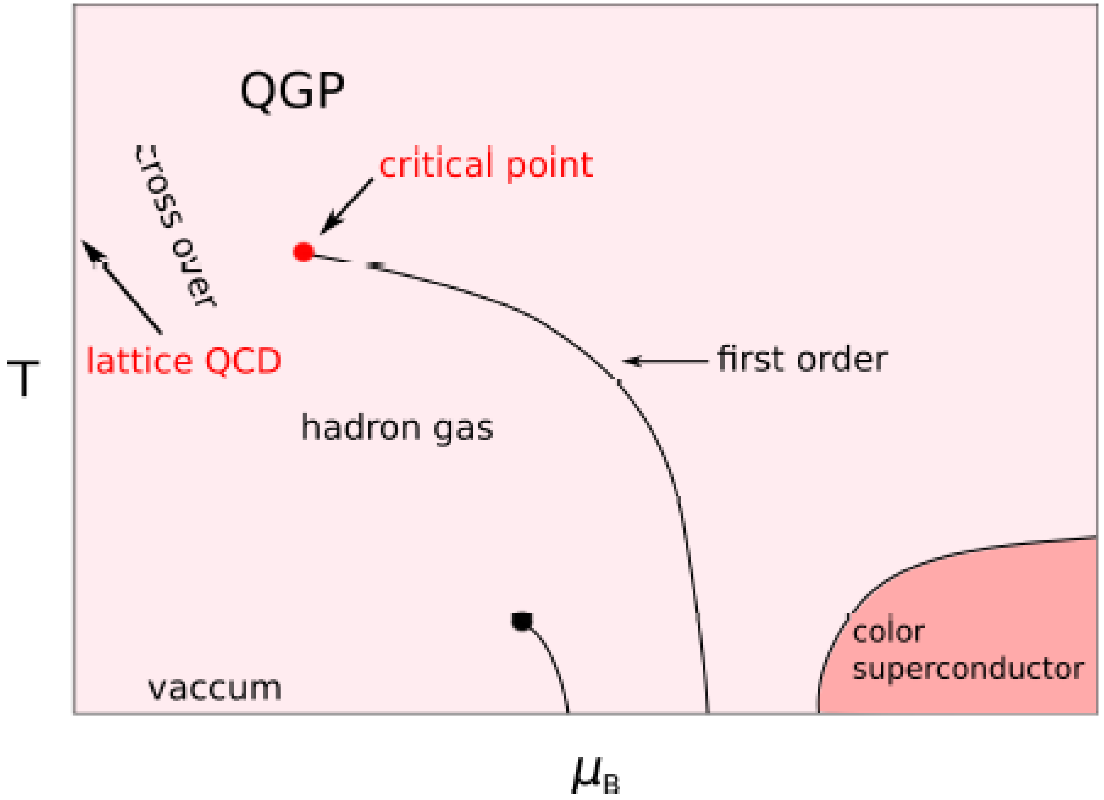
Finite Density

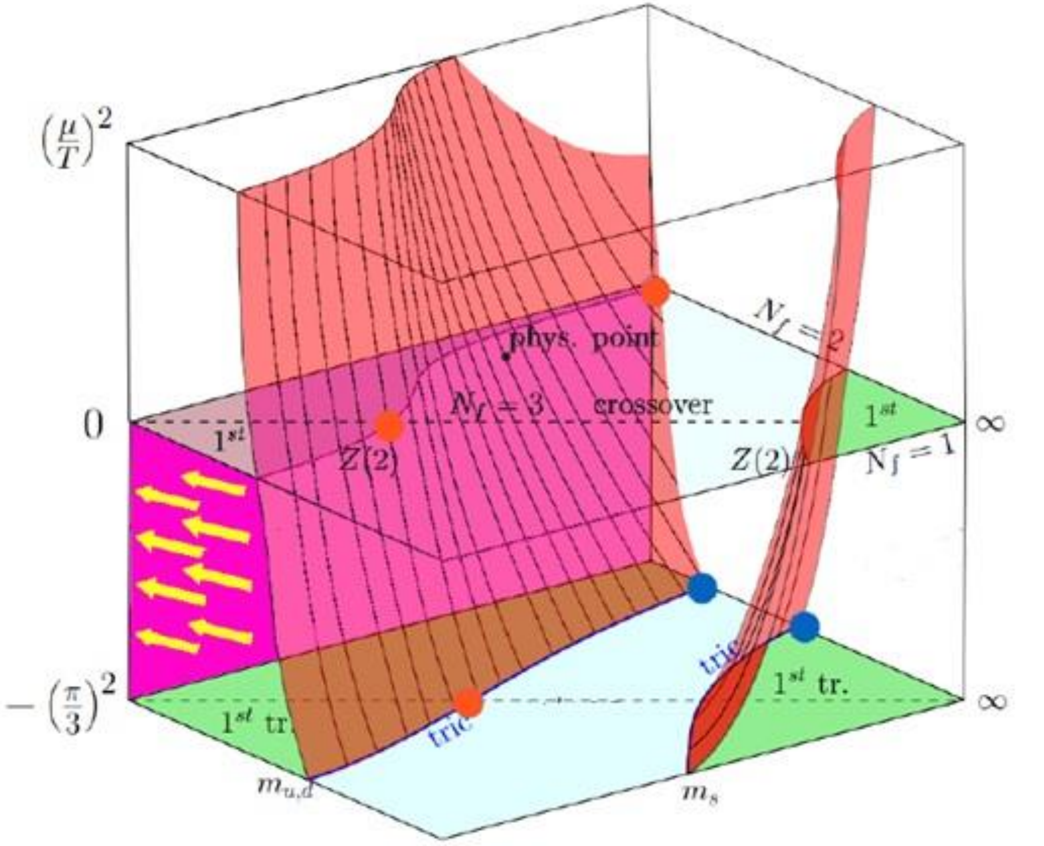
20230428

Conjectured QCD phase diagram



A. Li, et.al, Phys. Rev.D 82, 054502 (2010)

Columbia plot



A. D'Ambrosio, et.al, PoS LATTICE2022 (2022) 172

Grand canonical ensembles

- mu=0: HMC

$$\langle O \rangle = \frac{1}{Z} \int D\phi \, e^{-S_E(\phi)} O(\phi), \qquad Z = \int D\phi \, e^{-S_E(\phi)}$$

- Chemical potential in lattice QCD

$$\begin{aligned}(1+\gamma_4)U_4^\dagger(y) &\rightarrow (1+\gamma_4)U_4^\dagger(y)e^{\mu a}, \\ (1-\gamma_4)U_4(x) &\rightarrow (1-\gamma_4)U_4(x)e^{-\mu a}.\end{aligned}$$

P. Hasenfratz, et.al, Phys. Lett.B125 (1983) 308

- Real chemical potential: sign problem & reweighting

$$\langle O \rangle = \frac{\langle O e^{-i\,\mathrm{Im}\,S_E(\phi)} \rangle_0}{\langle e^{-i\,\mathrm{Im}\,S_E(\phi)} \rangle_0}, \qquad \langle O \rangle_0 = \int D\phi \, \frac{e^{-\mathrm{Re}\,S_E(\phi)}}{Z_0} O(\phi) \qquad Z_0 \equiv \int D\phi \, e^{-\mathrm{Re}\,S_E(\phi)}$$

Canonical partition function

- Fugacity expansion $$Z(V,T,\mu) = \sum_k Z_C(V,T,k) e^{\mu k/T}$$

- Fourier transform with an imaginary chemical potential

$$Z_C(V,T,k) = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\phi \, e^{-ik\phi} Z(V,T,\mu)|_{\mu=i\phi T}$$

- Integrate out the fermionic part

$$Z_C(V,T,k) = \int \mathcal{D}\mathrm{U} \, e^{-S_g(U)} \mathrm{det}_k M^{N_f}(U)$$

$$\mathrm{det}_k M^{N_f}(U) \equiv \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\phi \, e^{-ik\phi} \det M(m,\phi;U)^{N_f}$$

A. Hasenfratz, et.al, Nucl. Phys. B 371 (1992) 539

Canonical partition function

Charge conjugation symmetry

$$Z_C(V,T,k) = Z_C(V,T,-k)$$

$$Z_C(V,T,k) = \int \mathcal{D}\mathrm{U}\, e^{-S_g(U)} \mathrm{det}_k M^{N_f}(U)$$

$$\mathrm{det}_k M^{N_f}(U) \equiv \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\, e^{-ik\phi} \det M(m,\phi;U)^{N_f}$$

$$Z_C(V,T,k) = \int \mathcal{D}\mathrm{U}\, e^{-S_g(U)} \mathrm{Re}\,\mathrm{det}_k M^{N_f}(U)$$

Two-step simulation with HMC

$$Z_C(V,T,k) = \int \mathcal{D}\mathrm{U}\, e^{-S_g(U)} \mathrm{det} M^{N_f}(U) W(U) \alpha(U)$$

where

$$W(U) = \frac{|\mathrm{Re}\,\mathrm{det}_k M^{N_f}(U)|}{\mathrm{det} M^{N_f}(U)},$$

$$\alpha(U) = \mathrm{Sign}(\mathrm{Re}\,\mathrm{det}_k M^{N_f}(U)).$$

accept/reject method based on the weight W(U)

fold the phase factor α(U) into the measurements

A. Alexandru, et.al, Phys. Rev. D72, 114513 (2005)

A. Li, et.al, Phys. Rev.D 82, 054502 (2010)

Baryon chemical potential

- The difference of the free energy after adding one baryon

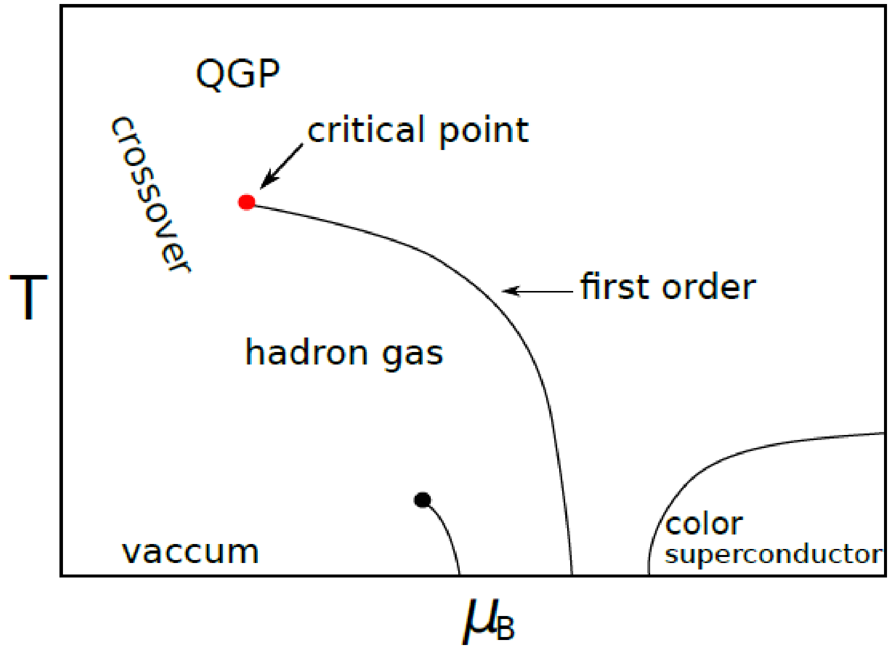
$$\langle\mu\rangle_{n_B} = \frac{F(n_B+1) - F(n_B)}{(n_B+1) - n_B} = -\frac{1}{\beta}\ln\frac{\langle\gamma(U)\rangle_o}{\langle\alpha(U)\rangle_o} \quad (7)$$

