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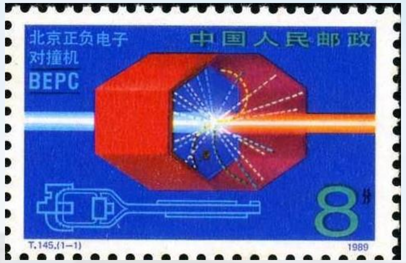

三体幺正耦合道模型研究 η(1405/1475)

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arXiv: 2212.07904

中国科学院大学 University of Chinese Academy of Sciences

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- 三体幺正耦合道模型
- 达利兹图拟合结果
- 理论分析
- 小结和展望

$\eta(1405/1475)$背景介绍（PDG）

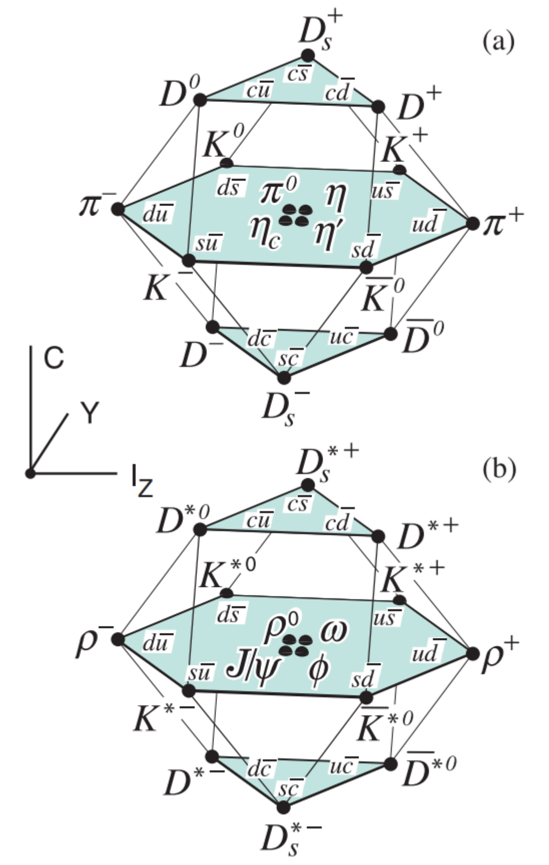
1 *15. Quark Model*

15. Quark Model

Revised August 2021 by C. Amsler (Stefan Meyer Inst.), T. DeGrand (Colorado U., Boulder) and B. Krusche (Basel U.).

$n^{2s+1}\ell_J$	J^{PC}	$I=1$ $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	$I=\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $\bar{u}s$	$I=0$ f'	$I=0$ f
1^1S_0	0^{-+}	π	K	η	$\eta'(958)$
1^3S_1	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}[a]	$h_1(1415)$	$h_1(1170)$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}[a]	$f_1(1420)$	$f_1(1285)$
1^3P_2	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$[a]	$\eta_2(1870)$	$\eta_2(1645)$
1^3D_1	1^{--}	$\rho(1700)$	$K^*(1680)$[b]	$\phi(2170)$[d]	$\omega(1650)$
1^3D_2	2^{--}		$K_2(1820)$[a]		
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$
1^3F_4	4^{++}	$a_4(1970)$	$K_4^*(2045)$	$f_4(2300)$	$f_4(2050)$
1^3G_5	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$		
2^1S_0	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$[c]	$\eta(1295)$
2^3S_1	1^{--}	$\rho(1450)$	$K^*(1410)$[b]	$\phi(1680)$	$\omega(1420)$
2^3P_1	1^{++}	$a_1(1640)$			
2^3P_2	2^{++}	$a_2(1700)$	$K_2^*(1980)$	$f_2(1950)$	$f_2(1640)$

[c] The $\eta(1475)$ and $\eta(1405)$ (not shown) may be manifestations of a single state [7].

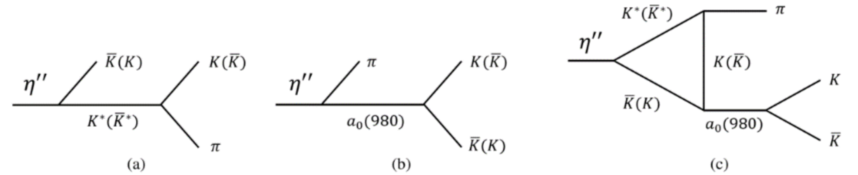


PHYSICAL REVIEW D **100**, 036005 (2019)

Internal particle width effects on the triangle singularity mechanism in the study of the $\eta(1405)$ and $\eta(1475)$ puzzle

Meng-Chuan Du[1,2,*] and Qiang Zhao[1,2,3,†]

Namely, the observed differences of η resonances within the mass region of 1.40–1.48 GeV are originated from the same state. For the isospin violated process $J/\psi \to \gamma\eta(1405/1475) \to f_0(980)\pi \to 3\pi$, we identify an additional contribution to the $a_0(980)-f_0(980)$ mixing via the TS mechanism.



PRL **108,** 081803 (2012) PHYSICAL REVIEW LETTERS week ending 24 FEBRUARY 2012

Puzzle of Anomalously Large Isospin Violations in $\eta(1405/1475) \to 3\pi$

Jia-Jun Wu,[1] Xiao-Hai Liu,[1] Qiang Zhao,[1,2,*] and Bing-Song Zou[1,2,†]
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(Received 18 August 2011; published 22 February 2012)

The BES-III Collaboration recently reported the observation of anomalously large isospin violations in $J/\psi \to \gamma\eta(1405/1475) \to \gamma\pi^0 f_0(980) \to \gamma + 3\pi$, where the $f_0(980)$ in the $\pi\pi$ invariant mass spectrum appears to be much narrower ($\sim$ 10 MeV) than the peak width ($\sim$ 50 MeV) measured in other processes. We show that a mechanism, named as triangle singularity (TS), can produce a narrow enhancement between the charged and neutral $K\bar{K}$ thresholds, i.e., $2m_{K^\pm} \sim 2m_{K^0}$. It can also lead to different invariant mass spectra for $\eta(1405/1475) \to a_0(980)\pi$ and $K\bar{K}^* +$ c.c., which can possibly explain the long-standing puzzle about the need for two close states $\eta(1405)$ and $\eta(1475)$ in $\eta\pi\pi$ and $K\bar{K}\pi$, respectively. The TS could be a key to our understanding of the nature of $\eta(1405/1475)$ and advance our knowledge about the mixing between $a_0(980)$ and $f_0(980)$.

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$\eta(1405/1475)$背景介绍（PDG）

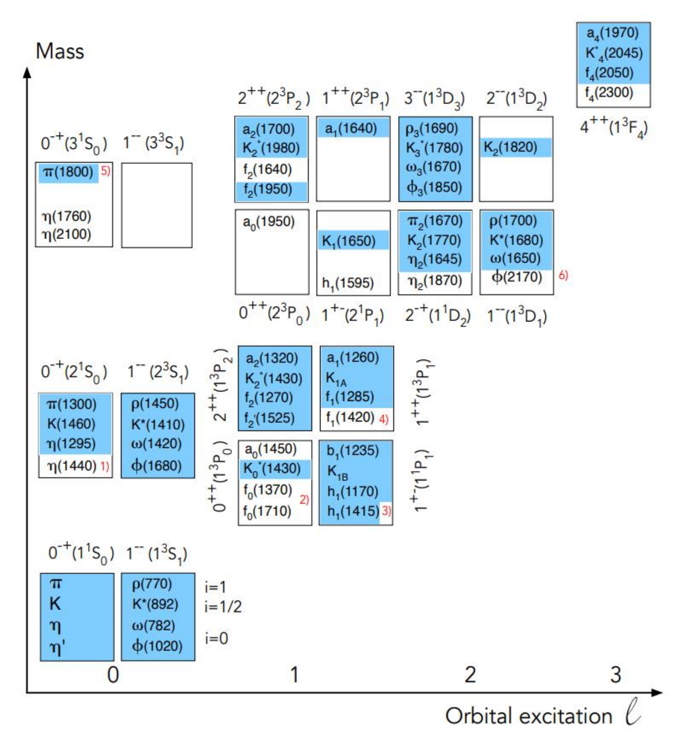
15. Quark Model

Revised August 2021 by C. Amsler (Stefan Meyer Inst.), T. DeGrand (Colorado U., Boulder) and B. Krusche (Basel U.).

63. Spectroscopy of Light Meson Resonances

Written August 2021 by C. Amsler (Stefan Meyer Inst.), S. Eidelman (Budker Inst., Novosibirsk; Novosibirsk U.), A. Masoni (INFN, Cagliari) and G. Venanzoni (INFN, Pisa).

$n^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}, \bar{u}d, \frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}, d\bar{s}; \bar{d}s, \bar{u}s$	I = 0 f'	I = 0 f
1^1S_0	0^{-+}	π	K	η	$\eta'(958)$
1^3S_1	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}[a]	$h_1(1415)$	$h_1(1170)$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}[a]	$f_1(1420)$	$f_1(1285)$
1^3P_2	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$[a]	$\eta_2(1870)$	$\eta_2(1645)$
1^3D_1	1^{--}	$\rho(1700)$	$K^*(1680)$[b]	$\phi(2170)$[d]	$\omega(1650)$
1^3D_2	2^{--}		$K_2(1820)$[a]		
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$
1^3F_4	4^{++}	$a_4(1970)$	$K_4^*(2045)$	$f_4(2300)$	$f_4(2050)$
1^3G_5	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$		
2^1S_0	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$[c]	$\eta(1295)$
2^3S_1	1^{--}	$\rho(1450)$	$K^*(1410)$[b]	$\phi(1680)$	$\omega(1420)$
2^3P_1	1^{++}	$a_1(1640)$			
2^3P_2	2^{++}	$a_2(1700)$	$K_2^*(1980)$	$f_2(1950)$	$f_2(1640)$



Hence, in radiative $J/\psi(1S)$ decay, $\pi^- p$ and $\bar{p}p$ annihilation at rest two isoscalar signals are observed in the 1400 – 1500 MeV mass region, while the $\eta(1405)$ is not seen in $\gamma\gamma$ interactions nor in B decays. The $\eta(1475)$ could be the first radial excitation of the η', with the $\eta(1295)$ being the first radial excitation of the η. Ideal mixing, suggested by the $\eta(1295)$ and $\pi(1300)$ mass degeneracy,

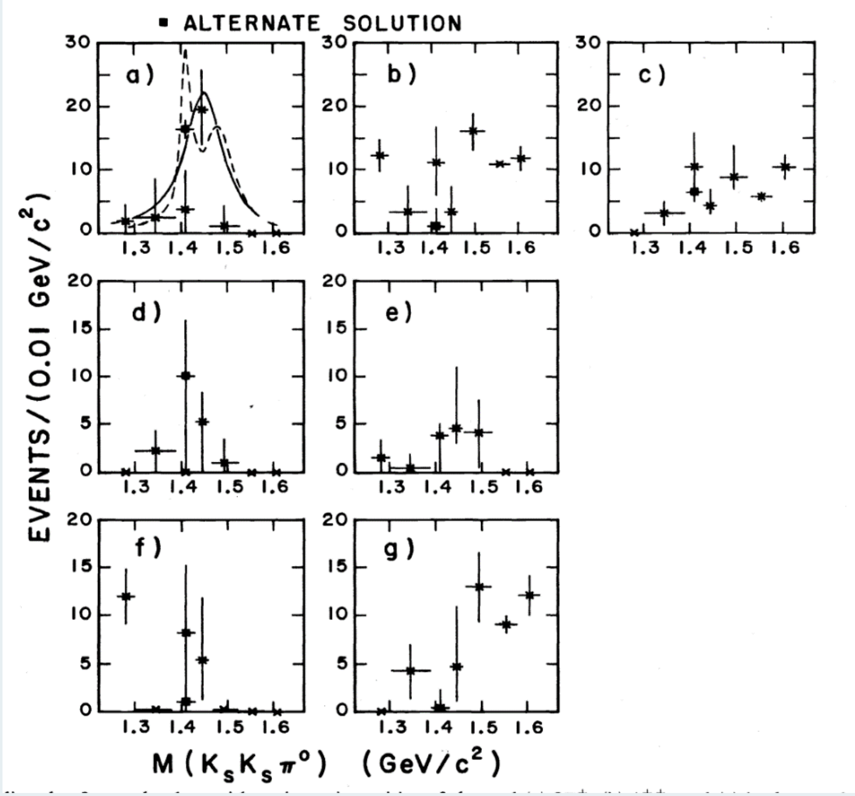
that there is sufficient evidence to consider the 0^{-+} nonet with the $\eta(1440)$ in fig. 63.1 as established. Whether one or two different states – $\eta(1405)$ and $\eta(1475)$ – exist is an open question, in which case the $\eta(1405)$ would be supernumerary. There is a wide number of experimental results indicating the presence of two separate states but, as mentioned above, data are also consistent with one state only. Theoretical interpretations of the most recent data are not able to lift the ambiguity.

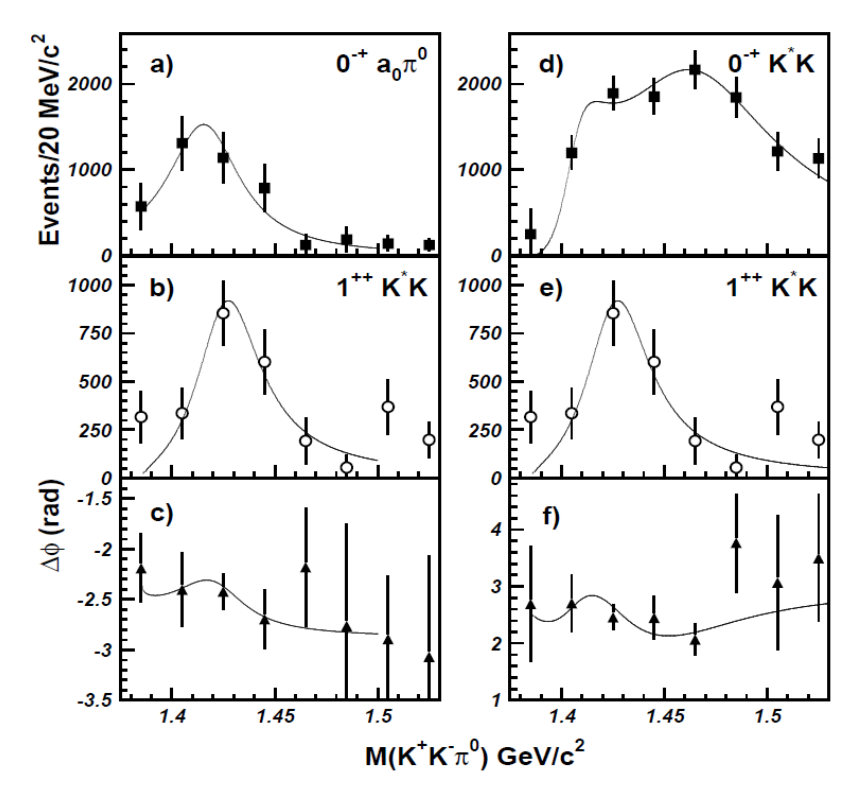
$\eta(1405/1475)$：确定应该存在！
人们不知道的：一个 或者 两个？
究竟是什么？

Open Question

$\eta(1405/1475)$背景介绍（实验）

- $\pi^- p$ 散射：PRD40,693(1989)【$KsKs\pi$ 1&2】, E852 PLB516, 264(2001)【$K^+K^-\pi$ 2】
- $\bar{p}p$ 湮灭实验
- ψ 衰变
- $\gamma\gamma$ 对撞
- B 衰变





$\eta(1405/1475)$背景介绍（实验）

- $\pi^{-}p$ 散射：PRD40,693(1989)【$KsKs\pi$ 1&2】, E852 PLB516, 264(2001)【$K^{+}K^{-}\pi$ 2】
- $\bar{p}p$ 湮灭实验：OBELX PLB361, 187（1995）; 400, 226（1997）; 462, 453（1999）; 545, 261（2002）【$K^{+}K^{-}\pi$ 2】, EPJC33, 23 (2004)【$\pi\pi\eta$ 1】
- ψ 衰变
- $\gamma\gamma$ 对撞
- B 衰变

