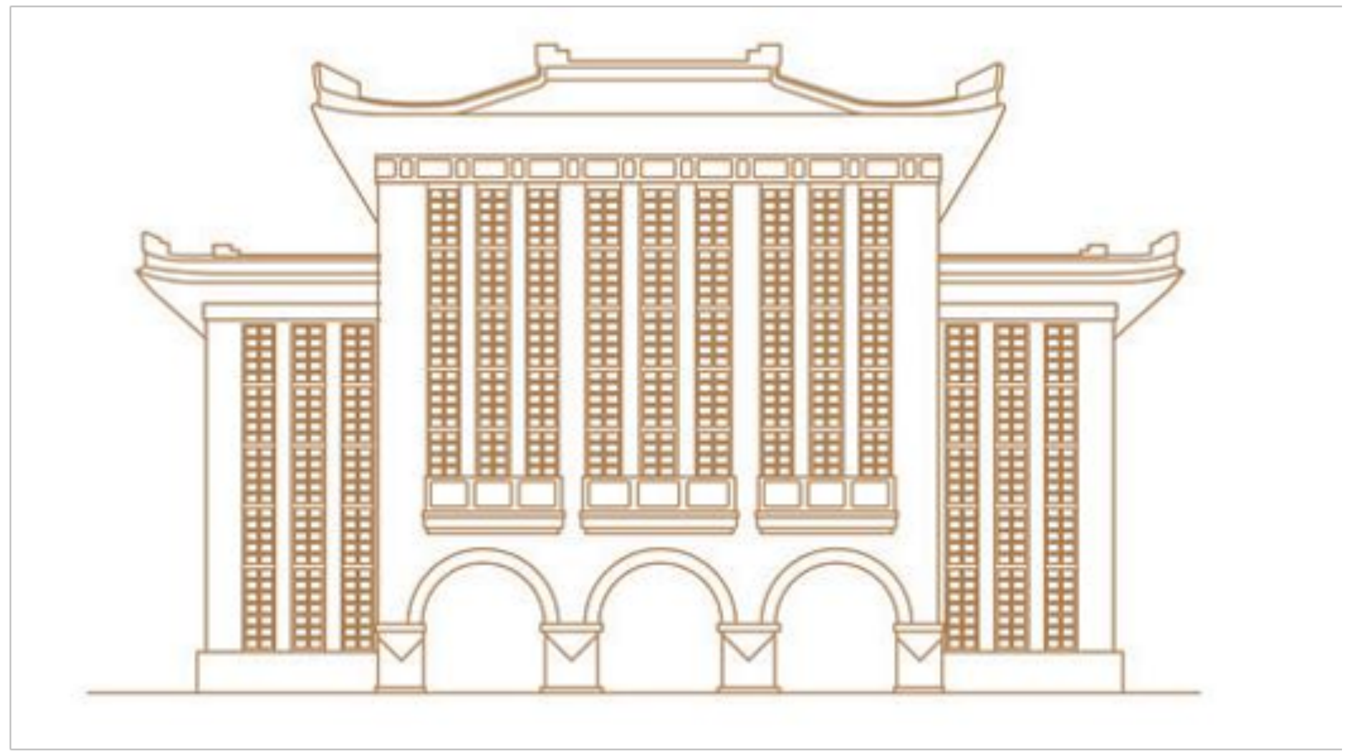
姚德良

Hunan University

July 23-29, 2023, Qingdao



OUTLINE



1. Introduction
2. Progress of BChPT with EOMS scheme
3. Prospect of BChPT
4. A two-loop calculation of nucleon mass
5. Summary and outlook


量子色动力学的有效场论
手征有效理论
非相对论性量子色动力学(NRQCD)
高密度有效理论
重强子手征微扰论
夸克偶素
有限温度
有限密度
软共线有效理论I
单重子
不稳定粒子有效场论
顶夸克
量子色动力学
喷注
软共线有效理论II
核力
模型无关
第一性原理出发

ChPT: EFT of QCD at low energies

✦ Facets of QCD — Asymptotic Freedom & Color Confinement

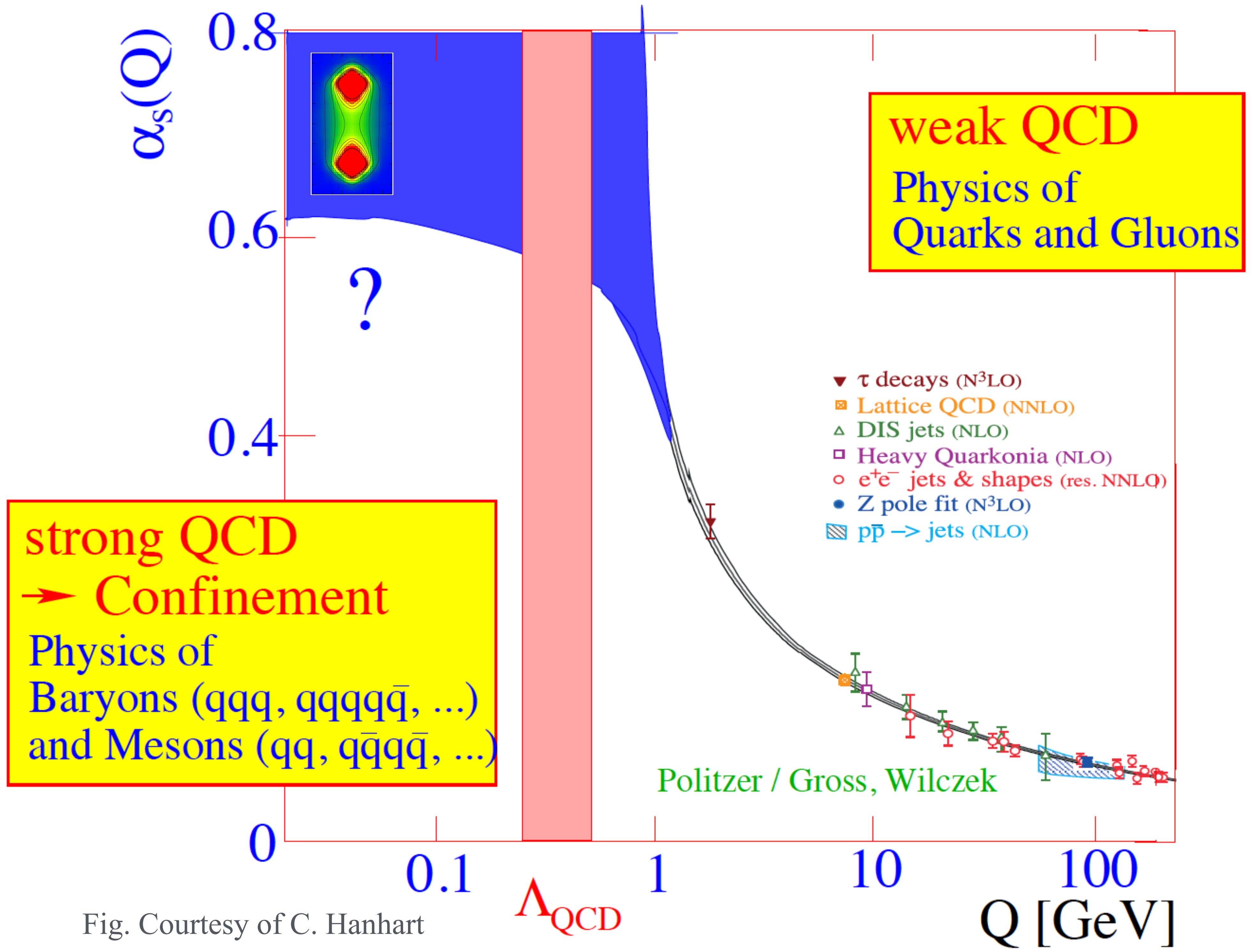
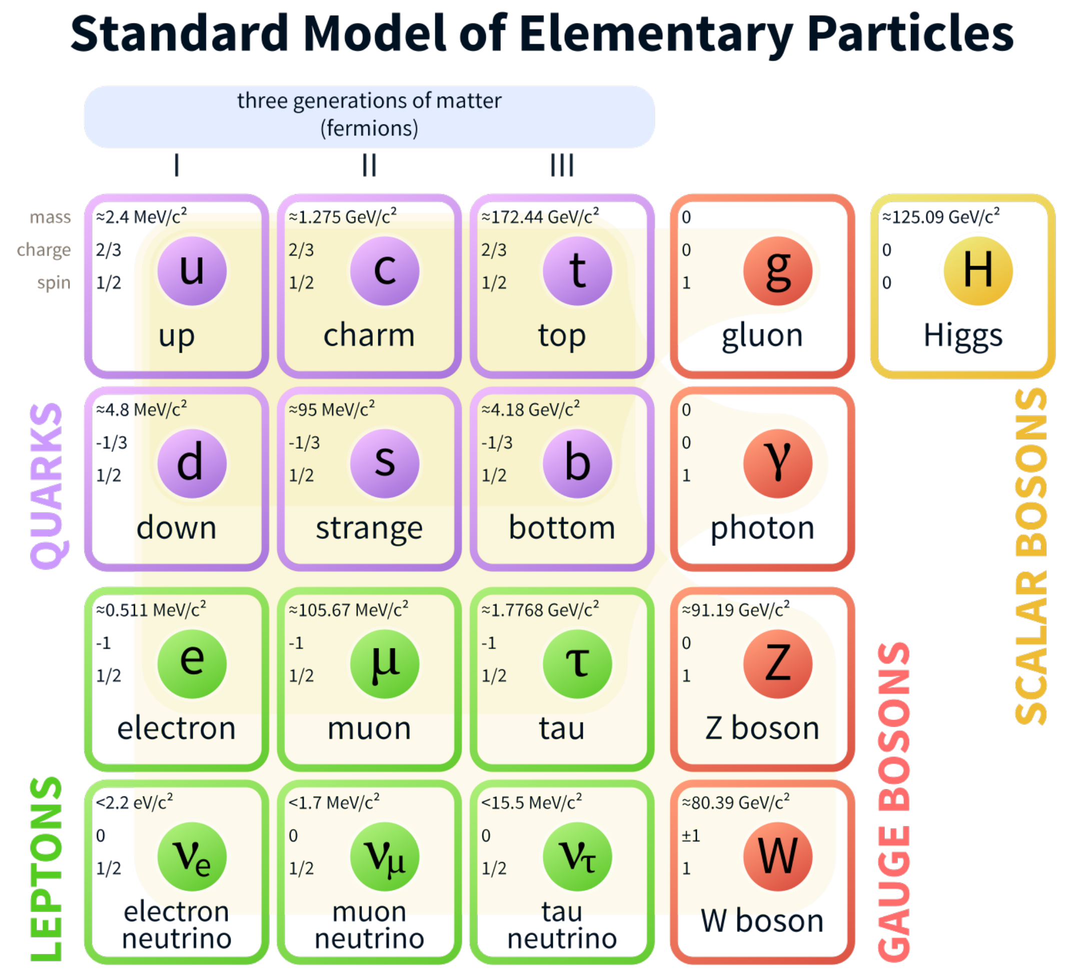
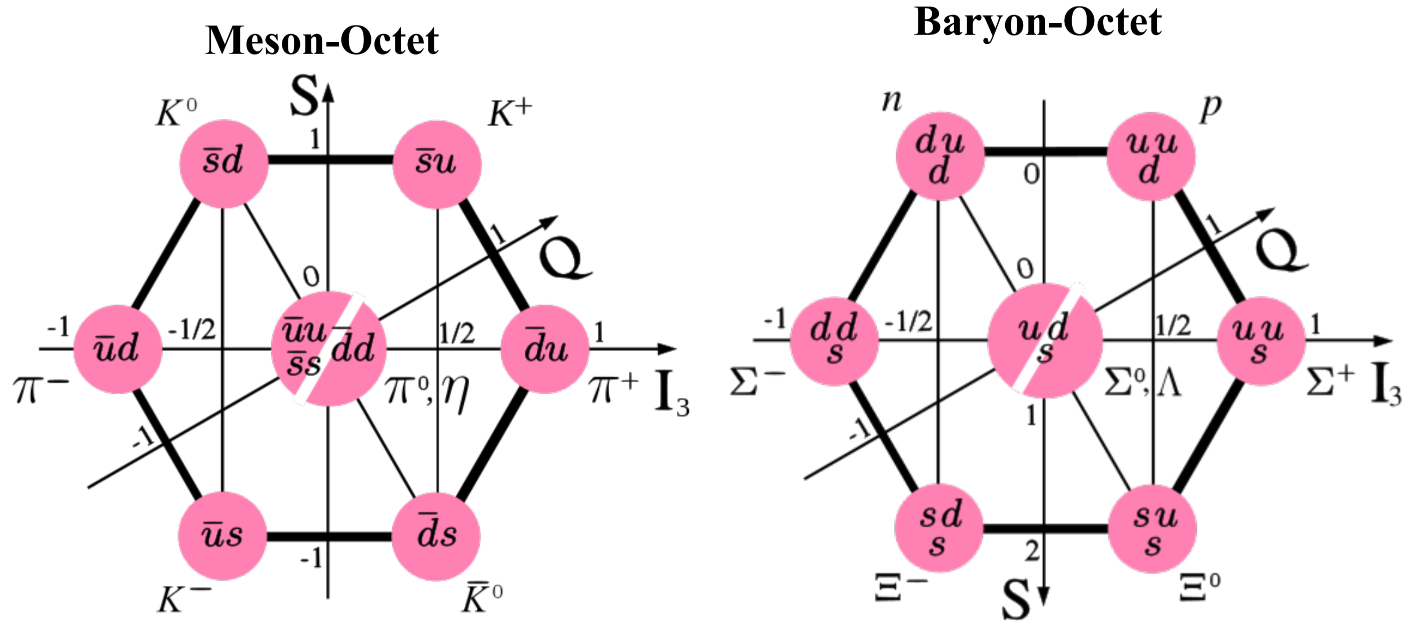


Fig. Courtesy of C. Hanhart

Quarks ⇨



Hadrons ⇨



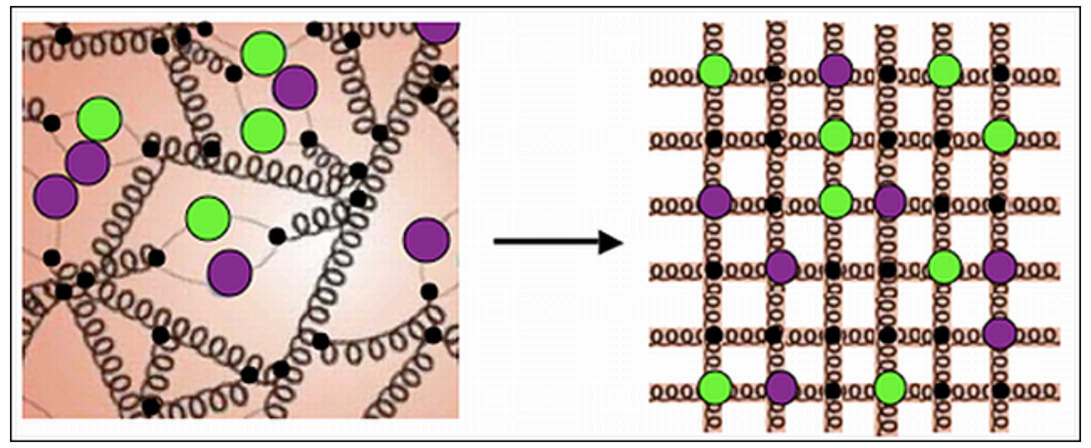
✦ Low-energy region — Quarks are glued together by gluons to form hadrons

Phenomenological Models

⇩

- **Linear Sigma Model**
- **Nambu-Jona-Lasinio Model**
- **Jülich Model**
- **…**

Lattice QCD



Effective Field Theories

⇩ Chiral Symmetry

- **Chiral Perturbation theory (ChPT)**

➡ hadron fields as degrees of freedom

➡ Expansion in masses and external momenta

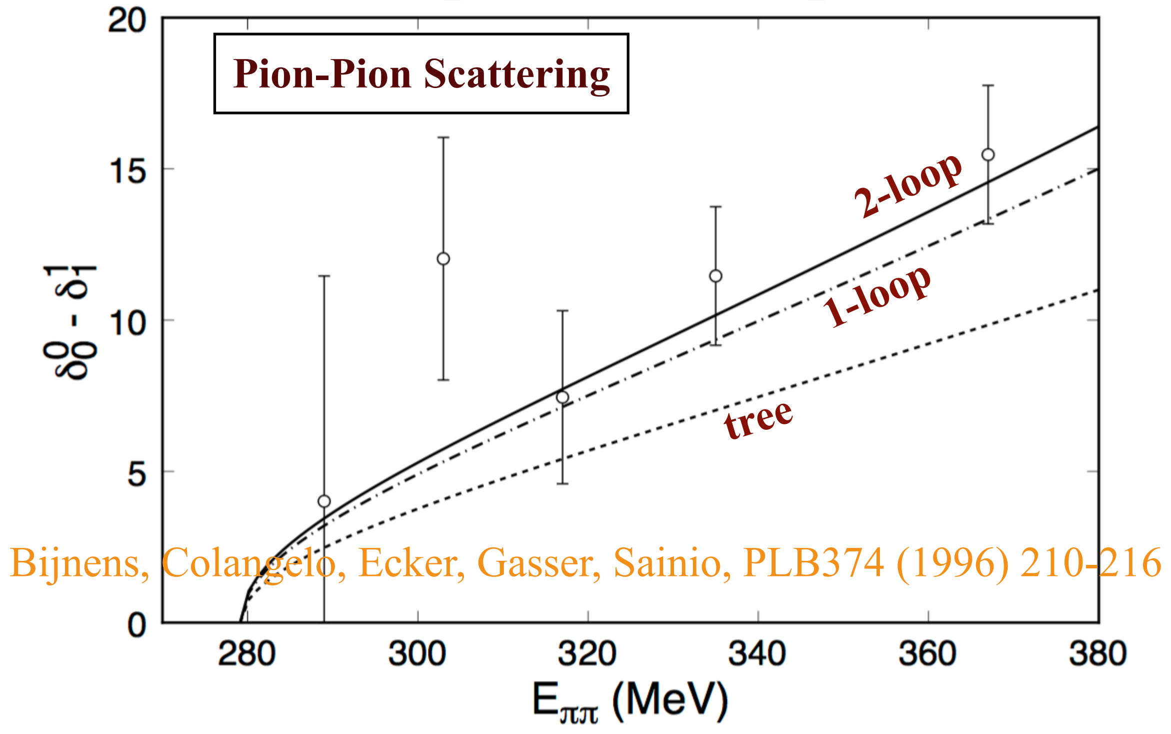
BChPT: Power Counting Breaking (PCB) problem

✦ Pure Goldstone Bosons: ChPT has gained great achievements

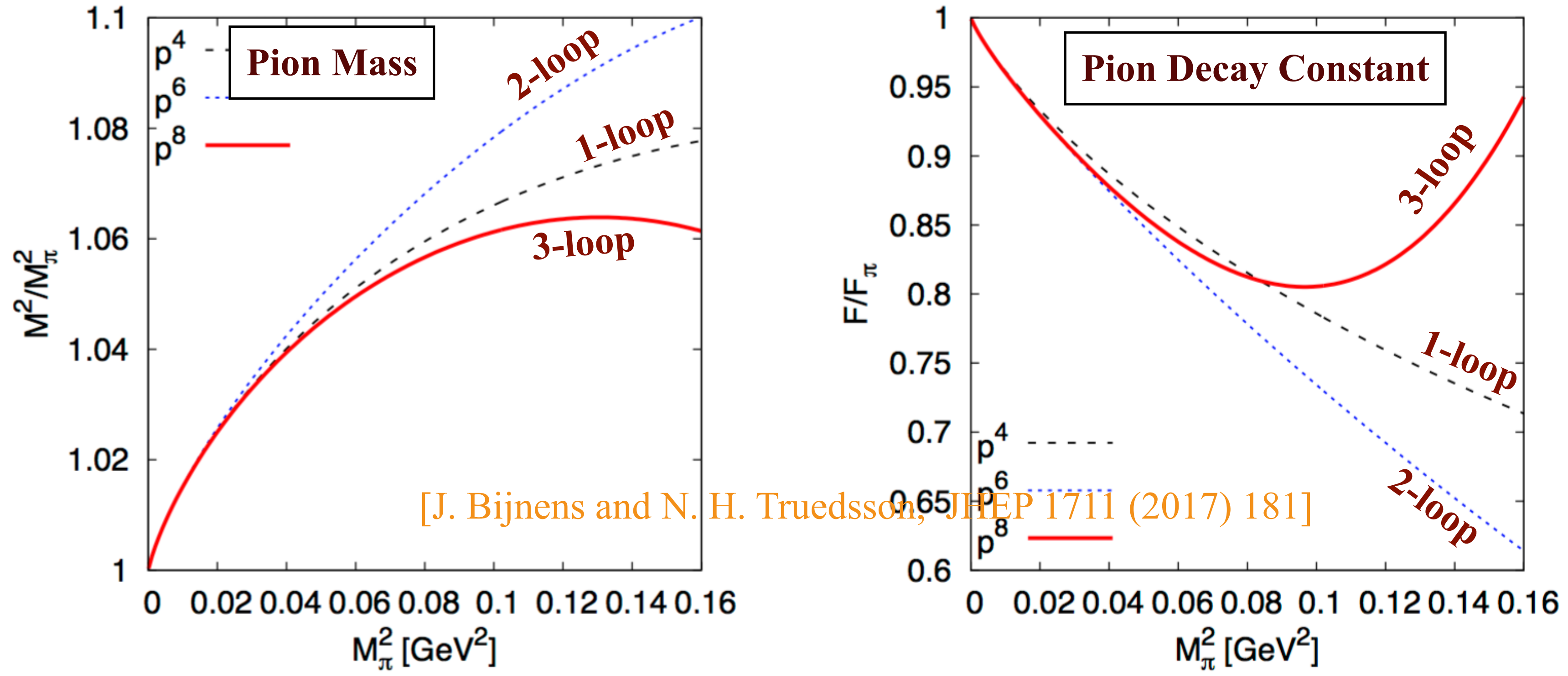
➡High-order calculations become standard

➡Fast convergence

up to Two-Loop



up to Three-Loop



✦ Covariant ChPT including matter fields (Baryons, D/B mesons)