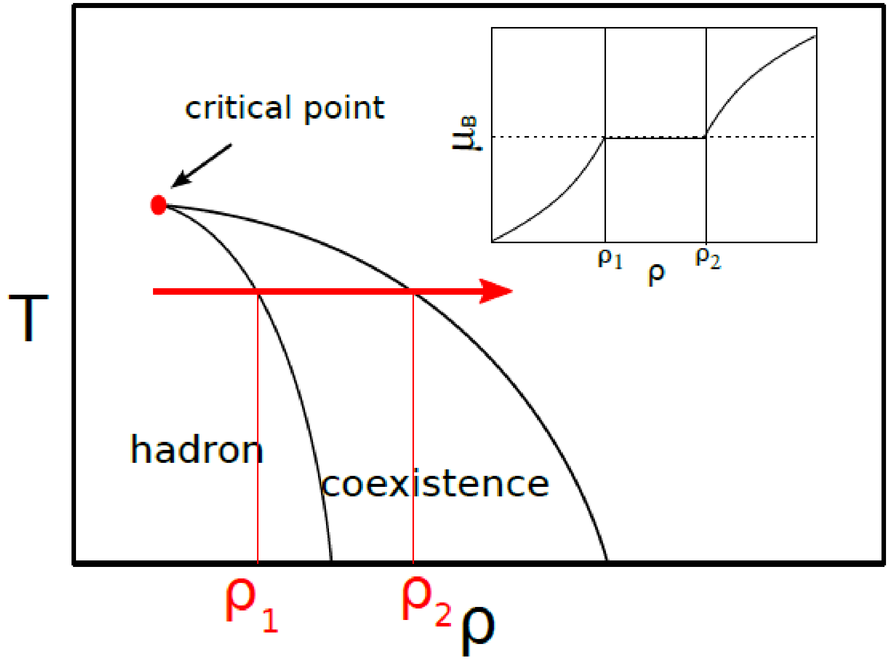
where

$$\gamma(U) = \frac{\mathrm{Re}\,\det_{3n_B+3} M^{n_f}(U)}{|\mathrm{Re}\,\det_{3n_B} M^{n_f}(U)|}. \quad (8)$$

$\langle\rangle_o$ in Eq. (7) stands for the average over the ensemble generated with the measure $|\mathrm{Re}\,\det_{3n_B} M^{n_f}(U)|$.

Phase diagram, grand canonical vs canonical





P. de Forcrand and S. Kratochvila, Nucl. Phys. Proc. Suppl. 153, 62 (2006)

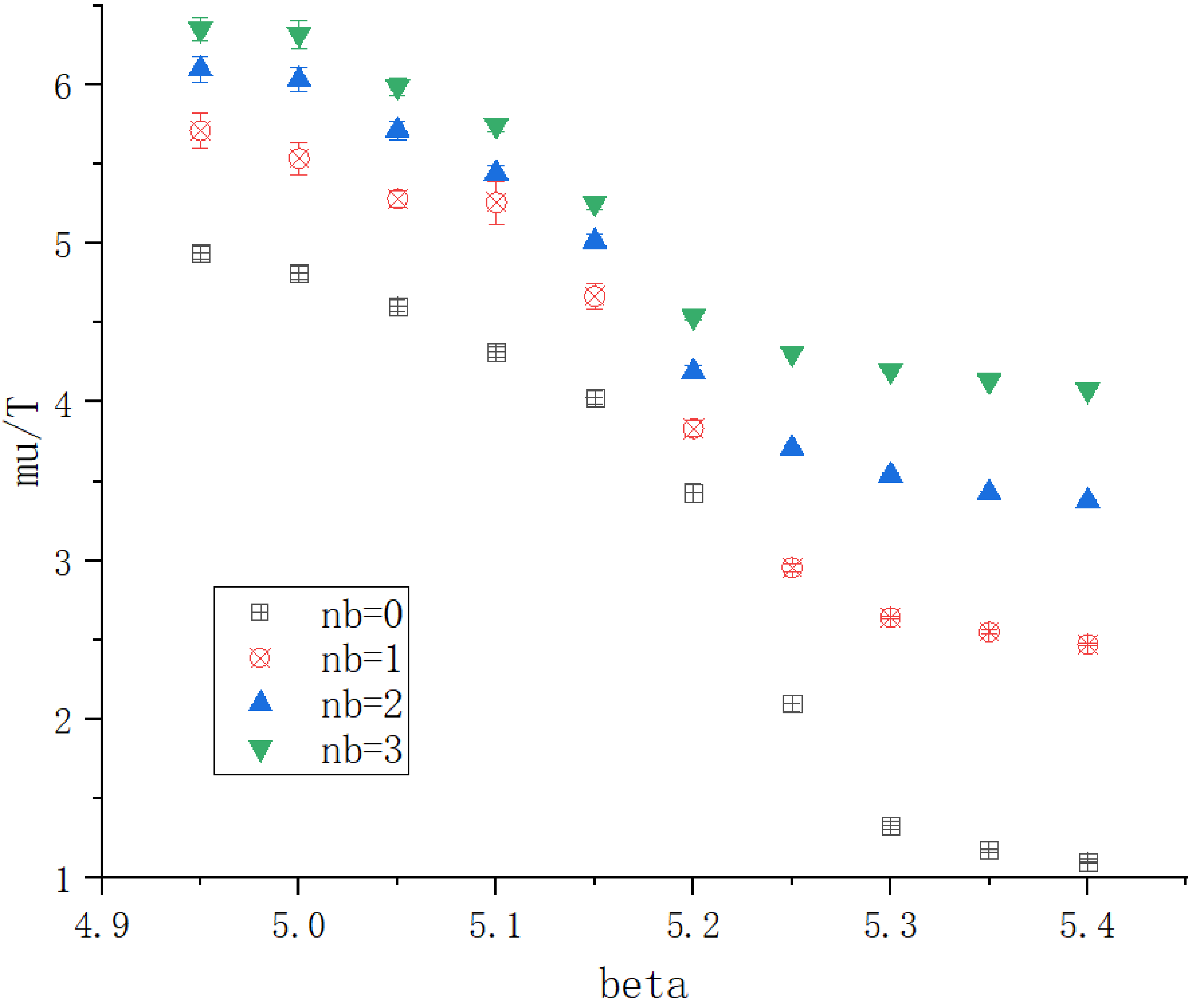
S. Ejiri, Phys. Rev. D78, 074507 (2008)

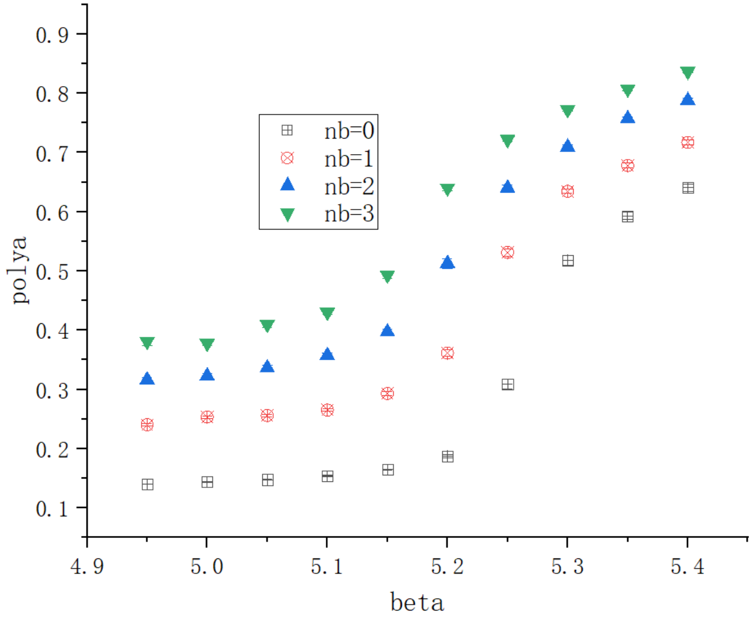
Lattice setup

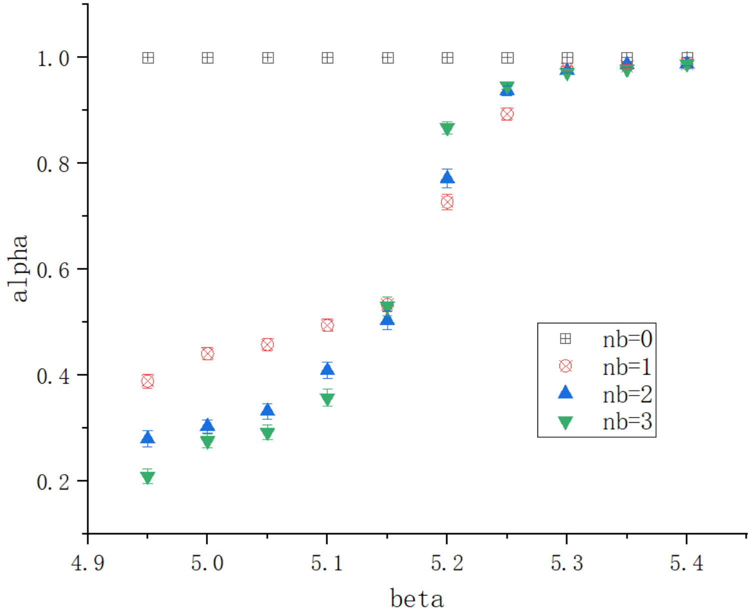
- Wilson gauge action + Wilson fermion action
- Lattice spacing: 12^3x24, w_0 scale (w_0=0.1755fm)
- N_f=2: m_π~900MeV; 4^3x4, 6^3x4
- N_f=3: m_π~900MeV, 700MeV, 500MeV; 4^3x4, 6^3x4; RHMC with Remez in Chroma
- **Exact determinate projection: compression** **A. Alexandru, et.al, Phys. Rev.D 83, 034502(2011)**

$$\det{}_k M^{N_f}(U) \equiv \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\, e^{-ik\phi} \det M(m,\phi;U)^{N_f}$$

