国产组态的参数标定

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Symanzik Action

In this work, we generate the 2+1 flavor full QCD ensembles using the tadpole improved tree level Symanzik gauge action

$$S_g = \frac{1}{N_c} \sum_x \mathrm{Re} \sum_{x,\mu<\nu} \mathrm{Tr}\left[1 - c_0 \mathcal{P}^U_{\mu,\nu}(x) - c_1 \mathcal{R}^U_{\mu,\nu}(x)\right]$$

rectangular term, higher order in a and suppress discretization errors

link variables representing gluon fields can acquire large fluctuations. Tadpole improvement is a technique used to mitigate these errors by rescaling the lattice coupling constant and the link variables.

$$c_0 = \frac{5}{3}\frac{6}{g_0^2 u_0^4} \equiv 10/g^2 \text{ with } g = g_0 u_0^2 \quad , c_1 = -\frac{c_0}{20 u_0^2}$$

$$u_0 = \left\langle \frac{1}{6 N_c V} \mathrm{Re}\,\mathrm{Tr} \sum_{x,\mu<\nu} \mathcal{P}^U_{\mu\nu}(x) \right\rangle^{1/4} \text{ with } V = L^3 \times T \quad v_0 = \left\langle \frac{1}{6 N_c V} \mathrm{Re}\,\mathrm{Tr} \sum_{x,\mu<\nu} \mathcal{P}^V_{\mu\nu}(x) \right\rangle^{1/4}$$

u_0 is a measure of the average size of the plaquette

stout smeared link V

Clover Action

$$S_{\text{Clover}} = S_W + \frac{1}{2}\sum_x \bar{\psi}(x)\frac{1}{v_0^3}\sigma^{\mu\nu}F^V_{\mu\nu}\psi(x)$$

Reduces discretization errors to order $\mathcal{O}(a^2)$ providing more accurate results

Stout Link Smearing in Lattice QCD

- Goal: Suppress short-distance noise and enhance signal-to-noise ratio for observables
- Process:
  1. Construct "staple" links, capturing local gauge field structure
  2. Combine original link variable and staple links using a weighted average, with smearing parameter ρ (e.g., ρ=0.125)
- Benefits: Improved accuracy in hadron masses, form factors, and other observables sensitive to short-distance noise

Lattice QCD in China

C11P29S: $a \approx 0.11$ fm,
$m_\pi \approx 0.29$ GeV
S: small

name	Volume	Lattice spacing	β	π mass	η_s mass	L	n_conf
C11P29Ss	$24^3\times62$	0.105fm	6.20	290MeV	640MeV	2.3fm	200
C11P29S	$24^3\times72$	0.105fm	6.20	290MeV	640MeV	2.6fm	900
C11P29M	$32^3\times64$	0.105fm	6.20	290MeV	640MeV	3.5fm	900
C11P22M	$32^3\times64$	0.105fm	6.20	220MeV	640MeV	3.5fm	450
C11P22L	$48^3\times96$	0.105fm	6.20	220MeV	640MeV	5.4fm	400
C11P14L	$48^3\times96$	0.105fm	6.20	135MeV	700MeV	5.4fm	100
C08P30S	$32^3\times96$	0.080fm	6.41	300MeV	650MeV	2.6fm	500
C08P30M	$48^3\times96$	0.080fm	6.41	300MeV	650MeV	3.8fm	400
C08P21S	$32^3\times64$	0.080fm	6.41	210MeV	650MeV	2.6fm	460
C08P21M	$48^3\times96$	0.080fm	6.41	210MeV	650MeV	3.8fm	450
C06P30S	$48^3\times144$	0.054fm	6.72	300MeV	650MeV	2.6fm	400

Parameters in this table are approximate, and should be determined later by fitting

LM Liu, M Gong, P Sun and YB Yang

[Chin.Phys.C 46 (2022)
,arXiv: 2207.00183
,arXiv: 2207.14132]

Parameters of the configurations

	C11P29Ss	C11P29S	C11P29M	C11P22M	C11P22L	C11P14L	C08P30S	C08P30M	C08P22S	C08P22M	C06P30S
$L^3 \times T$	$24^3 \times 64$	$24^3 \times 72$	$32^3 \times 64$	$32^3 \times 64$	$48^3 \times 96$	$48^3 \times 96$	$32^3 \times 96$	$48^3 \times 96$	$32^3 \times 64$	$48^3 \times 96$	$48^3 \times 144$
$10/g^2$	6.20						6.41				6.72
m_l^{b}						**-0.2825**					
						-0.2820					
						-0.2815					
				-0.2790	**-0.2790**				**-0.2320**	**-0.2320**	
			-0.2780	-0.2780	-0.2780			-0.2307	-0.2307	-0.2307	
	-0.2770	**-0.2770**	**-0.2770**	-0.2770	-0.2770		**-0.2295**	**-0.2295**	-0.2295	-0.2295	**-0.1850**
	-0.2760	-0.2760	-0.2760				-0.2288	-0.2288			-0.1845
	-0.2750	-0.2750					-0.2275				-0.1840
m_s^{b}	-0.2400	**-0.2400**	**-0.2400**	**-0.2400**	**-0.2400**	-0.2400	**-0.2050**	**-0.2050**	**-0.2050**	**-0.2050**	**-0.1700**
	-0.2355	-0.2355	-0.2355	-0.2355	-0.2355	-0.2355	-0.2030	-0.2030	-0.2030	-0.2030	-0.1694
	-0.2310	-0.2310	-0.2310	-0.2310	-0.2310	**-0.2310**	-0.2010	-0.2010	-0.2010	-0.2010	-0.1687
m_c^{b}	0.4780	0.4780	0.4780	0.4780	0.4780	0.4780	0.2326	0.2326	0.2326	0.2326	0.0770
	0.4800	0.4800	0.4800	0.4800	0.4800	0.4800	0.2340	0.2340	0.2340	0.2340	0.0780
	0.4820	0.4820	0.4820	0.4820	0.4820	0.4820	0.2354	0.2354	0.2354	0.2354	0.0790
δ_τ	1.0	0.7	0.7	0.7	0.7	1.0	0.5	0.5	0.5	0.5	1.0
$n_{\min}$		4050	11000	4100	1000	1600	1000	2690	13500	1600	1000
$n_{\max}$		48000	35050	26600	5050	2200	26200	6700	36400	6060	4070
u_0^{I}	0.855453	0.855453	0.855453	0.855520	0.855520	0.855548	0.863437	0.863473	0.863488	0.863499	0.873378
v_0^{I}	0.951479	0.951479	0.951479	0.951545	0.951545	0.951570	0.956942	0.956984	0.957017	0.957006	0.963137
u_0	0.8552548(68)	0.8554391(23)	0.8554294(19)	0.8555282(22)	0.85552338(94)	0.8555296(17)	0.8634595(14)	0.86345914(79)	0.8635185(18)	0.86351466(76)	0.87337220(70)
v_0	0.9512747(60)	0.9514611(21)	0.9514522(18)	0.9515501(20)	0.95154723(86)	0.9515537(16)	0.9569680(11)	0.95696728(62)	0.9570237(14)	0.95701949(59)	0.96313394(50)
a_{w_0}	0.10995(15)	0.106285(57)	0.106717(49)	0.104396(61)	0.104360(28)	0.103806(46)	0.079084(59)	0.078885(24)	0.076270(66)	0.076768(30)	0.053718(36)
$a_{\sqrt{t_0}}$	0.103578(97)	0.100952(34)	0.101202(30)	0.099702(37)	0.099665(15)	0.099448(25)	0.075872(35)	0.075800(10)	0.074393(36)	0.074631(15)	0.052514(21)
$a_{\sqrt{t_0}}/a_{w_0}$	0.94201(62)	0.94982(26)	0.94831(21)	0.95504(27)	0.95501(14)	0.95802(25)	0.95939(37)	0.96090(20)	0.97538(53)	0.97217(24)	0.97757(43)
a	0.10541(12)	0.10541(12)	0.10541(12)	0.10541(12)	0.10541(12)	0.10541(12)	0.07695(11)	0.07695(11)	0.07695(11)	0.07695(11)	0.05113(14)