462,453(1999)

Table 1
Partial widths (normalized to the $K\bar{K}\pi$ decay modes)

		$\Gamma_{K\bar{K}\pi}$	$\Gamma_{a_0\pi}$	$\Gamma_{K^*\bar{K}}$
Solution (A)	$\eta(1440)$	1	0.1 ± 0.01	0.8 ± 0.02
$L = 2550$; $\chi^2 = 3.0$	–	–	–	–
Solution (B)	$\eta(1400)$	1	0.2 ± 0.01	0.4 ± 0.01
$L = 2800$; $\chi^2 = 1.4$	$\eta(1500)$	1	0.1 ± 0.01	4 ± 0.2
Solution (C)	$\eta(1400)$	1	0.1 ± 0.01	0.8 ± 0.02
$L = 2800$; $\chi^2 = 1.4$	$\eta(1700–1800)$	1	0.1 ± 0.1	2 ± 0.1

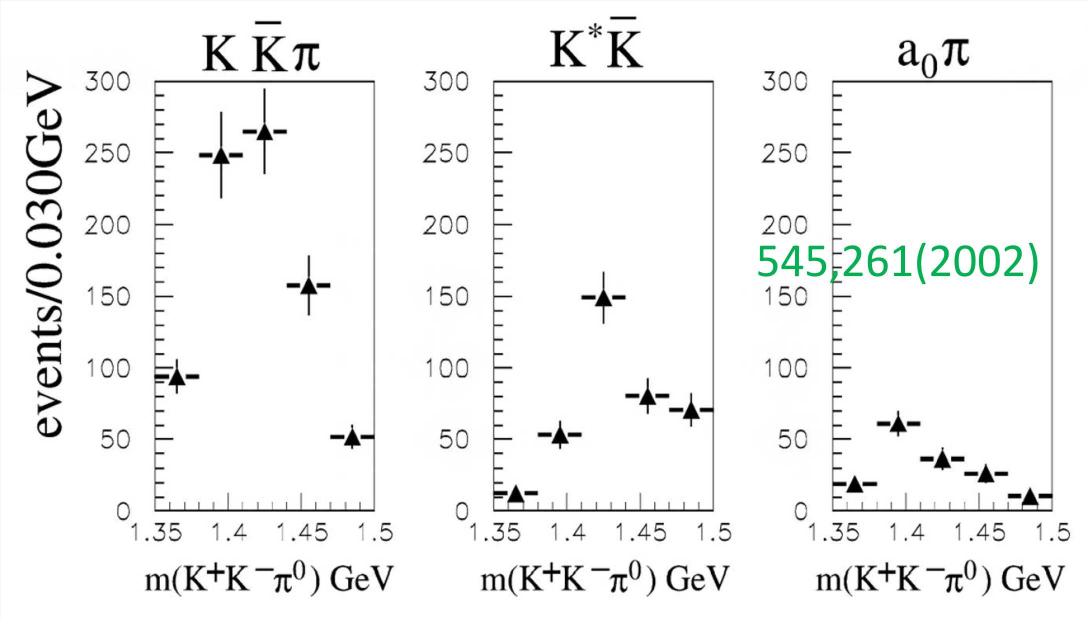


Fig. 5. $K^+K^-\pi^0$ intensity spectra from the partial wave analysis of $^1S_0$ initial protonium wave.

$\eta(1405/1475)$背景介绍（实验）

- $\pi^{-}p$ 散射：PRD40,693(1989)【$KsKs\pi$ 1&2】, E852 PLB516, 264(2001)【$K^{+}K^{-}\pi$ 2】
- $\bar{p}p$ 湮灭实验: OBELX PLB361,187(1995);400,226(1997);462,453(1999); 545,261(2002)【$K^{+}K^{-}\pi$ 2】, EPJC33, 23 (2004)【$\pi\pi\eta$ 1】
- ψ 衰变:MARKIII, PRL 65, 2507 (1990))【$J/\psi \to \gamma K^{+}K^{-}\pi$ 2】,PRL69, 1328 (1992) 【$J/\psi \to \gamma\pi\pi\eta$ 1】, DM2, PRD42, 10 (1990)【$J/\psi \to \gamma K^{+}K^{-}\pi$ 1】, PRD46, 1951 (1992) 【$J/\psi \to \gamma$ $K^{+}K^{-}\pi$ 2 $\gamma\pi\pi\eta$ 1 】, BES, PRD77, 032005 (2008) 【$J/\psi \to \omega K^{+}K^{-}\pi$ 1 $\phi K^{+}K^{-}\pi$ 0】, PRD87, 092006 (2013) 【$\psi(3686) \to \omega K^{+}K^{-}\pi$ 1】PLB 446, 356【$J/\psi \to \gamma\pi\pi\eta$ 1】PRL 108 182001(2012) 【$J/\psi \to \gamma\pi\pi\pi$ 1】 PLB 594 47(2004)【$J/\psi \to \gamma\gamma\rho$ 1】, PRD 97 051101(2018) 【$J/\psi \to \gamma\gamma\phi$ 1(1475)】
- $\gamma\gamma$ 对撞：
- B 衰变：

$\eta(1405/1475)$背景介绍（实验）

- $\pi^- p$ 散射：PRD40,693(1989)【$KsKs\pi$ 1&2】, E852 PLB516, 264(2001)【$K^+K^-\pi$ 2】
- $\bar{p}p$ 湮灭实验: OBELX PLB361,187(1995);400,226(1997);462,453(1999); 545,261(2002)【$K^+K^-\pi$ 2】, EPJC33, 23 (2004)【$\pi\pi\eta$ 1】
- ψ 衰变:MARKIII, PRL 65, 2507 (1990))【$J/\psi \to \gamma K^+K^-\pi$ 2】,PRL69, 1328 (1992) 【$J/\psi \to \gamma\pi\pi\eta$ 1】, DM2, PRD42, 10 (1990)【$J/\psi \to \gamma K^+K^-\pi$ 1】, PRD46, 1951 (1992) 【$J/\psi \to \gamma$ $K^+K^-\pi$ 2 $\gamma\pi\pi\eta$ 1 】, BES, PRD77, 032005 (2008) 【$J/\psi \to \omega K^+K^-\pi$ 1 $\phi K^+K^-\pi$ 0】, PRD87, 092006 (2013) 【$\psi(3686) \to \omega K^+K^-\pi$ 1】PLB 446, 356【$J/\psi \to \gamma\pi\pi\eta$ 1】PRL 108 182001(2012) 【$J/\psi \to \gamma\pi\pi\pi$ 1】 PLB 594 47(2004)【$J/\psi \to \gamma\gamma\rho$ 1】, PRD 97 051101(2018)【$J/\psi \to \gamma\gamma\phi$ 1(1475)】
- $\gamma\gamma$ 对撞：L3 PLB501 1(2001)【$\gamma\gamma \to$ KsK$^\pm\pi^\mp$ 1 $\pi\pi\eta$ 1 】 JHEP 03, 018 (2007)【$\gamma\gamma \to$ KsK$^\pm\pi^\mp$ 1(1475)】 CLEO-II, PRD71, 072001(2005)【$\gamma\gamma \to$ KsK$^\pm\pi^\mp$ 0 】
- B 衰变:

$\eta(1405/1475)$背景介绍（实验）

- $\pi^{-}p$ 散射：PRD40,693(1989)【$KsKs\pi$ 1&2】, E852 PLB516, 264(2001)【$K^{+}K^{-}\pi$ 2】
- $\bar{p}p$ 湮灭实验: OBELX PLB361,187(1995);400,226(1997);462,453(1999); 545,261(2002)【$K^{+}K^{-}\pi$ 2】, EPJC33, 23 (2004)【$\pi\pi\eta$ 1】
- ψ 衰变:MARKIII, PRL 65, 2507 (1990))【$J/\psi \to \gamma K^{+}K^{-}\pi$ 2】,PRL69, 1328 (1992)【$J/\psi \to \gamma\pi\pi\eta$ 1】, DM2, PRD42, 10 (1990)【$J/\psi \to \gamma K^{+}K^{-}\pi$ 1】, PRD46, 1951 (1992)【$J/\psi \to \gamma$ $K^{+}K^{-}\pi$ 2 $\gamma\pi\pi\eta$ 1】, BES, PRD77, 032005 (2008)【$J/\psi \to \omega K^{+}K^{-}\pi$ 1 $\phi K^{+}K^{-}\pi$ 0】, PRD87, 092006 (2013)【$\psi(3686) \to \omega K^{+}K^{-}\pi$ 1】PLB 446, 356【$J/\psi \to \gamma\pi\pi\eta$ 1】PRL 108 182001(2012)【$J/\psi \to \gamma\pi\pi\pi$ 1】 PLB 594 47(2004)【$J/\psi \to \gamma\gamma\rho$ 1】, PRD 97 051101(2018)【$J/\psi \to \gamma\gamma\phi$ 1(1475)】
- $\gamma\gamma$ 对撞：L3 PLB501 1(2001)【$\gamma\gamma \to \mathrm{KsK}^{\pm}\pi^{\mp}$ 1 $\pi\pi\eta$ 1】 JHEP 03, 018 (2007)【$\gamma\gamma \to \mathrm{KsK}^{\pm}\pi^{\mp}$ 1(1475)】 CLEO-II, PRD71, 072001(2005)【$\gamma\gamma \to \mathrm{KsK}^{\pm}\pi^{\mp}$ 0】
- B 衰变：BaBar, PRL101, 091801(2009)【$B^{+} \to (\bar{\mathrm{K}}^{*}\mathrm{K}/\pi\pi\eta)\mathrm{K}$ 1 (1405)】

$\eta(1405/1475)$背景介绍（实验）

- π^-p 散射：PRD40,693(1989)【$KsKs\pi$ 1&2】, E852 PLB516, 264(2001)【$K^+K^-\pi$ 2】
- $\bar{p}p$ 湮灭实验: OBELX PLB361,187(1995);400,226(1997);462,453(1999); 545,261(2002)【$K^+K^-\pi$ 2】, EPJC33, 23 (2004)【$\pi\pi\eta$ 1】
- ψ 衰变:MARKIII, PRL 65, 2507 (1990))【$J/\psi \to \gamma K^+K^-\pi$ 2】,PRL69, 1328 (1992)【$J/\psi \to \gamma\pi\pi\eta$ 1】, DM2, PRD42, 10 (1990)【$J/\psi \to \gamma K^+K^-\pi$ 1】, PRD46, 1951 (1992)【$J/\psi \to \gamma$ $K^+K^-\pi$ 2 $\gamma\pi\pi\eta$ 1 】, BES, PRD77, 032005 (2008)【$J/\psi \to \omega K^+K^-\pi$ 1 $\phi K^+K^-\pi$ 0】, PRD87, 092006 (2013)【$\psi(3686) \to \omega K^+K^-\pi$ 1】PLB 446, 356【$J/\psi \to \gamma\pi\pi\eta$ 1】PRL 108 182001(2012)【$J/\psi \to \gamma\pi\pi\pi$ 1】 PLB 594 47(2004)【$J/\psi \to \gamma\gamma\rho$ 1】, PRD 97 051101(2018)【$J/\psi \to \gamma\gamma\phi$ 1(1475)】
- $\gamma\gamma$ 对撞：L3 PLB501 1(2001)【$\gamma\gamma \to \mathrm{KsK}^{\pm}\pi^{\mp}$ 1 $\pi\pi\eta$ 1 】 JHEP 03, 018 (2007)【$\gamma\gamma \to \mathrm{KsK}^{\pm}\pi^{\mp}$ 1(1475)】 CLEO-II, PRD71, 072001(2005)【$\gamma\gamma \to \mathrm{KsK}^{\pm}\pi^{\mp}$ 0 】
- B 衰变：BaBar, PRL101, 091801(2009)【$B^+ \to (\bar{\mathrm{K}}^*\mathrm{K}/\pi\pi\eta)\mathrm{K}$ 1 (1405)】

$\eta(1405)$: $a_0\pi, \bar{K}K^*$
$\eta(1475)$: $\bar{K}K^*$

$\eta(1405/1475)$背景介绍(理论)

- 夸克模型：（η, η'）的径向激发态（$\eta(1295)$, $\eta(14??)$）
- 胶球：赝标胶球格点计算都在2.0 GeV以上【 UKQCD Collaboration, PLB 309, 378 (1993)/ C.J. Morningstar and M.J. Peardon, PRD 60, 034509 (1999)/ Y. Chen, A. Alexandru, S. J. Dong, T. Draper, I. Horva´th, F.X. Lee, K.F. Liu, N. Mathur et al., PRD 73, 014516 (2006) / UKQCD Collaboration, PRD 82, 034501 (2010)/ F. Chen, X. Jiang, Y. Chen, K.-F. Liu, W. Sun, and Y.-B. Yang, arXiv:2111.11929 】，有文献认为$\eta(1405)$是胶球【L. Faddeev, A. J. Niemi and U. Wiedner, PRD70, 114033 (2004) 】。
- 胶球夸克模型混合：H. Y. Cheng, H. n. Li and K. F. Liu, PRD 79 014024(2009) ,
- 四夸克态： J. D. Weinstein and N. Isgur, PRD 27 588(1983) .
- 分子态： $K\bar{K}\pi$ 分子态 R. S. Longacre, PRD 42 (1990) 874.
- ……

$\eta(1405/1475)$背景介绍

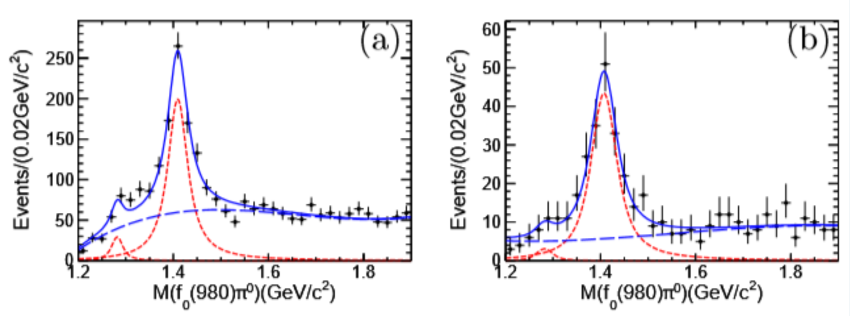
- $\eta(1405/1475)$的两个突出的问题

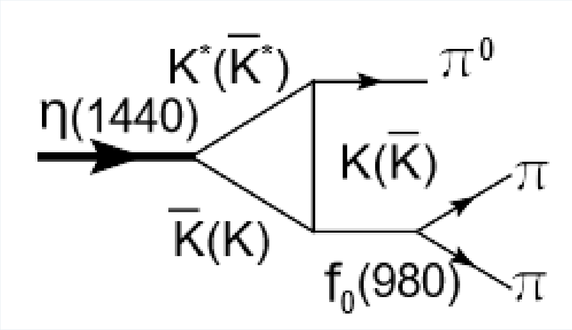
一个 或者 两个？

究竟是什么？

需要详实的实验数据和一个完整的分析

BESIII PRL 108 182001(2012) 【$J/\psi \rightarrow \gamma\pi\pi\pi$ 1】

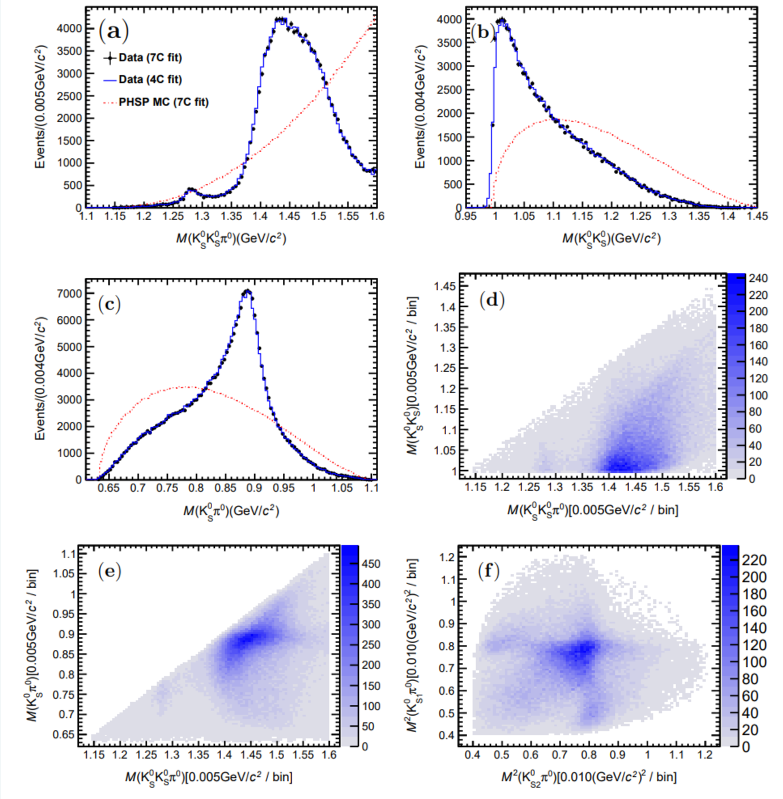




$J/\psi \to \gamma\eta(1405/1475) \to \gamma K_s K_s \pi$实验数据介绍

126436的$\gamma K_s K_s \pi$事例

- arXiv: 2209.11175 BESIII 100亿J/ψ事例数



对于$K_s K_s \pi$应用了
"Bin-by-bin analysis"