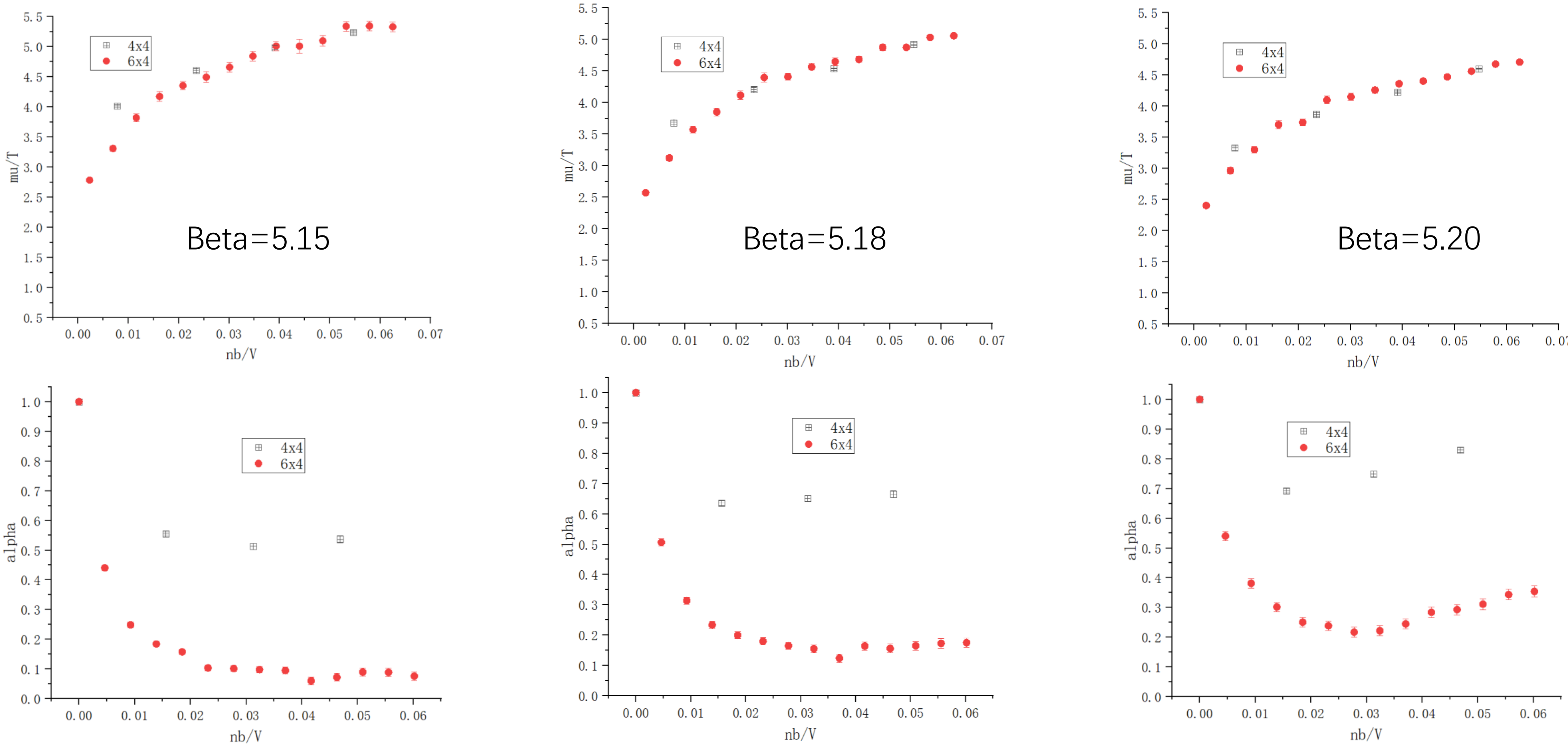
Two flavors (900MeV)



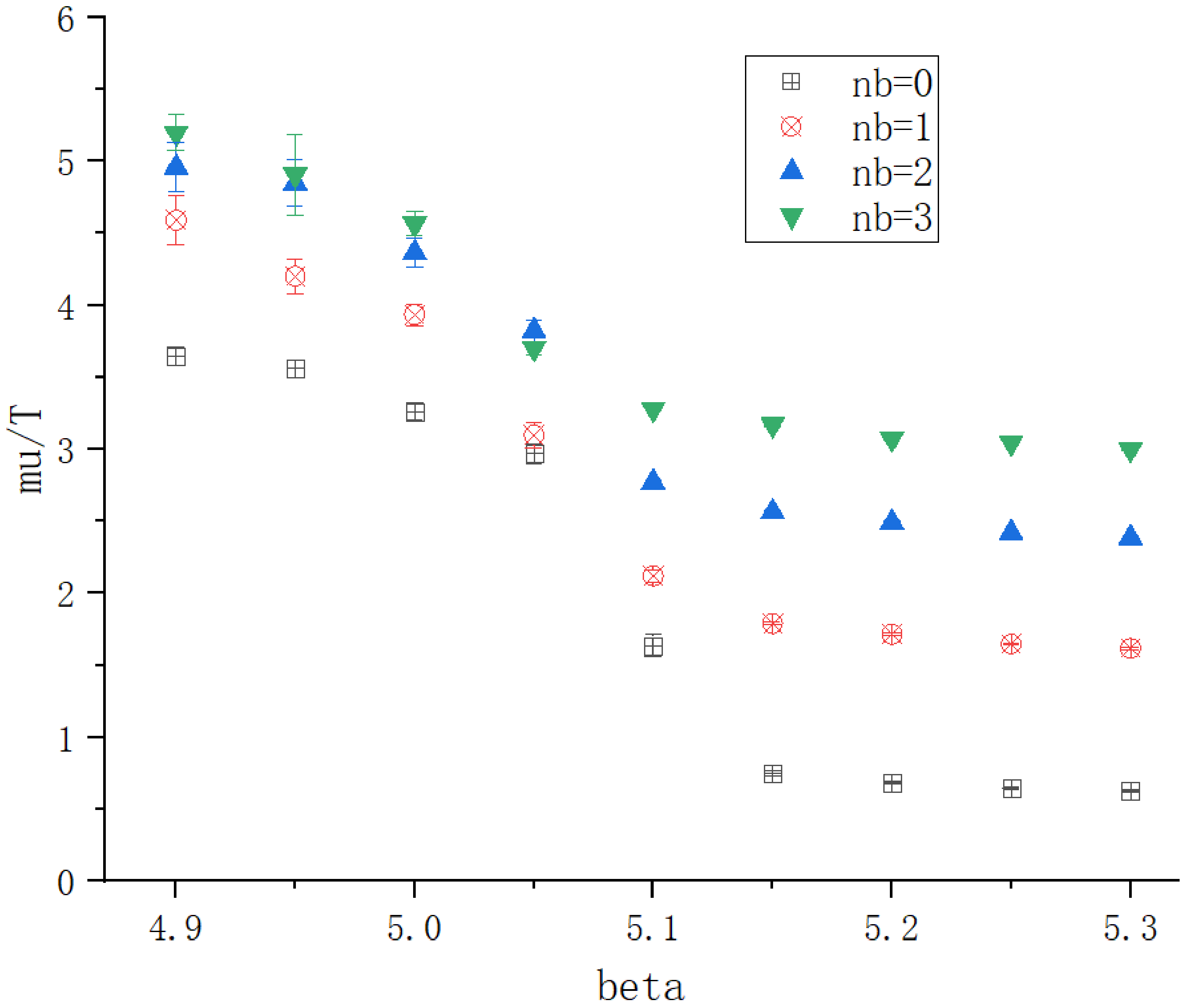


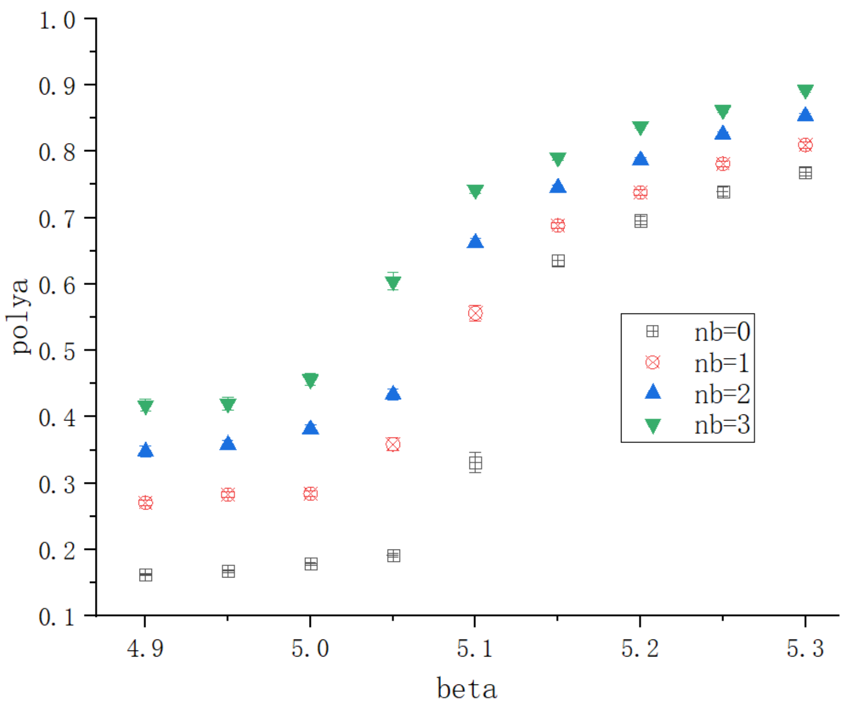


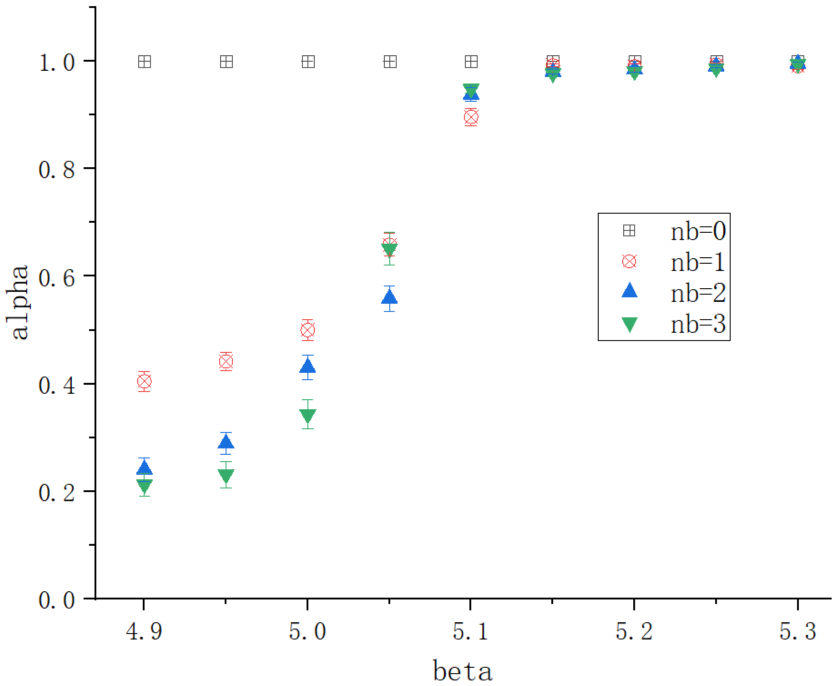
Kappa*2	beta	Lattice spacing (fm)	Temperature (GeV)	Pion mass (GeV)
0.158	5.15	0.31271(13)	0.1577	0.9409(14)
0.158	5.18	0.30803(13)	0.1601	0.9337(14)
0.158	5.20	0.30451(15)	0.1620	0.9307(16)



