A method to classify molecule type hadron

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强子非微扰质量的起源
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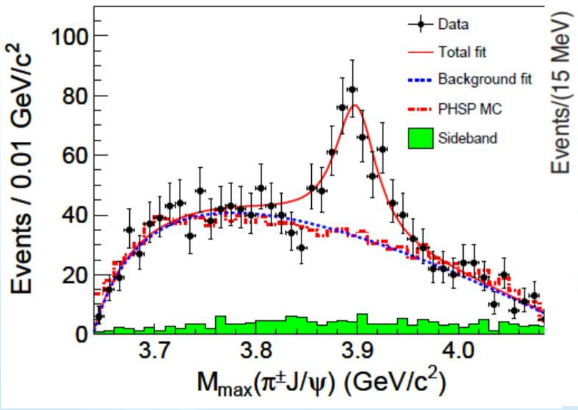
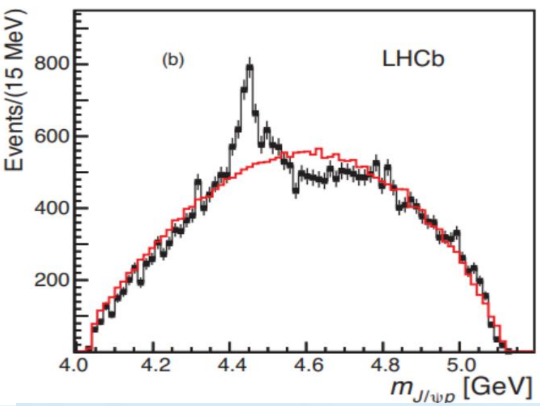
Outlines

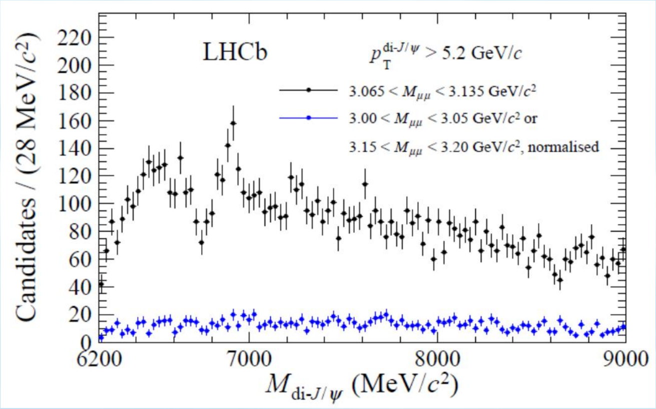
1. Introduction
2. Some old fashioned methods
3. A new method: scalars
4. Summary

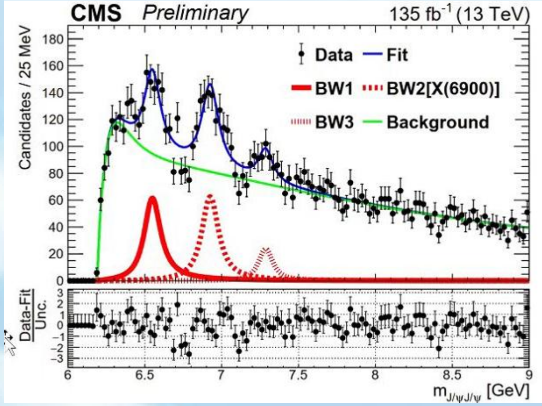
Introduction

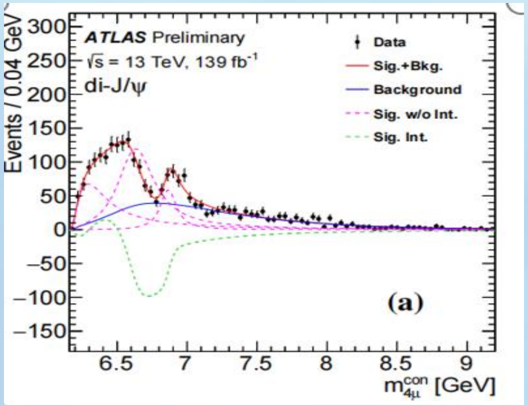
- $Z_c(3900)$ by BESIII and Belle, in 2013
- P_c states by LHCb, in2015
- T_{cc}, X(6900)....
- Their nature?
  - Quantum number?
  - The inner structure?
  - Hadronic molecules?







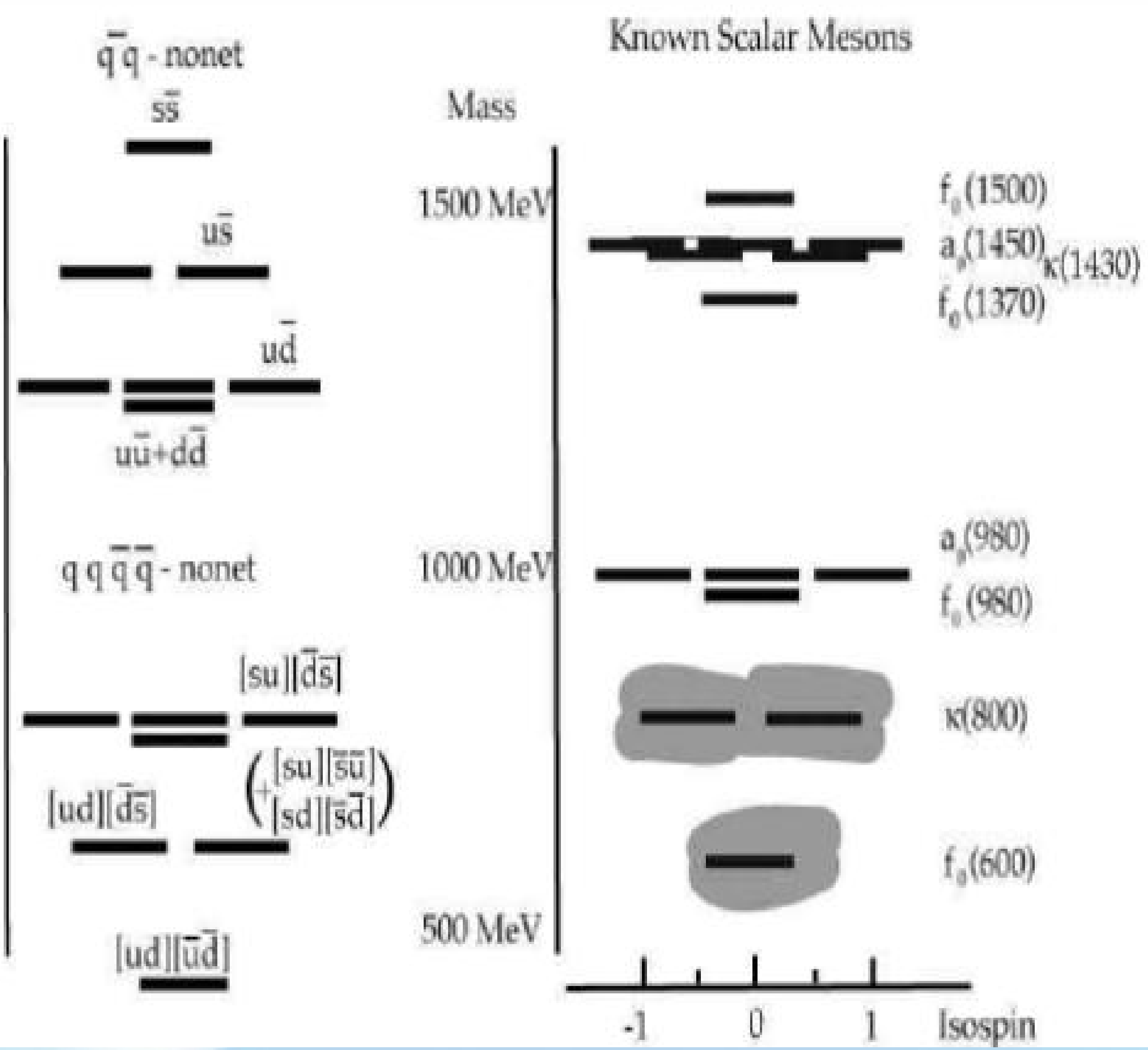




scalars

- The scalars and tensors appear in such processes are rather interesting.
- What is scalar? The same quantum number with QCD vacuum.

Jaffe
Phys. Rept. 409 (2005) 1



scalars&&mass of hadrons?

- Linear σ model: σ could be important for hadron getting mass

$$\mathcal{L} = \mathcal{L}_s + c\sigma ,$$
$$\mathcal{L}_s = \frac{1}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\pi)^2] - \frac{m^2}{2}[\sigma^2 + \pi^2] - \frac{\lambda}{4}[\sigma^2 + \pi^2]^2$$

$$\sigma = s + v \qquad {m_\pi}^2 = m^2 + \lambda v^2$$

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu\pi)^2 - {m_\pi}^2\pi^2] + \frac{1}{2}[(\partial_\mu s)^2 - {m_\sigma}^2 s^2] - \lambda v s(s^2 + \pi^2) - \frac{\lambda}{4}[s^2 + \pi^2]^2$$
$$+ s(c - m^2 v - \lambda v^3)$$

