CLQCD

# Program总结：
# 从强子能谱到强子结构

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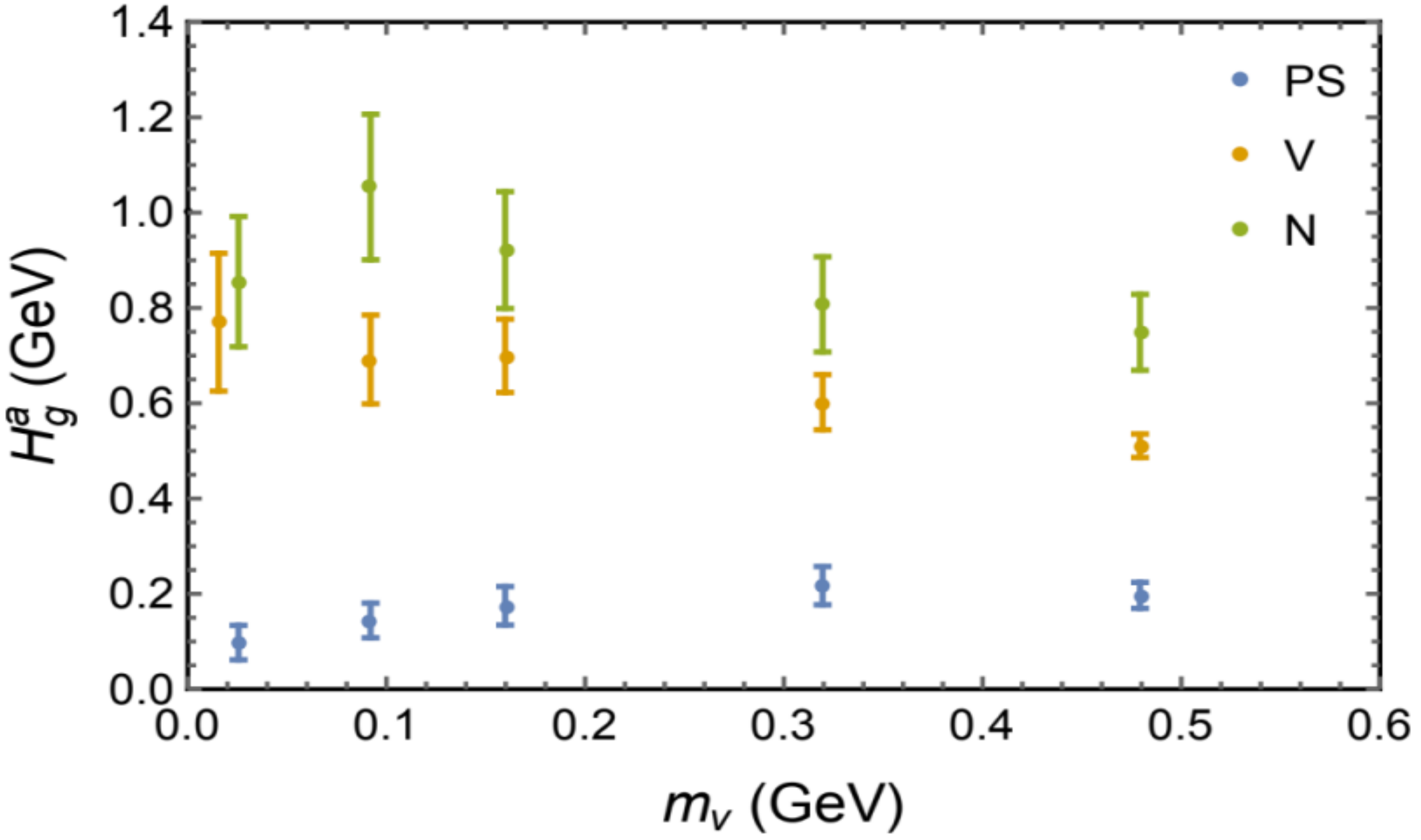
# Trace anomaly

## Gluon contribution

- The gluon contribution $\frac{\beta}{2g}\langle F^2\rangle_H$ to the hadron mass $m_H$.

- **$\frac{\beta}{2g}\langle F^2\rangle_N$ is around 800 MeV in the chiral limit $m_q \to 0$.**
- **$\frac{\beta}{2g}\langle F^2\rangle_\pi$ will be less than 100 MeV in the chiral limit $m_q \to 0$.**

- They are exact what QCD predict.




F. He, P. Sun, **YBY**, $\chi$QCD Collaboration, PRD104 (2021) 074507

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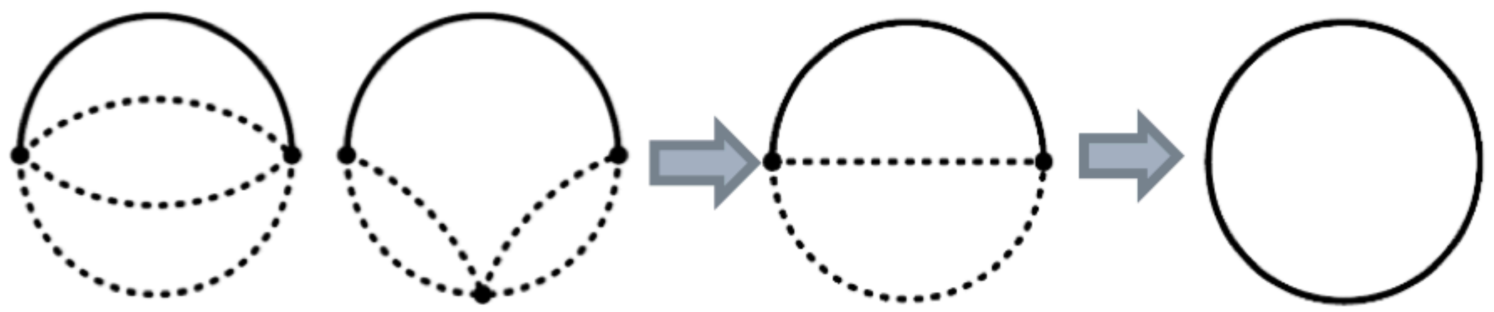
# AMFlow

# for Lattice PT

- Based on the idea of AMFlow, it is possible to obtain the Lattice PT beyond 1-loop level.
- It is essential to verify the factorization theorems under the lattice regularization.