Study of $\eta(1405)/\eta(1475)$ in $J/\psi \to \gamma K_S^0 K_S^0 \pi^0$ decay

The BESIII Collaboration

Components	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
(1). $J/\psi \to \gamma\mathrm{PHSP}(0^{-+}) \to \gamma K^*(892)^0 K_S^0 \to \gamma K_S^0 K_S^0 \pi^0$	3.4 ± 0.1	0.9 ± 0.1	9.0 ± 0.1	-2.7 ± 0.1	-1.2 ± 0.1	0.0 ± 0.0	0.3 ± 0.1	0.0 ± 0.0	1.4 ± 0.1	1.4 ± 0.1	0.0 ± 0.0	0.0 ± 0.0	0.2 ± 0.1	0.0 ± 0.0
(2). $J/\psi \to \gamma\mathrm{PHSP}(1^{++}) \to \gamma K^*(892)^0 K_S^0 \to \gamma K_S^0 K_S^0 \pi^0$		2.9 ± 0.1	1.2 ± 0.1	0.3 ± 0.1	-1.0 ± 0.2	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.2 ± 0.1	0.4 ± 0.1	1.4 ± 0.1	1.0 ± 0.2	0.0 ± 0.1	0.0 ± 0.0
(3). $J/\psi \to \gamma\eta(1405) \to \gamma K_S^0 (K_S^0\pi^0)_{\text{P-wave}} \to \gamma K_S^0 K_S^0 \pi^0$			9.5 ± 0.2	-1.0 ± 0.1	-1.6 ± 0.1	0.0 ± 0.0	-1.7 ± 0.1	0.0 ± 0.1	1.1 ± 0.1	2.4 ± 0.1	0.0 ± 0.0	0.0 ± 0.1	1.0 ± 0.1	0.0 ± 0.0
(4). $J/\psi \to \gamma\eta(1475) \to \gamma K_S^0 (K_S^0\pi^0)_{\text{P-wave}} \to \gamma K_S^0 K_S^0 \pi^0$				11.2 ± 0.2	2.0 ± 0.1	0.0 ± 0.0	-2.0 ± 0.1	0.0 ± 0.0	-1.9 ± 0.1	-1.0 ± 0.1	0.0 ± 0.0	0.0 ± 0.0	-0.8 ± 0.1	0.3 ± 0.1
(5). $J/\psi \to \gamma f_1(1420) \to \gamma K^*(892)^0 K_S^0 \to \gamma K_S^0 K_S^0 \pi^0$					18.9 ± 0.3	-0.1 ± 0.1	-0.7 ± 0.1	0.0 ± 0.0	-0.6 ± 0.1	0.0 ± 0.0	1.6 ± 0.1	-5.8 ± 0.3	-0.2 ± 0.1	0.0 ± 0.0
(6). $J/\psi \to \gamma f_2(1525) \to \gamma K^*(892)^0 K_S^0 \to \gamma K_S^0 K_S^0 \pi^0$						2.2 ± 0.1	0.0 ± 0.0	0.0 ± 0.1	0.0 ± 0.1	0.0 ± 0.0	0.0 ± 0.2	0.0 ± 0.1	0.0 ± 0.0	0.0 ± 0.1
(7). $J/\psi \to \gamma\mathrm{PHSP}(0^{-+}) \to \gamma a_0(980)^0 \pi^0 \to \gamma K_S^0 K_S^0 \pi^0$							22.9 ± 0.3	0.0 ± 0.0	9.1 ± 0.1	-2.1 ± 0.2	0.1 ± 0.1	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0
(8). $J/\psi \to \gamma\mathrm{PHSP}(2^{-+}) \to \gamma a_0(980)^0 \pi^0 \to \gamma K_S^0 K_S^0 \pi^0$								0.2 ± 0.1	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0
(9). $J/\psi \to \gamma\eta(1405) \to \gamma (K_S^0 K_S^0)_{\text{S-wave}} \pi^0 \to \gamma K_S^0 K_S^0 \pi^0$									7.2 ± 0.1	-0.9 ± 0.1	-0.1 ± 0.1	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0
(10). $J/\psi \to \gamma\eta(1475) \to \gamma (K_S^0 K_S^0)_{\text{S-wave}} \pi^0 \to \gamma K_S^0 K_S^0 \pi^0$										8.6 ± 0.2	0.1 ± 0.1	0.1 ± 0.1	0.0 ± 0.0	0.0 ± 0.0
(11). $J/\psi \to \gamma f_1(1285) \to \gamma a_0(980)^0 \pi^0 \to \gamma K_S^0 K_S^0 \pi^0$											2.1 ± 0.1	-0.8 ± 0.1	0.0 ± 0.0	0.0 ± 0.0
(12). $J/\psi \to \gamma f_1(1420) \to \gamma a_0(980)^0 \pi^0 \to \gamma K_S^0 K_S^0 \pi^0$												1.3 ± 0.1	0.0 ± 0.0	0.0 ± 0.0
(13). $J/\psi \to \gamma\eta(1405) \to \gamma a_2(1320)^0 \pi^0 \to \gamma K_S^0 K_S^0 \pi^0$													0.9 ± 0.1	-0.4 ± 0.1
(14). $J/\psi \to \gamma\eta(1475) \to \gamma a_2(1320)^0 \pi^0 \to \gamma K_S^0 K_S^0 \pi^0$														0.1 ± 0.1

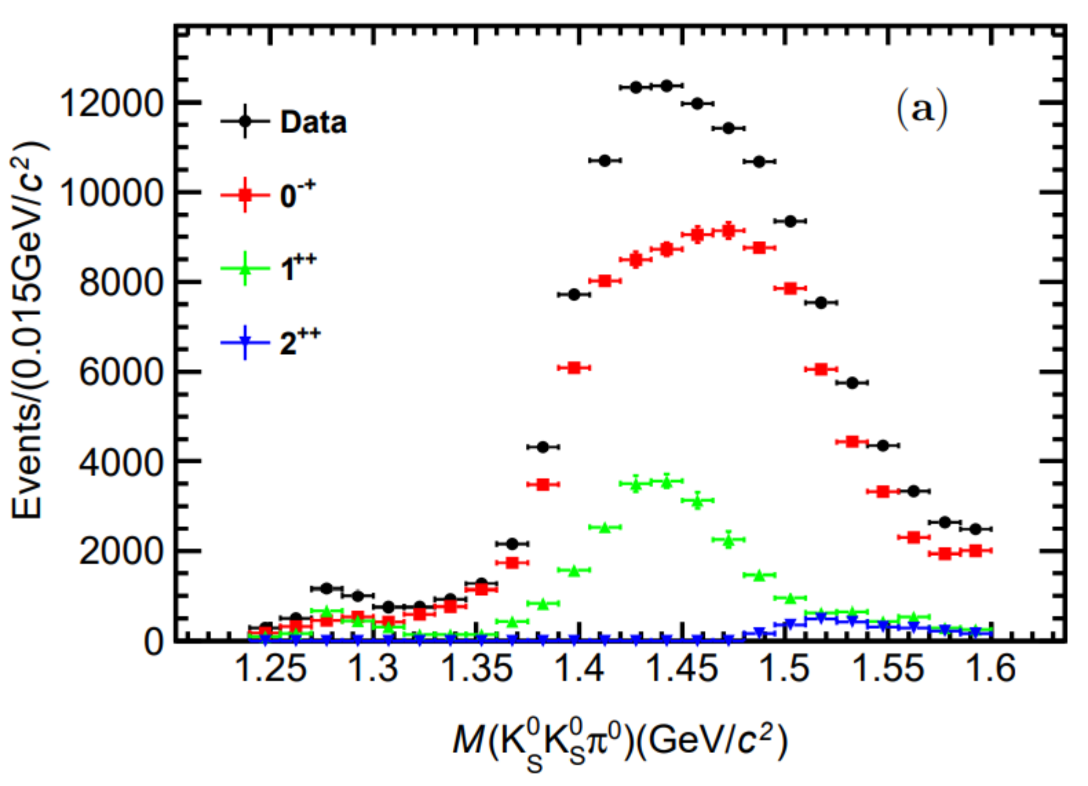
Table 3. Fraction of each component and interference fractions between two components (%) in the MD PWA nominal solution. The uncertainties are statistical only.

$J/\psi \to \gamma\eta(1405/1475) \to \gamma K_s K_s \pi$实验数据介绍

126436的$\gamma K_s K_s \pi$事例

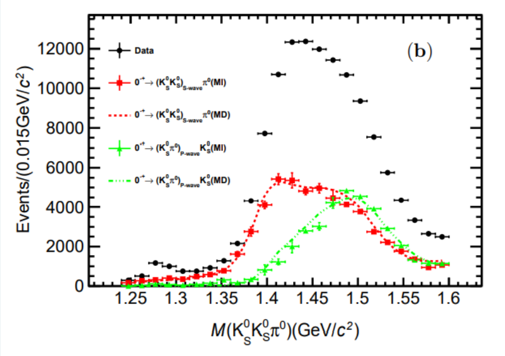
- arXiv: 2209.11175 BESIII 100亿J/ψ事例数

对于$K_s K_s \pi$应用了
"Bin-by-bin analysis"



$$BW(s) = \frac{1}{M^2 - s - iM\Gamma(s)},$$

$$\Gamma(s) = \Gamma(M^2)\left(\frac{M}{\sqrt{s}}\right)\left(\frac{\rho(s)}{\rho(M^2)}\right)^{2l+1} B_l^2(\rho(s)),$$



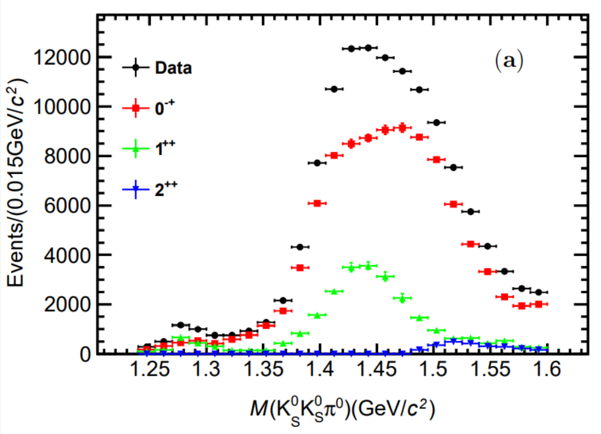
Source	$\eta(1405)$		$\eta(1475)$		$f_1(1285)$		$f_1(1420)$		$f_2(1525)$	
	ΔM	$\Delta\Gamma$	ΔM	$\Delta\Gamma$	ΔM	$\Delta\Gamma$	ΔM	$\Delta\Gamma$	ΔM	$\Delta\Gamma$
Breit-Wigner formula	$+11.0$	$^{+5.2}_{-11.8}$	$^{+15.5}_{-31.6}$	$+14.6$	$^{+0.8}_{-1.5}$	$^{+0.6}_{-2.9}$	$^{+27.9}_{-0.5}$	$^{+13.3}_{-9.9}$	$^{+2.6}_{-7.5}$	$^{+2.0}_{-5.1}$
Resonance parameters	$+2.5$	$+1.8$	-5.6	-10.8	$+0.3$	$+1.8$	-0.2	-2.4	$+1.6$	-3.0
Extra components	$^{+0.1}_{-0.3}$	$^{+0.6}_{-2.0}$	$^{+0.1}_{-2.2}$	$^{+2.3}_{-1.8}$	$^{+0.8}_{-0.2}$	$^{+5.2}_{-0.1}$	$^{+1.5}_{-0.5}$	$^{+2.7}_{-3.9}$	$+1.1$	-1.4
Total	$^{+11.3}_{-0.3}$	$^{+5.5}_{-12.0}$	$^{+15.5}_{-32.2}$	$^{+14.8}_{-10.9}$	$^{+1.2}_{-1.5}$	$^{+5.5}_{-2.9}$	$^{+27.9}_{-0.7}$	$^{+13.6}_{-10.9}$	$^{+3.2}_{-7.5}$	$^{+2.0}_{-6.1}$

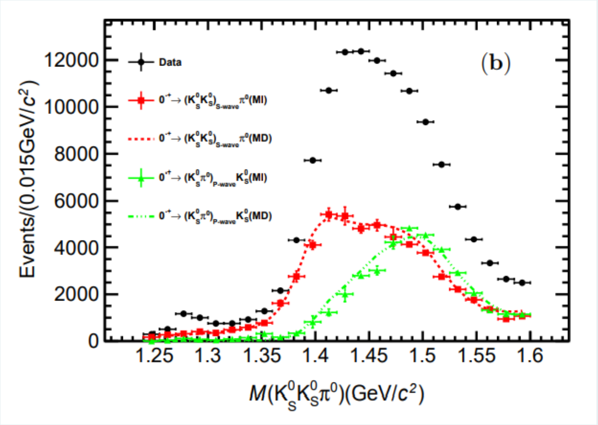
$J/\psi \to \gamma\eta(1405/1475) \to \gamma K_s K_s \pi$实验数据介绍

126436的$\gamma K_s K_s \pi$事例

- arXiv: 2209.11175 BESIII 100亿J/ψ事例数

对于$K_s K_s \pi$应用了 "Bin-by-bin analysis" + BW ? model





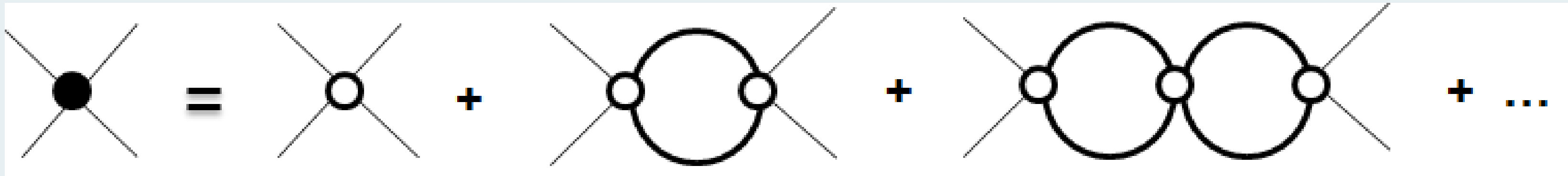
三体系统用了Bin by Bin的分析，使得我们可以比较放心使用实验分波以后的数据，但是对于单个分波，**BW形式**我们认为是不合适的。

1. η(1405/1475) 能区涉及不同的共振态，两个或两个以上的BW形式的共振态会破坏幺正性。
2. 在1400-1500 能区，有很强子道阈值效应，比如$K^*\bar{K}$。
3. 运动学效应复杂，比如之前提过的三角奇异性，圈图其实也会有相应的奇异性，这些贡献不能忽略。