$$\dot{B}_\mu = D_\nu G_{\nu\mu}, \quad B_\mu|_{t=0} = A_\mu,$$
$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu],$$
$$D_\mu = \partial_\mu + [B_\mu, \cdot]$$

Gradient flow scale

- one can define a scale by keeping a suitable gluonic observable defined at constant flow time t, e.g.,

$$t_0^2 \langle E(t_0) \rangle = 0.3 \,. \qquad E(t,x) = -\frac{1}{2} \,\mathrm{Tr}\, G_{\mu\nu}(t,x) G_{\mu\nu}(t,x)$$

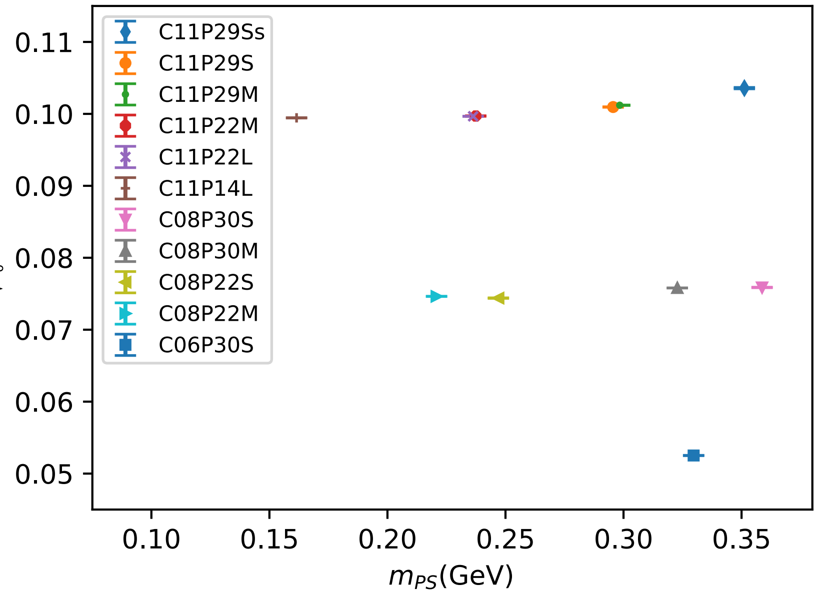
An alternative scale

$$W\left(w_0^2\right) = t_c \cdot \partial_t \left(t^2 \langle E(t) \rangle\right)_{t=t_c} = 0.3.$$

advantage: scale can be obtained in each configuration, not fit needed

[arXiv:2111.09849]

