Soft-Collinear Effective Theory:

review and recent progress

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For comprehensive review of SCET
Iain Stewart MIT lectures; Book by Becher et al. 1410.1892

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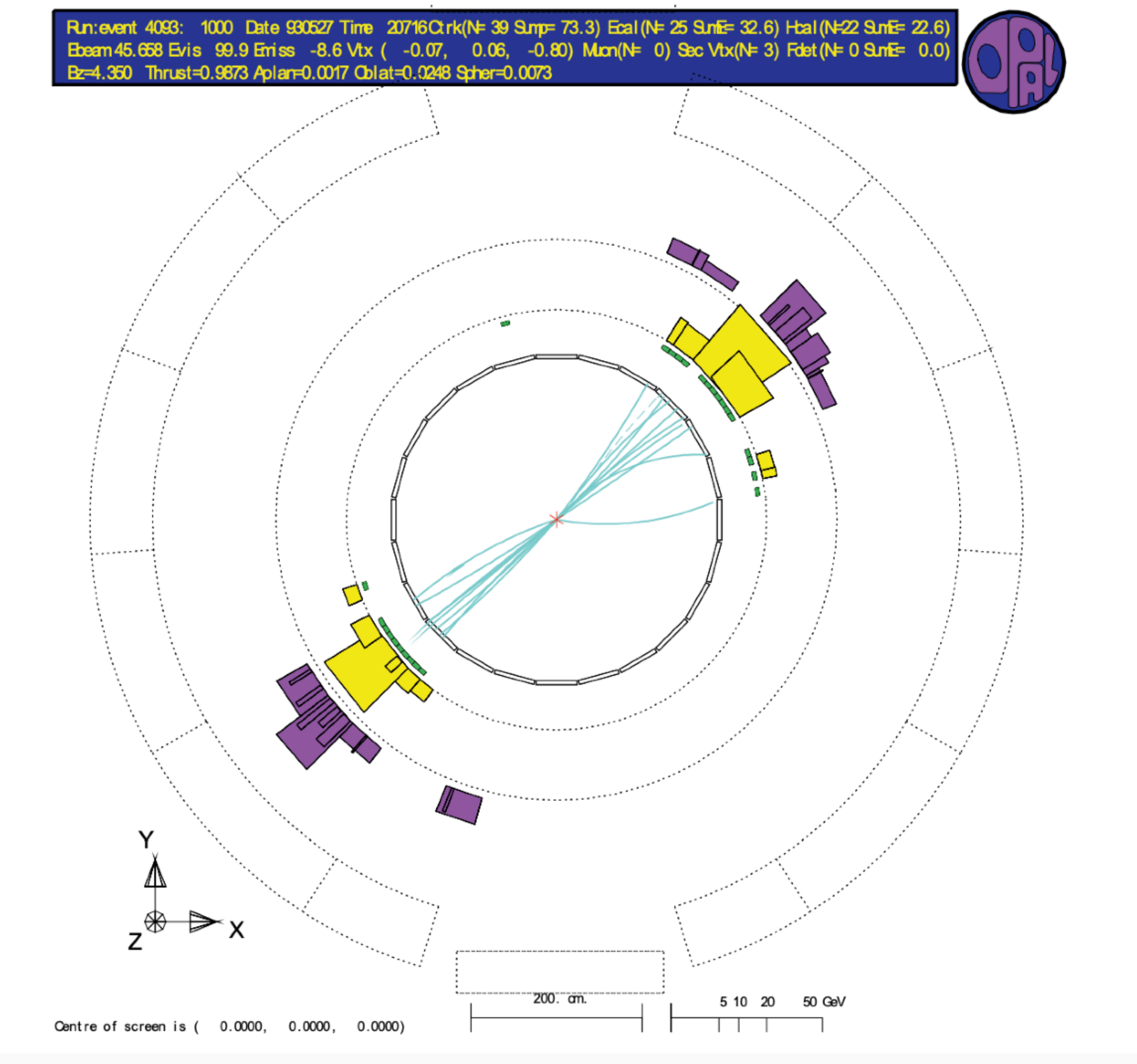
Outline

- Brief review on the construction of Soft-Collinear Effective Theory
- Applications to transverse momentum distribution of vector boson
- Power corrections in SCET
- Glauber gluon and factorization violation

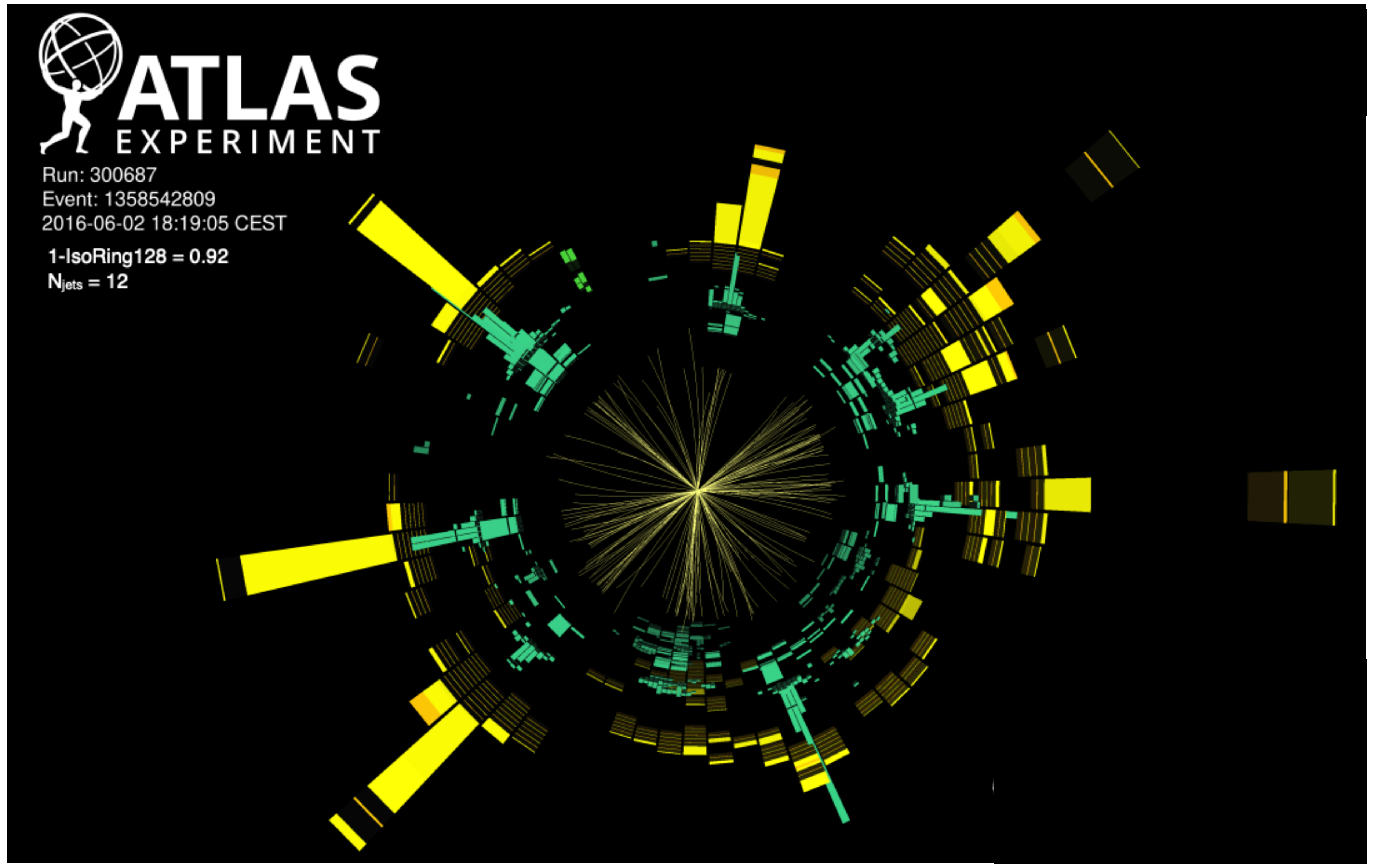
Soft-Collinear EFT (SCET)

SCET is an EFT for hard QCD process which contains collinear and soft particles

Bauer, Pirjol, Stewart, Fleming, Luke



ee



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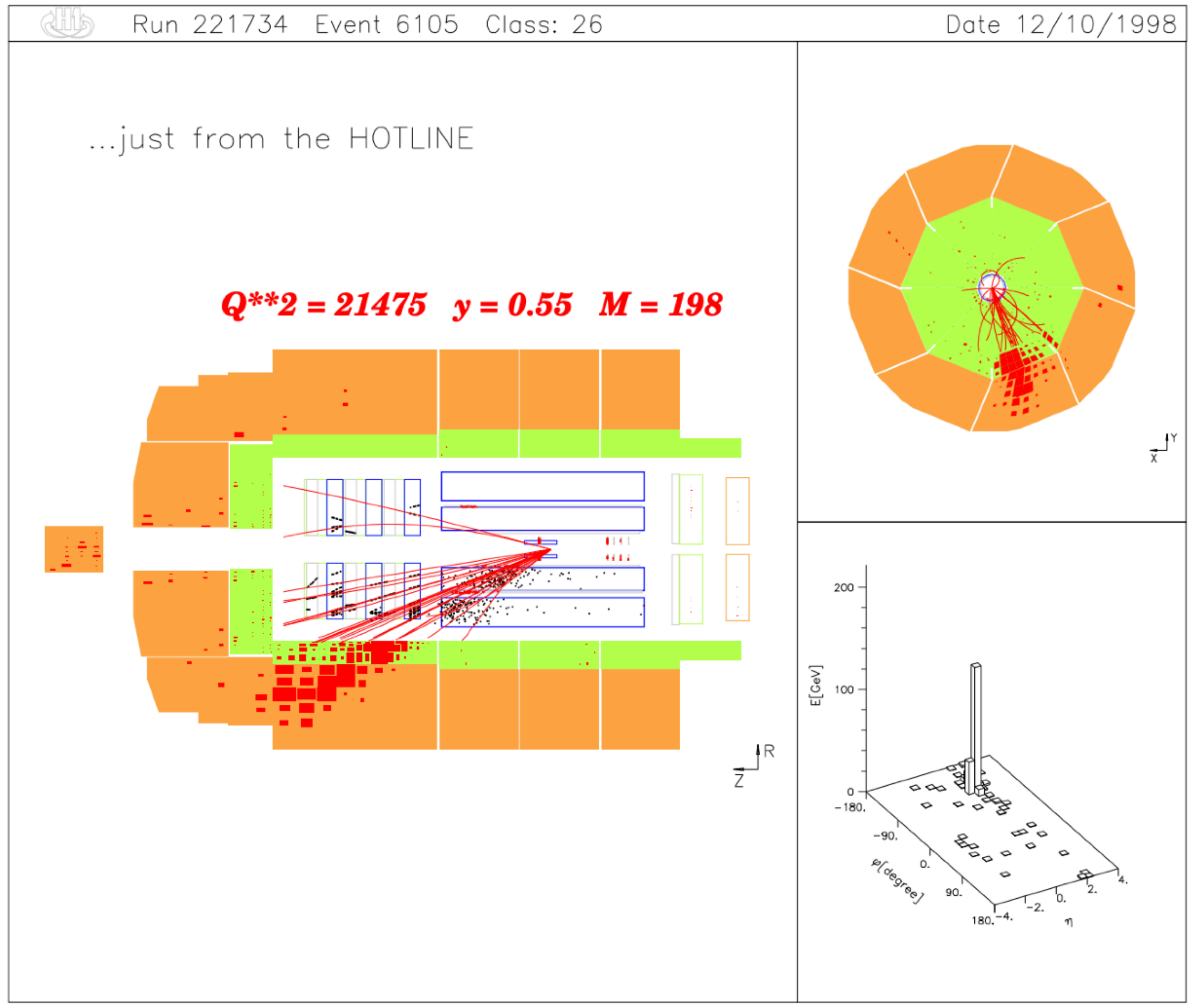
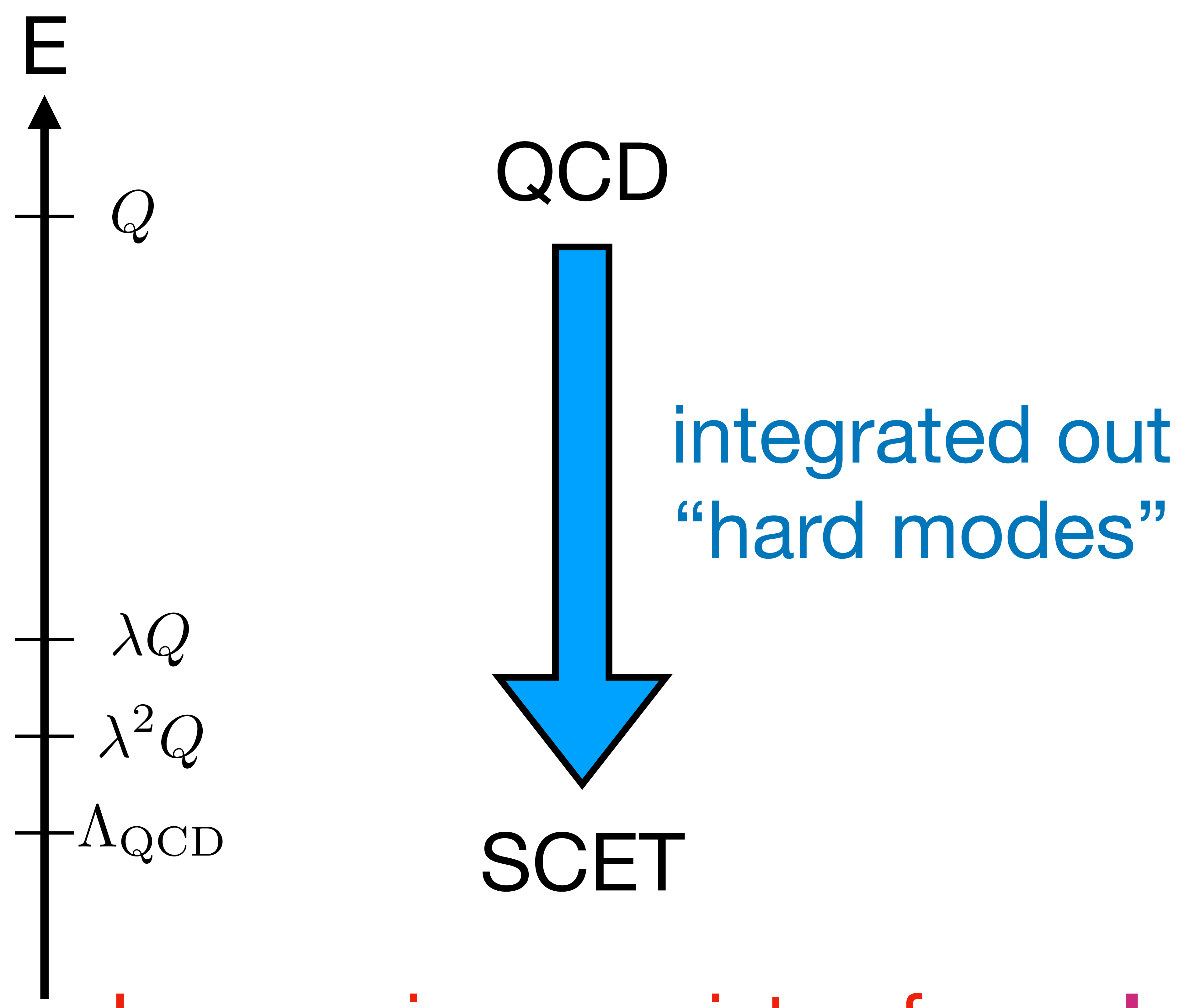


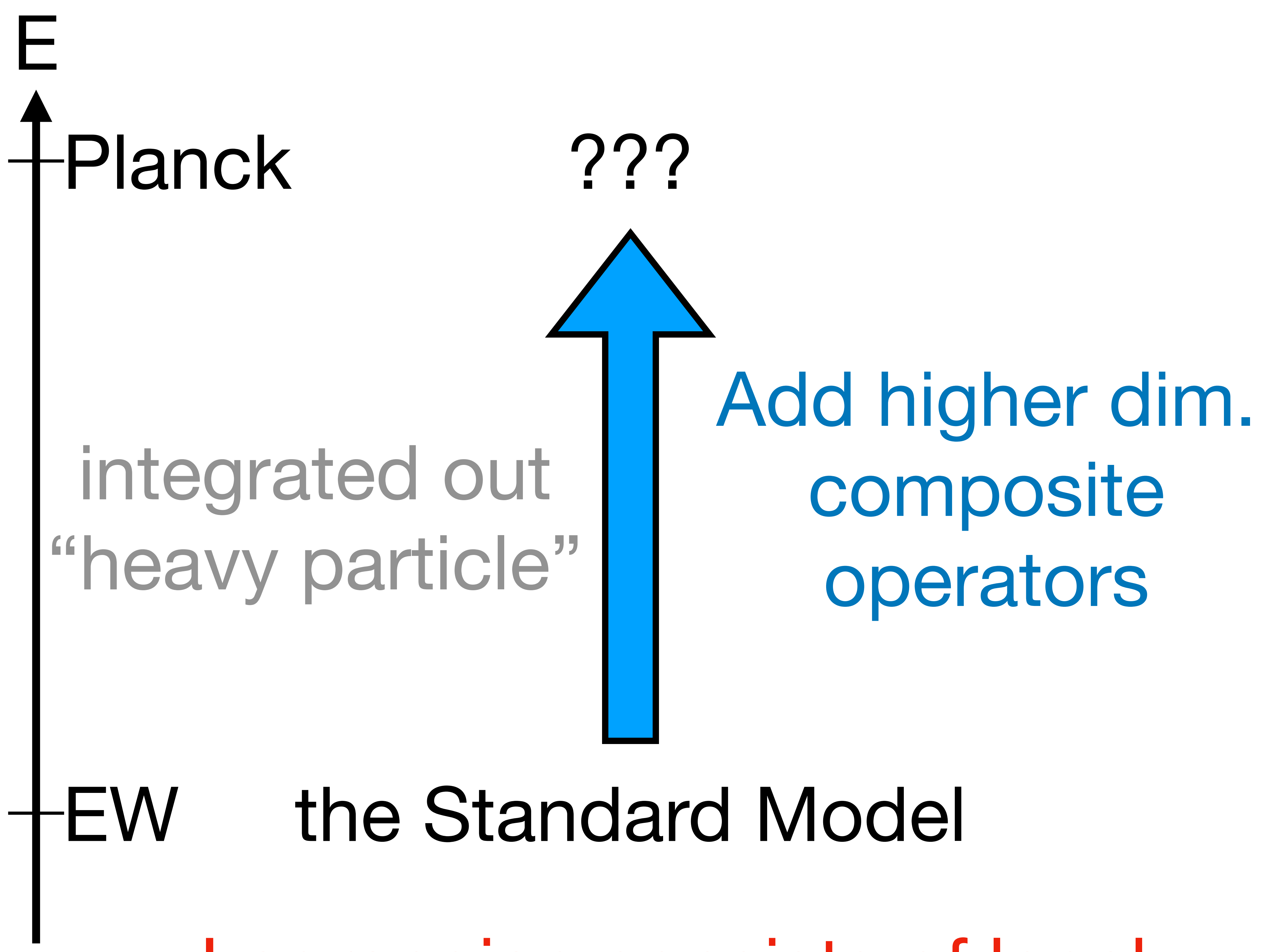
ep

Panorama view of SCET

Top down v.s. bottom up EFT



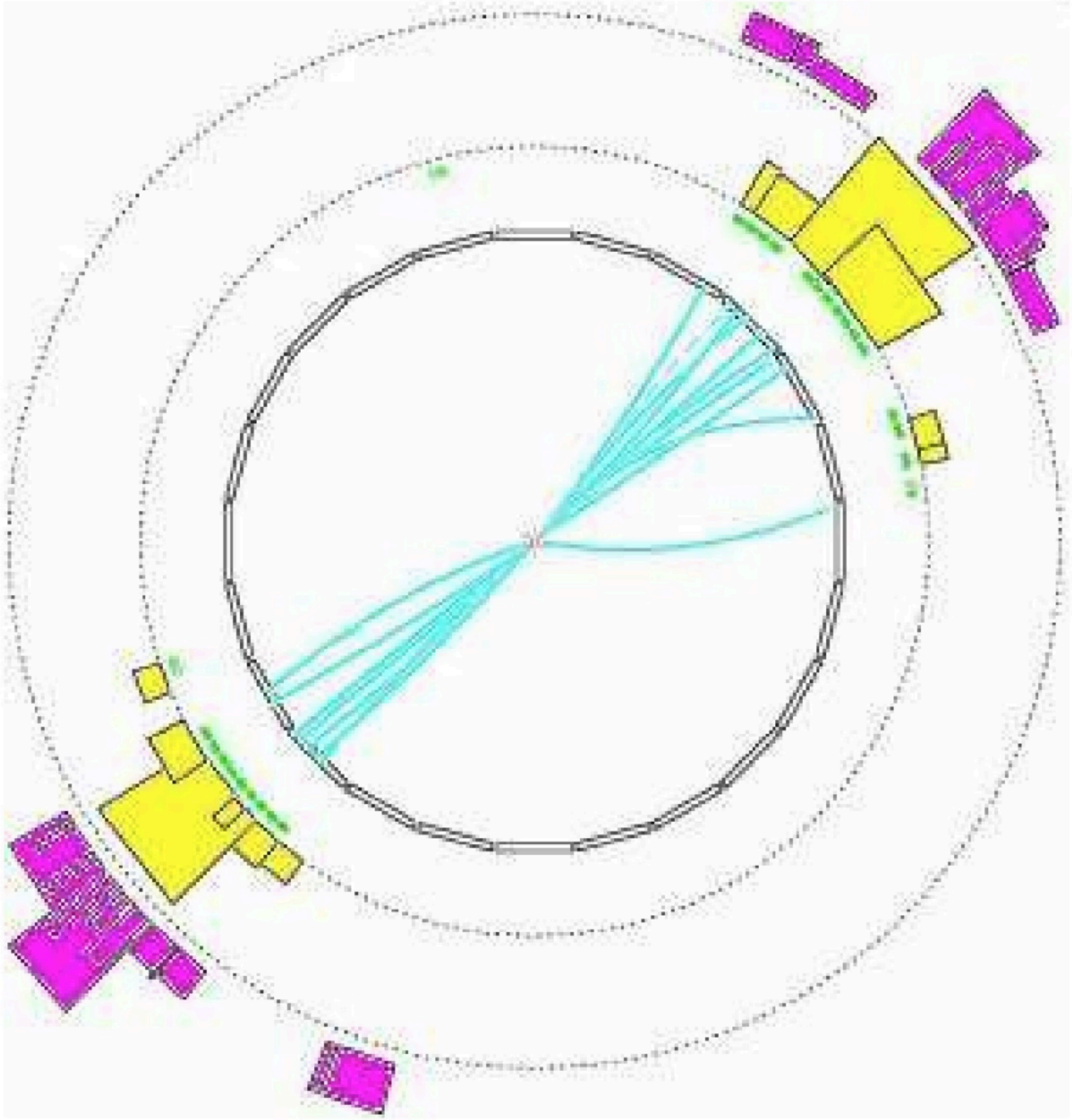
Lagrangian consists of **non-local** operators build out from collinear and soft fields

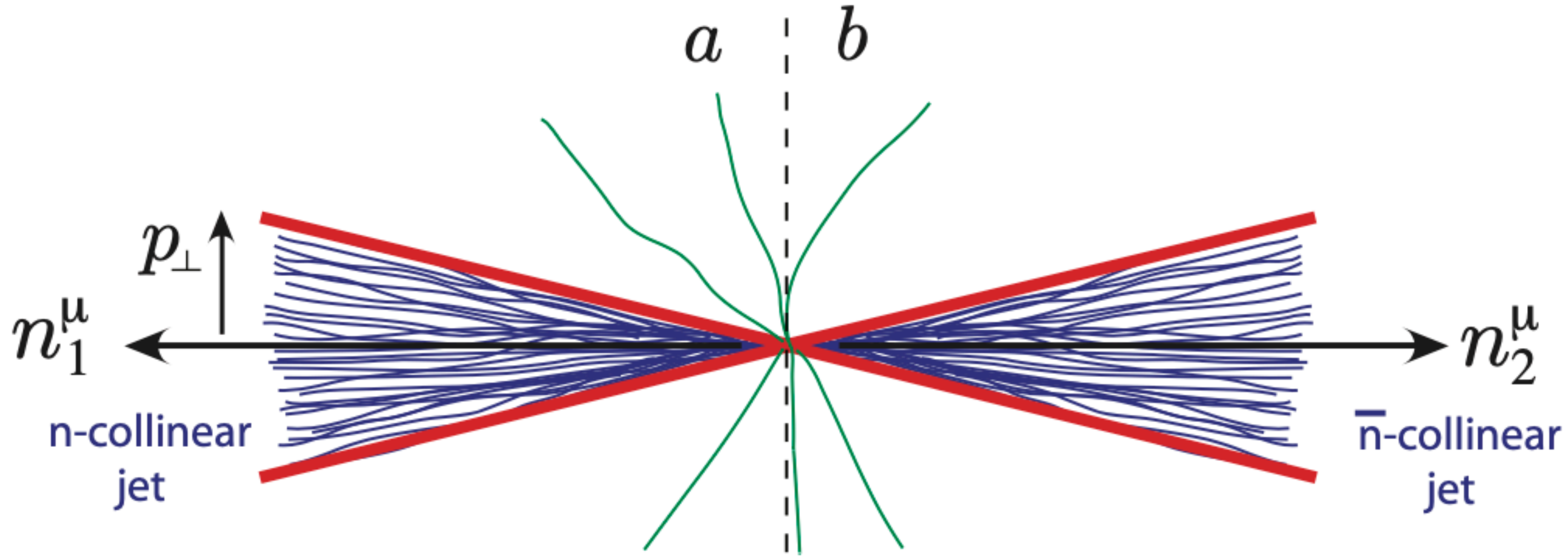


Lagrangian consists of local composite operator, ordered by dim, restricted by symmetry

SCET modes

dijet production in e+e-





SCET modes

Lightcone decomposition

$$n_1 = (1, \hat{n}_1) \quad \bar{n}_1 = (1, -\hat{n}_1)$$

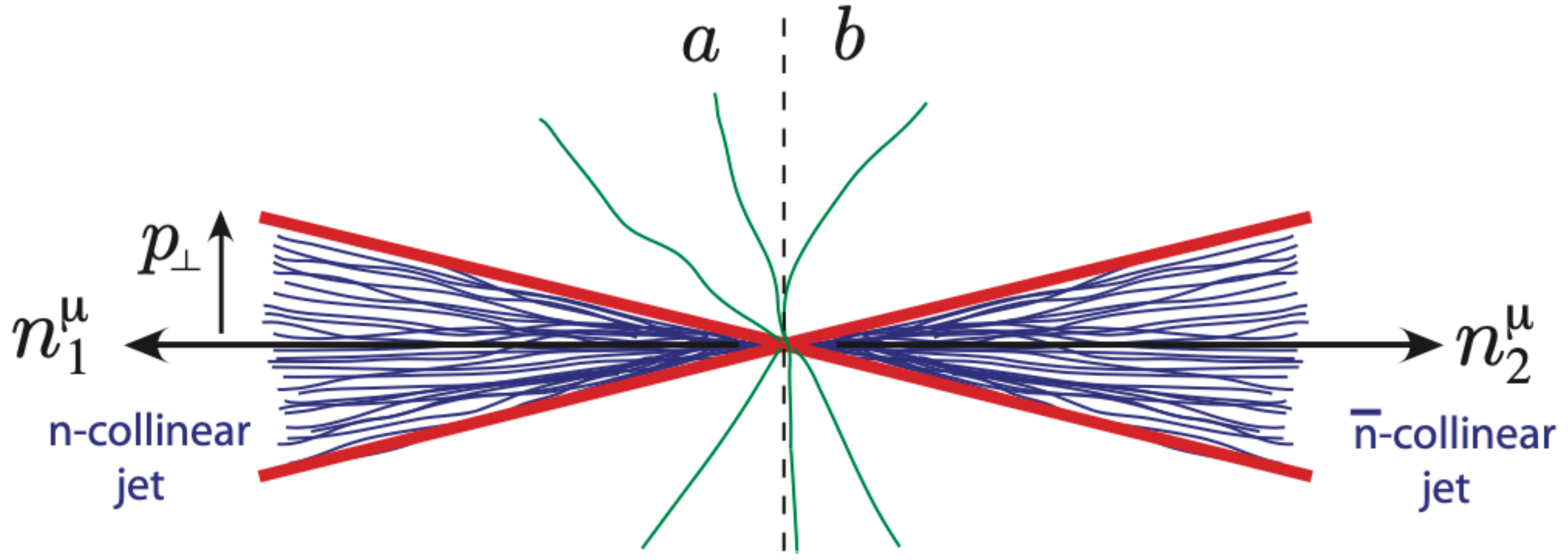
$$n_1^2 = \bar{n}_1^2 = 0$$

$$n_1 \cdot \bar{n}_1 = 2$$

$$p^\mu = \underbrace{\bar{n}_1 \cdot p}_{p^-} \frac{n_1^\mu}{2} + \underbrace{n_1 \cdot p}_{p^+} \frac{\bar{n}_1^\mu}{2} + p_\perp^\mu$$

$$p^\mu = (p^+, p^-, p_\perp)$$

$$p^2 = p^+ p^- + p_\perp^2$$



hard mode: $p_h^\mu = (Q, Q, Q)$

coll. mode: $p_c^\mu = Q(\lambda^2, 1, \lambda)$

anti-coll. mode: $p_{\bar{c}}^\mu = Q(1, \lambda^2, \lambda)$

soft mode: $p_s^\mu = Q(\lambda, \lambda, \lambda)$ relevant for jet broadening

ultra-soft mode: $p_{us}^\mu = Q(\lambda^2, \lambda^2, \lambda^2)$ relevant for jet mass