因此，我们需要一个满足**幺正性**的三体模型才能准确的抽取$\eta(1405/1475)$ 的极点！

两体幺正耦合道模型

T=V+VGT



单道单共振：耦合道 && BW 形式

BS 方程：$T(p_f,p_i;P)=V(p_f,p_i;P)+\frac{1}{i}\int\frac{d^4k}{(2\pi)^4}V(p_f,k;P)G(k,P)T(k,p_i;P)$, $\quad G(k,P)=\frac{1}{k^2-m_1^2+i\epsilon}\frac{1}{(P-k)^2-m_1^2+i\epsilon}$

1. 在质心系考虑该方程，并且出末粒子在壳，$P=\left(E,\vec{0}\right), p_{f/i}=\left(\omega_1(\vec{p}_{f/i}),\vec{p}_{f/i}\right)$
2. 对k_0积分，$G(k,P)dk_0\sim\frac{1}{4\omega_1(\vec{k})\omega_2(\vec{k})}\frac{1}{P_0-\omega_1(\vec{k})-\omega_2(\vec{k})+i\epsilon}$
3. 对V和T分波，$\mathrm{d}\Omega 4\pi Y_{00}Y_{00}\sim 4\pi$

$$V(p_f,p_i;E)=\frac{g(p_f)g(p_i)}{E^2-m_0^2}$$

1维散射方程：$T(p_f,p_i;E)=V(p_f,p_i;E)+\int\frac{k^2dk}{8\pi^2}V(p_f,k;P)\frac{1}{\omega_1(\vec{k})\omega_2(\vec{k})}\frac{1}{P_0-\omega_1(\vec{k})-\omega_2(\vec{k})+i\epsilon}T(k,p_i;P)$

$$T(p_f,p_i;E)=\frac{g(p_f)g(p_i)}{E^2-m_0^2-\Sigma(E)}$$

$$T(q,q;E)=\frac{g^2(q)}{E^2-m_0^2-\mathrm{Re}\left(\Sigma(E)\right)+\mathrm{i}\Gamma(\mathrm{E})E}$$

能量依赖性在阈附近有重要贡献，如$f_0(980)$

$$\Sigma(E)=\int\frac{k^2dk}{8\pi^2}\frac{g^2(k)}{\omega_1\left(\vec{k}\right)\omega_2(\vec{k})}\frac{1}{P_0-\omega_1\left(\vec{k}\right)-\omega_2(\vec{k})+i\epsilon}$$

$$=\mathrm{Re}\left(\Sigma(E)\right)-\mathrm{i}\frac{qg^2(q)}{8\pi^2E^2}E=\mathrm{Re}\left(\Sigma(E)\right)-\mathrm{i}\Gamma(\mathrm{E})E$$

满足幺正性

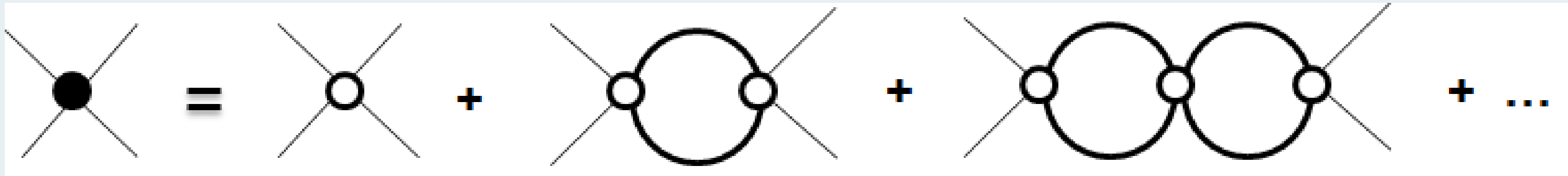
$$S=1+2i\frac{q}{8\pi^2E}T$$

$$|S|=1$$

$$Im(T)=\frac{-q}{8\pi^2E}|T|^2$$

两体幺正耦合道模型

T=V+VGT



单道双共振峰：耦合道 && BW 形式

1维散射方程：$T(p_f,p_i;E)=\sum_n V_n(p_f,p_i;E)+\int\frac{k^2dk}{8\pi^2}\sum_n V_n(p_f,k;E)\frac{1}{\omega_1(\vec{k})\omega_2(\vec{k})}\frac{1}{P_0-\omega_1(\vec{k})-\omega_2(\vec{k})+i\epsilon}T(k,p_i;P)$

$$V_n(p_f,p_i;E)=\frac{g_n(p_f)g_n(p_i)}{E^2-m_n^2}$$

$$T(q,q;E)=(g_1(q),g_2(q))\begin{pmatrix}E^2-m_1^2-\Sigma_{11}(E) & -\Sigma_{12}(E)\\ -\Sigma_{21}(E) & E^2-m_2^2-\Sigma_{22}(E)\end{pmatrix}\begin{pmatrix}g_1(q)\\ g_2(q)\end{pmatrix}$$

满足幺正性 $S=1+2i\frac{q}{8\pi^2E}T$ $|S|=1$ or $Im(T)=\frac{-q}{8\pi^2E}|T|^2$

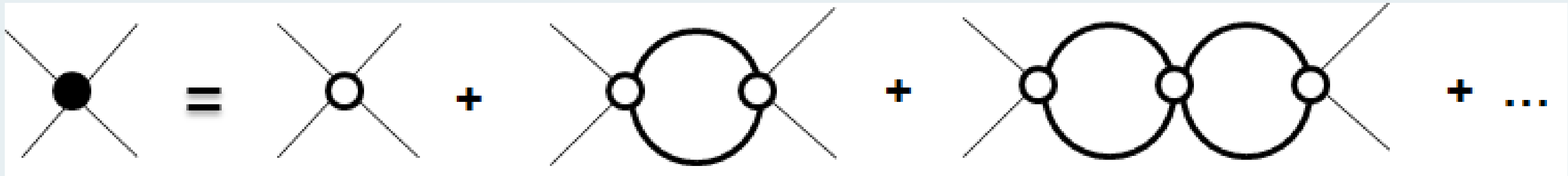
BW 形式 $T(q,q;E)=\frac{g_1^2(q)}{E^2-\tilde{m}_1^2+\mathrm{i}\Gamma_1(\mathrm{E})E}+\frac{g_2^2(q)}{E^2-\tilde{m}_2^2+\mathrm{i}\Gamma_2(\mathrm{E})E}$

不可能满足幺正性，尤其是$\tilde{m}_1^2\sim\tilde{m}_2^2$的时候

单道的时候不满足幺正性其实是一件很奇怪的事情，在这个系统中初态的两个粒子AB散射最后的稳定态一定是AB，但是这个T有表示几率不归一，那么损失的或者多余两粒子去哪里了呢？显然是不物理的。

两体幺正耦合道模型

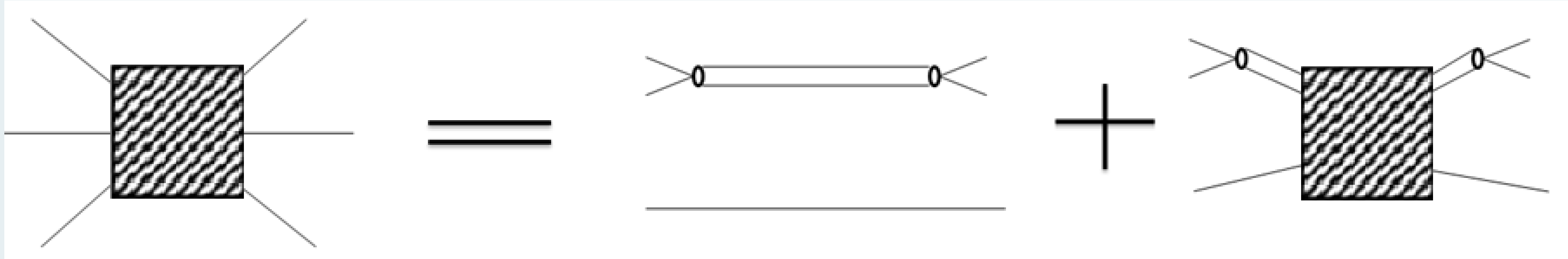
T=V+VGT

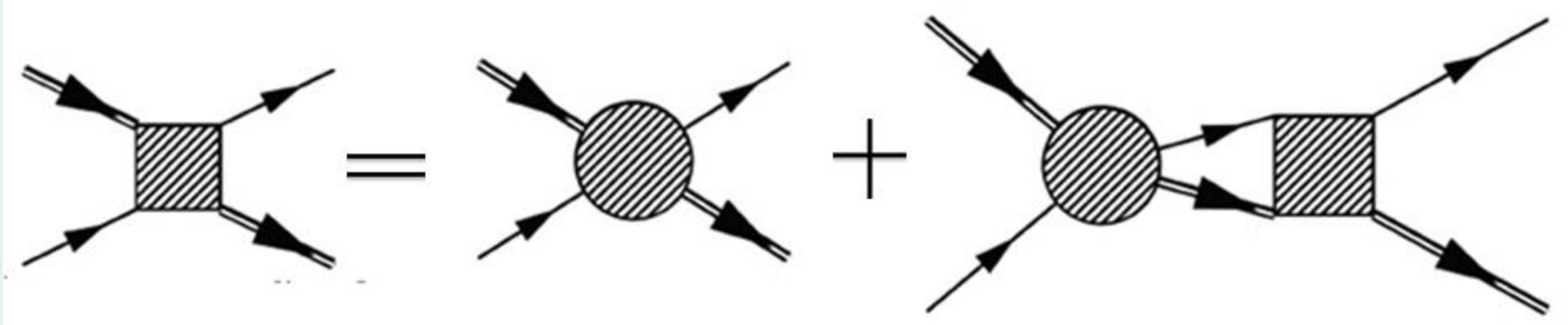


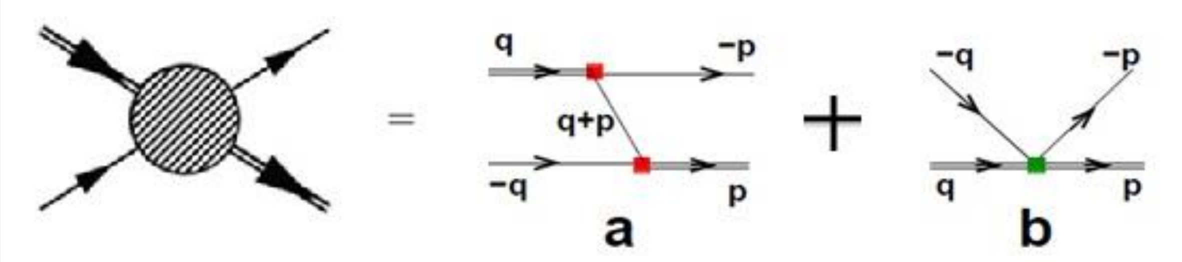
S=1-iT => |S|=1

比较通俗的理解就是在有树图阶顶点时，需要把所有重散射的效应都计算，就能保证幺正性，必然会涉及解散射方程的问题。

三体幺正耦合道模型



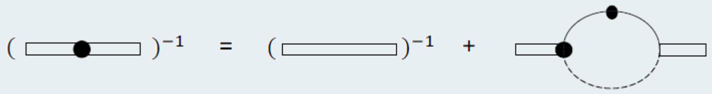




三体幺正耦合道模型

$$f_{ab,R}(q) = g_{ab,R}\frac{(1+q^2/c_{ab,R}^2)^{-2-\frac{L}{2}}}{\sqrt{m_R E_a(q) E_b(q)}}\frac{q^L}{m_\pi^{L-1}}$$

$$F_{(cR)_l,\eta_i^*}(q) = C_{(cR)_l}^i\frac{[1+q^2/(\Lambda_{(cR)_l}^i)^2]^{-2-\frac{l}{2}}}{\sqrt{8E_c(q)E_R(q)\,m_{\eta_i^*}}}\frac{q^l}{m_\pi^{l-1}}$$



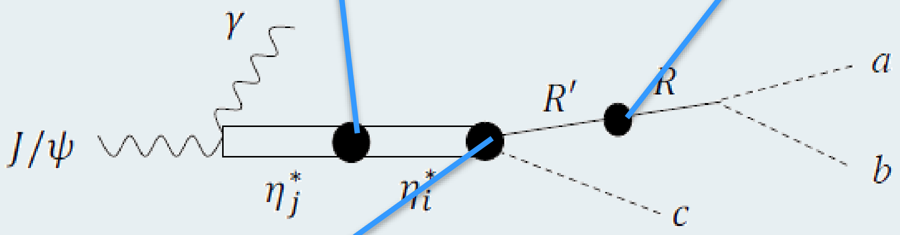
η^*的完整传播子

R的完整传播子

$$\Gamma_{\gamma\eta_j^*,J/\psi} = \frac{g_{J/\psi\eta_j^*\gamma}\,(\boldsymbol{\epsilon}_{J/\psi}\times\boldsymbol{\epsilon}_\gamma)\cdot\boldsymbol{p}_\gamma}{\sqrt{8E_\gamma\,m_{J/\psi}\,m_{\eta_j^*}}}$$

裸顶点

$$\Gamma_{ab,R}(\boldsymbol{q}) = (t_a t_a^z t_b t_b^z|t_R t_R^z)Y_{s_R,s_R^z}(\hat{q})f_{ab,R}(q)$$



$$A_{\gamma abc,J/\psi} = \sum_{abc}^{\text{cyclic}}\sum_{RR's_R^z}\sum_{ij}\Gamma_{ab,R}(\boldsymbol{p}_a^*)\,\tau_{R,R'}(p_c,E-E_c)\,\bar{\Gamma}_{cR',\eta_i^*}(\boldsymbol{p}_c,E)\,\bar{G}_{ij}(E)\,\Gamma_{\gamma\eta_j^*,J/\psi}$$

$$[\tau^{-1}(p,E)]_{R,R'} = [E-E_R(p)]\delta_{R,R'} - \Sigma_{R,R'}(p,E)$$

R的完整传播子

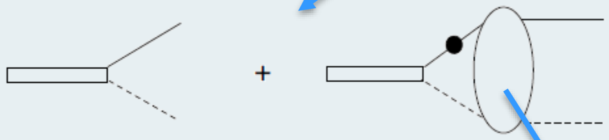
$$[\bar{G}^{-1}(E)]_{ij} = (E-m_{\eta_i^*})\delta_{ij} - [\Sigma_{\eta^*}(E)]_{ij}$$

η^*的完整传播子

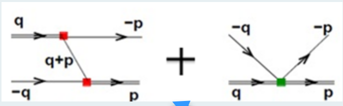
R 的自能

$$\Sigma_{R,R'}(p,E) = \sum_{ab}(t_a t_a^z t_b t_b^z|t_R,t_a^z+t_b^z)(t_a t_a^z t_b t_b^z|t_{R'},t_a^z+t_b^z) \times\sqrt{\frac{m_R m_{R'}}{E_R(p)E_{R'}(p)}}\int q^2dq\frac{M_{ab}(q)}{\sqrt{M_{ab}^2(q)+p^2}} \times\frac{\mathcal{B}_{ab}\,f_{R,ab}(q)f_{ab,R'}(q)}{E-\sqrt{M_{ab}^2(q)+p^2}+i\epsilon}$$

η^*的自能

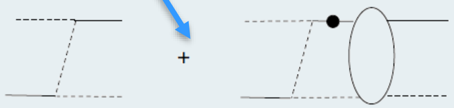


$\eta^* \to Rc$的完全顶点，包括重散射



$$\bar{\Gamma}_{cR,\eta_i^*}(\boldsymbol{p}_c,E) = \sum_{l,l^z}(ll^z s_R s_R^z|s_{\eta^*}s_{\eta^*}^z)(t_R t_R^z t_c t_c^z|t_{\eta^*}t_{\eta^*}^z) \times Y_{l,l^z}(-\hat{p}_c)\bar{F}_{(cR)_l,\eta_i^*}(p_c,E)$$

$$[\Sigma_{\eta^*}(E)]_{ij} = \mathcal{B}_{Rc}\sum_{cRR'l}\int q^2dq\,F_{(cR)_l,\eta_i^*}(q) \times\tau_{R,R'}(q,E-E_c(q))\bar{F}_{(cR')_l,\eta_j^*}(q,E)$$



$$X_{(c'R')_{l'},(cR)_l}(p',p;E) = V_{(c'R')_{l'},(cR)_l}(p',p;E) + \sum_{c'',R'',R''',l''}\int q^2dq\,V_{(c'R')_{l'},(c''R''')_{l''}}(p',q;E) \times\tau_{R''',R''}(q,E)X_{(c''R'')_{l''},(cR)_l}(q,p;E)\ ,$$

$$\bar{F}_{(cR)_l,\eta_i^*}(p_c,E) = F_{(cR)_l,\eta_i^*}(p_c) + \sum_{c'R'R''l'}\int q^2dq \times X_{(cR)_l,(c'R'')_{l'}}(p_c,q;E) \times\tau_{R'',R'}(q,E-E_{c'})F_{(c'R')_{l'},\eta_i^*}(q)$$

