arXiv:2207.14132

The "distance" between various discretized fermion actions

2023.04.28

Dian-Jun Zhao

ITP, CAS

In Collaboration with Gen Wang, Fangcheng He, Kuan Zhang, Luchang Jin, Peng Sun, and Yi-Bo Yang

CONTENT

1. Introduction

Hybrid Monte Carlo

Generate configuration

CG, MG, ...

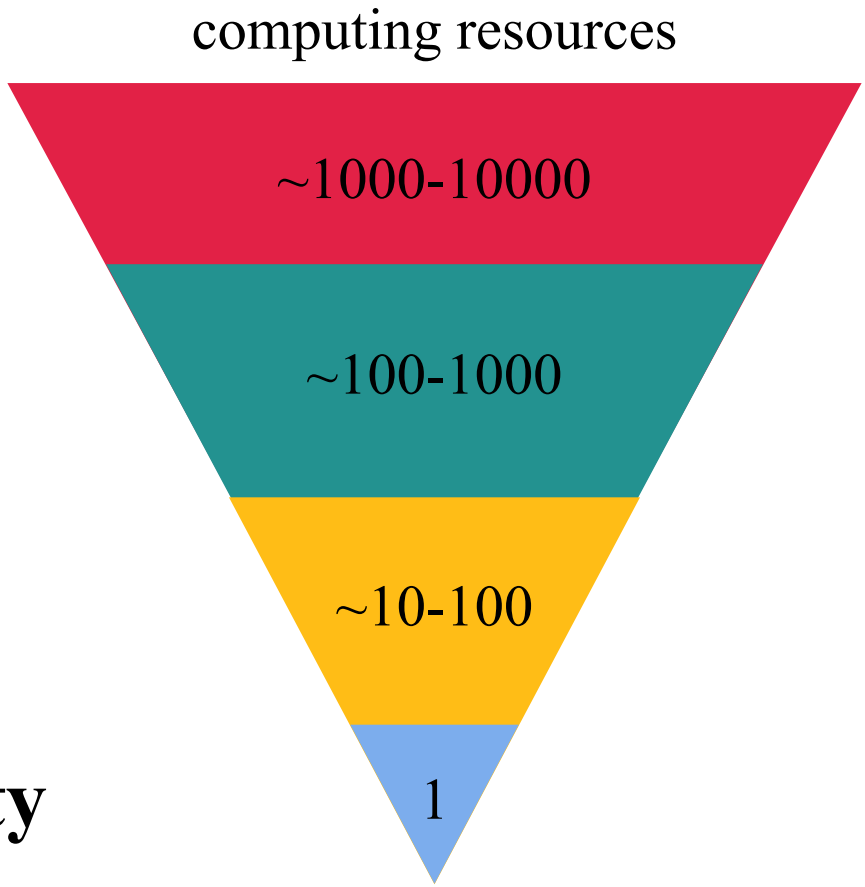
Calculate propagator

Contraction

Calculate correlator

Re-sampling, Non-linear fit, ...

Fit to the physical quantity



1. Introduction

Staggered fermion (MILC)

$$S_{\mathrm{st}}(m) = \sum_{x,y} \bar{\psi}_{\mathrm{st}}(x) D_{\mathrm{st}}(x,y,m) \psi_{\mathrm{st}}(y)$$

$$\psi_{st}(x) = \gamma_4^{x_4} \gamma_1^{x_1} \gamma_2^{x_2} \gamma_3^{x_3} \psi(x)$$

Fermion doubling ❌ 👉HISQ
Chrial symmetry ✅

Clover fermion (CLQCD)

$$S_w(m) = \sum_{x,y} \bar{\psi}_w(x) D_w(x,y,m) \psi_w(y)$$

$$S_{\mathrm{cl}}(m) = S_{\mathrm{w}}(m) + c_{\mathrm{sw}} \sum_x \bar{\psi}(x) \sigma_{\mu\nu} F^{\mu\nu} \psi(x)$$