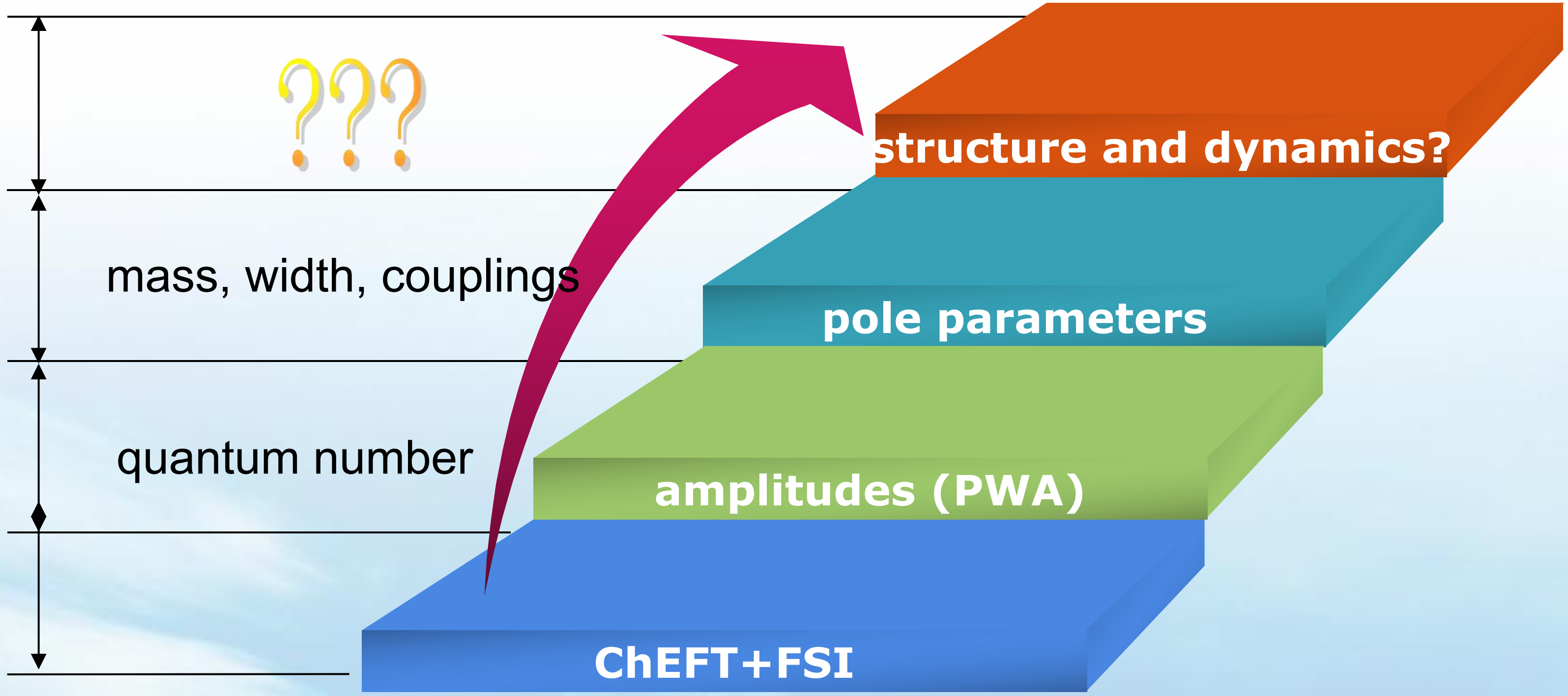
- σ v.s. Higgs?

Low energy strong interactions	Higgs sector of electroweak theory
$\Phi = \begin{pmatrix} i\pi^+ \\ \frac{\sigma - i\pi^0}{\sqrt{2}} \end{pmatrix}$ mesons	$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ Higgs
$\langle\sigma\rangle = \frac{F_\pi}{\sqrt{2}} = 93\,\text{MeV}$	$\langle\sigma\rangle = v = 246\,\text{GeV}$

Abdel-Rehim, *et.al.* AIP Conference Proceedings 688, 99–109 (2003)

Inner structure?

- Inner structure of a resonance?



2、Some old-fashioned methods

- Wave renormalization factor.

$$1-Z=\int dE\sum_n \frac{|\langle E,n|V|\mathfrak{B}\rangle|^2}{E+B}$$

Weinbreg, PR130 (1963) 776
See also Guo's talk

- Z=1 elementary, Z=0, composite
- Applied to light scalars

Baru, et.al.,
Phys.Lett.B 586 (2004) 53-61

- Define states?

Pole counting

- Pole counting rule: distinguish molecule and BW resonance. Shadow poles in different Riemann sheets.
- At the very beginning, applied to light mesons

Morgan NPA543 (1992) 632.
Dai,Wang and Zheng
CTP57 (2012) 841, CTP58 (2012) 410

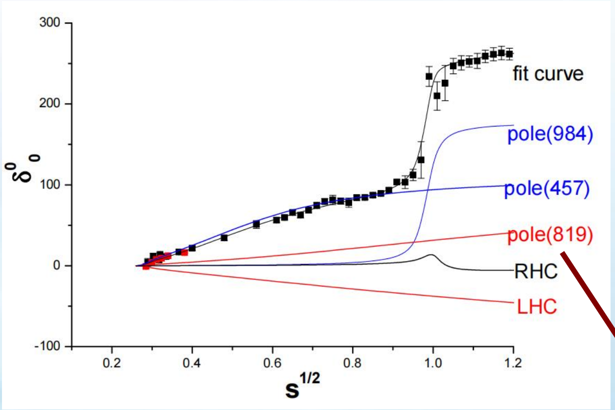
- Applied to heavier mesons

Dai, Sun, Kang, Szczepaniak, Yu,
PRD 105 (2022) 5, L051507
Kuang,**Dai**,Kang,Yao,
EPJC 80 (2020) 5, 433
Dai, Shi, Tang, Zheng,
PRD 92 (2015) 1, 014020

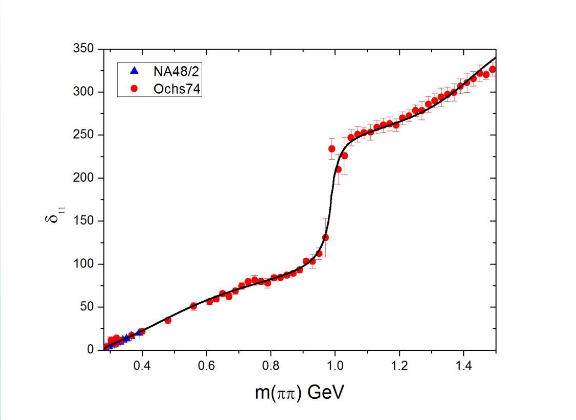
- Any tools else?

phase shifts?

- Phase shifts help to study hadronic scatterings and resonances therein
- Successful in study of light scalars: PKU,Roy equation, Omnes representations, etc.



Zhou, Qin, Zhang, Xiao, Zheng,JHEP 02 (2005)



Dai,Pennington, PLB736(2014)11; PRD90 (2014) 036004;

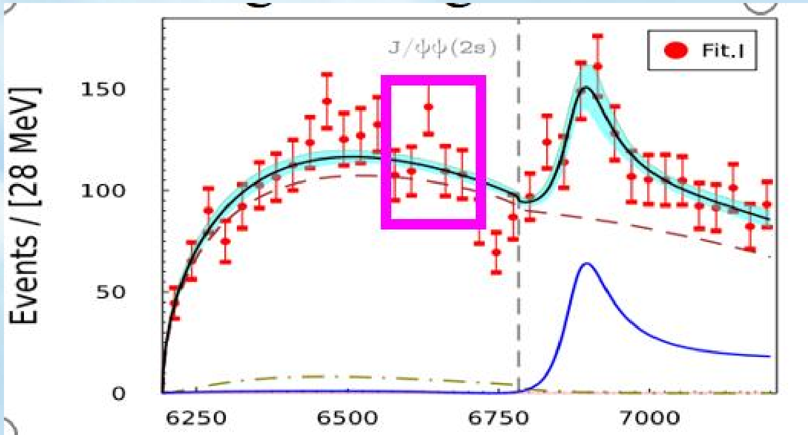
- The phase shifts of a narrow BW resonance rise from 0 to 180 degrees, crossing 90 degrees at Ecm=M

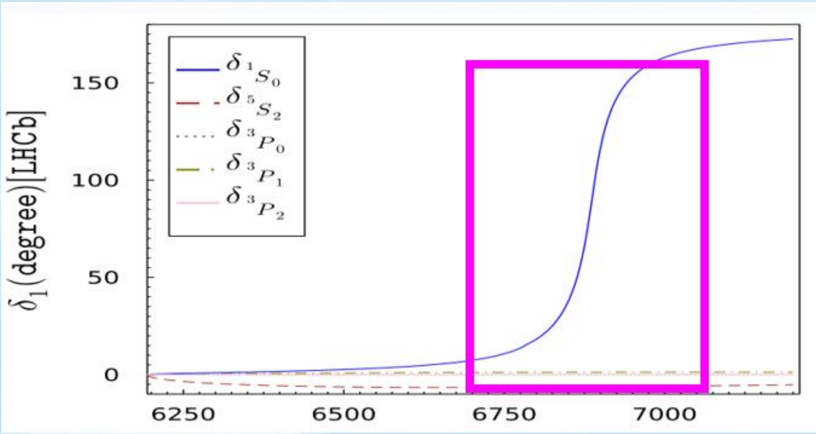
PWA: S-wave for molecules

- X(6900)
- PWA: J/ψJ/ψ-ψ(2s)J/ψ-ψ(3770)J/ψ scatterings

$$T^{ij}_{\mu_1\mu_2;\mu_3\mu_4}(s,z_s) = 16\pi N_{ij}\sum_J (2J+1)T^{J,ij}_{\mu_1\mu_2;\mu_3\mu_4}(s)d^J_{\mu\mu'}(z_s).$$

L	$S=0$	$S=1$	$S=2$
0	$\mathbf{0^{++}}$ ($^1\mathbf{S}_0$)	1^{+-}	$\mathbf{2^{++}}$ ($^5\mathbf{S}_2$)
1	1^{--}	$\mathbf{0^{-+}}$ ($^3\mathbf{P}_0$) $\mathbf{1^{-+}}$ ($^3\mathbf{P}_1$) $\mathbf{2^{-+}}$ ($^3\mathbf{P}_2$)	1^{--} 2^{--} 3^{--}
2	2^{++}	1^{+-} 2^{+-} 3^{+-}	0^{++} 1^{++} 2^{++} 3^{++} 4^{++}
⋮	⋮	⋮	⋮





+Pole counting

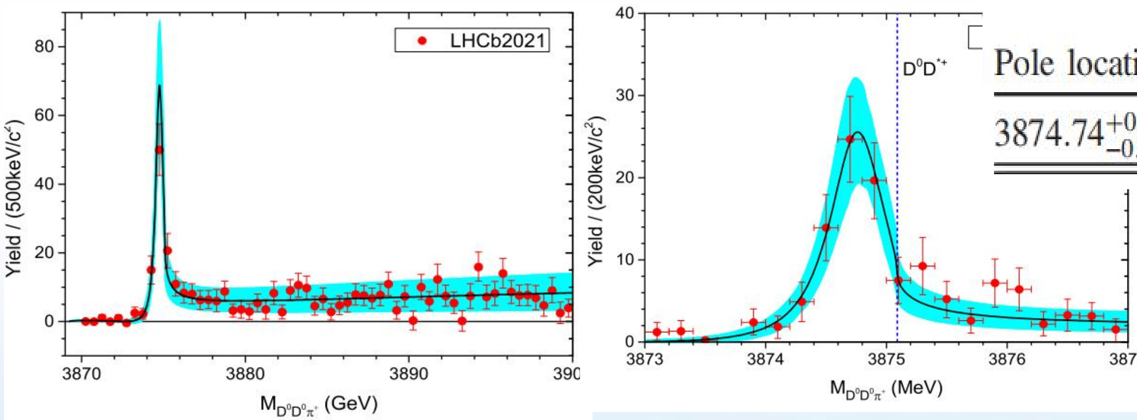
- One resonace found in $^1S_0$ wave: X(6900)----0^{++}
- Couple channels case: a pair of accompanying poles
- Triple channels case: Four poles in unphysical sheets, implying the BW origin.