Gradient flow scale $a_{\sqrt{t0}}$

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C11P29Ss 112 confs a_ttFF 0.103578(97) a_tDFF 0.10995(15) a_ttFF/a_tDFF 0.94201(62)
C11P29S 879 confs a_ttFF 0.100952(34) a_tDFF 0.106285(57) a_ttFF/a_tDFF 0.94982(26)
C11P29M 601 confs a_ttFF 0.101202(30) a_tDFF 0.106717(49) a_ttFF/a_tDFF 0.94831(21)
C11P22M 451 confs a_ttFF 0.099702(37) a_tDFF 0.104396(61) a_ttFF/a_tDFF 0.95504(27)
C11P22L 405 confs a_ttFF 0.099665(15) a_tDFF 0.104360(28) a_ttFF/a_tDFF 0.95501(14)
C11P14L 107 confs a_ttFF 0.099448(25) a_tDFF 0.103806(46) a_ttFF/a_tDFF 0.95802(25)
C08P30S 483 confs a_ttFF 0.075872(35) a_tDFF 0.079084(59) a_ttFF/a_tDFF 0.95939(37)
C08P30M 403 confs a_ttFF 0.075800(10) a_tDFF 0.078885(24) a_ttFF/a_tDFF 0.96090(20)
C08P22S 459 confs a_ttFF 0.074393(36) a_tDFF 0.076270(66) a_ttFF/a_tDFF 0.97538(53)
C08P22M 447 confs a_ttFF 0.074631(15) a_tDFF 0.076768(30) a_ttFF/a_tDFF 0.97217(24)
C06P30S 257 confs a_ttFF 0.052514(21) a_tDFF 0.053718(36) a_ttFF/a_tDFF 0.97757(43)
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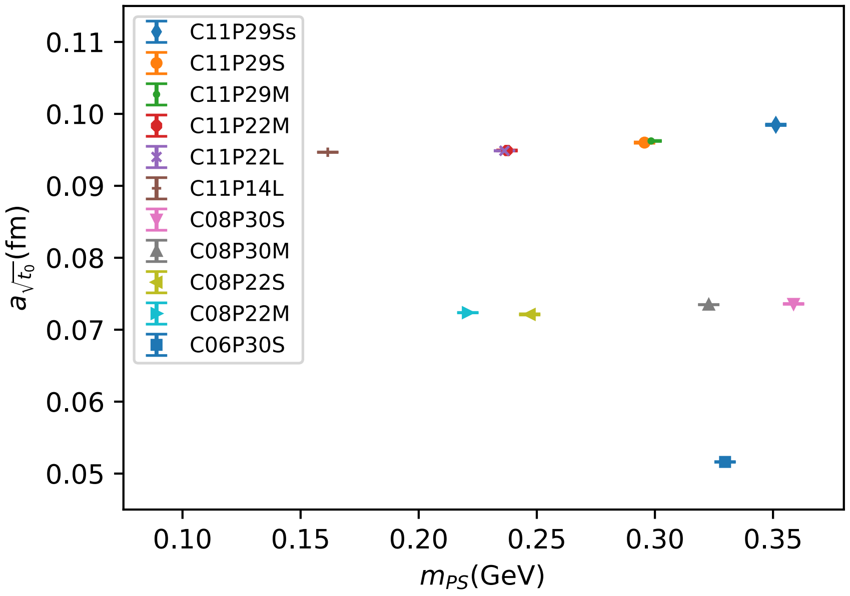
Wilson flow

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C11P29S 128 confs a_ttFF 0.096006(88) a_tDFF 0.10742(15) a_ttFF/a_tDFF 0.89373(55)
C11P29M 118 confs a_ttFF 0.096234(66) a_tDFF 0.10789(11) a_ttFF/a_tDFF 0.89197(39)
C11P22M 102 confs a_ttFF 0.094908(76) a_tDFF 0.10570(13) a_ttFF/a_tDFF 0.89793(51)
C11P22L 81 confs a_ttFF 0.094874(33) a_tDFF 0.105639(63) a_ttFF/a_tDFF 0.89809(27)
C11P14L 61 confs a_ttFF 0.094669(32) a_tDFF 0.105158(63) a_ttFF/a_tDFF 0.90026(28)
C08P30S 144 confs a_ttFF 0.073586(57) a_tDFF 0.079930(98) a_ttFF/a_tDFF 0.92062(58)
C08P30M 81 confs a_ttFF 0.073481(22) a_tDFF 0.079664(52) a_ttFF/a_tDFF 0.92239(42)
C08P22S 101 confs a_ttFF 0.072125(83) a_tDFF 0.07709(15) a_ttFF/a_tDFF 0.9356(10)
C08P22M 101 confs a_ttFF 0.072369(31) a_tDFF 0.077611(65) a_ttFF/a_tDFF 0.93246(46)
C06P30S 69 confs a_ttFF 0.051619(45) a_tDFF 0.053914(85) a_ttFF/a_tDFF 0.95744(93)
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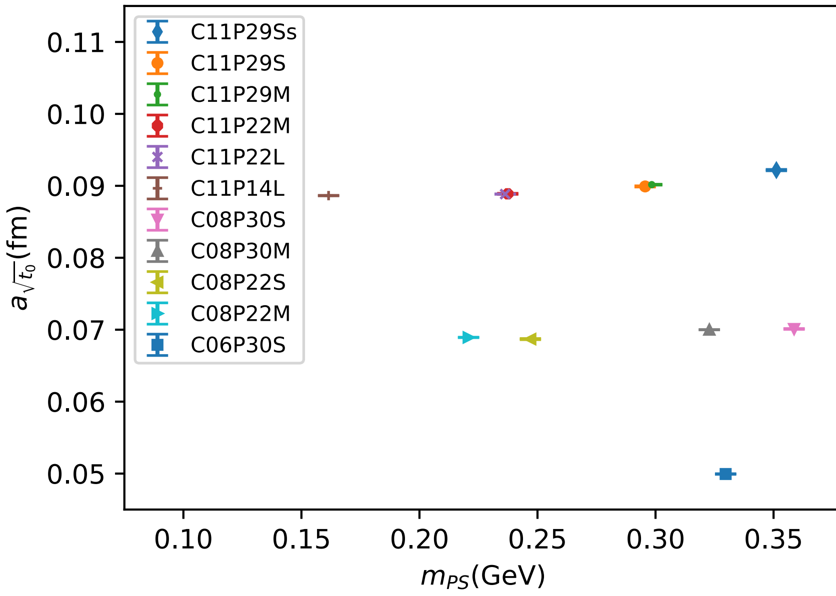
Symanzik flow