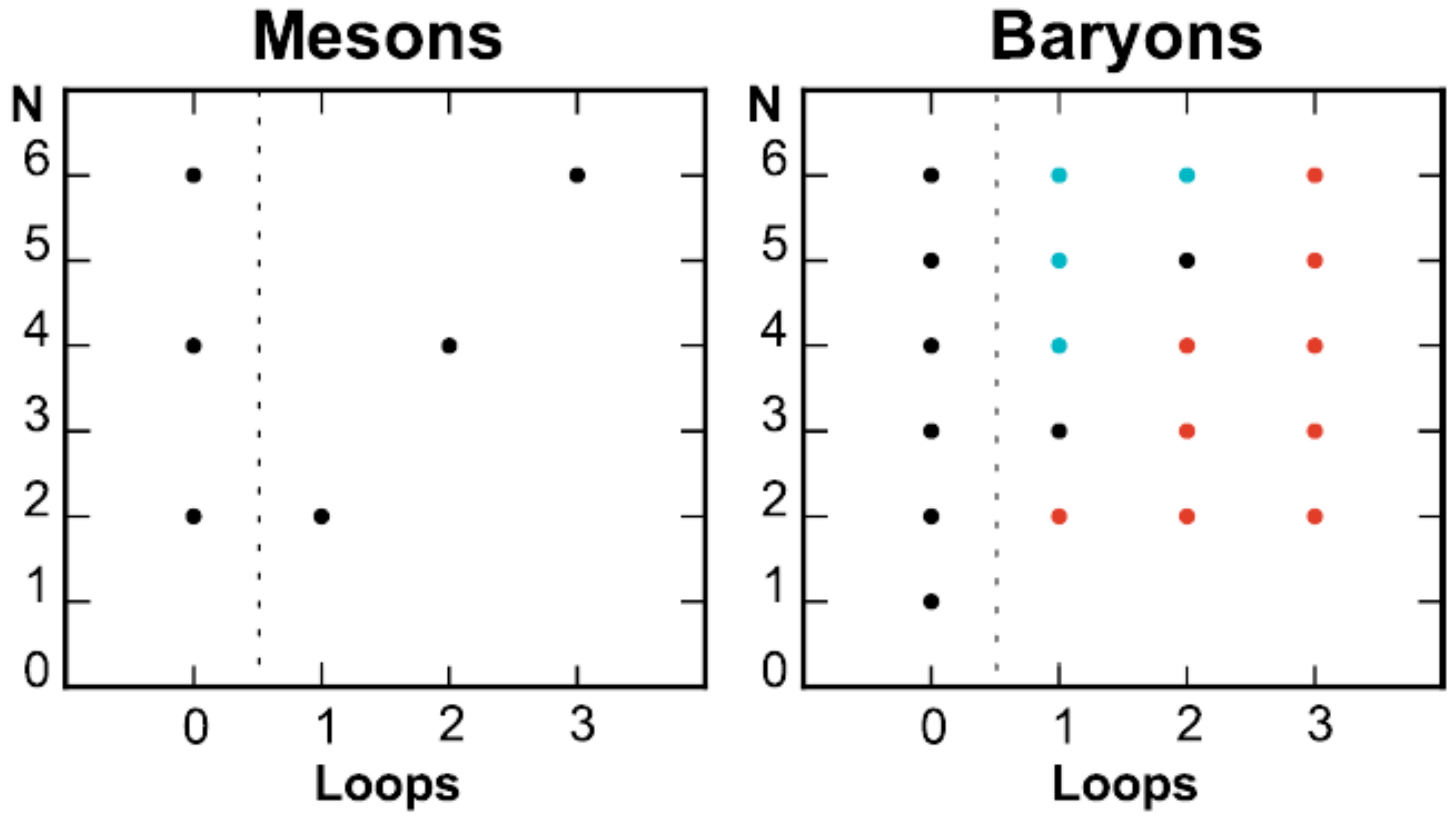
- **Dimensional Regularization (DR) with standard MSbar-1 subtraction**
- **A systematic power counting rule is lost due to the non-zero mass of matter fields in the chiral limit**

Feynman Diagram

⇩

$\mathcal{O}(p^N)$

How important?

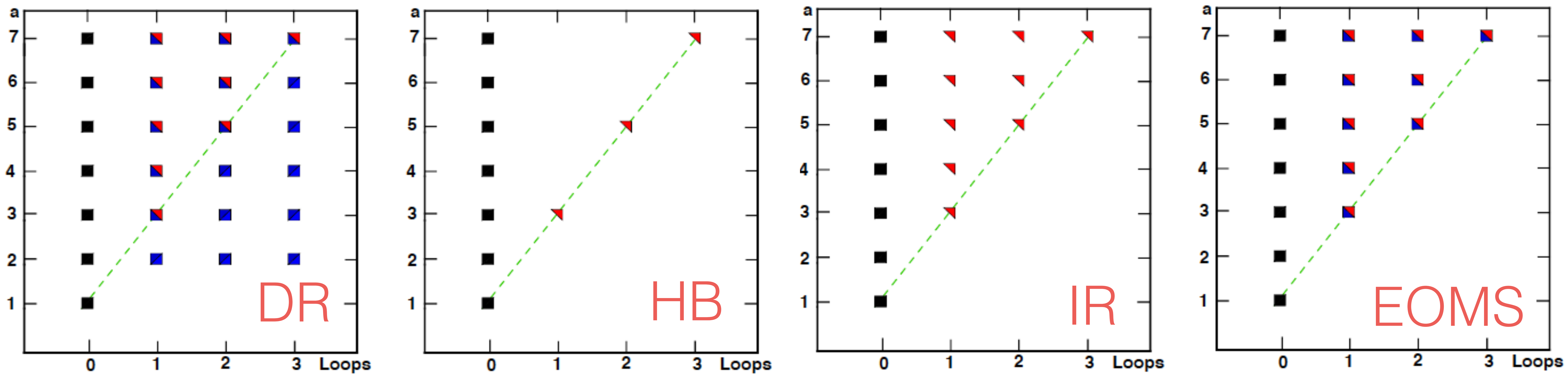


Chiral order: $N = 4L - 2N_M - N_B + \Sigma_k k V_k$

red dots denote possible PCB terms

➡Is there a solution?

BChPT: HB approach, IR prescription & EOMS scheme



✦ Heavy Baryon ChPT (HBChPT):

[Jenkins and Manohar,PLB255'91]

A simultaneous expansion in external momenta and 1/mB.

— Non-covariant and slowly convergent in the threshold region.

[N.Fettes,Ulf-G.Meissner and S.Steininger, NPA'98], [M.Mojˇziˇs,Eur.Phys.J.C2'98]

— Even divergent in the sub-threshold region (e.g. scalar form factor).

[V.Bernard,N.Kaiser and Ulf-G.Meissner,Int.J.Mod.Phys.E4'95], [T.Becher and H.Leutwyler,Eur.Phys.J.C9'99]

✦ Infrared Regularization (IR):

[T.Becher and H.Leutwyler,Eur.Phys.J.C9'99]

The full integral is separated into Infared singular and ~~Regular parts.~~

—Scale-dependence: amplitude and observables. [T.Becher and H.Leutwyler,JHEP0106'01]

—Unphysical cuts(u=0) [J.M.Alarcon,J.Martin Camalich,J.A.Oller and L.Alvarez-Ruso,PRC83'11]

—Bad predictions: e.g., huge Goldberger-Treiman relation violation (20-30%).

[J.M.Alarcon,J.Martin Camalich,J.A.Oller and L.Alvarez-Ruso,PRC83'11]

✦ Extended-on-mass-shell (EOMS): [T.Fuchs,J.Gegelia,G.Japaridze and S.Scherer,PRD68'03]

Is it an approach to solve the above problems?

EOMS scheme

✦ **EOMS is a two-step renormalization scheme**

step 1: **Ultraviolet (UV) divergence**
Dimensional regularization
MSbar-1 subtraction

→ **finite ren.** →

step 2: **PCB terms**
(polynomials)
removed by redefinition of LECs
in Effective Lagrangians

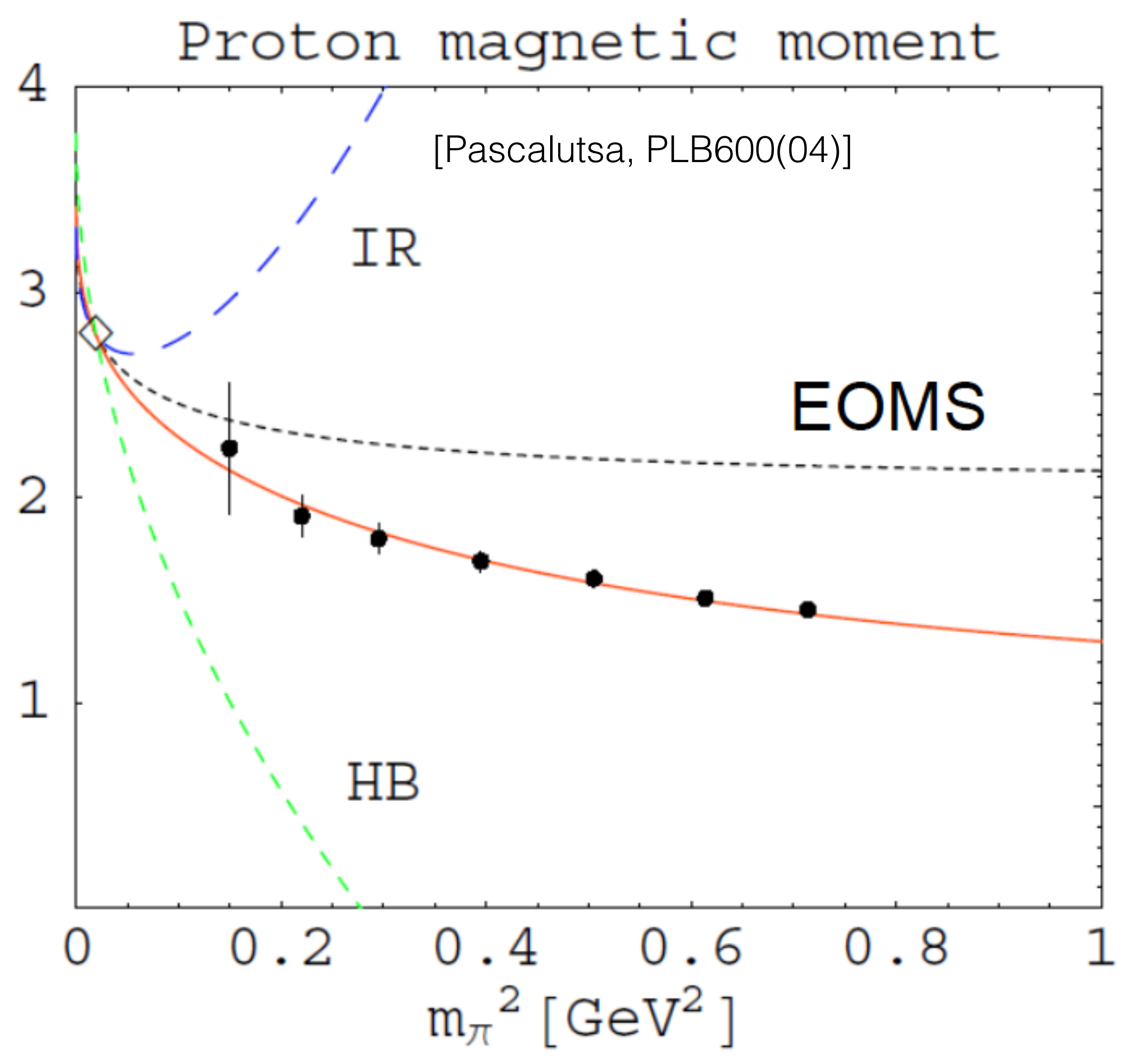
↓ Scale independent

↓ Correct power counting
(respectively faster convergence)

↓ keep original analyticity
and all assumed symmetries

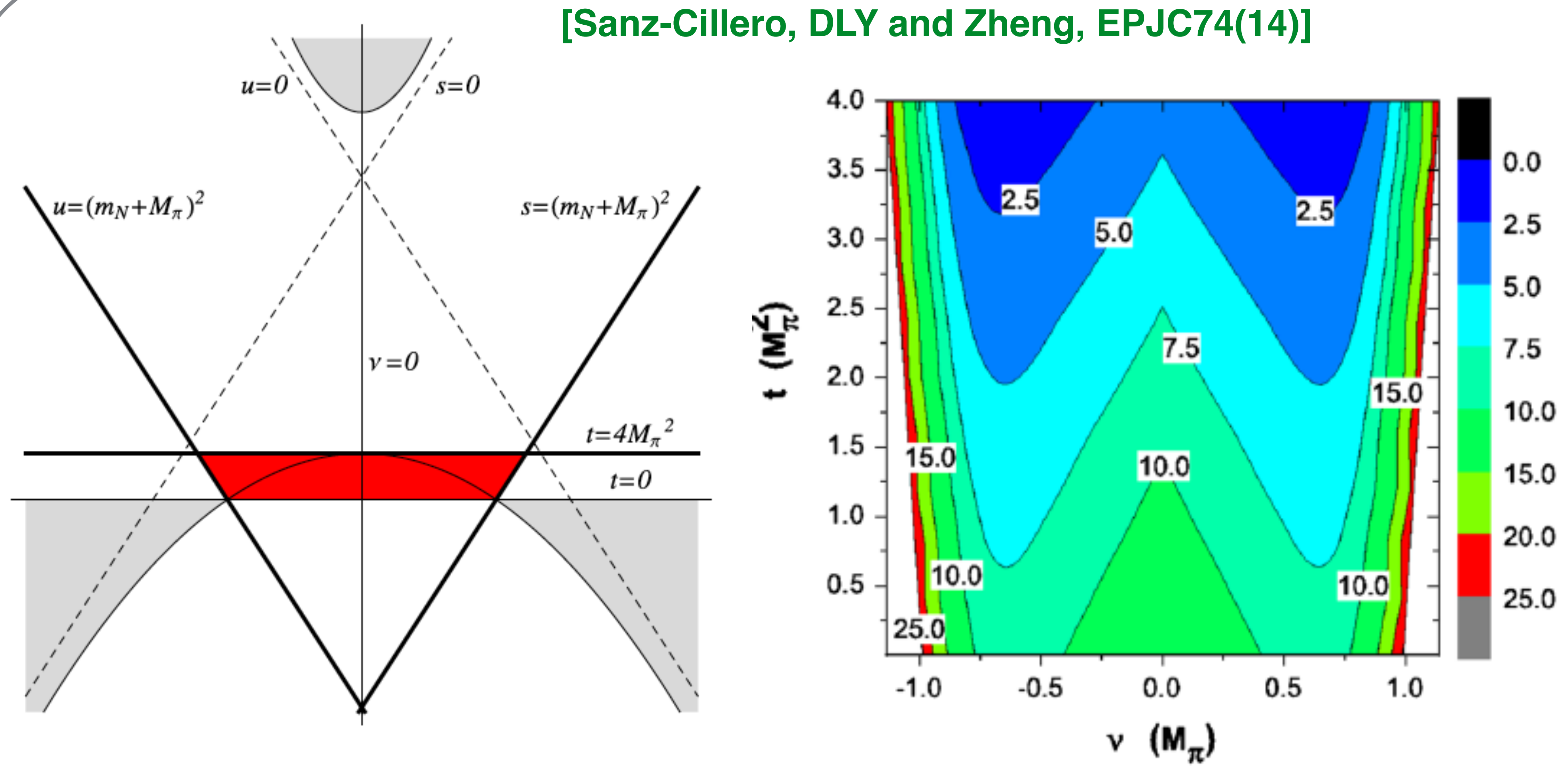
✦ **…related to other theories:**

Lattice QCD



Chiral extrapolation

Dispersion Theory



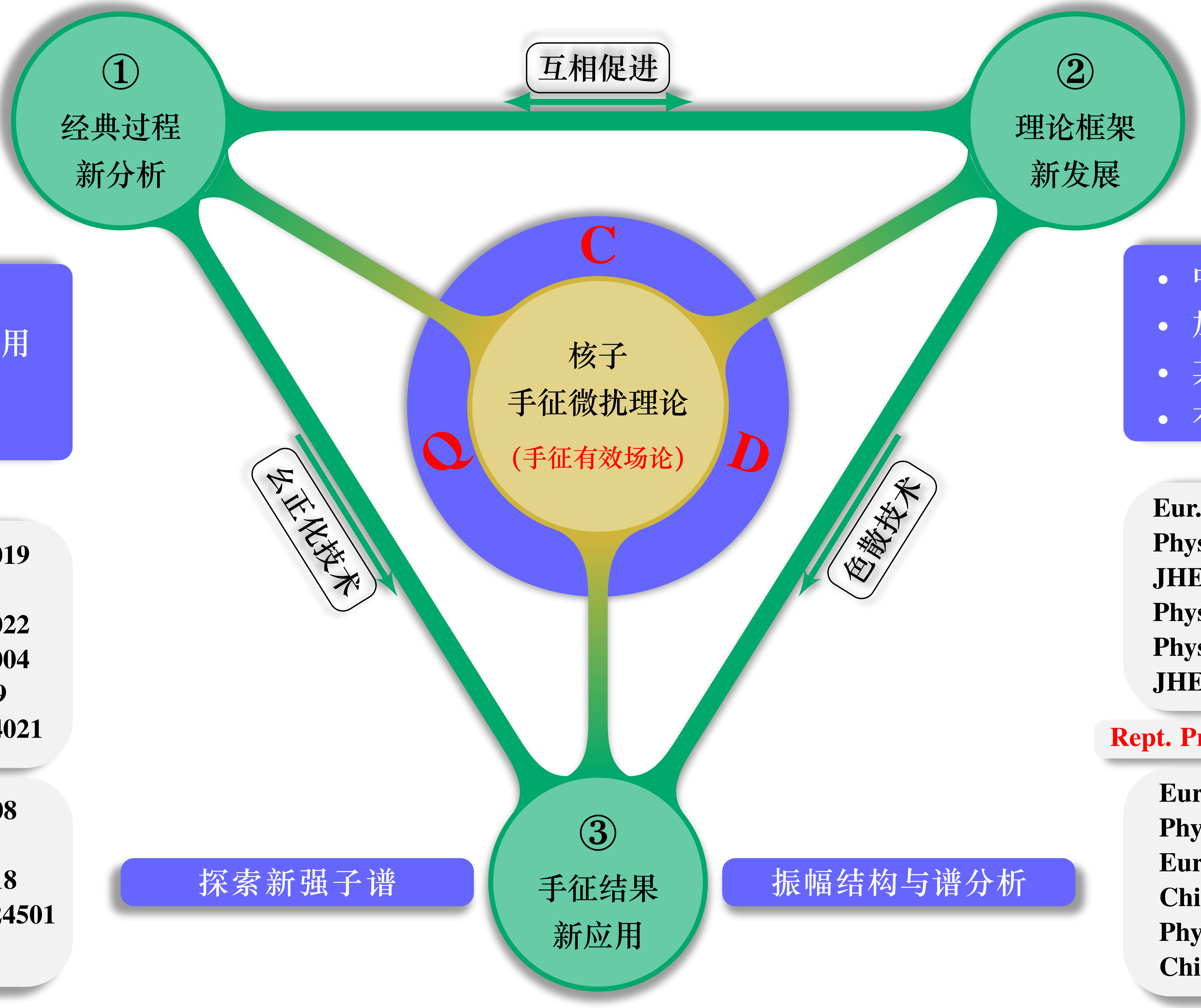
Dispersion Relation -> Positivity Bounds

Progress of BChPT with EOMS scheme

- 介子核子散射
- 核子电磁相互作用
- 中微子核子散射
- 介子光生过程

Phys.Rev.D87(2013)054019
JHEP1605(2016)038
Phys.Rev.D96(2017)116022
Phys.Rev.D98(2018)076004
Phys.Lett.B794(2019)109
Phys.Rev.D100(2019)094021

Phys.Rev.D92(2015)094008
Eur.Phys.J.C77(2017)728
Phys.Rev.D98(2018)094018
Front.Phys.(BJ)14(2019)24501
Eur.Phys.J.C80(2020)433



- 中高能区域推广
- 加入不稳定粒子
- 其它重轻系统
- 有限体积修正效应

Eur.Phys.J.C75(2015)499
Phys.Lett.B760(2016)736
JHEP11(2015)058
Phys.Rev.D97(2018)034012
Phys.Rev.D103(2021)034006
JHEP12(2022)029

Rept. Prog. Phys. 84, 076201 (2021)

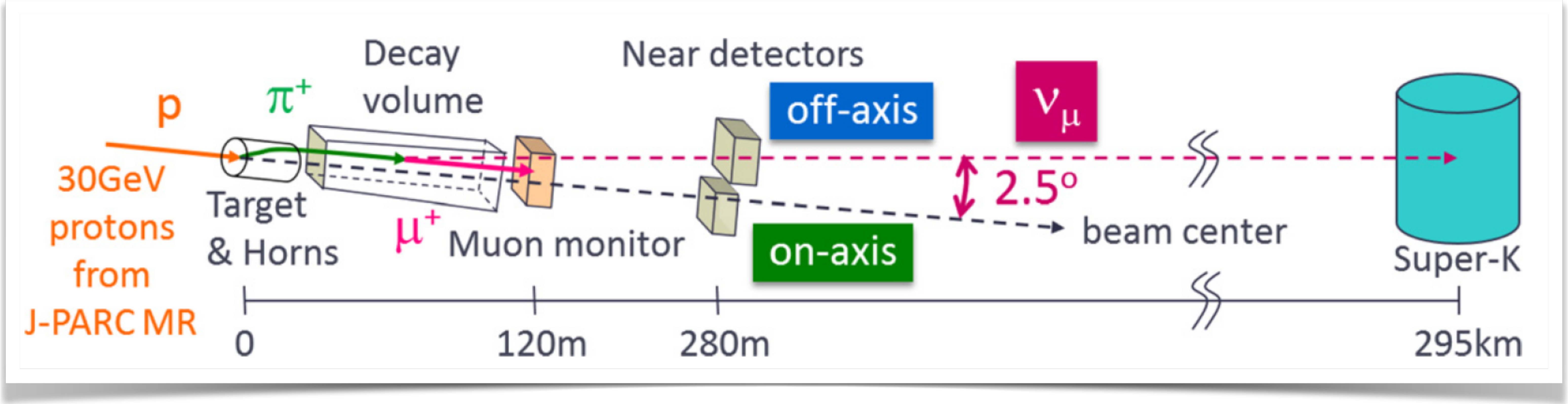
Eur.Phys.J.C74(2014)2763
Phys.Rev.D94(2016)094037
Eur.Phys. J.C78(2018)310
Chin.Phys.C43(2019)064110
Phys.Rev.D101 (2020)034014
Chin.Phys.C45(2021)014104

Selected Progress I: Applications in Neutrino Physics

✦ Oscillation experiments (e.g T2K)