From QCD fields to SCET fields

For illustration, consider decomposing full quark field into collinear and usoft quark field

$$\psi(x) \Rightarrow \psi_c(x) + \psi_s(x)$$

$$\psi_c(x) = P_+\psi_c + P_-\psi_c = \xi + \eta$$

Projection operator:

$$P_+ = \frac{\not{n}\,\not{\bar{n}}}{4}\,, \qquad P_- = \frac{\not{\bar{n}}\,\not{n}}{4}$$

$$P_+^2 = P_+\,, \qquad P_-^2 = P_-\,, \qquad P_+ + P_= = \mathbf{1}$$

Note that:

$$\not{n}\xi = 0\,, \qquad \not{\bar{n}}\eta = 0$$

Fermion propagator:

$$\langle 0|\mathbf{T}\left[\xi(x)\bar{\xi}(0)\right]|0\rangle = \frac{\not{n}\bar{\not{n}}}{4}\langle 0|\mathbf{T}\left[\psi_c(x)\bar{\psi}_c(0)\right]|0\rangle\frac{\bar{\not{n}}\not{n}}{4}$$

$$= \int \frac{d^4p}{(2\pi)^4}\frac{i}{p^2+i\epsilon}e^{-ipx}\underbrace{\frac{\not{n}\bar{\not{n}}}{4}\not{p}\frac{\bar{\not{n}}\not{n}}{4}}_{\frac{\not{n}}{2}\bar{n}\cdot p} \sim \lambda^4\frac{1}{\lambda^2}$$

This completely determine the scaling of different component

$$\xi(x) \sim \lambda$$
$$\eta(x) \sim \lambda^2$$
$$\psi_s(x) \sim \lambda^{3/2}$$

Summary of field power counting

Type	$(p^+, p^-, p^\perp)$	Fields	Field Scaling
collinear	$(\lambda^2, 1, \lambda)$	$\xi_{n,p}$	λ
		$(A^+_{n,p}, A^-_{n,p}, A^\perp_{n,p})$	$(\lambda^2, 1, \lambda)$
soft	$(\lambda, \lambda, \lambda)$	$q_{s,p}$	$\lambda^{3/2}$
		$A^\mu_{s,p}$	λ
usoft	$(\lambda^2, \lambda^2, \lambda^2)$	q_{us}	λ^3
		A^μ_{us}	λ^2

Collinear Lagrangian

subleading power or factor out

$$\mathcal{L}_\psi = \bar{\psi} i \not{D} \psi = (\bar{\xi} + \bar{\eta} + \bar{\psi}_s) i \not{D} (\xi + \eta + \psi_s)$$

$$= (\bar{\xi} + \bar{\eta}) \left[\frac{\not{\bar{n}}}{2} i n \cdot D + \frac{\not{n}}{2} i \bar{n} \cdot D + i \not{D}_\perp \right] (\xi + \eta)$$

$$= \bar{\xi} \frac{\not{\bar{n}}}{2} i n \cdot D \xi + \bar{\xi} i \not{D}_\perp \eta + \bar{\eta} i \not{D}_\perp \xi + \bar{\eta} \frac{\not{n}}{2} i \bar{n} \cdot D \eta$$

homogeneous in power counting

$$p^\mu = \bar{n}_1 \cdot p \frac{n_1^\mu}{2} + n_1 \cdot p \frac{\bar{n}_1^\mu}{2} + p_\perp^\mu$$

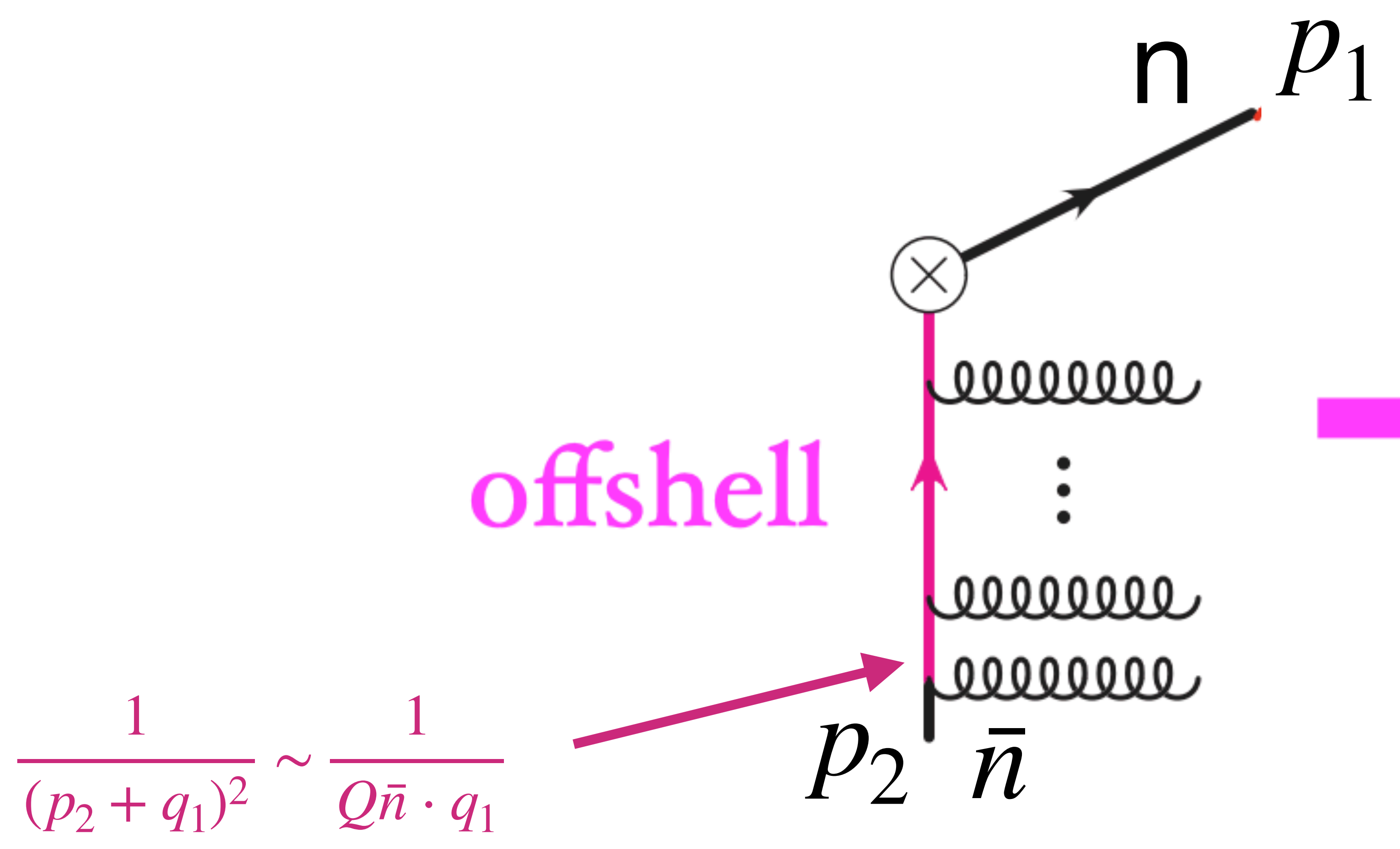
$$\not{n} \xi = 0 \,, \qquad \not{\bar{n}} \eta = 0$$

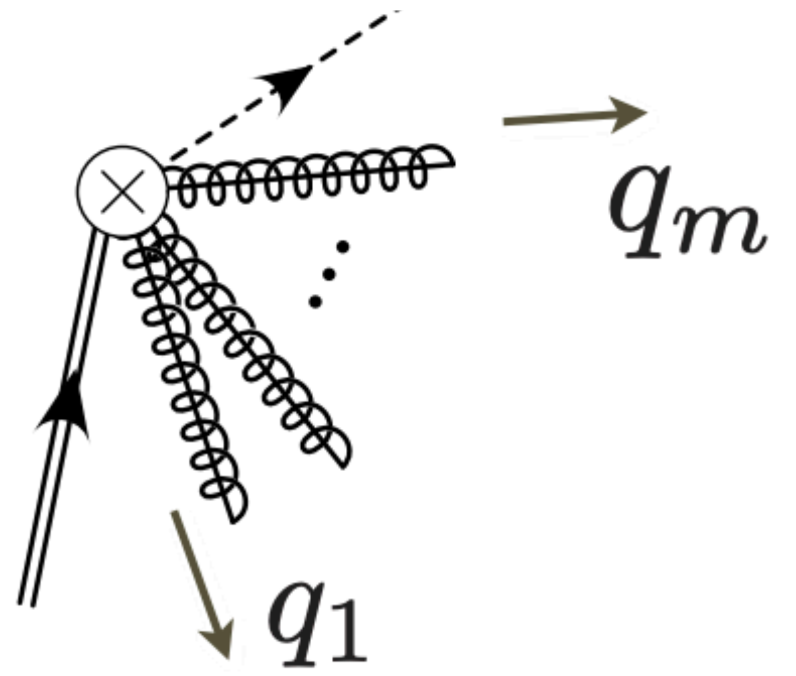
η can be completely integrated out suing EOM

$$\eta = \frac{1}{i \bar{n} \cdot D} i \not{D}_\perp \frac{\not{\bar{n}}}{2} \xi$$

$$\mathcal{L}_c = \bar{\xi} \left[\frac{\not{\bar{n}}}{2} i n \cdot D - i \not{D}_\perp \frac{\not{\bar{n}}}{2} \frac{1}{i \bar{n} \cdot D} i \not{D}_\perp \right] \xi$$

Hard-collinear factorization





$\bar{n}\cdot A_n$ is $\mathcal{O}(\lambda^0)$. Can add any number of collinear gluon

$$= g^m \sum_{\text{perms}} \frac{(\bar{n}^{\mu_m} T^{A_m})\cdots(\bar{n}^{\mu_1} T^{A_1})}{[\bar{n}\cdot q_1][\bar{n}\cdot(q_1+q_2)]\cdots[\bar{n}\cdot\sum_{i=1}^m q_i]}$$

Sum into collinear Wilson line:

$$W_n = \sum_k \sum_{\text{perm}} \frac{(-g)^k}{k!}\left(\frac{\bar{n}\cdot A_n(q_1)\cdots\bar{n}\cdot A_n(q_k)}{[\bar{n}\cdot q_1][\bar{n}\cdot(q_1+q_2)]\cdots[\bar{n}\cdot\sum_{i=1}^k q_i]}\right)$$ mom. space

$$W(0,-\infty) = \mathrm{P}\exp\left(ig\int_{-\infty}^0 ds\,\bar{n}\cdot A_n(\bar{n}s)\right)$$ position space

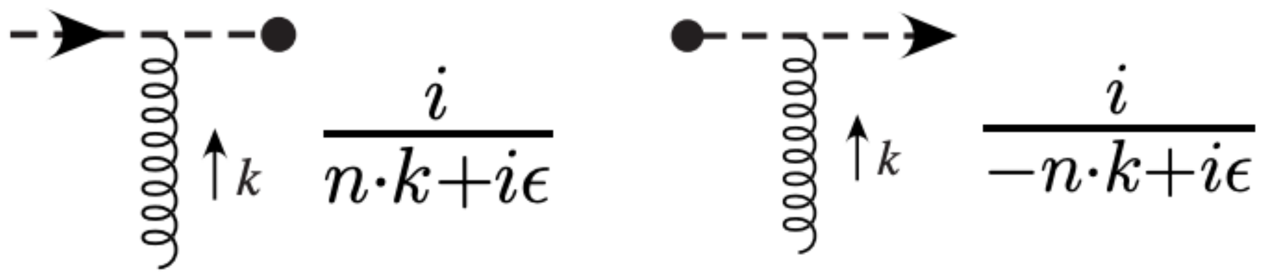
Collinear building block: $\chi_n = W_n^\dagger \xi_n$ $\quad \bar{\chi}_n = \bar{\xi}_n W_n$ $\quad \mathcal{B}_{n\perp}^\mu = \frac{1}{g}[W_n^\dagger i D_\perp^\mu W_n]$

(U)Soft-collinear factorization

Multi-pole expansion of collinear lagraigian:

$$\mathcal{L}_c = \bar{\xi}\left[n\cdot iD_{us} + gn\cdot A_n + i\not{D}_\perp^c \frac{1}{i\bar{n}\cdot D_c} i\not{D}_\perp^c\right]\frac{\not{\bar{n}}}{2}\xi$$

eikonal Feynman rule:



(u)soft-collinear interaction remove by BPS field redefinition

$$\xi \to Y_n \xi^{(0)}\,, \qquad A_c \to Y_n A_c^{(0)} Y_n^\dagger$$

$$Y_n(x) = P\exp\left(ig\int_{-\infty}^{0} ds\, n\cdot A_{us}(x+ns)\right) \qquad n\cdot D_{us} Y_n = 0\,, \quad Y_n^\dagger Y_n = 1$$

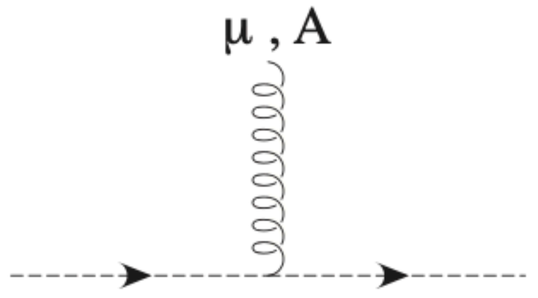
$$\mathcal{L}_c \to \mathcal{L}_c^{(0)} = \bar{\xi}^{(0)}\left[n\cdot iD_c + i\not{D}_{c\perp}\frac{1}{i\bar{n}\cdot D_c} i\not{D}_{c\perp}\right]\frac{\not{\bar{n}}}{2}\xi^{(0)}$$

complete decoupling between collinear and soft sector

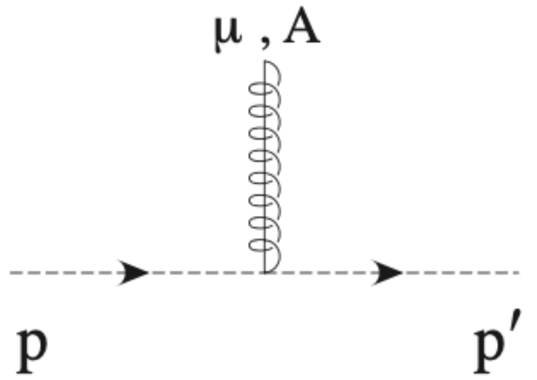
SCET Feynman rules

(p, p_r)

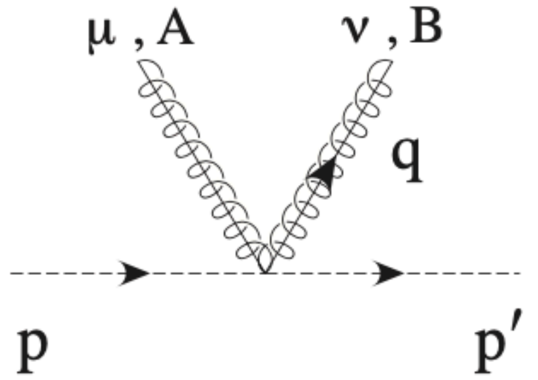
$$= i\,\frac{\not n}{2}\,\frac{\bar n\cdot p}{n\cdot p_r\,\bar n\cdot p + p_\perp^2 + i0}$$



$$= ig\,T^A\, n_\mu\, \frac{\not{\bar n}}{2}$$



$$= ig\,T^A \left[n_\mu + \frac{\gamma_\mu^\perp \not p_\perp}{\bar n\cdot p} + \frac{\not p'_\perp \gamma_\mu^\perp}{\bar n\cdot p'} - \frac{\not p'_\perp \not p_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n_\mu \right] \frac{\not{\bar n}}{2}$$



$$= \frac{ig^2\,T^A\,T^B}{\bar n\cdot(p-q)} \left[\gamma_\mu^\perp \gamma_\nu^\perp - \frac{\gamma_\mu^\perp \not p_\perp}{\bar n\cdot p}\bar n_\nu - \frac{\not p'_\perp \gamma_\nu^\perp}{\bar n\cdot p'}\bar n_\mu + \frac{\not p'_\perp \not p_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n_\mu \bar n_\nu \right] \frac{\not{\bar n}}{2}$$

$$+ \frac{ig^2\,T^B\,T^A}{\bar n\cdot(q+p')} \left[\gamma_\nu^\perp \gamma_\mu^\perp - \frac{\gamma_\nu^\perp \not p_\perp}{\bar n\cdot p}\bar n_\mu - \frac{\not p'_\perp \gamma_\mu^\perp}{\bar n\cdot p'}\bar n_\nu + \frac{\not p'_\perp \not p_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n_\mu \bar n_\nu \right] \frac{\not{\bar n}}{2}$$

In reality, collinear sector of SCET can be considered as boosted version of QCD. Since amplitudes are Lorentz invariant, it is more convenient to use QCD Feynman rule for calculation

Factorized SCET Lagrangian

$$\mathcal{L} = \sum_{p\geq 0} \mathcal{L}^{(p)}_{\rm dyn} + \sum_{p} \mathcal{L}^{(p)}_{\rm hard} + \mathcal{L}^{(0)}_{\rm G}$$

Dynamics of infrared modes

Hard Scattering operators (often only once)

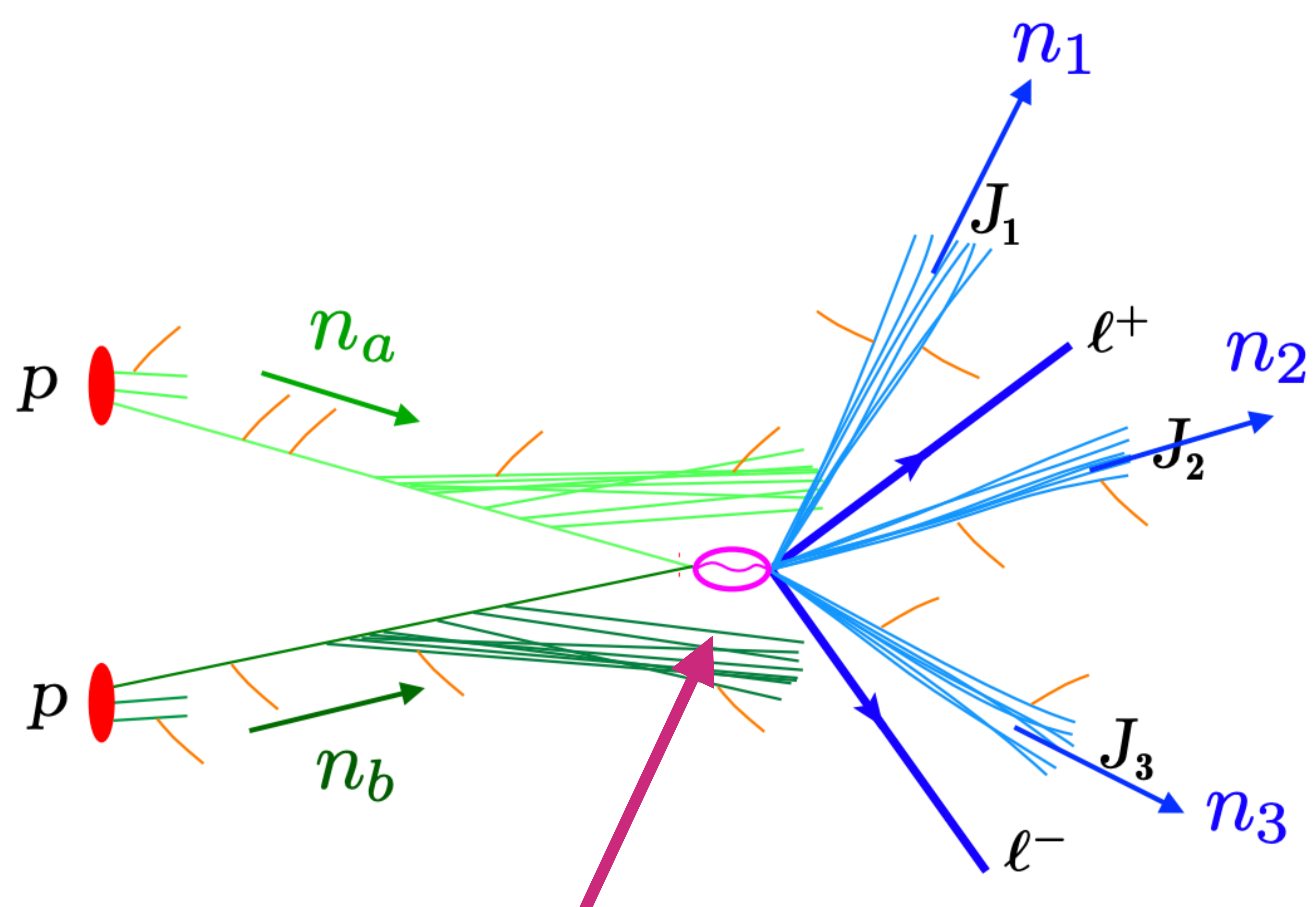
Glauber gluon exchange (only factorization violating term)

At leading power in expansion parameter, only need

$$\mathcal{L}^{(0)}_{\rm hard} = \sum_i C_i^{(0)} \mathcal{O}_i^{(0)}$$

$$\mathcal{L}^{(0)}_{\rm dyn} = \sum_n \mathcal{L}^{(0)}_n + \mathcal{L}^{(0)}_{\rm soft}$$

Glauber "usually" cancel out, but not always!



hard operator build out from

$$\chi_n = W_n^\dagger \xi_n \quad \bar{\chi}_n = \bar{\xi}_n W_n \quad \mathcal{B}^\mu_{n\perp} = \frac{1}{g}[W_n^\dagger i D^\mu_\perp W_n]$$

Wilson coefficient obtained from matching to full QCD

Schematic matching and evolving in SCET

Consider the jets production in hadronic collision

step 1: match from QCD to SCET hard operators

hard function: $H = |C|^2$

step 2: compute collinear matrix element; evolving from hard scale to collinear scale