pole counting rule:
Morgan,
NPA543 (1992) 632.
Dai,Wang and Zheng
CTP57 (2012) 841

Data	RS	pole location (MeV)	$g_{J/\psi J/\psi} = \|g\|e^{i\varphi}$		$g_{J/\psi\psi(2S)} = \|g\|e^{i\varphi}$		$g_{J/\psi\psi(3770)} = \|g\|e^{i\varphi}$	
			$\|g_1\|$(MeV)	$\varphi_1(^\circ)$	$\|g_2\|$(MeV)	$\varphi_2(^\circ)$	$\|g_3\|$(MeV)	$\varphi_3(^\circ)$
LHCb(Fit.IV)	II(- + +)	$6874.8^{+5.0}_{-5.8}$-$i50.4^{+1.7}_{-1.1}$	$1398.5^{+21.6}_{-15.8}$	$85.9^{+0.3}_{-0.1}$	$962.1^{+14.9}_{-10.9}$	$84.6^{+0.1}_{-0.1}$	$18.2^{+0.7}_{-0.4}$	$-79.9^{+1.2}_{-0.2}$
	III(- - +)	$6862.0^{+4.3}_{-6.2}$ -$i68.9^{+1.9}_{-2.0}$	$1364.7^{+20.1}_{-12.2}$	$80.6^{+0.3}_{-0.1}$	$927.4^{+13.4}_{-8.3}$	$77.5^{+0.5}_{-0.1}$	$19.3^{+0.7}_{-0.4}$	$-79.0^{+0.6}_{-0.3}$
	IV(- - -)	$6862.0^{+4.3}_{-6.2}$ -$i68.9^{+1.9}_{-2.0}$	$1361.6^{+19.0}_{-12.4}$	$80.7^{+0.2}_{-0.1}$	$925.3^{+12.5}_{-8.7}$	$77.5^{+0.4}_{-0.1}$	$19.4^{+0.7}_{-0.4}$	$-78.6^{+0.5}_{-0.2}$
	VII(- + -)	$6874.8^{+5.0}_{-5.8}$ -$i50.4^{+1.7}_{-1.1}$	$1394.3^{+17.7}_{-17.5}$	$85.9^{+0.2}_{-0.1}$	$959.2^{+11.7}_{-12.1}$	$84.5^{+0.1}_{-0.1}$	$18.4^{+0.7}_{-0.4}$	$-79.2^{+1.0}_{-0.3}$
CMS(Fit.V)	II(- + +)	$6888.4^{+11.3}_{-7.2}$-$i59.4^{+1.7}_{-0.5}$	$1452.8^{+23.1}_{-6.8}$	$85.6^{+0.1}_{-0.1}$	$795.8^{+12.2}_{-4.3}$	$83.3^{+0.1}_{-0.1}$	$38.8^{+2.1}_{-0.1}$	$82.2^{+0.3}_{-0.1}$
	III(- - +)	$6878.9^{+11.3}_{-7.4}$ -$i73.1^{+2.6}_{-1.1}$	$1430.3^{+29.4}_{-5.7}$	$82.0^{+0.1}_{-0.1}$	$773.9^{+15.5}_{-4.2}$	$77.8^{+0.2}_{-0.1}$	$36.4^{+2.2}_{-0.1}$	$65.0^{+1.6}_{-0.4}$
	IV(- - -)	$6878.9^{+11.3}_{-7.4}$ -$i73.1^{+2.6}_{-1.1}$	$1430.5^{+18.8}_{-5.0}$	$82.0^{+0.1}_{-0.1}$	$773.8^{+8.7}_{-3.1}$	$77.8^{+0.2}_{-0.1}$	$36.7^{+2.1}_{-0.1}$	$65.6^{+1.6}_{-0.4}$
	VII(- + -)	$6888.4^{+11.5}_{-7.2}$ -$i59.4^{+1.7}_{-0.5}$	$1452.3^{+24.4}_{-5.6}$	$85.6^{+0.1}_{-0.1}$	$795.4^{+13.6}_{-3.4}$	$83.3^{+0.1}_{-0.1}$	$39.4^{+2.2}_{-0.1}$	$83.4^{+0.4}_{-0.2}$
ATLAS(Fit.VI)	II(- + +)	$6897.7^{+19.1}_{-4.3}$-$i50.9^{+0.9}_{-0.2}$	$1409.8^{+12.0}_{-1.9}$	$86.2^{+0.1}_{-0.1}$	$997.0^{+8.8}_{-1.8}$	$85.0^{+0.1}_{-0.1}$	$5.7^{+0.1}_{-0.1}$	$56.7^{+0.8}_{-0.3}$
	III(- - +)	$6883.8^{+18.3}_{-4.0}$ -$i73.4^{+2.8}_{-0.7}$	$1373.6^{+7.3}_{-2.7}$	$80.8^{+0.1}_{-0.1}$	$960.0^{+5.6}_{-1.3}$	$77.5^{+0.1}_{-0.2}$	$7.2^{+0.1}_{-0.1}$	$21.6^{+1.1}_{-1.0}$
	IV(- - -)	$6883.8^{+18.3}_{-4.0}$ -$i73.4^{+2.8}_{-0.7}$	$1379.0^{+10.0}_{-2.0}$	$80.8^{+0.1}_{-0.1}$	$963.8^{+7.1}_{-1.2}$	$77.5^{+0.1}_{-0.1}$	$7.3^{+0.1}_{-0.1}$	$22.1^{+1.1}_{-1.0}$
	VII(- + -)	$6897.7^{+19.1}_{-4.3}$ -$i50.9^{+0.9}_{-0.2}$	$1406.7^{+10.4}_{-2.0}$	$86.2^{+0.1}_{-0.1}$	$994.9^{+7.4}_{-2.3}$	$85.0^{+0.1}_{-0.1}$	$5.8^{+0.2}_{-0.1}$	$57.6^{+0.9}_{-0.2}$

Pole analysis

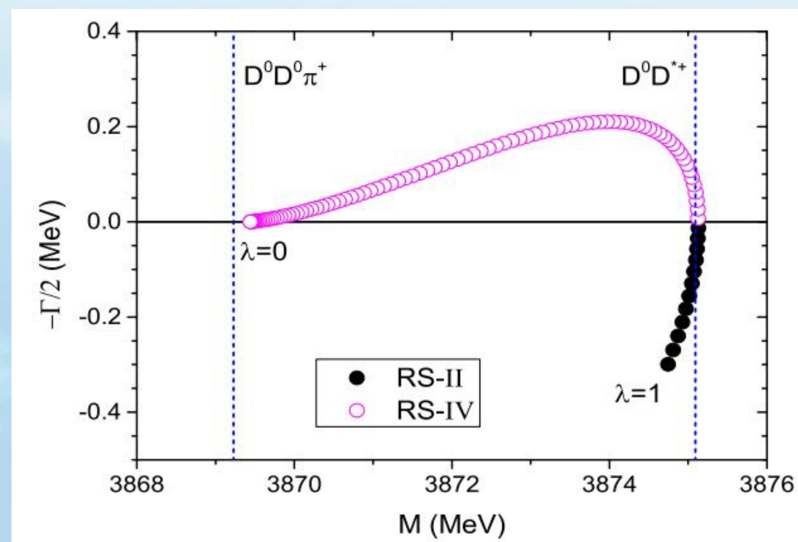
- Tcc



Pole location (MeV)	$g^{II}_{D^0D^0\pi^+} = \lvert g\rvert e^{i\varphi}$		$g^{II}_{D^0D^{*+}} = \lvert g\rvert e^{i\varphi}$	
	$\lvert g_1\rvert$ (GeV)	φ_1 (°)	$\lvert g_2\rvert$ (GeV)	φ_2 (°)
$3874.74^{+0.11}_{-0.04} - i0.30^{+0.05}_{-0.09}$	$0.22^{+0.03}_{-0.04}$	9^{+11}_{-5}	$0.69^{+0.04}_{-0.02}$	10^{+11}_{-5}

- Pole analysis by switching the inelastic channel
- pole-counting, one pole,molecule?
- λ=0 RS-IV corresponds to RS-II in D^0D^{*+}single channel, virtual state origin!

Ling, Geng, Xie *et.al.*, PLB 826 (2022) 136897



Pole analysis: Pc states

- $J/\psi p - \overline{D}^0\Sigma_c^+ - \overline{D}^{*0}\Sigma_c^+$

- Fit 1: no pole in KM

- Fit 2: a pole in KM around Pc(4440)

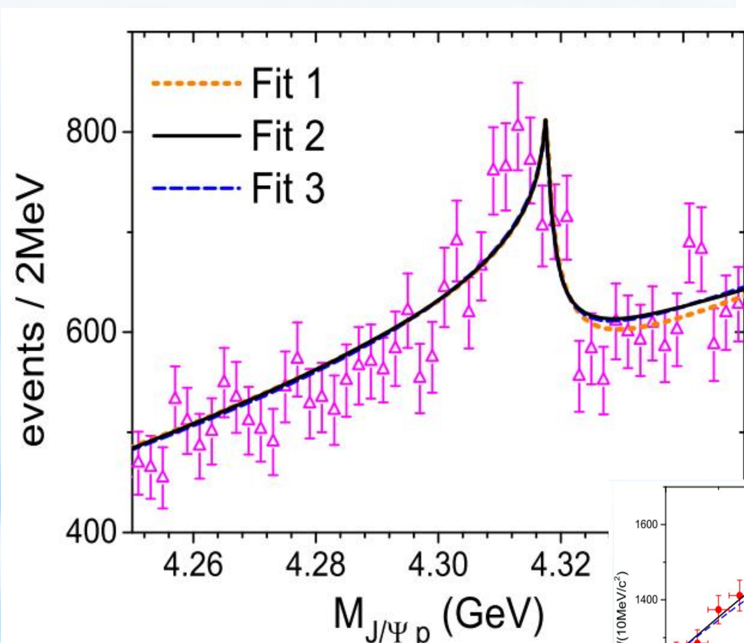
- Fit 3: P-wave instead of KM pole around Pc(4440)