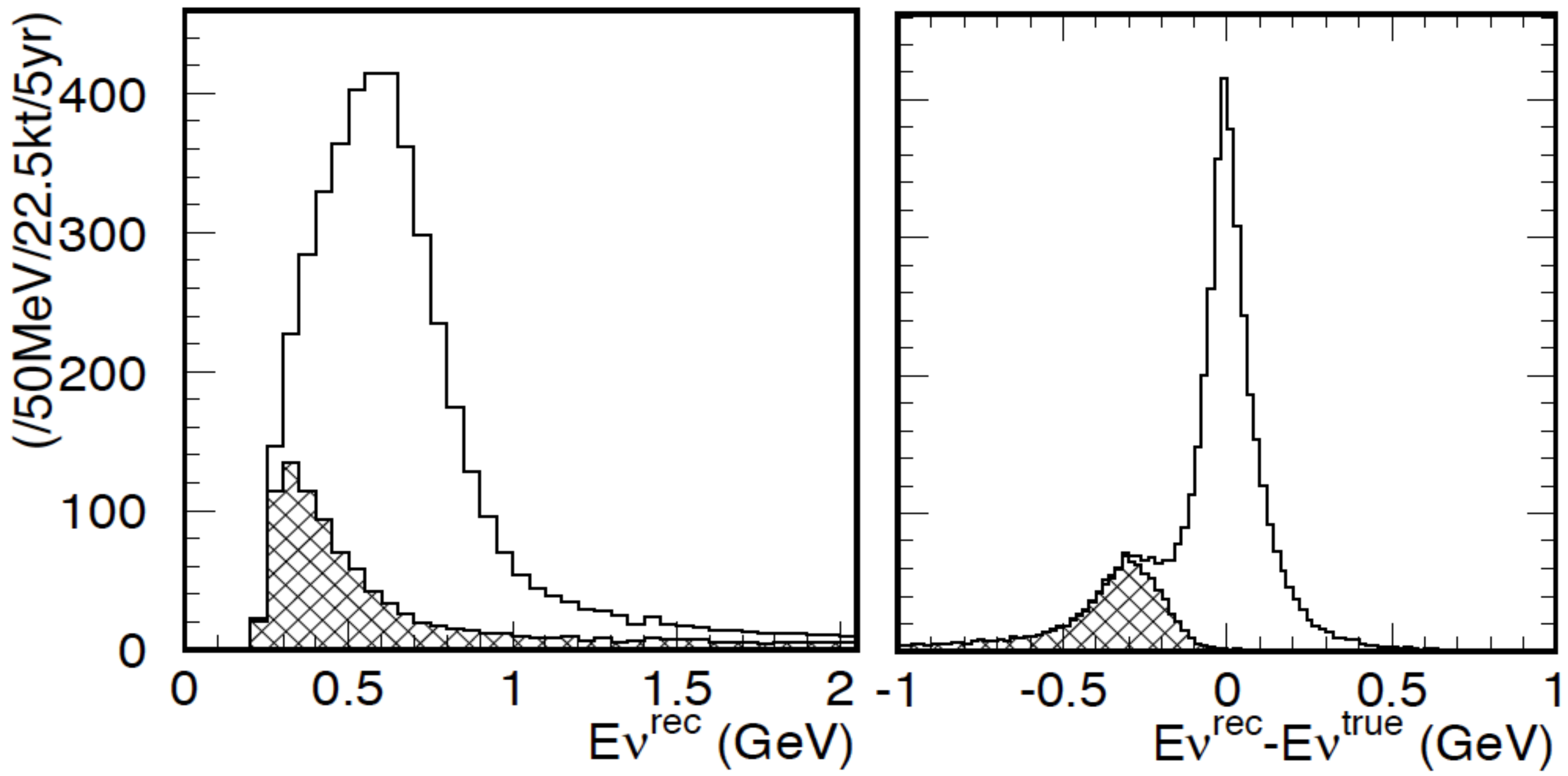
survival probability of ν_μ

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta_{\mu\tau} \cdot \sin^2 \frac{\Delta m_{23} L}{E_\nu}$$



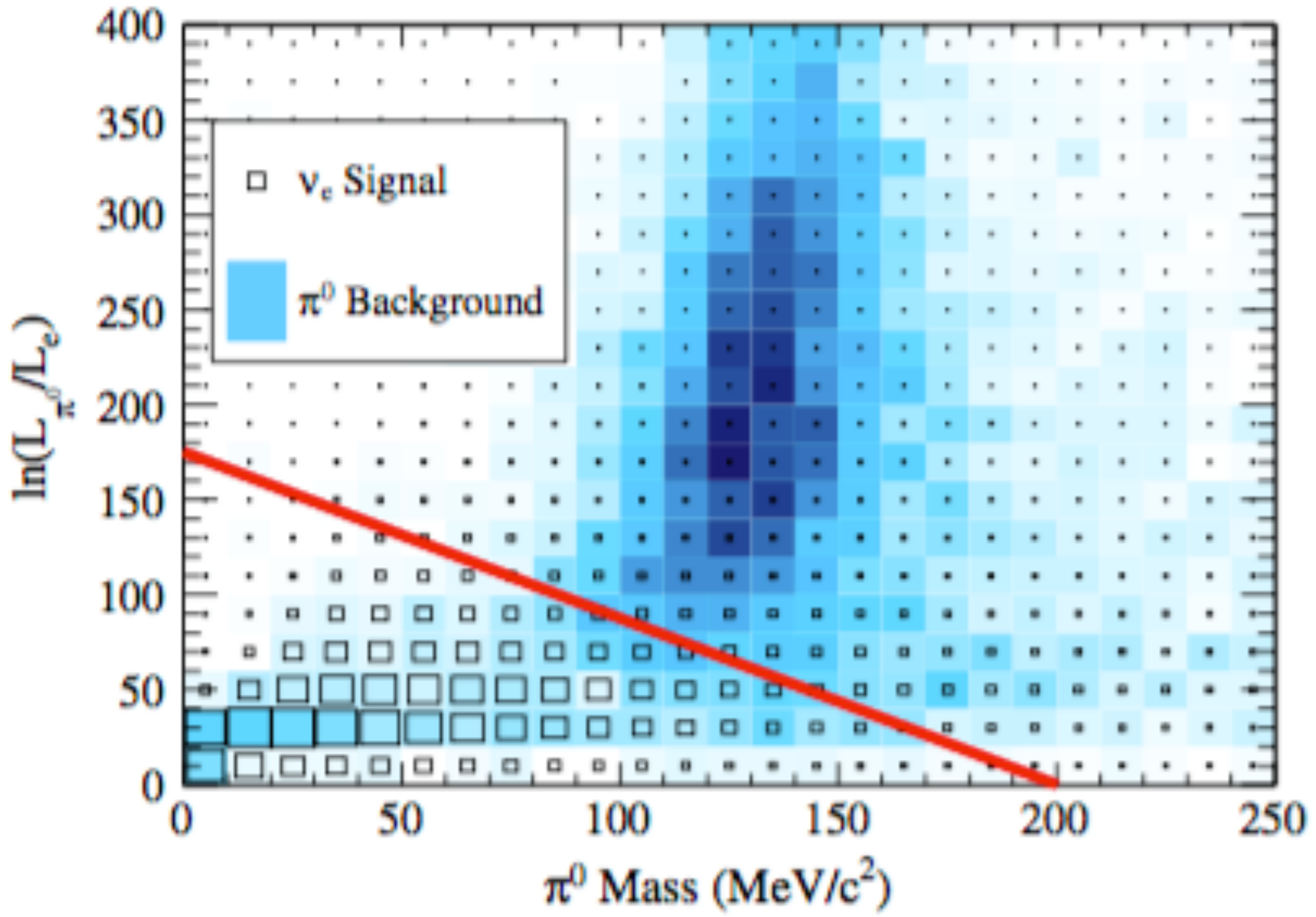
✦ CC1π:

- Source of CCQE-like events
  - ➡Misidentification of pion
- to be subtracted for a good Eν reconstruction



✦ NC1π:

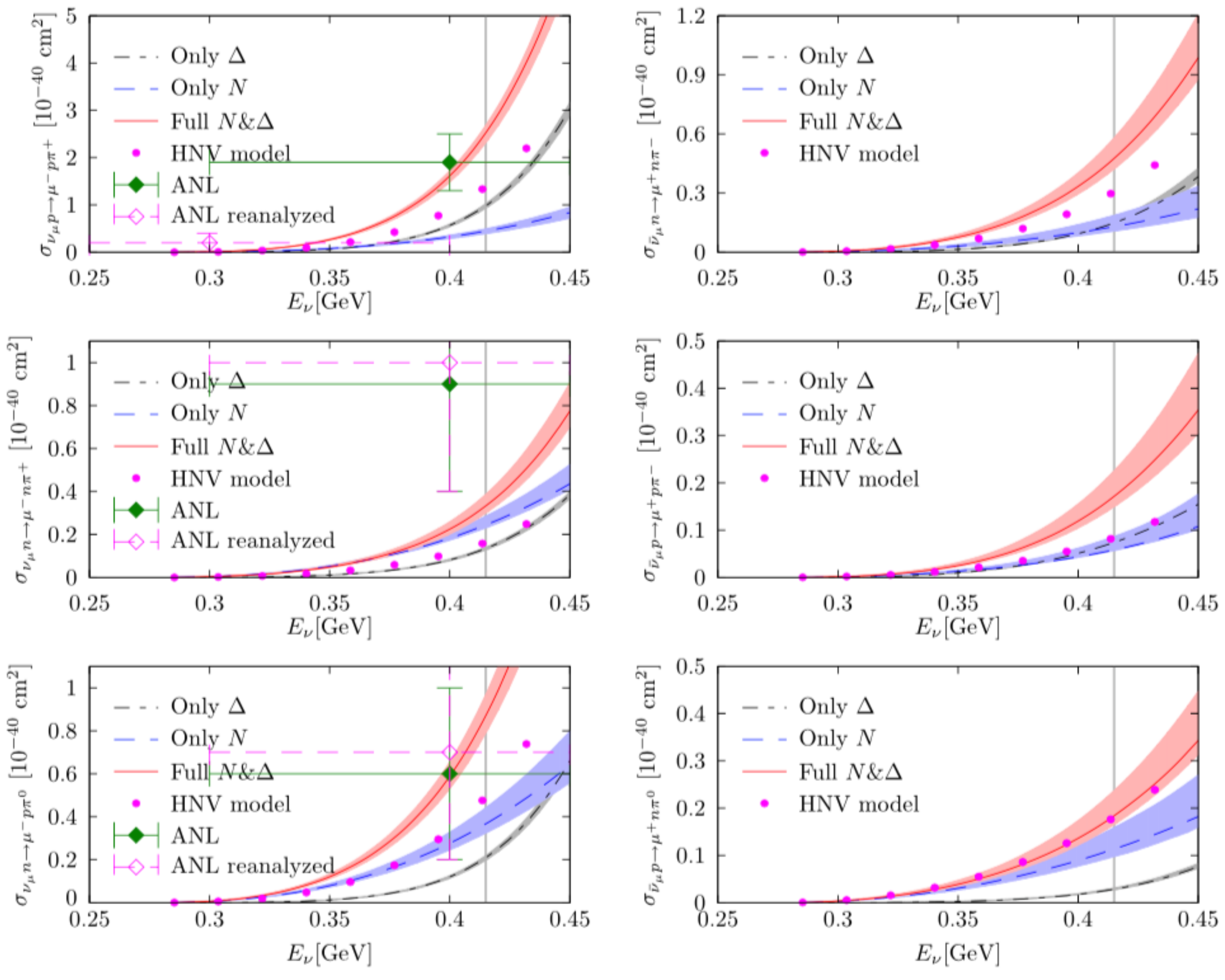
- e-like background to $\nu_\mu \to \nu_e$ searches
- Improved at T2K with a π^0 rejection cut



Selected Progress I: Applications in Neutrino Physics

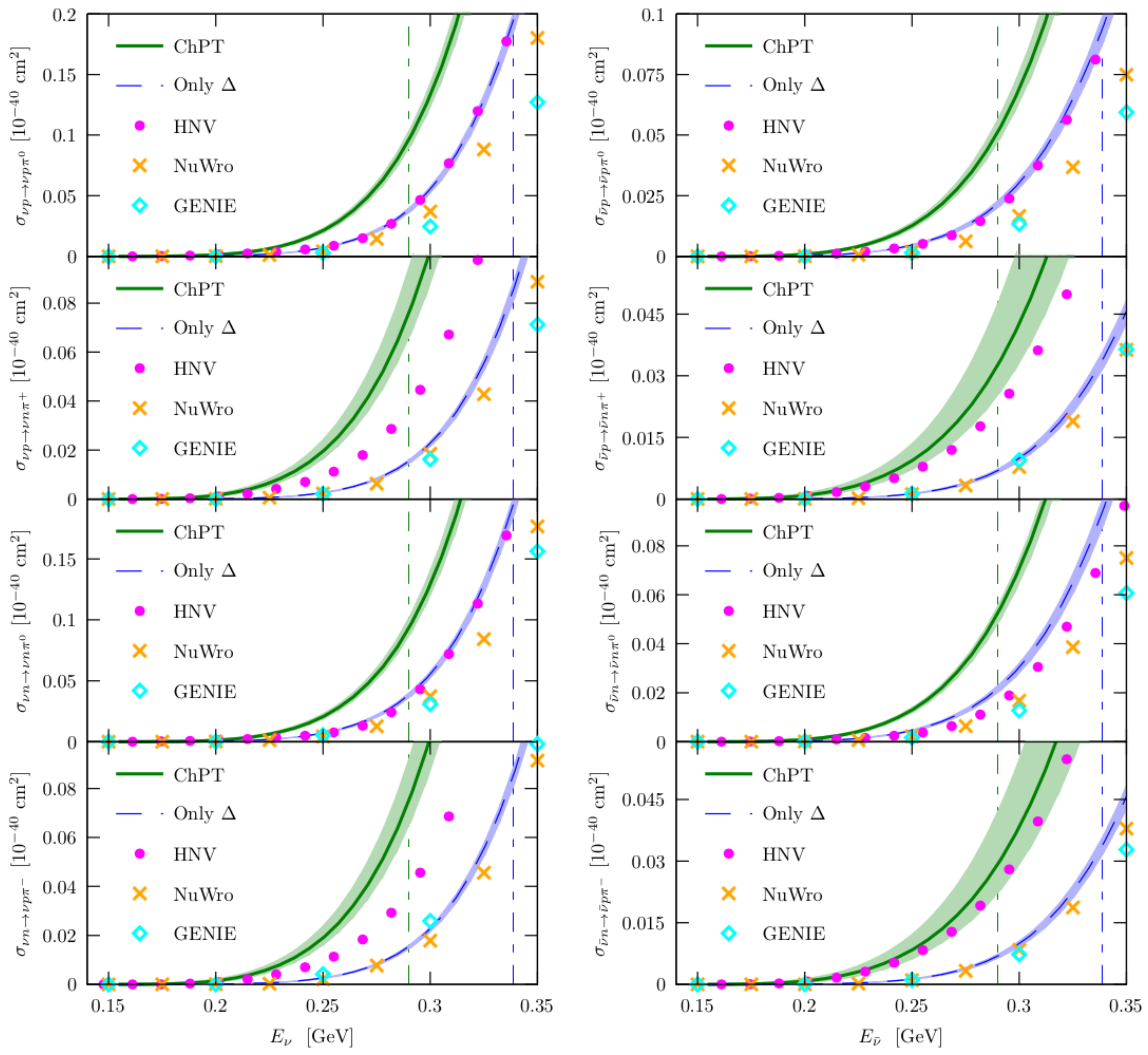
✦ First results from relativistic BChPT

ν-CC1π **anti-ν-CC1π**



[DLY, Alvarez-Ruso, Hiller-Blin and Vicente-Vacas, PRD98(2018)076004]

ν-NC1π **anti-ν-NC1π**



[DLY, Alvarez-Ruso and Vicente-Vacas, PRD794(2019)109]

✦ More outcomes from BChPT:

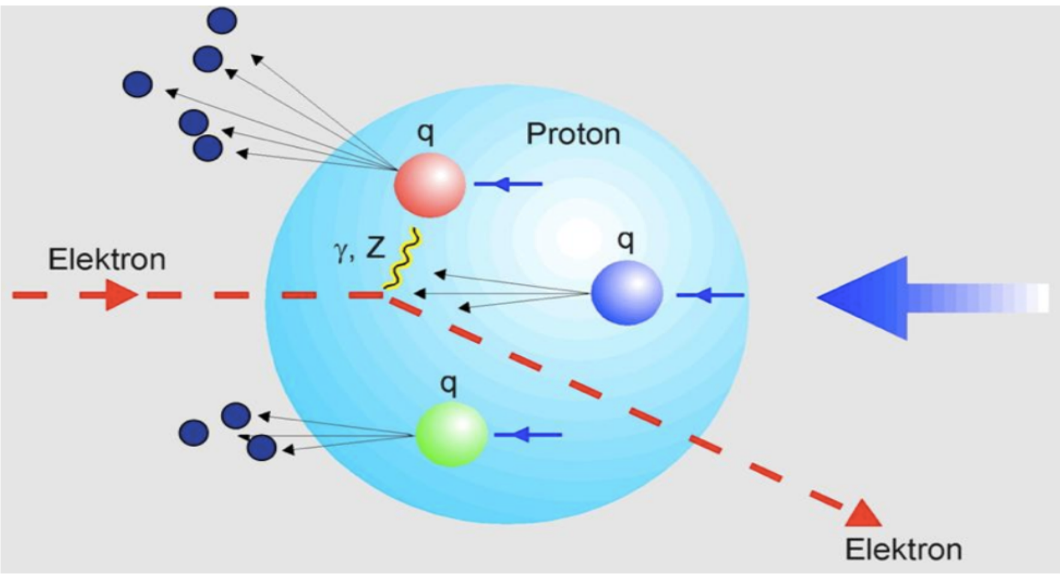
➡Extended to intermediate energies.

➡Multi pion productions & Coherent pion productions (nuclear effects)

➡Production of pseudo-scalar with strangeness

➡The same game can be played for neutinoless beta decays?

➡Strangeness of the nucleon



Selected Progress II: Extensions in Heavy Flavour Physics

✦ Chiral potentials for charmed meson interactions from ChPT & Unitarization

✓ **NLO:** [Kolomeitsev & Lutz PLB582 (2004) 39] [Guo, et al, PLB641 (2006) 278] [Liu, et al, PRD87, 014508 (2013)] [Altenbuchinger, et al, PRD89, 014026 (2014)] ……

✓ **NNLO:** [Geng, et al, PRD82, 054022 (2010)] [DLY, Du, Guo & Meissner, JHEP11(2015)058 & EPJC77(2017)]

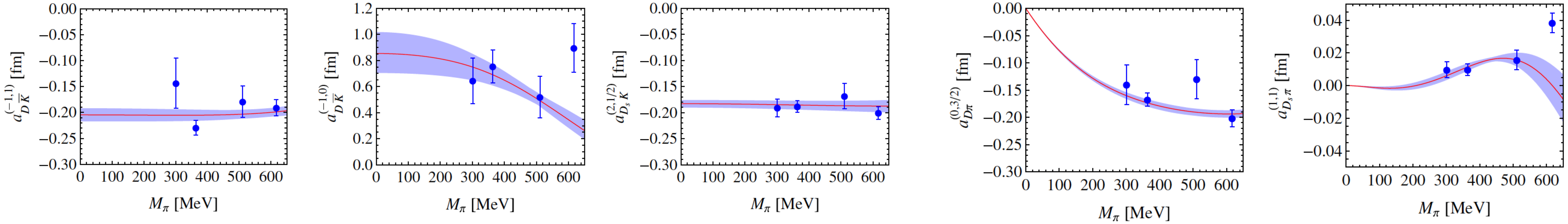
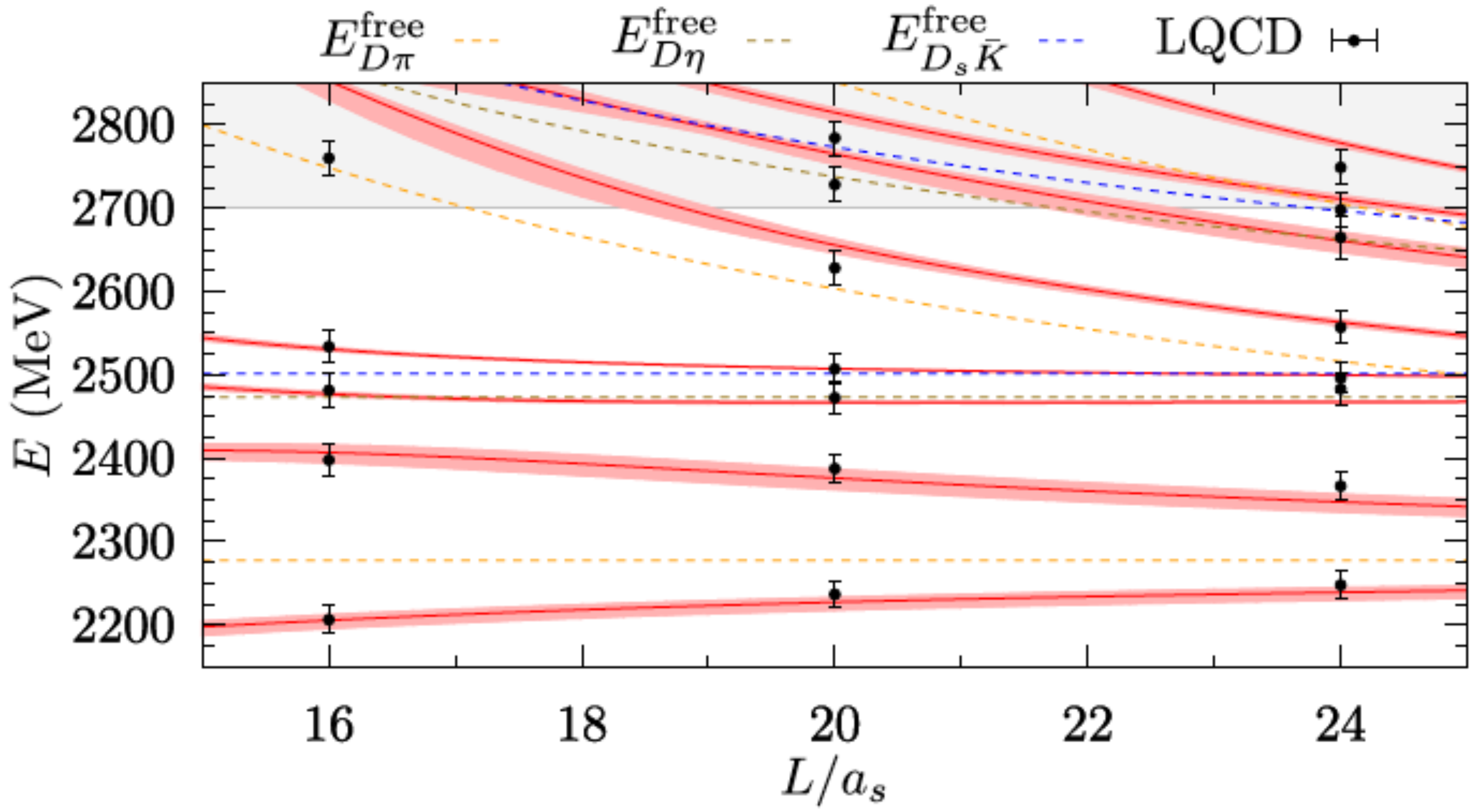