Fermion doubling ✅
Chrial symmetry ❌

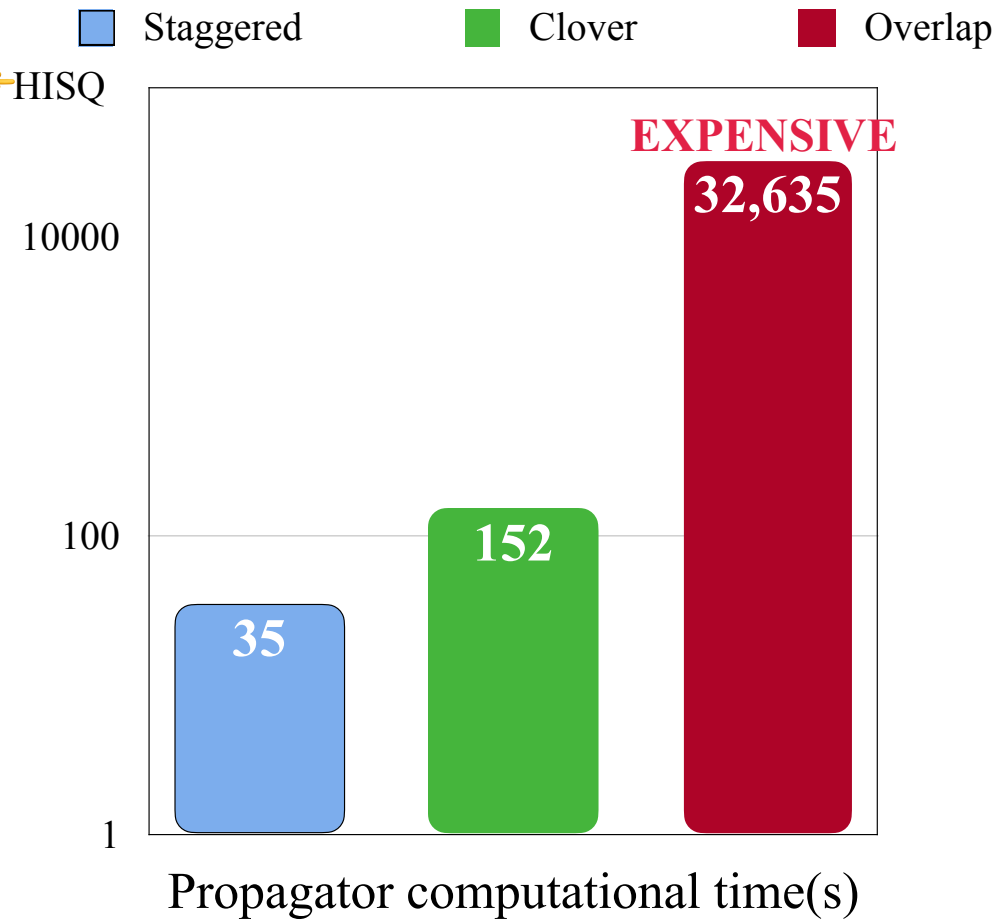
Overlap fermion (JLQCD)

$$S_{\mathrm{c}}(m) = \sum_x \bar{\psi}(x) [(1 - \frac{m}{2\rho}) D_{\mathrm{ov}}(x,y,m) + \delta_{x,y} m] \psi(y)$$

DW fermion (RBC, …)

$$S^5_{\mathrm{GDW}}(m) = \sum_{x,y} \bar{\psi}(x) D_{\mathrm{GDW}}(x,y,m) \psi(y)$$

Fermion doubling ✅
Chrial symmetry ✅



1. Introduction

Definition

MAPQχPT

Val_Val pion mass: $m_{\pi,vv}$

Sea_Sea pion mass: $m_{\pi,ss}$

Val_Sea pion mass: $m_{\pi,vs}$

$$\Delta_{\mathrm{mix}}^{B/A}(m_{\pi,\mathrm{vv}}, m_{\pi,\mathrm{ss}}, a) \equiv m_{\pi,\mathrm{vs}}^2 - \frac{m_{\pi,\mathrm{vv}}^2 + m_{\pi,\mathrm{ss}}^2}{2}$$