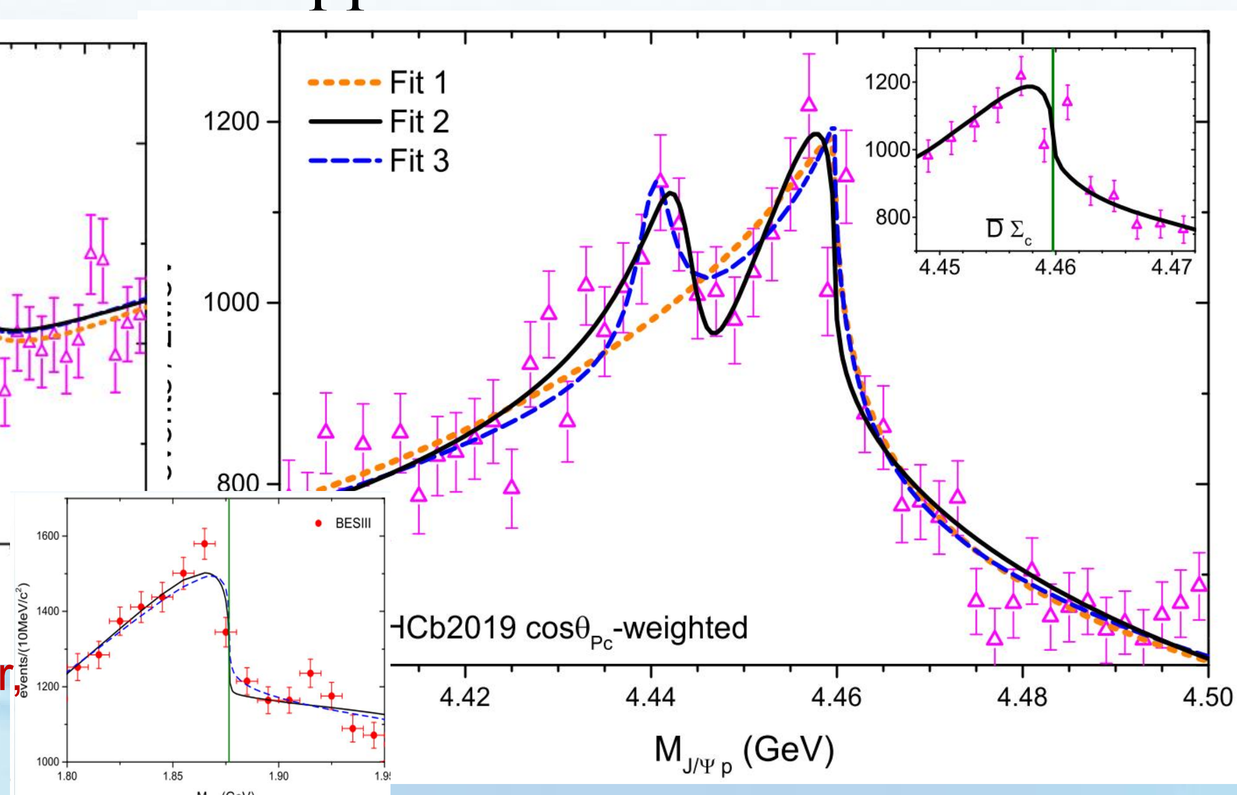
State	Pole locations (MeV)					
	RS	Fit.1	RS	Fit.2	RS	Fit.3
$P_c(4312)$	III	$4296.93^{+2.48}_{-3.00}$	III	⋯	**V**⋆	$4313.38^{+2.52}_{-5.73}$
		$-i5.12^{+2.44}_{-1.06}$		⋯		$-i2.05^{+1.65}_{-0.75}$
	V⋆	$4312.74^{+1.69}_{-0.67}$	**V**⋆	$4314.31^{+2.06}_{-1.10}$	VIII	$4313.11^{+3.86}_{-4.76}$
		$-i3.33^{+2.91}_{-1.25}$		$-i1.43^{+1.50}_{-0.57}$		$-i3.11^{+1.63}_{-2.02}$
$P_c(4440)$	⋯	⋯	**III**⋆	$4444.09^{+2.53}_{-1.48}$	**III**⋆	$4440.53^{+0.47}_{-0.31}$
		⋯		$-i3.10^{+0.53}_{-1.33}$		$-i2.42^{+0.22}_{-0.22}$
	⋯	⋯	IV	$4443.69^{+2.89}_{-1.34}$	IV	$4440.38^{+0.41}_{-0.19}$
		⋯		$-i0.32^{+1.23}_{-0.04}$		$-i1.40^{+0.59}_{-0.50}$
	⋯	⋯	V	$4444.22^{+2.72}_{-1.41}$	V	$4440.53^{+0.37}_{-0.30}$
		⋯		$-i2.48^{+0.57}_{-0.67}$		$-i2.32^{+0.27}_{-0.61}$
	⋯	⋯	VII	$4443.84^{+1.93}_{-1.91}$	VIII	$4440.38^{+3.31}_{-0.52}$
		⋯		$-i1.02^{+1.05}_{-0.92}$		$-i1.30^{+4.45}_{-0.50}$
$P_c(4457)$	⋯	⋯	III	$4466.53^{+2.13}_{-4.75}$	⋯	⋯
		⋯		$-i3.88^{+6.95}_{-0.93}$		⋯
	⋯	⋯	VII	$4456.77^{+3.10}_{-8.89}$	VIII	$4453.44^{+7.11}_{-3.34}$
		⋯		$-i7.77^{+11.07}_{-4.41}$		$-i21.58^{+8.01}_{-6.36}$

Kuang, Dai#, Kang, Yao, EPJC 80 (2020) 433

Pc states: inner structure?

- pole-counting: Pc(4312), molecule?
- Fit 2 better than Fit 3, Pc(4440) prefers to be S-wave,compact tetraquark
- Pc(4457): quite similar to ppbar threshold behaviour

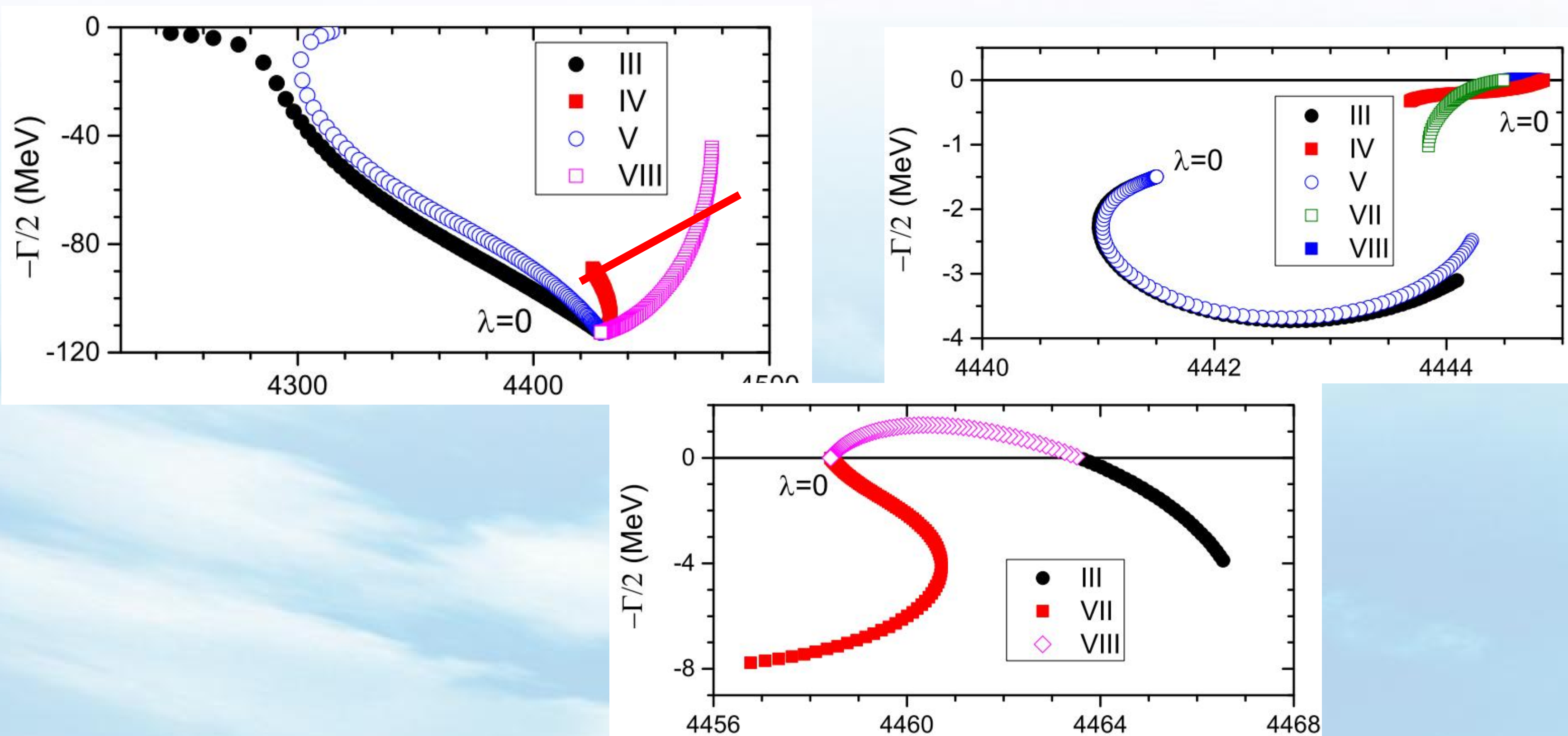




Dai, Haidenbauer, Meissner,
PRD 98 (2018) 1, 014005

Pole trajectories?