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C11P29Ss 100 confs a_ttFF 0.092185(93) a_tDFF 0.11237(17) a_ttFF/a_tDFF 0.82034(
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C11P29M 118 confs a_ttFF 0.090160(61) a_tDFF 0.10927(10) a_ttFF/a_tDFF 0.82509(
C11P22M 102 confs a_ttFF 0.088889(73) a_tDFF 0.10711(13) a_ttFF/a_tDFF 0.82986(
C11P22L 80 confs a_ttFF 0.088850(31) a_tDFF 0.107048(58) a_ttFF/a_tDFF 0.83001(
C11P14L 61 confs a_ttFF 0.088640(34) a_tDFF 0.106592(65) a_ttFF/a_tDFF 0.83158(
C08P30S 144 confs a_ttFF 0.070104(55) a_tDFF 0.081250(98) a_ttFF/a_tDFF 0.86281(
C08P30M 81 confs a_ttFF 0.069998(23) a_tDFF 0.081018(53) a_ttFF/a_tDFF 0.86399(
C08P22S 101 confs a_ttFF 0.068695(79) a_tDFF 0.07857(15) a_ttFF/a_tDFF 0.87434(
C08P22M 102 confs a_ttFF 0.068925(30) a_tDFF 0.079033(61) a_ttFF/a_tDFF 0.87210(
C06P30S 76 confs a_ttFF 0.049940(41) a_tDFF 0.054428(76) a_ttFF/a_tDFF 0.91755(
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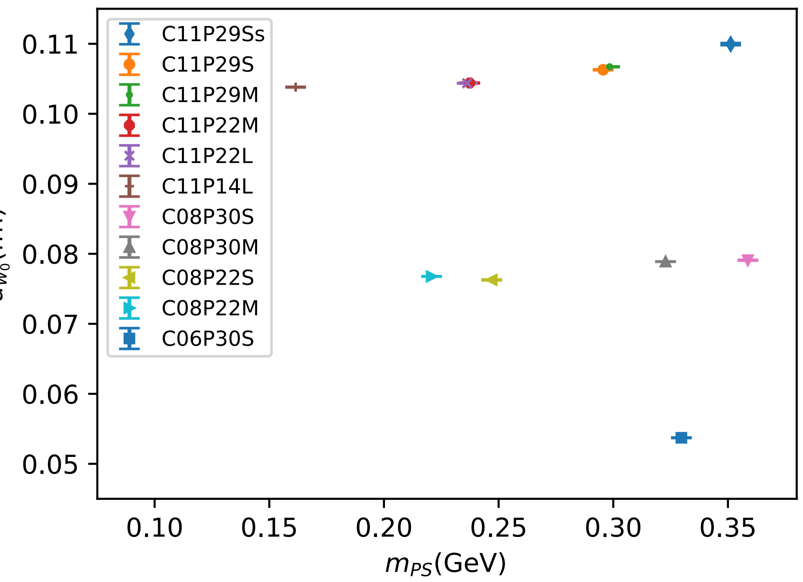
iwasaki flow

Gradient flow scale a_{w0}

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1P29Ss 112 confs a_ttFF 0.103578(97) a_tDFF 0.10995(15) a_ttFF/a_tDFF 0.94201(62)
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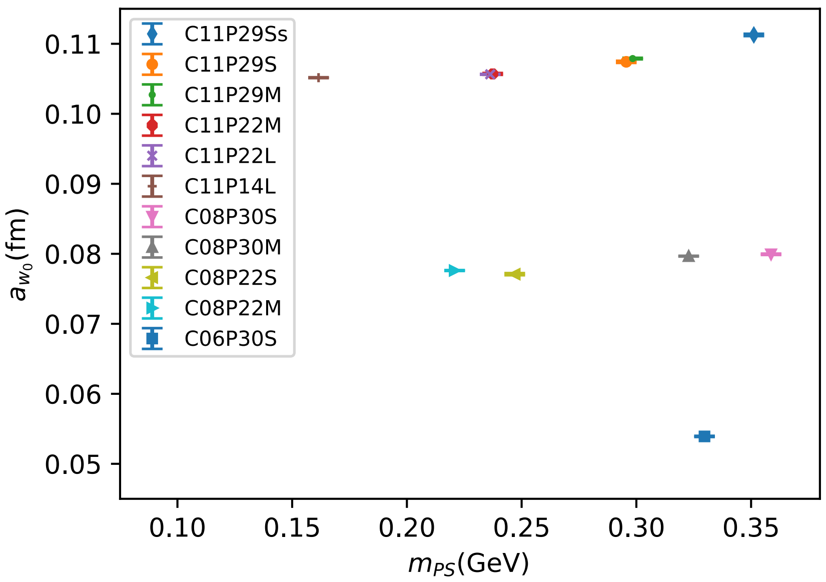
Wilson flow

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C06P30S 69 confs a_ttFF 0.051619(45) a_tDFF 0.053914(85) a_ttFF/a_tDFF 0.95744(93)
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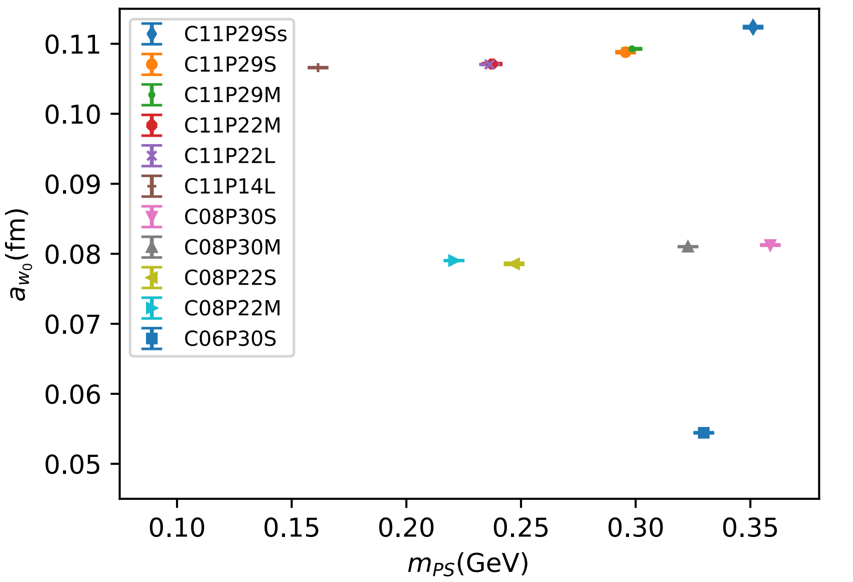
Symanzik flow