Previous works

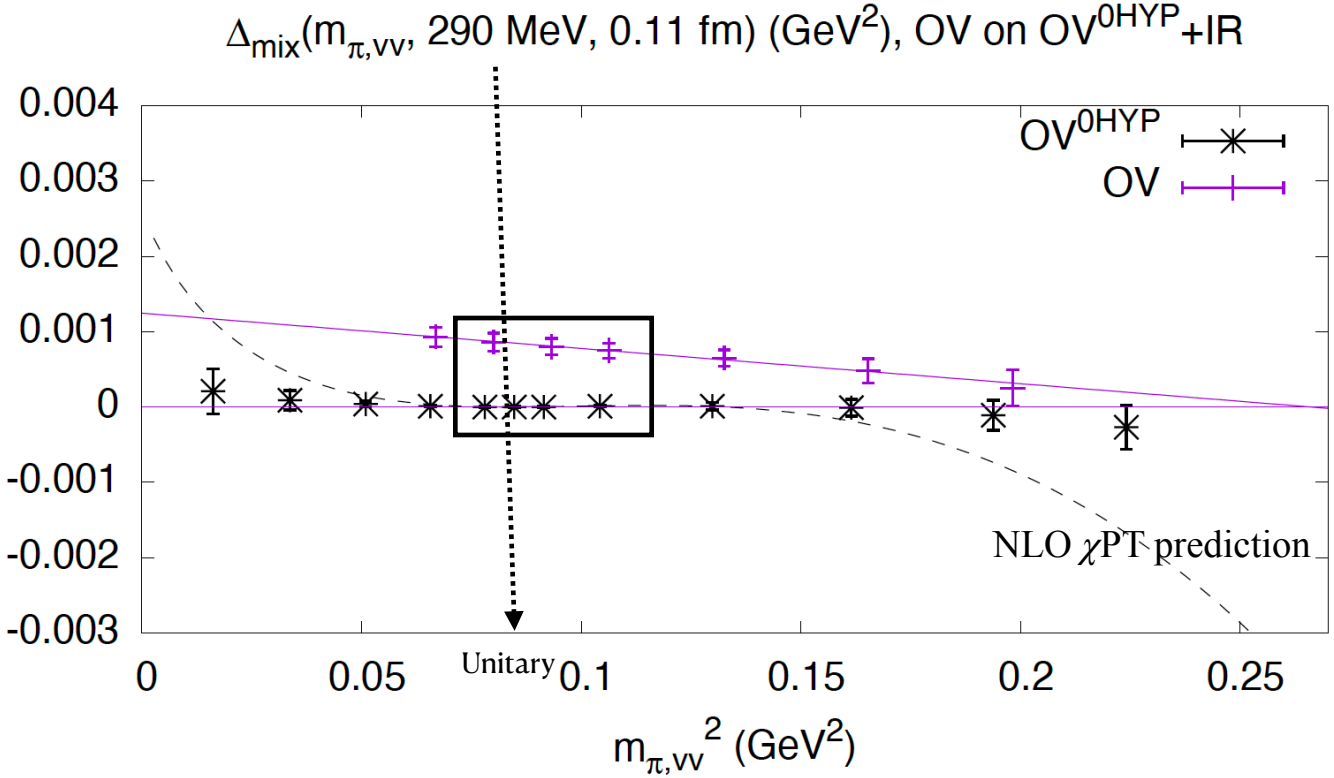
Valence	**Sea**	$\delta m_\pi = m_{\pi,vs} - m_{\pi,ss}$ (MeV)	a(fm)	$m_{\pi,ss}$(MeV)	
Overlap	Clover	153	0.09	300	PoS LATTICE2007, 115 (2007)
DW	Staggered	30-60	0.13-0.09	310	Phys. Rev. D86, 014501 (2012)
Overlap	DW	~10	0.11-0.08	300-400	Phys. Rev. D77, 094505 (2008) Phys. Rev. D77, 114501 (2008) Phys. Rev. D96, 054513 (2017)

Innovation

1. based on the calculation at different lattice spacing
2. pion mass $m_{\pi,ss}$ is not limited to the case of ~300MeV
3. mixed action effect with kinds of the valence and sea fermion combinations

2. Methodology and setup

Contribution of the delta mix definition to the NLO at $m_{\pi,\mathrm{vv}} \neq m_{\pi,\mathrm{ss}}$



$$\Delta_{\mathrm{mix}}(m_{\pi,\mathrm{vv}}, m_{\pi,\mathrm{ss}}, a) \equiv m^2_{\pi,\mathrm{vs}} - \frac{m^2_{\pi,\mathrm{vv}} + m^2_{\pi,\mathrm{ss}}}{2}$$

$$\downarrow m_{\pi,\mathrm{vv}} = m_{\pi,\mathrm{ss}}$$

$$\Delta_{\mathrm{mix,uni}}(m_\pi, a) \equiv \Delta_{\mathrm{mix}}(m_\pi, m_\pi, a)$$