Fig from [Liu, et al, PRD87, 014508 (2013)]

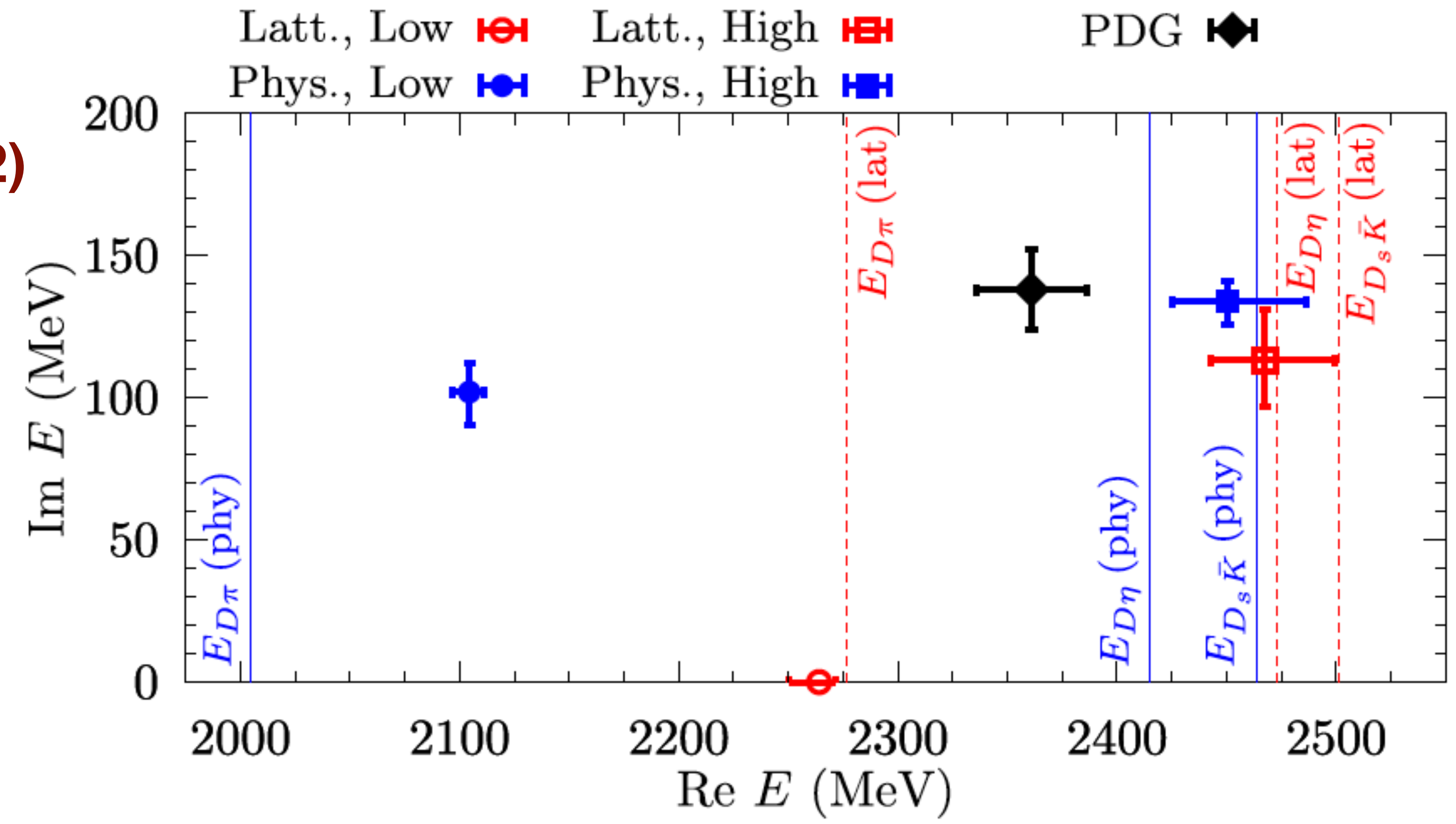
✦ Applications of NLO potentials by Liu, et al

Energy Level



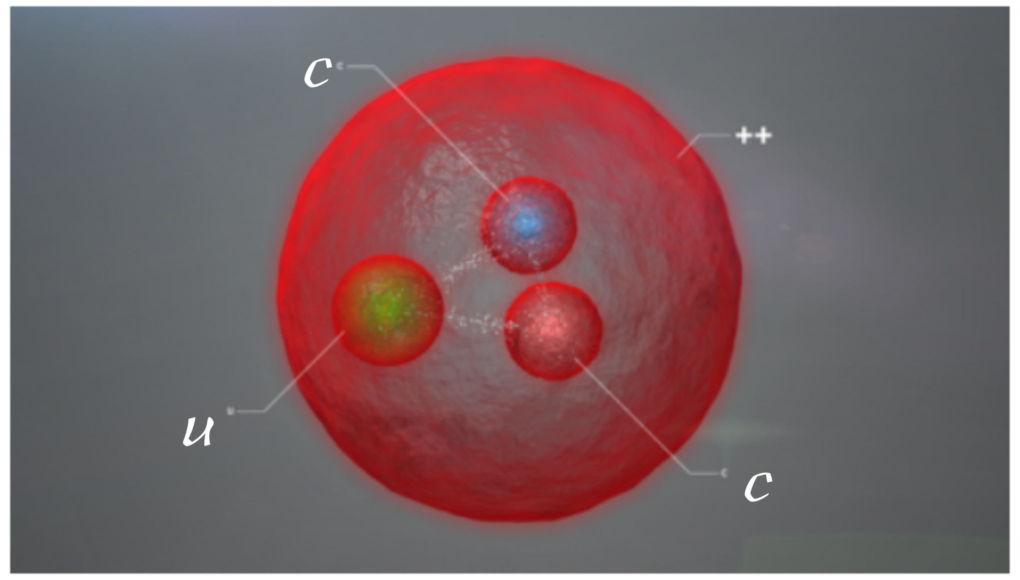
(S,I)=(0,1/2)

Two-pole structure of D*(2400)



Selected Progress II: Extensions in Heavy Flavour Physics

✦ **Time to study doubly charmed baryons (DCB)**



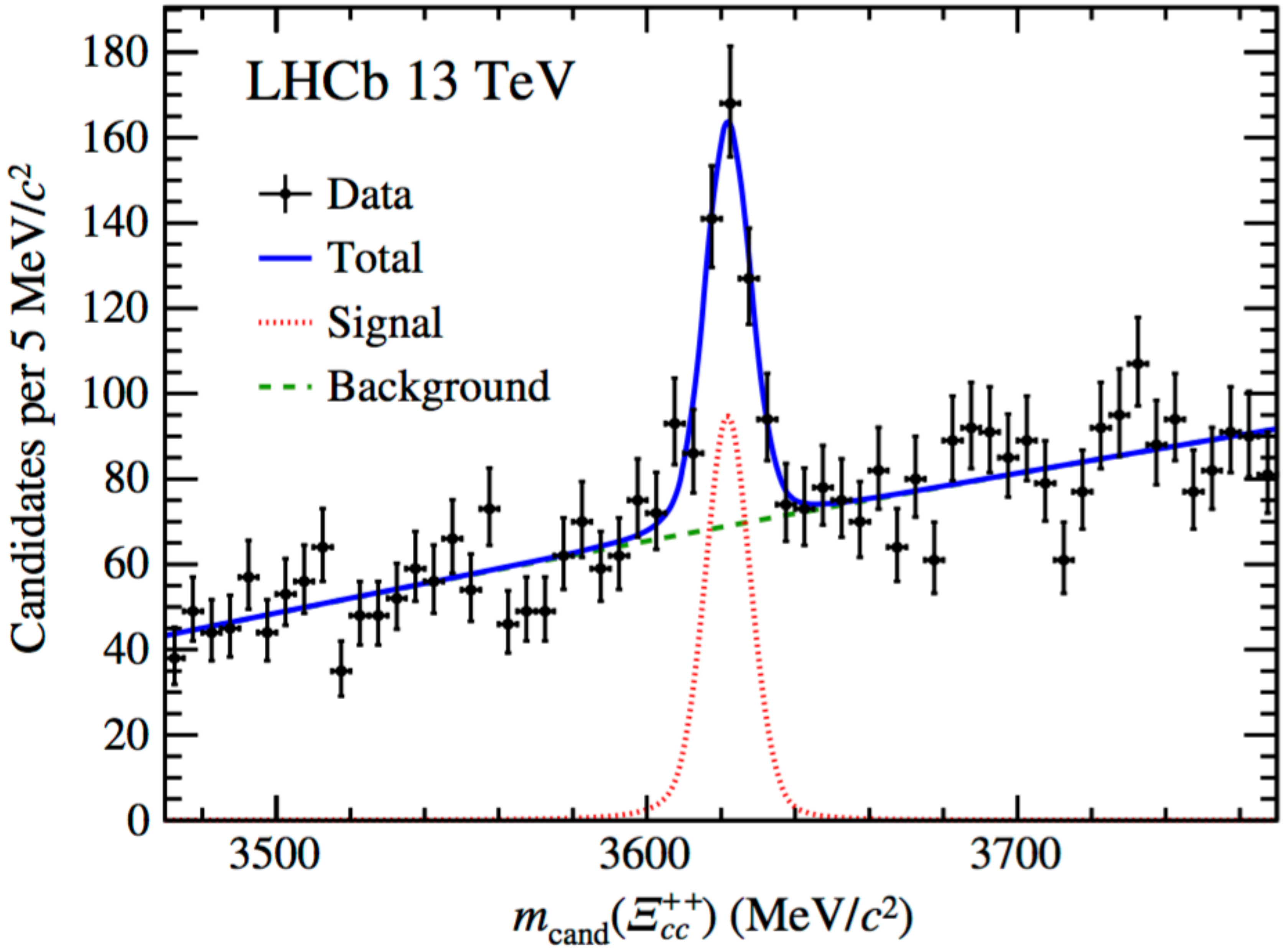
LHCb announces a charming new particle

The LHCb experiment has reported the observation of a baryon containing two charm quarks and one up quark

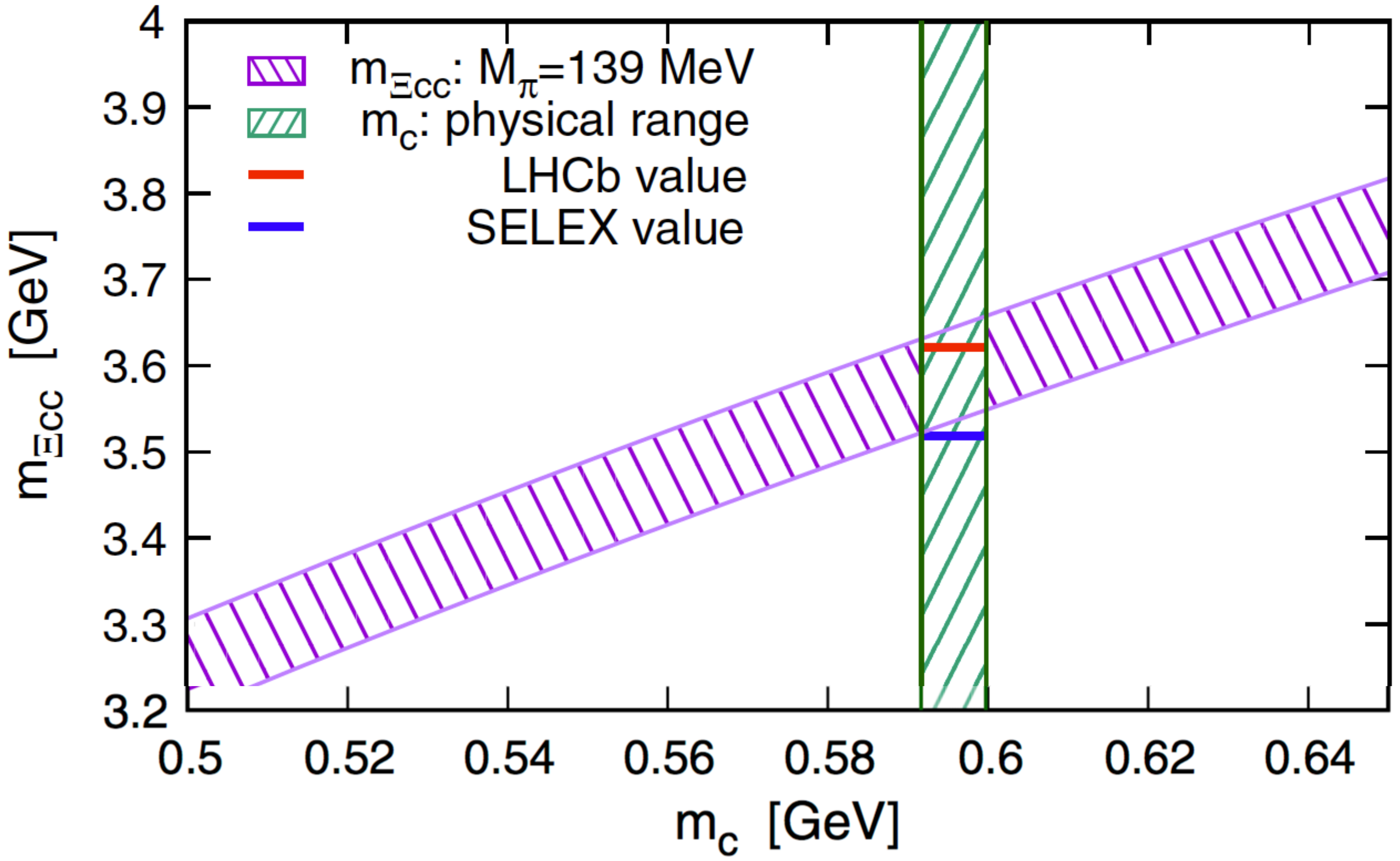
News | Experiments | 06 July, 2017

LHCb-2017: $m(\Xi_{cc}^{++}) = (3621.4 \pm 0.78)$ MeV

SELEX-2002: $m(\Xi_{cc}^{+}) = (3519 \pm 2)$ MeV



[LHCb, PRL119(2017)112001]



[DLY, PRD97(2018)034012]

✦ **Towards a new paradigm for negative-parity doubly charmed baryons?**

➡**Interactions between Goldstone Bosons and DCB**

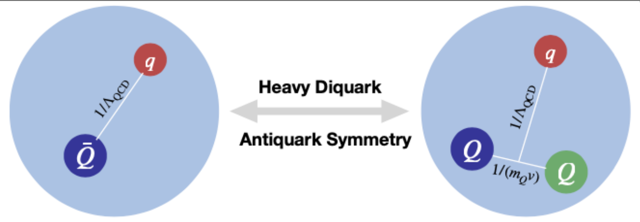
Selected Progress II: Extensions in Heavy Flavour Physics

✦ Interactions between Goldstone Bosons and DCB

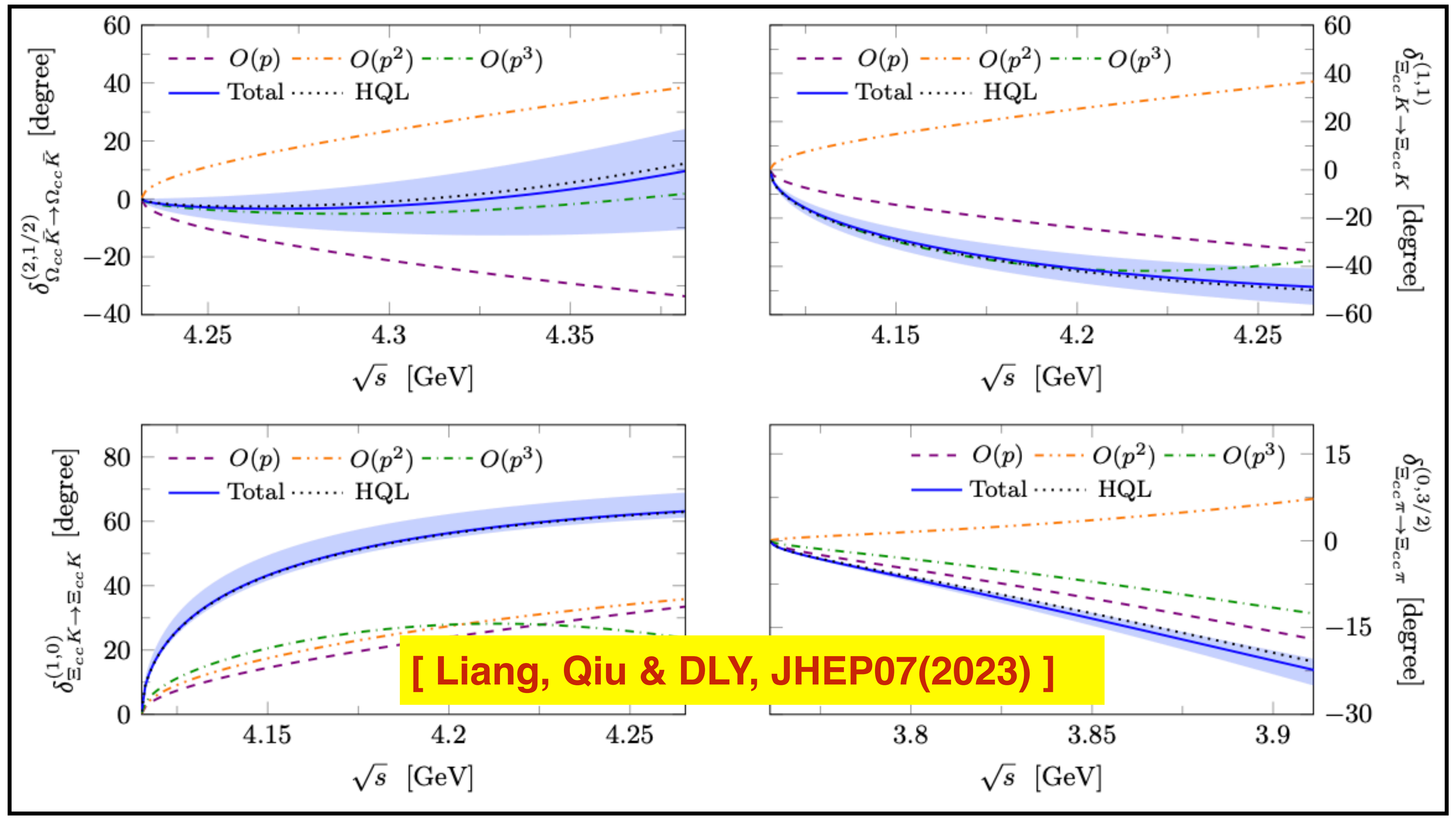
$$\mathcal{L}_{M\psi} = \bar{\psi}\Big\{\Big[(i\not{D} - m) + \frac{g_A}{2}\not{u}\gamma_5\Big] + \sum_{i=1}^{8} b_i \mathcal{O}_i^{(2)} + \sum_{j=1}^{32} c_j \mathcal{O}_j^{(3)} + \sum_{k=1}^{218} d_k \mathcal{O}_k^{(4)} + \cdots\Big\}\psi$$

		Relativistic	Non-relativistic
$O(p^2)$	1	$\langle\chi_+\rangle$	$\langle\chi_+\rangle$
	2	$\tilde{\chi}_+$	$\tilde{\chi}_+$
	3	u^2	u^2
	4	$\langle u^2\rangle$	$\langle u^2\rangle$
	5	$\{u^\mu, u^\nu\}D_{\mu\nu} + h.c.$	$-8m^2(v\cdot u)^2$
	6	$\langle u^\mu u^\nu\rangle D_{\mu\nu} + h.c.$	$-4m^2\langle(v\cdot u)^2\rangle$
	7	$i[u^\mu, u^\nu]\sigma_{\mu\nu}$	$2[S_\mu, S_\nu][u^\mu, u^\nu]$
	8		$S_\nu]f_+^{\mu\nu}$

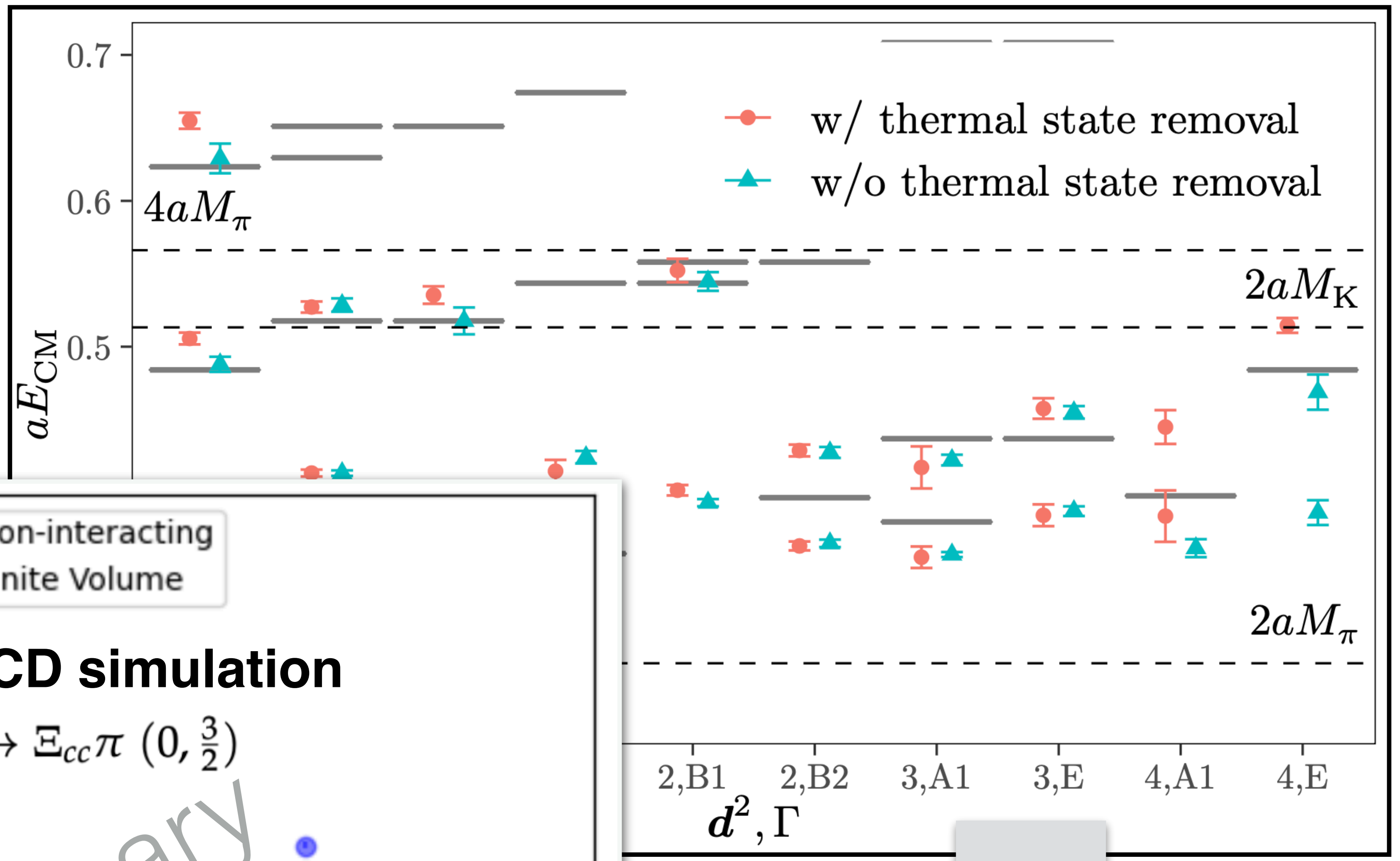
[Qiu & DLY, PRD103(2021)]