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C08P22S 101 confs a_ttFF 0.068695(79) a_tDFF 0.07857(15) a_ttFF/a_tDFF 0.87434(82)
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C06P30S 76 confs a_ttFF 0.049940(41) a_tDFF 0.054428(76) a_ttFF/a_tDFF 0.91755(73)
```

iwasaki flow

$$a(\beta, m_\pi, \bar{m}_{\eta_s}) = a(\beta)\left(1 + c_1\left(m_\pi^2 - 0.135^2\right) + c_2\left[\left(\frac{\bar{m}_{\eta_s} * 0.1973}{a(\beta)}\right)^2 - 0.69^2\right] + c_3 \exp(-m_\pi a(\beta)/0.1973 * L)\right)$$

Wilson flow

```
Least Square Fit (no prior):
  chi2/dof [dof] = 9.4 [5]    Q = 5.9e-09

Parameters:
             c1     0.610 (15)      [    1 +- inf ]
             c2    0.2086 (95)      [  0.5 +- inf ]
             c3    -0.126 (30)      [  0.5 +- inf ]
         a(6.2)   0.10323 (11)      [  0.1 +- inf ]
        a(6.41)   0.07558 (11)      [ 0.08 +- inf ]
        a(6.72)   0.05061 (14)      [ 0.05 +- inf ]

Fit:
      key          y[key]       f(p)[key]
-------------------------------------------
      a 0     0.10995 (17)     0.10934 (12)  ***
        1     0.10608 (16)    0.106534 (89)  **
        2     0.10654 (11)    0.106683 (65)  *
        3     0.10437 (14)    0.103926 (68)  ***
        4    0.104305 (66)    0.104231 (55)  *
        5    0.103785 (62)    0.103909 (58)  **
        6    0.079144 (97)    0.079020 (50)  *
        7    0.078837 (56)    0.078945 (46)  *
        8     0.07622 (16)     0.07629 (12)
        9    0.076786 (68)    0.076673 (48)  *
       10    0.053638 (83)    0.053638 (83)

Settings:
  svdcut/n = 1e-12/0    tol = (1e-08,1e-10,1e-10*)    (itns/time = 6/0.0)
  fitter = scipy_least_squares    method = trf
```

Intepolated to $-1/(12u_0^2)$

```
Least Square Fit (no prior):
  chi2/dof [dof] = 8.3 [5]    Q = 8.2e-08

Parameters:
             c1     0.589 (15)      [    1 +- inf ]
             c2    0.2110 (96)      [  0.5 +- inf ]
             c3    -0.124 (30)      [  0.5 +- inf ]
         a(6.2)   0.10541 (12)      [  0.1 +- inf ]
        a(6.41)   0.07695 (11)      [ 0.08 +- inf ]
        a(6.72)   0.05113 (14)      [ 0.05 +- inf ]

Fit:
      key          y[key]       f(p)[key]
-------------------------------------------
      a 0     0.11163 (17)     0.11103 (12)  ***
        1     0.10780 (15)    0.108246 (87)  **
        2     0.10827 (11)    0.108382 (64)
        3     0.10608 (13)    0.105674 (66)  ***
        4    0.106017 (63)    0.105962 (53)
        5    0.105548 (63)    0.105669 (59)  *
        6    0.080159 (98)    0.080070 (48)
        7    0.079903 (52)    0.079986 (44)  *
        8     0.07734 (15)     0.07739 (12)
        9    0.077852 (64)    0.077756 (46)  *
       10    0.053989 (85)    0.053989 (85)

Settings:
  svdcut/n = 1e-12/0    tol = (1e-08,1e-10,1e-10*)    (itns/tim
  fitter = scipy_least_squares    method = trf
```