- Pc(4312): support molecule.
- Pc(4440): compact tetraquark
- Pc(4457): threshold behaviour

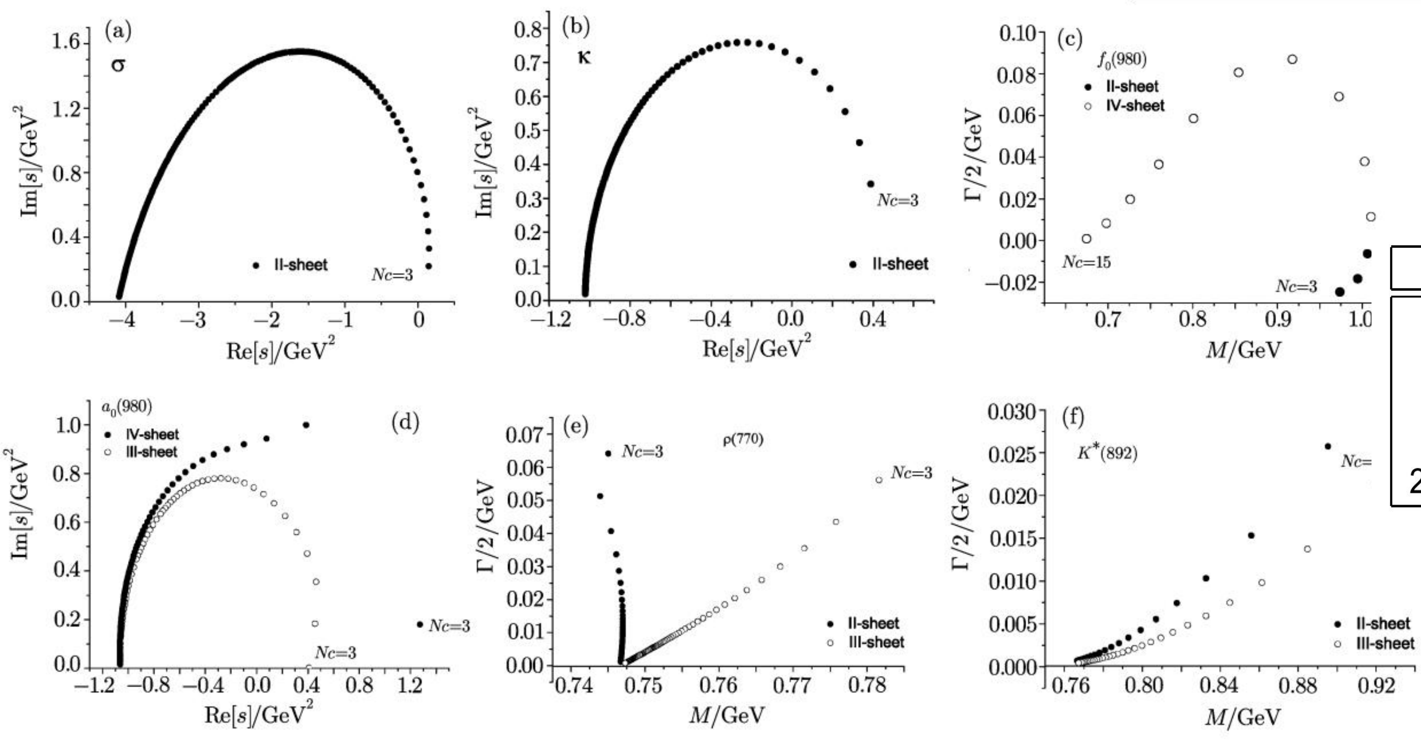


Nc Trajectories+pole counting

- BW: twin poles will meet each other on the real axis in large Nc limit

Table 3 Resonance pole positions on $\sqrt{s}$ plane in unit of GeV, and virtual pole position on s plane.

Resonance	II	III	IV
σ	$0.457 - i0.242$		
$f_0(980)$	$0.974 - i0.025$		
$a_0(980)$		$0.640 - i0.002$	$1.131 - i0.079$
$\rho(770)$	$0.740 - i0.069$	$0.782 - i0.056$	
$(I, J) = (2, 0)$	$0.045 m_\pi^2$		
$\kappa(800)$	$0.673 - i0.254$		
$K^*(892)$	$0.895 - i0.026$	$0.921 - i0.021$	

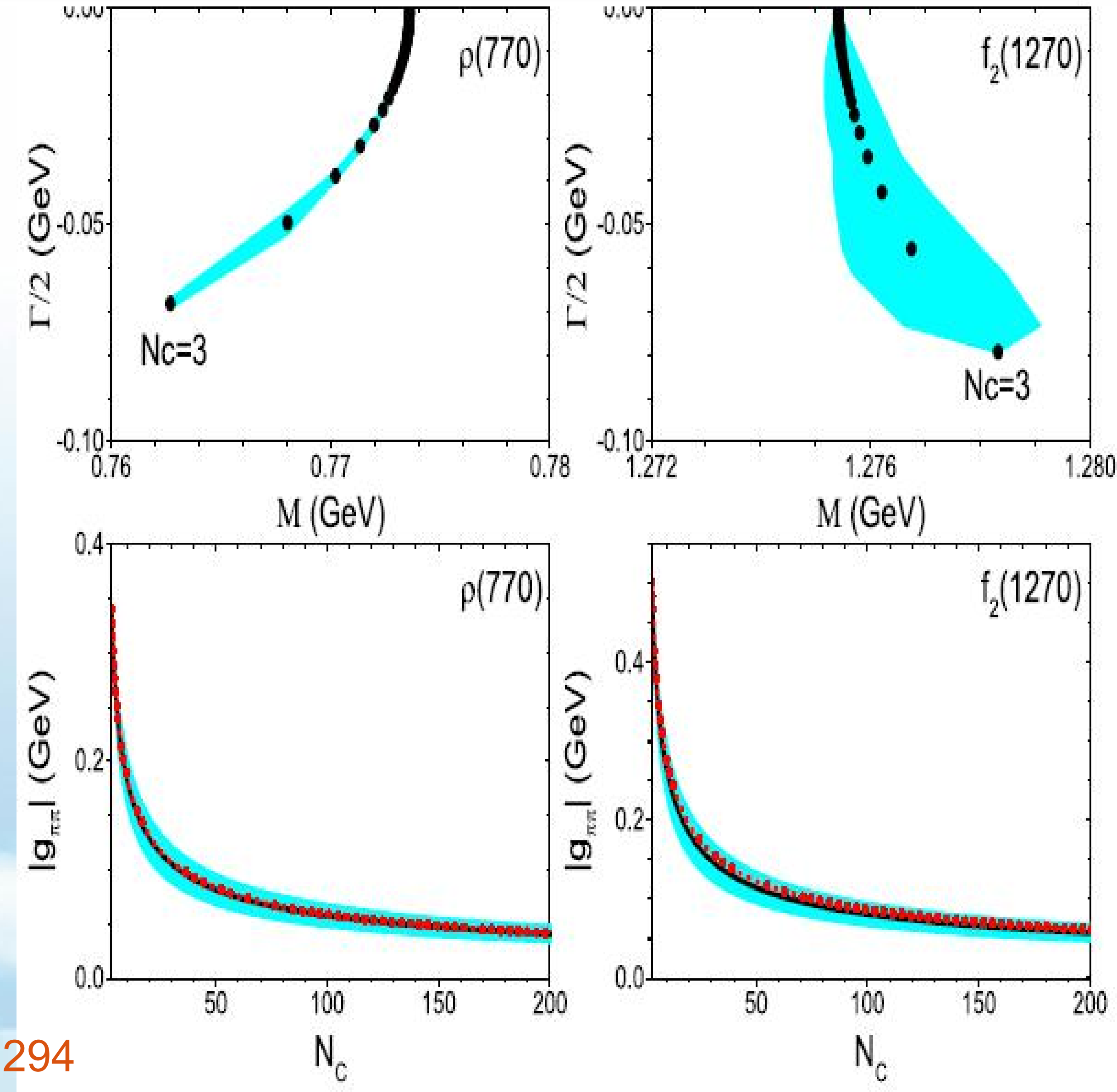


	Nc order		Nc order
L_1	$\mathcal{O}(N_C)$	L_5	$\mathcal{O}(N_C)$
L_2	$\mathcal{O}(N_C)$	L_6	$\mathcal{O}(1)$
L_3	$\mathcal{O}(N_C)$	L_7	$\mathcal{O}(1)$
L_4	$\mathcal{O}(1)$	L_8	$\mathcal{O}(N_C)$
$2L_2 - L_1$	$\mathcal{O}(1)$		

Dai, Wang and Zheng CTP57 (2012) 841, CTP58 (2012) 410

Nc Trajectories of Dispersion relation

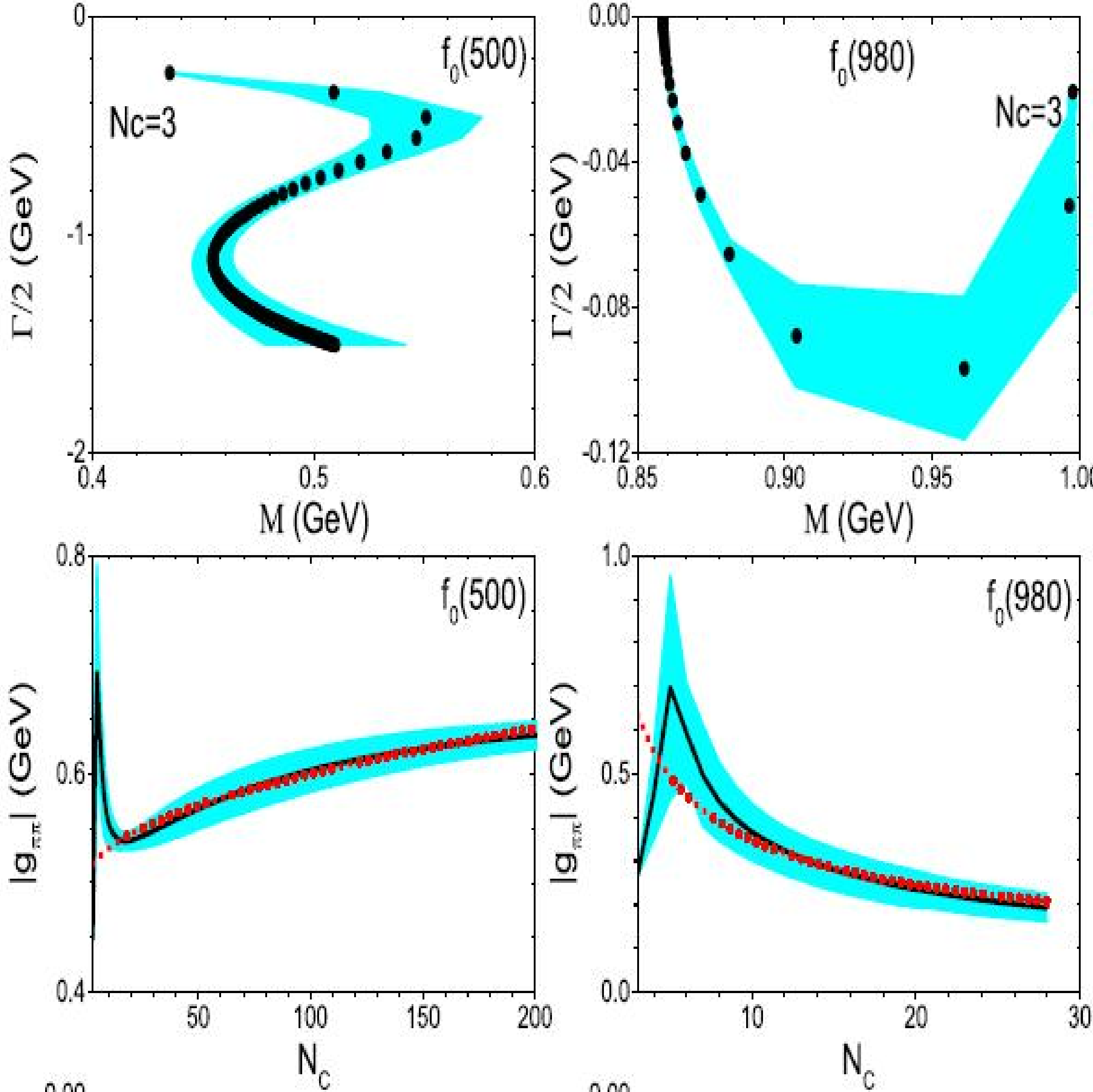
- ρ, f_2 behave as BW resonances. The masses are of O(1) and widths of O(Nc^{-1}). The couplings to ππ is O($Nc^{-1/2}$).
- This confirms the introduce of Nc dependence is properly.