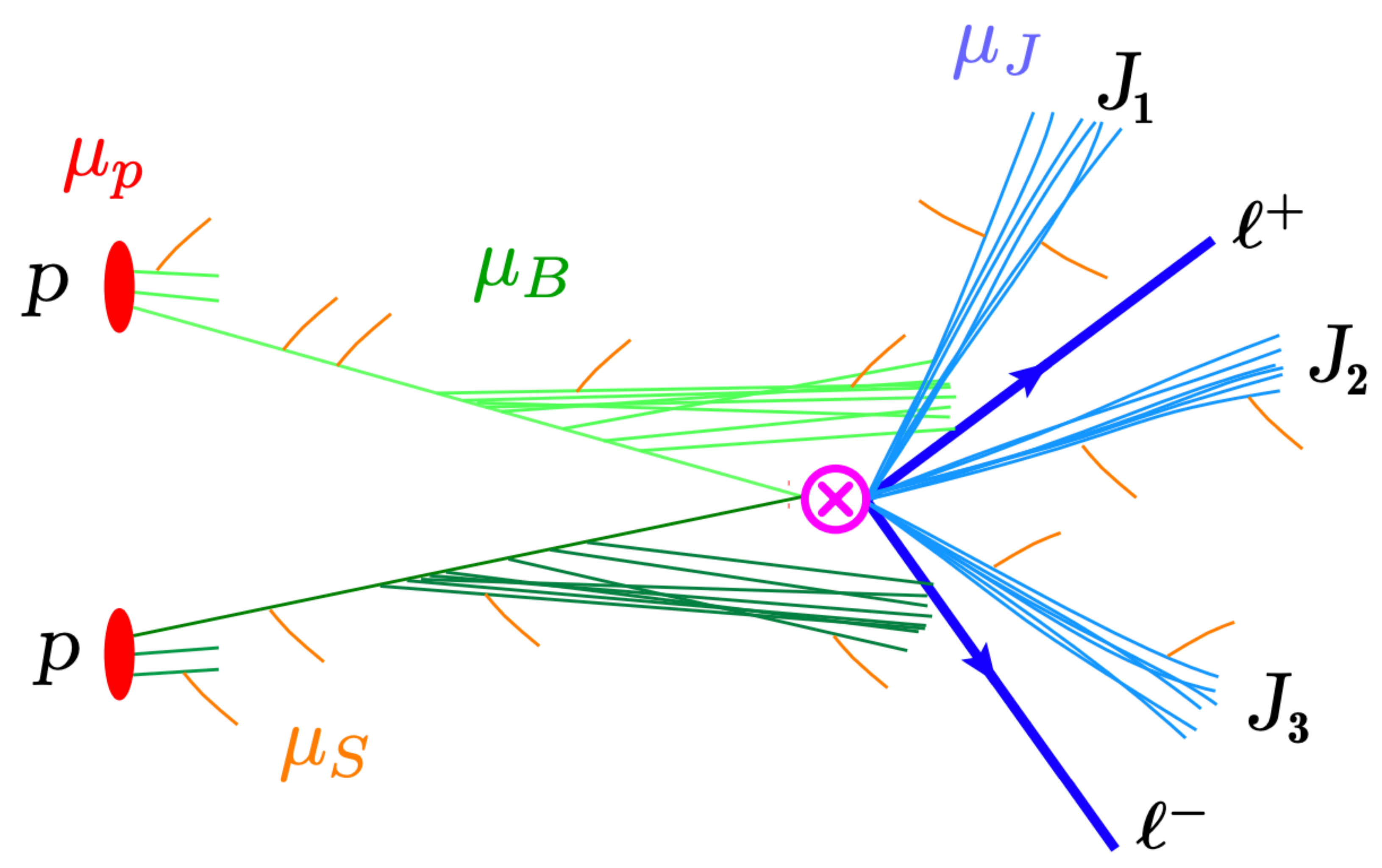
Beam and jet function: $B_{a,b}, J_i$

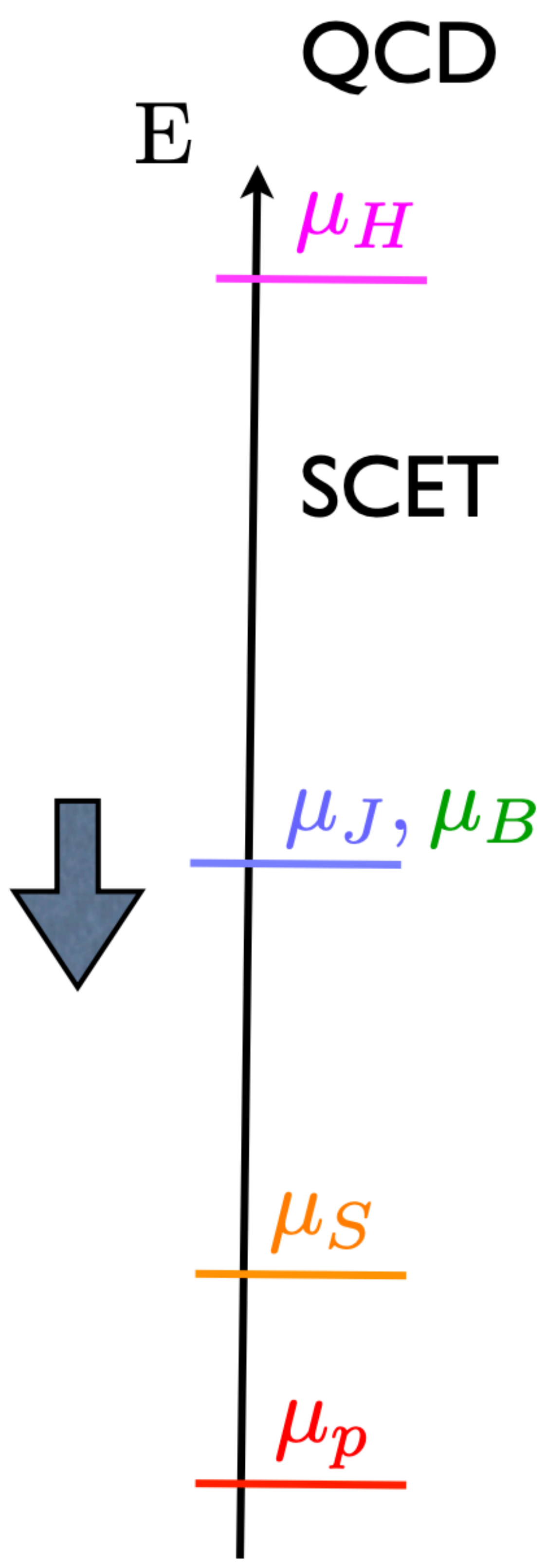
large logs: $\log(\mu_H^2/\mu_J^2)$

step 3: compute soft matrix element; evolving from jet scale to soft scale

Soft function: $S = |\langle Y_a Y_b Y_1 Y_2 Y_3 \rangle|^2$

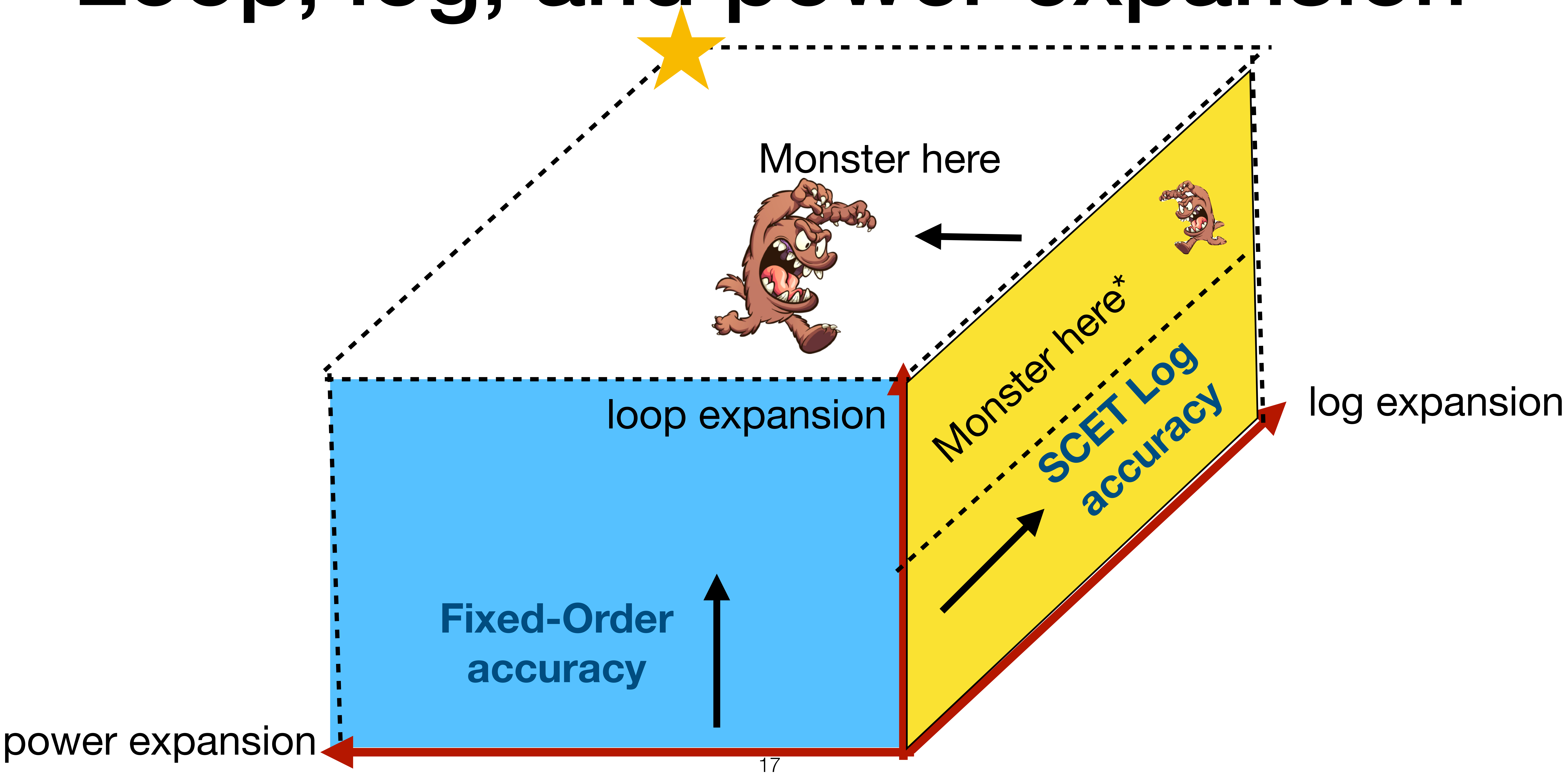
large logs: $\log(\mu_H^2/\mu_S^2),\ \log(\mu_J^2/\mu_S^2)$



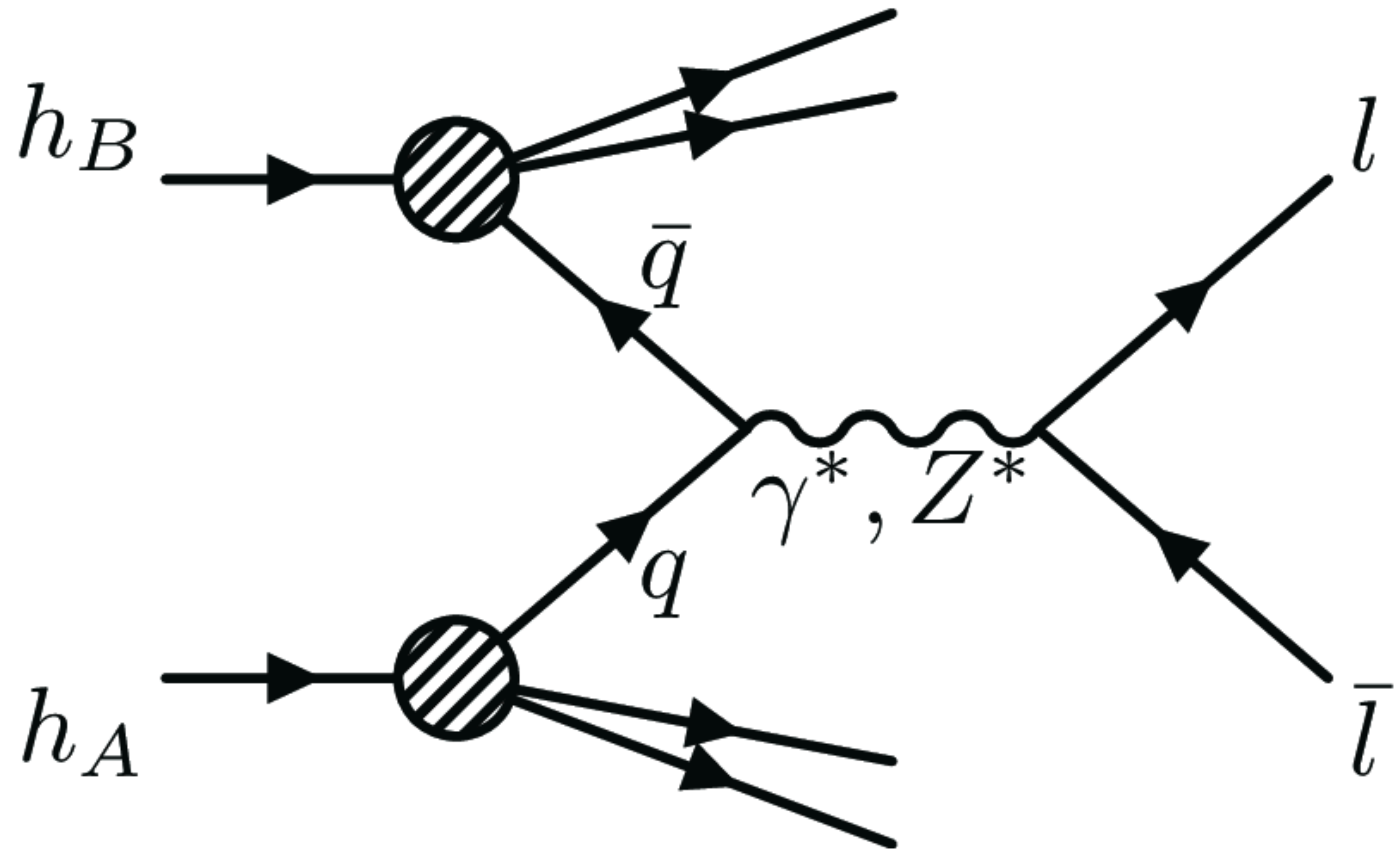


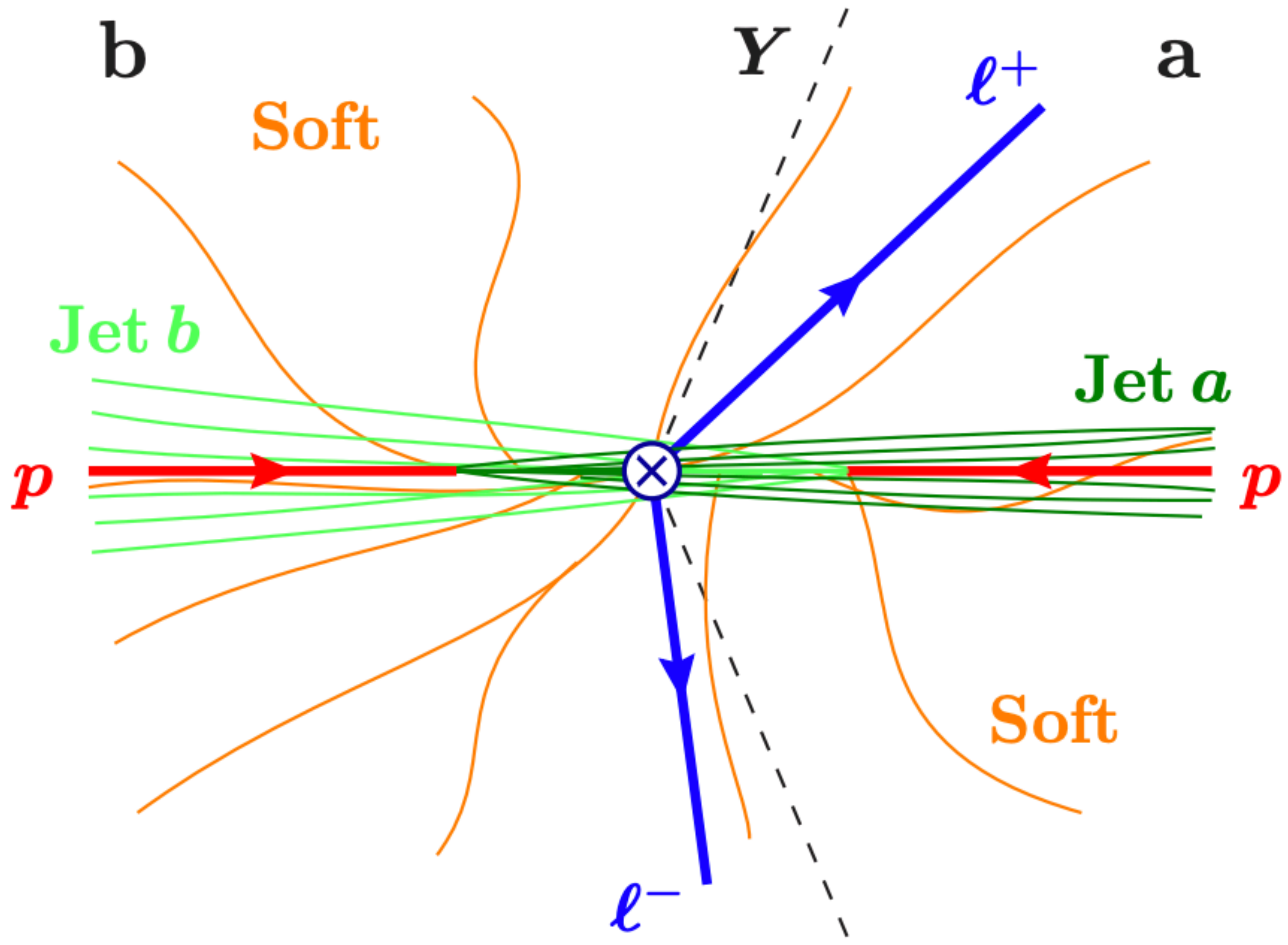
step 4: match onto non-perturbative matrix element: Parton Distribution Functions and Fragmentation Functions

Loop, log, and power expansion

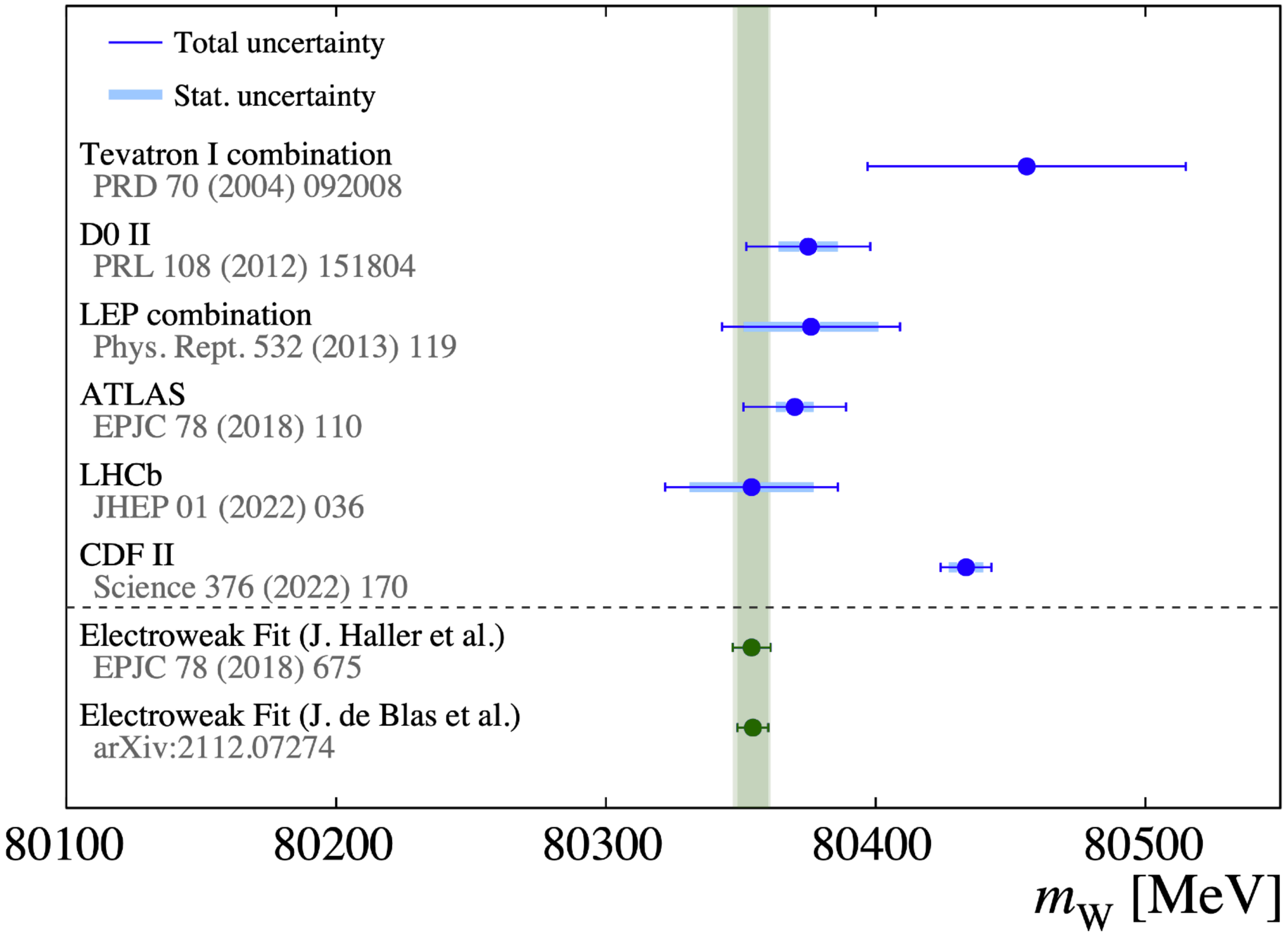


Application to Drell-Yan (Higgs) pT distribution

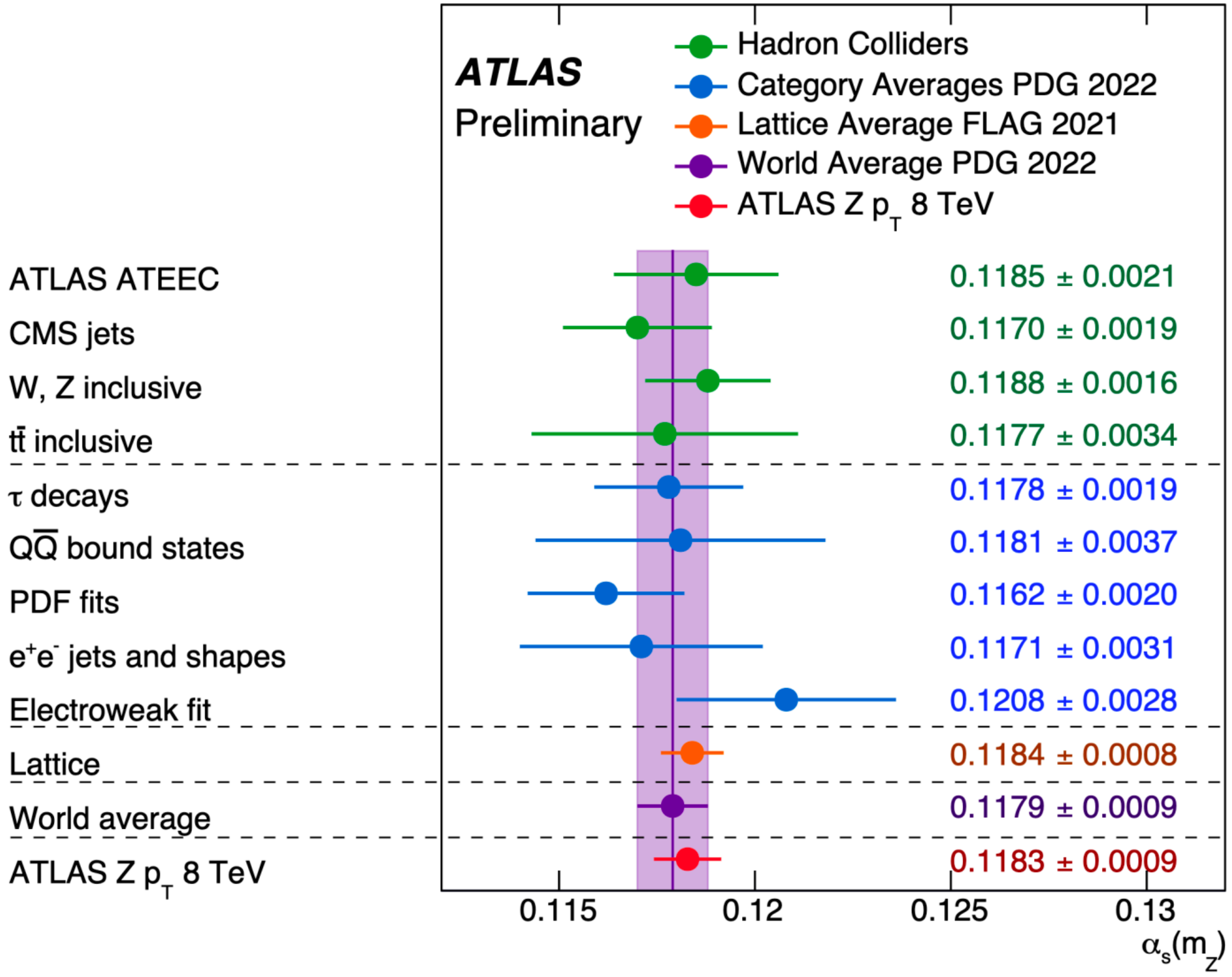




Importance of pT distribution

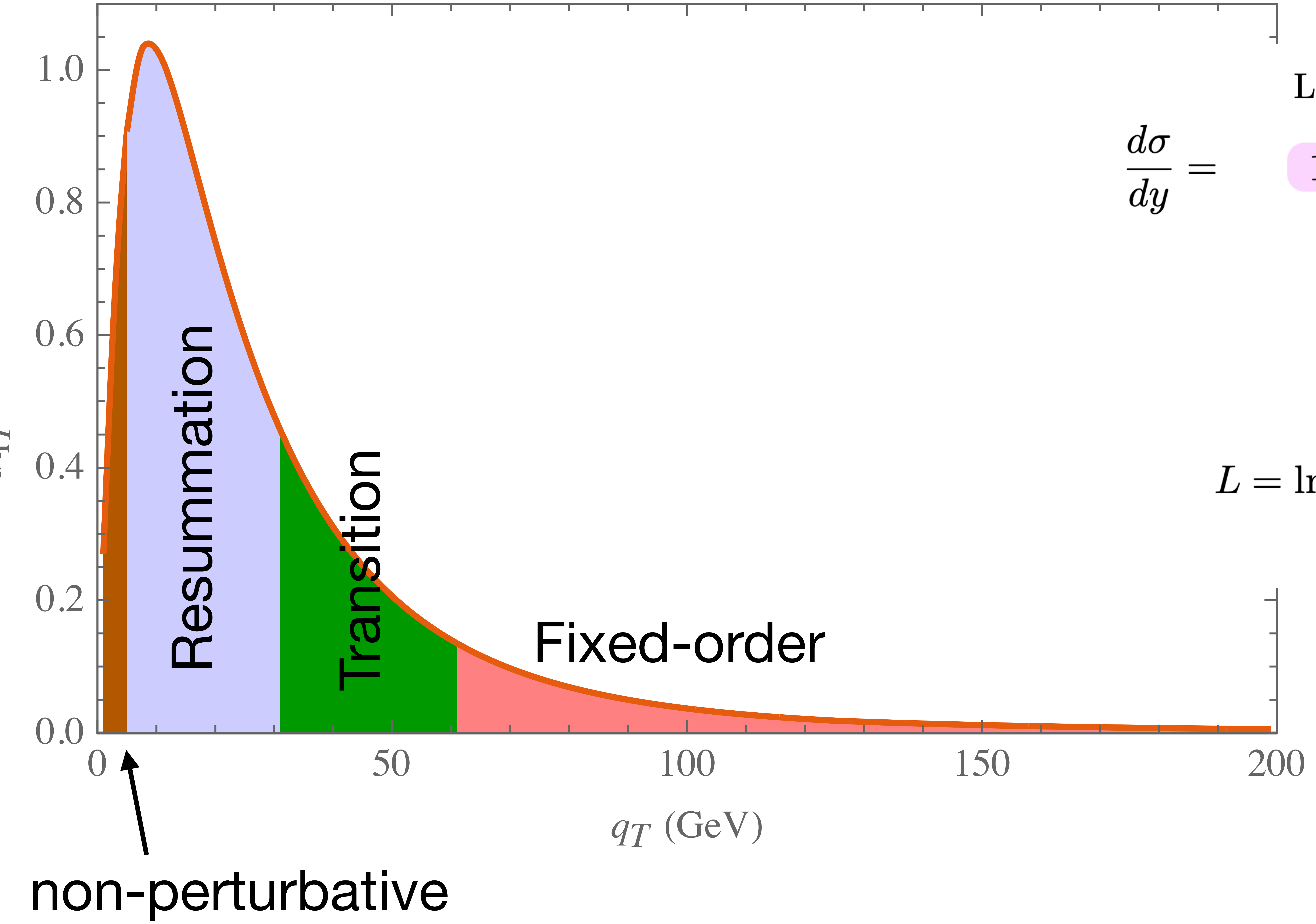


CDF measurement of W mass



ATLAS measurement of strong coupling

Large logs



	LO	NLO	NNLO	N^{3}LO		
$\frac{d\sigma}{dy} =$	1	$+\,\alpha_s L^2$	$+\,\alpha_s^2 L^4$	$+\,\alpha_s^3 L^6$	$+\ldots$	LL
		$+\,\alpha_s L$	$+\,\alpha_s^2 L^3$	$+\,\alpha_s^3 L^5$	$+\ldots$	NLL
		$+\,\alpha_s$	$+\,\alpha_s^2 L^2$	$+\,\alpha_s^3 L^4$	$+\ldots$	NNLL
			$+\,\alpha_s^2 L$	$+\,\alpha_s^3 L^3$	$+\ldots$	NNLL
			$+\,\alpha_s^2$	$+\,\alpha_s^3 L^2$	$+\ldots$	N^{3}LL
				$+\,\alpha_s^3 L$	$+\ldots$	N^{3}LL
				$+\,\alpha_s^3$	$+\ldots$	

$L = \ln y$

y is the Fourier conjugate of qT

Counting of logarithmic accuracy

	matching (singular)	nonsingular	γ_x	$\Gamma_{\rm cusp}$	β	PDF
LO	LO	LO	-	-	1-loop	LO
NLO	NLO	NLO	-	-	2-loop	NLO
NNLO	NNLO	NNLO	-	-	3-loop	NNLO
LL	LO	-	-	1-loop	1-loop	LO
NLL	LO	-	1-loop	2-loop	2-loop	LO
NNLL	NLO	-	2-loop	3-loop	3-loop	NLO
NLL′+NLO	NLO	NLO	1-loop	2-loop	2-loop	NLO
NNLL+NNLO	(N)NLO	NNLO	2-loop	3-loop	3-loop	NNLO
NNLL′+NNLO	NNLO	NNLO	2-loop	3-loop	3-loop	NNLO
N^3LL+NNLO	NNLO	NNLO	3-loop	4-loop	4-loop	NNLO

Davies, Webber, Stirling, 1985

Y. Li, HXZ, 2016

NNNLO

M.X. Luo, T.Z. Yang, Y.J. Zhu, HXZ, 2019

Ebert, Mistlberger, Vita, 2020

4-loop

Moult, Y.J. Zhu, HXZ, 2022

Duhr, Mistlberger, Vita, 2022

Naive qT factorization in SCET

$$d\sigma \sim \int d^4x e^{-iq\cdot x}\langle N_1N_2|J^{\mu\dagger}(x)J_\mu(0)|N_1N_2\rangle \qquad J^\mu = \bar{\psi}\gamma^\mu\psi$$

Match on SCET operator $\quad J^\mu \to C_V(-q^2)\bar{\chi}_{\bar{n}}Y_{\bar{n}}^\dagger\gamma^\mu Y_n\chi_n$

$$d\sigma \sim \int d^4x e^{-iq\cdot x}|C_V(-q^2)|^2\langle 0|\mathrm{Tr}[\overline{T}(Y_n^\dagger(x_\perp)Y_{\bar{n}}(x_\perp))T(Y_{\bar{n}}^\dagger(0)Y_n(0))]|0\rangle$$

$$\times\langle N_1|\bar{\chi}_n(x_+ + x_\perp)\frac{\not{\bar{n}}}{2}\chi_n(0)|N_1\rangle\langle N_2|\bar{\chi}_{\bar{n}}(x_- + x_\perp)\frac{\not{n}}{2}\chi_{\bar{n}}(0)|N_2\rangle$$

$$d\sigma \sim \int d^2x_\perp e^{-i(q^+x^-/2+q^-x^+/2+q_\perp\cdot x_\perp)}H(q^2)S(x_\perp)B_1(x_+,x_\perp)B_2(x_-,x_\perp)$$

$$\frac{dH(q^2,\mu)}{d\log\mu} = \left[\Gamma_{\mathrm{cusp}}(\alpha_s)\log\frac{-q^2}{\mu^2} + 2\gamma^q(\alpha_s)\right]H(q^2,\mu)$$