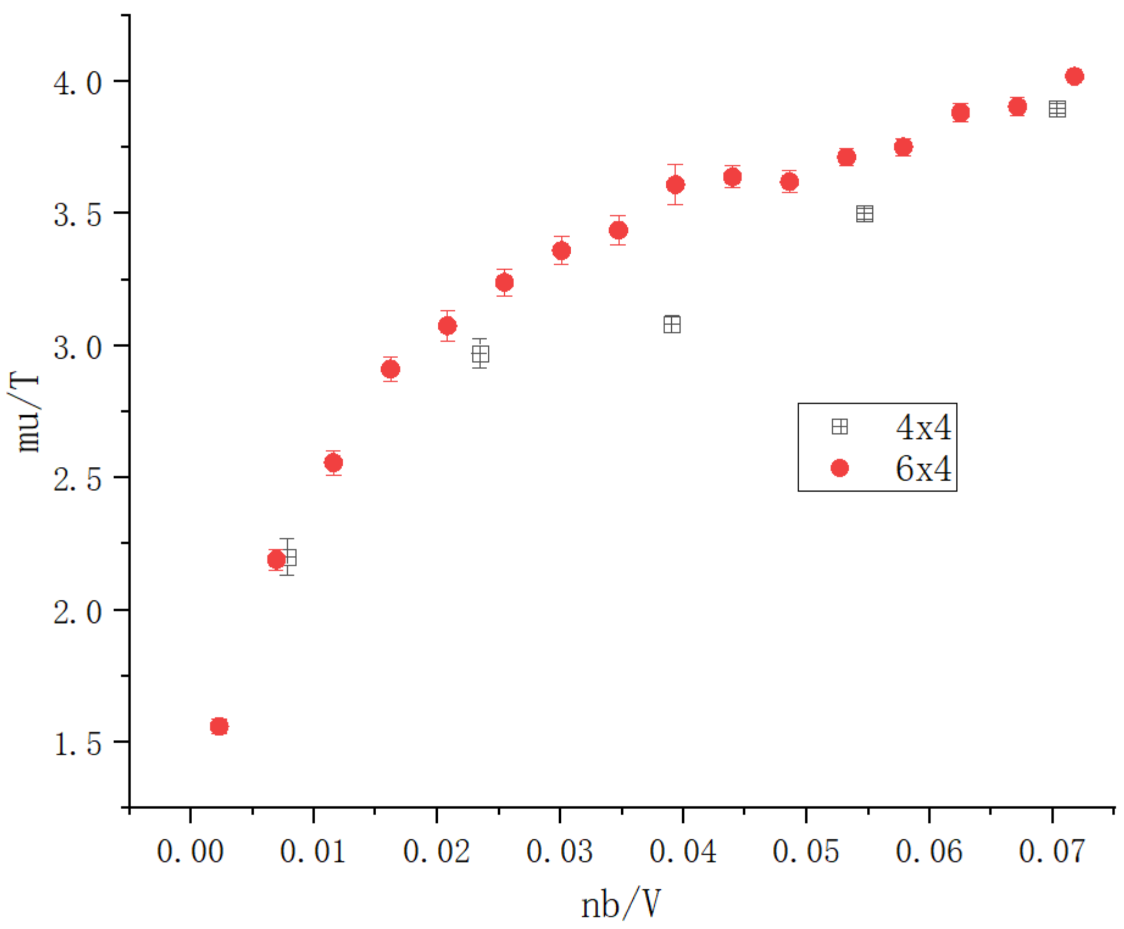
0.158*3	5.07	5.05	5.03
a	0.31242(10)	0.31504(12)	0.31770(10)
m_π	0.9421(15)	0.9447(14)	0.9471(15)

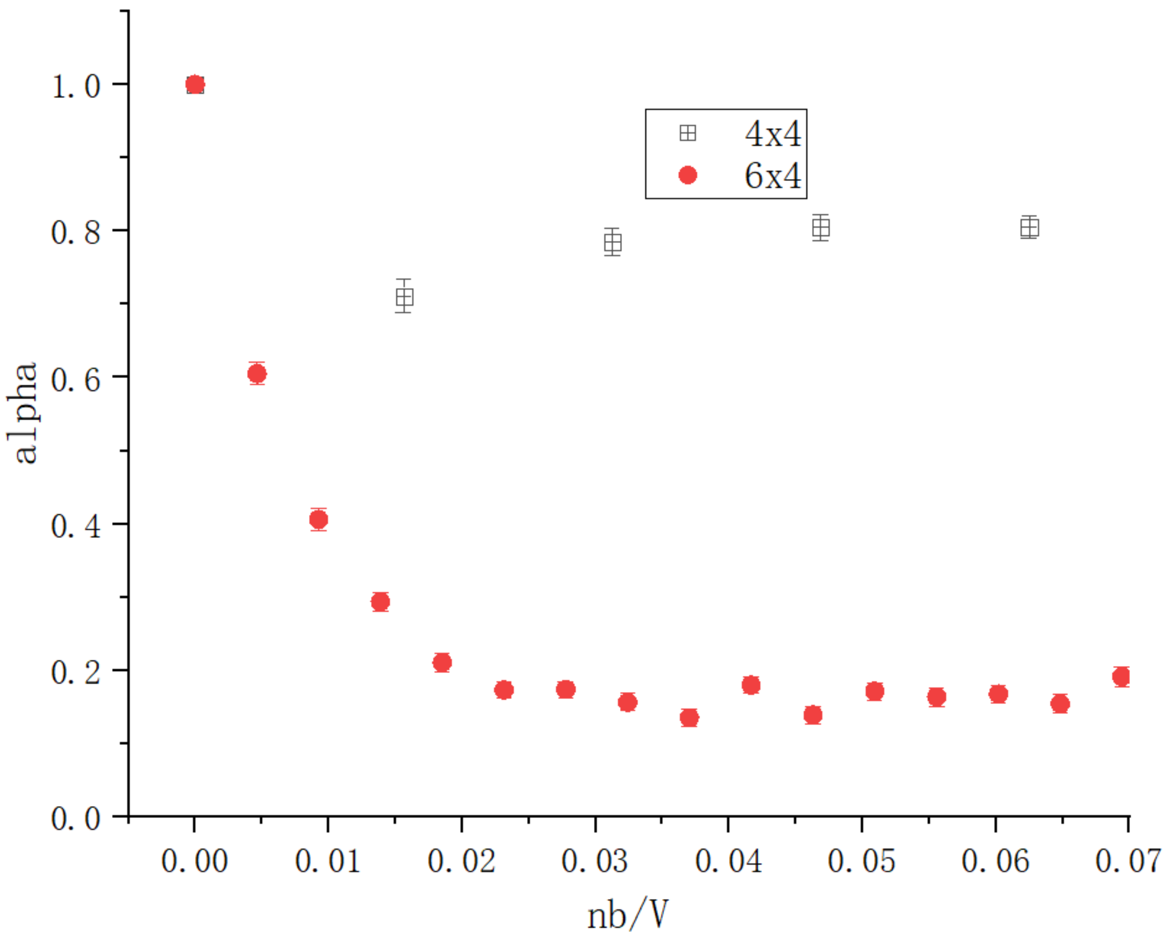




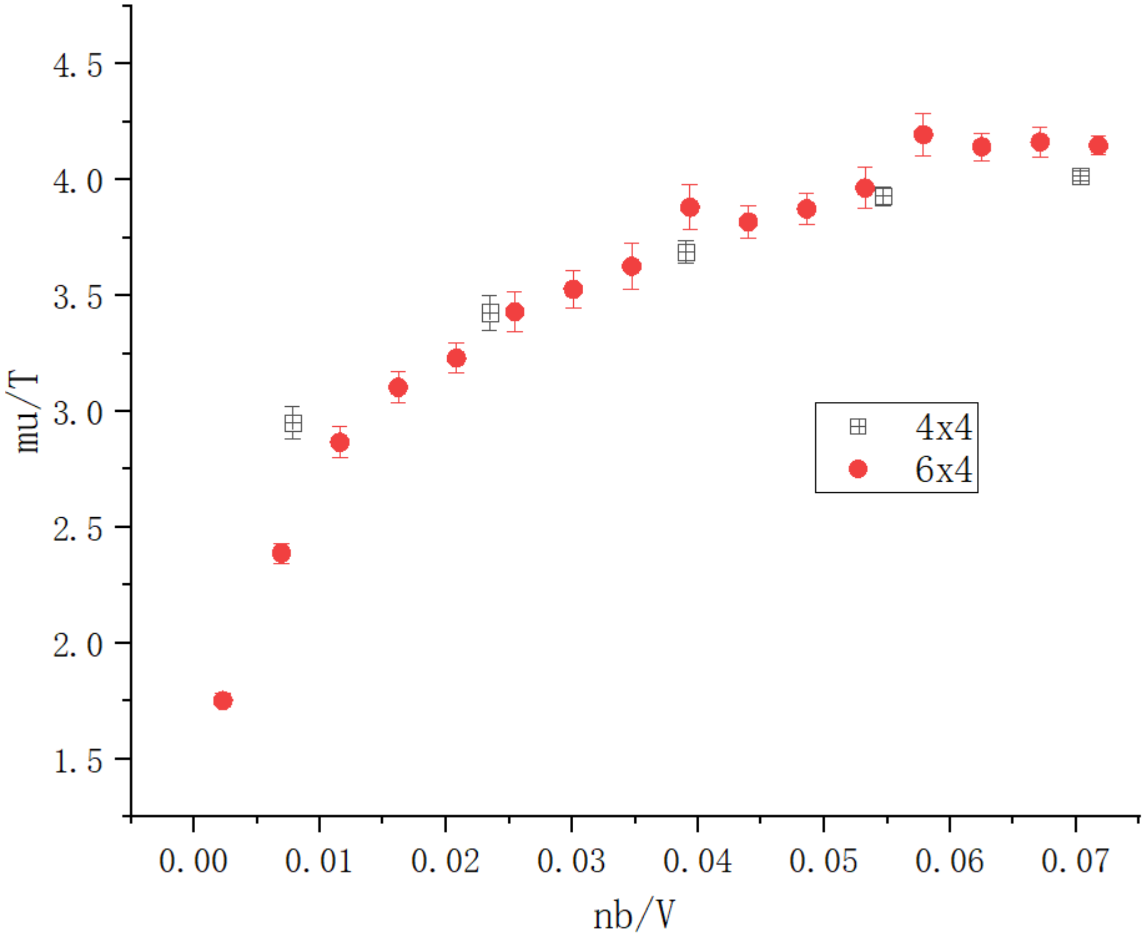


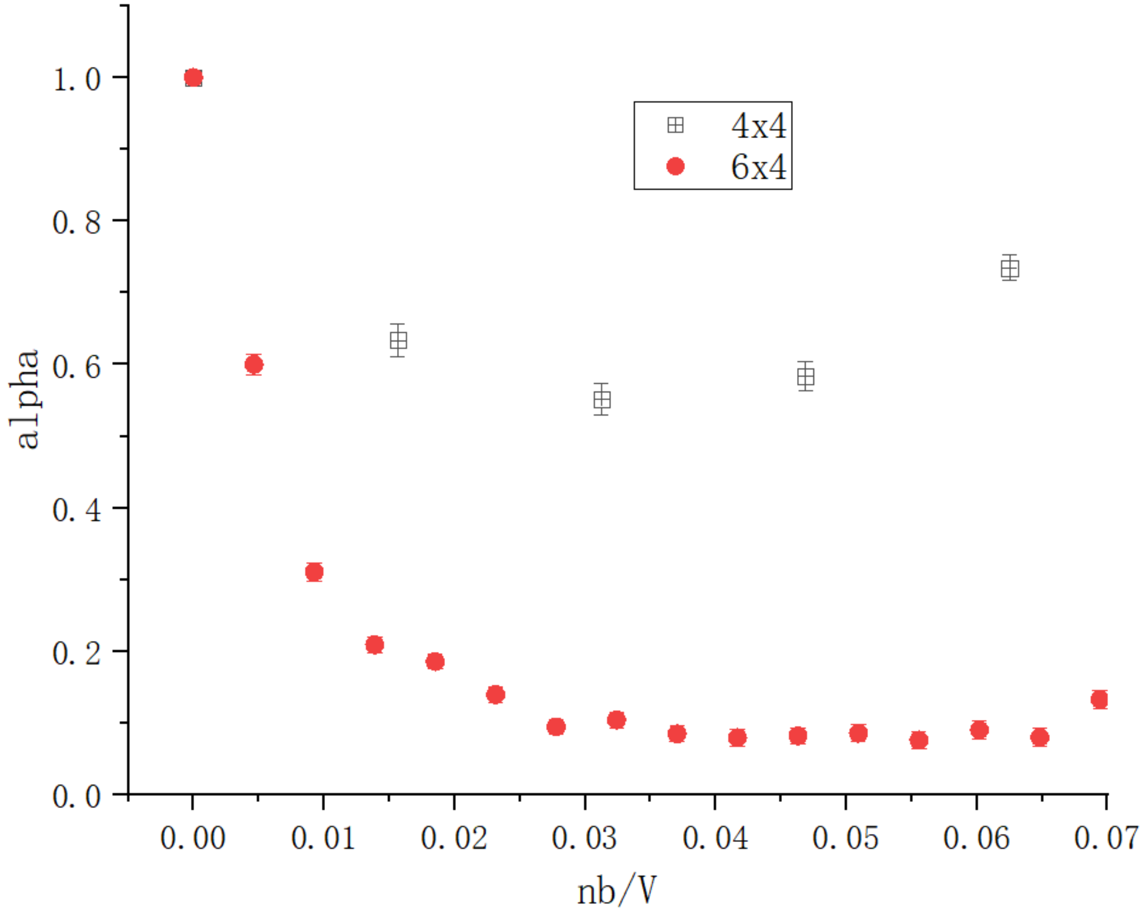
Kappa*3	beta	Lattice spacing (fm)	Temperature (GeV)	Pion mass (GeV)
0.158	5.07	0.31242(10)	0.1579	0.9421(15)