```
metas_arr=gv.gvar(['0.39956(77)','0.3520(11)','0.35037(76)','0.3452(14)','0.34577(37)','0.37787(43)','0.2707(11)','0.26590(37)','0.2636(16)','0.26
193(54)','0.18397(88)'])
```

Joint jit

Based on PCAC relation, wp-pp ratio and form of pion 2pt, we do the joint fit

$$\frac{C_{2,wp}^{A_4P}(t-a;m_q)-C_{2,wp}^{A_4P}(t+a;m_q)}{C_{2,wp}^{PP}(t;m_q)}\Big|_{0\ll t\ll T}=\frac{4Z_P(\mu)}{Z_A}\frac{\mathrm{Sinh}(m_\pi a)}{m_\pi}m_q^R(\mu),$$

$$\sqrt{\frac{C_{2,wp}^{PP}(t;m_q)}{C_{2,ww}^{PP}(t;m_q)}}\Big|_{0\ll t\ll T}=\frac{m_\pi^2}{2Z_P(\mu)m_q^R\sqrt{Z_{wp}}}f_\pi(m_q),$$

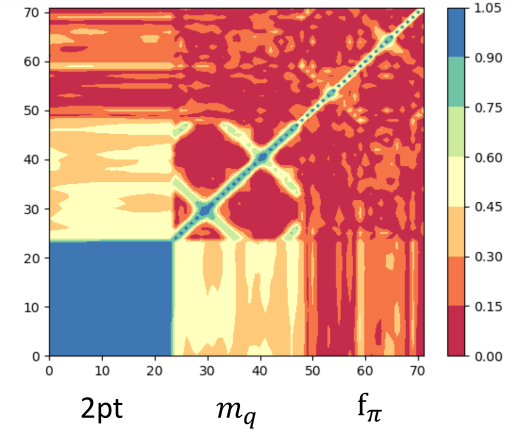
$$C_{2,wp}^{PP}(t;m_q)|_{0\ll t\ll T}=\frac{Z_{wp}}{2m_\pi}\exp\left(-\frac{T}{2}m_\pi\right)\cosh\,(t-\frac{T}{2})m_\pi$$

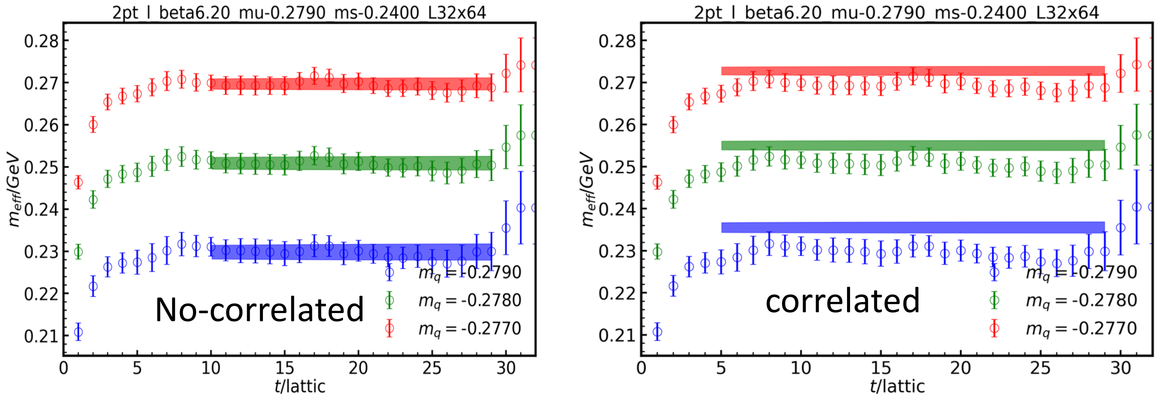