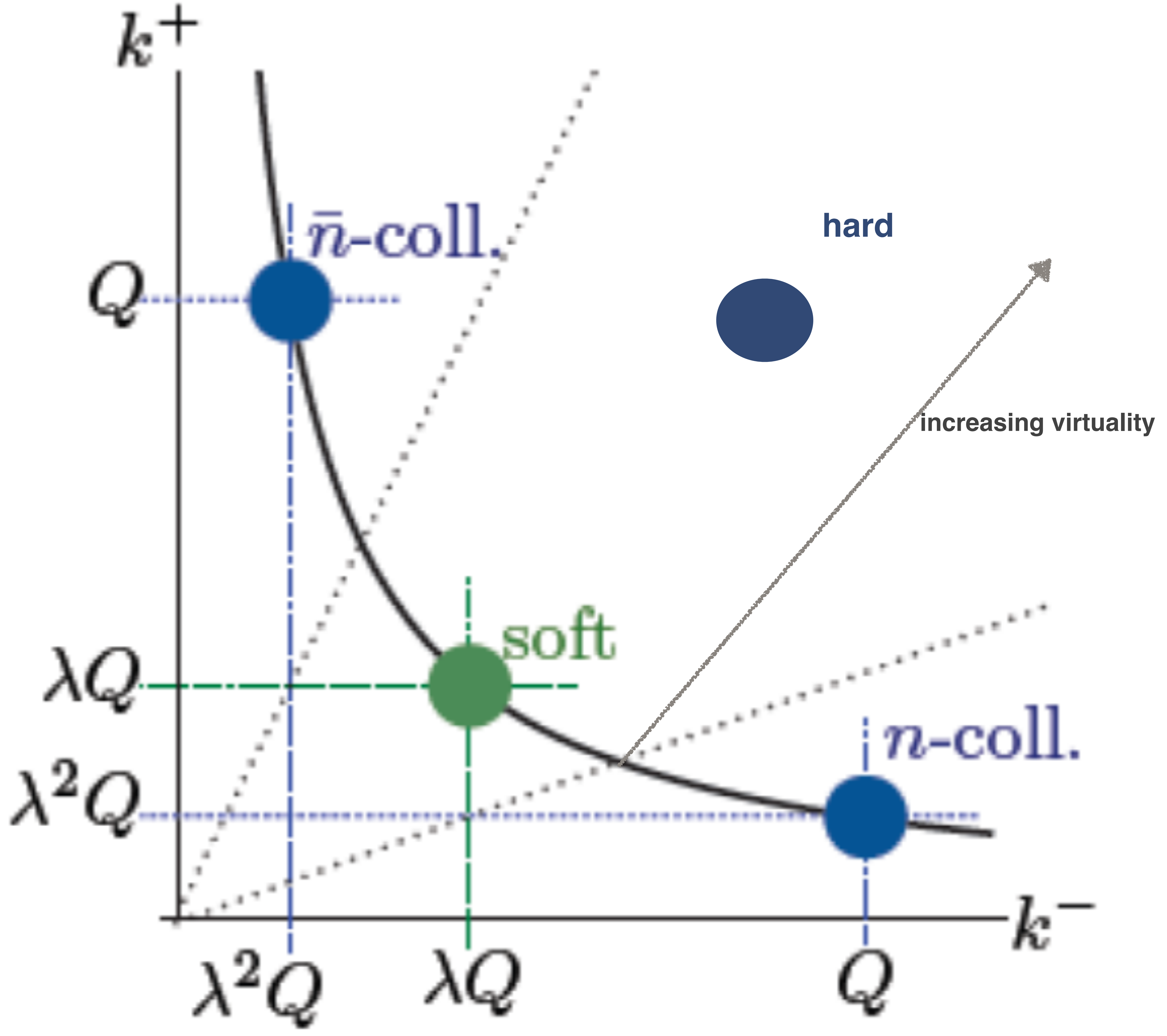
Degeneracy of soft and (anti) collinear modes

$$p_h^\mu = (Q, Q, Q)$$

$$p_c^\mu = Q(\lambda^2, 1, \lambda)$$

$$p_{\bar{c}}^\mu = Q(1, \lambda^2, \lambda)$$

$$p_s^\mu = Q(\lambda, \lambda, \lambda)$$



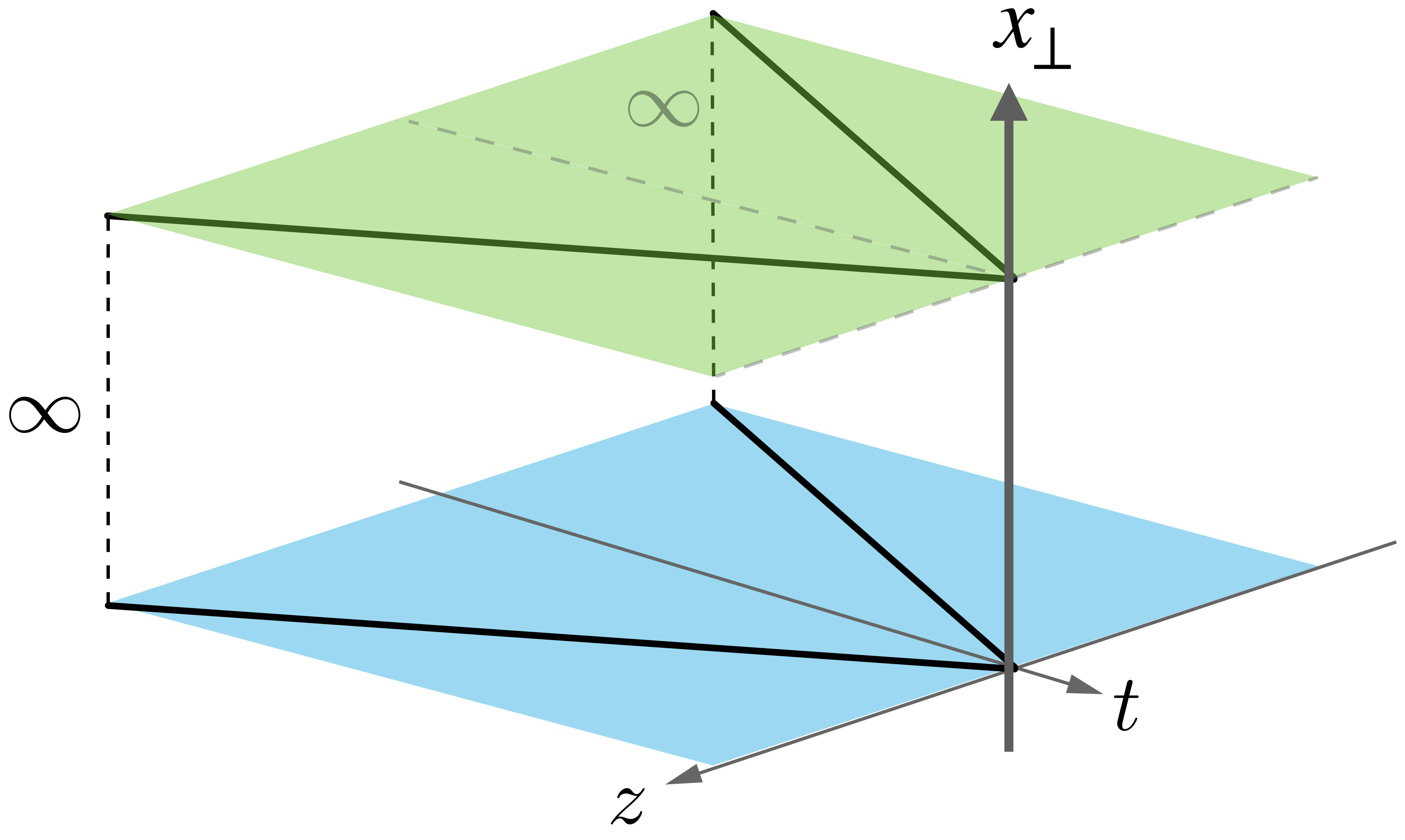
Lightcone/rapidity divergence

naive soft function

$$S(x_\perp) = \langle 0|\mathrm{Tr}[\overline{T}(Y_n^\dagger(x_\perp)Y_{\bar{n}}(x_\perp))T(Y_{\bar{n}}^\dagger(0)Y_n(0))]|0\rangle$$

$$Y_n(x) = P\exp\left(ig\int_{-\infty}^0 ds\, n\cdot A_{us}(x+ns)\right)$$

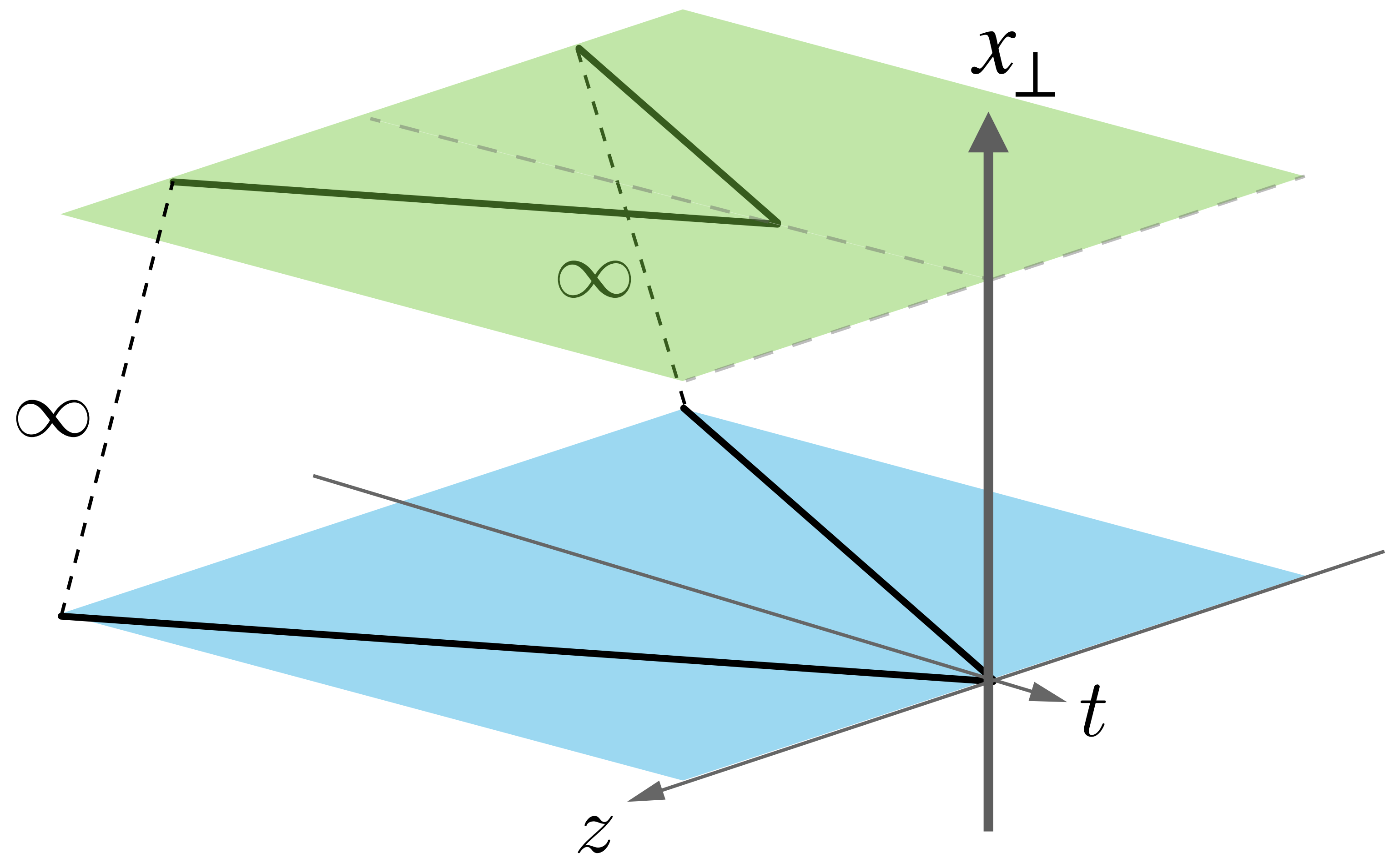
The configuration is invariant under Lorentz boost in z



Exponential regulated soft function

Y. Li, Neill, HXZ, 2016

$$S(x_\perp) = \langle 0|\mathrm{Tr}[\overline{T}(Y_n^\dagger(x_0+x_\perp)Y_{\bar{n}}(x_0+x_\perp))T(Y_{\bar{n}}^\dagger(0)Y_n(0))]|0\rangle$$



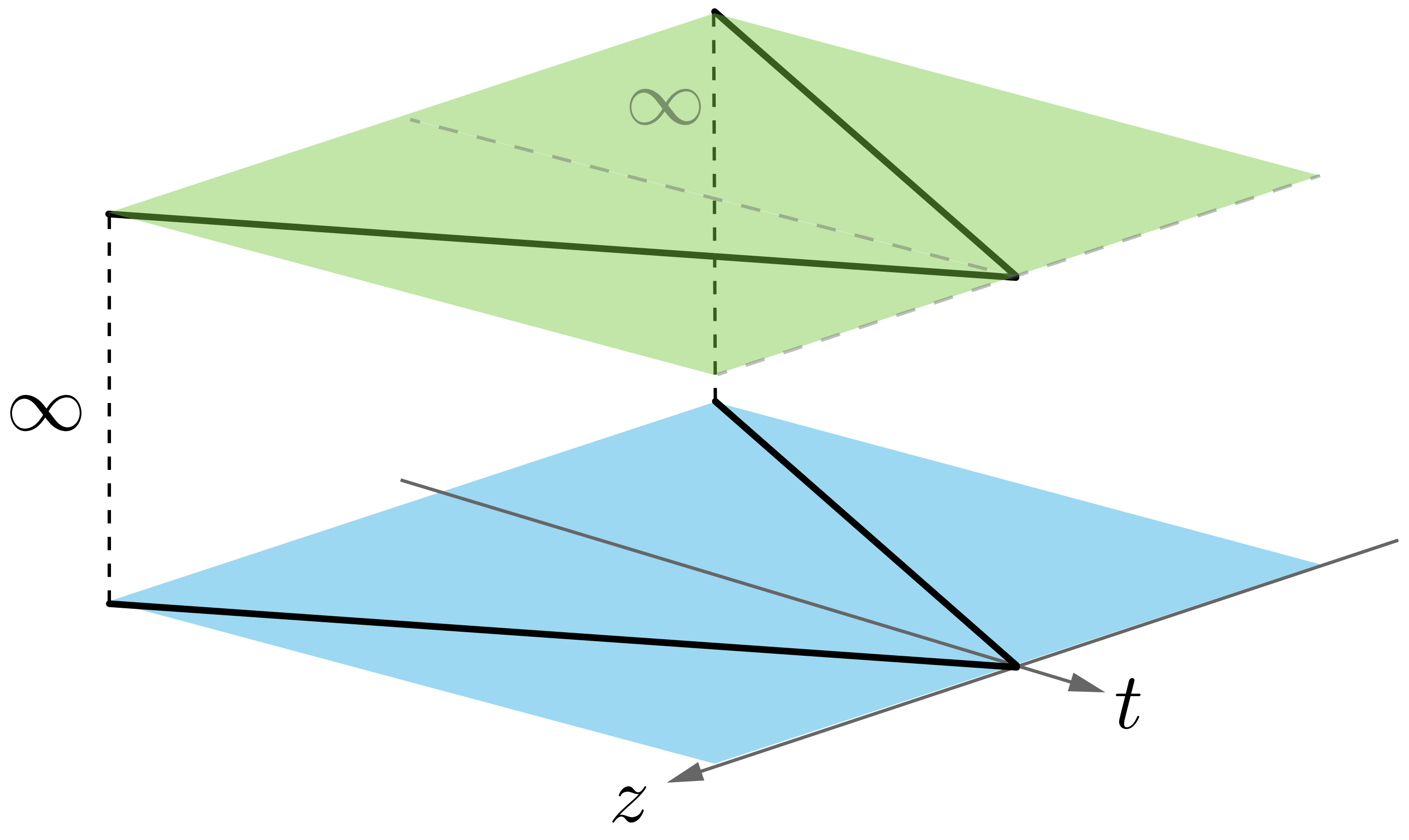
Three-loop TMD soft function Y. Li, HXZ, 2016

Three-loop TMD beam function M.X. Luo, T.Z. Yang, Y.J. Zhu, HXZ, 2019

Ebert, Mistlberger, Vita, 2020

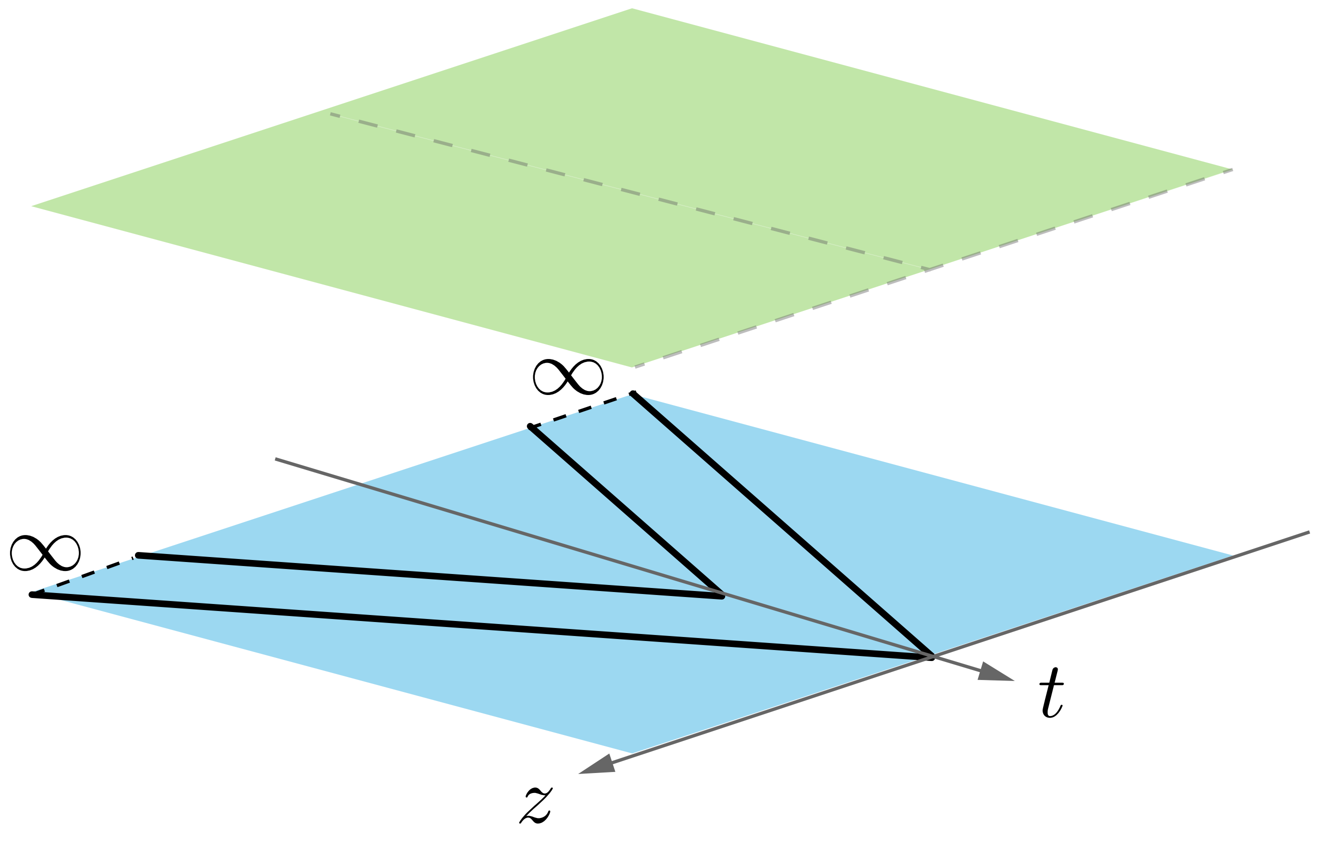
Conformal symmetry of Collins-Soper kernel

$$\frac{d\log S(x_\perp,\mu,\nu)}{d\log\nu^2} = \left[\int_{\mu^2}^{x_\perp^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\Gamma_{\rm cusp}[\alpha_s(\bar{\mu})] + \gamma_r[\alpha_s(x_\perp^{-1})]\right] S(x_\perp,\mu,\nu)$$



Y. Li, HXZ, 2016

$$\gamma_0^r = \gamma_0^s$$
$$\gamma_1^r = \gamma_1^s - \beta_0 c_1^s$$
$$\gamma_2^r = \gamma_2^s - 2\beta_0 c_2^s - \beta_1 c_1^s + 2C_a C_A \beta_0 \zeta_4$$



Proved by conformal transformation connection the two configuration Vladimirov, 2017

Proved using EEC in the back-to-back limit Moult, Y.J. Zhu, HXZ, 2022

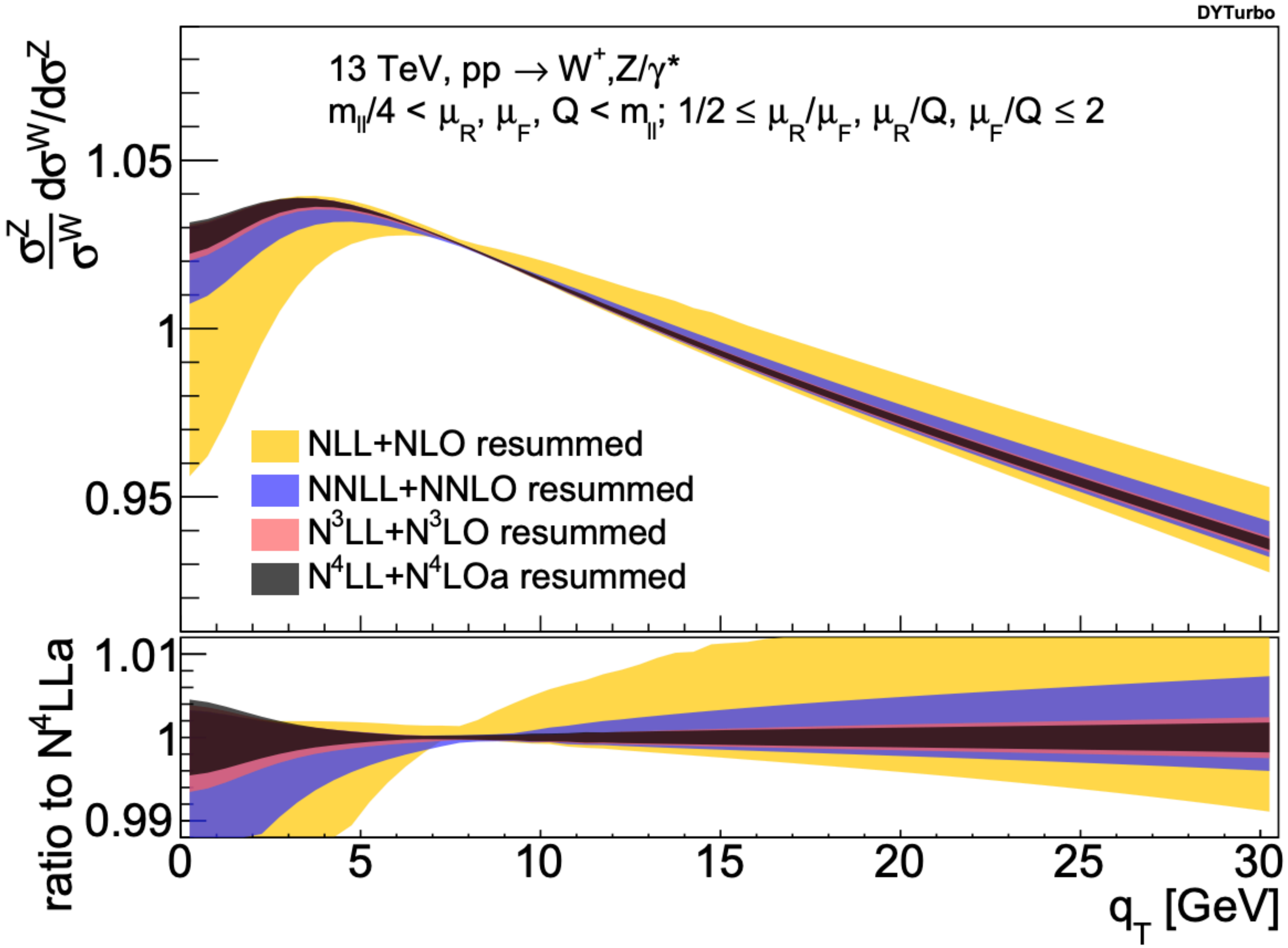
$$\frac{d\alpha_s(\mu)}{d\log\mu^2} = -2\epsilon + 2\beta(\alpha_s)$$ ⇒ perturbative QCD is conformal in $\epsilon^* = \beta(\alpha)$

$$\gamma_r(\alpha,\epsilon^*) = \gamma_s(\alpha)$$ ⇒ Four-loop rapidity Collines-Soper Kernel