$$\begin{aligned}
&\Gamma^{ABCD}_{\mu\nu\lambda\rho}(k,q,r,s) = \\
&-g_0^2\Big[\sum_E f_{ABE}f_{CDE}\Big\{\delta_{\mu\lambda}\delta_{\nu\rho}[\cos\frac{1}{2}a(q-s)_\mu\cos\frac{1}{2}a(k-r)_\nu-\frac{a^4}{12}\tilde{k}_\nu\tilde{q}_\mu\tilde{r}_\nu\tilde{s}_\mu] \\
&\qquad -\delta_{\mu\rho}\delta_{\nu\lambda}[\cos\frac{1}{2}a(q-r)_\mu\cos\frac{1}{2}a(k-s)_\nu-\frac{a^4}{12}\tilde{k}_\nu\tilde{q}_\mu\tilde{r}_\mu\tilde{s}_\nu] \\
&\qquad +\frac{1}{6}\delta_{\nu\lambda}\delta_{\nu\rho}a^2\widetilde{(s-r)}_\mu\tilde{k}_\nu\cos(\frac{1}{2}aq_\mu) \\
&\qquad -\frac{1}{6}\delta_{\mu\lambda}\delta_{\mu\rho}a^2\widetilde{(s-r)}_\nu\tilde{q}_\mu\cos(\frac{1}{2}ak_\nu) \\
&\qquad +\frac{1}{6}\delta_{\mu\nu}\delta_{\mu\rho}a^2\widetilde{(q-k)}_\lambda\tilde{r}_\rho\cos(\frac{1}{2}as_\lambda) \\
&\qquad -\frac{1}{6}\delta_{\mu\nu}\delta_{\mu\lambda}a^2\widetilde{(q-k)}_\rho\tilde{s}_\lambda\cos(\frac{1}{2}ar_\rho) \\
&\qquad +\frac{1}{12}\delta_{\mu\nu}\delta_{\mu\lambda}\delta_{\mu\rho}a^2\sum_\sigma\widetilde{(q-k)}_\sigma\widetilde{(s-r)}_\sigma\Big\} \\
&\qquad +(B\leftrightarrow C,\nu\leftrightarrow\lambda,q\leftrightarrow r)+(B\leftrightarrow D,\nu\leftrightarrow\rho,q\leftrightarrow s)\Big] \\
&\qquad +\frac{g_0^2}{12}a^4\Big\{\frac{2}{3}(\delta_{AB}\delta_{CD}+\delta_{AC}\delta_{BD}+\delta_{AD}\delta_{BC}) \\
&\qquad +\sum_E(d_{ABE}d_{CDE}+d_{ACE}d_{BDE}+d_{ADE}d_{BCE})\Big\} \\
&\qquad \times\Big\{\delta_{\mu\nu}\delta_{\mu\lambda}\delta_{\mu\rho}\sum_\sigma\tilde{k}_\sigma\,\tilde{q}_\sigma\tilde{r}_\sigma\tilde{s}_\sigma-\delta_{\mu\nu}\delta_{\mu\lambda}\tilde{k}_\rho\tilde{q}_\rho\tilde{r}_\rho\tilde{s}_\mu \\
&\qquad -\delta_{\mu\nu}\delta_{\mu\rho}\tilde{k}_\lambda\tilde{q}_\lambda\tilde{s}_\lambda\tilde{r}_\mu-\delta_{\mu\lambda}\delta_{\mu\rho}\tilde{k}_\nu\tilde{r}_\nu\tilde{s}_\nu\tilde{q}_\mu-\delta_{\nu\lambda}\delta_{\nu\rho}\tilde{q}_\mu\tilde{r}_\mu\tilde{s}_\mu\tilde{k}_\nu \\
&\qquad +\delta_{\mu\nu}\delta_{\lambda\rho}\tilde{k}_\lambda\tilde{q}_\lambda\tilde{r}_\mu\tilde{s}_\mu+\delta_{\mu\lambda}\delta_{\nu\rho}\tilde{k}_\nu\tilde{r}_\nu\tilde{q}_\mu\tilde{s}_\mu+\delta_{\mu\rho}\delta_{\nu\lambda}\tilde{k}_\nu\tilde{s}_\nu\tilde{q}_\mu\tilde{r}_\mu\Big\}
\end{aligned}$$

$$(S_G)_{A^4} = -\frac{1}{4!}\int_{BZ}\frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{d^4r}{(2\pi)^4}\frac{d^4s}{(2\pi)^4} \times\sum_{\mu\nu\lambda\rho}\sum_{ABCD}\Gamma^{ABCD}_{\mu\nu\lambda\rho}(k,q,r,s)\tilde{A}^A_\mu(k)\tilde{A}^B_\nu(q)\tilde{A}^C_\lambda(r)\tilde{A}^D_\rho(s)\ .$$