and obtain the renormalized quark mass m_q^R and pion decay constant f_π

$$\frac{C^{A_4P}_{2,wp}(t-a;m_q)-C^{A_4P}_{2,wp}(t+a;m_q)}{C^{PP}_{2,wp}(t;m_q)}\Big|_{0\ll t\ll T}=\frac{4Z_P(\mu)}{Z_A}\frac{\mathrm{Sinh}(m_\pi a)}{m_\pi}m_q^R(\mu),$$

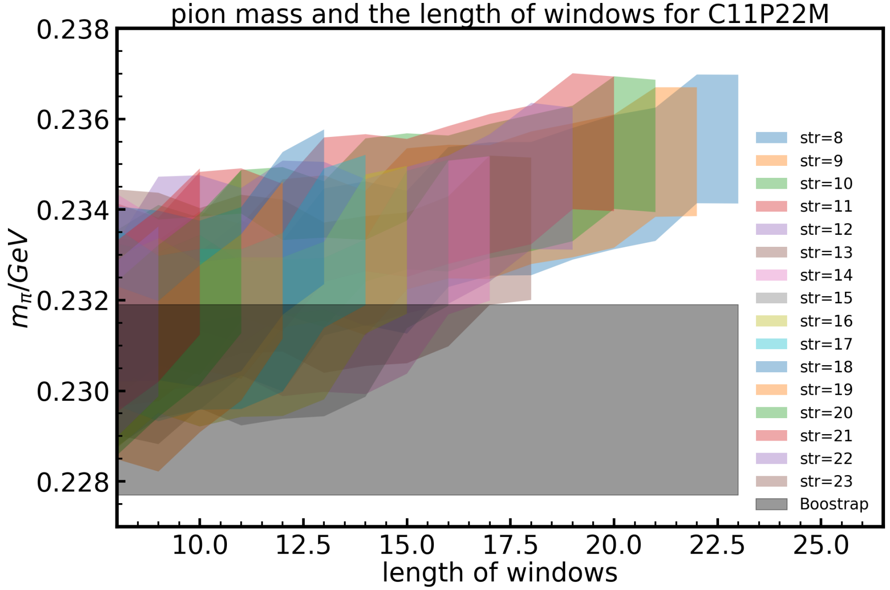
$$\sqrt{\frac{C^{PP}_{2,wp}(t;m_q)}{C^{PP}_{2,ww}(t;m_q)}}\Big|_{0\ll t\ll T}=\frac{m_\pi^2}{2Z_P(\mu)m_q^R\sqrt{Z_{wp}}}f_\pi(m_q),$$

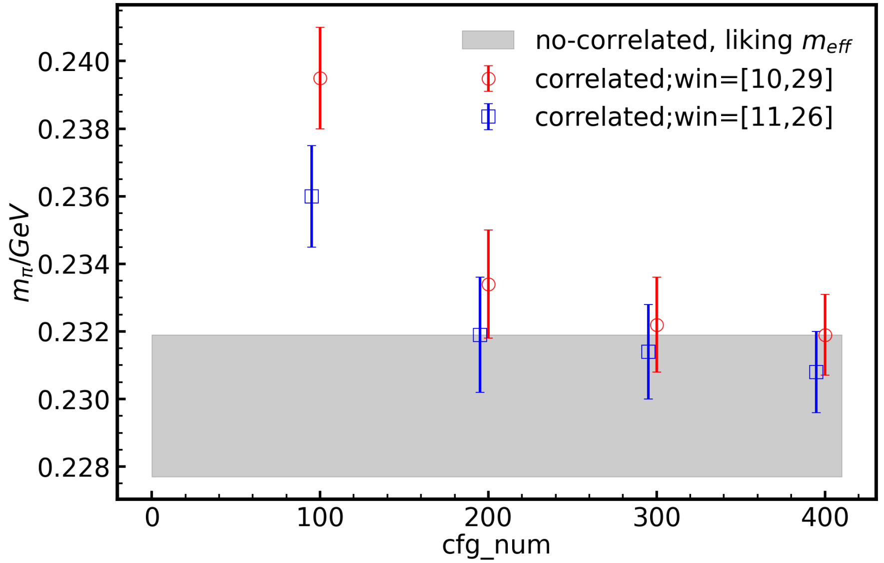
$$C^{PP}_{2,wp}(t;m_q)|_{0\ll t\ll T}=\frac{Z_{wp}}{2m_\pi}\exp\left(-\frac{T}{2}m_\pi\right)\cosh\left(t-\frac{T}{2}\right)m_\pi$$





a=0.105fm, mPS=0.229GeV





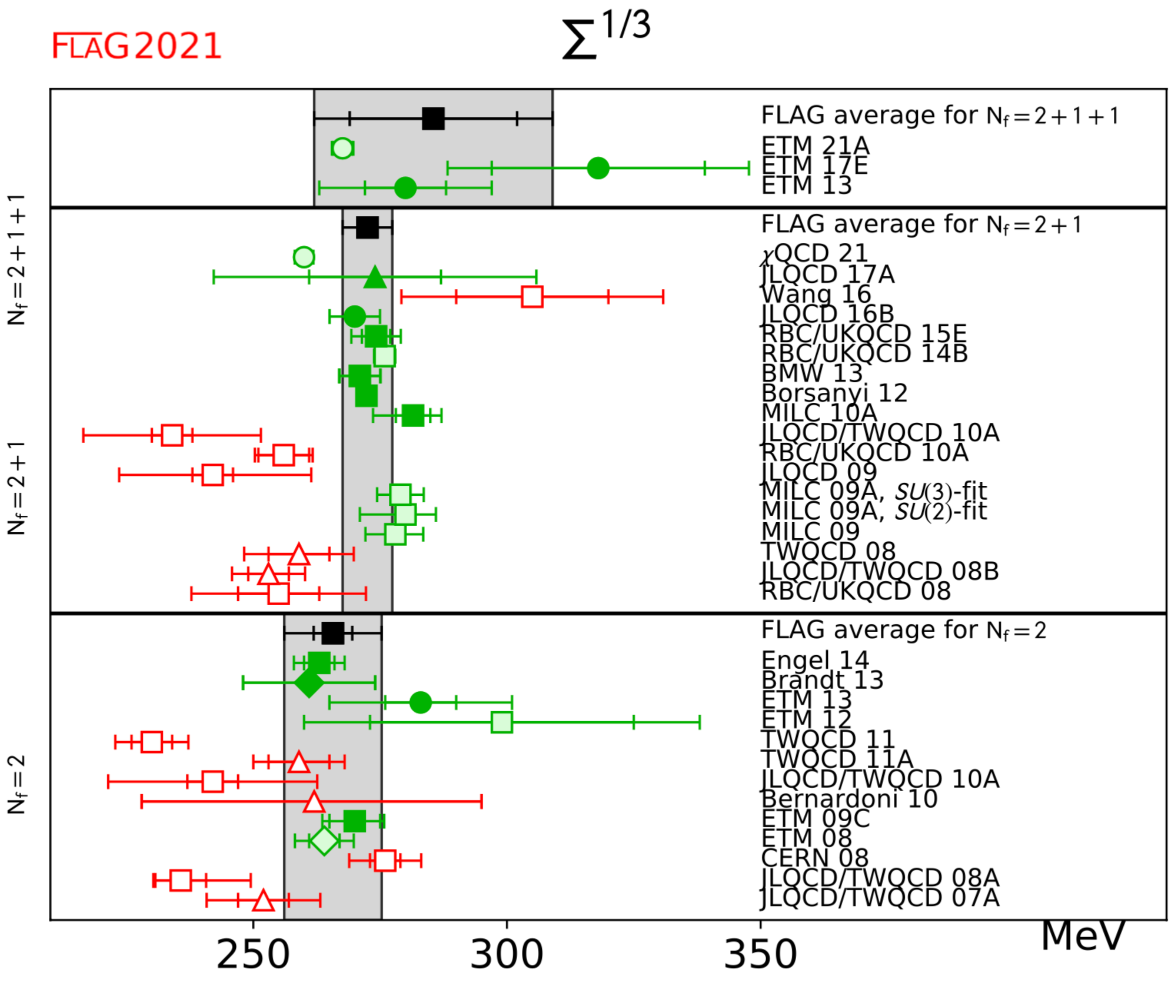
Two way to make correlated fit reliable:

1. Reduce the number of correlated point.
2. Increase the statistics.