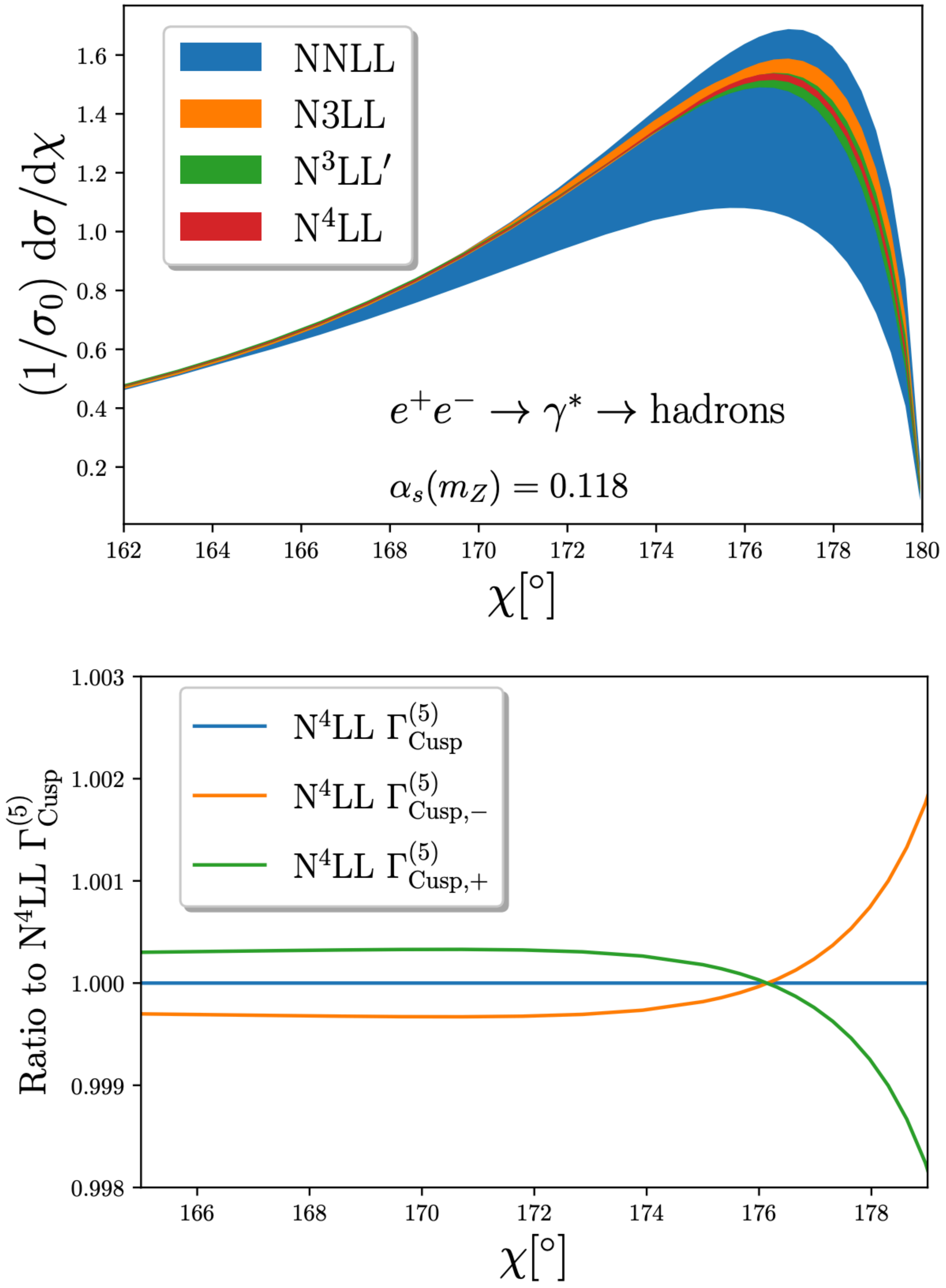
Moult, Y.J. Zhu, HXZ, 2022

Ebert, Mistlberger, Vita, 2020

N4LL uncertainties

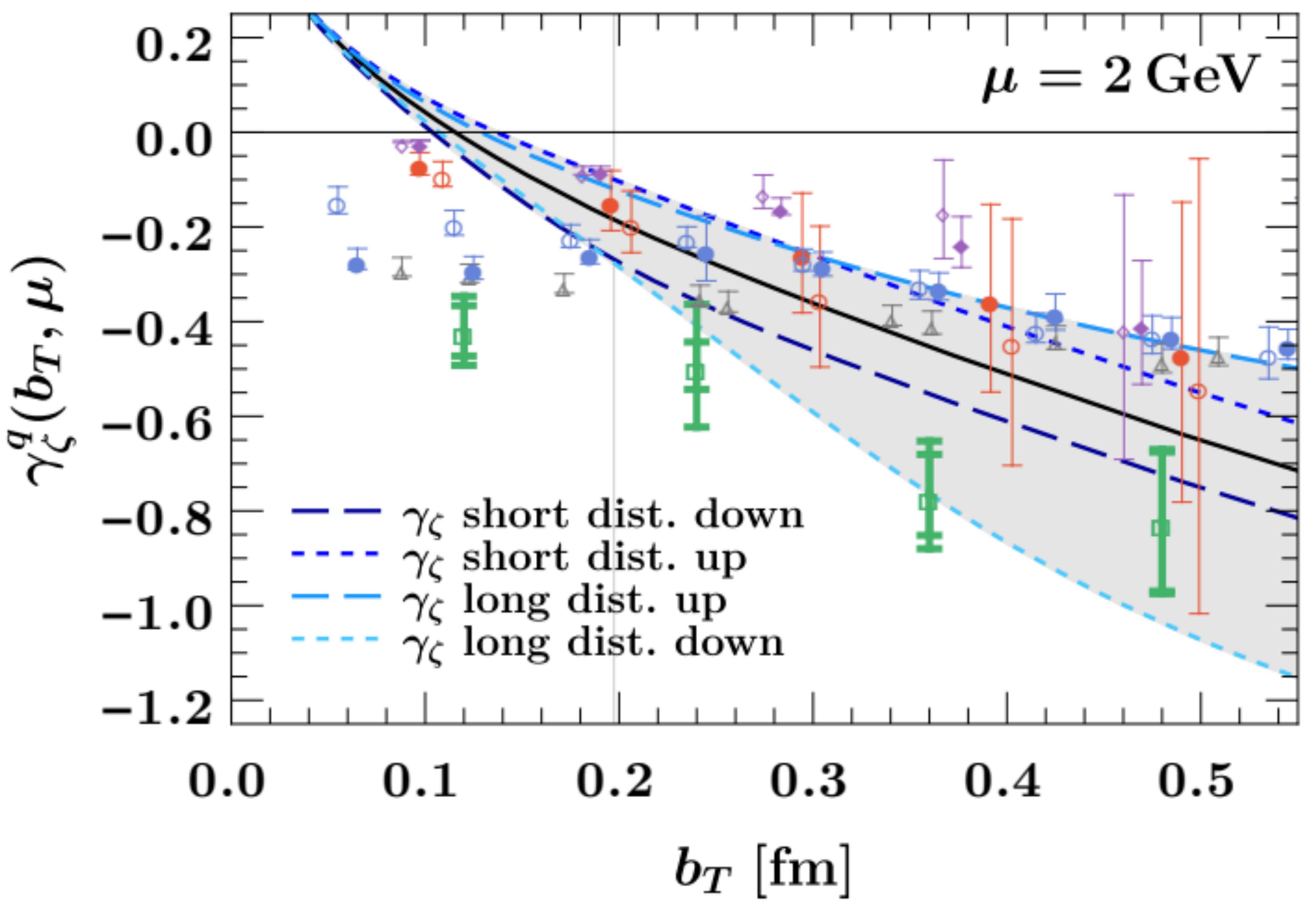


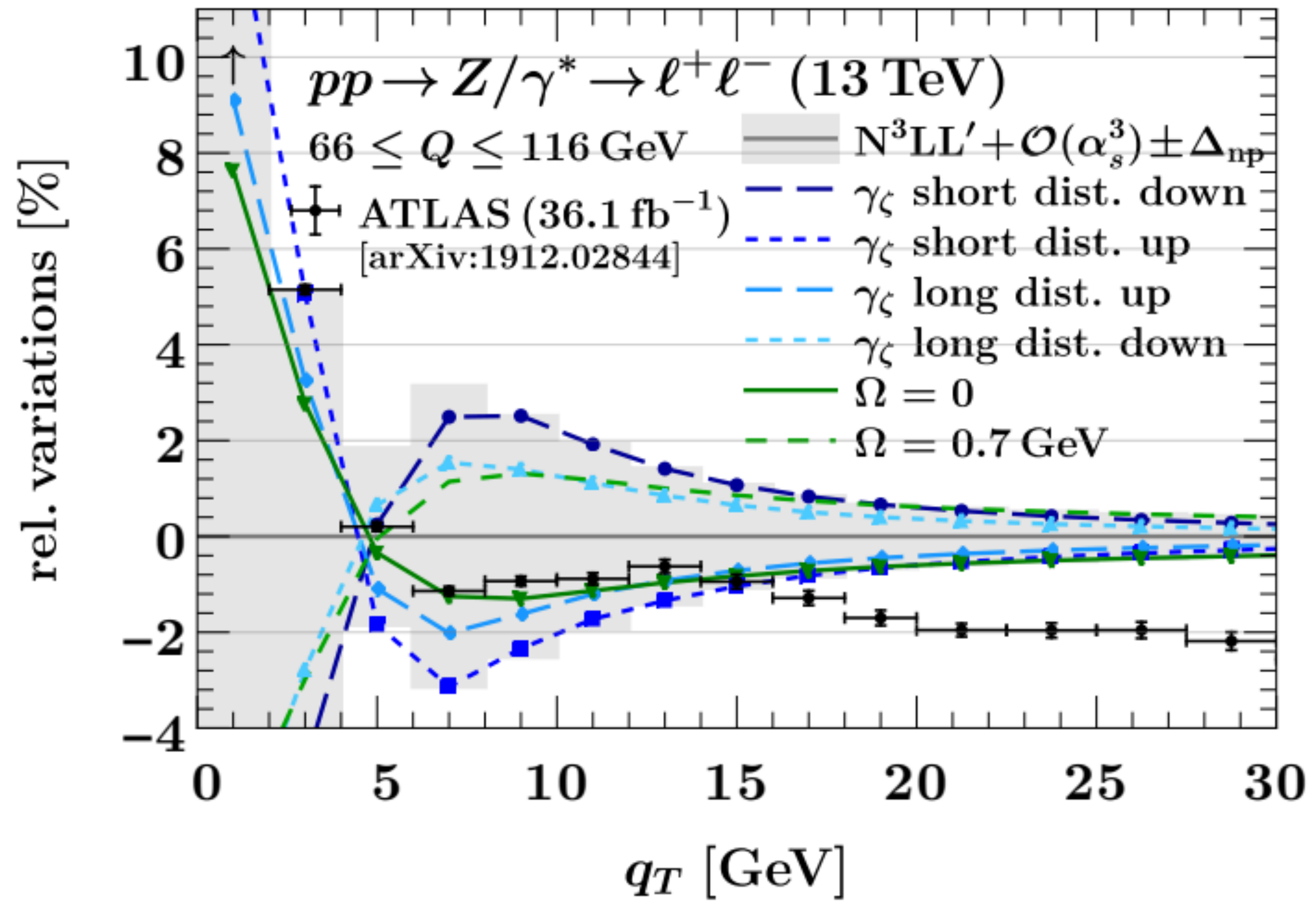
Carmarda, Cieri, Ferrera, 2023



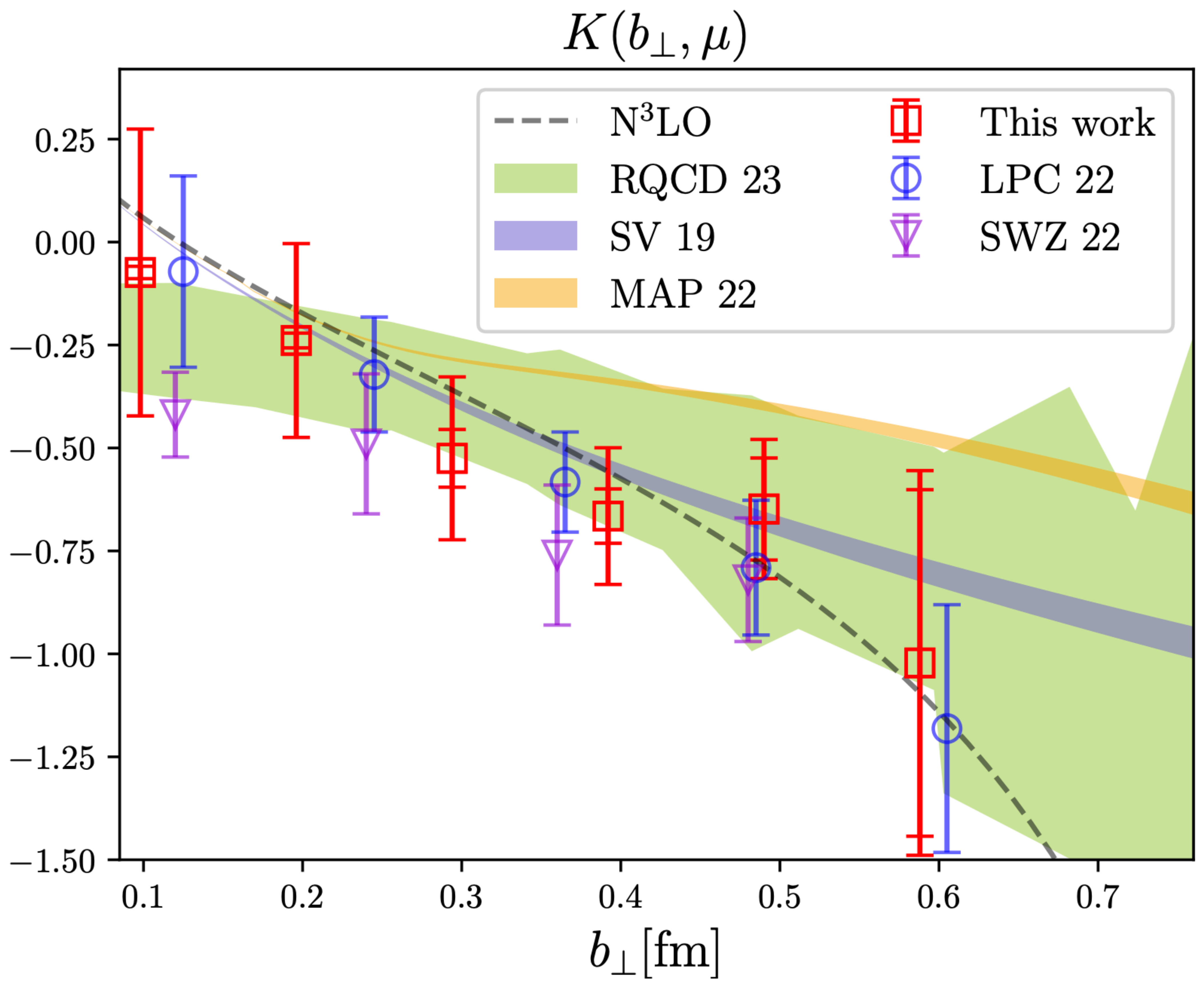
Duhr, Mistlberger, Vita, 2022

Shanahan, Wagman, Y. Zhao, 2021





Lattice/exp. extraction of Collins-Soper kernel



M.H. Chu, et al., LPC, 2306.06488

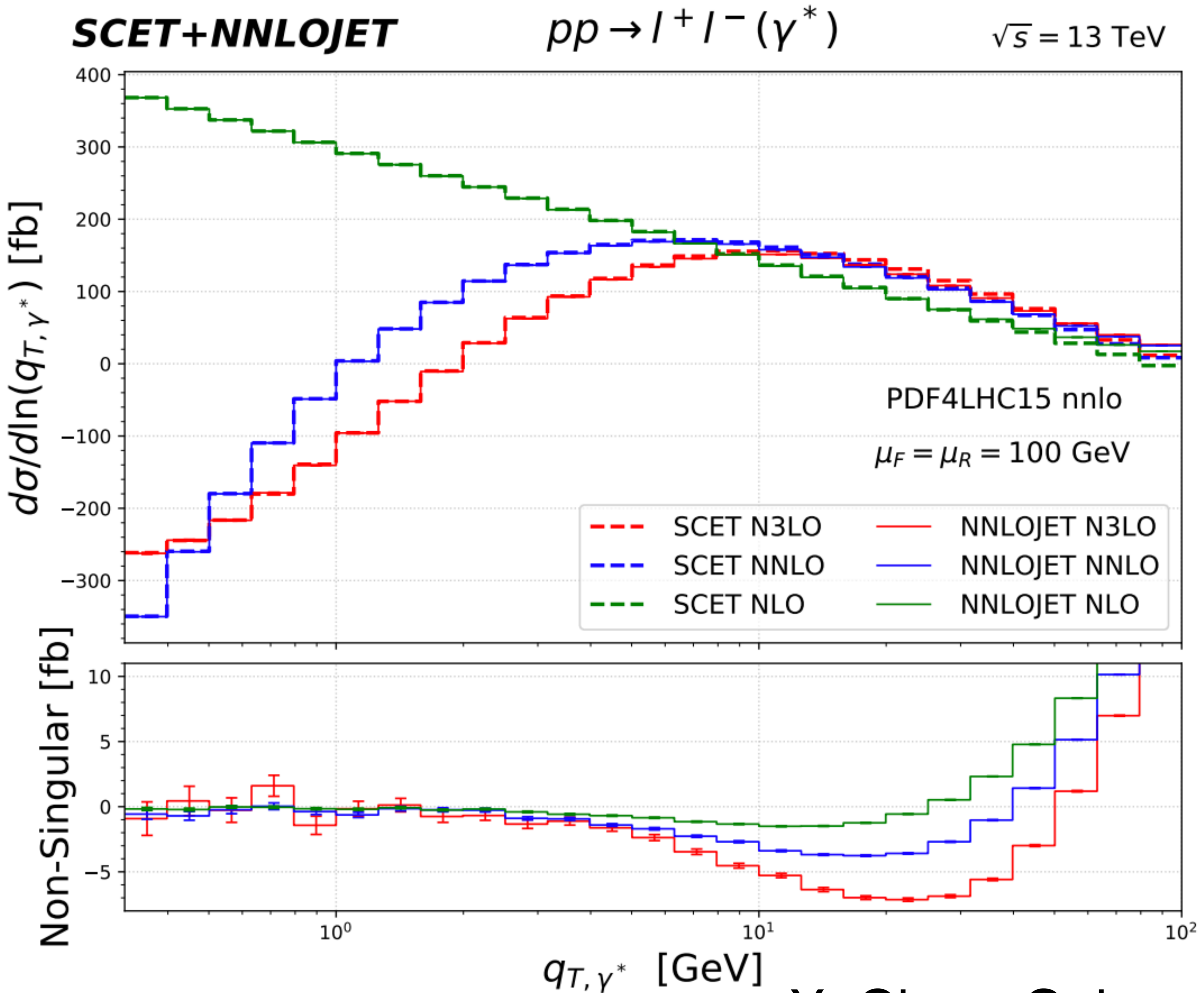
Drell-Yan production at N3LO

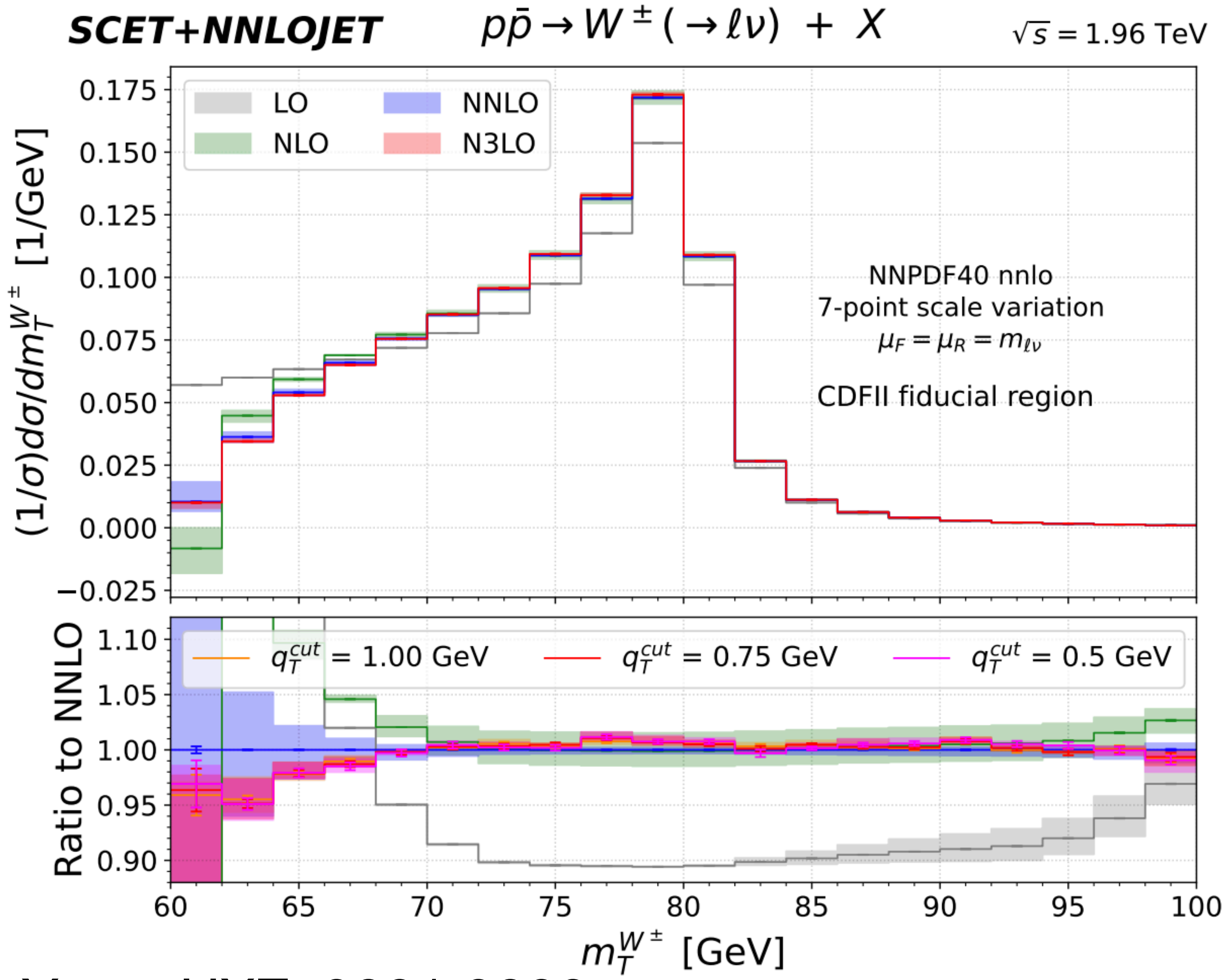
qT subtraction Catani, Grazzini, 2002

$$\frac{d^2\sigma_{\gamma^*}}{dQ^2 dy} = \int_0^{q_T^{\text{cut}}} d^2\boldsymbol{q}_T \frac{d^4\sigma_{\gamma^*}}{d^2\boldsymbol{q}_T dQ^2 dy} + \int_{q_T^{\text{cut}}} d^2\boldsymbol{q}_T \frac{d^4\sigma_{\gamma^*}}{d^2\boldsymbol{q}_T dQ^2 dy}$$

approximated by $H(q^2)S(x_\perp)B_1(x_+, x_\perp)B_2(x_-, x_\perp)$

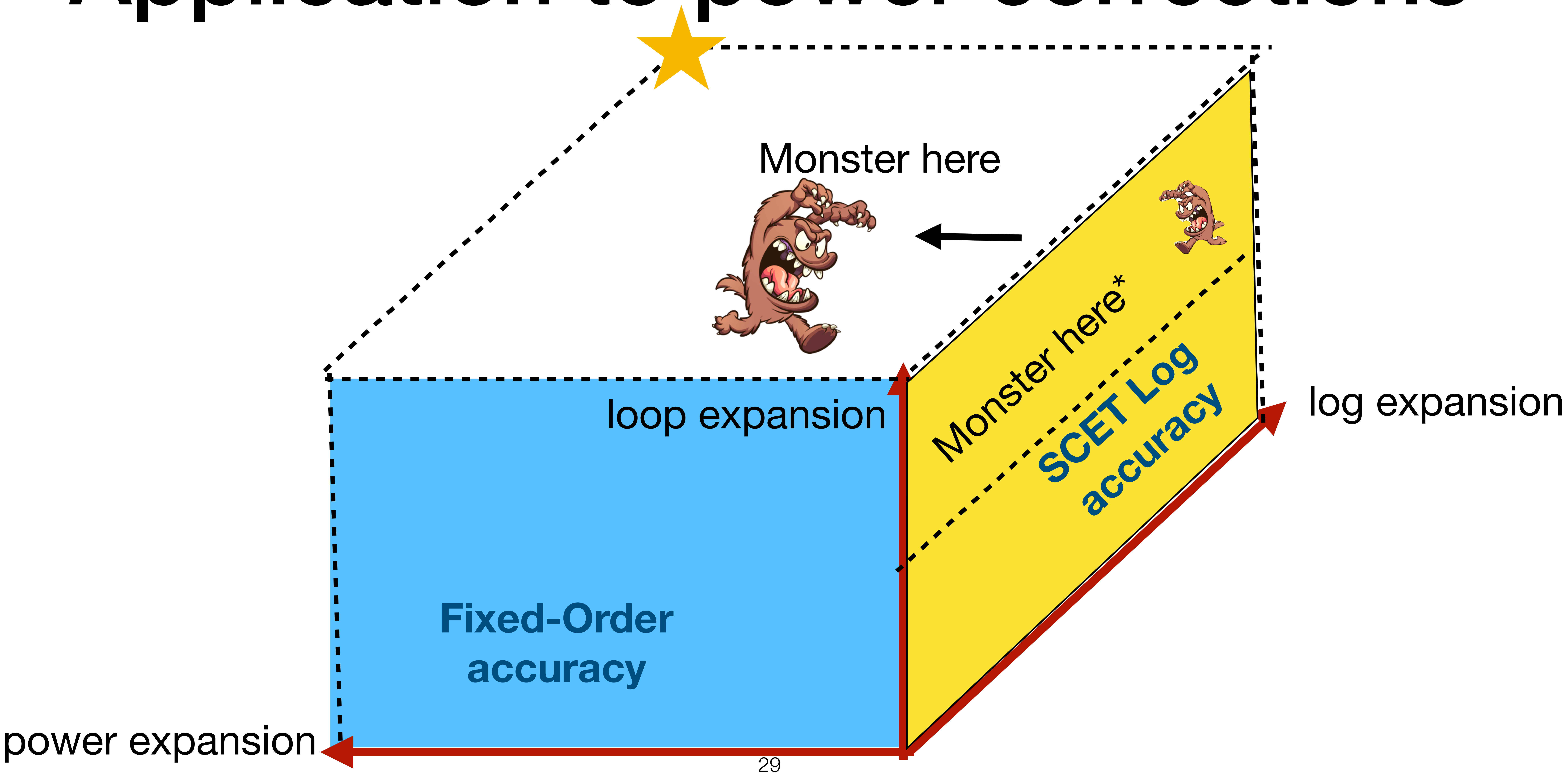
	LO	NLO	NNLO	N^3LO		
$\frac{d\sigma}{dy} =$	1	$+\alpha_s L^2$	$+\alpha_s^2 L^4$	$+\alpha_s^3 L^6$	$+\dots$	LL
		$+\alpha_s L$	$+\alpha_s^2 L^3$	$+\alpha_s^3 L^5$	$+\dots$	NLL
		$+\alpha_s$	$+\alpha_s^2 L^2$	$+\alpha_s^3 L^4$	$+\dots$	NNLL
			$+\alpha_s^2 L$	$+\alpha_s^3 L^3$	$+\dots$	
			$+\alpha_s^2$	$+\alpha_s^3 L^2$	$+\dots$	N^3LL
$L = \ln y$				$+\alpha_s^3 L$	$+\dots$	
				$+\alpha_s^3$	$+\dots$	