2. Methodology and setup

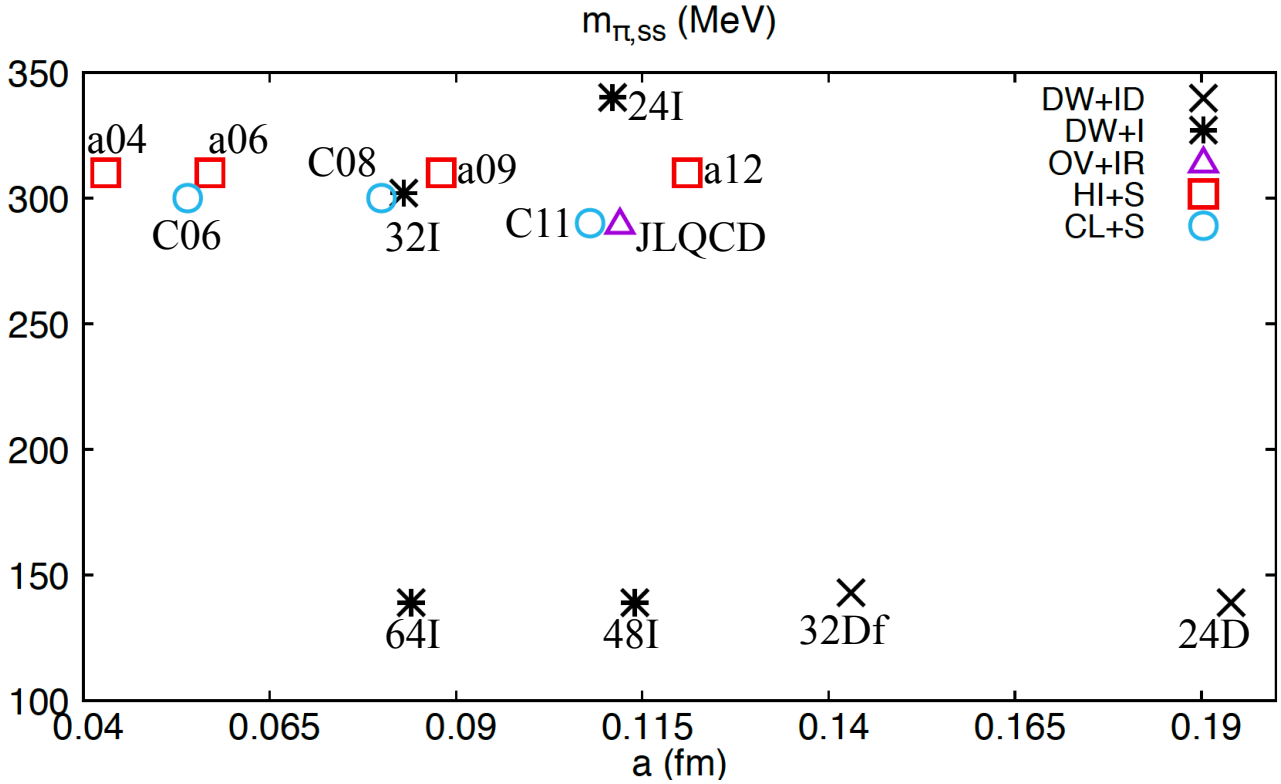
Valence fermion actions

HI: HISQ action without any additional smearing on the gauge link;

CL: Clover fermion with 1-step HYP smearing and tree level tadpole improved clover coefficient c_{sw};

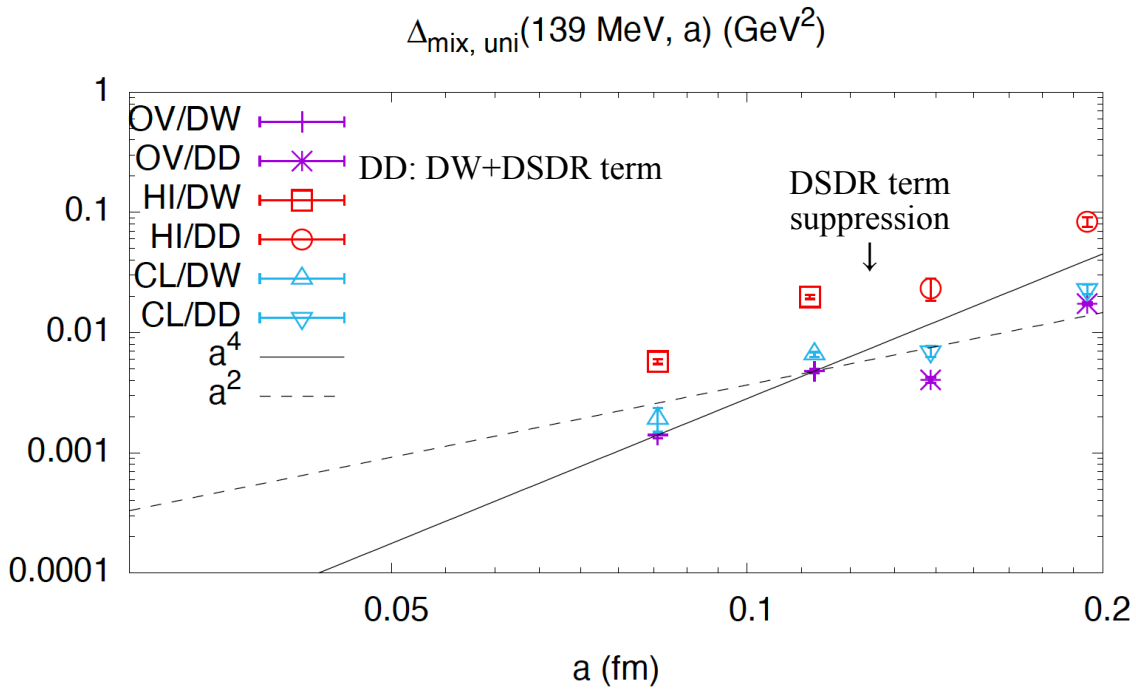
OV: Overlap fermion with 1-step HYP smearing and $\rho = 1.5$.