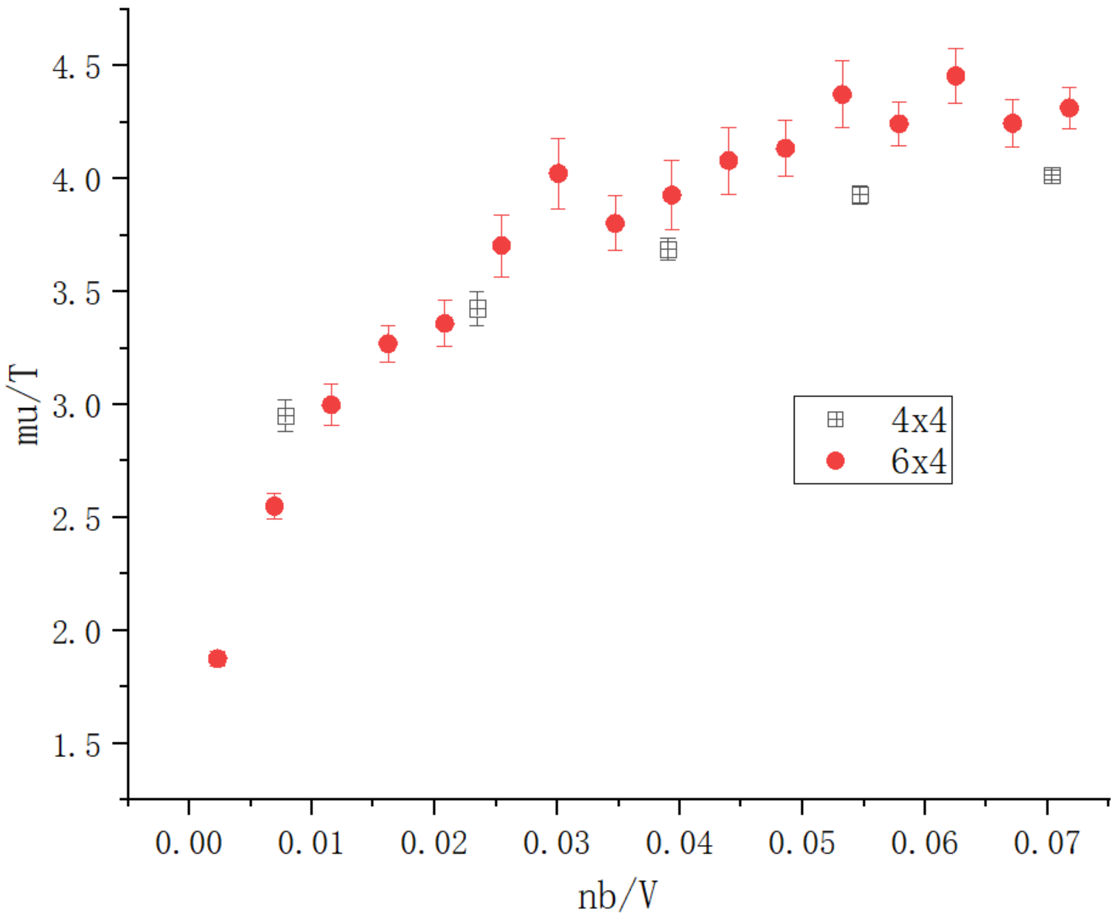


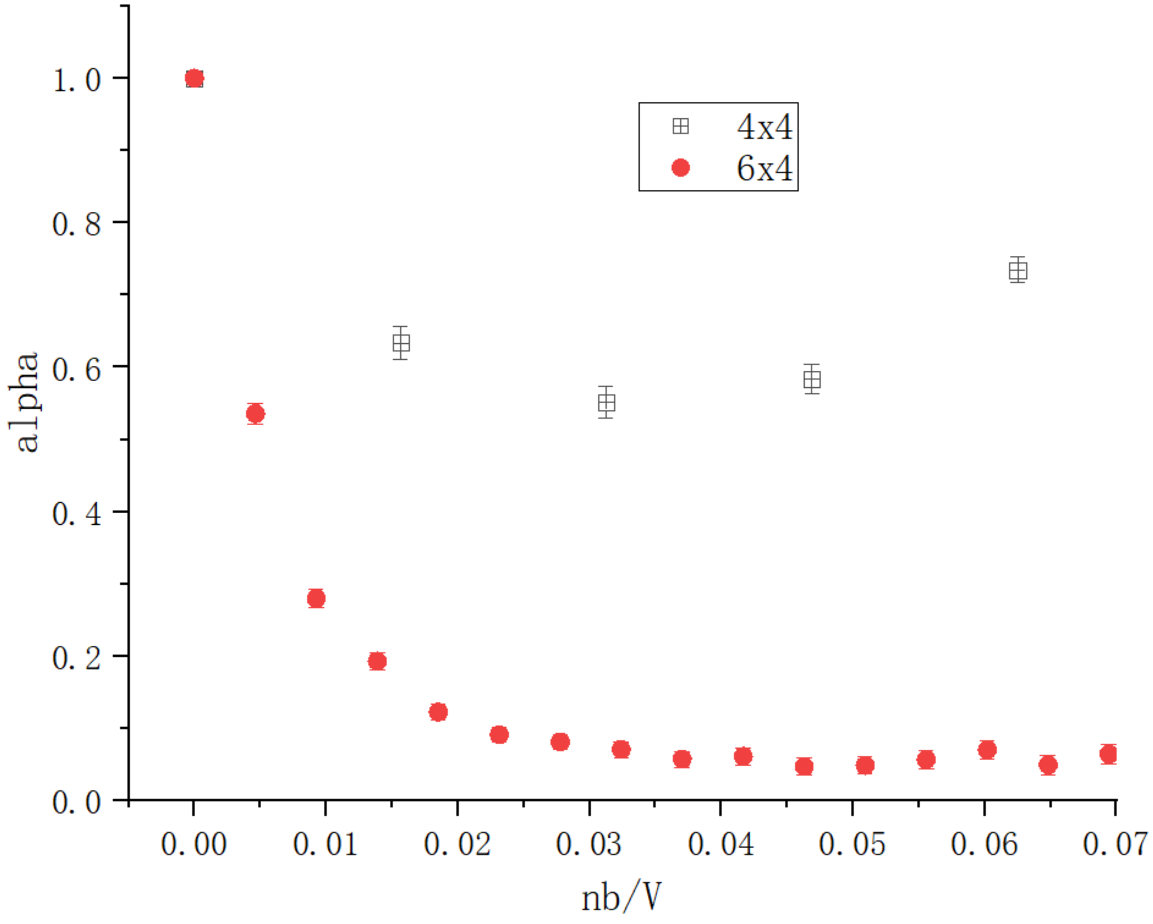
Kappa*3	beta	Lattice spacing (fm)	Temperature (GeV)	Pion mass (GeV)
0.158	5.05	0.31504(12)	0.1566	0.9447(15)