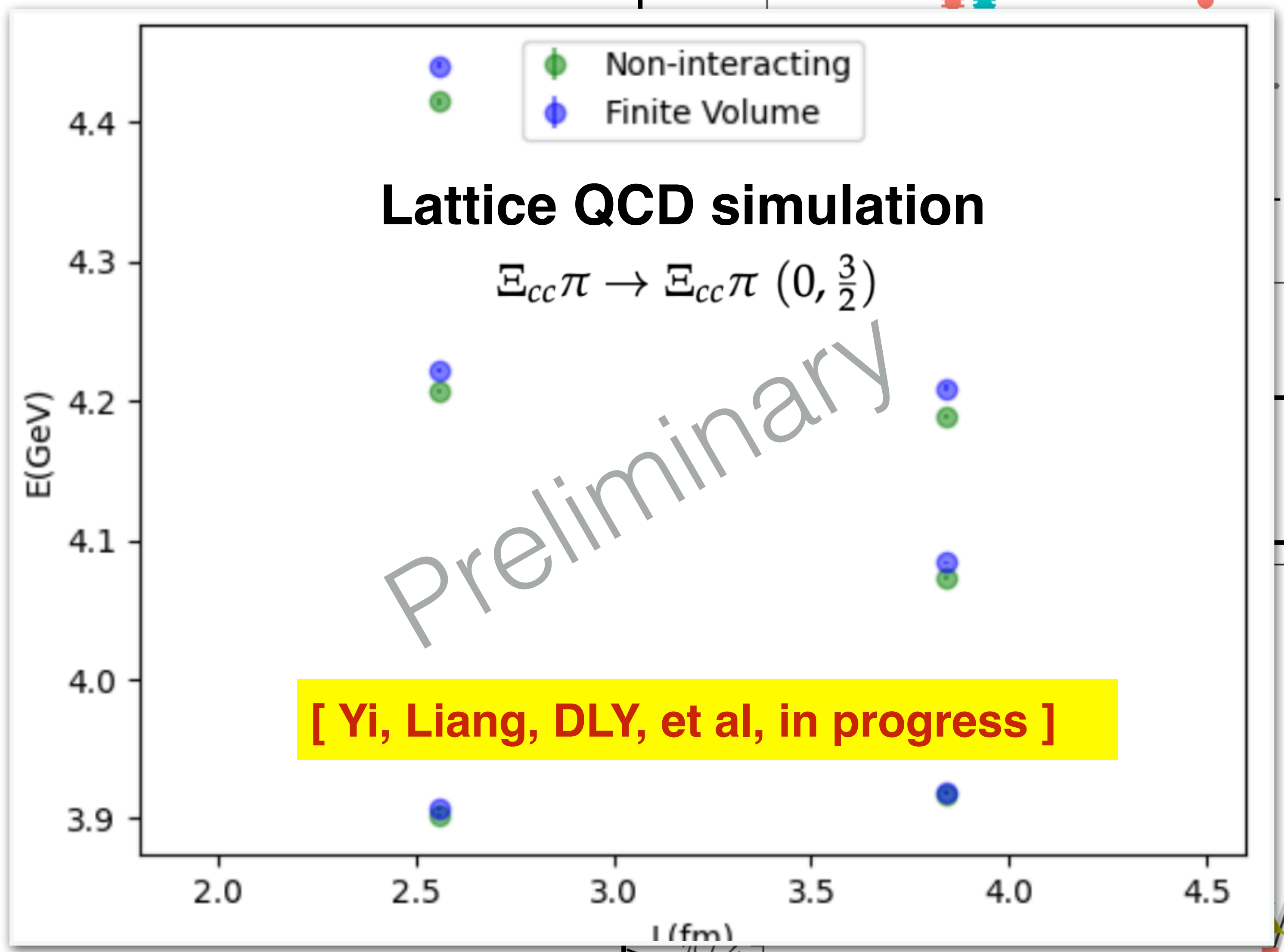


Heavy Diquark-Antiquark Symmetry

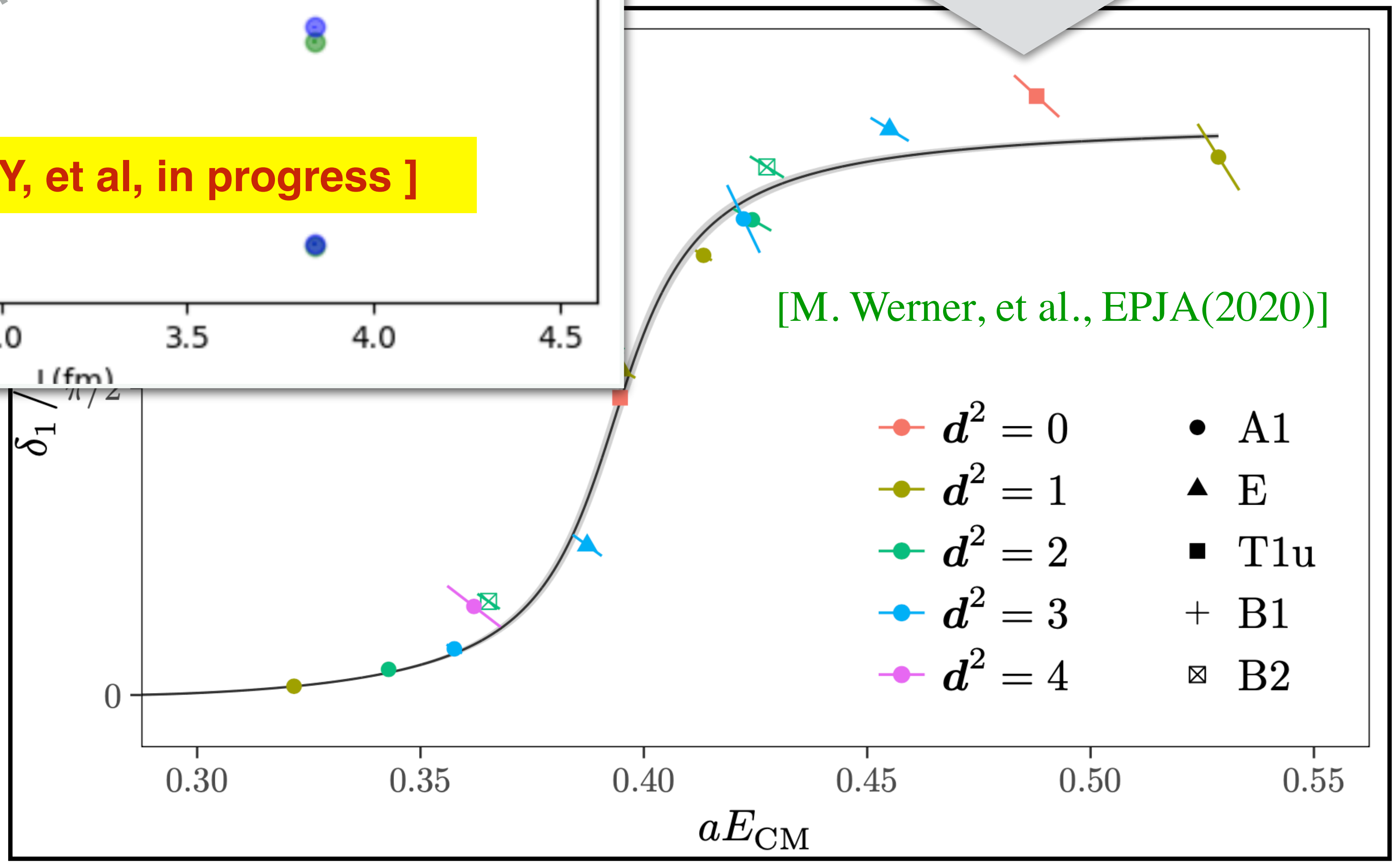


[Liang, Qiu & DLY, JHEP07(2023)]





[Yi, Liang, DLY, et al, in progress]

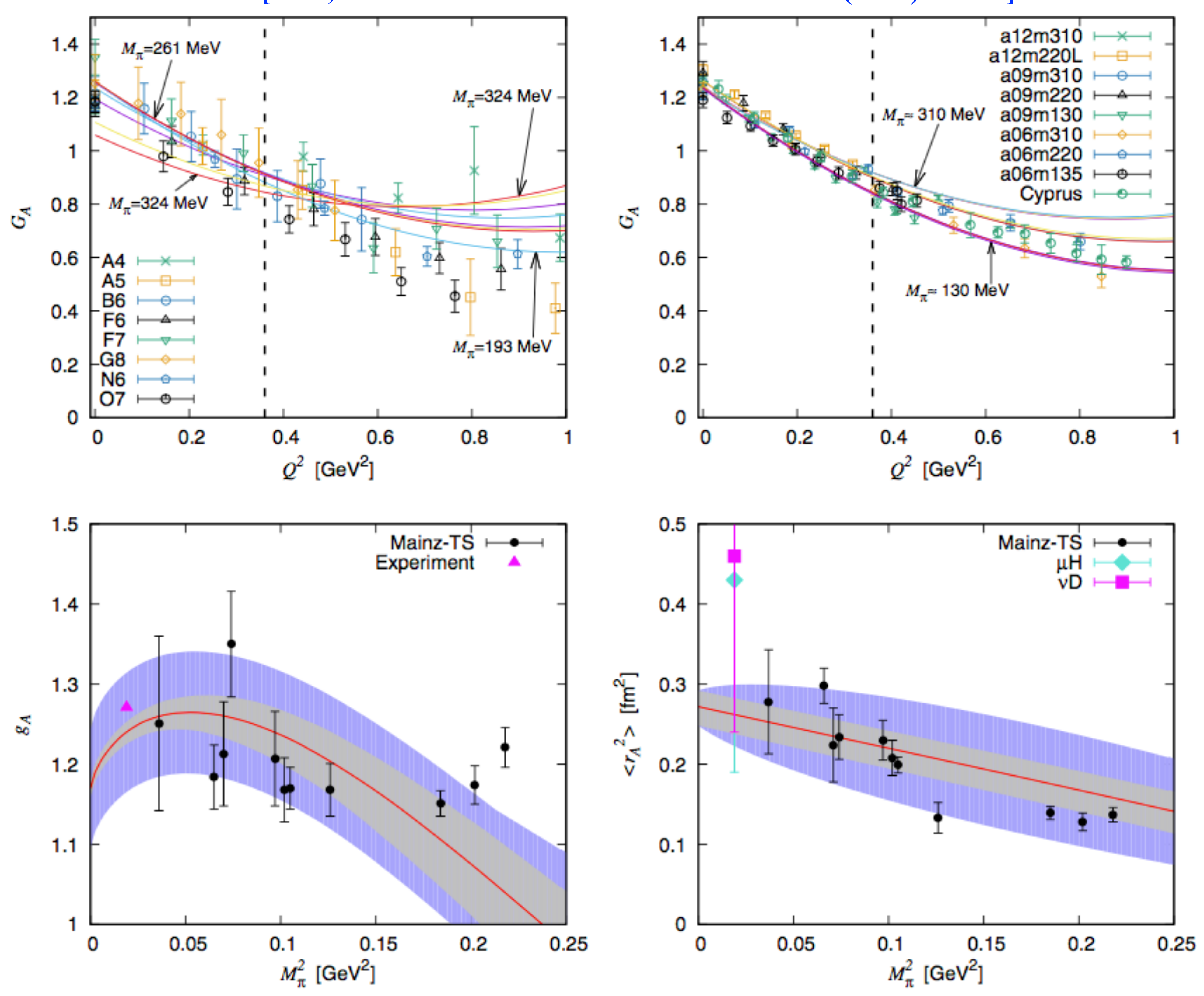


[M. Werner, et al., EPJA(2020)]

Selected Progress III: Combinations with Lattice Techniques

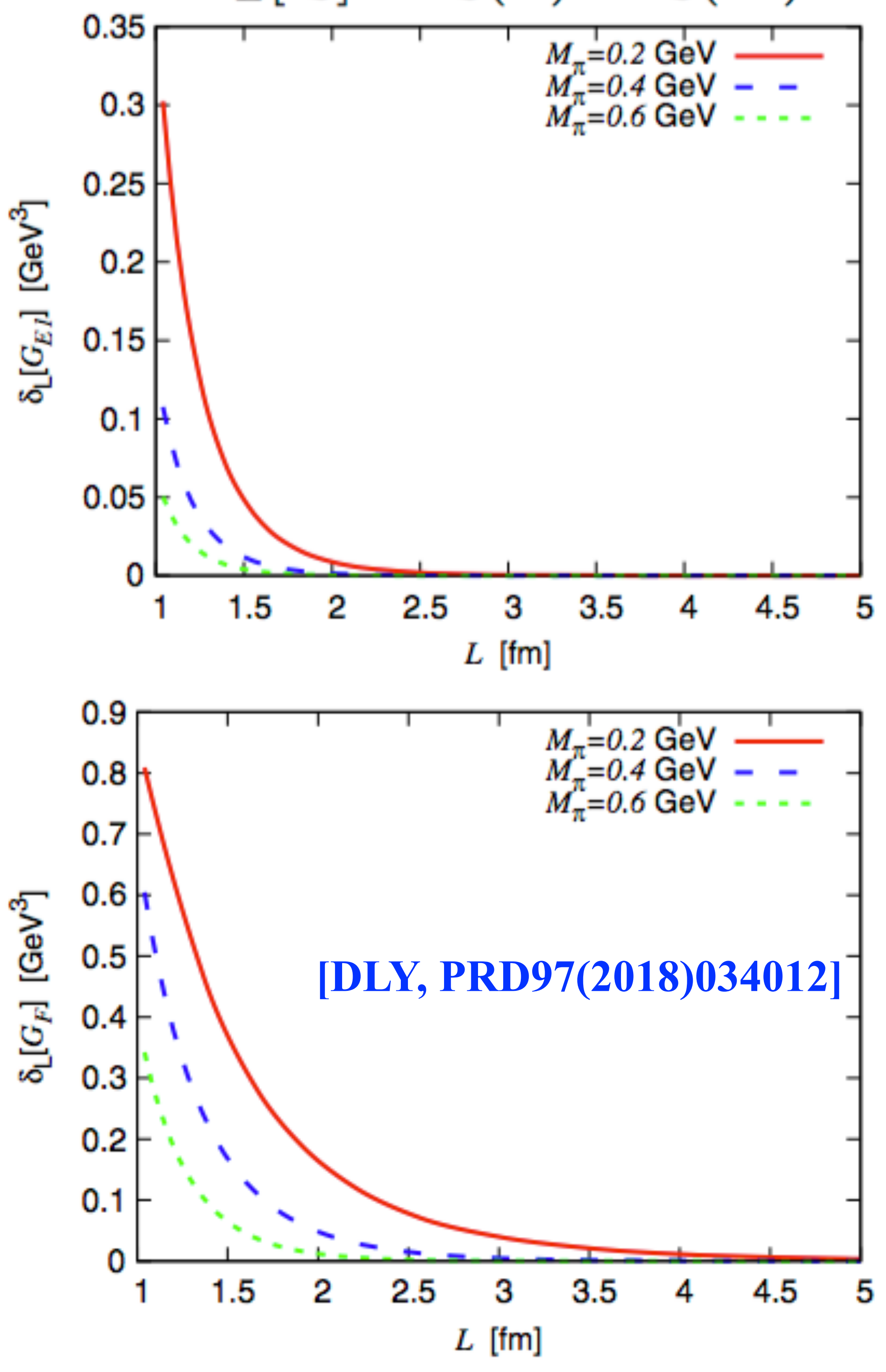
✦ Chiral Extrapolation & Finite volume correction…

[DLY, Alvarez-Ruso and Vicente-Vacas PRD96(2017)116022]



Axial coupling and radius of the nucleon

$$\delta_L[\mathcal{Q}] = \mathcal{Q}(L) - \mathcal{Q}(\infty)$$



[DLY, PRD97(2018)034012]

Lattice-size (L) dependence

Finite volume correction has to be computed case by case

A unified formulation of one-loop tensor integrals for finite volume effects

[Liang, DLY, JHEP12(2022)]

$$\mathcal{A}_a(L) = \frac{3g_A^2 m_N}{4F_\pi^2}\left\{\widetilde{A}_0(m_N^2;L) + M_\pi^2 \widetilde{B}_0(m_N^2, m_N^2, M_\pi^2;L)\right\},$$

$$\mathcal{B}_a(L) = \frac{1}{m_N}\mathcal{A}_a(L), \quad m_N^{\rm FVC}(L) = [\mathcal{A}(L) + m_N\mathcal{B}(L)]$$

Systematical computation of FVC in ChPT

Automatization

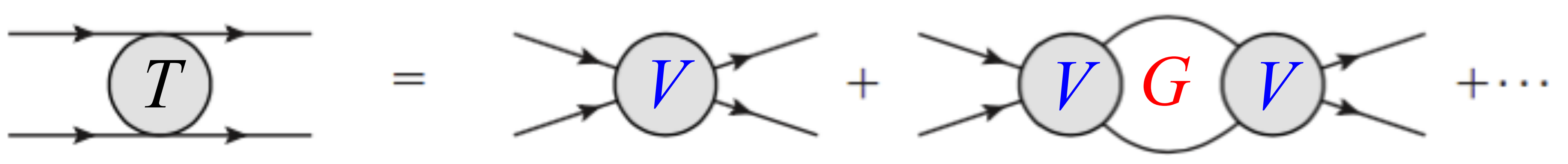
✦ Unitarized ChPT in a finite volume

$$T = V \bullet (1 - V \bullet G)^{-1}$$

[Doring, Meissner, Oset and Rusetsky, EPJA(2011)139]

ChPT

Discretization



Selected Progress III: Combinations with Lattice Techniques

✦ Nuclear Lattice Chiral Effective Field Theory (NLEFT)