Sea fermion actions



Information of the ensembles used in this calculation

3. Result



Effect of DSDR term on the delta mix on the DW sea

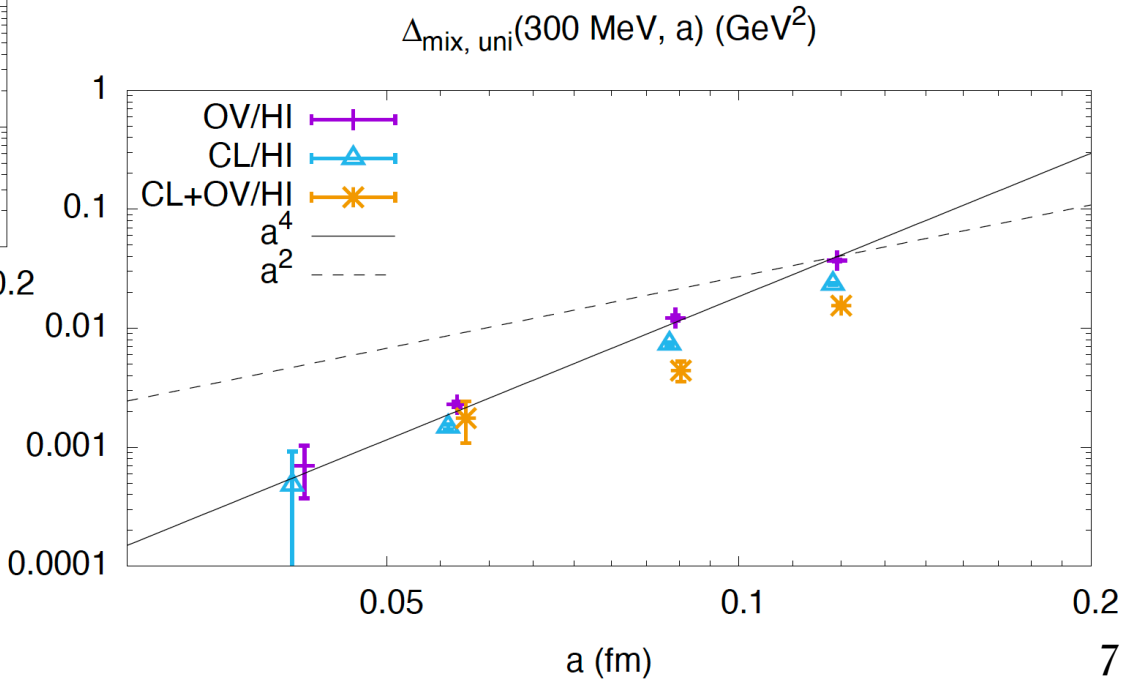
$$|\Delta^{B/A}_{mix,uni} - \Delta^{C/A}_{mix,uni}| \le \bar{\Delta}^{B+C/A}_{mix,uni} \le \Delta^{B/A}_{mix,uni} + \Delta^{C/A}_{mix,uni}$$

$$\downarrow$$

$$|\Delta^{OV/HI}_{mix,uni} - \Delta^{CL/HI}_{mix,uni}| \le \bar{\Delta}^{OV+CL/HI}_{mix,uni} \le \Delta^{OV/HI}_{mix,uni} + \Delta^{CL/HI}_{mix,uni} \quad \rightarrow$$

$$\bar{\Delta}^{B+C/A}_{mix,uni}(m_\pi, a) \equiv m^2_{\pi,BC} - \frac{m^2_{\pi,BB} + m^2_{\pi,CC}}{2}\Big|_{m_{\pi,BB}=m_{\pi,CC}=m_{\pi,AA}=m_\pi}$$