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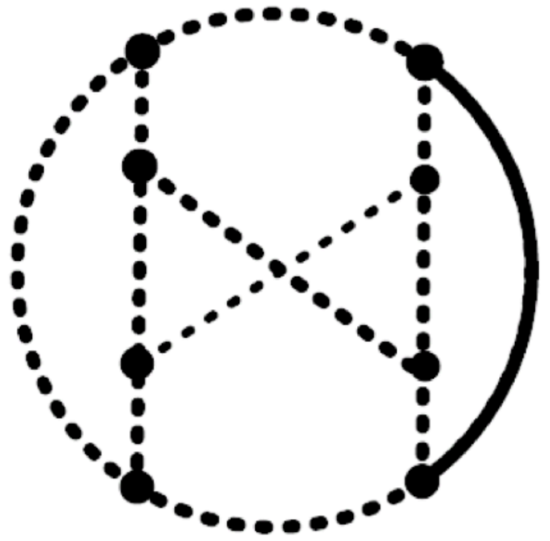
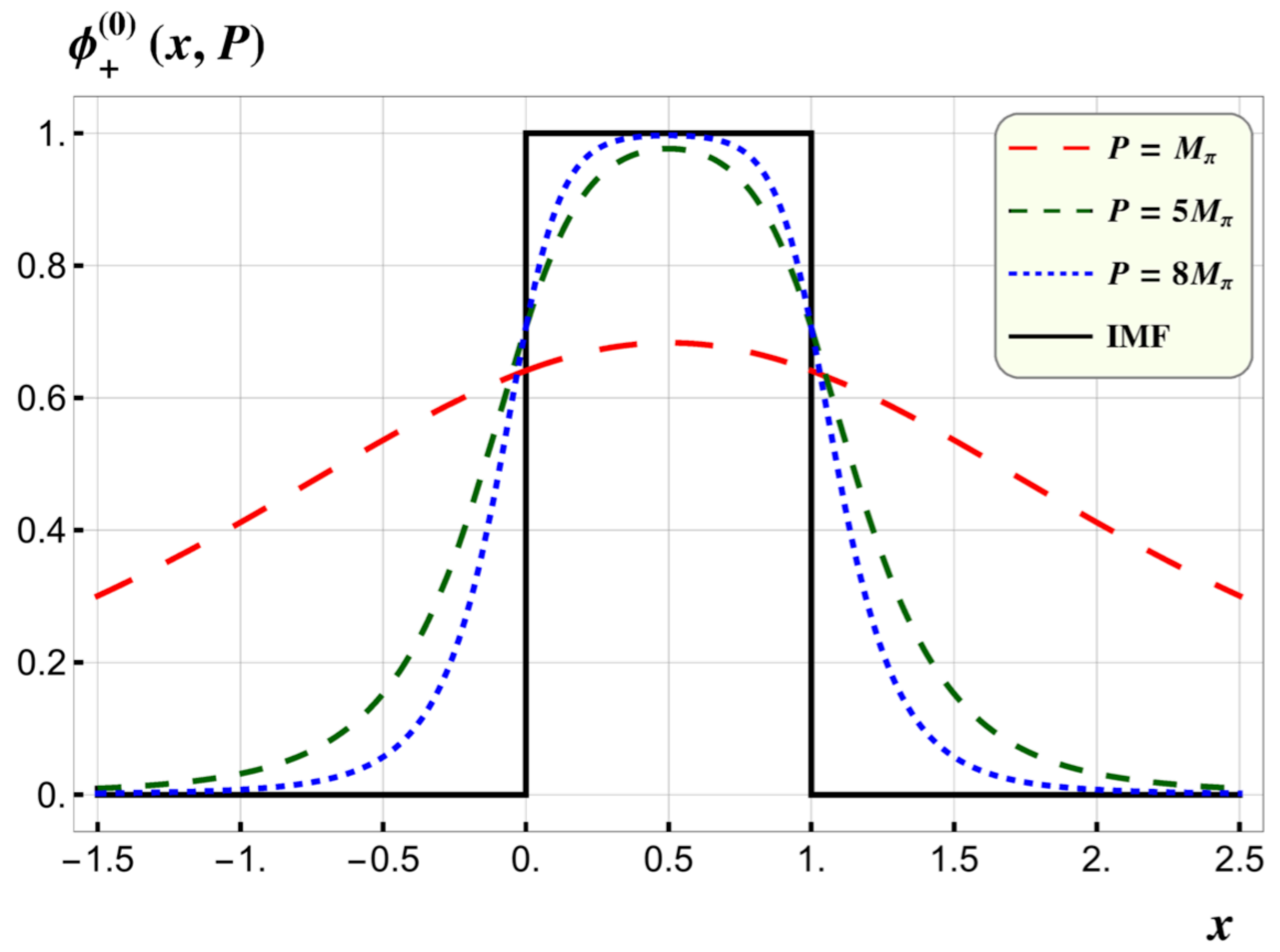
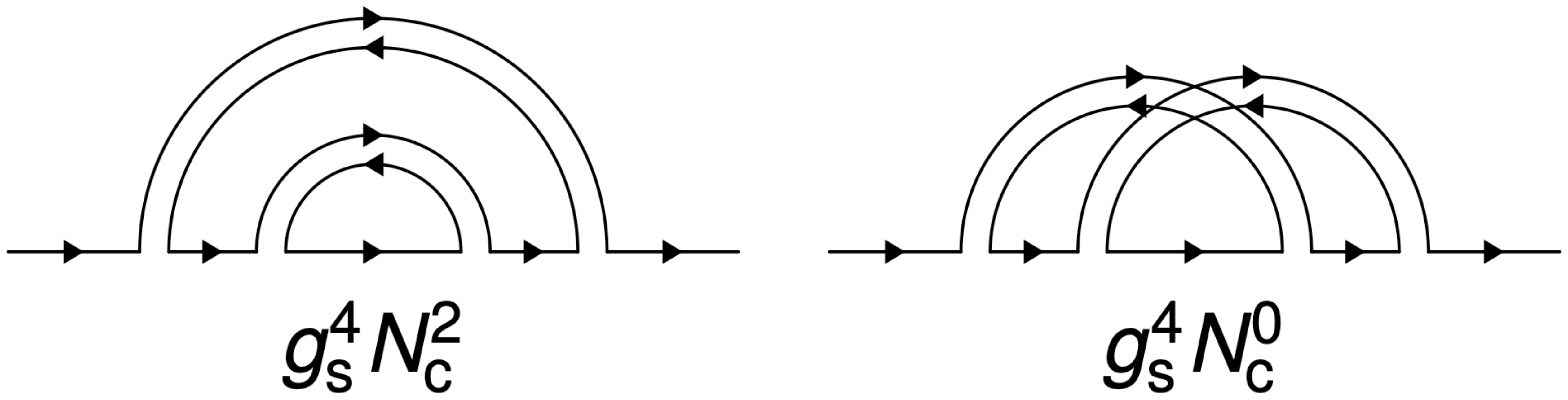
H. Rothe, 《Lattice Gauge Theories, An Introduction》, Eq.15.53