Dai,Meissner, PLB 783 (2018) 294

Light scalars

- The mass of σ is of O(1) and the coupling to ππ has O($Nc^{1/2}$).
- The coupling of $f_0(980)$ contains O($Nc^{-1/2}$).
- They all have a peak around Nc=5. Implying mixing structure.



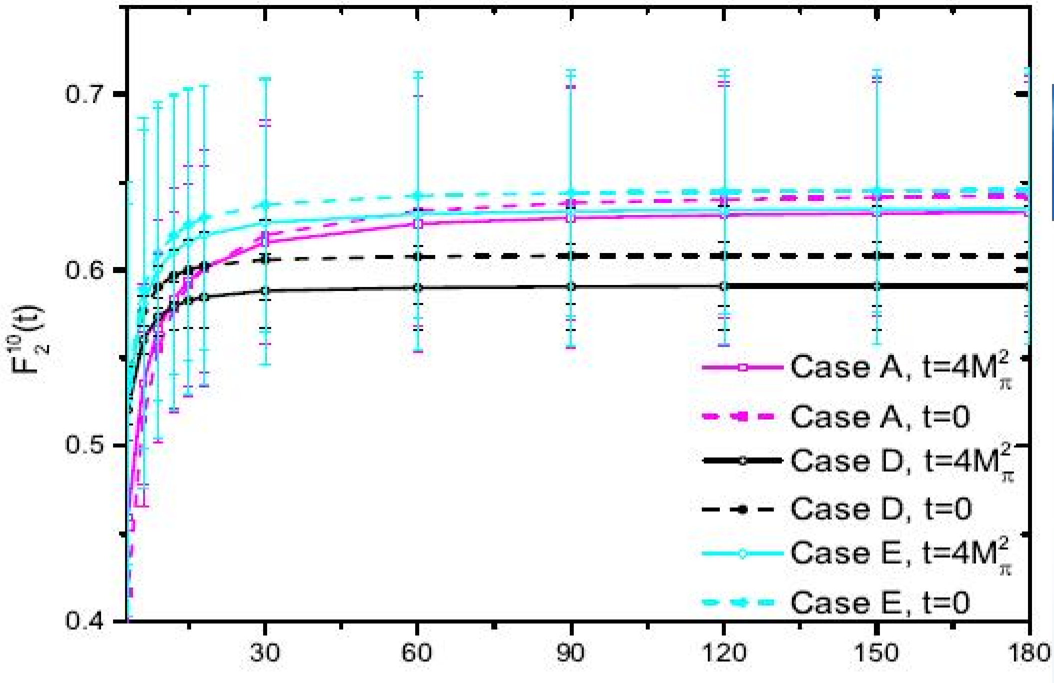
Semi-local duality

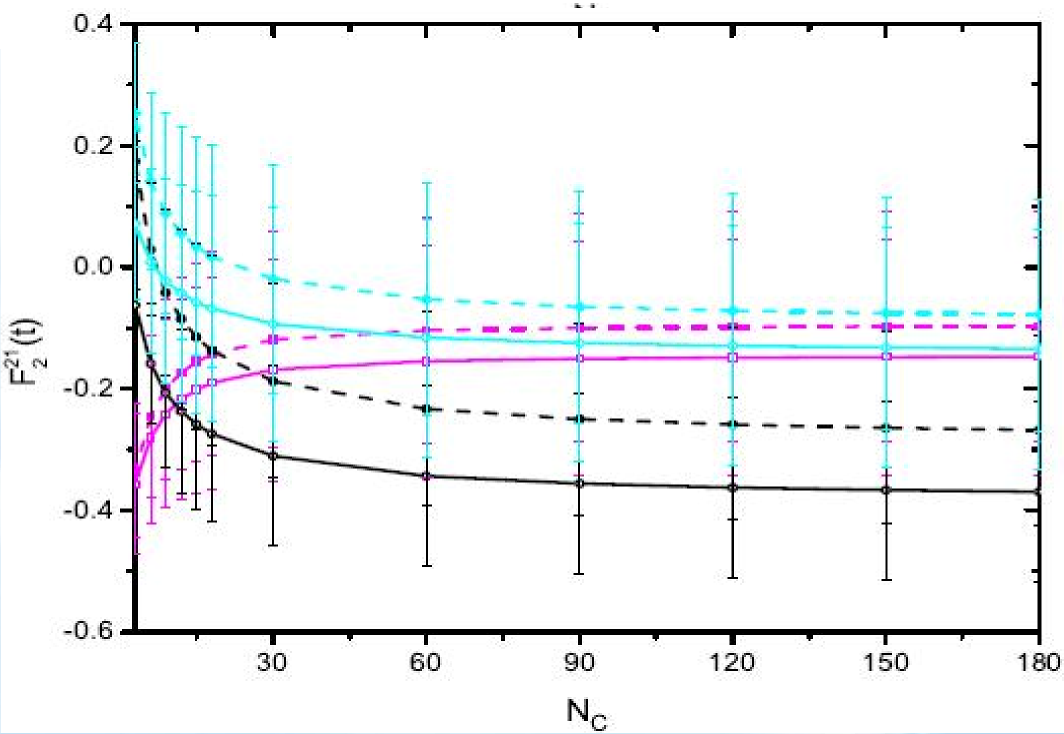
Case	$F_2^{10}(t)$		$F_2^{21}(t)$	
	$t=4M_\pi^2$	$t=0$	$t=4M_\pi^2$	$t=0$
O	0.64(5)	0.65(4)	−0.14(16)	−0.06(15)
A	0.63(7)	0.64(7)	−0.15(20)	−0.10(19)
B	0.59(8)	0.59(7)	−0.36(20)	−0.34(19)
C	0.62(8)	0.63(7)	−0.23(20)	−0.19(19)
D	0.59(3)	0.61(3)	−0.37(15)	−0.27(16)
E	0.63(8)	0.65(7)	−0.13(20)	−0.08(19)

Semi-local duality implies that $F^{10}_n=2/3$ and $F^{21}_n=0$

- O: original one
- A: no σ
- B: no $f_0(980)$
- C: no $f_0(1370)$
- D: no $f_2(1270)$
- E: include [2,4GeV^2]

Dai,Kang,Meissner,
PRD 98 (2018) 7, 074033





- σ contributes a lot at Nc=3 but not at large Nc.
- Heavier resonances such as ρ(1450), ρ(1700) have small contribution.
- σ contains molecular or tetraquark, $f_0(980)$ contains some ss component, they are mixing states.

3、A new method

- Some ways more simple?
- The simplest way, satisfying intuition: For a molecule, its mass should increase/decrease as that of the constituent hadrons!
- How to make sure the trend of the amplitudes is right in unphysical region?
- In the physical region, constrained by data and also ensured by ChEFT.

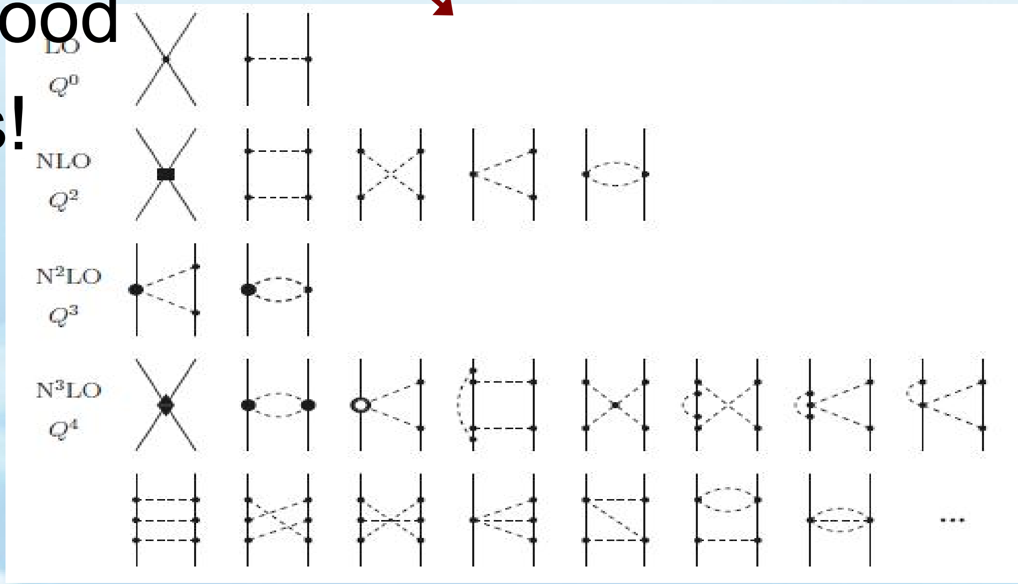
$$\begin{aligned}
\mathscr{L}_2 = \quad & \frac{f_0^2}{4}\langle \partial_\mu U^\dagger \partial^\mu U + \mathscr{M}(U+U^\dagger)\rangle, \\
\mathscr{L}_4 = \quad & L_1 \langle \partial_\mu U^\dagger \partial^\mu U\rangle^2 + L_2 \langle \partial_\mu U^\dagger \partial_\nu U\rangle \langle \partial^\mu U^\dagger \partial^\nu U\rangle \\
& + L_3 \langle \partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\rangle + L_4 \langle \partial_\mu U^\dagger \partial^\mu U\rangle \langle U^\dagger \mathscr{M} + \mathscr{M}^\dagger U\rangle \\
& + L_5 \langle \partial_\mu U^\dagger \partial^\mu U (U^\dagger \mathscr{M} + \mathscr{M}^\dagger U)\rangle + L_6 \langle U^\dagger \mathscr{M} + \mathscr{M}^\dagger U\rangle^2 \\
& + L_7 \langle U^\dagger \mathscr{M} - \mathscr{M}^\dagger U\rangle^2 + L_8 \langle U^\dagger \mathscr{M} U^\dagger \mathscr{M} + \mathscr{M}^\dagger U \mathscr{M}^\dagger U\rangle,
\end{aligned}$$

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

ChEFT

- Supplies dynamics
- Isospin symmetry: The mass difference between charged and neutral particles is ignored in ChEFT
- Describe the physics in low energy region successfully.
- Isospin symmetry is good

  for strong interactions!