Lattice OCD

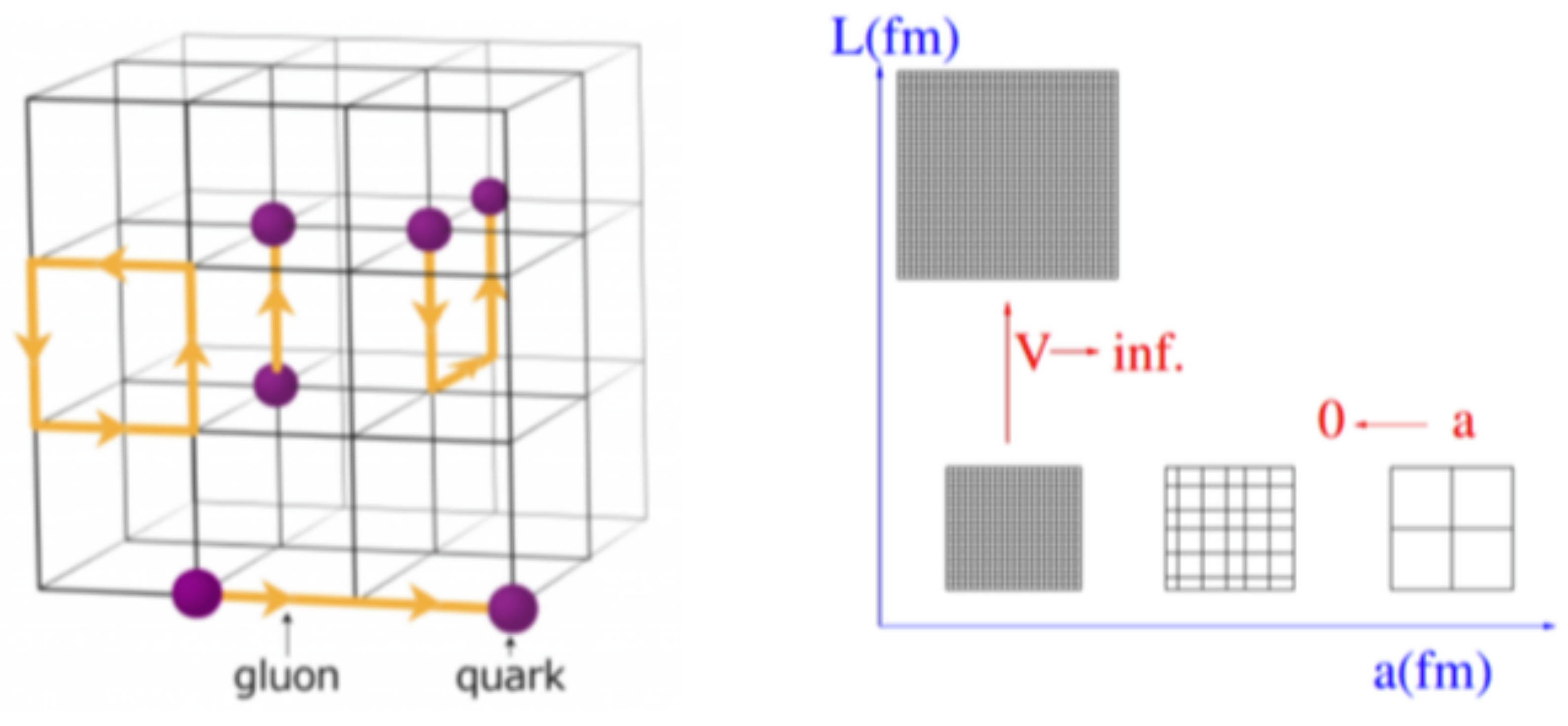
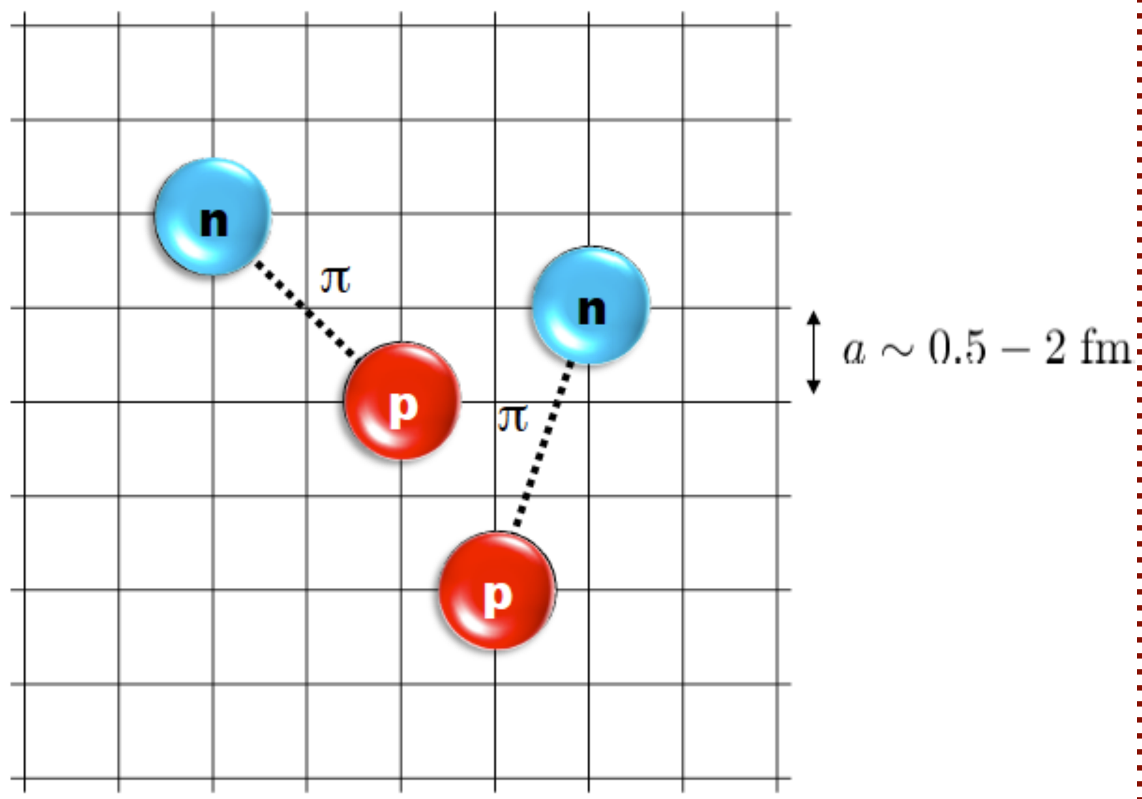
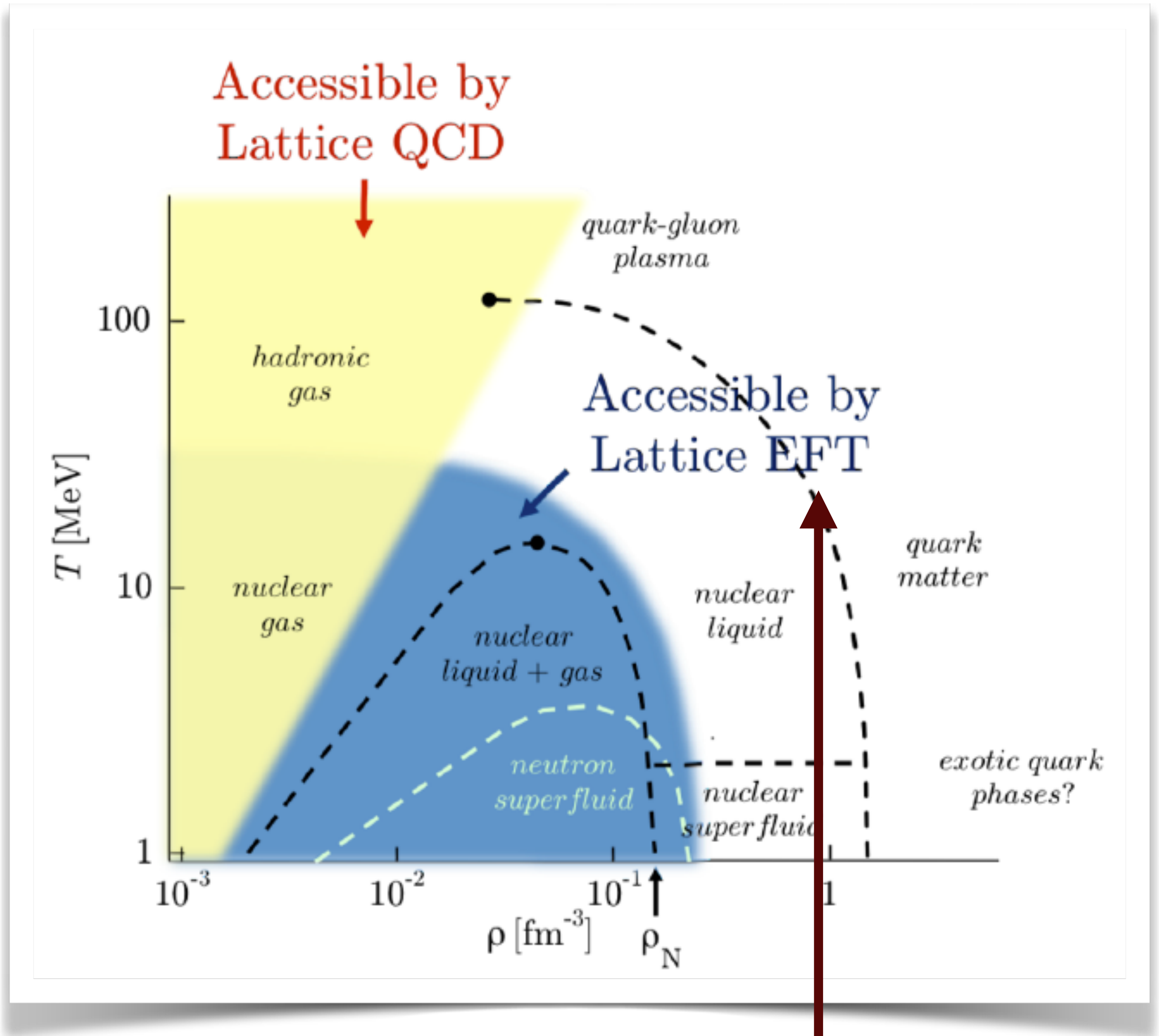


Figure: Taken from S.Ryan MESON2016

Nuclear Lattice Simulation



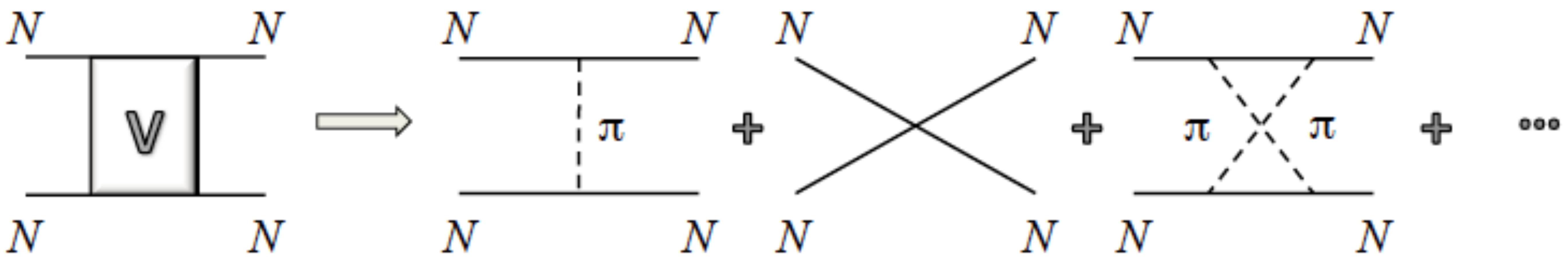
Review: D.L, Prog. Part. Nucl. Phys. 63 117-154 (2009)
TALENT summer school lectures: qmc2016.wordpress.ncsu.edu



BChPT is a widely-used EFT to be used more widely

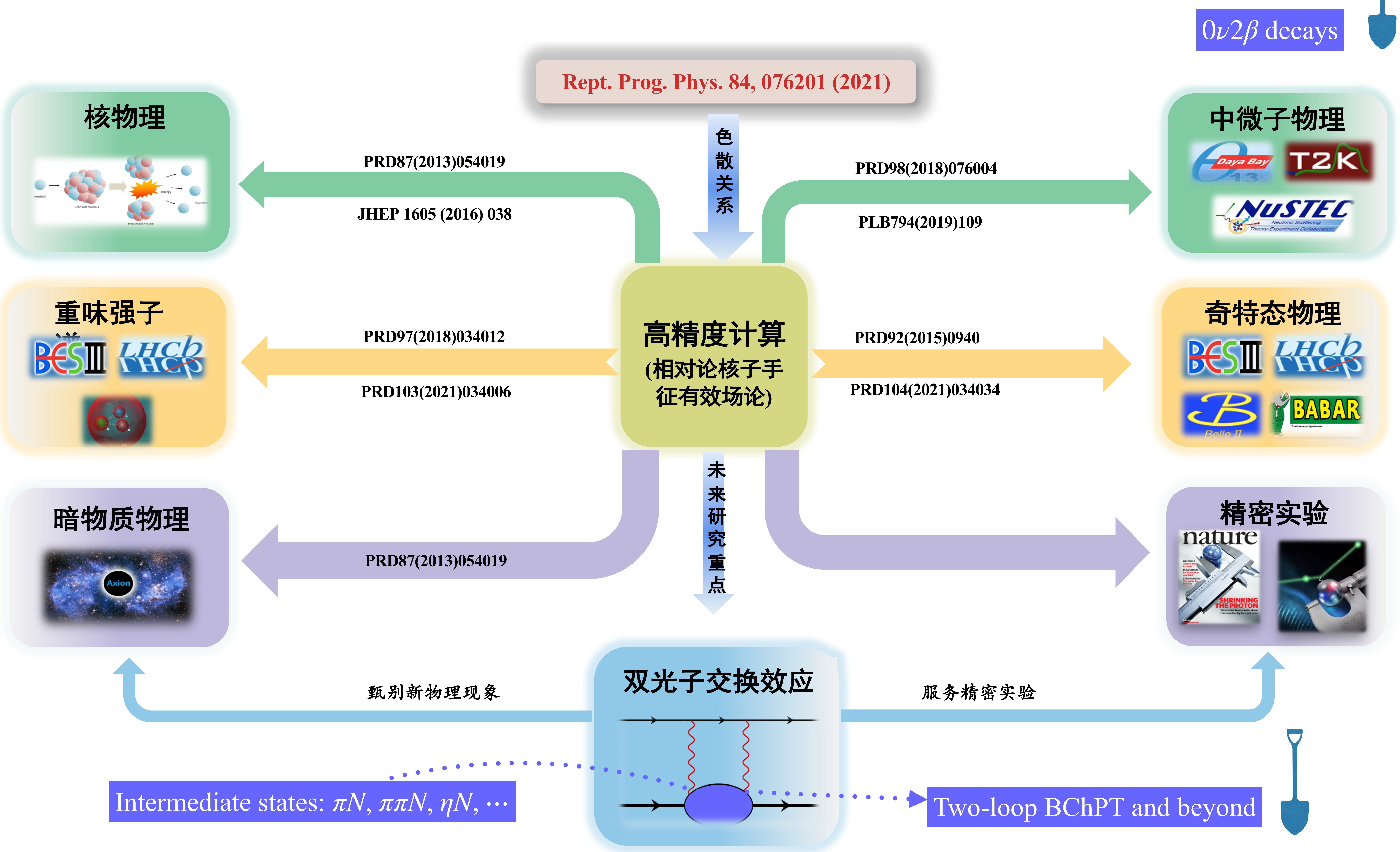
✦ Inputs: chiral effective Lagrangians

Effective Potential:

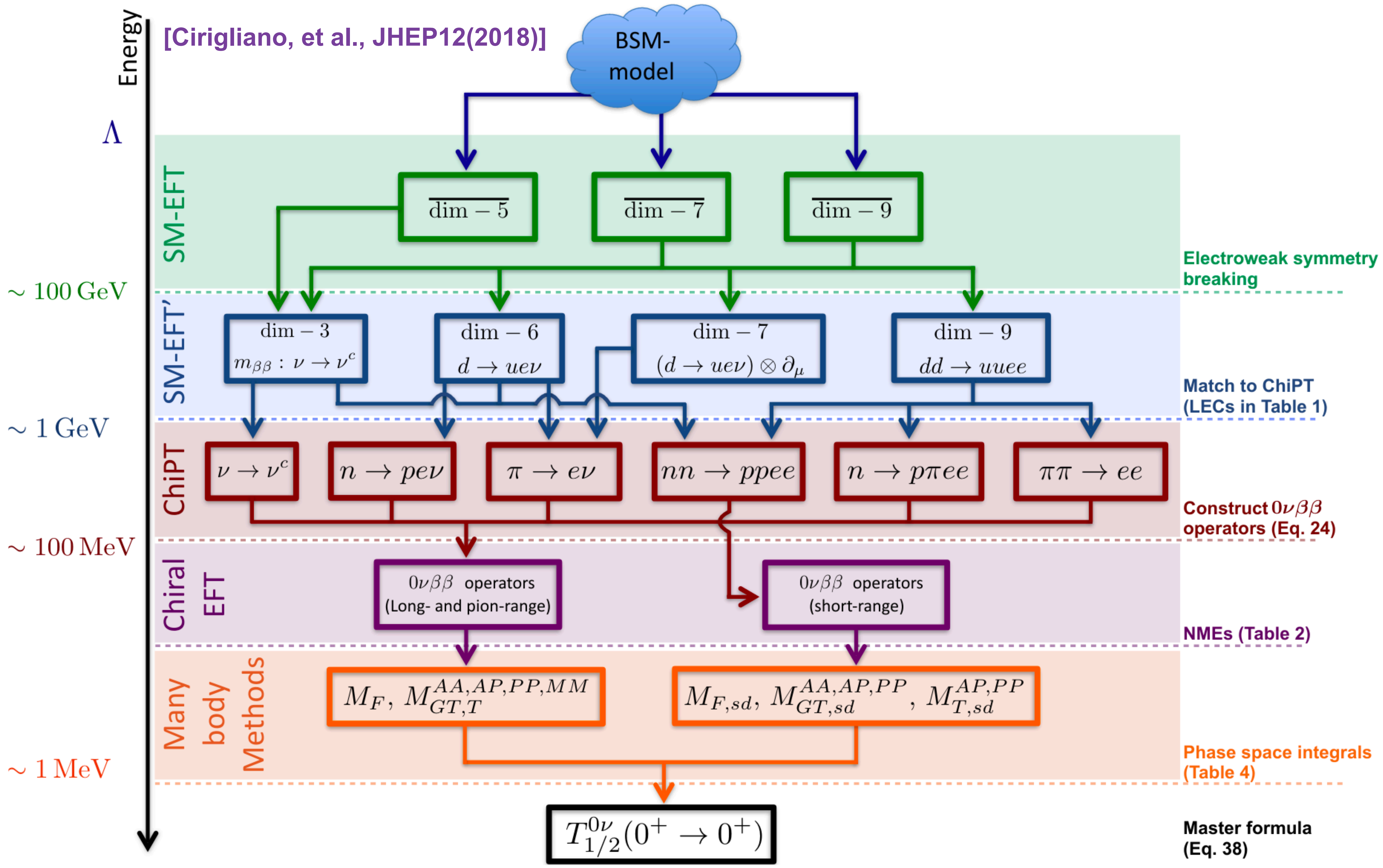


➡ πN scatterings are sub-processes ➡ one-nucleon sector to two- or multi-nucleon sectors ➡ study also strangeness?…

Overview of Prospect of BChPT



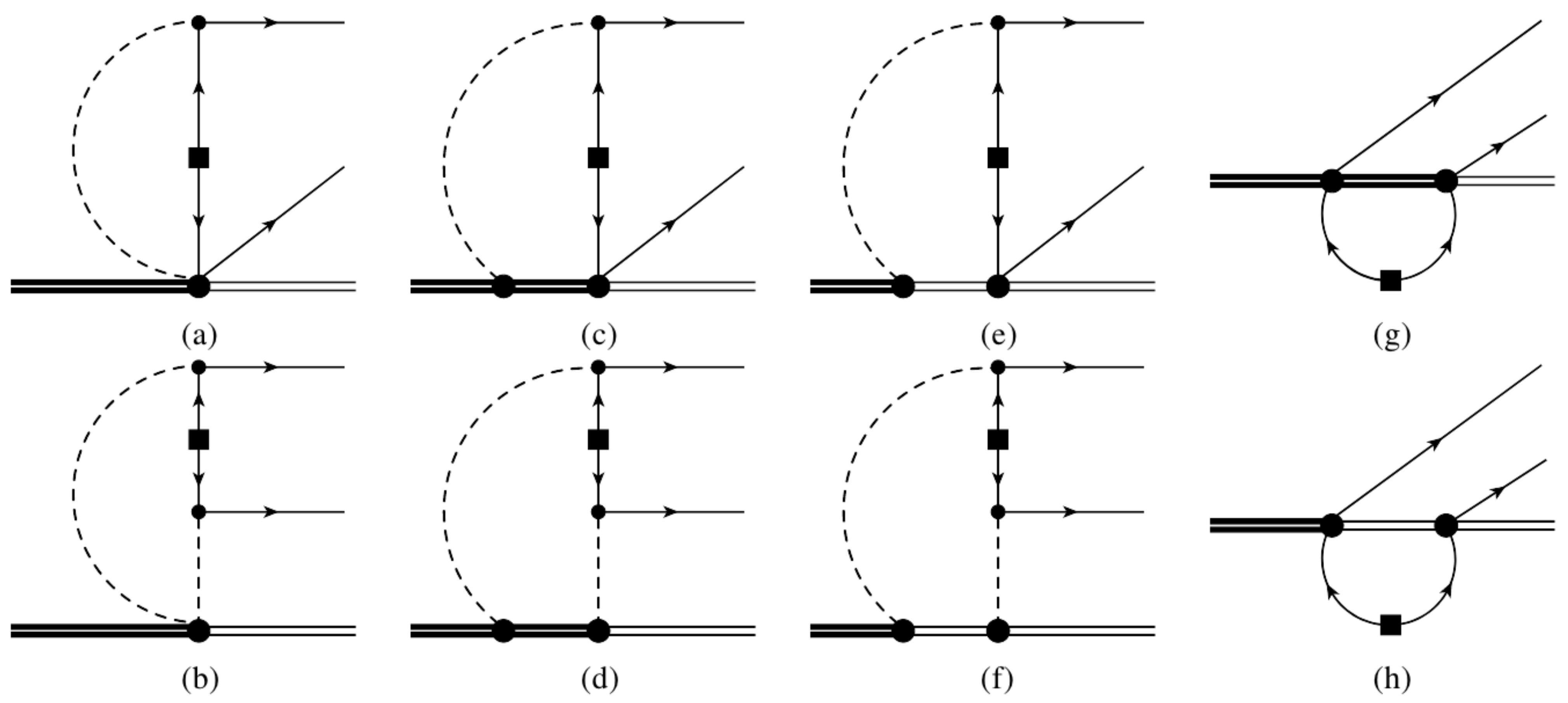
Prospect I: neutrinoless double beta decay



- $\Delta^- \to p e^- e^-$
- $\Sigma^- \to p e^- e^- \ldots$

$\Delta L = 2$ operator

$$\mathscr{L}_{\nu\nu} = -\frac{1}{2} m_{\beta\beta} \left(\nu_L^T C \nu_L + \bar{\nu}_L C^\dagger \bar{\nu}_L^T\right)$$



work in progress

- $K^\pm \to \pi^\mp l_\alpha^\pm l_\beta^\pm$ [Liao, Ma and Wang, JHEP03(2020)]
- $\pi^-\pi^- \to e^-e^-$ [Cirigliano, et al, PRC97(2018)]

Light neutrino, heavy neutrino,….