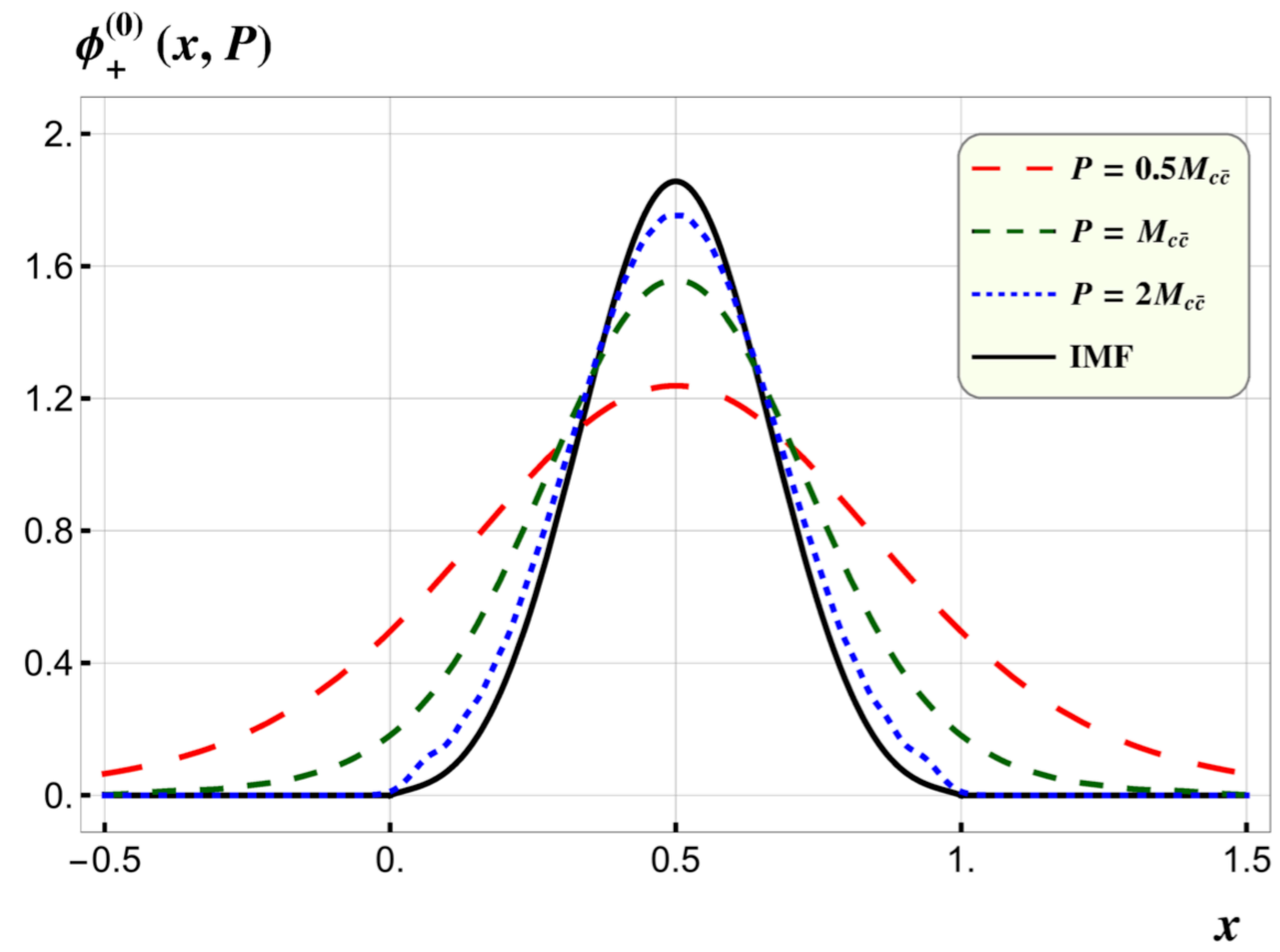


$$\begin{aligned}
= &-2.073855510286740\epsilon^{-2}-7.812755312590133\epsilon^{-1} \\
&-17.25882864945875+717.6808845492140\epsilon \\
&+8190.876448160049\epsilon^2+78840.29598046500\epsilon^3 \\
&+566649.1116484678\epsilon^4+3901713.802716081\epsilon^5 \\
&+23702384.71086095\epsilon^6+142142936.8205112\epsilon^7,
\end{aligned}$$