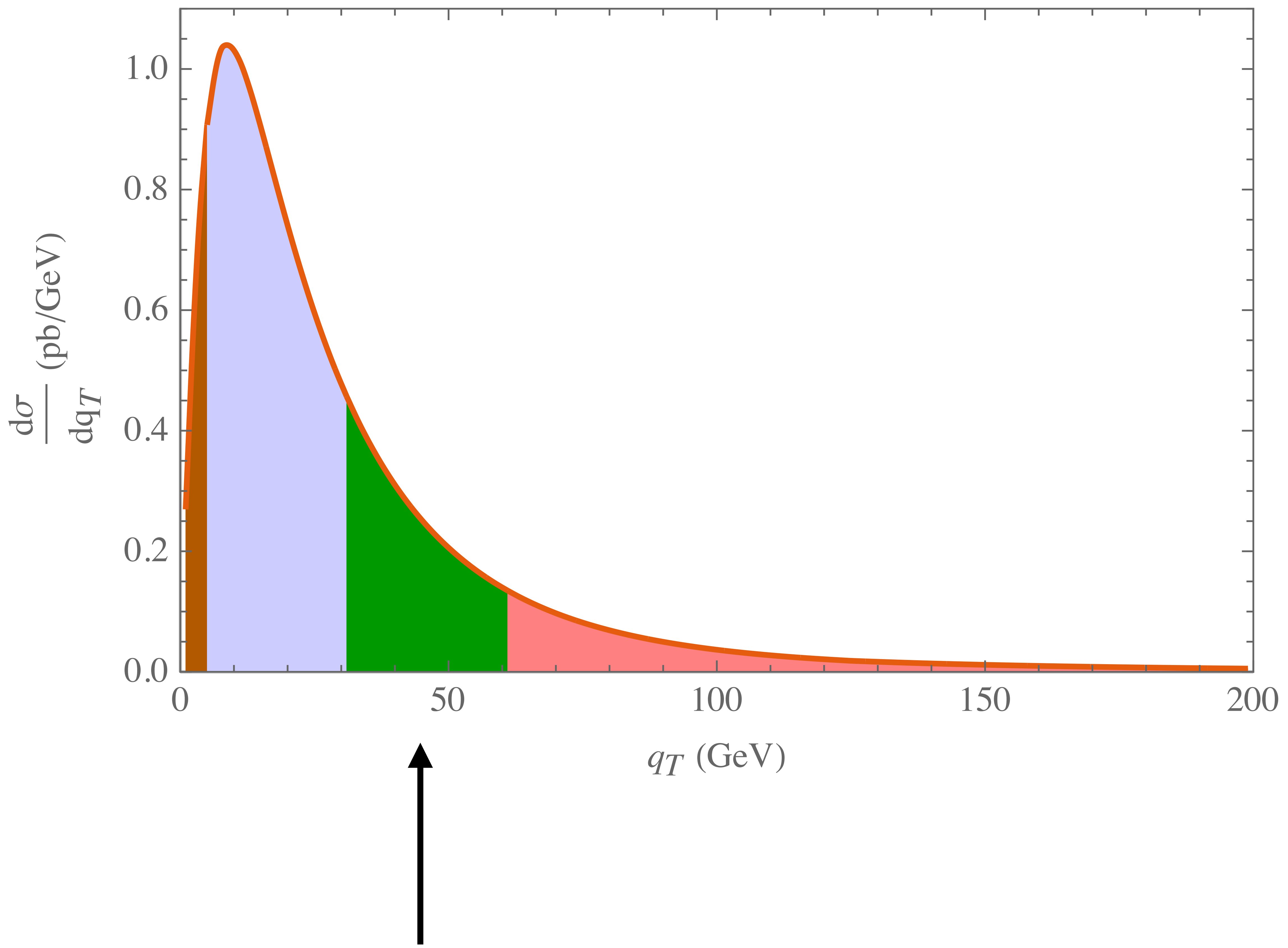


X. Chen, Gehrmann, Glover, Huss,T.Z. Yang, HXZ, 2021,2022

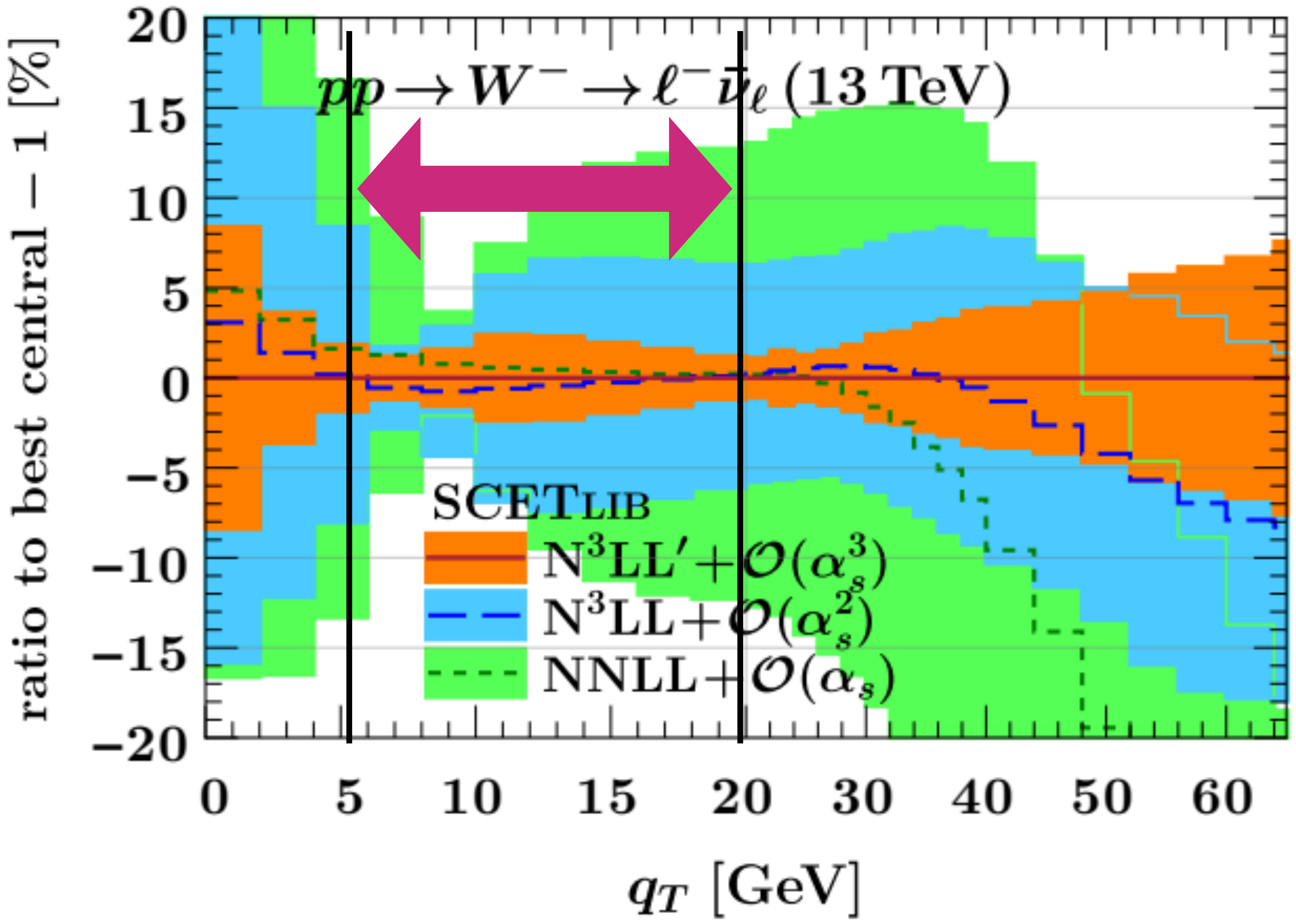
Application to power corrections



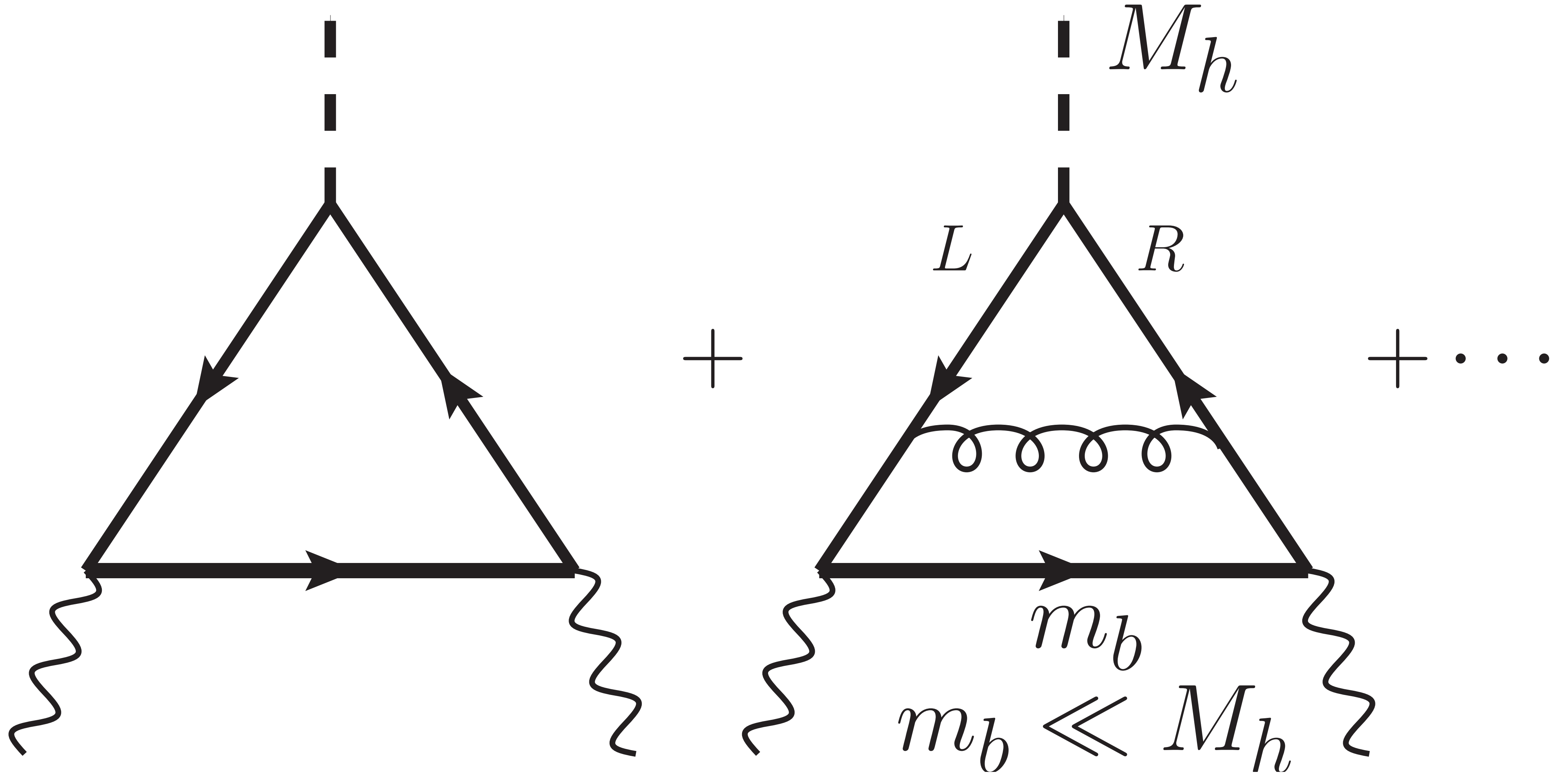
Why power corrections?



No natural matching scheme in the transition region, due to mismatch in the order of coupling constant expansion



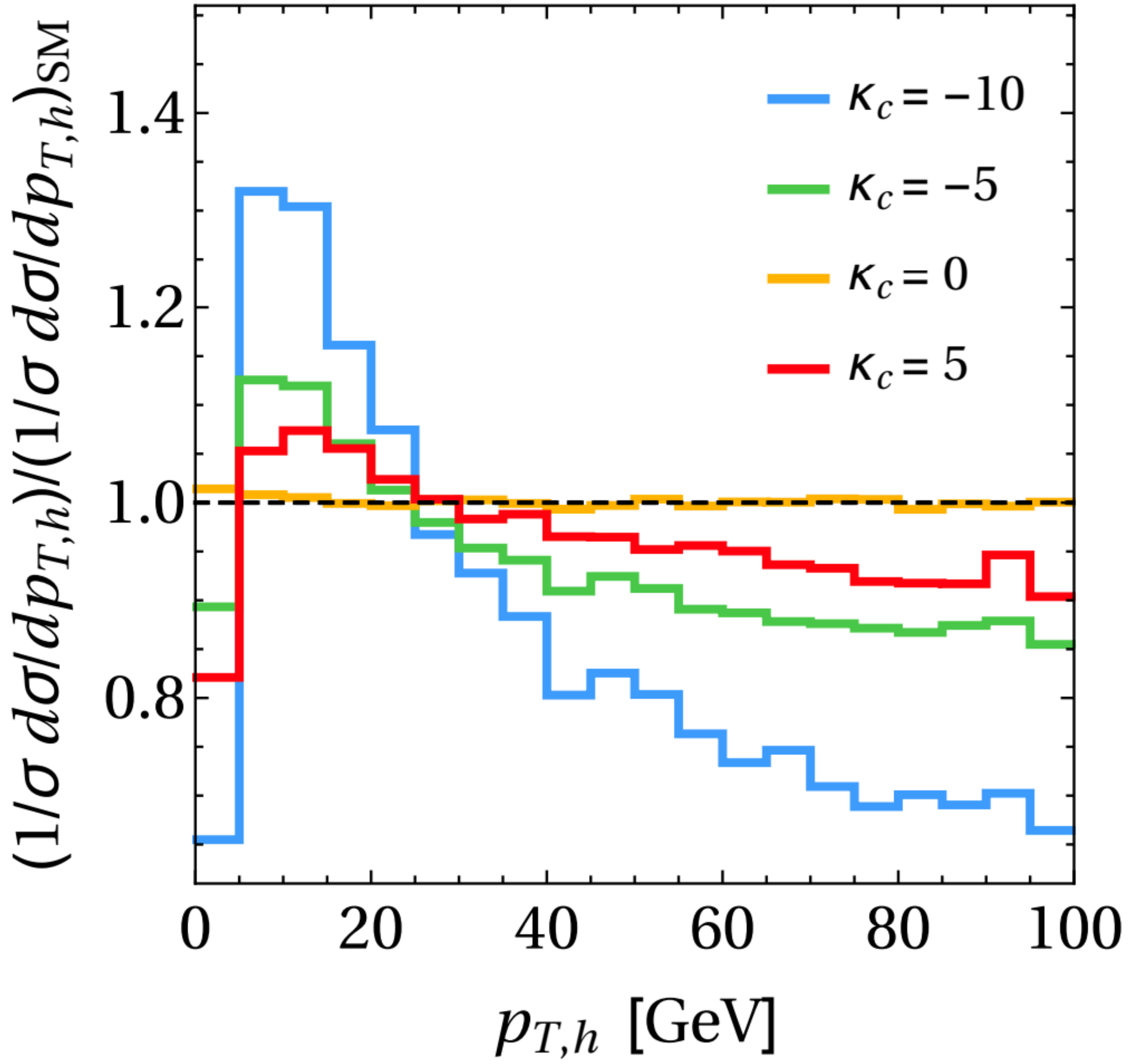
Higgs production with light-quark loop



$$\lambda \sim \frac{m_b}{M_h}$$

$$L = \ln \frac{-M_h^2}{m_b^2}$$

$$\sim y_b m_b \left\{ \left(\frac{L^2}{2} - 2 \right) + \frac{C_F \alpha_s}{4\pi} \left[-\frac{L^4}{12} - L^3 + \cdots \right] \right\} + \mathcal{O}(\alpha_s^2)$$

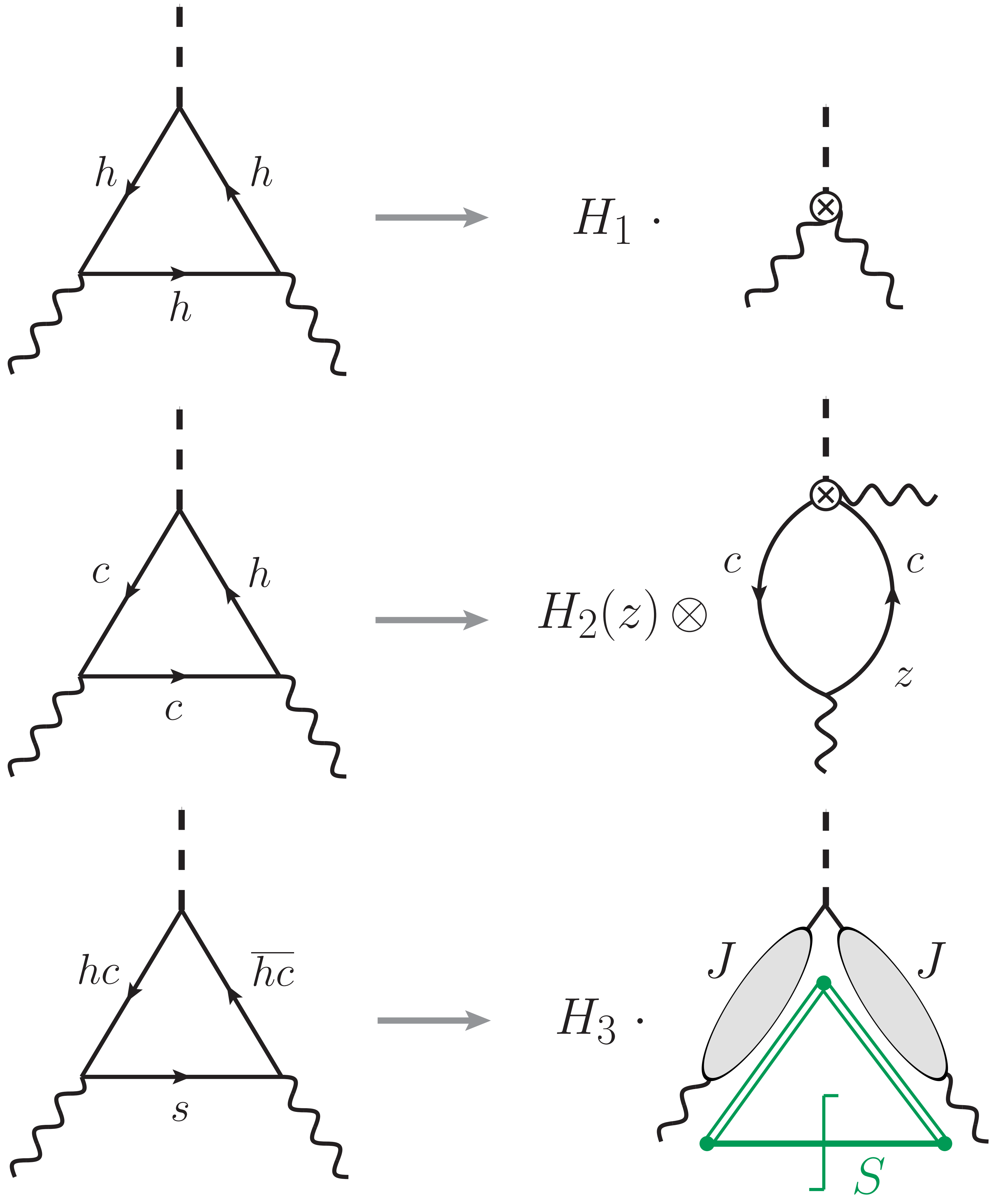


pT spectrum sensitive to light particle in the loops

Bishara, Haisch, Monni, Re, 2016

End point singularity

Z.L. Liu, Neubert, 2019



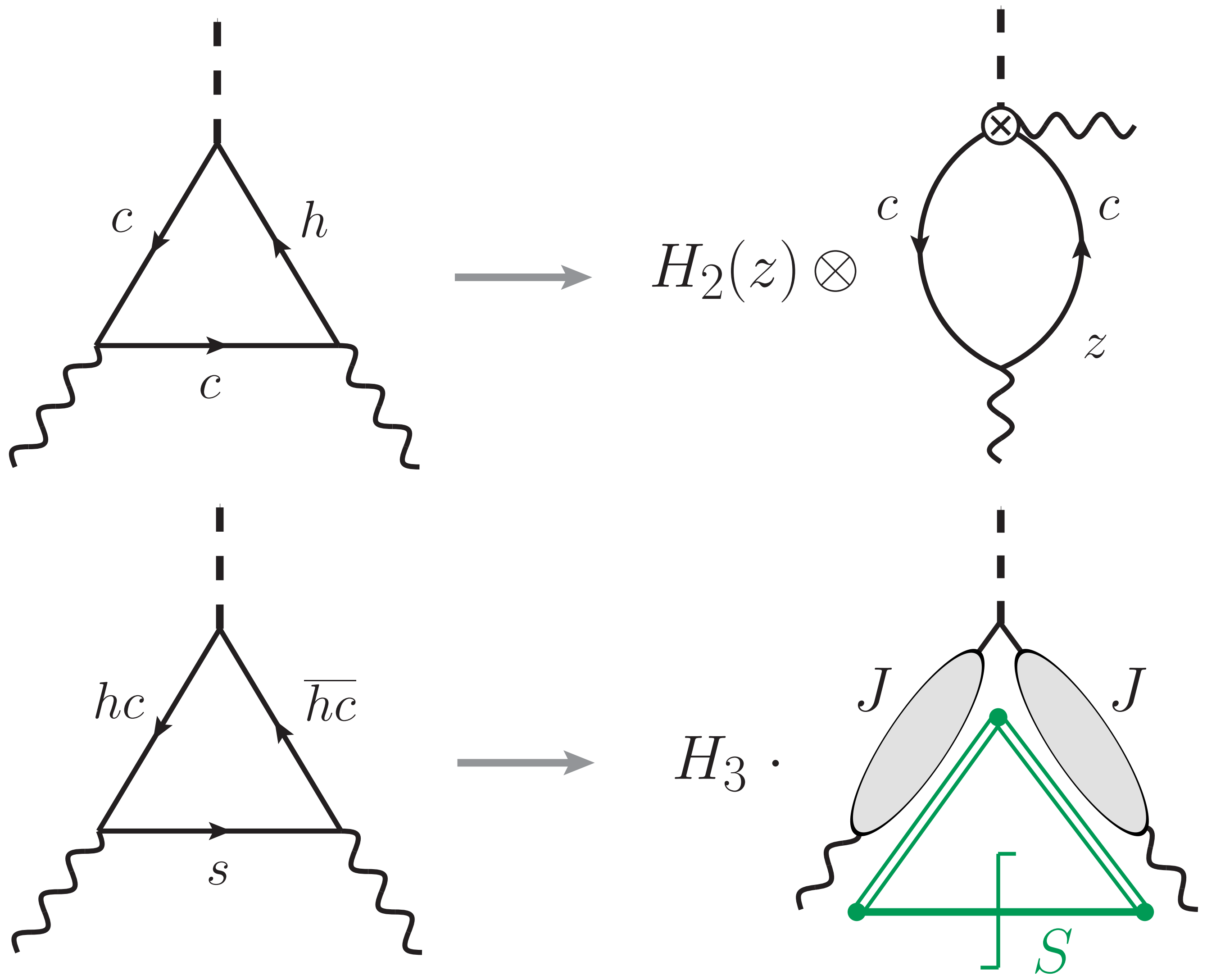
$$\mathcal{M}_b(h \to \gamma\gamma) = H_{1,\gamma}^{(0)} \langle O_{1,\gamma}^{(0)} \rangle + 2 \int_0^1 \mathrm{d}z\, H_{2,\gamma}^{(0)}(z) \langle O_{2,\gamma}^{(0)}(z) \rangle + H_{3,\gamma}^{(0)} \langle O_{3,\gamma}^{(0)} \rangle$$

$\mathcal{O}(\alpha_s)$	$\lim_{z \to 0} H_2(z)$	$\lim_{z \to 0} O_2(z)$
LO	$\frac{1}{z}$	$\frac{1}{\epsilon} - L_m$
NLO	$\frac{c_0}{z} + \frac{c_1}{z^{1+\epsilon}}$	$a_0 + a_1 z^{\epsilon}$
NNLO	$\frac{c_0'}{z} + \frac{c_1'}{z^{1+\epsilon}} + \frac{c_2'}{z^{1+2\epsilon}}$	$a_0' + a_1' z^{\epsilon} + a_2' z^{2\epsilon}$

Generic feature of end point divergence

A subtraction scheme based on refactorization condition

Z.L. Liu, Neubert, 2019



$$[\![\bar{H}^{(0)}_{2,\gamma}(z)]\!] = -H^{(0)}_{3,\gamma} J^{(0)}_{\gamma}(z M_h^2)\,,$$

$$[\![\langle O^{(0)}_{2,\gamma}(z)\rangle]\!] = -\frac{\varepsilon_1^{\perp}(k_1)\cdot\varepsilon_2^{\perp}(k_2)}{2}\int_0^{\infty}\frac{\mathrm{d}\ell_+}{\ell_+} J^{(0)}_{\gamma}(-M_h\ell_+) S^{(0)}_{\gamma}(z M_h \ell_+)\,.$$

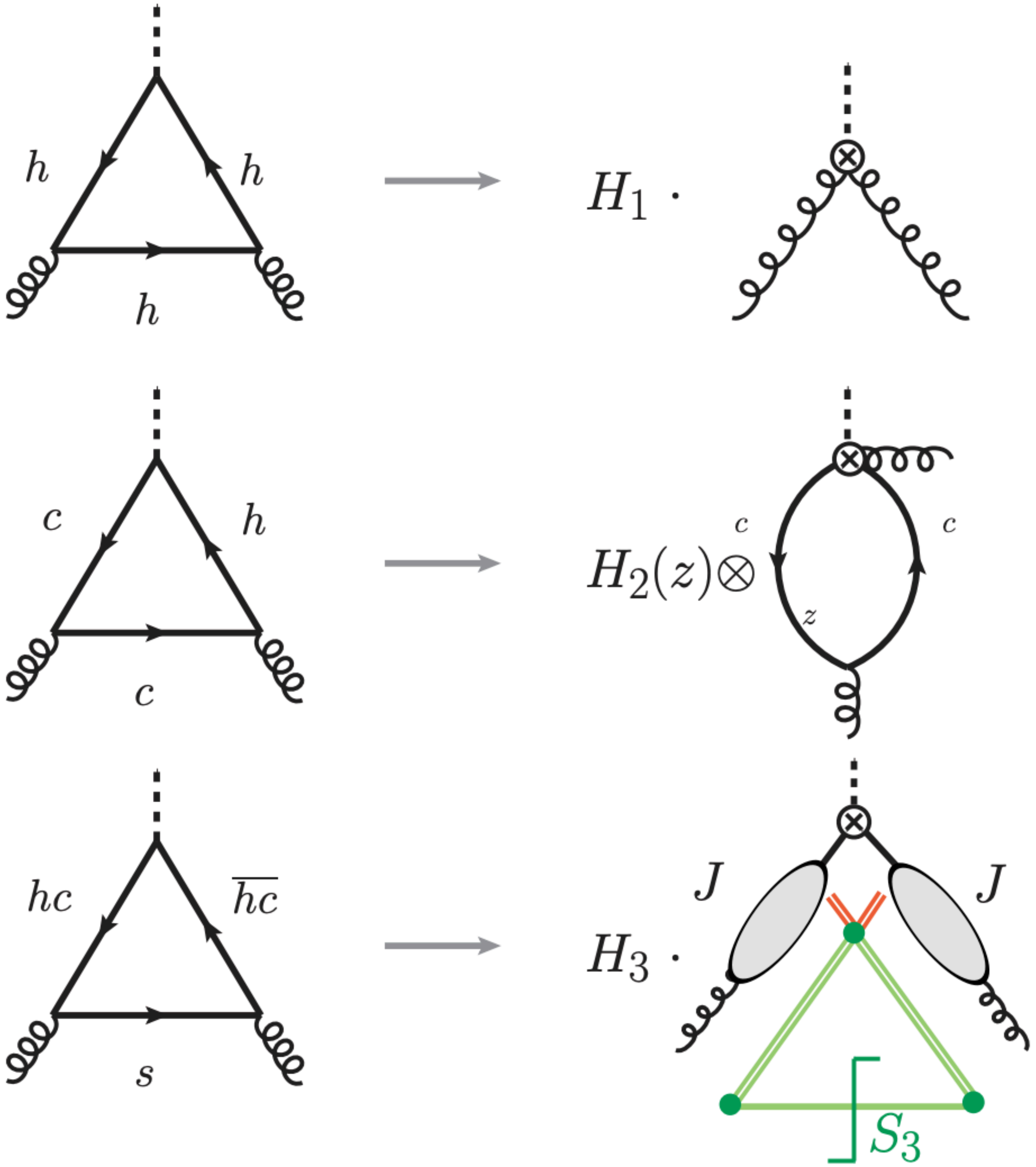
$$\mathcal{M}_b(h\to\gamma\gamma) = \left(H^{(0)}_{1,\gamma} + \Delta H^{(0)}_{1,\gamma}\right)\langle O^{(0)}_{1,\gamma}\rangle$$

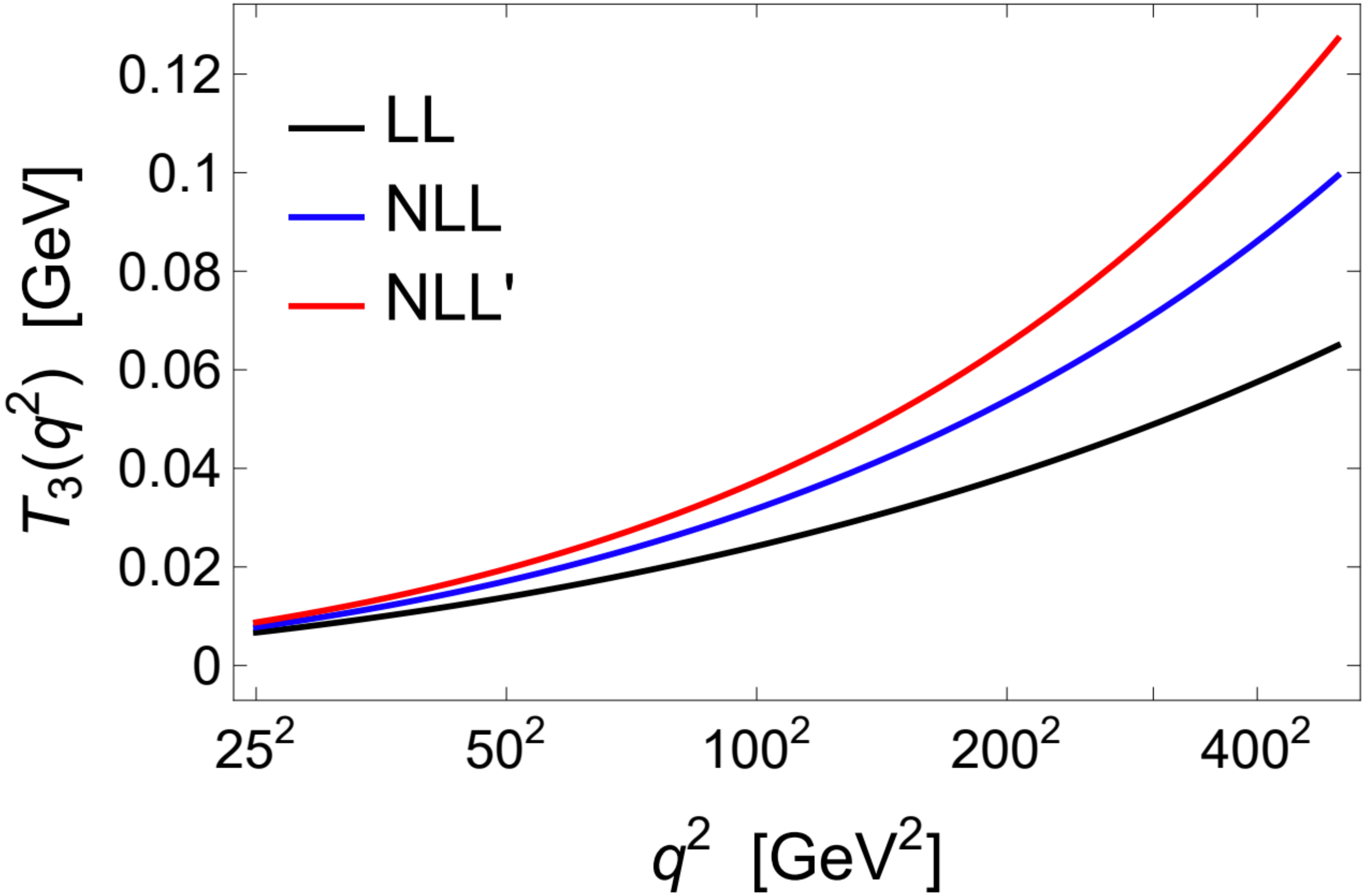
$$+\,2\int_0^1 \mathrm{d}z\left[H^{(0)}_{2,\gamma}(z)\langle O^{(0)}_{2,\gamma}(z)\rangle - \frac{[\![\bar{H}^{(0)}_{2}(z)]\!]}{z}[\![\langle O^{(0)}_{2,\gamma}(z)\rangle]\!] - \frac{[\![\bar{H}^{(0)}_{2}(1-z)]\!]}{1-z}[\![\langle O^{(0)}_{2,\gamma}(z)\rangle]\!]\right]$$

$$+\,\varepsilon_1^{\perp}\cdot\varepsilon_2^{\perp}\lim_{\sigma\to -1} H^{(0)}_{3,\gamma}\int_0^{M_h}\frac{\mathrm{d}\ell_-}{\ell_-}\int_0^{\sigma M_h}\frac{\mathrm{d}\ell_+}{\ell_+} J^{(0)}_{\gamma}(M_h\ell_-) J^{(0)}_{\gamma}(-M_h\ell_+) S^{(0)}_{\gamma}(\ell_-\ell_+)\Big|_{\text{leading power}}\,.$$

gg to H with light quark loop

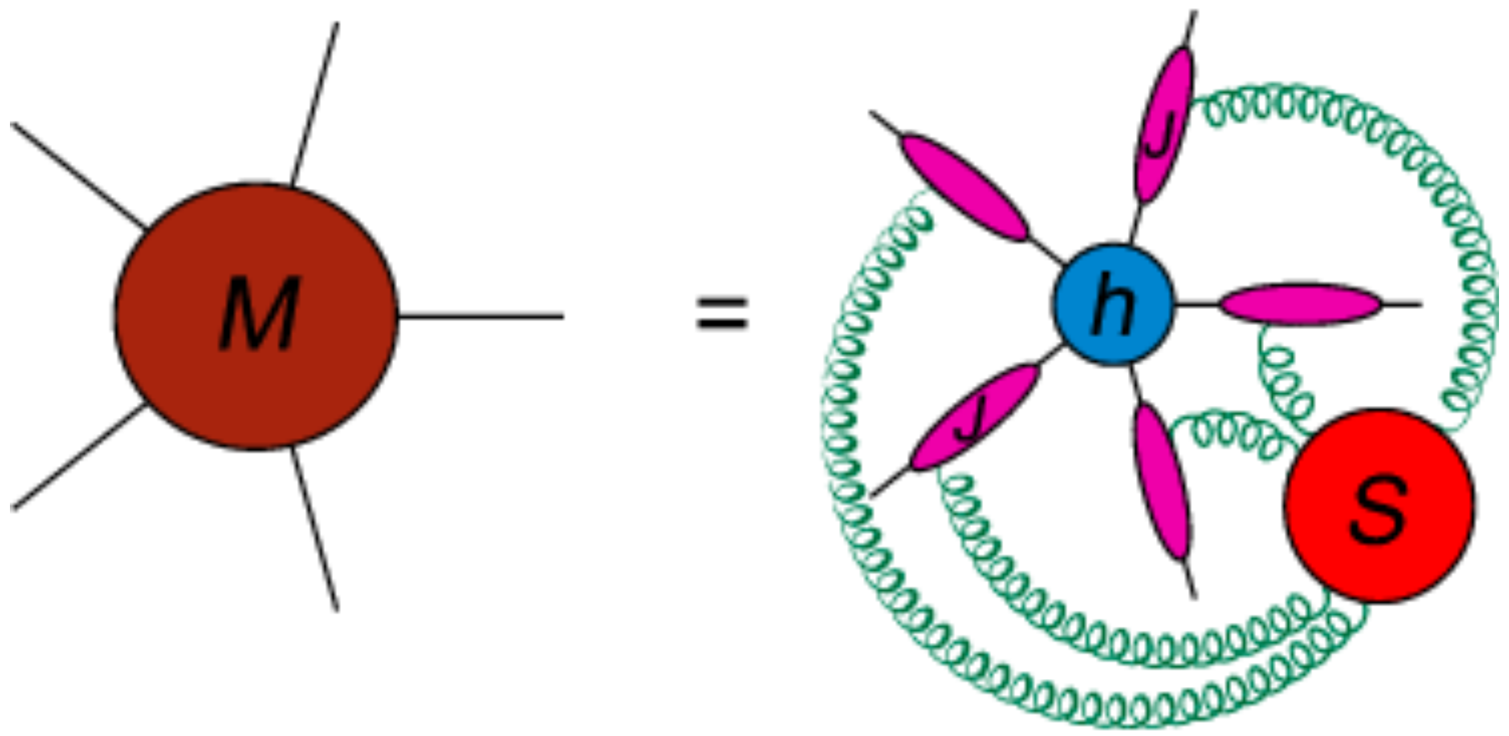
Z.L. Liu, Neubert, Schnubel, X. Wang, 2019