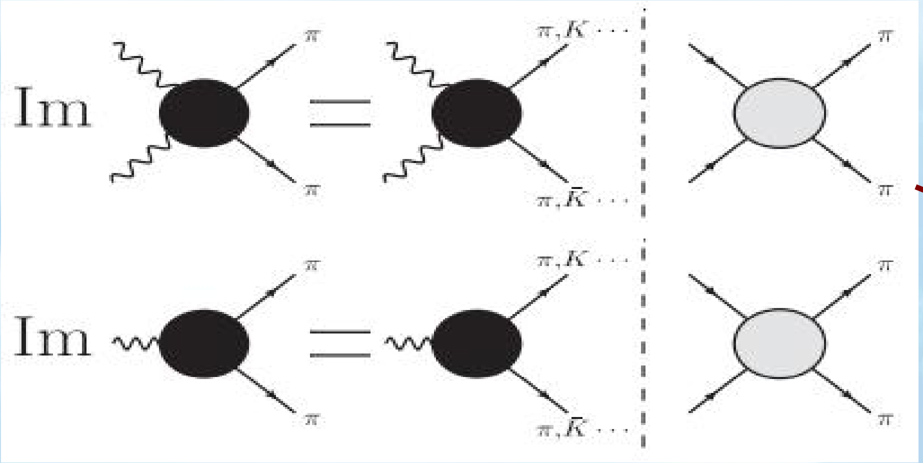


Towards pole extraction

- FSI needs to be taken into account to perform an amplitude analysis------> right amplitudes

- Methods: Pade, KM, N/D, AMP, Roy equation, PKU, Pade, LSE, BSE, ChEFT, *et.al.*

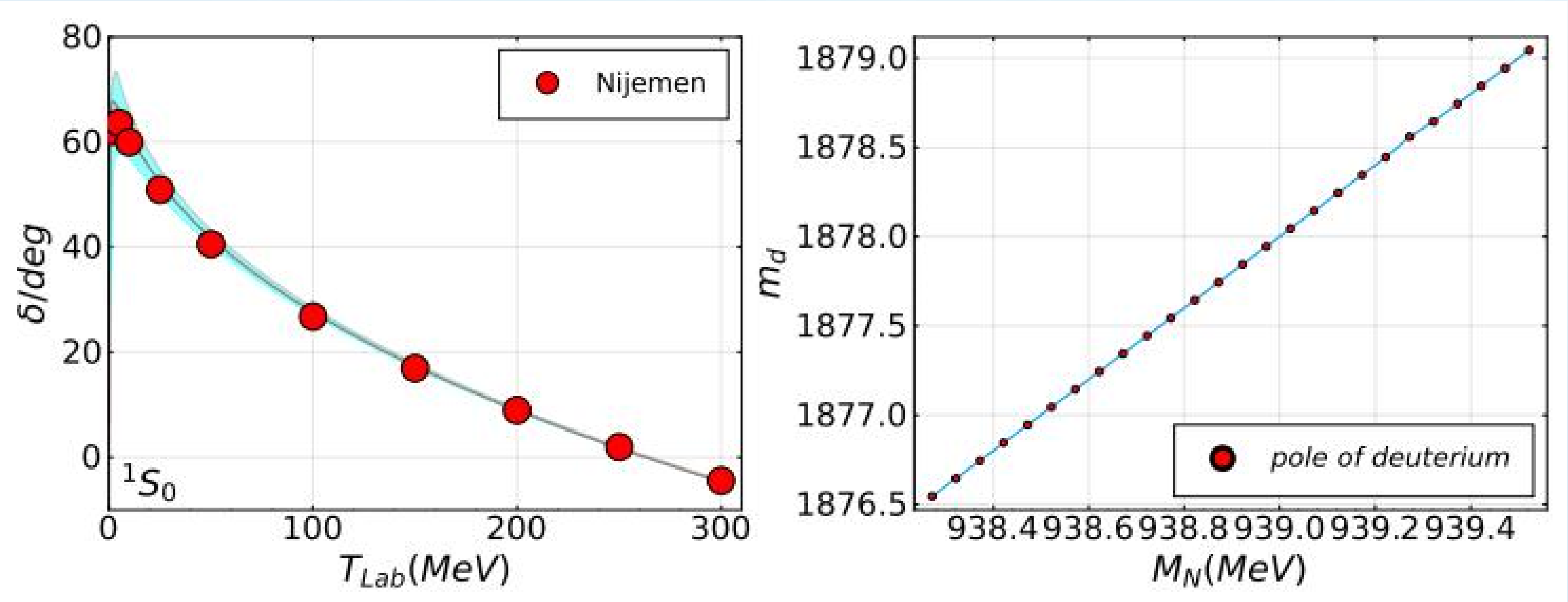
- Fixed scattering amplitudes: extracting resonance information



Yao, Dai#, Zheng, Zhou, RPP84(2021)076201

deuteron

- Deuteron: Maybe the only undoubted molecule.
- Varying the masses within the range allowed by isospin symmetry. The amplitudes still fit rather well to the ‘data’.
- Mass of deuteron increases as that of nucleons.

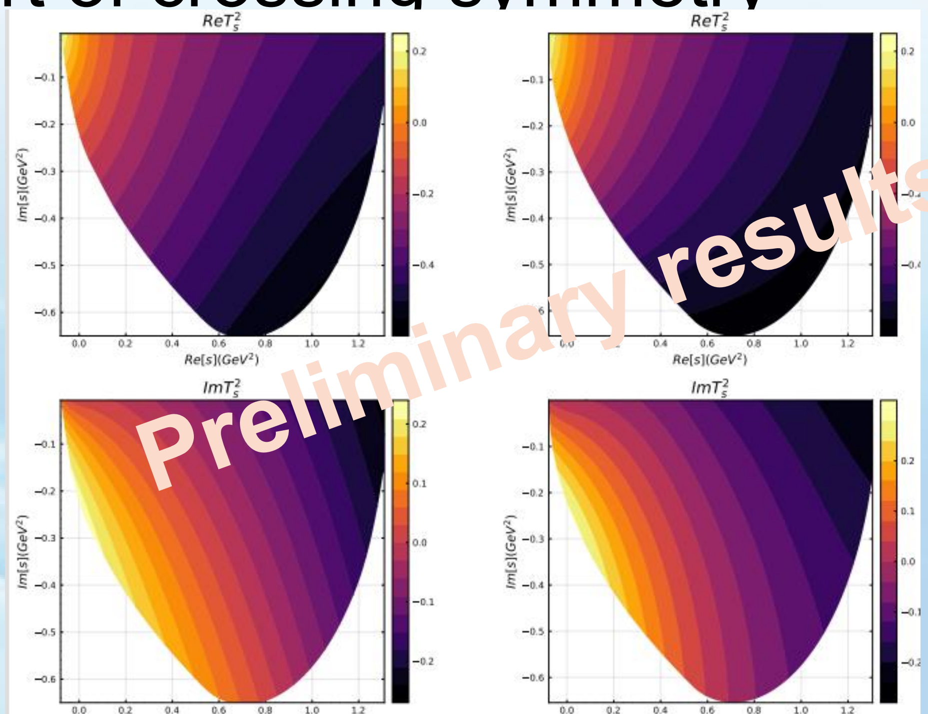


amplitudes

- ChPT for the dynamics
- Unitarization to restore unitarity

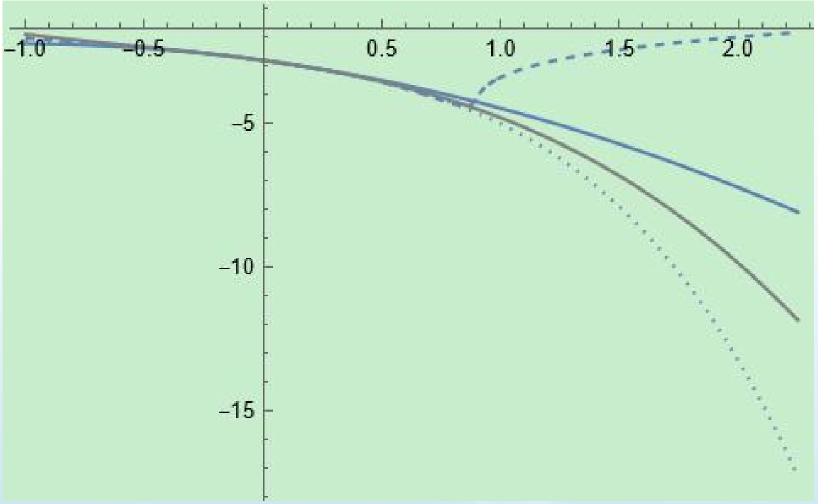
$$T^{(I,J)} = T_2^{(I,J)} \cdot [T_2^{(I,J)} - T_4^{(I,J)}]^{-1} T_2^{(I,J)}$$