# 1+1 dimension QCD with large $N_c$

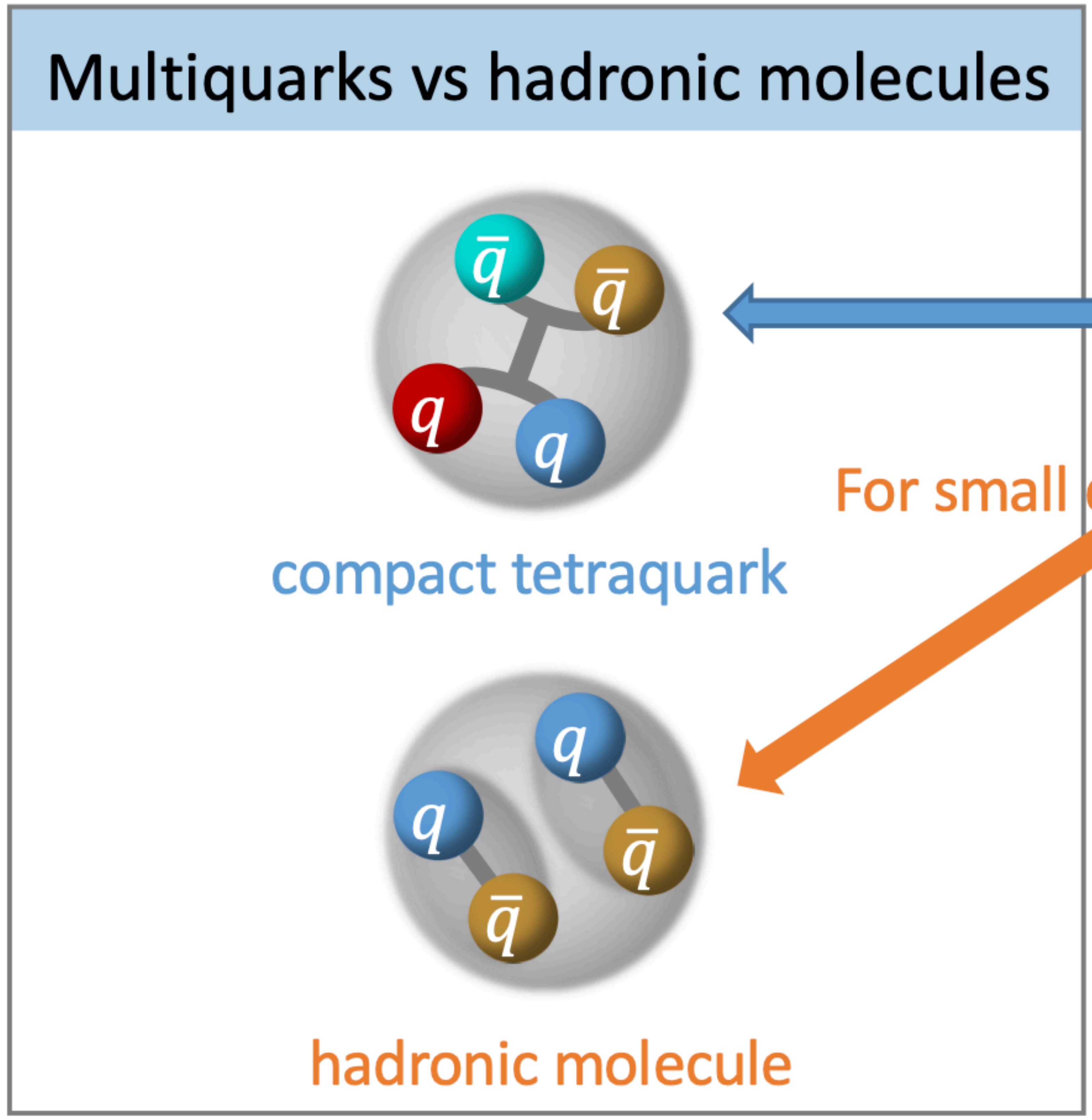


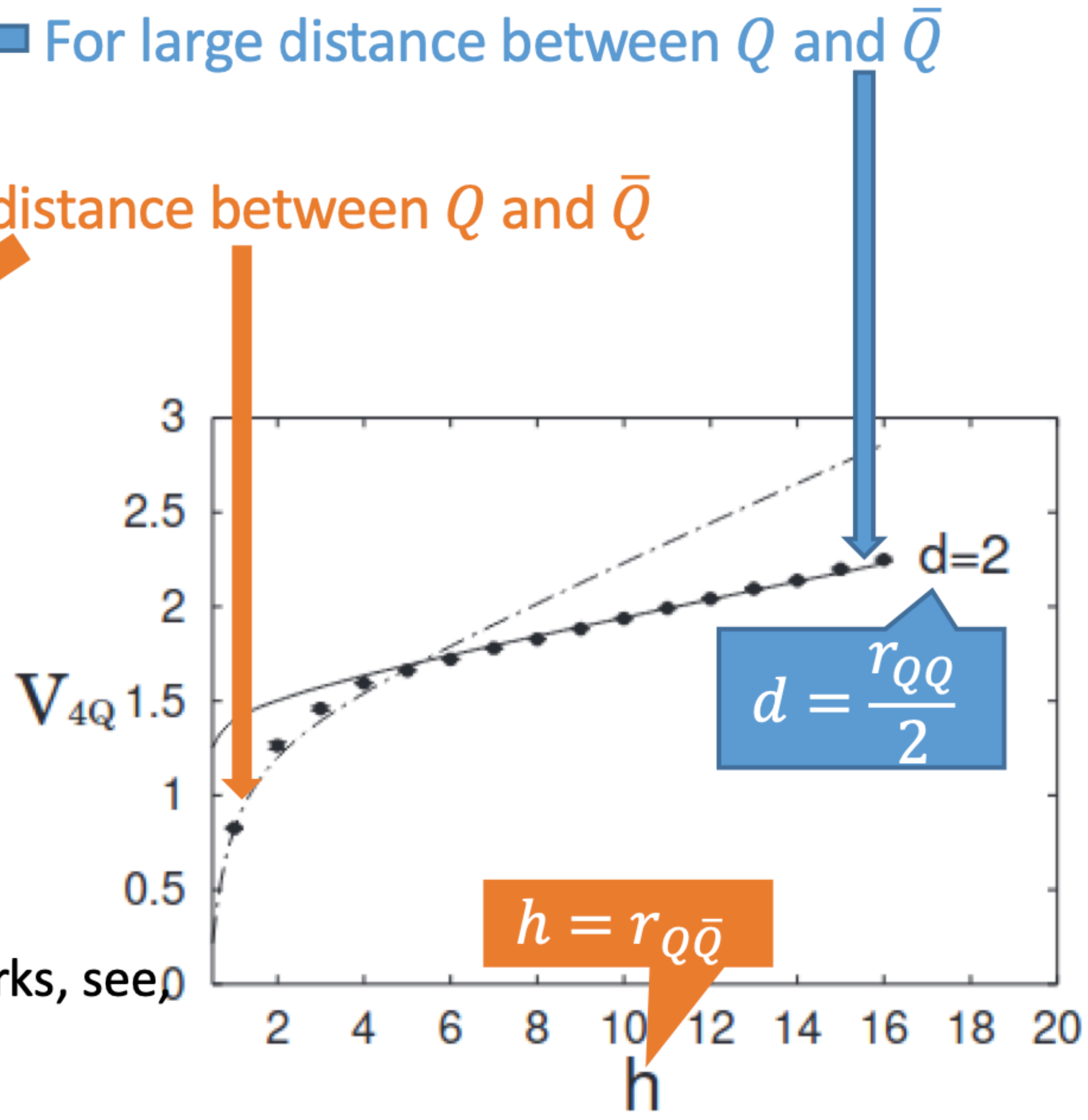




- Kinds of the features which would be similar to the 4D case but not exactly the same;
- Confinement and “almost” spontaneously break the chiral symmetry (in the weak coupling limit);
- Should be valuable to implement it with lattice field theory.

# Tetraquark and/or hadronic molecule?




F. Okiharu et al., PRD72(2005)014505

For lattice studies of flux tube picture of multiquarks, see e.g., F. Okiharu, H. Suganuma, T. T. Takahashi, PRD72(2005)014505; PRL94(2005)192001
An overview: H. Suganuma et al., arXiv:1103.4015