A different angle on power corrections

factorization



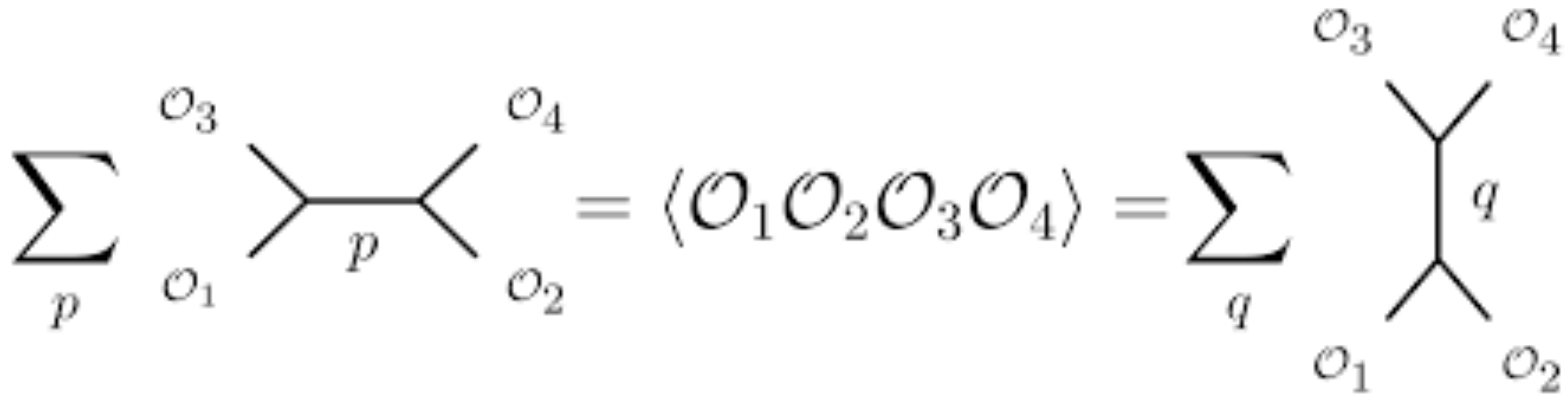
collinear gluon/quark

soft gluon

leading power: local factorization

subleading power: non-local factorization
end-point singularity

operator product expansion



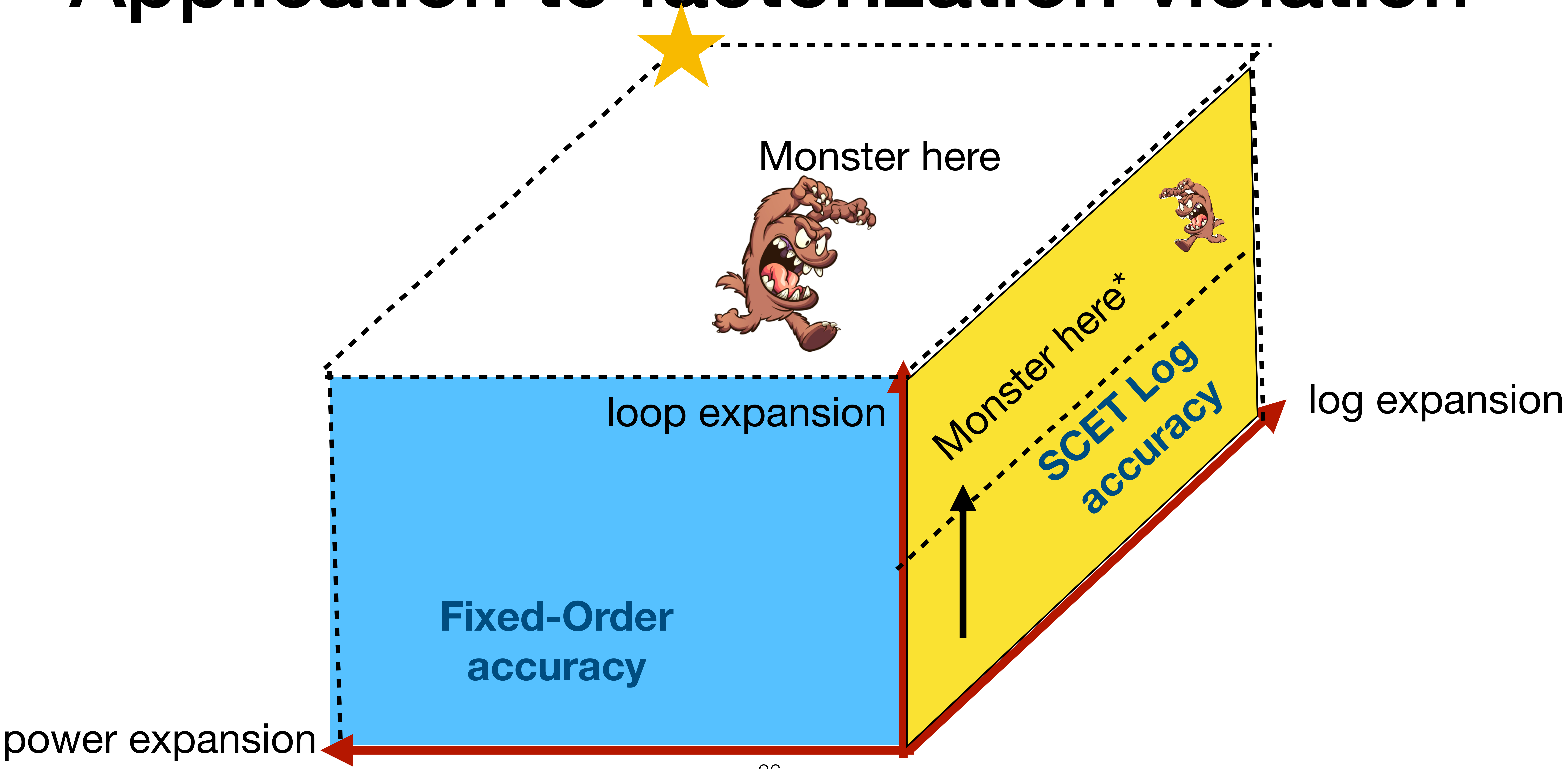
twist operator expansion

large spin limit

leading power: leading twist and spin

subleading power: large spin expansion
higher twist

Application to factorization violation



Glauber operator

$$\mathcal{L}_G^{(0)} = \sum_{n,\bar{n}} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} + \sum_{n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

(3 rapidity sectors) (2 rapidity sectors)

↑ sum pairwise on all collinears

↑ sum on all collinears

$$\mathcal{O}_n^{qB} = \bar{\chi}_n T^B \frac{\bar{\not{n}}}{2} \chi_n \qquad \mathcal{O}_n^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{n\perp\mu}^C \frac{\bar{n}}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{n\perp}^{D\mu}$$

$$\mathcal{O}_{\bar{n}}^{qB} = \bar{\chi}_{\bar{n}} T^B \frac{\not{n}}{2} \chi_{\bar{n}} \qquad \mathcal{O}_{\bar{n}}^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{\bar{n}\perp\mu}^C \frac{n}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{\bar{n}\perp}^{D\mu}$$

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \Big\{ \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp\mu} - \mathcal{P}_\mu^\perp g \widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} - \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \widetilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu} \mathcal{P}_\mu^\perp - g \widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \widetilde{\mathcal{B}}_{S\perp\mu}^{\bar{n}} - \frac{n_\mu \bar{n}_\nu}{2} \mathcal{S}_n^T i g \widetilde{G}_s^{\mu\nu} \mathcal{S}_{\bar{n}} \Big\}^{BC}$$

$$\mathcal{O}_s^{q_n B} = 8\pi\alpha_s \left(\bar{\psi}_S^n T^B \frac{\not{n}}{2} \psi_S^n \right) \qquad \mathcal{O}_s^{g_n B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{nC} \frac{n}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{nD\mu} \right)$$

$$\mathcal{O}_s^{q_{\bar{n}} B} = 8\pi\alpha_s \left(\bar{\psi}_S^{\bar{n}} T^B \frac{\bar{\not{n}}}{2} \psi_S^{\bar{n}} \right) \qquad \mathcal{O}_s^{g_{\bar{n}} B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{\bar{n}C} \frac{\bar{n}}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{\bar{n}D\mu} \right)$$

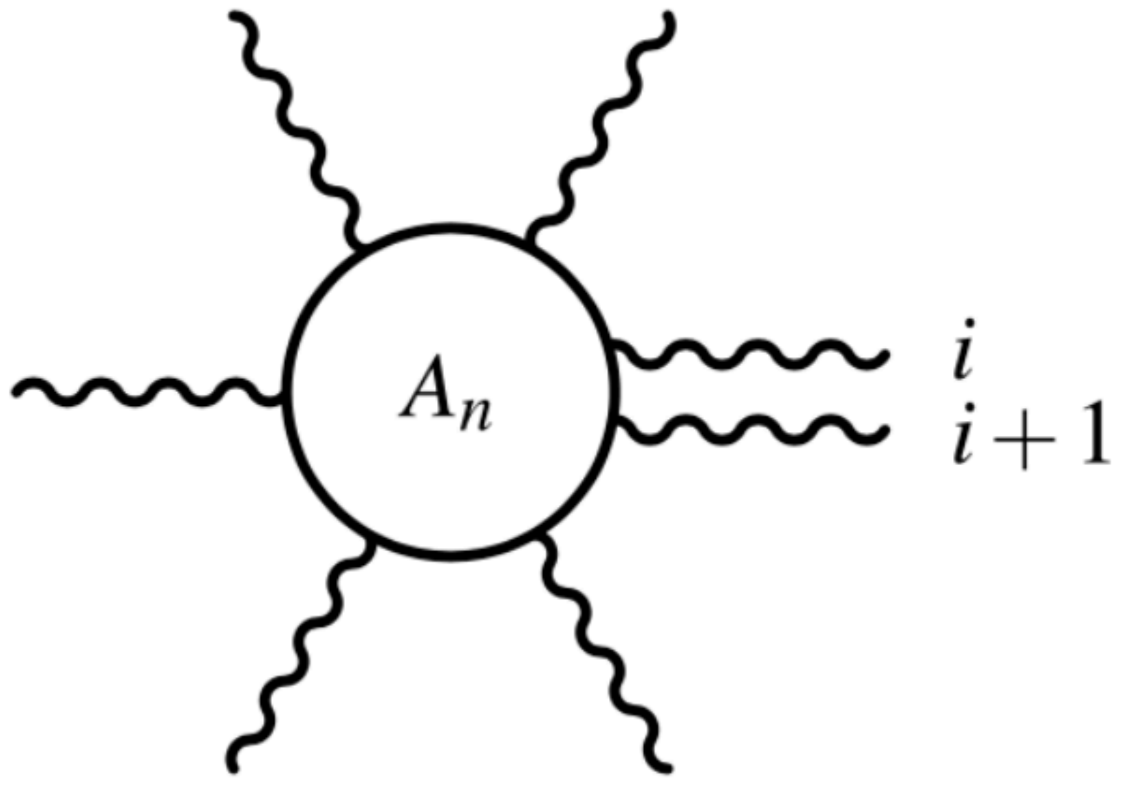
Glauber Lagrangian spoils factorization by coupling different sectors in SCET

Often its effects exponentiate into overall phase, or sum to zero for sufficient inclusive observables

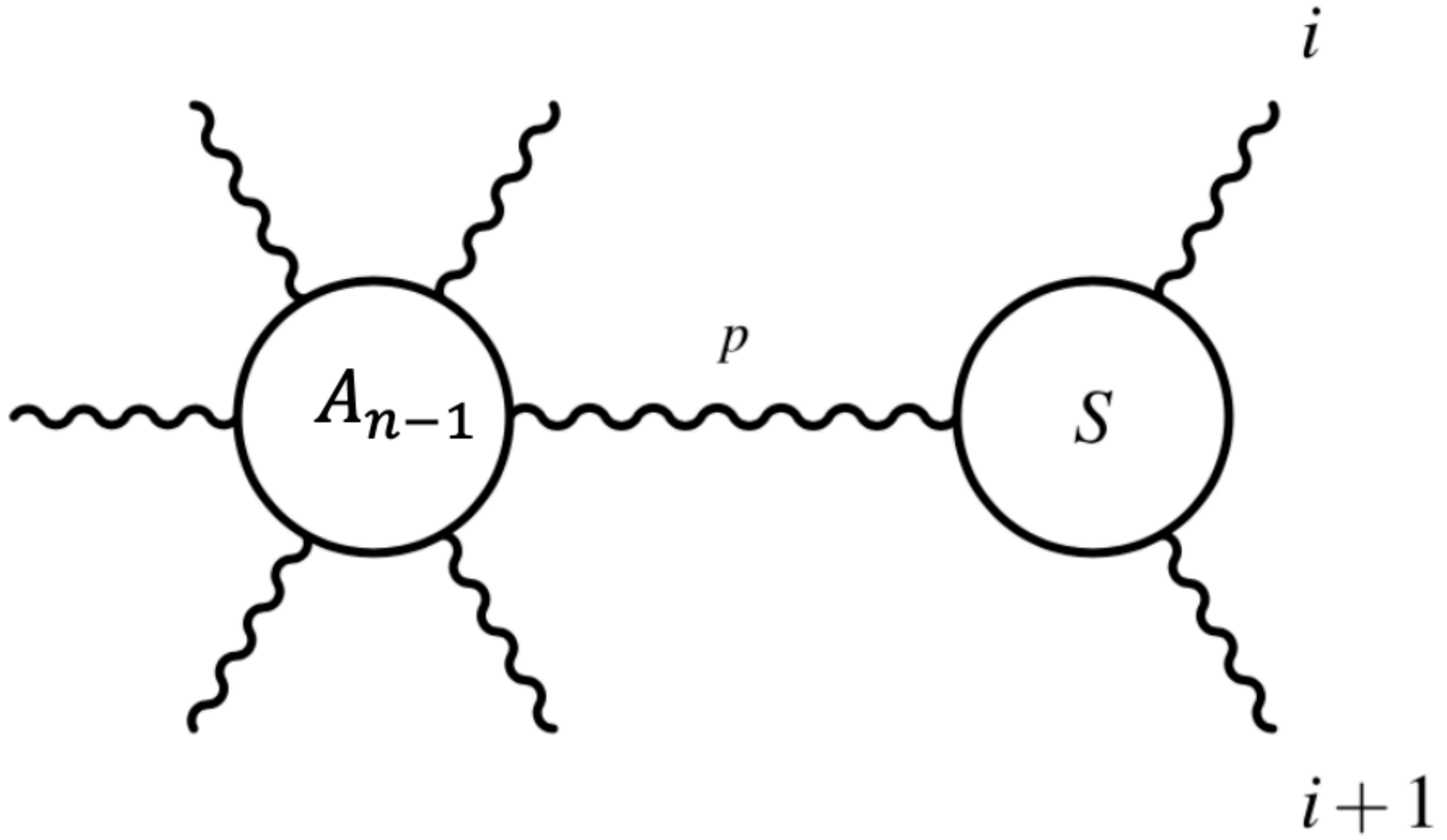
Strict collinear factorization

Scattering amplitudes obey factorization in two-particle collinear limit

$$A_n(\dots, i, i+1, \dots) \xrightarrow{i \parallel i+1} \sum_{\lambda} Split_{-\lambda}\,(z;\ i, i\ +\ 1)\ A_{n-1}(\dots, P^{\lambda}, \dots)$$



$$\lambda_i = \sqrt{z}\,\lambda_P,$$
$$\lambda_{i+1} = \sqrt{1-z}\,\lambda_P$$

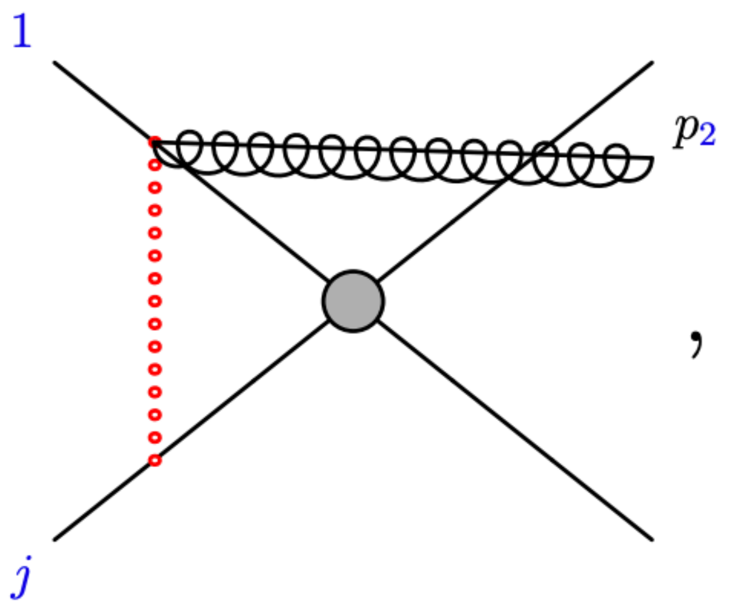


Strict collinear fact.: splitting amplitude ONLY depends of color and kinematics of the collinear pair

Violation of strict collinear factorization by Glauber

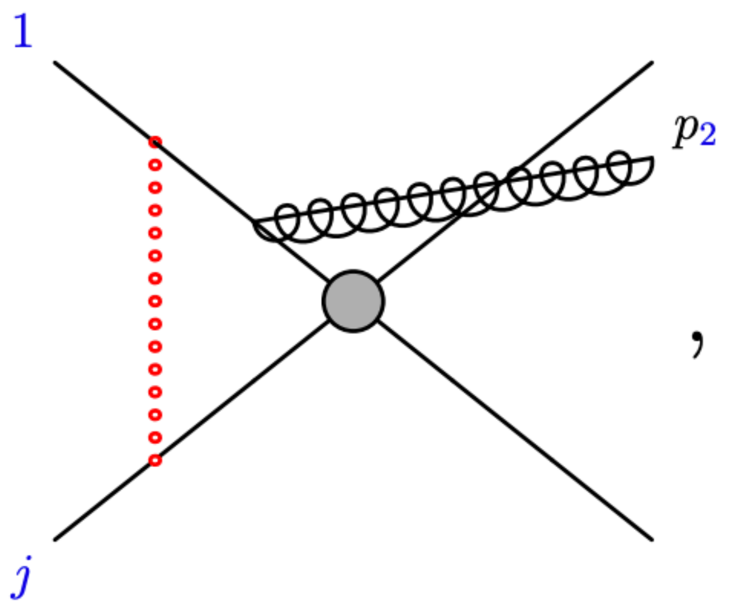
Schwartz, K. Yan, HXZ, 2017

$I^{j}_{\mathrm{Gc}} =$



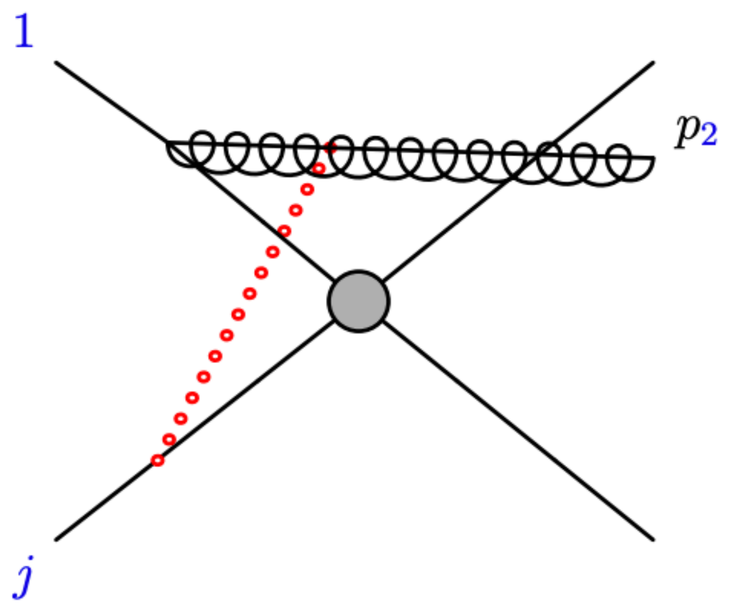
,

$I^{j}_{\mathrm{Gd}} =$



,

$I^{j}_{\mathrm{Ge}} =$



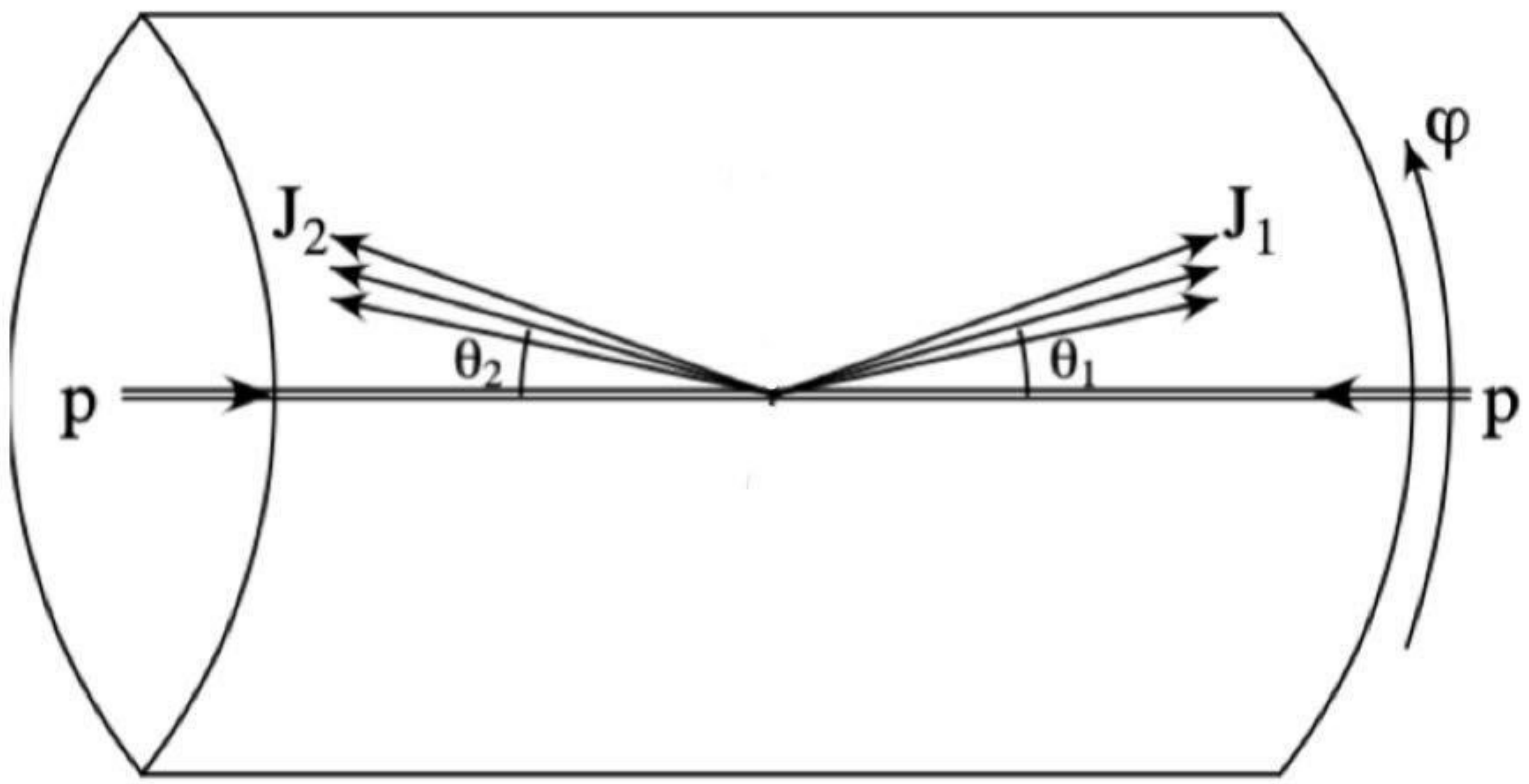
$$\sum_{n,\bar{n}} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC}$$

$$\mathbf{Sp}^{1,\text{non-fact}} = \frac{\alpha_s}{2\pi}(4\pi e^{-\gamma_E})^\epsilon (i\pi)\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{-2p_1\cdot p_2} + \ln\frac{z-1}{z}\right)\left(-\mathbf{T}_2\cdot\mathbf{T}_3 + \sum_{j>3}\mathbf{T}_2\cdot\mathbf{T}_j\right)\mathbf{Sp}^0$$

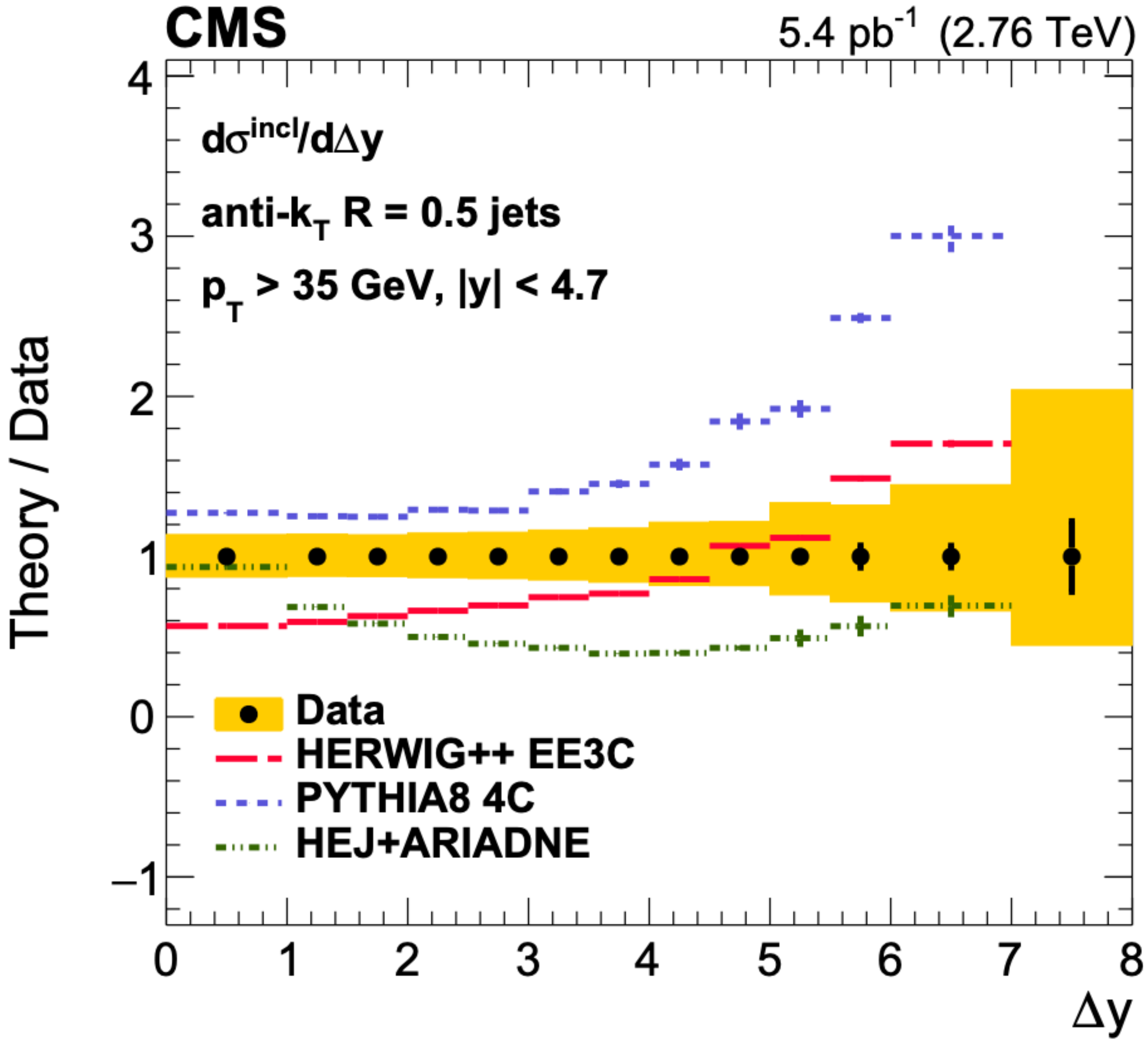
Color entangled with non-collinear color flow
Cancelled after adding conjugate diagram
Also cancel if particle 1 is in final state using color conservation

At least at NNLO in cross section, we don't need to worry about them!