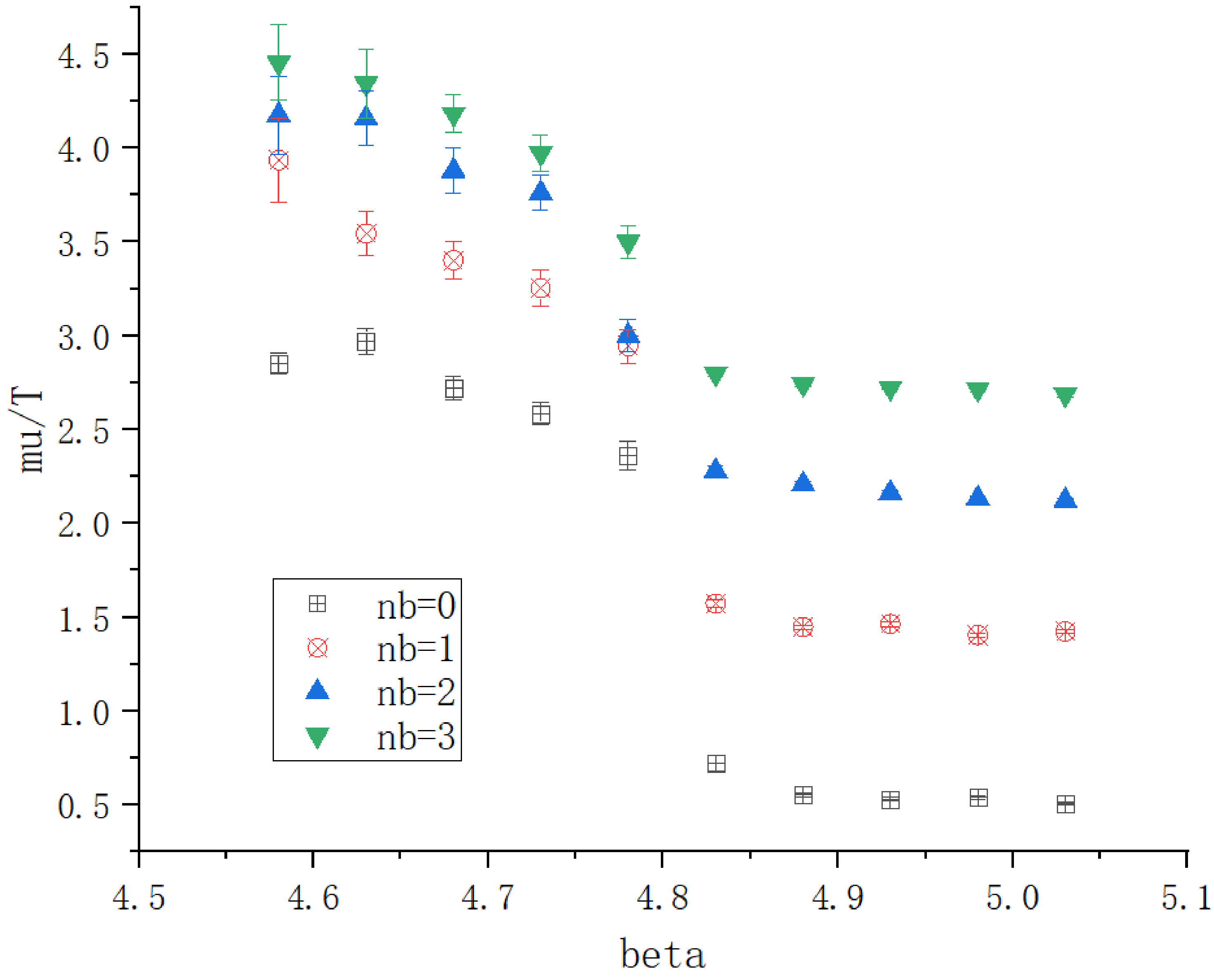


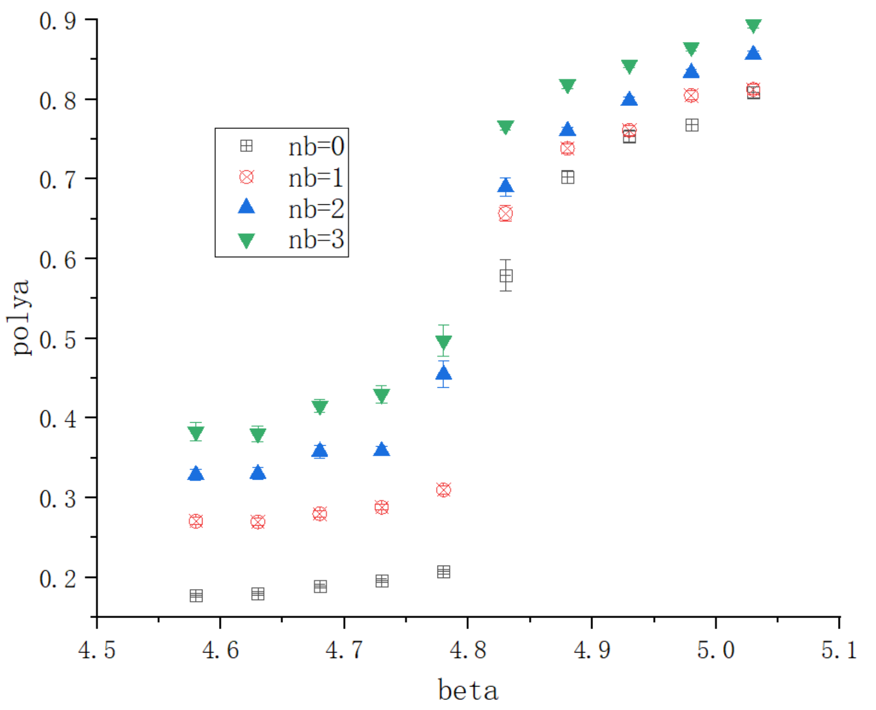
Kappa*3	beta	Lattice spacing (fm)	Temperature (GeV)	Pion mass (GeV)
0.158	5.03	0.31770(10)	0.1553	0.9471(15)

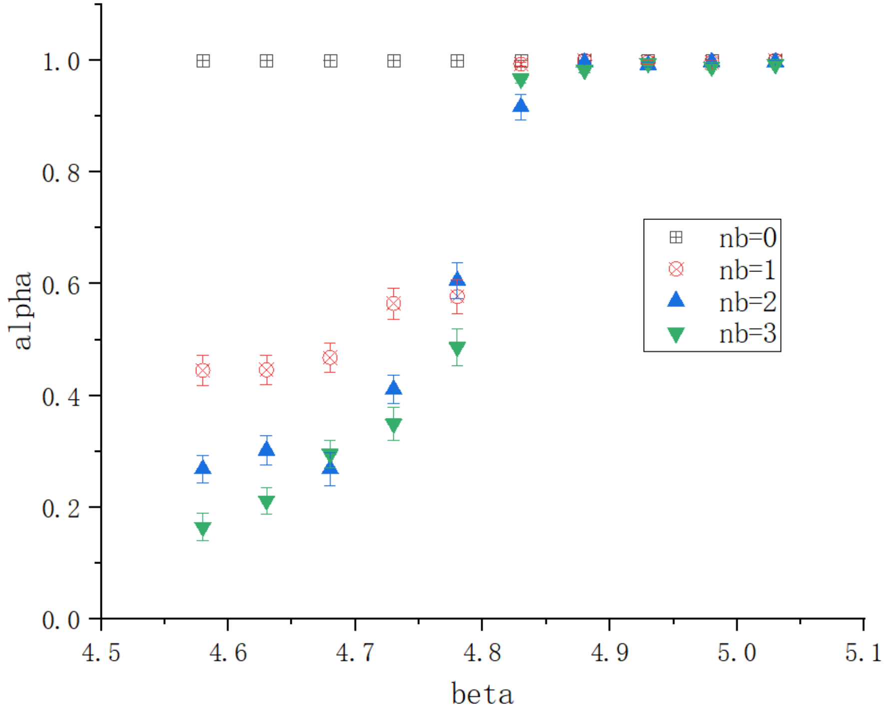




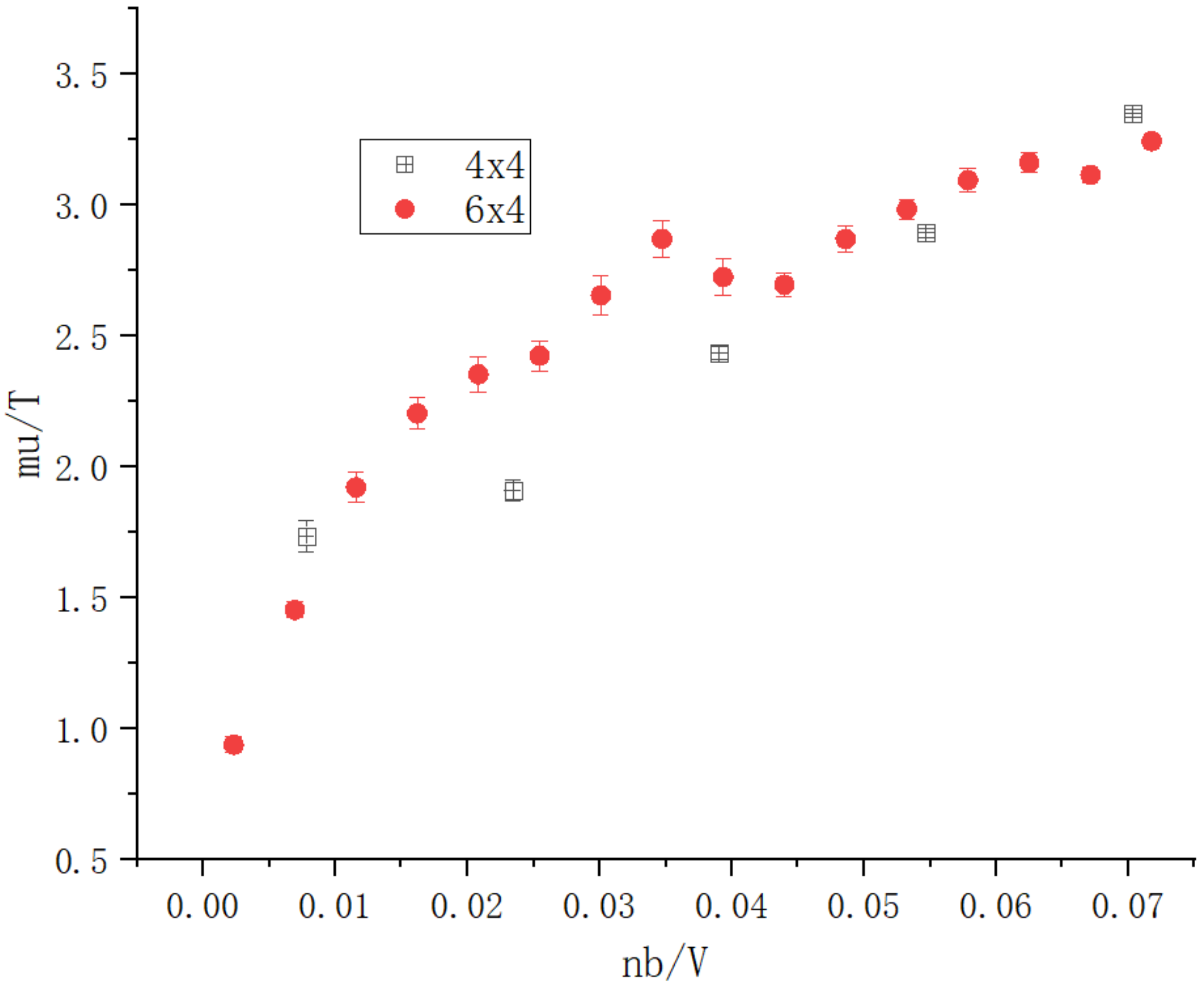
0.173*3	4.80	4.78	4.76
a	0.32286(11)	0.32519(8)	0.32713(8)
m_π	0.7485(13)	0.7542(13)	0.7587(10)