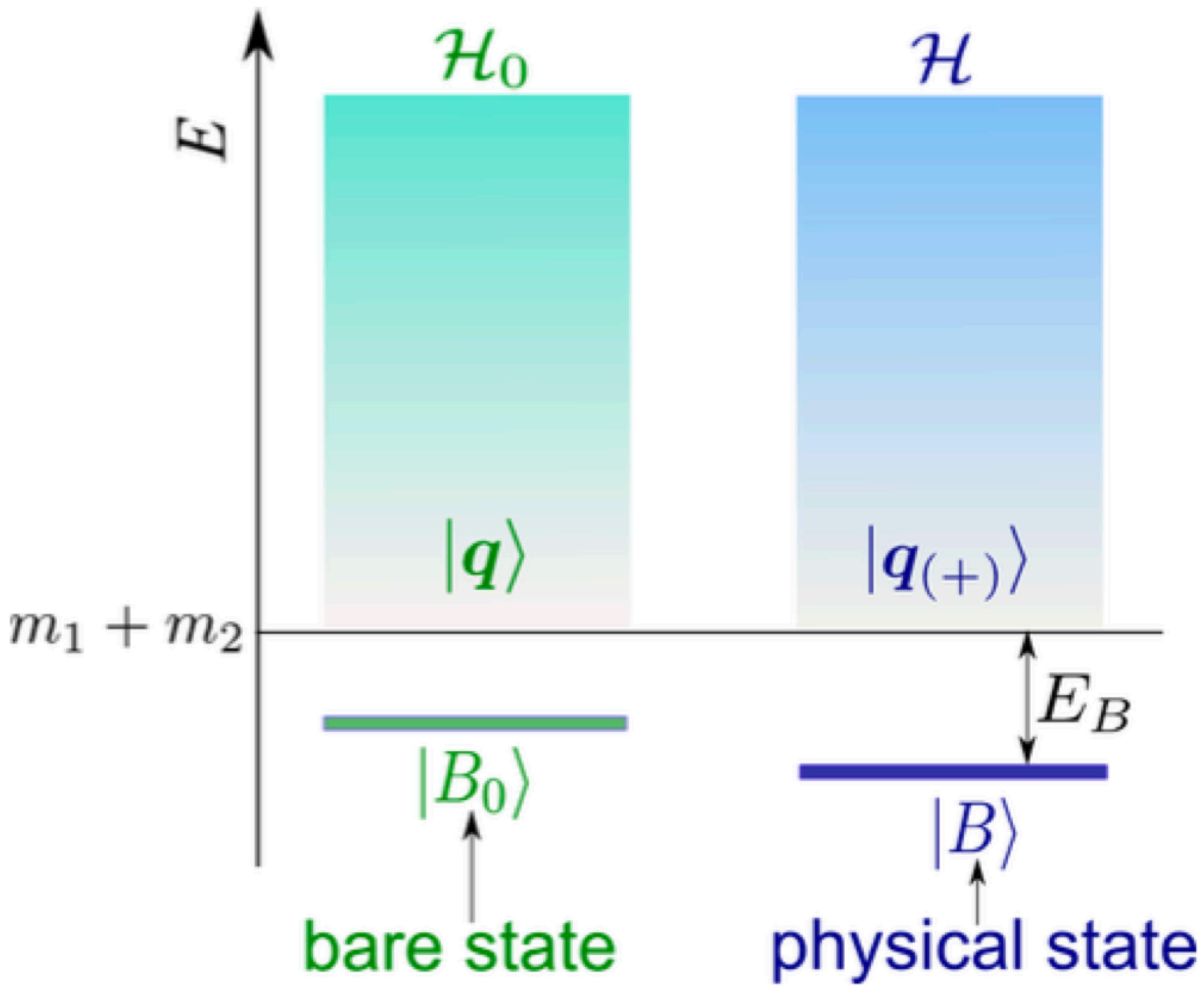
- QCD would alway prefer to bind the closer objects first;
- Such a case provides a lower energy level.
- How to say more about the intrinsic structure of those particles?

# Compositeness critica

$$\color{red}{1-Z} = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} |\langle \boldsymbol{q}|B\rangle|^2$$

$Z = |\langle B_0|B\rangle|^2, \qquad 0 \le (1-Z) \le 1$

- ☞ $Z=0$: pure bound (composite) state
- ☞ $Z=1$: pure elementary state




- Compositeness criteria provides some suggestions on whether the state is elementary or not.
- Deuteron as $pn$ bound state gives $Z \ll 1$;
- Most of the Tetraquark/Pentquark states correspond relatively small $Z$.

# QCD sum rule

$$\Pi(p^2) = i\int d^4x e^{ip\cdot x}\langle 0|T\left\{J(x)J^\dagger(0)\right\}|0\rangle$$
$$= \frac{\lambda_Z^2}{M_Z^2 - p^2} + \underbrace{\frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\Pi_H(s)}{s-p^2}}$$

The vacuum condensates are taken to be the standard values $\langle\bar{q}q\rangle = -(0.24\pm 0.01\,\mathrm{GeV})^3$, $\langle\bar{s}s\rangle = (0.8\pm 0.1)\langle\bar{q}q\rangle$, $\langle\bar{q}g_s\sigma Gq\rangle = m_0^2\langle\bar{q}q\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle = m_0^2\langle\bar{s}s\rangle$, $m_0^2 = (0.8\pm 0.1)\,\mathrm{GeV}^2$, $\langle\frac{\alpha_s GG}{\pi}\rangle = (0.33\,\mathrm{GeV})^4$ at the energy scale $\mu = 1\,\mathrm{GeV}$.

取两个极限：单纯介子对的贡献，不能满足求和规则；单纯四夸克态的贡献，可以满足求和规则。(Phys.Rev. D101 (2020) 074011)

- QCD sum rule introduces modeling on the quark propagator, and decompose the hadron correlator into the contributions from ground state pole and continuous state spectrum.
- And has different criteria for Tetraquark/Pentquark states.