$Rc \to R'c'$的完整散射振幅，需要解方程

三体幺正耦合道模型

$$f_{ab,R}(q) = g_{ab,R}\frac{(1+q^2/c^2_{ab,R})^{-2-\frac{L}{2}}}{\sqrt{m_R E_a(q) E_b(q)}}\frac{q^L}{m_\pi^{L-1}}$$

$$F_{(cR)_l,\eta_i^*}(q) = C^i_{(cR)_l}\frac{[1+q^2/(\Lambda^i_{(cR)_l})^2]^{-2-\frac{l}{2}}}{\sqrt{8E_c(q)E_R(q)\,m_{\eta_i^*}}}\frac{q^l}{m_\pi^{l-1}}$$

$$\Gamma_{\gamma\eta_j^*,J/\psi} = \frac{g_{J/\psi\eta_j^*\gamma}\,(\boldsymbol{\epsilon}_{J/\psi}\times\boldsymbol{\epsilon}_\gamma)\cdot\boldsymbol{p}_\gamma}{\sqrt{8E_\gamma\, m_{J/\psi}\, m_{\eta_j^*}}}$$

裸顶点

只和两体相关的都可以通过两体散射确定，拟合只要针对$\eta^* \to Rc$和$Rc \to R'c'$中的非Z图贡献。

$Rc \ = K^*(892)\overline{K}, \kappa\overline{K}, a0(980)\pi,$
$a2(1320)\pi, f_0\eta, \rho\rho, f_0\pi$

我们还考虑了下面两个比值来限制参数：

$$R_{exp1} = \frac{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(KK\pi^-)]}{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(\eta\pi+\pi-)]} \sim 6.8 - 11.9$$

$$R_{exp2} = \frac{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(\rho 0\gamma)]}{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(KK\pi^-)]} \sim 0.015 - 0.043$$

$$\Gamma_{ab,R}(\boldsymbol{q}) = (t_a t_a^z t_b t_b^z|t_R t_R^z)Y_{s_R,s_R^z}(\hat{q})f_{ab,R}(q)$$

$$A_{\gamma abc,J/\psi} = \sum_{abc}^{\text{cyclic}}\sum_{RR's_R^z}\sum_{ij}\Gamma_{ab,R}(\boldsymbol{p}_a^*)\,\tau_{R,R'}(p_c,E-E_c)\,\bar{\Gamma}_{cR',\eta_i^*}(\boldsymbol{p}_c,E)\,\bar{G}_{ij}(E)\,\Gamma_{\gamma\eta_j^*,J/\psi}$$

$$[\tau^{-1}(p,E)]_{R,R'} = [E-E_R(p)]\delta_{R,R'} - \Sigma_{R,R'}(p,E)$$

R的完整传播子

$$[\bar{G}^{-1}(E)]_{ij} = (E-m_{\eta_i^*})\delta_{ij} - [\Sigma_{\eta^*}(E)]_{ij}$$

η^*的完整传播子

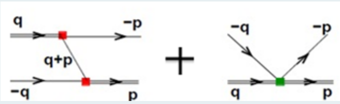
R 的自能

$$\Sigma_{R,R'}(p,E) = \sum_{ab}(t_a t_a^z t_b t_b^z|t_R,t_a^z+t_b^z)(t_a t_a^z t_b t_b^z|t_{R'},t_a^z+t_b^z) \times\sqrt{\frac{m_R m_{R'}}{E_R(p)E_{R'}(p)}}\int q^2dq\frac{M_{ab}(q)}{\sqrt{M^2_{ab}(q)+p^2}} \times\frac{\mathcal{B}_{ab}\,f_{R,ab}(q)f_{ab,R'}(q)}{E-\sqrt{M^2_{ab}(q)+p^2}+i\epsilon}$$

η^*的自能

$$\bar{\Gamma}_{cR,\eta_i^*}(\boldsymbol{p}_c,E) = \sum_{l,l^z}(ll^z s_R s_R^z|s_{\eta^*}s^z_{\eta^*})(t_R t_R^z t_c t_c^z|t_{\eta^*}t^z_{\eta^*}) \times Y_{l,l^z}(-\hat{p}_c)\bar{F}_{(cR)_l,\eta_i^*}(p_c,E)$$

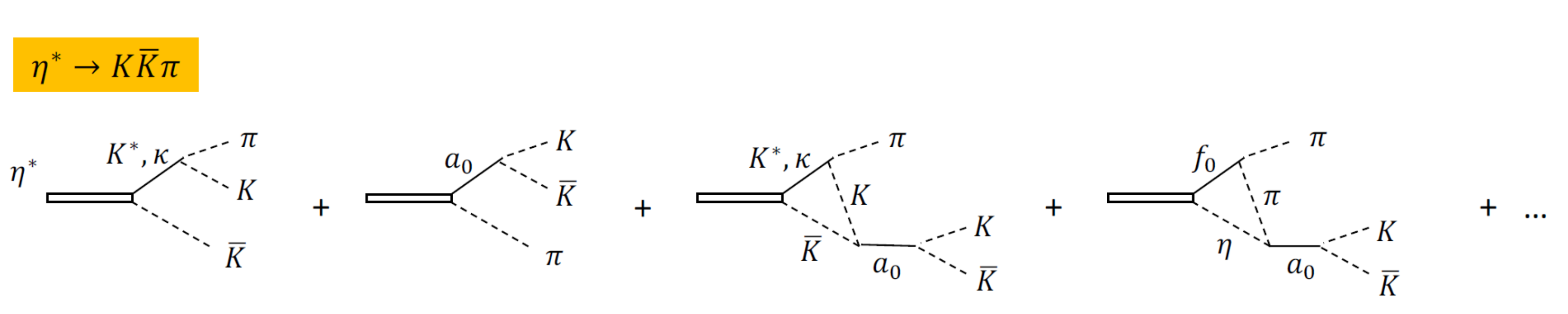
$$[\Sigma_{\eta^*}(E)]_{ij} = \mathcal{B}_{Rc}\sum_{cRR'l}\int q^2dq\,F_{(cR)_l,\eta_i^*}(q) \times\tau_{R,R'}(q,E-E_c(q))\bar{F}_{(cR')_l,\eta_j^*}(q,E)$$

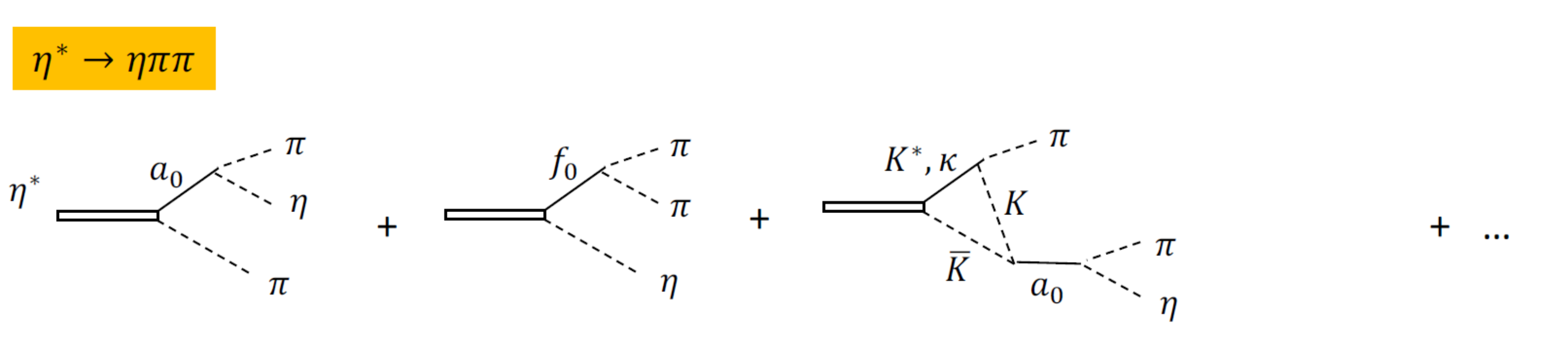


$$X_{(c'R')_{l'},(cR)_l}(p',p;E) = V_{(c'R')_{l'},(cR)_l}(p',p;E) + \sum_{c'',R'',R''',l''}\int q^2dq\,V_{(c'R')_{l'},(c''R''')_{l''}}(p',q;E) \times\tau_{R''',R''}(q,E)X_{(c''R'')_{l''},(cR)_l}(q,p;E)\ ,$$

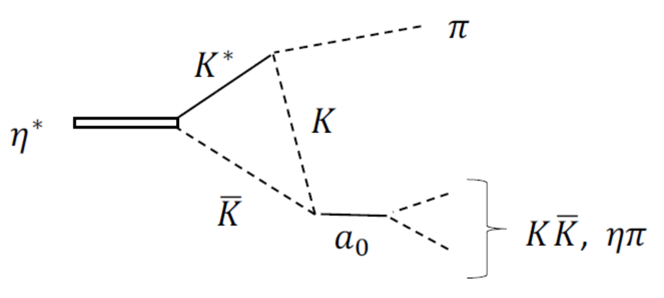
$$\bar{F}_{(cR)_l,\eta_i^*}(p_c,E) = F_{(cR)_l,\eta_i^*}(p_c) + \sum_{c'R'R''l'}\int q^2dq \times X_{(cR)_l,(c'R'')_{l'}}(p_c,q;E) \times\tau_{R'',R'}(q,E-E_{c'})F_{(c'R')_{l'},\eta_i^*}(q)$$