Delta mix on the HISQ sea

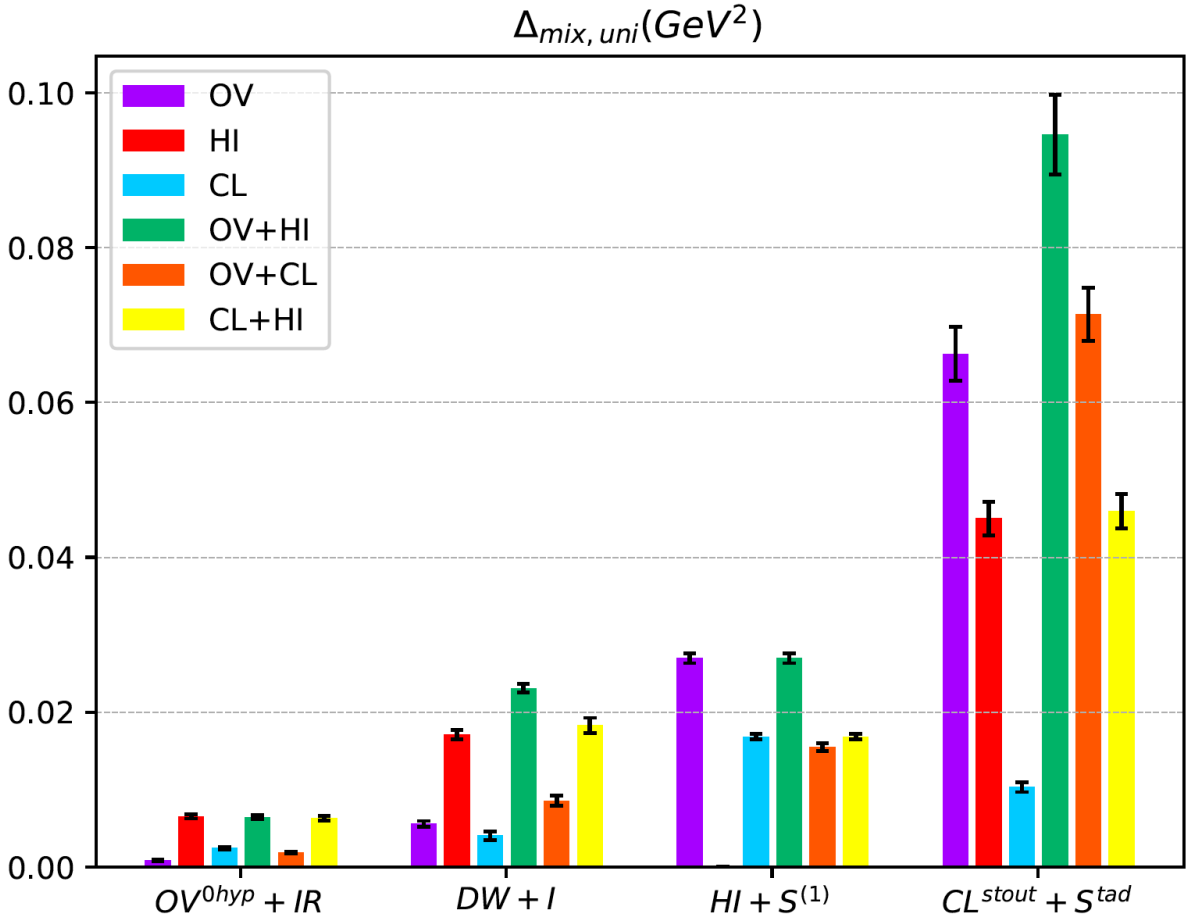


3. Result

$$\Delta_{\mathrm{mix}}(m_{\pi,\mathrm{vv}}, m_{\pi,\mathrm{ss}}, a) \equiv m^2_{\pi,\mathrm{vs}} - \frac{m^2_{\pi,\mathrm{vv}} + m^2_{\pi,\mathrm{ss}}}{2}$$

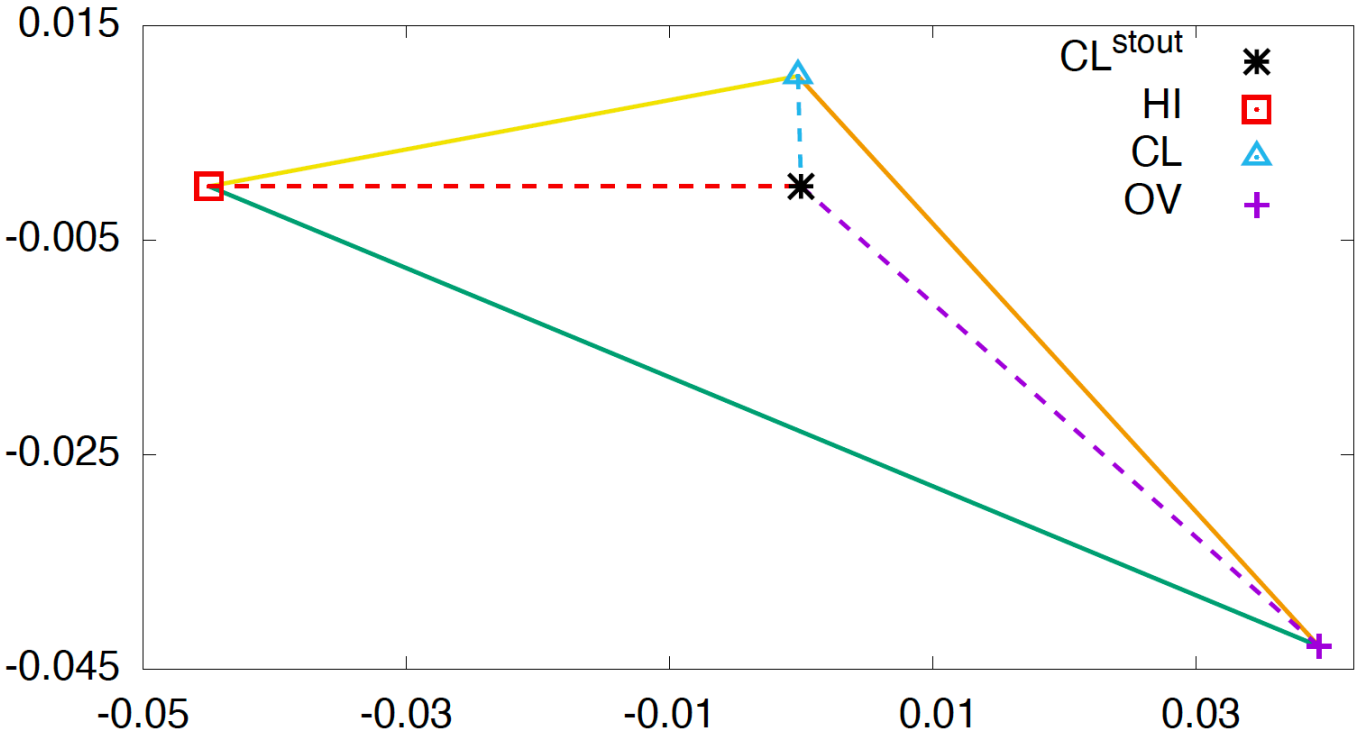
$$\Delta_{\mathrm{mix,uni}}(m_\pi, a) \equiv \Delta_{\mathrm{mix}}(m_\pi, m_\pi, a)$$