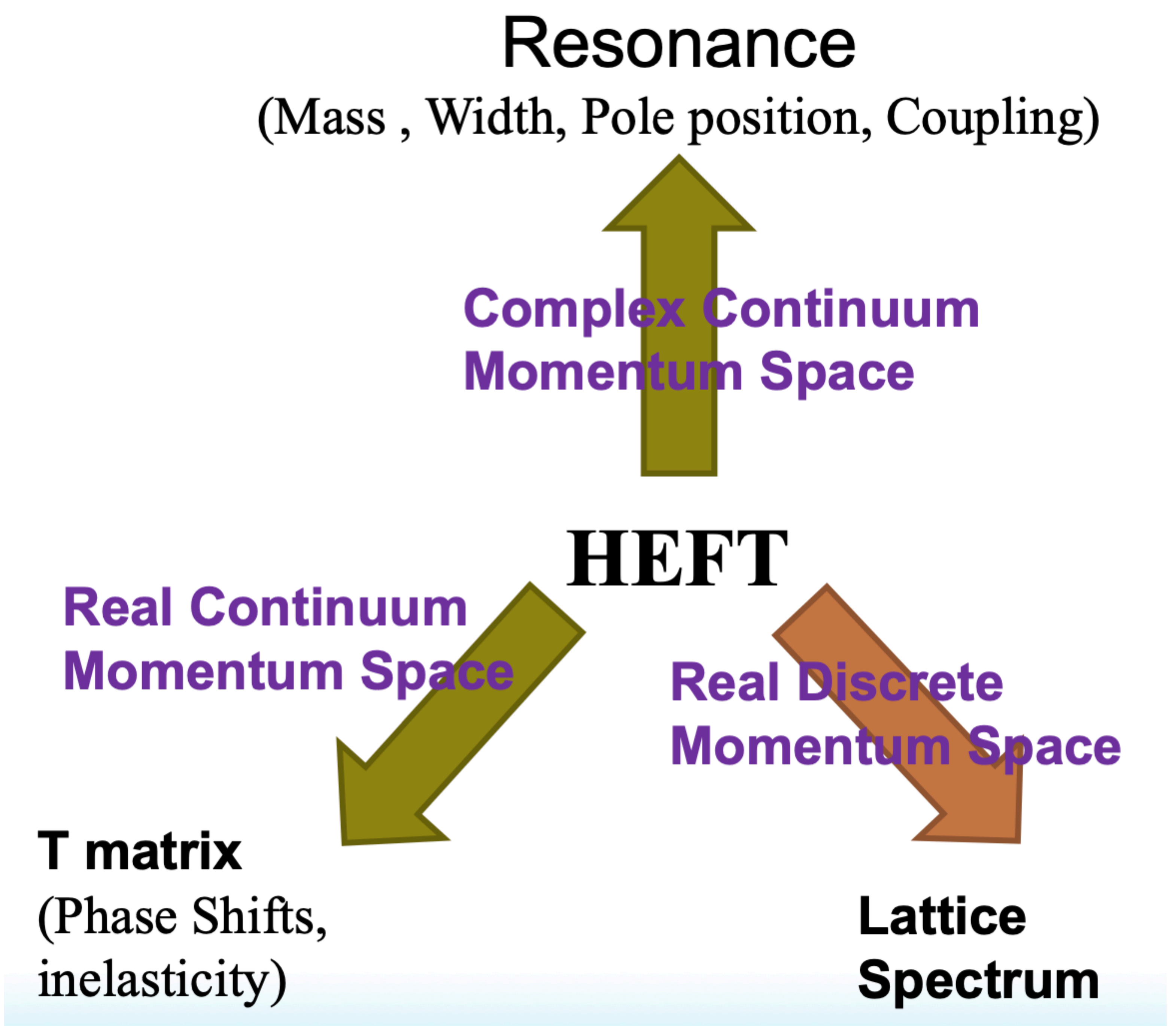
# Hamiltonian effective field theory

$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha,\mathrm{i}} \left|\vec{k}_\mathrm{i}, -\vec{k}_\mathrm{i}\right\rangle_\alpha \left[\sqrt{m_{\alpha_B}^2 + k_\alpha^2} + \sqrt{m_{\alpha_M}^2 + k_\alpha^2}\right]_\alpha \left\langle \vec{k}_\mathrm{i}, -\vec{k}_\mathrm{i}\right|$$

$$H_I = \sum_\mathrm{j} \left(2\pi/\mathrm{L}\right)^{3/2} \sum_\alpha \sum_{i=1,n} \left[\left|\vec{k}_\mathrm{j}, -\vec{k}_\mathrm{j}\right\rangle_\alpha g_{i,\alpha}^+ \langle B_i| + |B_i\rangle g_{i,\alpha}\ {}_\alpha\left\langle \vec{k}_\mathrm{j}, -\vec{k}_\mathrm{j}\right|\right] + \sum_{\mathrm{i,j}} \left(2\pi/\mathrm{L}\right)^3 \sum_{\alpha,\beta} \left|\vec{k}_\mathrm{i}, -\vec{k}_\mathrm{i}\right\rangle_\alpha v_{\alpha,\beta}\ {}_\beta\left\langle \vec{k}_\mathrm{j}, -\vec{k}_\mathrm{j}\right|$$

$$[H_0]_{N_c+1} = \begin{pmatrix} m_0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & \epsilon_1(k_0) & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & \epsilon_2(k_0) & \cdots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \ddots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots & \epsilon_{n_c}(k_0) & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & \epsilon_1(k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