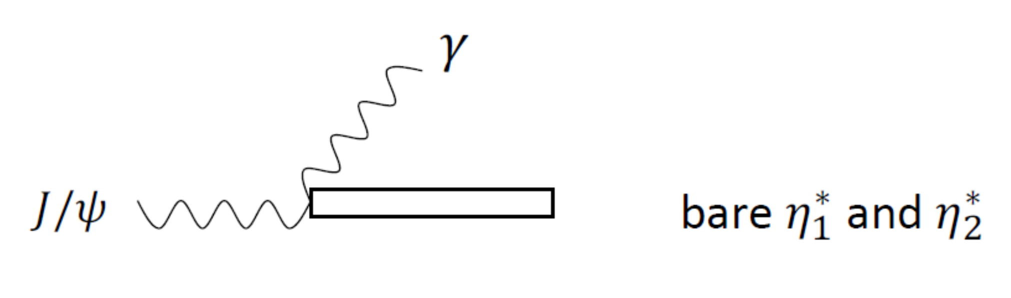
三体幺正耦合道模型



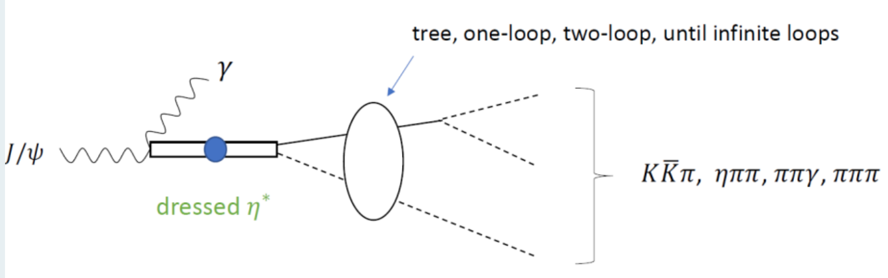


单圈的三角奇点贡献自动包含在该模型中

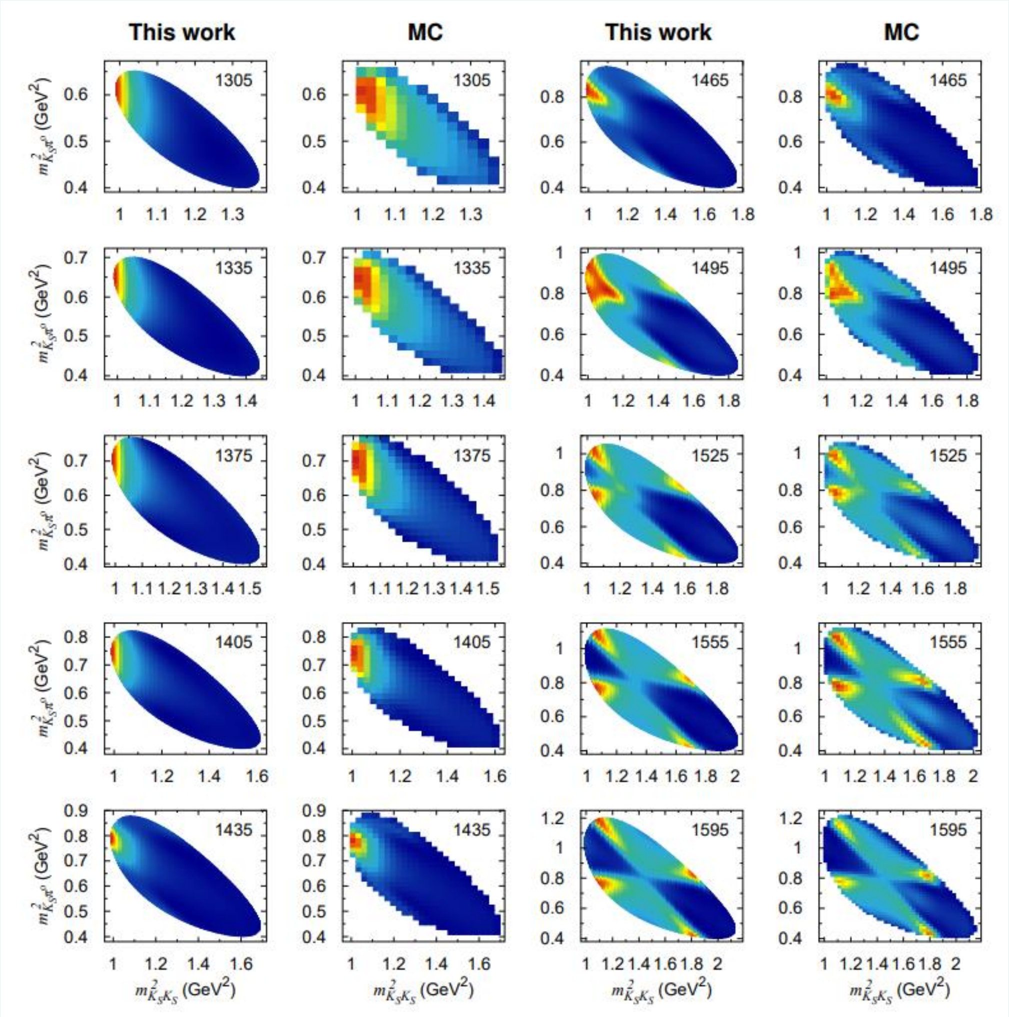




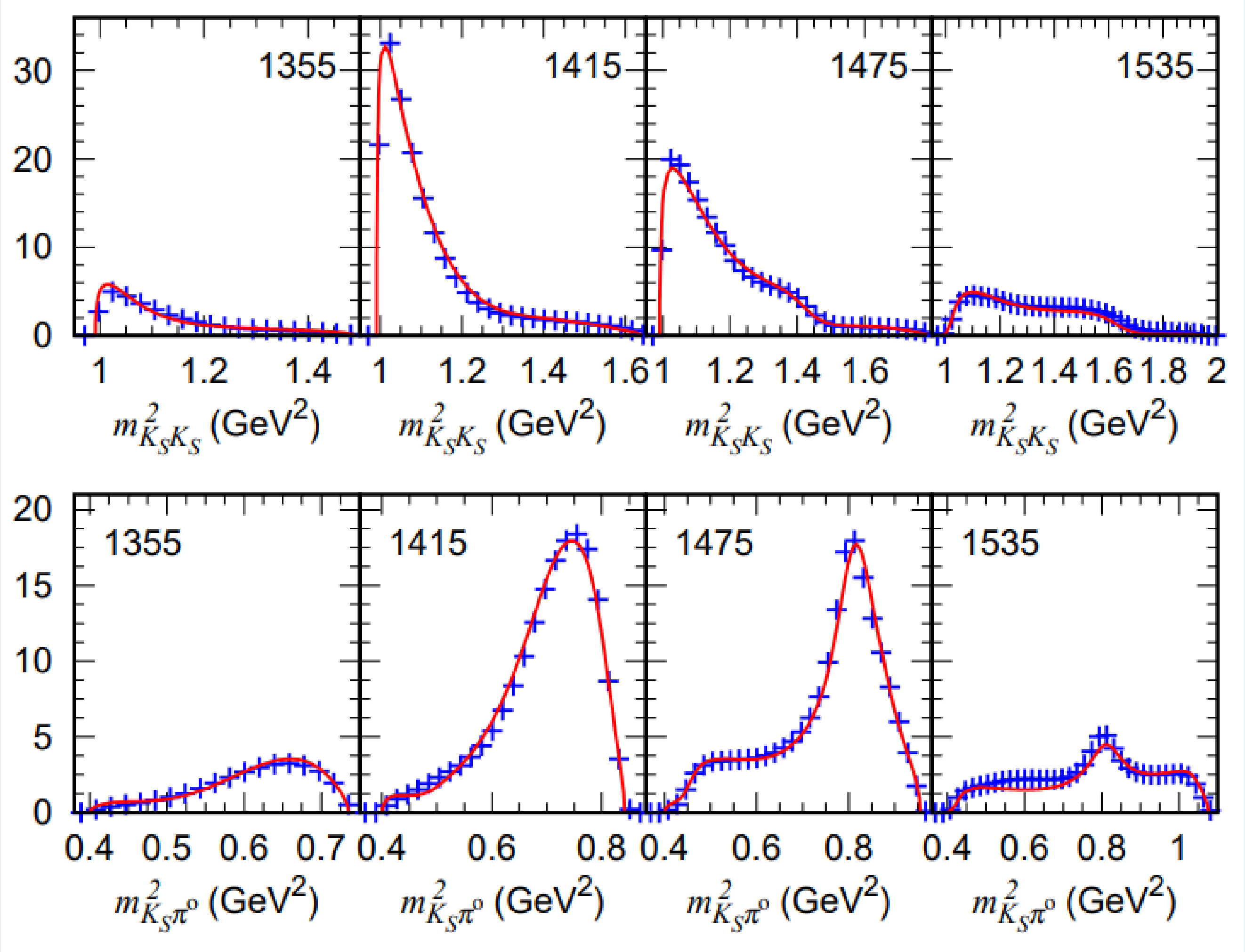
唯2不受耦合道约束的顶点是电磁作用的产生顶点。



达利兹图拟合结果

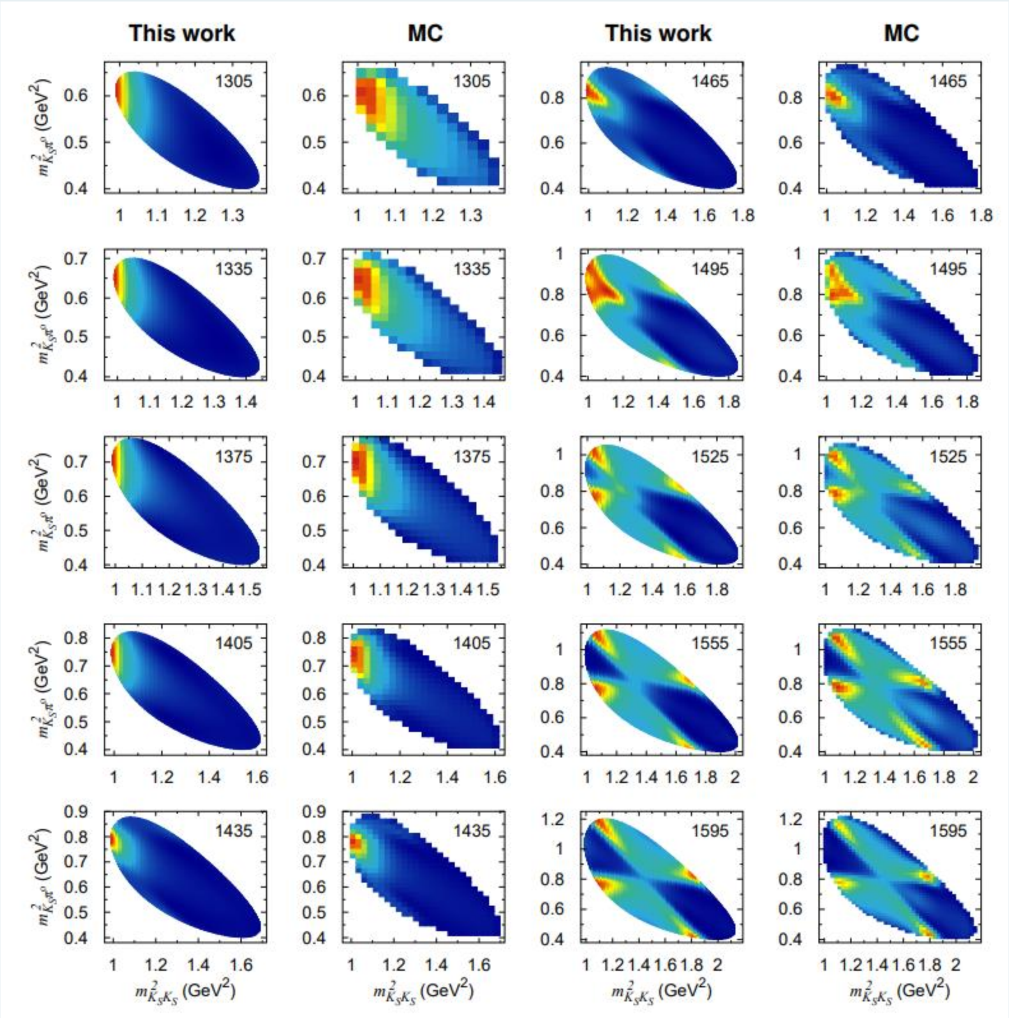


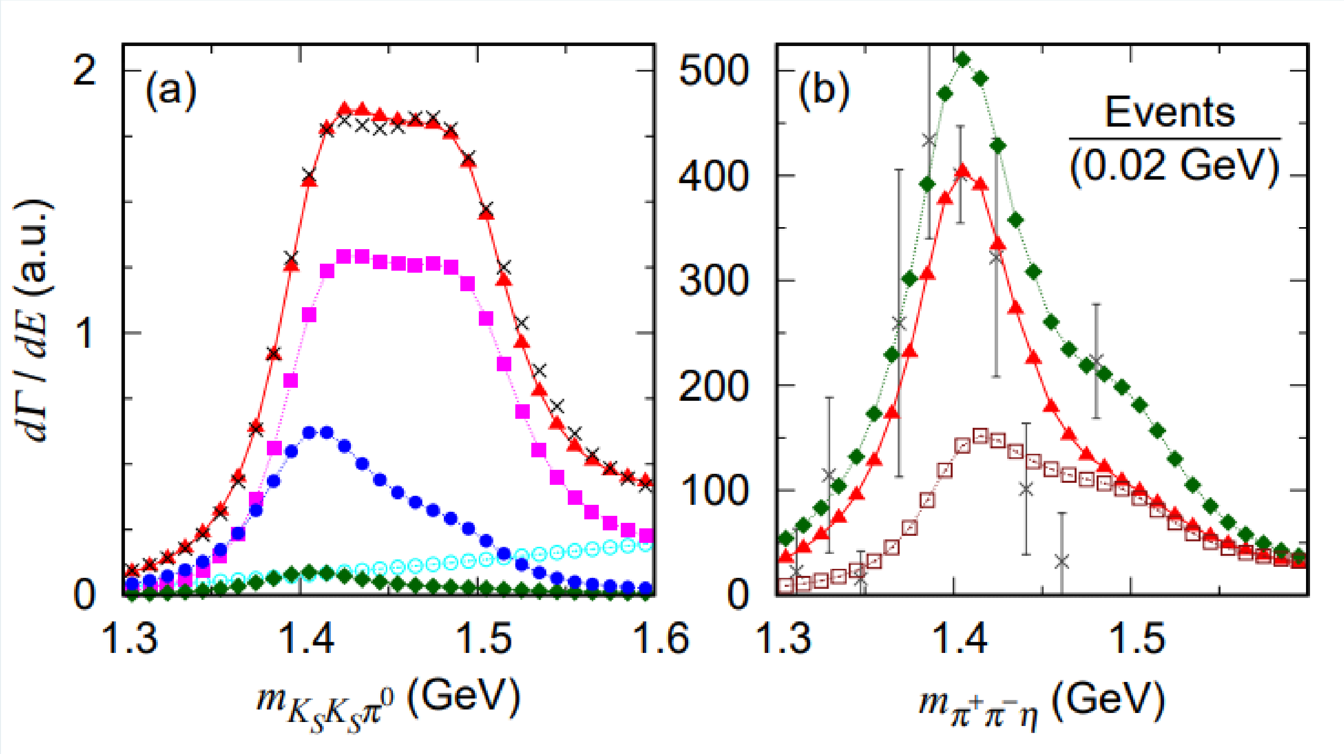
This work
MC
This work
MC
1305
1335
1375
1405
1435
1465
1495
1525
1555
1595
$m^2_{K_S\pi^0}$ (GeV^2)
$m^2_{K_SK_S}$ (GeV^2)



1355
1415
1475
1535
$m^2_{K_SK_S}$ (GeV^2)
$m^2_{K_S\pi^0}$ (GeV^2)

达利兹图拟合结果



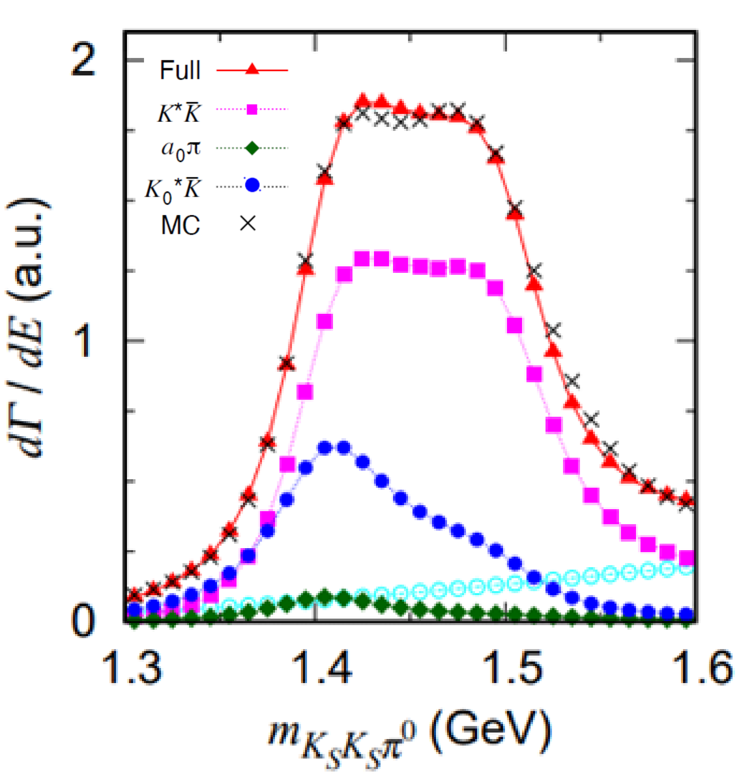


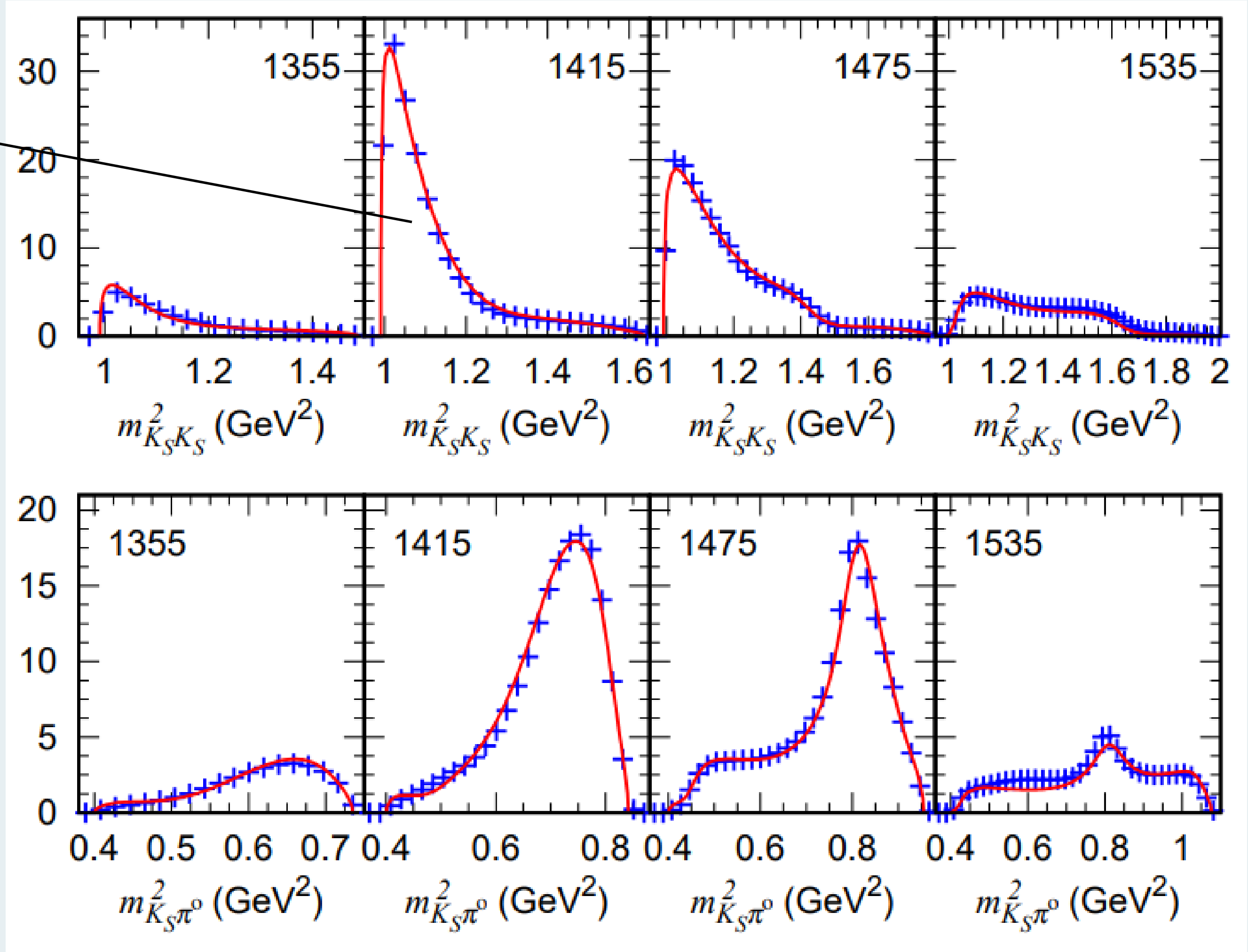
$$R_{exp1} = \frac{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(KK\pi^-)]}{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(\eta\pi+\pi-)]} \sim 6.8 - 11.9 \qquad 6.9$$

$$R_{exp2} = \frac{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(\rho\ 0\gamma)]}{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(KK\pi^-)]} \sim 0.015 - 0.043 \qquad 0.026$$