$\nu_e = U_{ei}\nu_i + R_{ej}N_j \qquad i = 1, 2, 3$

- ν_i is massive active neutrino, N sterile neutrino
- U is PMNS matrix, R mixing matrix of N

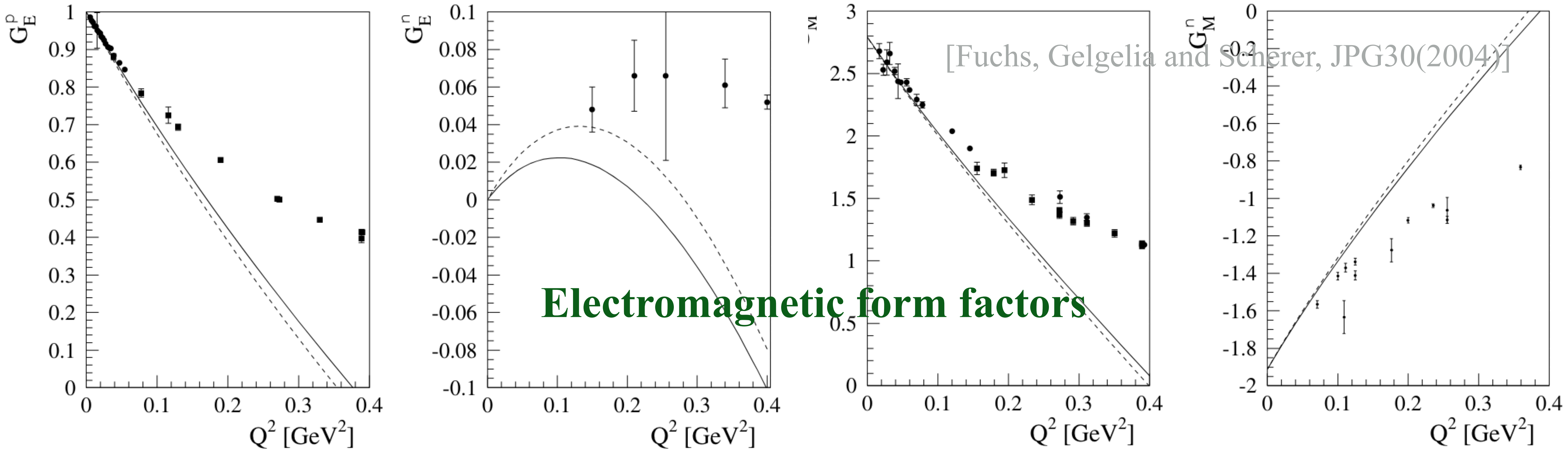
- **Pion-range coupling via resonance saturation + lattice QCD.**
- **Hyperon factory @ BESIII & STCF** $J/\psi \to \Sigma\bar{\Sigma}$

Prospect II: Two-loop BChPT

✦ Meson sector

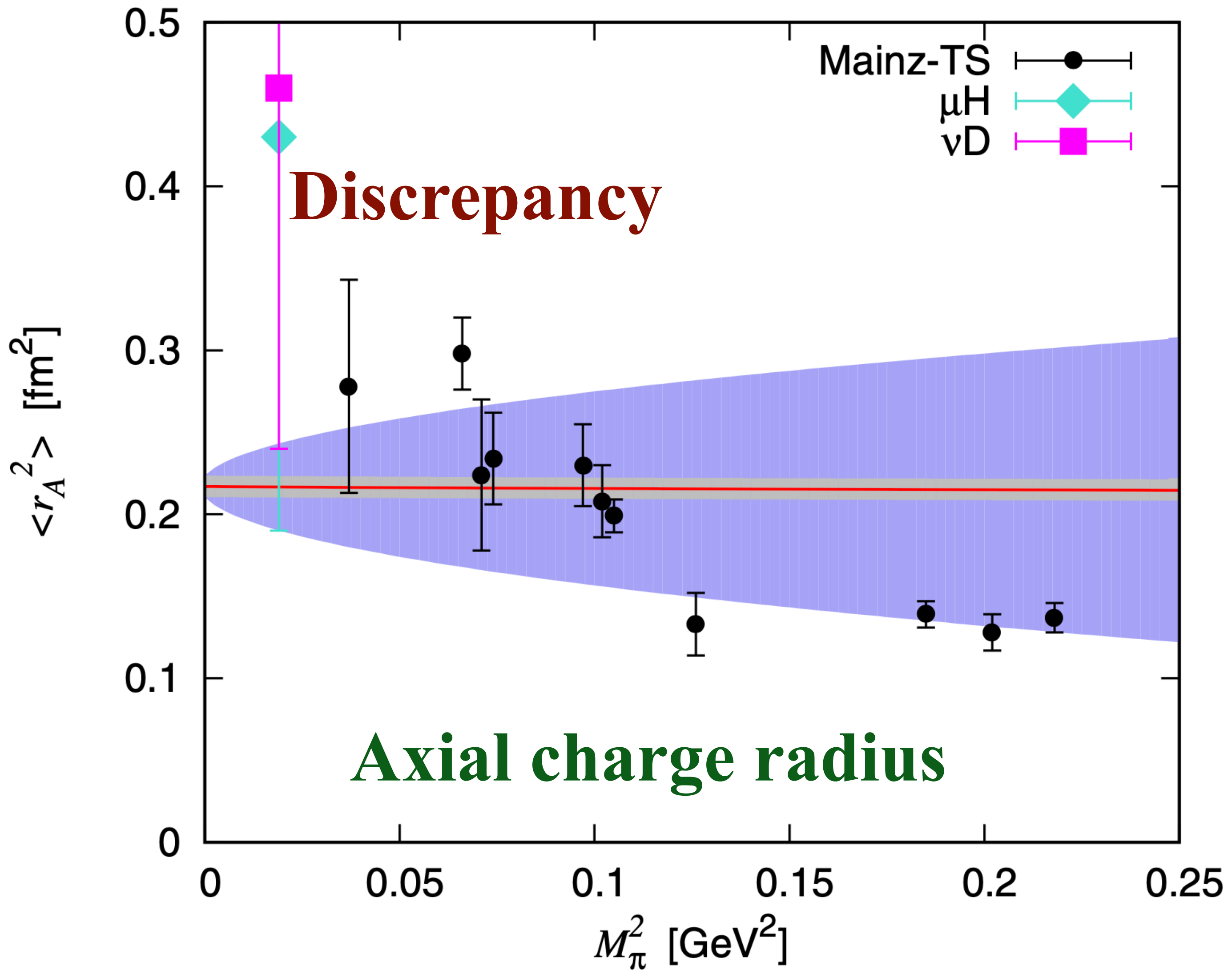
- $\pi\pi$ scattering, πK scattering, pion mass, pion decay constant, $K_{\ell 4}$, Electromagnetic form factor, …

[Bijnens, Colangelo, Ecker, Gasser and Sainio, PLB374(1996)]
[Bijnens, Colangelo, Ecker, Gasser and Sainio, NPB508(1997)]
[Bijnens, Talavera, JHEP03(2002)]
[Kaiser, Schweizer, JHEP06(2006)]
[Kaiser, JHEP09(2007)]
[Bijnens,Truedsson, JHEP11(2017)]
…



✦ Baryon sector

- Nucleon mass up to $O(p^5)$ in HBChPT [McGovern & Birse, PLB446(1999)]
- Nucleon axial-vector coupling beyond one-loop (renormalization group techniques) [Bernard & Meissner, PLB639(2006)]
- Nucleon mass to $O(p^6)$ in IR-BChPT [Schindeler, Djukanovic, Gegelia & Scherer, PLB649(2007), NPA803(2008)]



The full one-loop chiral results can only describe data at very low energies

Two-loop contributions should play a important role!

$$G_A(t) = g_A\left[1 + \frac{1}{6}\langle r_A^2\rangle t + \mathcal{O}(t^2)\right]$$

LO contribution is of one-loop order!

✦ Long-standing challenges

1. Computation of two-loop integrals

Auxiliary mass flow (AMFlow) [Liu and Ma, Comput.Phys.Commun. 283 (2023)]

2. Power counting breaking issue

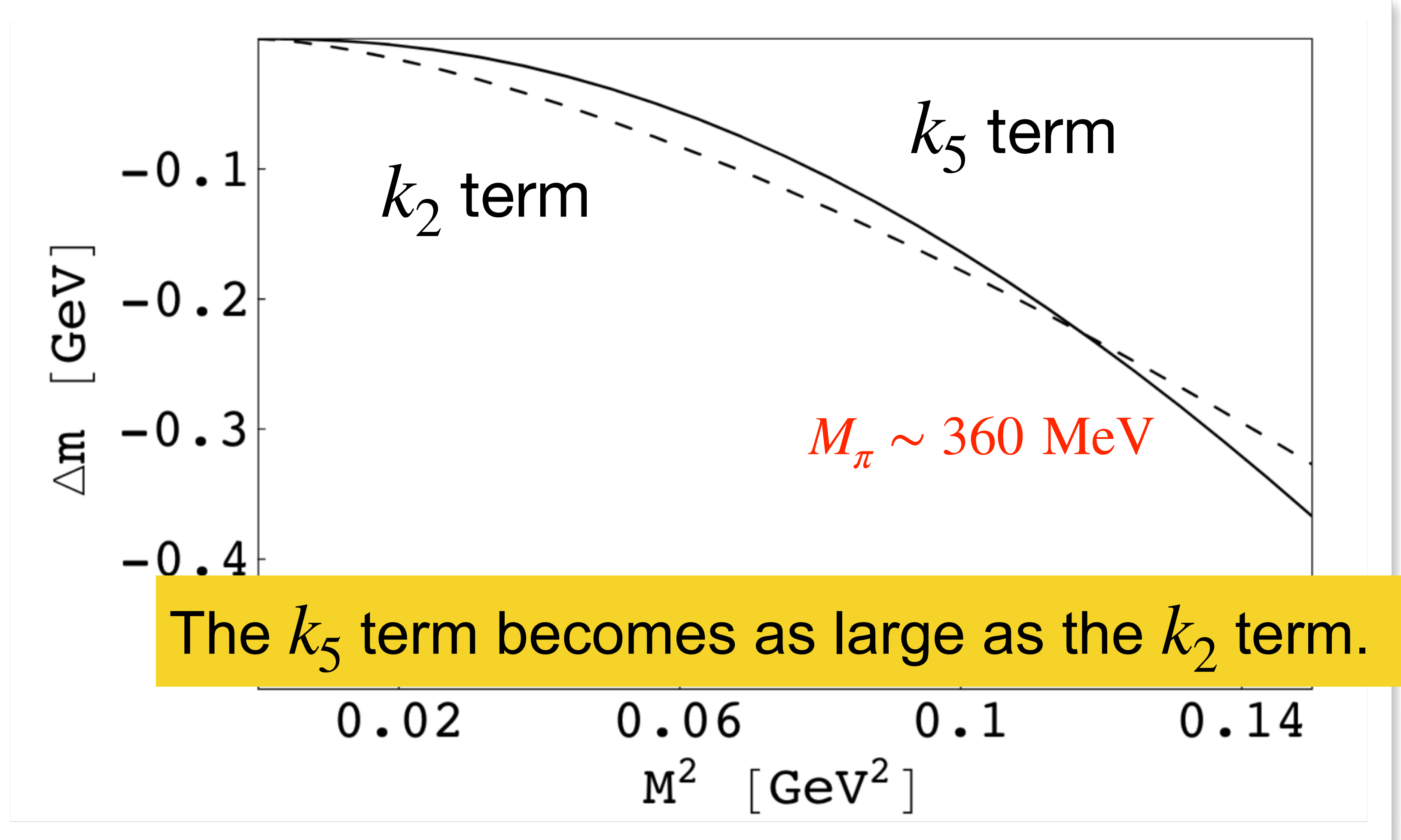
Applicability of EOMS at two-loop order?

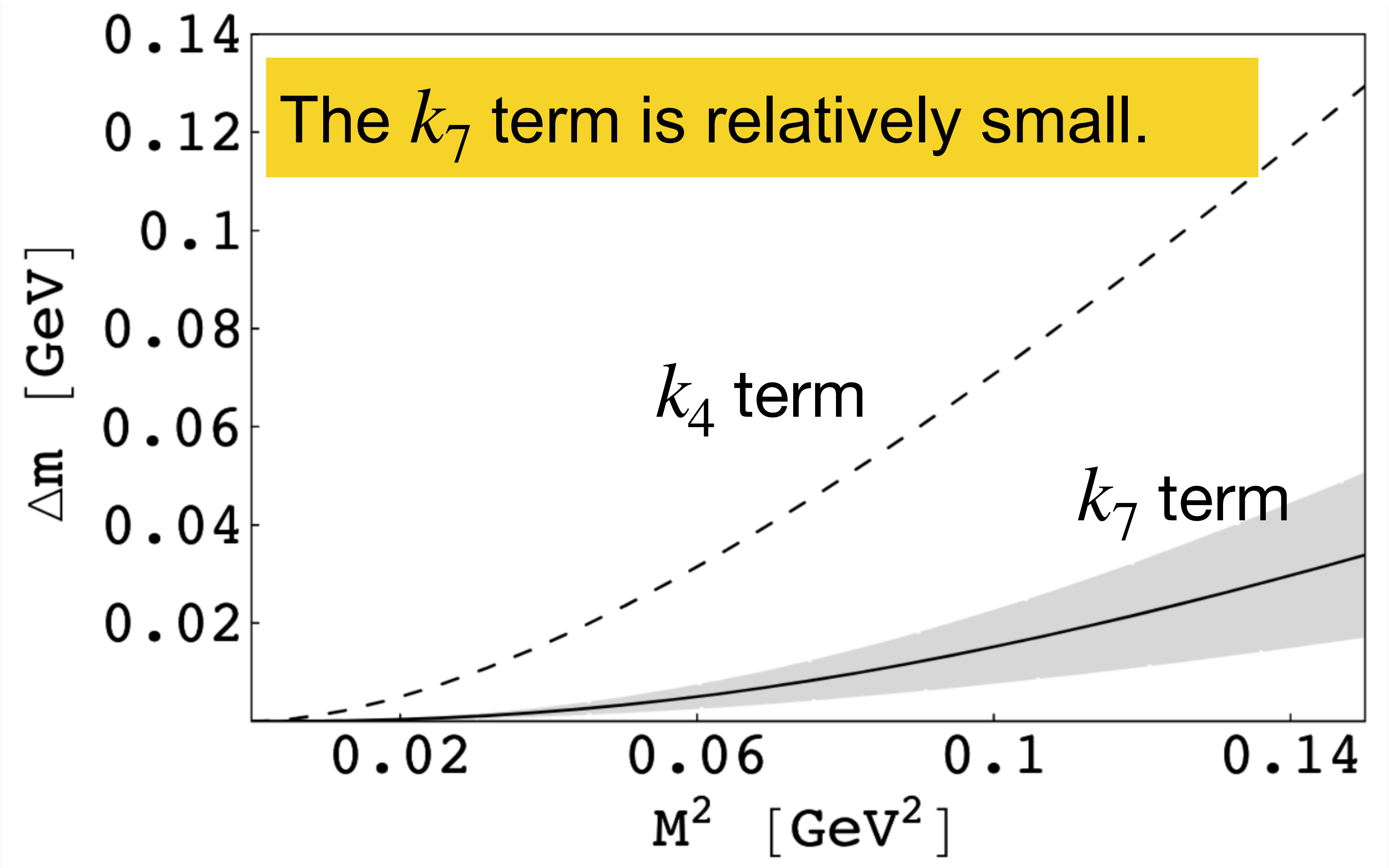
Nucleon mass at two-loop order

✦ Previous works

- **Heavy baryon formalism up to $O(p^5)$** [McGovern & Birse, PLB446(1999)]
- **Infrared Regularisation prescription up to $O(p^6)$** [Schindeler, Djukanovic, Gegelia & Scherer, PLB649(2007), NPA803(2008)]

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln\frac{M}{\mu} + k_4 M^4 + k_5 M^5 \ln\frac{M}{\mu} + k_6 M^5 + k_7 M^6 \ln^2\frac{M}{\mu} + k_8 M^6 \ln\frac{M}{\mu} + k_9 M^6$$





✦ Necessity of a leading two-loop study with EOMS scheme

- **Leading two-loop contribution [$O(p^5)$] is sizeable, while the next one [$O(p^6)$] is slight.**
- **A reliable chiral extrapolation relies on correct analytic property.**

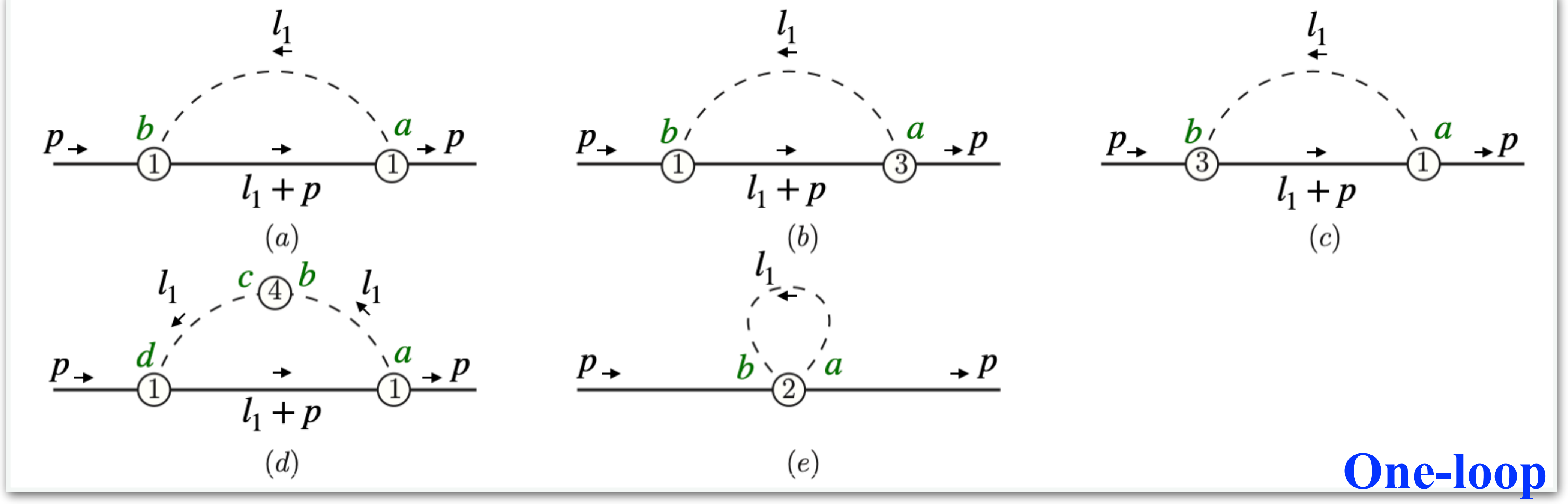
EOMS at $O(p^5)$!

Nucleon mass at two-loop order

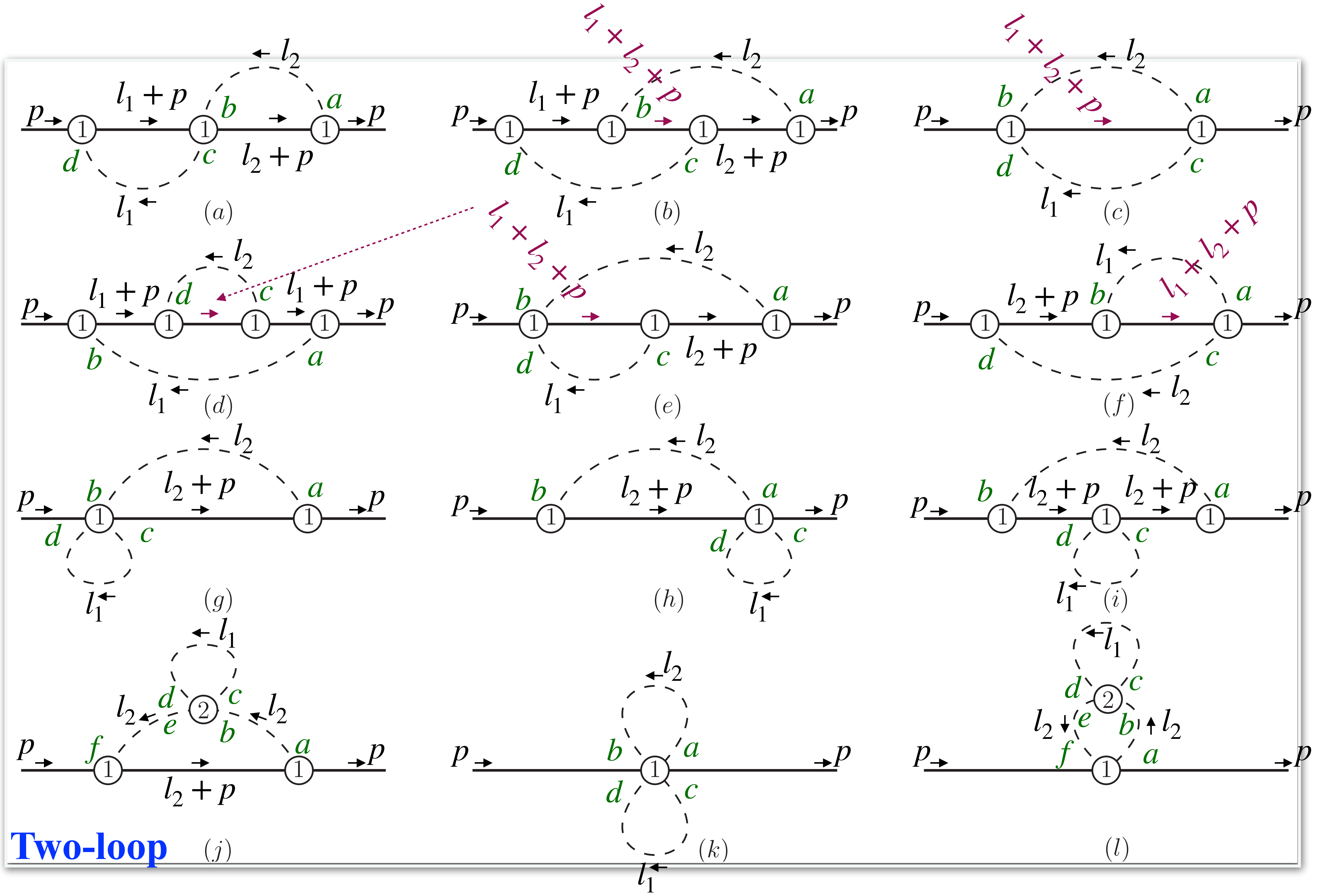
✦ Chiral effective Lagrangians and Feynman Diagrams

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)}$$

Trees



One-loop



Two-loop

✦ Calculation of self-energies

xAmplCalc — A mathematica package

$$I_{\nu_1\nu_2\nu_3\nu_4\nu_5} = \int\int \frac{\mathrm{d}^d\ell_1}{(i\pi^{d/2})} \frac{\mathrm{d}^d\ell_2}{(i\pi^{d/2})} \frac{1}{\mathscr{D}_1^{\nu_1}\mathscr{D}_2^{\nu_2}\mathscr{D}_3^{\nu_3}\mathscr{D}_4^{\nu_4}\mathscr{D}_5^{\nu_5}}$$

ν_i's are integers

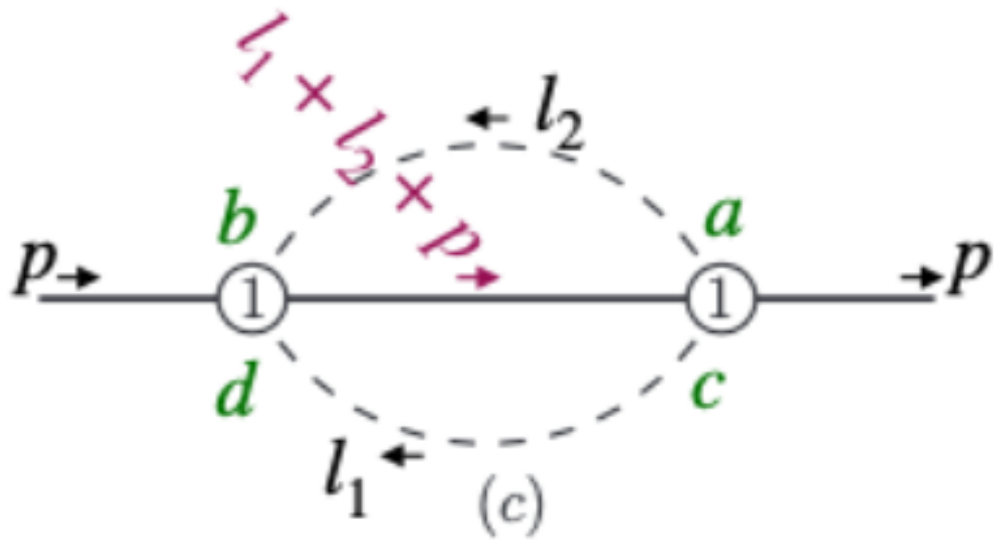
Number of independent scalar products

$$N = L \times E + \frac{1}{2}L(L+1)$$

$$\mathscr{D}_1 = \ell_1^2 - M^2\,,\mathscr{D}_2 = \ell_2^2 - M^2\,,\mathscr{D}_3 = (p+\ell_1+\ell_2)^2 - m^2\,,\mathscr{D}_4 = (p+\ell_1)^2 - m^2\,,\mathscr{D}_5 = (p+\ell_2)^2 - m^2\,.$$

IBP procedure and DSR relations

✦ Reduction to master integrals



$$\Sigma_c = -\frac{1}{32F_0^4}\int \frac{\mathrm{d}^d\ell_1}{(i\pi^{d/2})}\frac{\mathrm{d}^d\ell_2}{(i\pi^{d/2})}\left[\frac{(\not\ell_2-\not\ell_1)((\not p+\not\ell_1+\not\ell_2)+m)(\not\ell_2-\not\ell_1)}{[\ell_1^2-M^2][\ell_2^2-M^2][(p+\ell_1+\ell_2)^2-m^2]}\right]$$

$$\Sigma_c = -\frac{1}{32F_0^4}\int \frac{\mathrm{d}^d\ell_1}{(i\pi^{d/2})}\frac{\mathrm{d}^d\ell_2}{(i\pi^{d/2})}\frac{1}{\mathcal{D}_1\mathcal{D}_2\mathcal{D}_3}\Big[m[\mathcal{D}_1+\mathcal{D}_2-\mathcal{D}_3+\mathcal{D}_4+\mathcal{D}_5+(m^2+2M^2-s)]$$
$$+[\mathcal{D}_3-\mathcal{D}_4-\mathcal{D}_5-\mathcal{D}_1-\mathcal{D}_2-(m^2+2M^2-s)]\not p-[2\mathcal{D}_2-\mathcal{D}_3+2\mathcal{D}_5+(m^2+2M^2-s)]\not\ell_1$$
$$-[2\mathcal{D}_1-\mathcal{D}_3+2\mathcal{D}_4+(m^2+2M^2-s)]\not\ell_2\Big] \quad = A+\not p B$$

$$m_N^{(2c)} = -\frac{3}{16m_NF^4}\Big|2M_\pi^2\left(2I_{11100}M_\pi^2+2I_{01100}+2I_{10100}-I_{11000}\right)+4I_{00100}-I_{01000}-I_{10000}+I_{110(-1)0}+I_{1100(-1)}-4I_{111(-1)(-1)}\Big| ,$$

Scalar integrals in d dim.