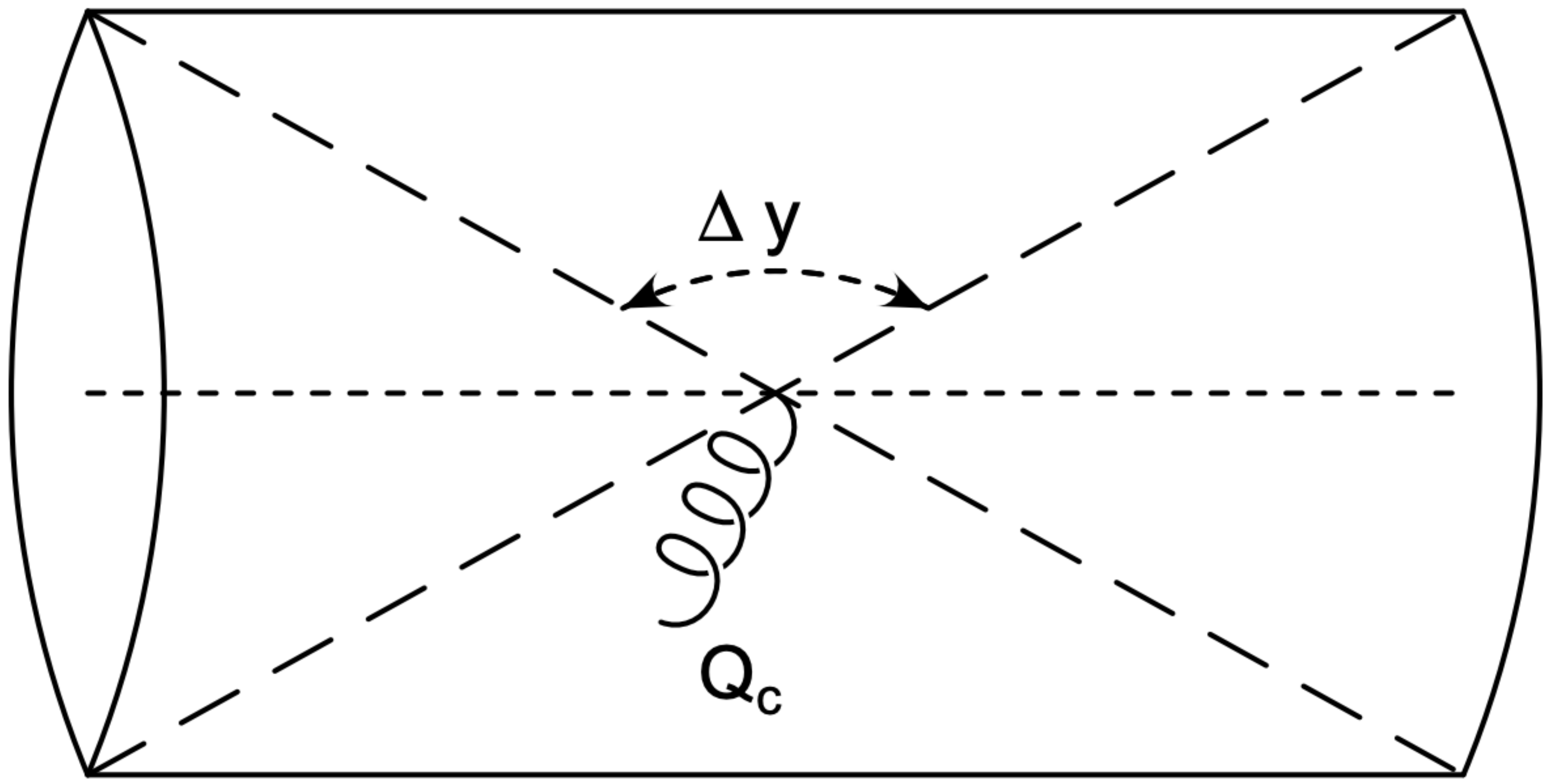
Glauber at observable level



None of the DGLAP-based Monte Carlo generators using LO or NLO calculations can provide a complete description of all measured cross sections and their ratios



Super leading logarithms



leading logs:

super leading

$$e^+e^-, ep: \quad \alpha_s^n \ln^n\left(\frac{Q}{Q_0}\right)$$

$$pp: \quad \cdots \quad + \alpha_s^3 (i\pi)^2 \ln^3\left(\frac{Q}{Q_0}\right) \times \alpha_s^n \ln^{2n}\left(\frac{Q}{Q_0}\right)$$

Gap-between-jet observable first proposed by Bjorken in 1993

Normal leading logarithms first analyzed by Oderda, Sterman, 1998

Super leading log first appear at four loops: Forshaw, Kyrieleis, Seymour, 2008

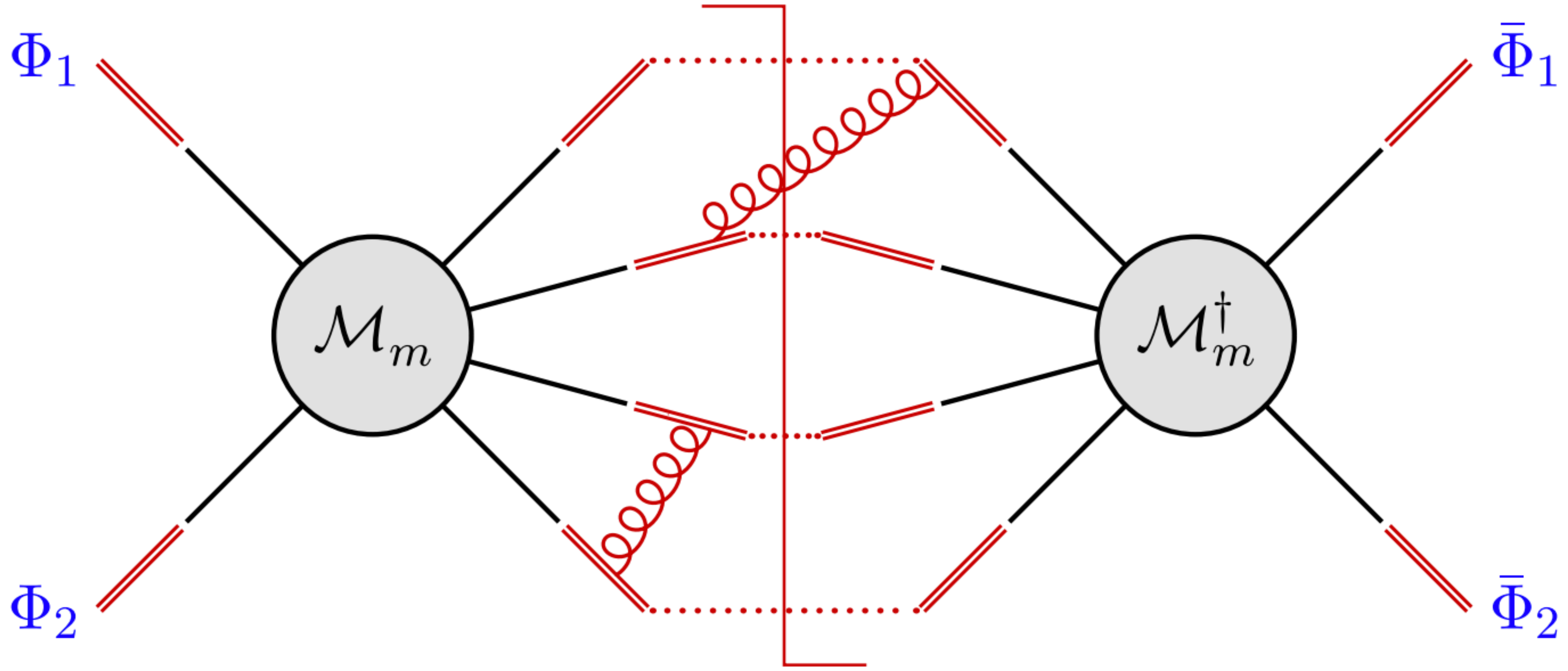
No all-order resummation formula available for a long time

Non-global evolution

Becher, Neubert, D.Y. Shao, 2021

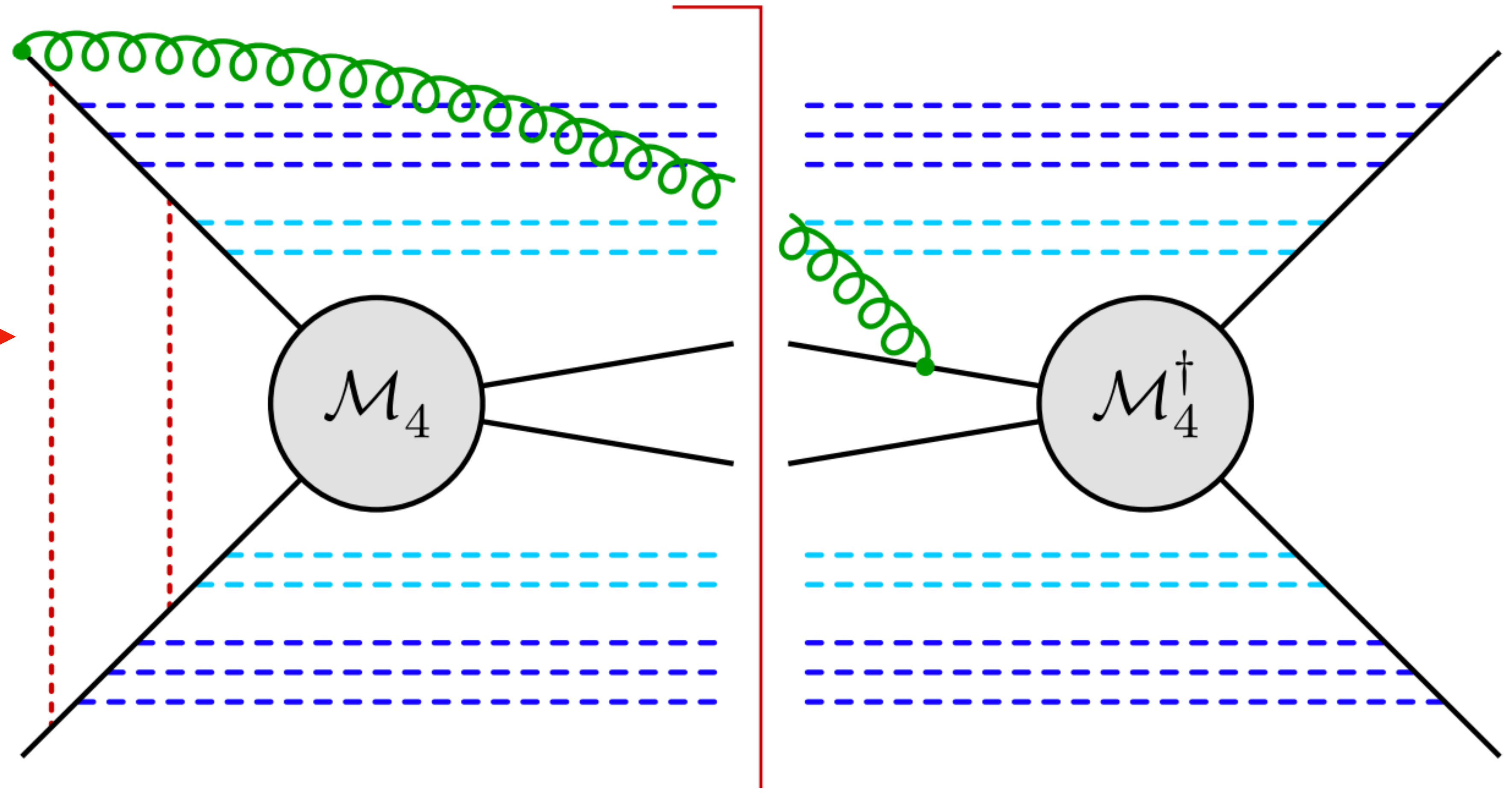
$$\sigma_{2\to M}(Q_0) = \int dx_1 \int dx_2 \sum_{m=2+M}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, s, x_1, x_2, \mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, x_1, x_2, \mu) \rangle$$

$$\frac{d}{d\ln\mu} \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, s, \mu) = -\sum_{l=2+M}^{m} \boldsymbol{\mathcal{H}}_l(\{\underline{n}\}, s, \mu) \star \boldsymbol{\Gamma}^H_{lm}(\{\underline{n}\}, s, \mu)$$



Origin of super leading logs

Glauber gluon →



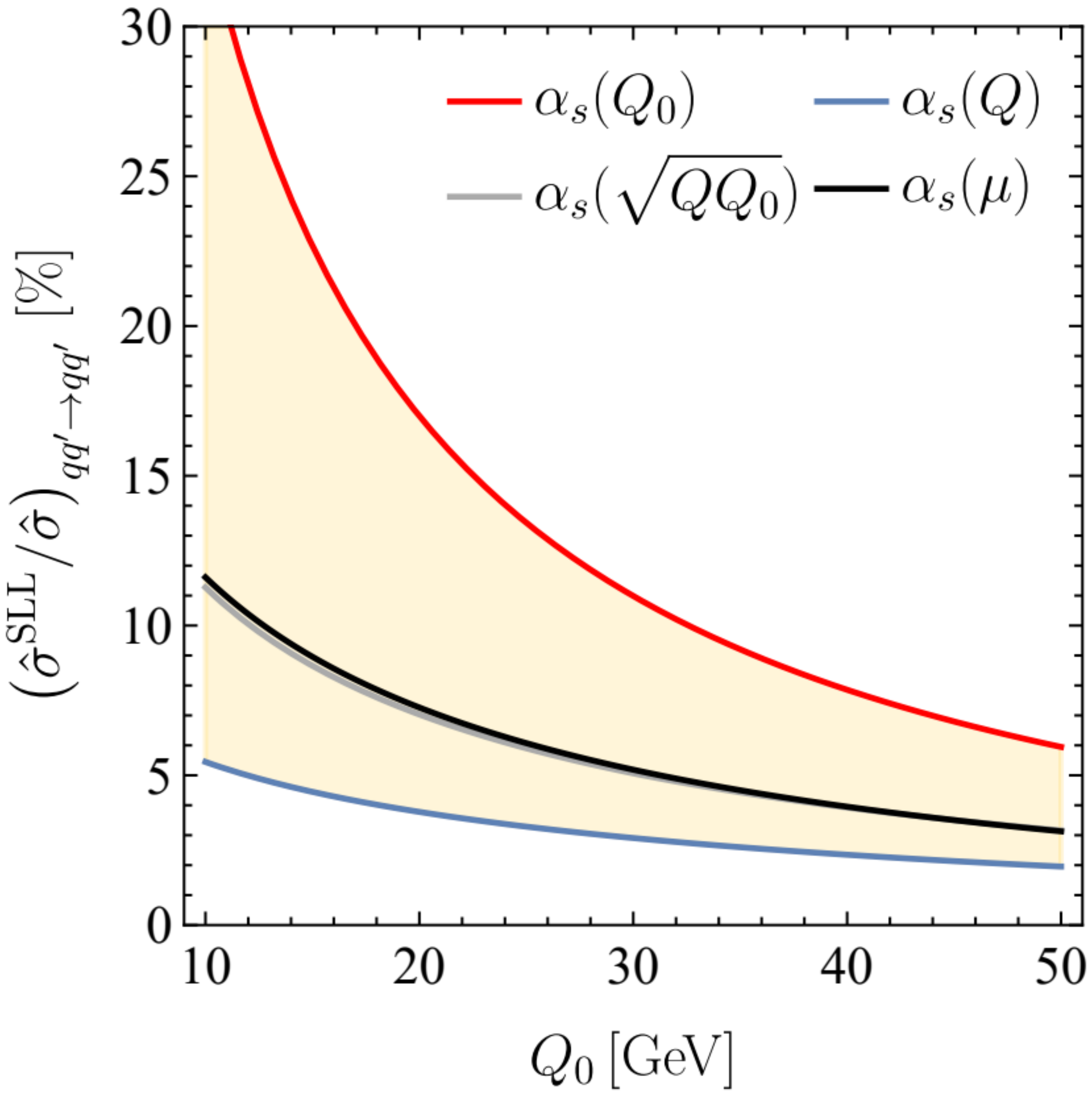
$$\sigma \sim \sum_{n=0}^{\infty} \left[c_{0,n} \left(\frac{\alpha_s}{\pi} L \right)^n + c_{1,n} \left(\frac{\alpha_s}{\pi} L \right) \left(\frac{\alpha_s}{\pi} i\pi L \right)^2 \left(\frac{\alpha_s}{\pi} L^2 \right)^n + \ldots \right]$$

$$L \sim \ln(Q/Q_0) \gg 1$$

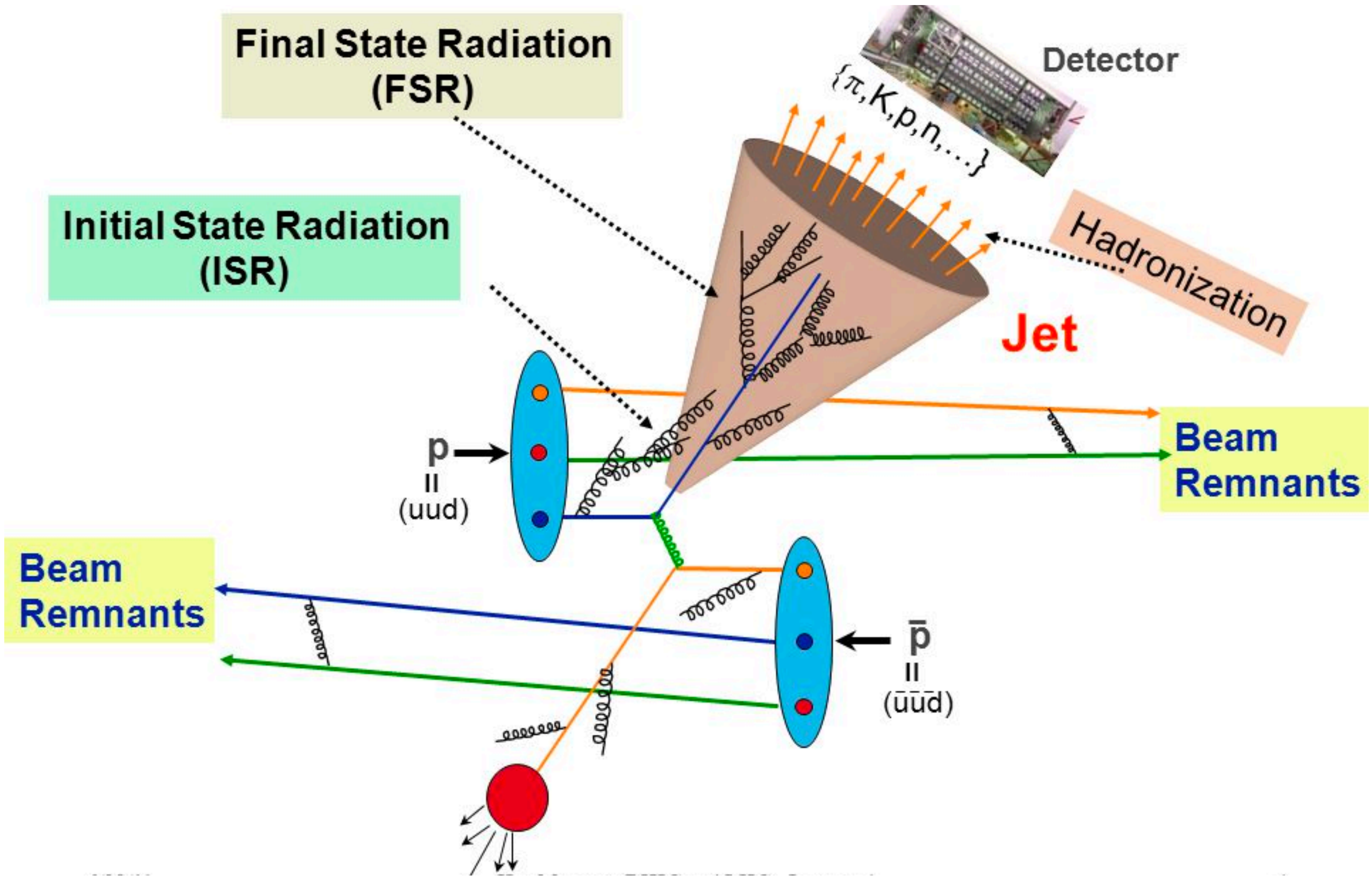
Sudakov logs: $e^{-\alpha_s L^2}$

Super leading logs: $\dfrac{\log(\alpha_s L^2)}{\alpha_s L^2}$

Becher, Neubert, D.Y. Shao, Stillger, 2023



Question: when will we first see factorization breaking in jet cross section?



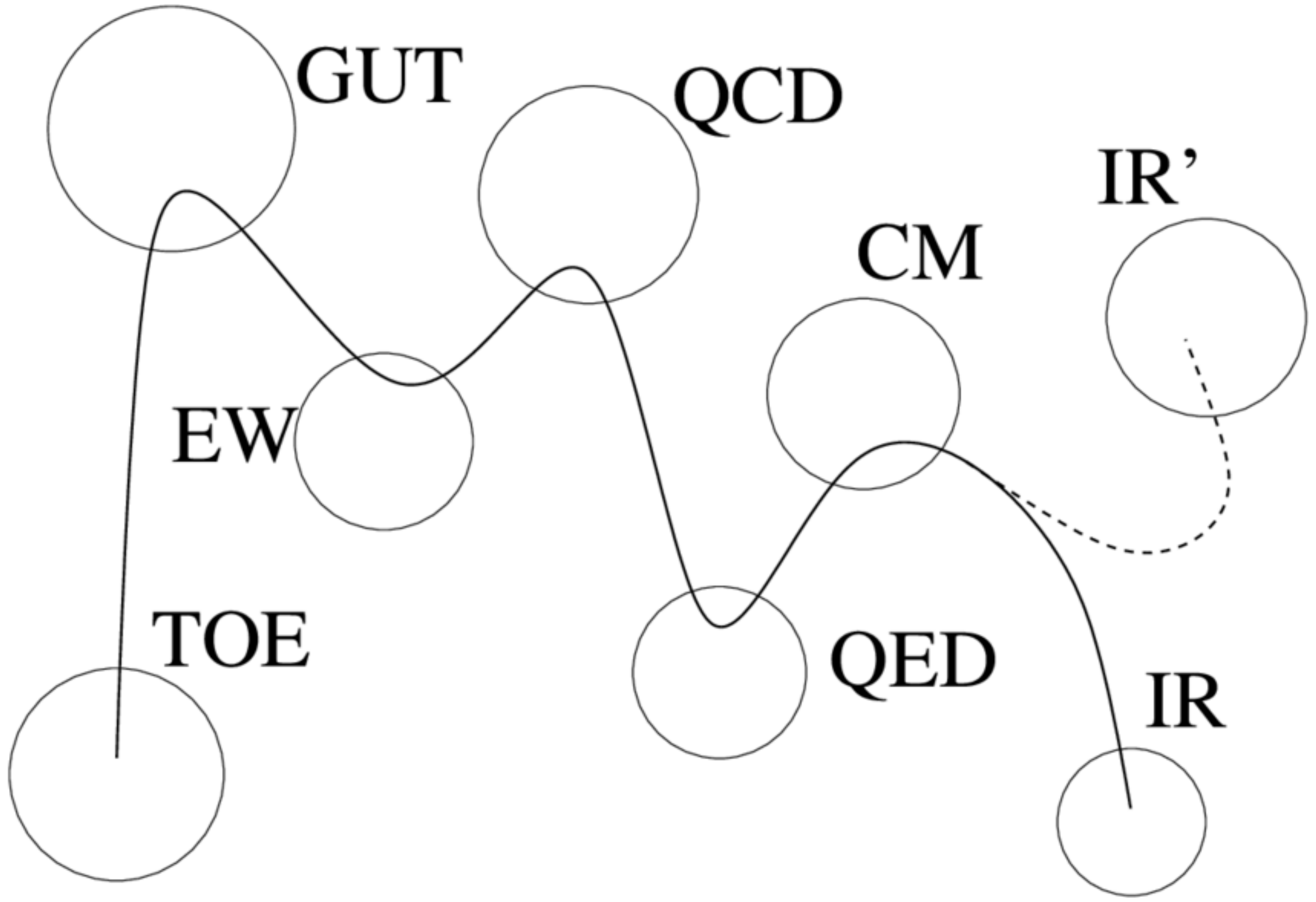
NNNLO? NNNNLO?

Summary

- SCET is indispensable for precision collider physics
- A few examples:
  - Transverse momentum distribution of W/Z/H
  - Power corrections to Higgs amplitudes
  - Super leading logarithms in gap cross section
- Only a small facet of SCET. For a list of active topics:

SCET 2023 workshop https://indico.physics.lbl.gov/event/2384/

A central theme in physics is to understand the renormalization group flow QFTs



SCET provides a toolbox in exploring the varied and rich RG flows of QCD

- Sudakov problem
- Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equation
- Balitsky-Fadin-Kuraev-Lipatov equation
- Balitsky-Kovchegov equation
- Banfi-Marchesini-Syme equation

Thank you for your attention!