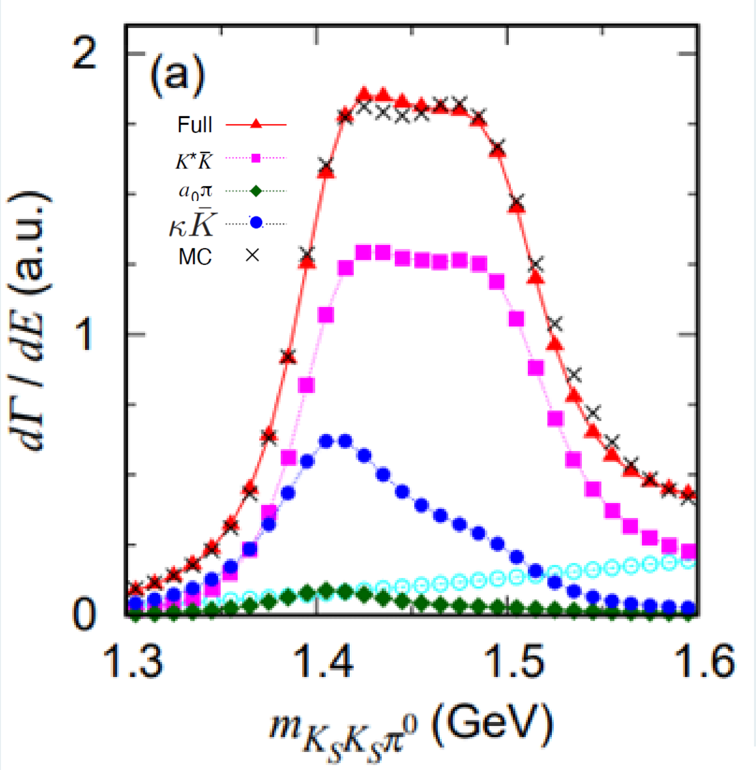
理论分析

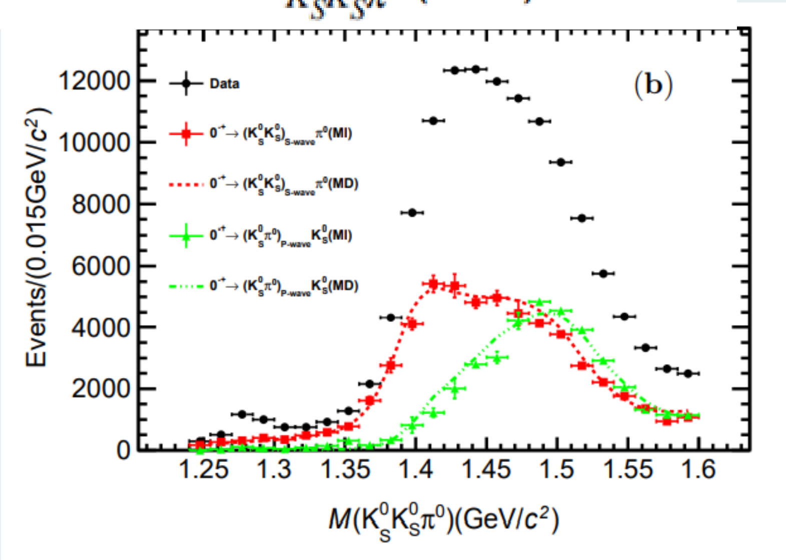
这个峰来自$\bar{K}K^*$的反射，而不是$a_0\pi$





理论分析



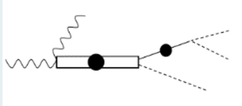
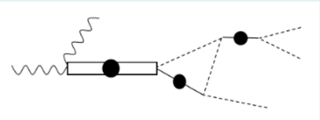


和BW的结果不同，虽然达利兹图相同，但是内部的解释是不一样的。

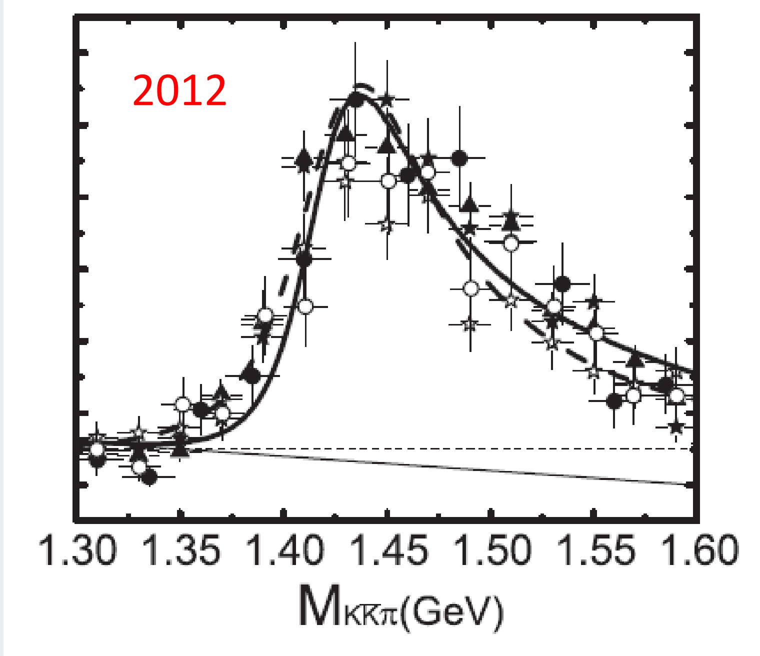
1. 包含了$\kappa\bar{K}$的贡献。
2. 同时拟合了R_{exp1}，这个比值告诉我们$a_0\pi$的贡献不能很大。这也是为什么$\kappa\bar{K}$的贡献比较重要的原因。
3. 包括了耦合道效应。

从这个拟合的结论可知，

data

1. 平台结构主要来自于$\bar{K}K^*$的贡献。
2. $\bar{K}K^*$的贡献主要来自直接产生。
3. $a_0\pi$和$\kappa\bar{K}$的主要来自单圈图中的三角奇异性贡献。
4. 需要两个裸态η^*，质量为1.6 GeV和2.0 GeV以上。

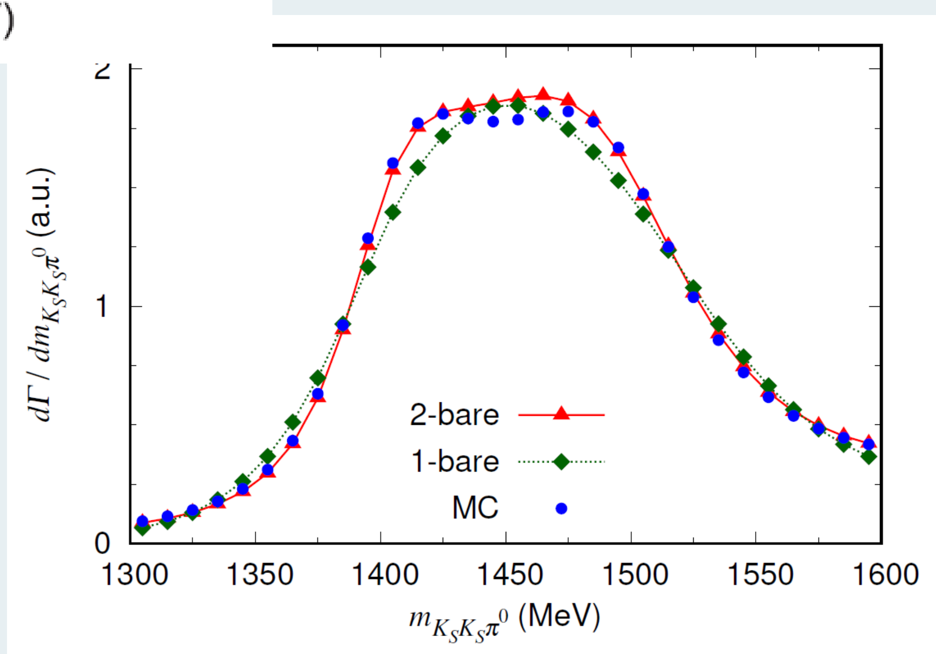
理论分析

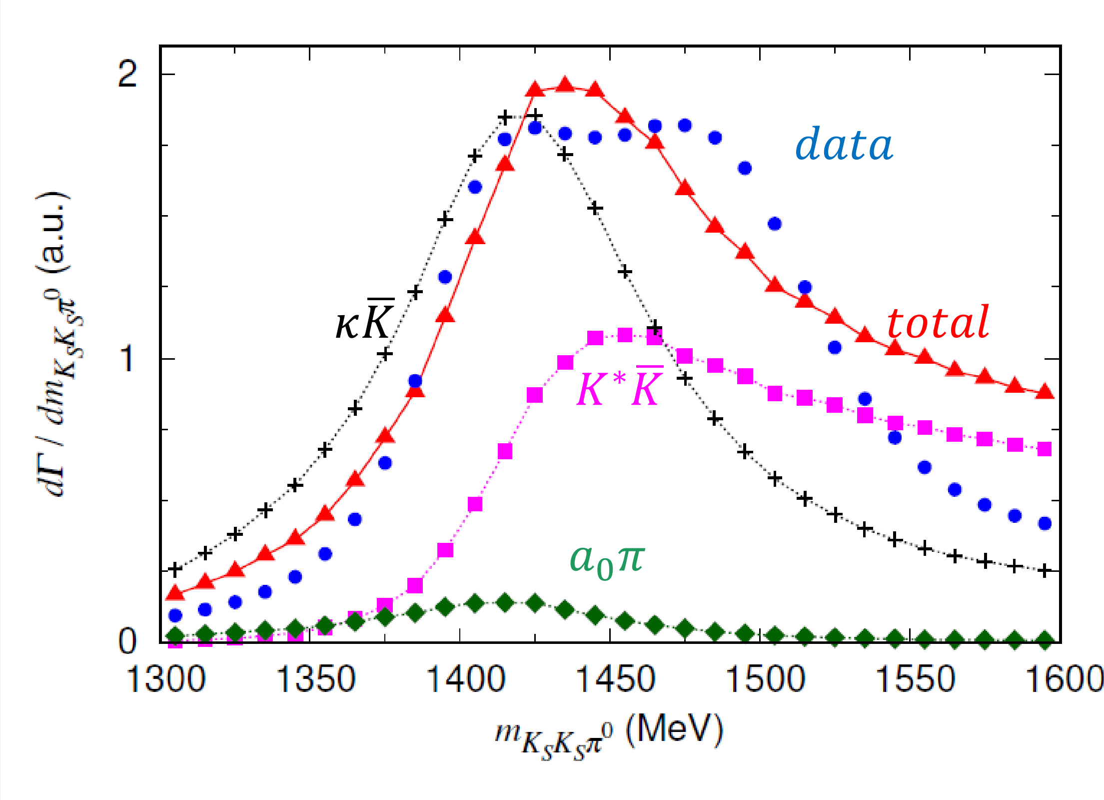


从这个拟合的结论可知，

1. 平台结构主要来自于$\bar{K}K^*$的贡献。
2. $\bar{K}K^*$的贡献主要来自直接产生。
3. $a_0\pi$和$\kappa\bar{K}$的主要来自单圈图中的三角奇异性贡献。
4. 需要两个裸态η^*。

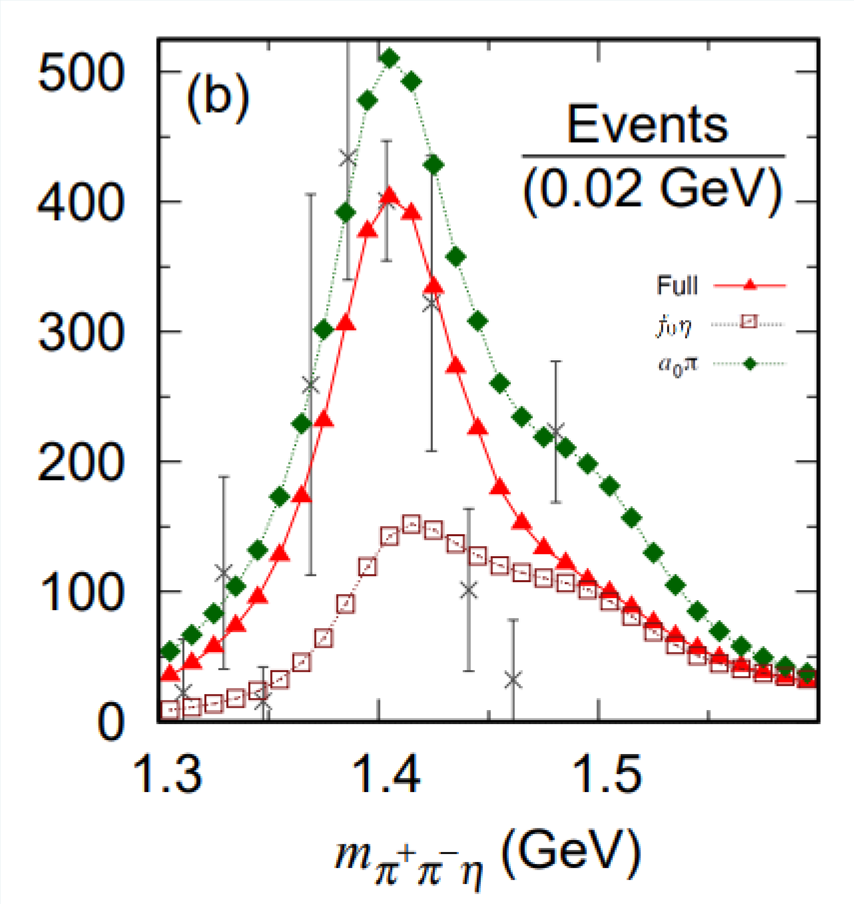
如果只有一个裸态，即使我们不拟合达利兹图，**就拟合不变质量谱**，也无法出现100 MeV 宽的平台结构。所以我们相信要得到100 MeV的宽峰应该做不到。