$$\bar{\Delta}^{\mathrm{B+C/A}}_{\mathrm{mix,uni}}(m_\pi, a) \equiv m^2_{\pi,\mathrm{BC}} - \frac{m^2_{\pi,\mathrm{BB}} + m^2_{\pi,\mathrm{CC}}}{2}\Big|_{m_{\pi,\mathrm{BB}}=m_{\pi,\mathrm{CC}}=m_{\pi,\mathrm{AA}}=m_\pi}$$



3. Result

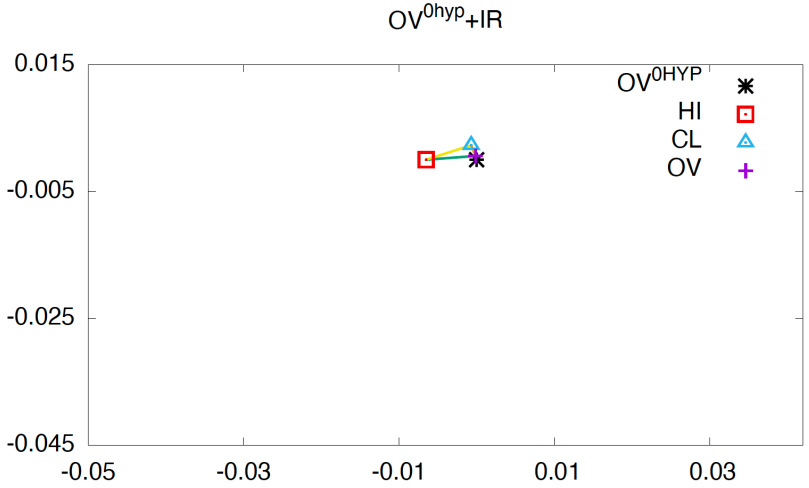
$$\bar{\Delta}^{\mathrm{B+C/A}}_{\mathrm{mix,uni}}(m_\pi, a) \equiv m^2_{\pi,\mathrm{BC}} - \frac{m^2_{\pi,\mathrm{BB}} + m^2_{\pi,\mathrm{CC}}}{2}\Big|_{m_{\pi,\mathrm{BB}}=m_{\pi,\mathrm{CC}}=m_{\pi,\mathrm{AA}}=m_\pi}$$