Integration by parts (IBP) — Laporta algorithm

Master integrals

ISP-basis

Dot-basis ($\nu_i \geq 0$) → suited for implementing EOMS scheme!

Tensor integrals in d dim.

Scalar integrals in d+2 dim.

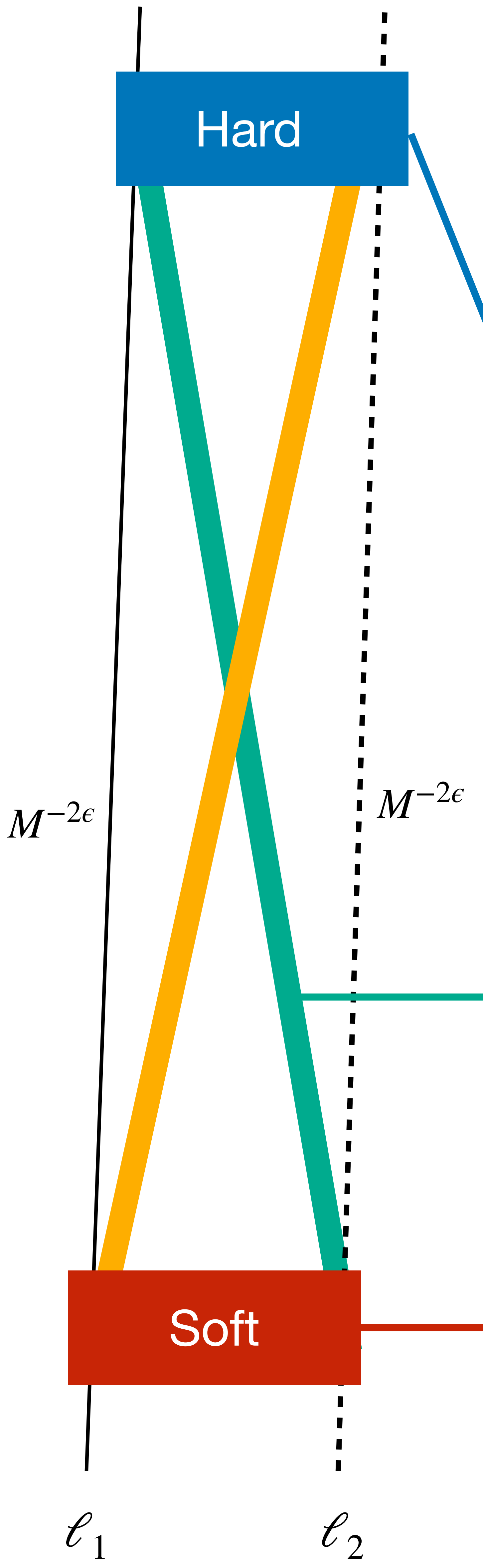
Dimensional Shift Relations (DSR)

Dimensional counting method

Strategy of region:

[Beneke and Smirnov, NPB522(1998)]

[Gegelia, Japaridze and Turashvili, Theor. Math. Phys. 101(1994)]



- $$I_{11111} = \iint \frac{\mathrm{d}^d\ell_1}{(i\pi^{d/2})}\frac{\mathrm{d}^d\ell_2}{(i\pi^{d/2})}\frac{1}{[\ell_1^2-M_\pi^2][\ell_2^2-M_\pi^2][(\ell_1+\ell_2+p)^2-m_N^2][(\ell_1+p)^2-m_N^2][(\ell_2+p)^2-m_N^2]}$$

- $$I_{11111} = I_{11111}^{(0,0)} + I_{11111}^{(0,1)} + I_{11111}^{(1,0)} + I_{11111}^{(1,1)} \qquad \ell_1 \to \tilde{\ell}_1\left(\frac{M_\pi}{m_N}\right)^{\alpha} \qquad \ell_2 \to \tilde{\ell}_2\left(\frac{M_\pi}{m_N}\right)^{\beta}$$

Infrared regular part

$$I_{11111}^{(0,0)} = \sum_{i,j} M_\pi^{2i+2j}\iint \frac{\mathrm{d}^d\ell_1}{(i\pi^{d/2})}\frac{\mathrm{d}^d\ell_2}{(i\pi^{d/2})}\frac{1}{[\ell_1^2+i0^+]^{1+i}[\ell_2^2+i0^+]^{1+j}\mathscr{D}_3\mathscr{D}_4\mathscr{D}_5}$$

Polynomial of M_π^2 → **PCB issue occurs!**

$M_\pi^{-2\epsilon} \times$ Polynomial of M_π^2

$$I_{11111}^{(0,1)} = \sum_{i,j,l}(-1)^{j+l}M_\pi^{d-3+2i+j+l}m_N^{3-d-j-l}\iint \frac{\mathrm{d}^d\tilde{\ell}_1}{(i\pi^{d/2})}\frac{\mathrm{d}^d\ell_2}{(i\pi^{d/2})}\frac{(\tilde{\ell}_1^2)^j(\frac{M_\pi}{m_N}\tilde{\ell}_1^2+2p\cdot\tilde{\ell}_1+2\tilde{\ell}_1\cdot\ell_2)^l}{[\tilde{\ell}_1^2-m_N^2+i0^+][\ell_2^2+i0^+]^{1+i}[2p\cdot\tilde{\ell}_1+i0^+]^{1+i}[\ell_2^2+2p\cdot\ell_2+i0^+]^{2+l}}$$

$$I_{11111}^{(1,1)} = \sum_{i,j,l}(-1)^{i+j+l}\left(\frac{M_\pi}{m_N}\right)^{2d-7+i+j+l}\iint \frac{\mathrm{d}^d\tilde{\ell}_1}{(i\pi^{d/2})}\frac{\mathrm{d}^d\ell_2}{(i\pi^{d/2})}\frac{(\tilde{\ell}_1^2)^i(\tilde{\ell}_2^2)^j(\tilde{\ell}_1^2+2\tilde{\ell}_1\cdot\tilde{\ell}_2+\tilde{\ell}_2^2)^l}{[\tilde{\ell}_1^2-m_N^2+i0^+][(\tilde{\ell}_2^2-m_N^2+i0^+][2p\cdot\tilde{\ell}_1+i0^+]^{1+i}[2p\cdot\tilde{\ell}_2+i0^+]^{1+j}[2p\cdot\ell_1+2p\cdot\ell_2+i0^+]^{2+l}}$$

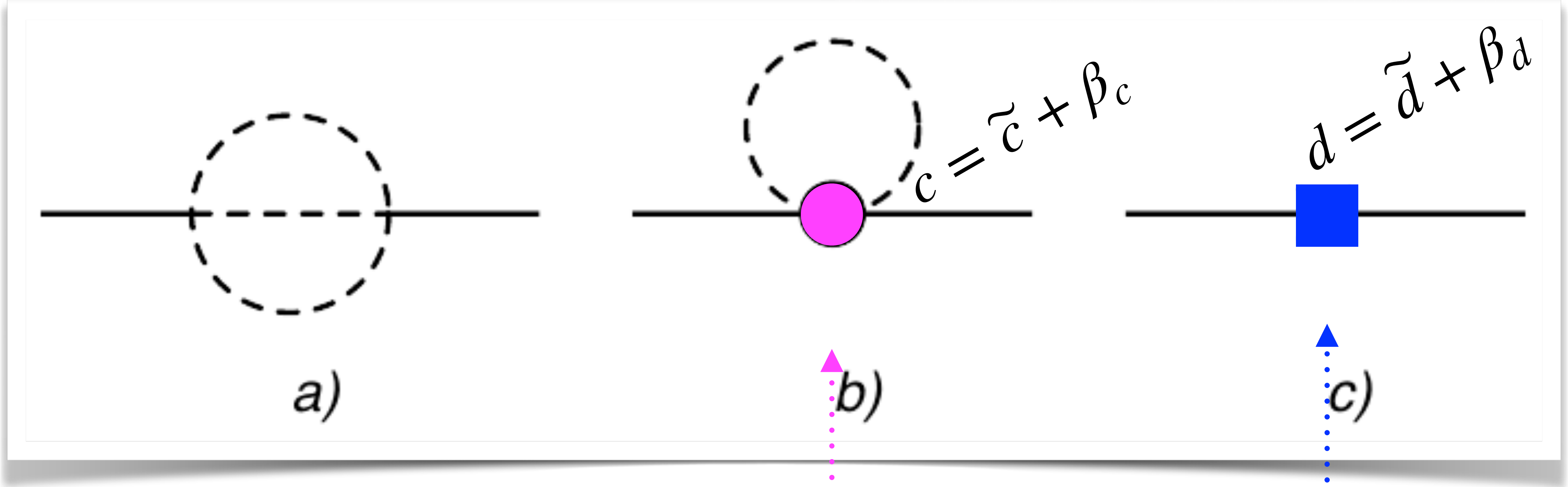
Infrared singular part

No PCB problem!

Applicability of (complex) EOMS scheme at two-loop order

✦ **A EFT toy model for the sake of easy illustration:**

$$\mathscr{L}_{\rm EFT} = \frac{1}{2}(\partial_\mu \pi \partial^\mu \pi - M^2\pi^2) + \frac{1}{2}(\partial_\mu \Psi \partial^\mu \Psi - m^2\Psi^2) - \frac{g}{3!}\pi^3\Psi + \mathscr{L}_c$$



Unrenormalized two-loop contribution

$$\Sigma_a = -\frac{g^2}{6(2\pi)^{2n}} \int\int \frac{{\rm d}^n k_1 {\rm d}^n k_2}{[k_1^2 - M^2 + i0^+][k_2^2 - M^2 + i0^+][(k_1+k_2+p)^2 - M^2 + i0^+]}$$

$$= M^{2n-4} H(p^2, M^2; n) + M^{n-2} G(p^2, M^2; n) + F(p^2, M^2; n)$$

No PCB

Taylor series of M^2 multiplied by M^{n-2}

Taylor series of M^2

Analytical demonstration

PCB term by G in diagram (a)

$$M^{n-2}\Delta G = \frac{g^2 M^{n-2}(-m^2 - i0^+)^{\frac{n}{2}-5}}{96(4\pi)^n}\Gamma\left(1-\frac{n}{2}\right)\Gamma\left(2-\frac{n}{2}\right)\Gamma\left(\frac{n}{2}-1\right)^2 \left\{\frac{48\, m^6}{\Gamma(n-2)} + 6m^2\left[\frac{(m^2-p^2)^2(n-4)(n-6)}{\Gamma(n-2)} - \frac{8m^2M^2(n+4)}{n\Gamma(n-3)}\right] - \frac{24m^4(m^2-p^2)(n-4)}{\Gamma(n-2)} - (n-6)(m^2-p^2)\left(\frac{(m^2-p^2)^2(n-4)(n-8)}{\Gamma(n-2)} - \frac{24m^2M^2(n+4)}{n\Gamma(n-3)}\right)\right\}. \quad (13)$$

Cancel with each other

Counter term by diagram (b)

$$-i\Sigma_{CT} = -\frac{ig^2\lambda(m,n)}{48m^6(n-4)n}\Big\{m^6(n-10)(n-8)(n-6)n - 3m^4(n-8)n\left[M^2(n-2)(n+4) + (n-10)(n-4)p^2\right] + 3m^2(n-6)\Big[M^4(n-2)n(n+4) + 2M^2(n+4)((n-8)n+20)p^2 + (n-10)(n-4)np^4\Big] - (n-8)(n-6)(n-4)(M^2+p^2)(M^4 n + 2M^2(n+6)p^2 + np^4)\Big\} I_\pi$$

Applicability also holds true for nucleon mass!

Nucleon-mass Physics

✦ Pion-nucleon sigma term

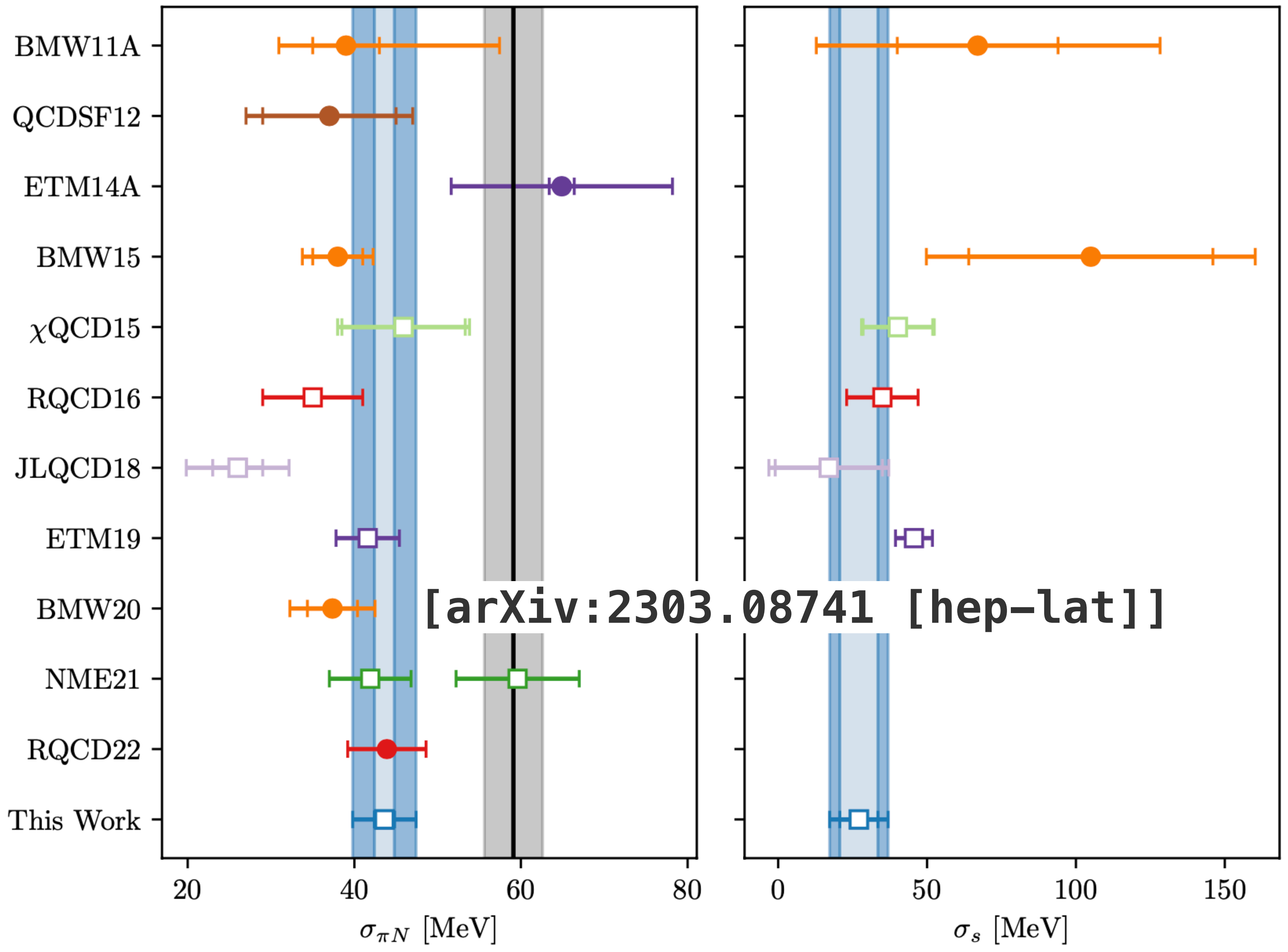
$$\sigma_{\pi N} = \frac{\hat{m}}{2m_N}\langle N|\bar{u}u + \bar{d}d|N\rangle = M_\pi^2 \frac{\partial m_N}{\partial M_\pi^2}$$

Feynman—Hellmann Theorem

$$m_N = \langle N(p)|\theta^\mu_\mu|N(p)\rangle$$ **Energy—Momentum Tensor**

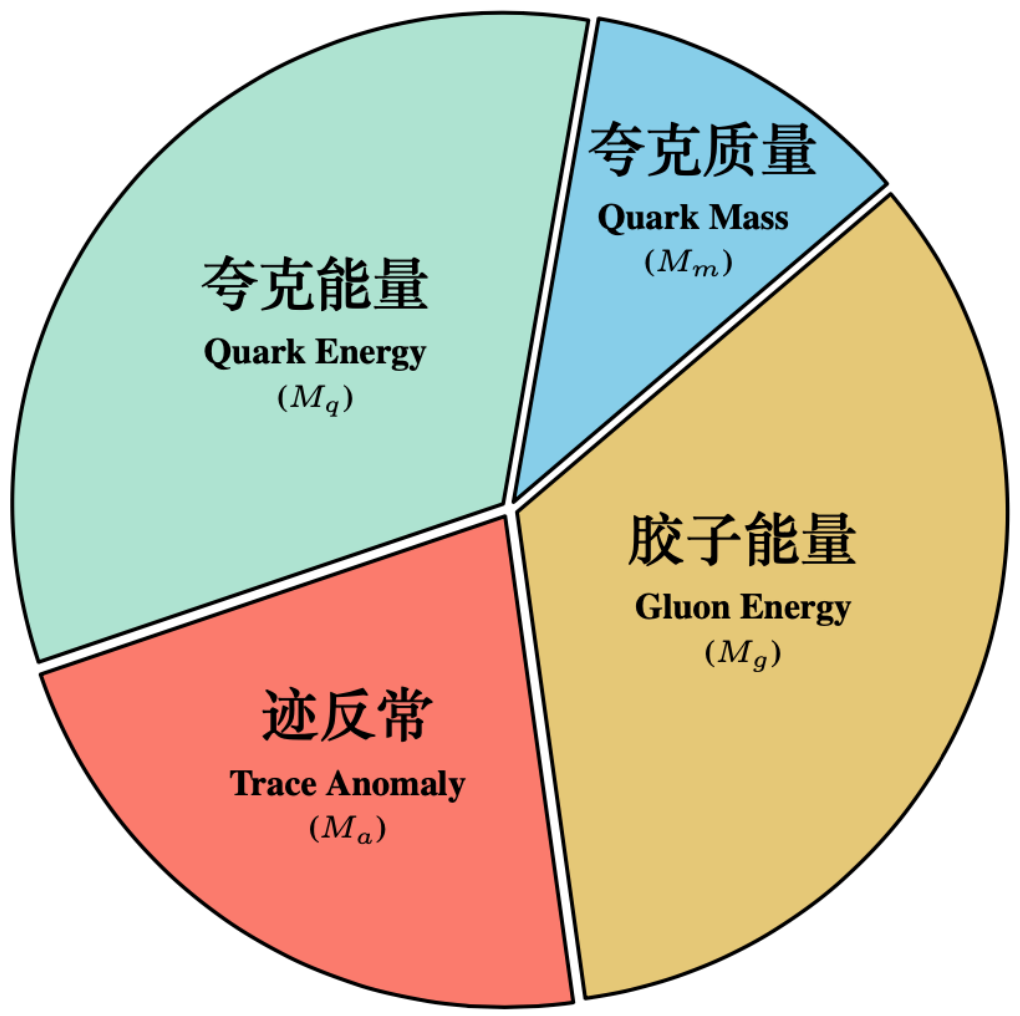
$$= \langle N(p)|\underbrace{\frac{\beta_{QCD}}{2g}G^a_{\mu\nu}G^{\mu\nu}_a}_{\text{field energy}} + \underbrace{m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s}_{\text{Higgs}}|N(p)\rangle$$

➡ **Tension between lattice QCD and phenomenological results**



[arXiv:2303.08741 [hep-lat]]

➡ **The proton mass decomposition**



[X. Ji, PRL74(1995) & PRD52(1995)]

- $M_q = \frac{3}{4}\left(a - \frac{b}{1+\gamma_m}\right)M$
- $M_a = \frac{1}{4}(1-b)M$
- $M_m = \frac{4+\gamma_m}{4(1+\gamma_m)}bM$
- $M_g = \frac{3}{4}(1-a)M$

➡ **Inputs to the study of WIMP dark matter**

- **scalar coupling of the nucleon**

A two-loop study of nucleon mass may shed new light on the related physics!

Summary and Outlook

✦ Successfulness of covariant BChPT at one-loop order

✓ **EOMS BChPT has been successfully applied to classic processes**

- pion-nucleon scattering
- pion photo-/electro-production off the nucleon
- neutral and charged current weak pion production off the nucleon

✓ **Extended versions for heavy hadrons and for unstable particles**

- Δ(1232) resonance, Roper resonance, ….
- Charmed mesons → Exotic heavy meson spectrum
- Doubly charmed baryons → Aiming at predicting new negative-parity 1/2 DC baryons

✓ **Interplay with Lattice techniques :**

- Chiral extrapolation
- Finite volume correction → Systematical computation
- Unitarized ChPT in finite volume → Energy level
- Nuclear Lattice Chiral EFT

Thank you for your attention!

✦ Perspective of BChPT

✓ **Era of two-loop accuracy:** nucleon mass, axial radius, electromagnetic form factors, pion-nucleon scattering, ….

✓ **New physics:** neutrinoless double beta decays, dark matter, …