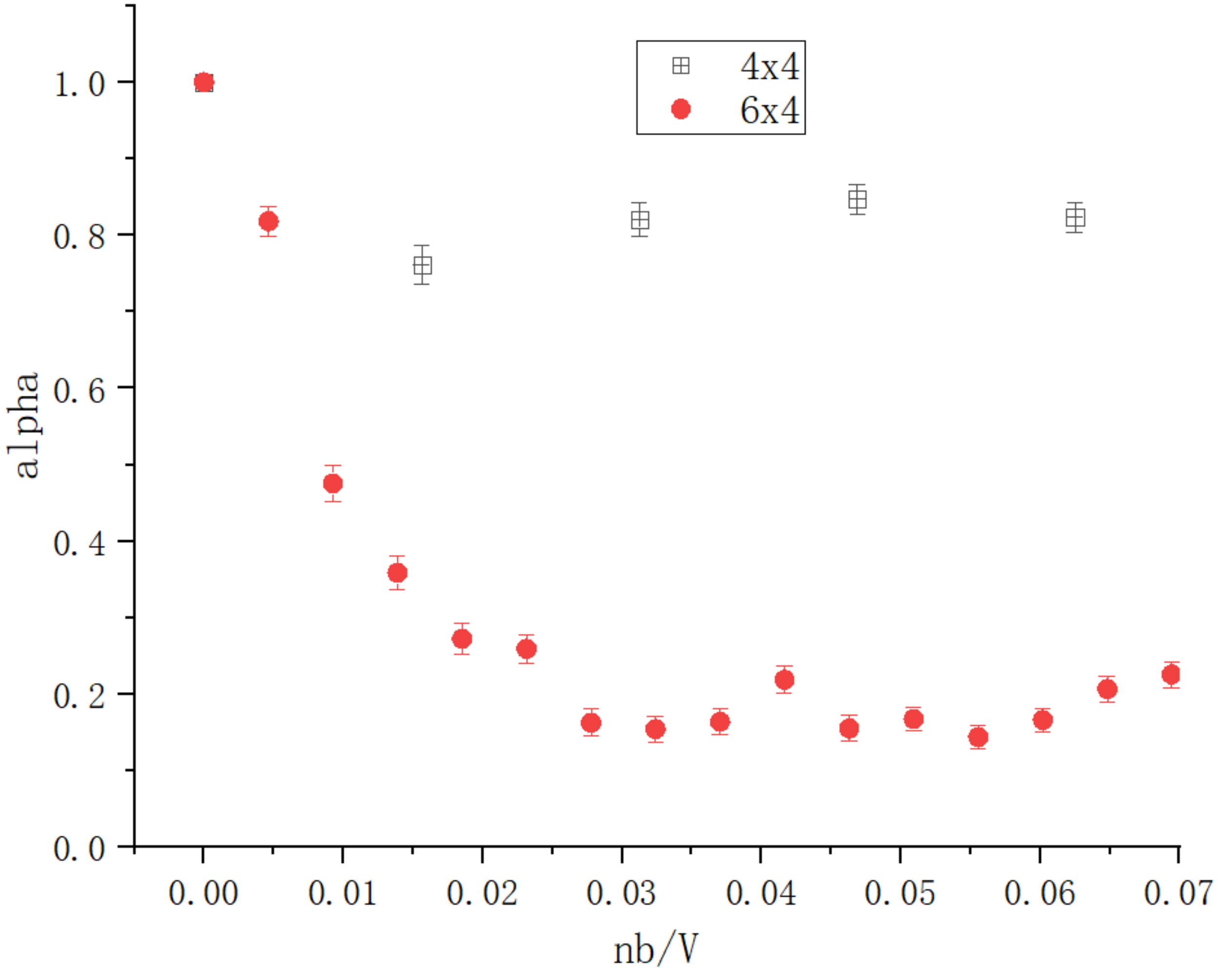




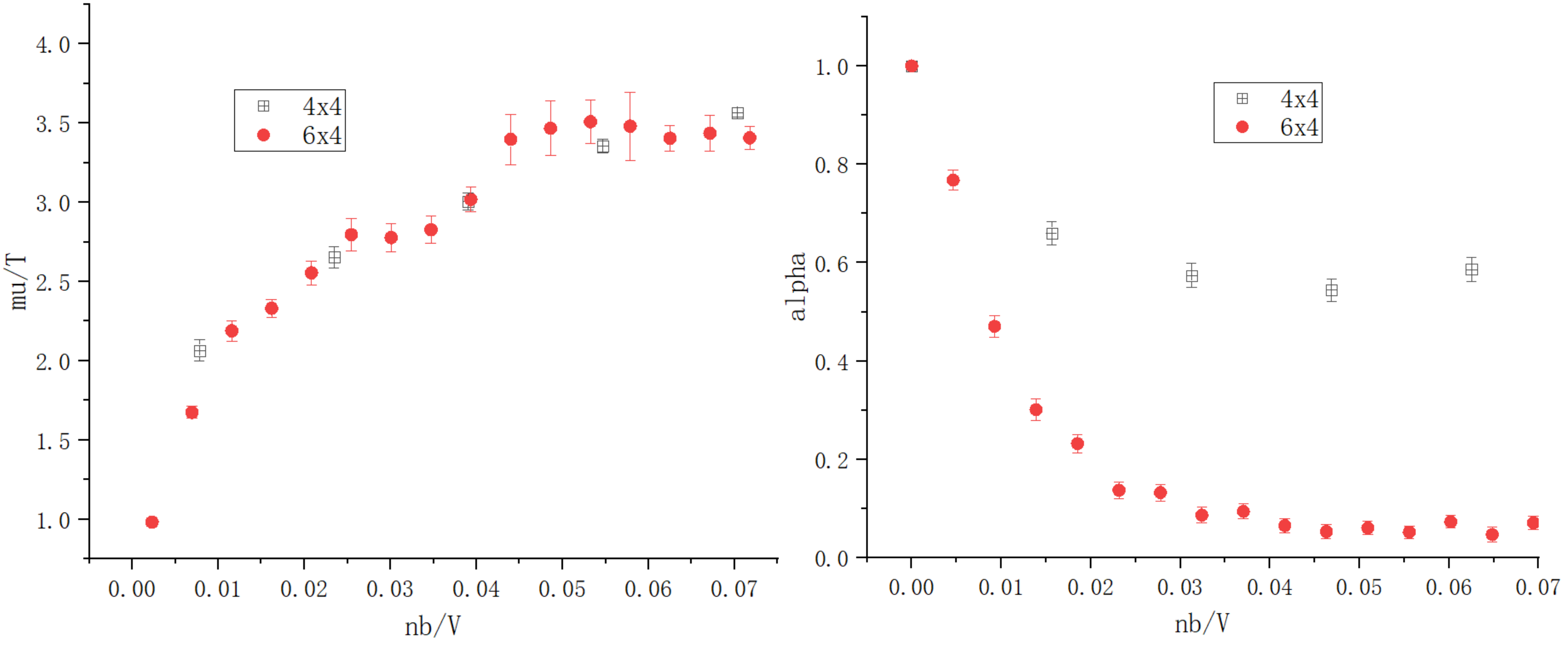


Kappa*3	beta	Lattice spacing (fm)	Temperature (GeV)	Pion mass (GeV)
0.173	4.80	0.32286(11)	0.1528	0.7485(13)

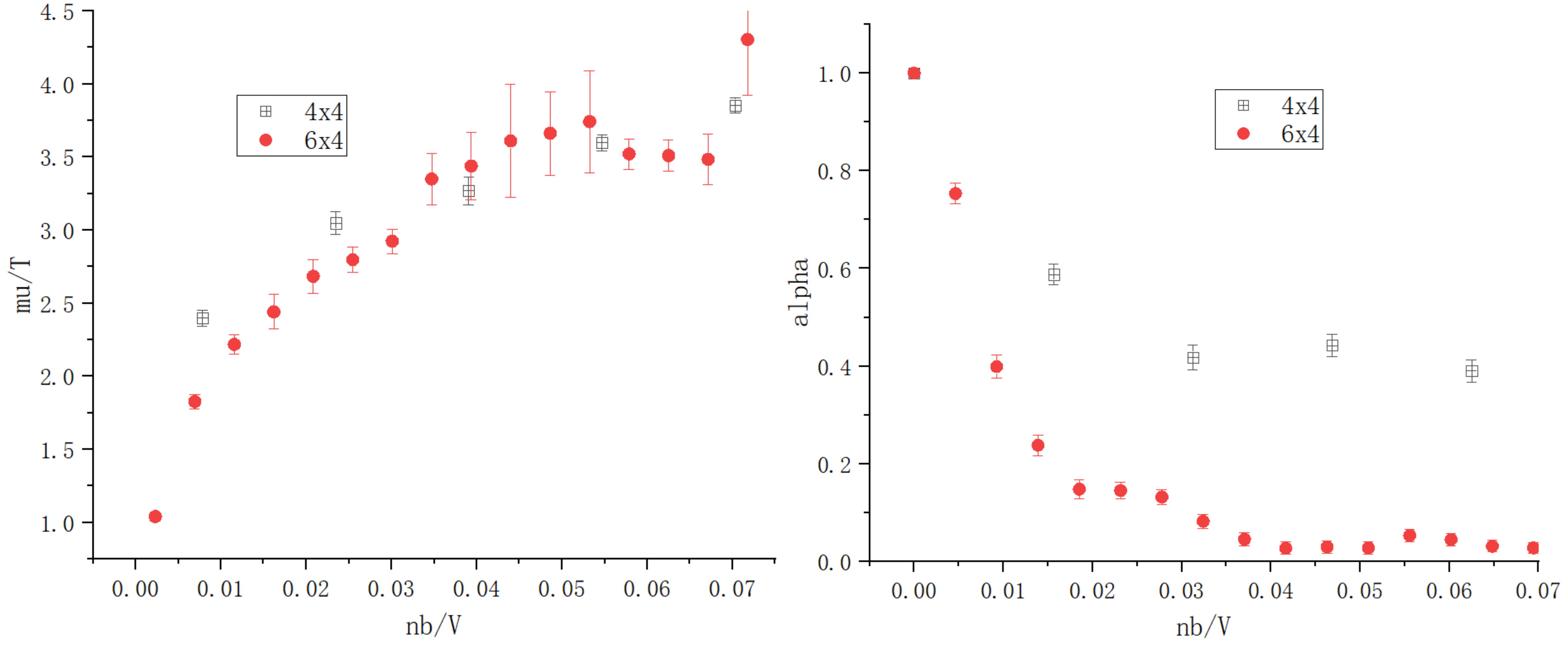




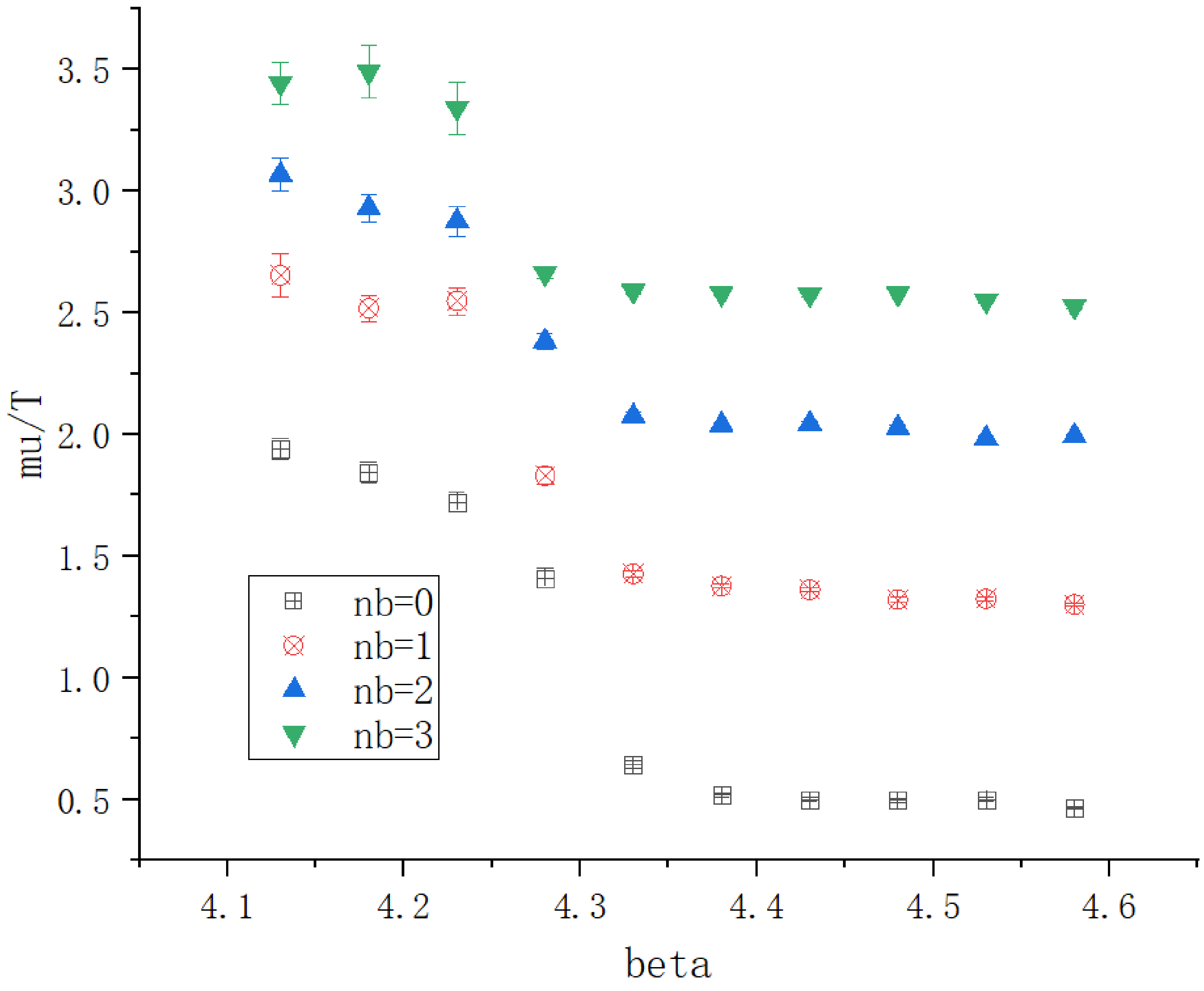
Kappa*3	beta	Lattice spacing (fm)	Temperature (GeV)	Pion mass (GeV)
0.173	4.78	0.32519(8)	0.1517	0.7542(13)