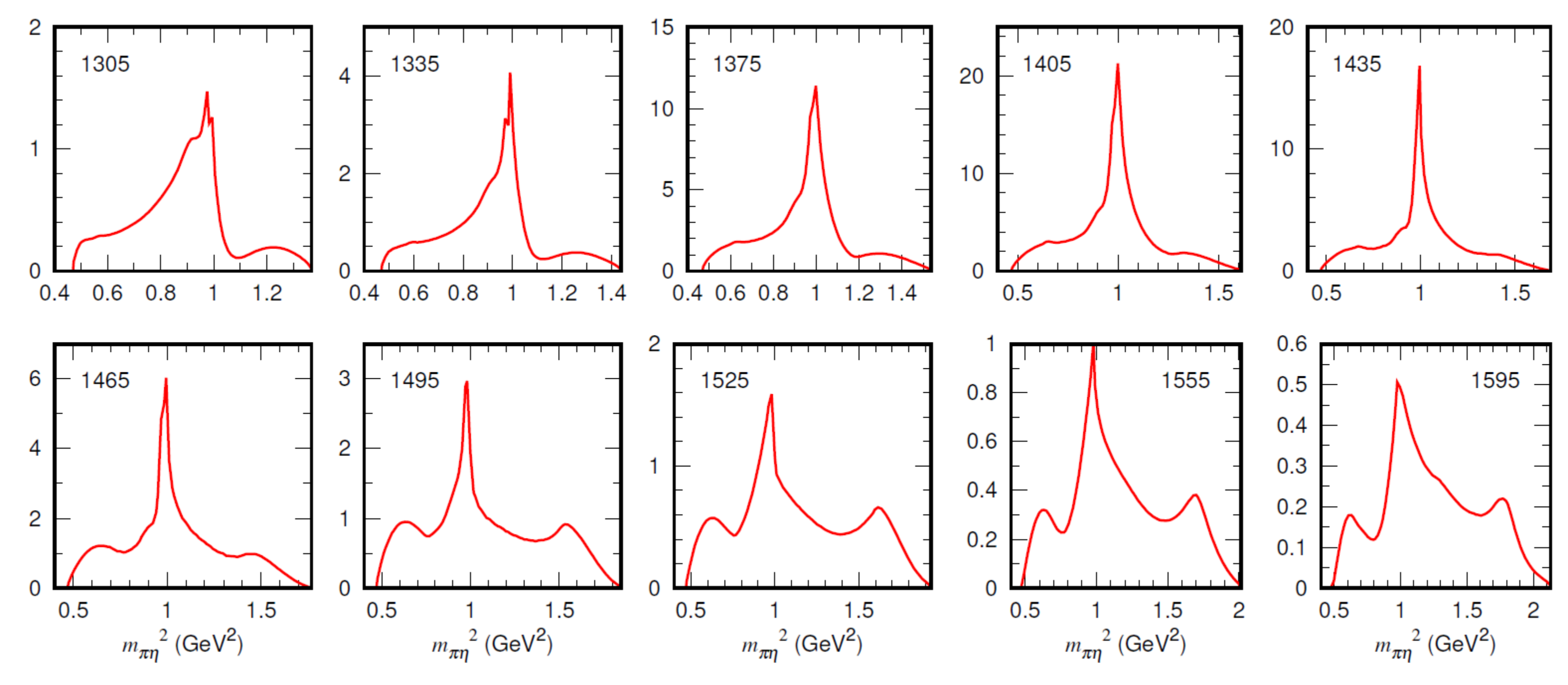


理论分析

下面我们利用得到的模型post预言了$\eta(1405/1475) \to \pi\pi\eta$ 和 $\pi\pi\pi$的质量谱。



主要贡献来自于$a_0\pi$



理论分析

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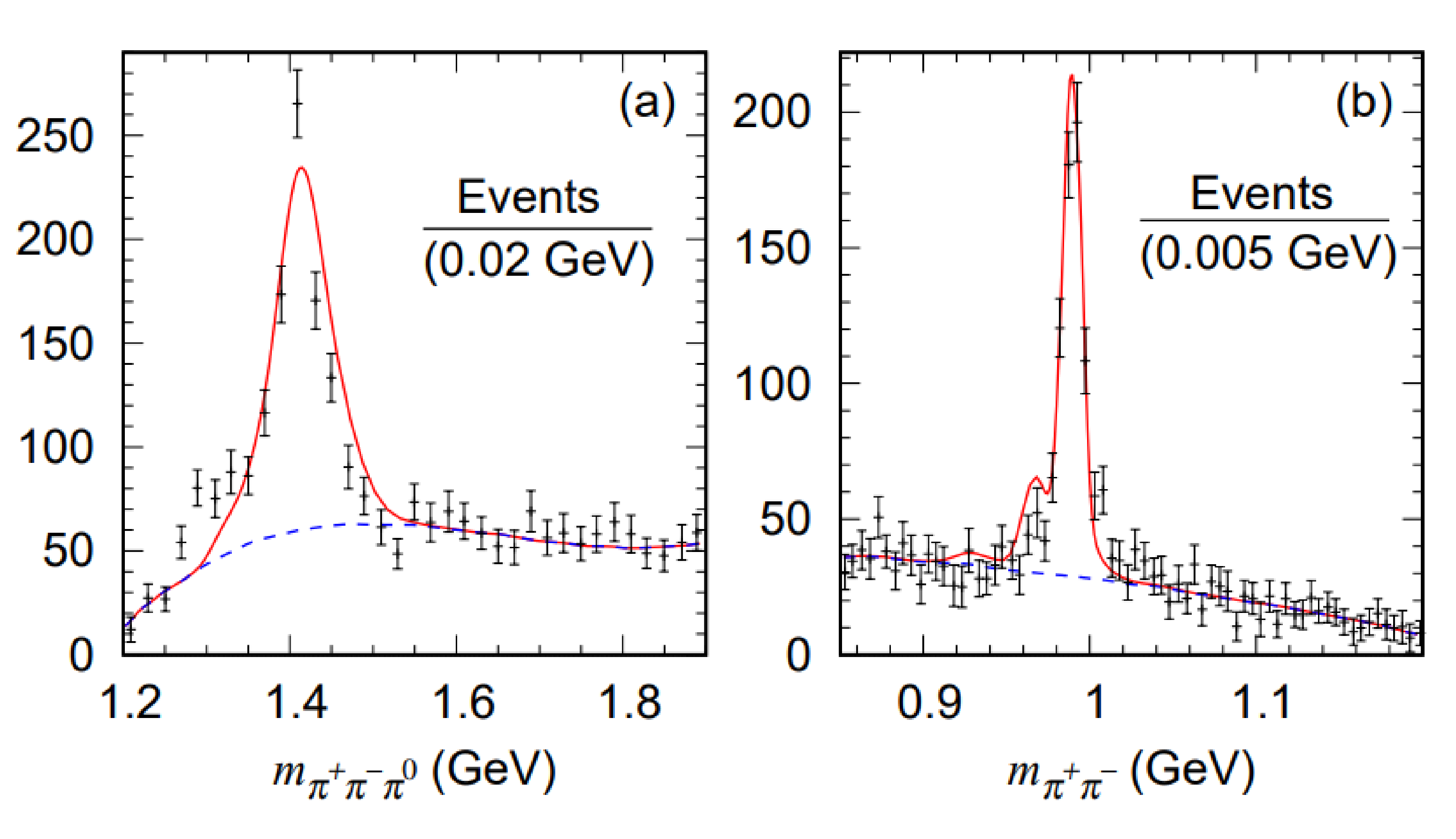
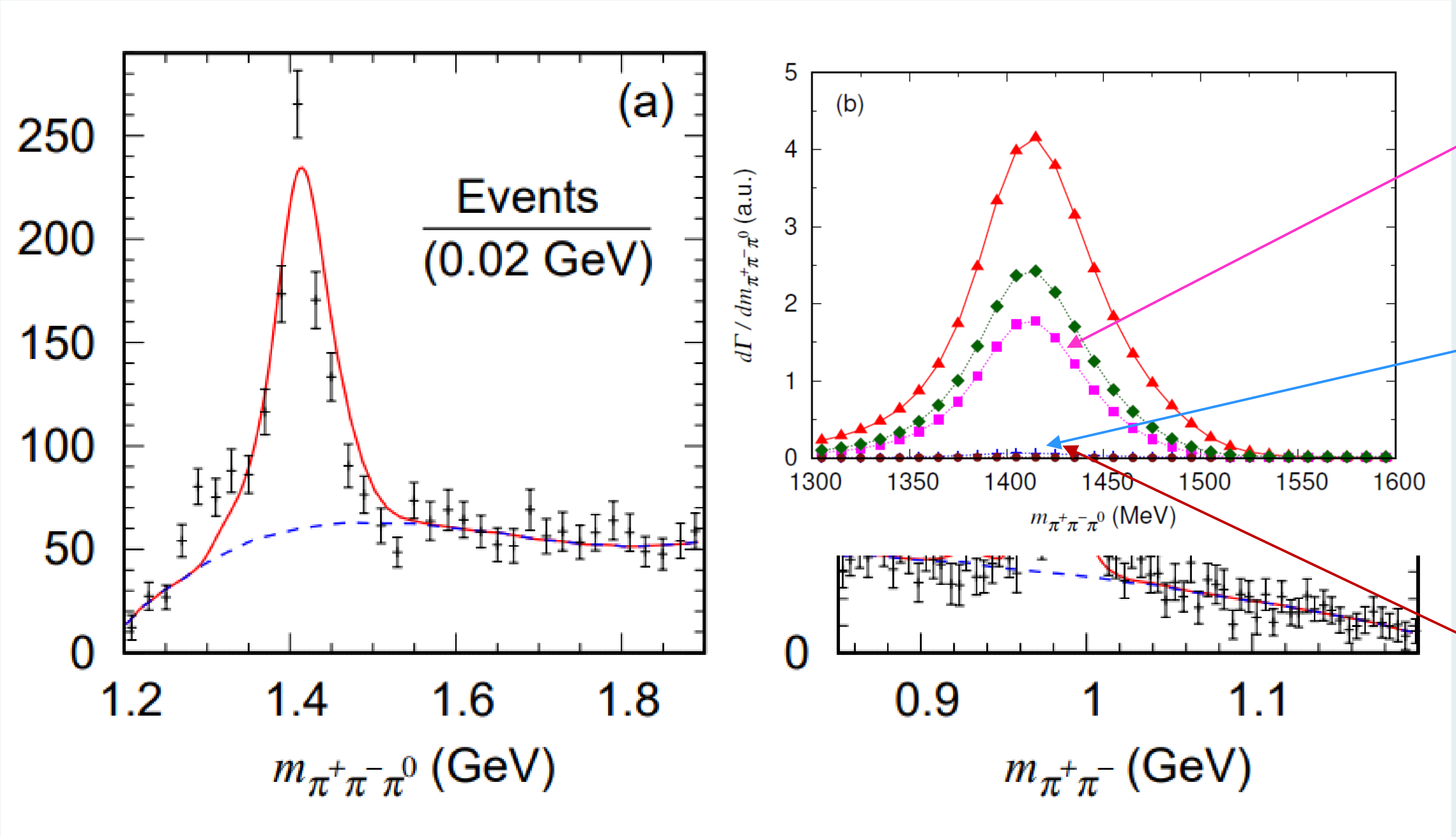
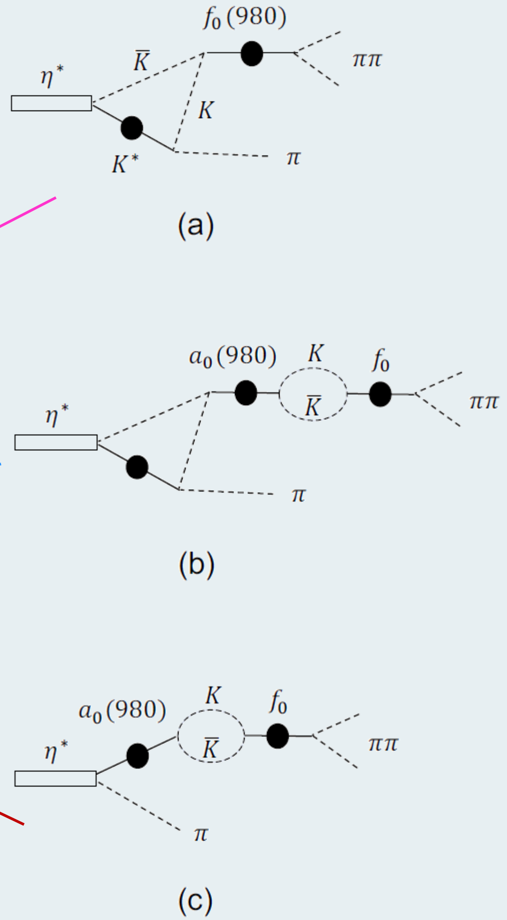


理论分析

下面我们利用得到的模型post预言了$\eta(1405/1475) \to \pi\pi\eta$ 和 $\pi\pi\pi$的质量谱。

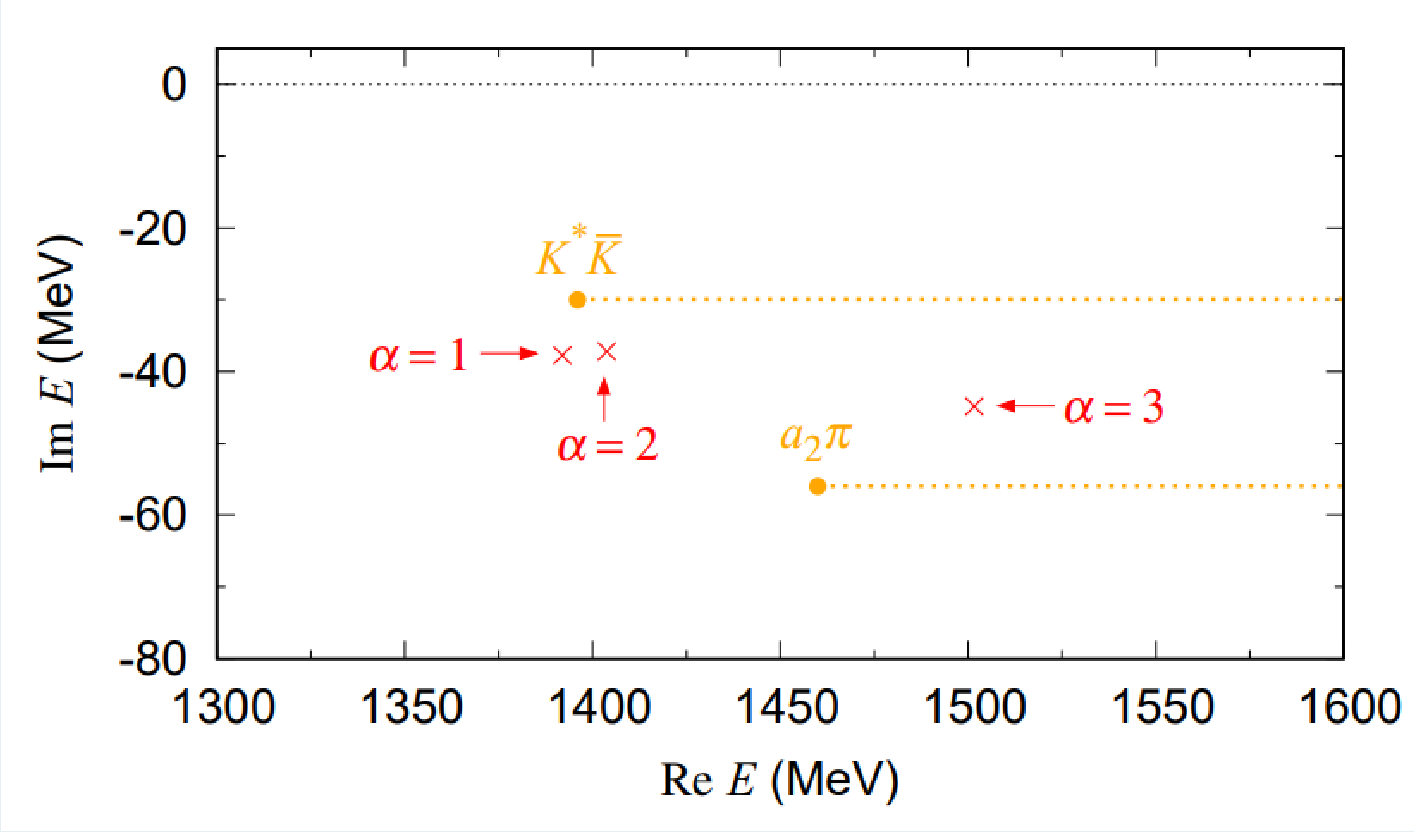




主要贡献来自于三角奇异性

理论分析

最后我们利用得到的模型可以计算$\eta(1405/1475)$的极点位置。



	$\alpha=1$	$\alpha=2$	$\alpha=3$	BESIII [27]	
M	1392	1404	1502	1391.7±0.7	1507.6±1.6
Γ	76	74	90	60.8±1.2	115.8±2.4
RS	(pp)	(up)	(up)		

两个裸态的意义：实际物理观测到的共振态是夸克胶子层次相互作用和强子层次相互作用共同作用的结果，而一般的夸克模型并没有包含强子层次的相互作用，因而通过耦合道相互作用可以极大的改变质量。

1.6GeV的η裸态比较像 $q\bar{q}$ 模型的径向激发态，而2.0 GeV以上的态应该可以有多种解释，胶球，混杂态， $q\bar{q}$高激发态， $qq\overline{qq}$态都有可能，如果要确定这些，我们需要更多的实验数据。

小结和展望

在现有的实验MC数据下和三体幺正模型下，

$\eta(1405/1475)$的两个突出的问题：

一个 或者 两个？ 应该是两个

究竟是什么？ 暂时还不知道，在考察了强子层次的耦合道相互作用以后，高激发态可以移动到低能处。

我们需要：更多的实验数据。

迫切期待高统计量的$J/\psi \to \gamma\eta(1405/1475) \to \gamma\pi\pi\eta$数据；

更多的$X \to Y\ \eta(1405/1475) \to Y(K\bar{K}\pi, \pi\pi\eta, \gamma\pi\pi, \pi\pi\pi)$等数据。

期待耦合道模型能够应用于实验分析。

谢谢!