- Fitting Roy's amplitudes in the complex plane to include part of crossing symmetry



amplitudes

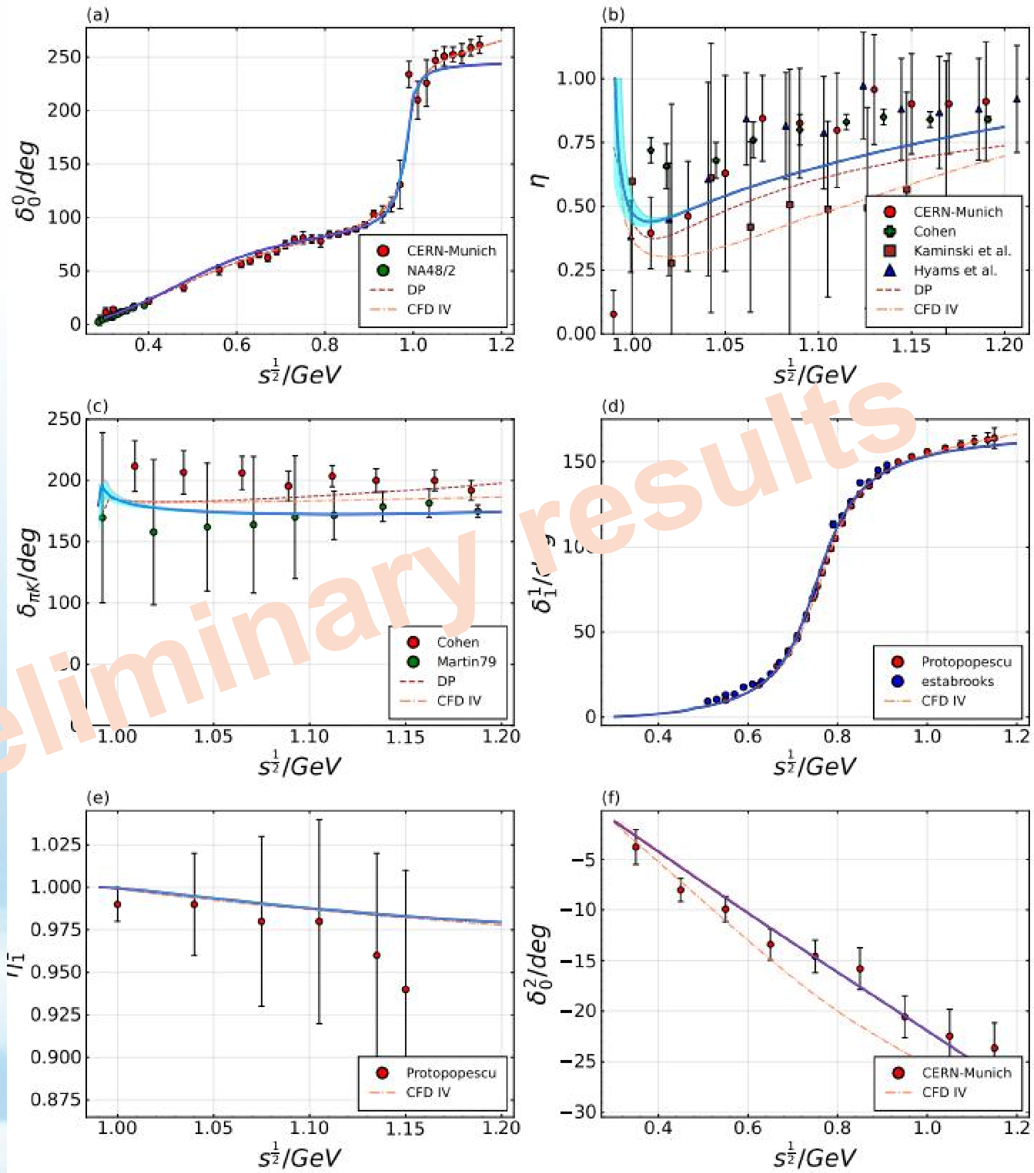
- l.h.c. caused by KK scattering is removed, to strictly restore unitarity



- Random forest method is applied to get more reliable LECs from minimum χ^2

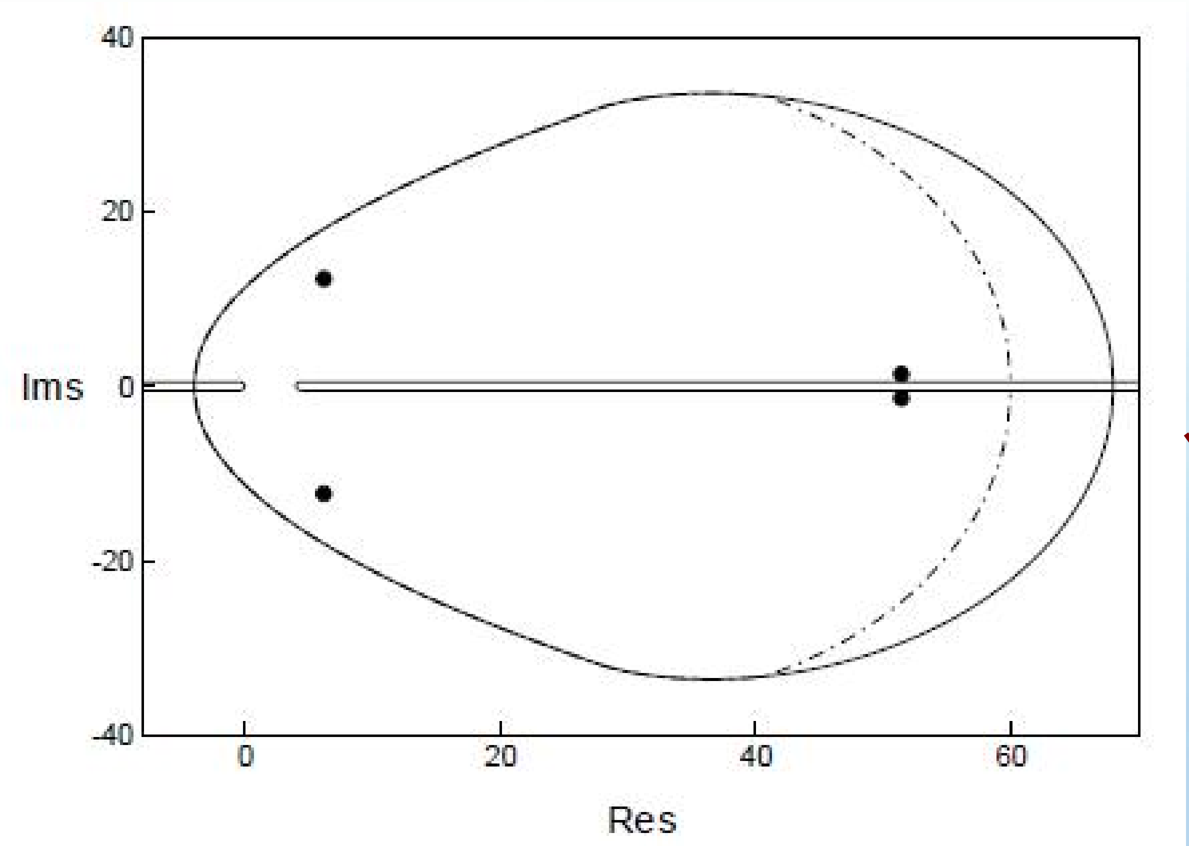
scalars

- Varying the masses of pseudoscalars, the amplitudes are almost not changed



Poles are not bound states?

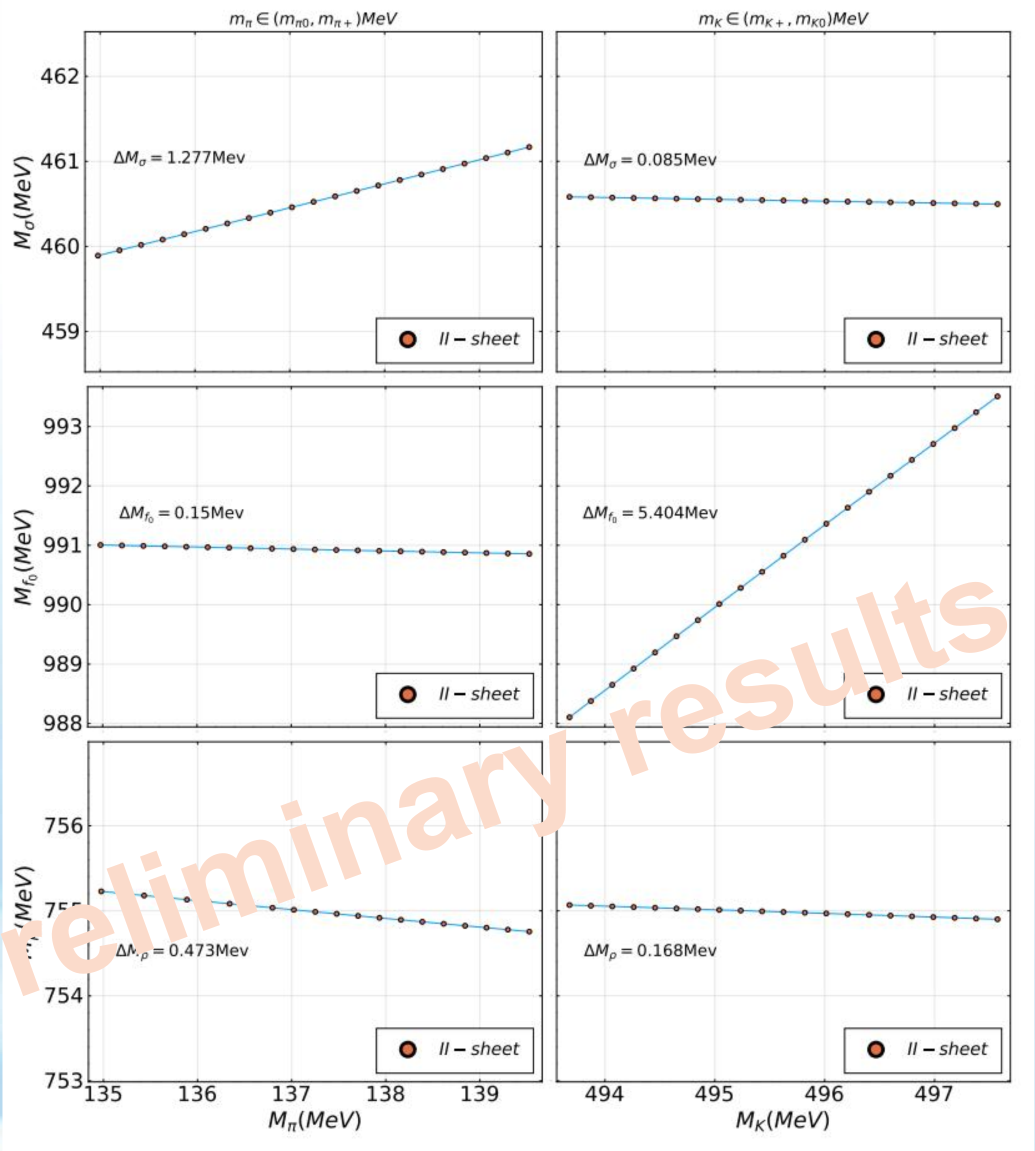
- Cuts will drive the single channle bound state into the complex plane,



I. Caprini, G. Colangelo and H. Leutwyler,
PRL96 (2006) 132001

scalars

- Pole counting:
  - σ RS-II, III;
  - $f_0(980)$, RS-II
  - ρ(770), RS-II, RS-III
- σ, not ordinary qq, not molecule
- $f_0(980)$, dominated by KK molecule!



4、Summary

molecule

We propose a mthod to classify the molecule type hadron: its mass should increas as that of the constituent hadrons

Amplitudes

ChEFT supplies dynamics, isospin symmetry ensures that the amplitudes are still in the right region and can be checked by exp.

structure

$f_0(980)$ is a KK molecule;
σ is an non-ordinary resonance;

Next?

Other resonances? Comapring with LQCD's?

Thank You For your patience !