```
chi2: 2.11(37)
Q: 0.002(11)
Sigma_cubic_root: 0.2769(20)
F: 0.08757(70)
F_pi/F: 1.0531(84)
alpha_4: 0.31(10)
alpha_5: -0.21(22)
alpha_6: 0.072(72)
alpha_8: 0.57(16)
c_m: -5.25(61)
c_f: -9.44(29)
c_fl: -0.937(92)
c_ml: 0.93(22)
```

```
F : 0.08763(78)
sigma : 0.02105(31)
a4 : 0.297(78)
a5 : -0.06(14)
a6 : 0.090(36)
a8 : 0.642(60)
cm : -4.80(47)
cml : 0.99(16)
cf : -9.81(29)
cfl : -0.97(11)
```

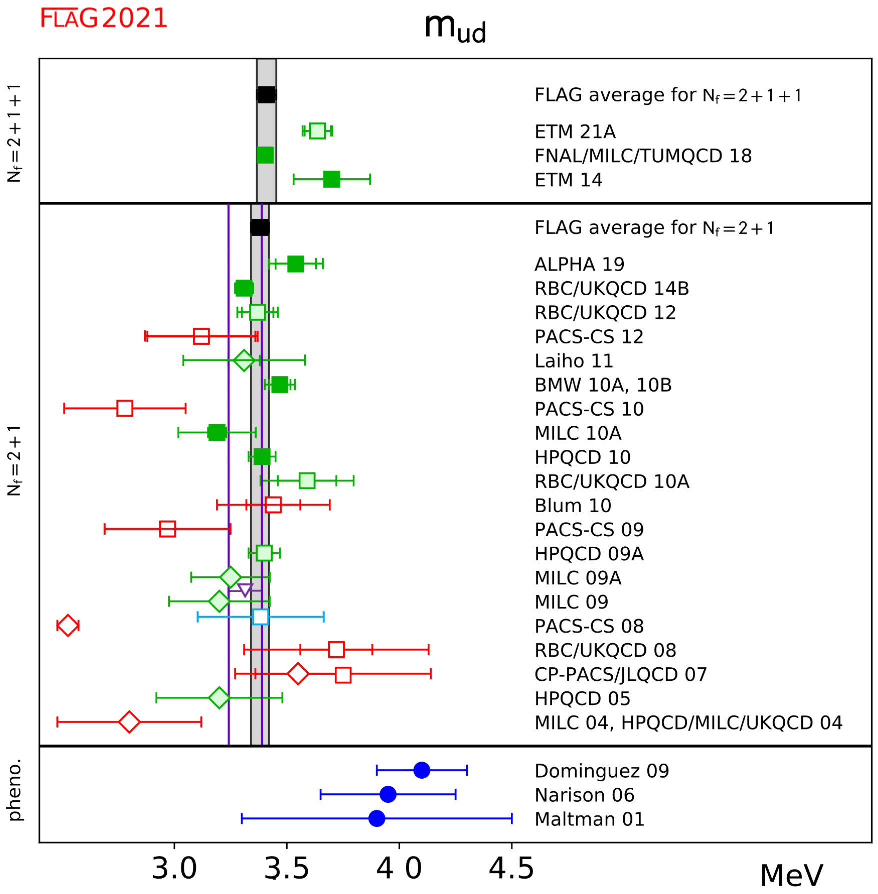
$$
\begin{aligned}
m^2_{\pi,\mathrm{vv}} =& \Lambda_\chi^2 2y_\mathrm{v}\left\{1 + \frac{2}{N_f}\left[(2y_\mathrm{v} - y_\mathrm{s})\ln(2y_\mathrm{v}) + (y_\mathrm{v} - y_\mathrm{s})\right]\right. \\
&\left. + 2y_\mathrm{v}(2\alpha_8 - \alpha_5) + 2y_\mathrm{s}N_f(2\alpha_6 - \alpha_4)\right\} \quad \left(1 + c_m a^2 + c_{ml}e^{-m_\pi L}\right) \\
F_{\pi,\mathrm{vv}} =& F\left(1 - \frac{N_f}{2}(y_\mathrm{v} + y_\mathrm{s})\ln(y_\mathrm{v} + y_\mathrm{s}) + \alpha_5 y_\mathrm{v} + \alpha_4 N_f y_\mathrm{s}\right) \quad \left(1 + c_f a^2 + c_{fl}e^{-m_\pi L}\right)
\end{aligned}
$$

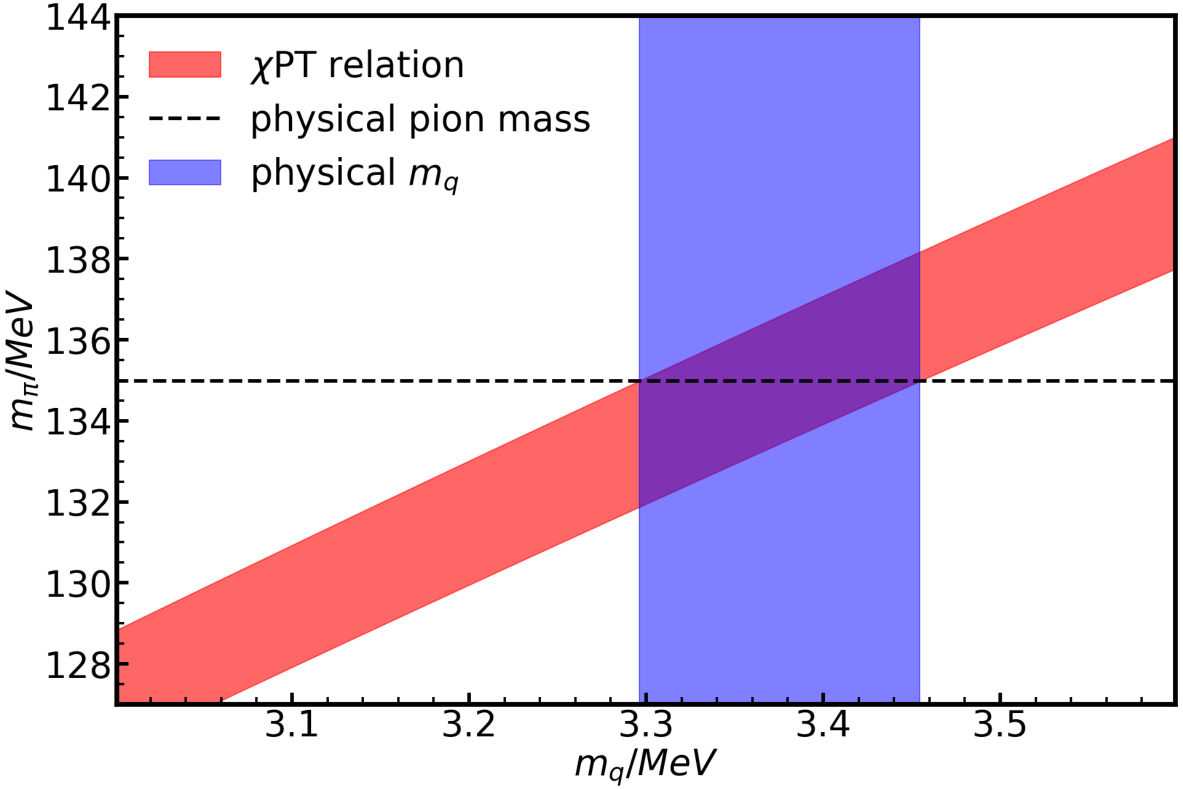


With Continuous limit and infinite volume, $a \to 0,\ L \to +\infty$

Inverse solve the equation: $$m^2_{\pi,\mathrm{vv}} = \Lambda^2_\chi 2y\{1 + \frac{2}{N_f} y\ln(2y) + 2y(2\alpha_8 - \alpha_5) + 2yN_f(2\alpha_6 - \alpha_4)\}$$

Physical point: $m_\pi = 134.98\mathrm{MeV} \longrightarrow \mathrm{m_{ud}} = 3.375(79)\mathrm{MeV}$





Future Plans

- Addressing Lattice Spacing and Renormalization Errors:
  - In our calculation, the actual error of quark mass is expected to be larger due to:
  - Errors in the lattice spacing estimation
  - Errors in renormalization constants
  - Plan to refine these estimates and account for their impact on quark mass errors

Future Plans

- Mass Difference of u and d Quarks:
  - Study the kaon meson to calculate the mass difference between up (u) and down (d) quarks
  - Investigate the role of this mass difference in the properties of the kaon and its interactions
  - Compare the results with other methods and experimental data to validate our findings

Future Plans

- Tidy Python Codes for Data:
  - Improve the organization and readability of the Python codes used for data analysis
  - Ensure the code is modular, reusable, and well-documented
  - Implement error handling and validation to increase the robustness of the analysis pipeline

Thank you

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