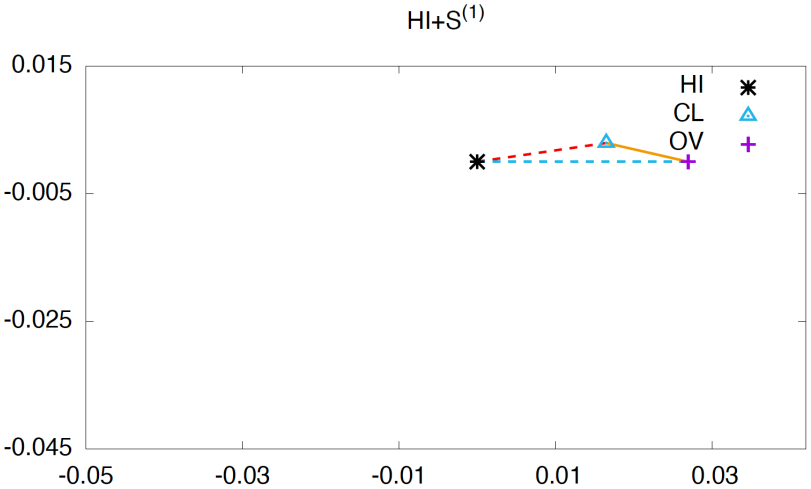


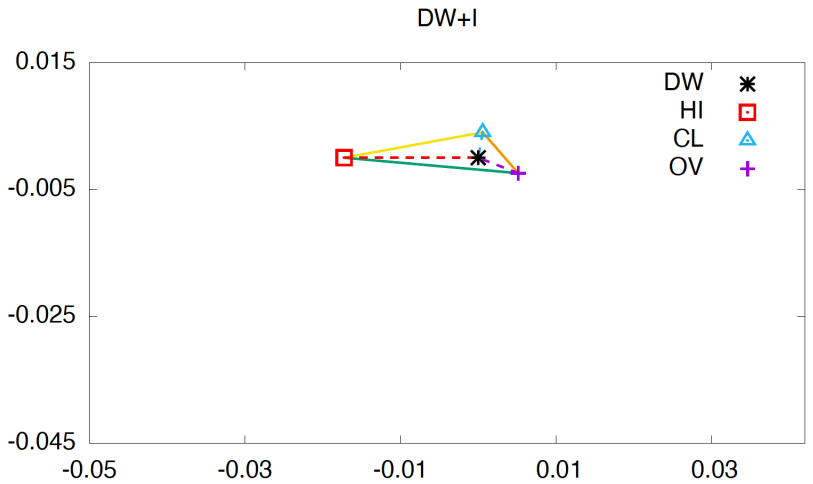
$$|\Delta^{\mathrm{B/A}}_{\mathrm{mix,uni}} - \Delta^{\mathrm{C/A}}_{\mathrm{mix,uni}}| \le \bar{\Delta}^{\mathrm{B+C/A}}_{\mathrm{mix,uni}} \le \Delta^{\mathrm{B/A}}_{\mathrm{mix,uni}} + \Delta^{\mathrm{C/A}}_{\mathrm{mix,uni}} \qquad |\bar{\Delta}^{\mathrm{B+D/A}}_{\mathrm{mix,uni}} - \bar{\Delta}^{\mathrm{C+D/A}}_{\mathrm{mix,uni}}| \le \bar{\Delta}^{\mathrm{B+C/A}}_{\mathrm{mix,uni}} \le \bar{\Delta}^{\mathrm{B+D/A}}_{\mathrm{mix,uni}} + \bar{\Delta}^{\mathrm{C+D/A}}_{\mathrm{mix,uni}}$$

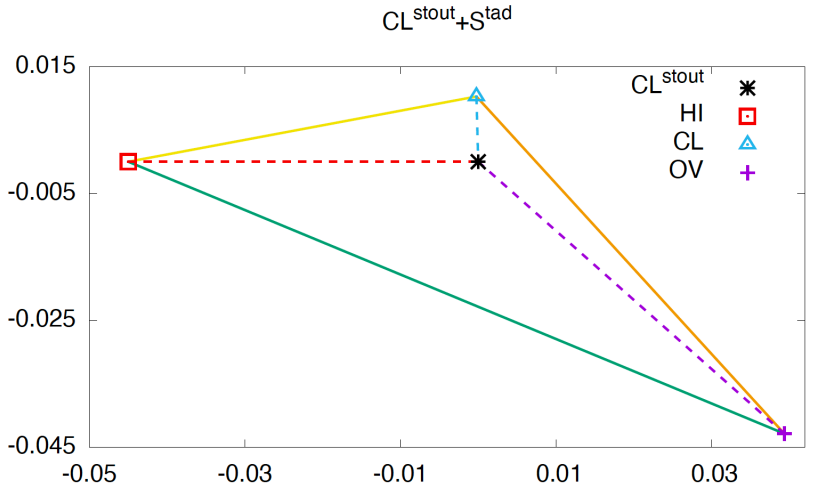
3. Result

$$\bar{\Delta}^{\mathrm{B+C/A}}_{\mathrm{mix,uni}}(m_\pi, a) \equiv m^2_{\pi,\mathrm{BC}} - \frac{m^2_{\pi,\mathrm{BB}} + m^2_{\pi,\mathrm{CC}}}{2}\Big|_{m_{\pi,\mathrm{BB}}=m_{\pi,\mathrm{CC}}=m_{\pi,\mathrm{AA}}=m_\pi}$$









$$|\Delta^{\mathrm{B/A}}_{\mathrm{mix,uni}} - \Delta^{\mathrm{C/A}}_{\mathrm{mix,uni}}| \le \bar{\Delta}^{\mathrm{B+C/A}}_{\mathrm{mix,uni}} \le \Delta^{\mathrm{B/A}}_{\mathrm{mix,uni}} + \Delta^{\mathrm{C/A}}_{\mathrm{mix,uni}}$$

$$|\bar{\Delta}^{\mathrm{B+D/A}}_{\mathrm{mix,uni}} - \bar{\Delta}^{\mathrm{C+D/A}}_{\mathrm{mix,uni}}| \le \bar{\Delta}^{\mathrm{B+C/A}}_{\mathrm{mix,uni}} \le \bar{\Delta}^{\mathrm{B+D/A}}_{\mathrm{mix,uni}} + \bar{\Delta}^{\mathrm{C+D/A}}_{\mathrm{mix,uni}}$$

arXiv:2207.14132

4. Summary and outlook

1. Leading mixed action effect would be a^4, and are small when the sea fermion action has the chiral symmetry (page 7). So it is closer than the previous estimate, especially at smaller lattice spacings;
2. Ginsburg-Wilson relation fermion+Iwasaki gauge – smallest Δ_{mix} , $CL_{stout} + S_{tad}$ – largest Δ_{mix} (page 8). The strong sensitivity of Δ_{mix} to sea fermion chirality (page 3);
3. Mechanism of a^4 scaling of Δ_{mix} and whether the Δ_{mix} can display a^2 scaling in other observables need further studies.

THANKS FOR YOUR ATTENTION!