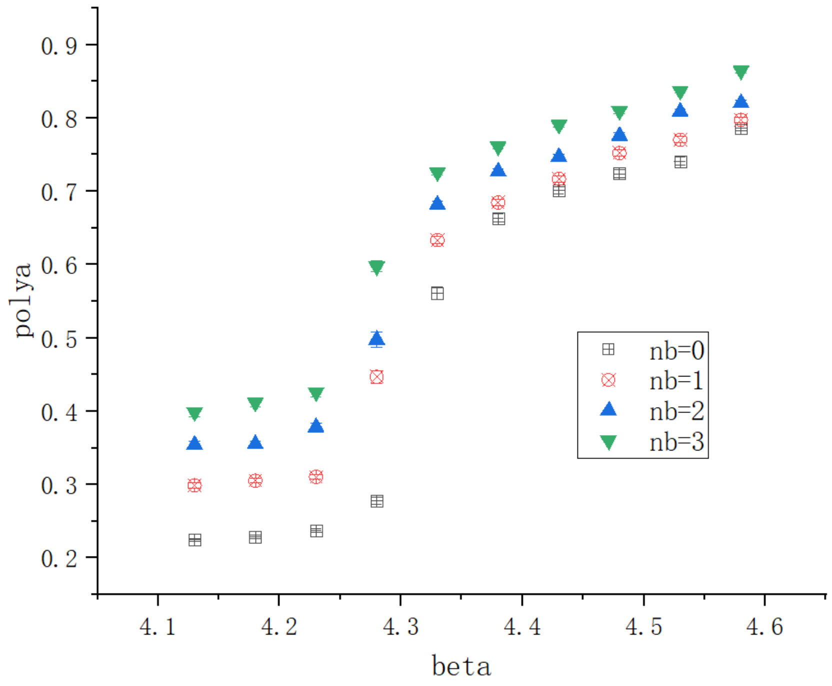


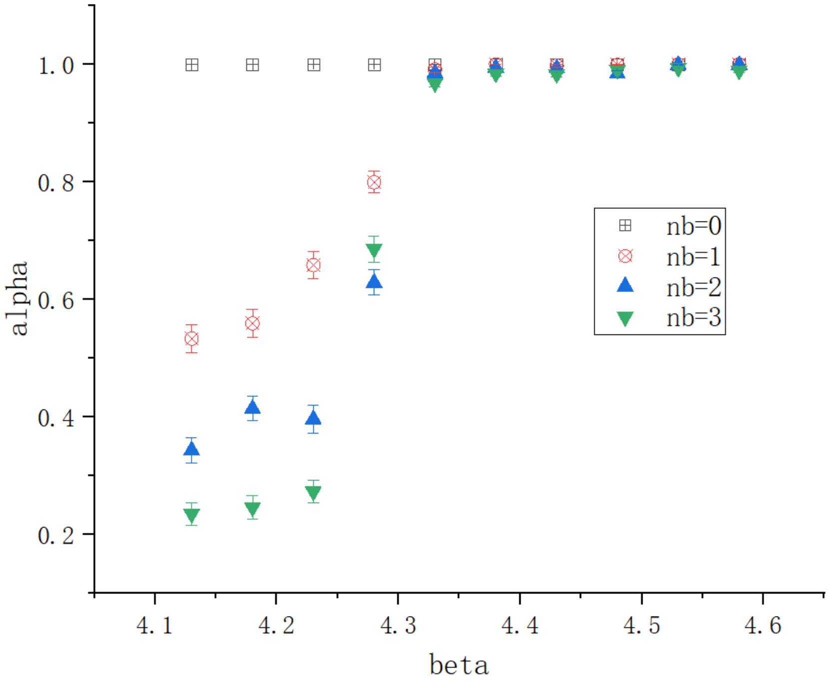
Kappa*3	beta	Lattice spacing (fm)	Temperature (GeV)	Pion mass (GeV)
0.173	4.76	0.32713(8)	0.1508	0.7587(10)



0.194*3	4.30	4.28	4.26
a	0.33258(11)	0.33453(3)	0.33559(4)
m_{π}	0.4842(13)	0.5009(12)	0.5103(13)







Summary and outlook

- We use **canonical ensembles** with **exact projection** to investigate the phase transition in **real chemical potential** and **different mass** on 4^3x4 and 6^3x4 lattice.

- It looks like there is **no first order phase transition** in the region we investigate.

- We plan to generate more configurations in 700MeV and 500MeV pion mass case.

BACKUP

Step size and accept rate

- The length of the trajectories: 0.5 with $\Delta\tau = 0.01$, accept rate>90%

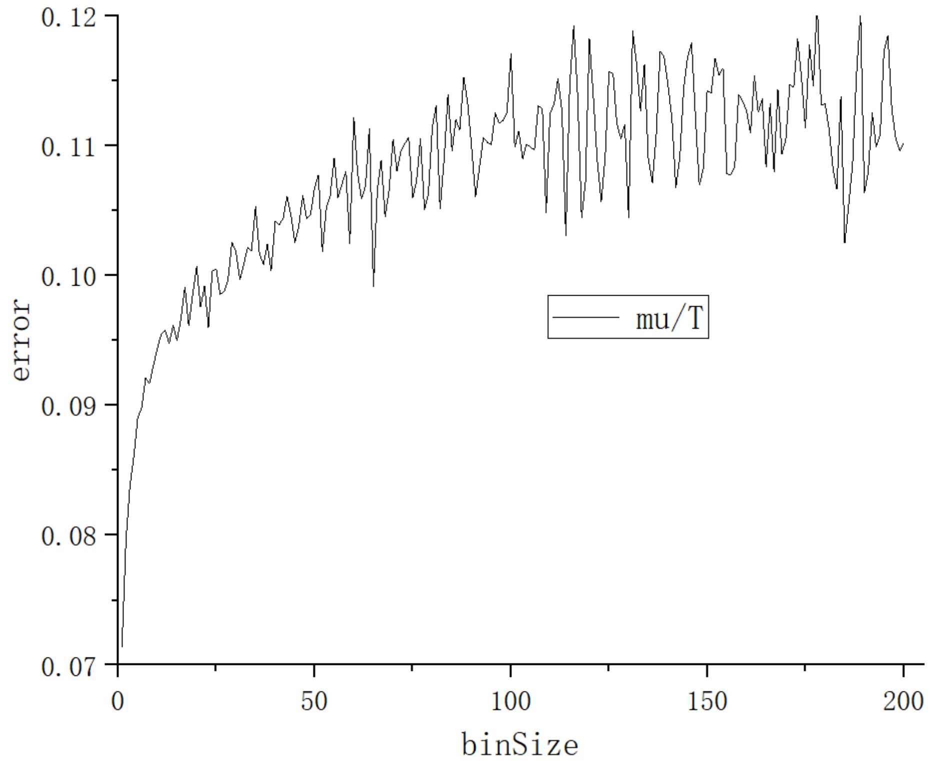
- Acceptance rate

6^3x4, nf3

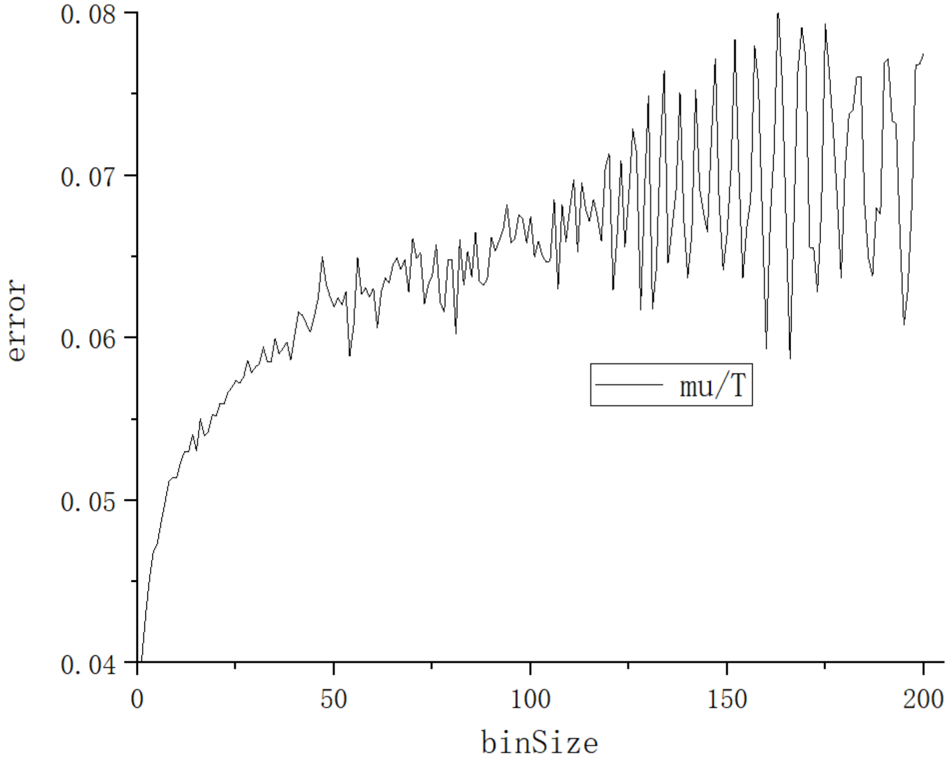
Kappa	Beta	Baryon number	Accept rate
0.173	4.80	3	39%
0.173	4.80	12	36%
0.173	4.76	3	43%
0.173	4.76	12	28%

Auto-correlation and binning

6^3x4, kappa=0.158, beta = 5.15, n_b=12



6^3x4, kappa=0.173, beta = 4.80, n_b=3



Cost

$$\det_k M^{N_f}(U) \equiv \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\, e^{-ik\phi} \det M(m,\phi;U)^{N_f}$$

- 4^3x4 one step projection, one core, 6 seconds

One step	Trajectories	Baryon num	Beta	Mass	total
6s	6×10^4	5	3	3	4.5×10^3 core*hrs.

- 6^3X4 one step projection, one core, 250 seconds

One step	Trajectories	Baryon num	Beta	Mass	total
250s	2×10^5	16	3	3	2.0×10^6 core*hrs.

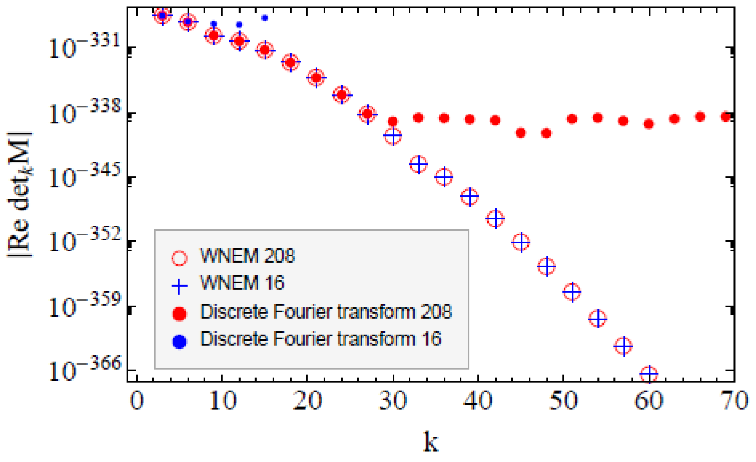
$$\det_k M^{N_f}(U) \equiv \frac{1}{2\pi}\int_0^{2\pi} d\phi\, e^{-ik\phi} \det M(m,\phi;U)^{N_f}$$

Projection of the determinant

Discrete Fourier transform

Winding number expansion → **Matrix reduction technique**

$$\det_k M^{N_f}(U) \approx \frac{1}{N}\sum_{j=0}^{N-1} e^{-ik\phi_j} \det M(U_{\phi_j})^{N_f}, \qquad \phi_j = \frac{2\pi j}{N}$$



- Exact projection of the determinant
- The size of the reduced matrix is independent of the temporal lattice extent
- Factor out the chemical potential