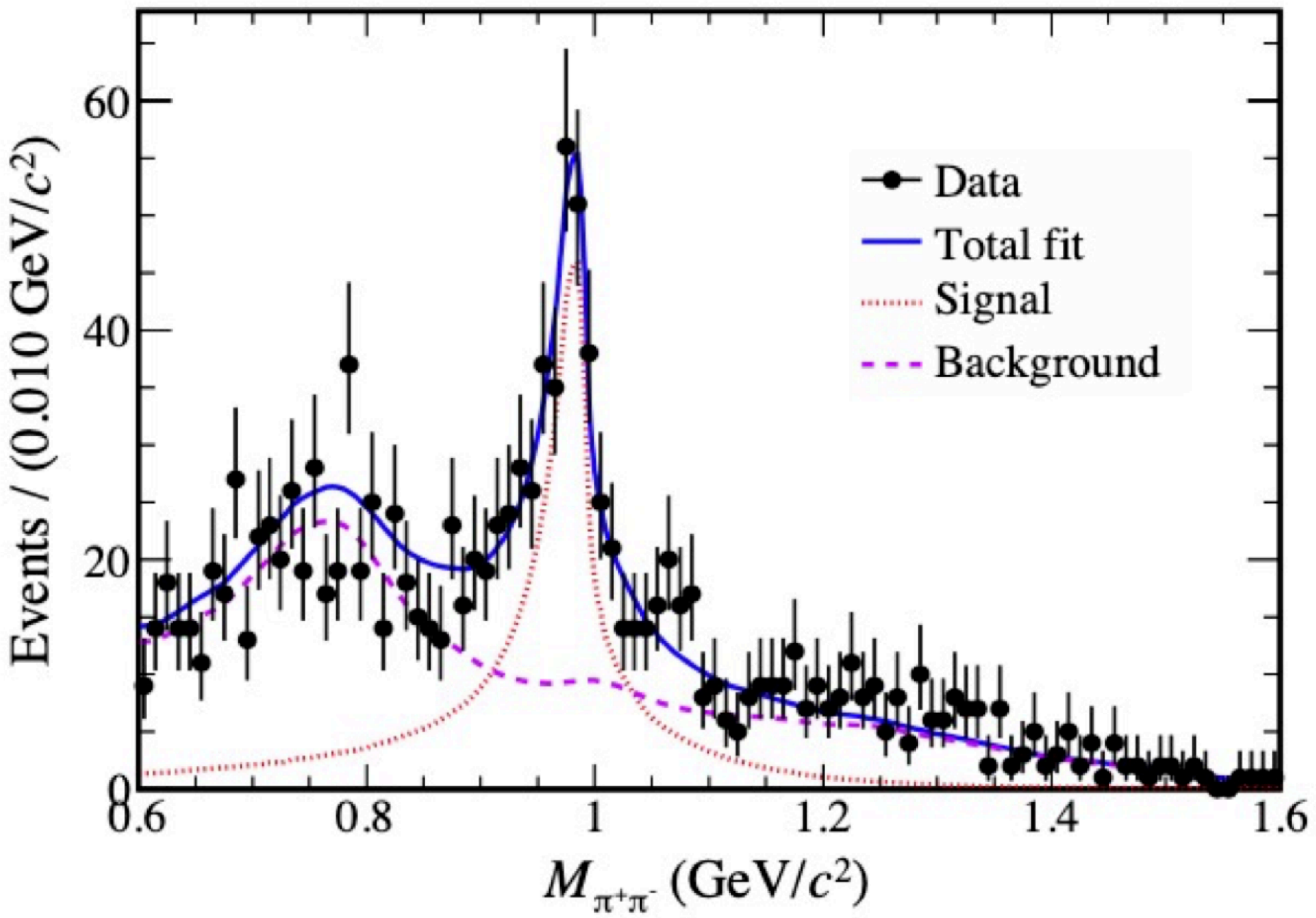
$$[H_I]_{N_c+1} = \begin{pmatrix} 0 & g_1^V(k_0) & g_2^V(k_0) & \cdots & g_{n_c}^V(k_0) & g_1^V(k_1) & \cdots \\ g_1^V(k_0) & v_{1,1}^V(k_0,k_0) & v_{1,2}^V(k_0,k_0) & \cdots & v_{1,n_c}^V(k_0,k_0) & v_{1,1}^V(k_0,k_1) & \cdots \\ g_2^V(k_0) & v_{2,1}^V(k_0,k_0) & v_{2,2}^V(k_0,k_0) & \cdots & v_{2,n_c}^V(k_0,k_0) & v_{2,1}^V(k_0,k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ g_{n_c}^V(k_0) & v_{n_c,1}^V(k_0,k_0) & v_{n_c,2}^V(k_0,k_0) & \cdots & v_{n_c,n_c}^V(k_0,k_0) & v_{n_c,1}^V(k_0,k_1) & \cdots \\ g_1^V(k_1) & v_{1,1}^V(k_1,k_0) & v_{1,2}^V(k_1,k_0) & \cdots & v_{1,n_c}^V(k_1,k_0) & v_{1,1}^V(k_1,k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

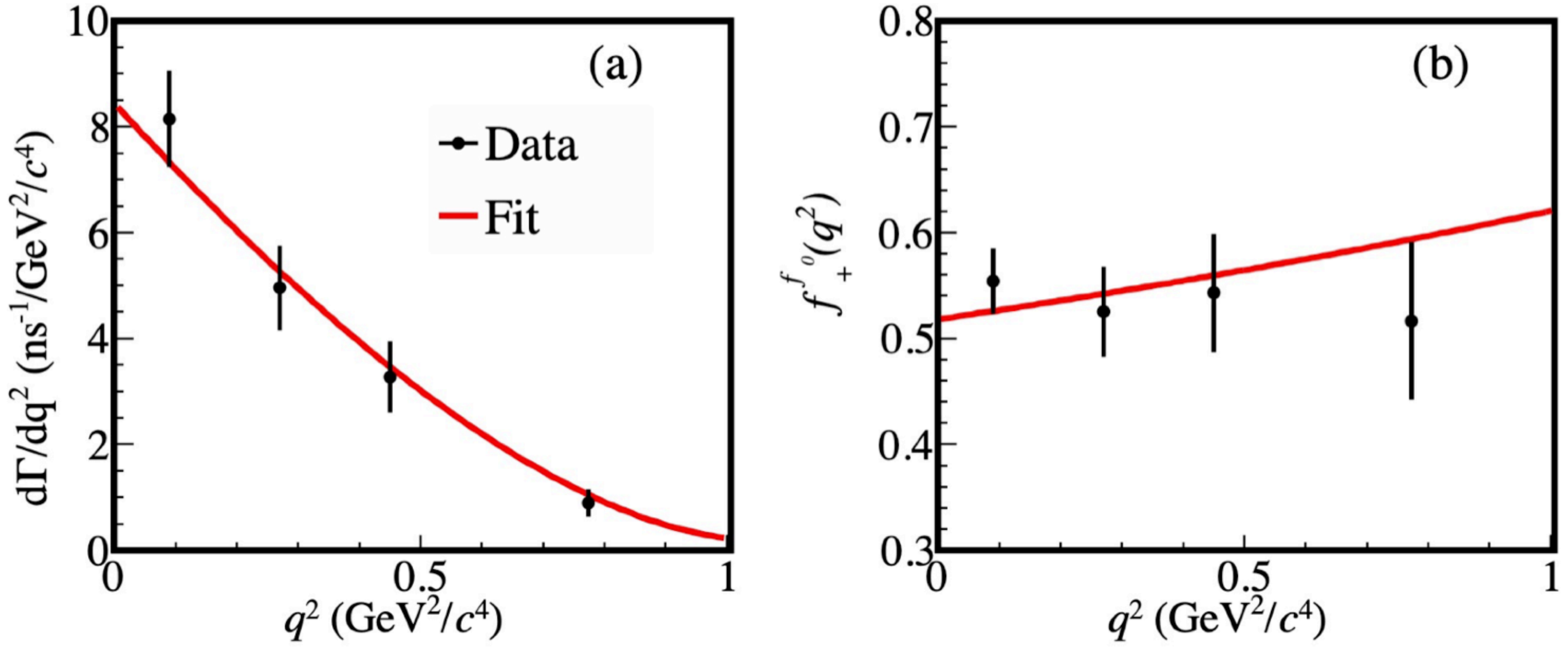



$$m_\rho = m_0 + c_l^{val} m_l^{val} + c_l^{sea} m_l^{sea} + c_s^{sea} m_s^{sea} + \Sigma_{\pi\pi}(m_l^{val} + m_l^{sea})$$

- Can obtain the valence/sea quark mass dependences trough lattice QCD
- Would be able to verify whether $\rho$ and the other states is four quark state or not.

# light scalar meson form factor

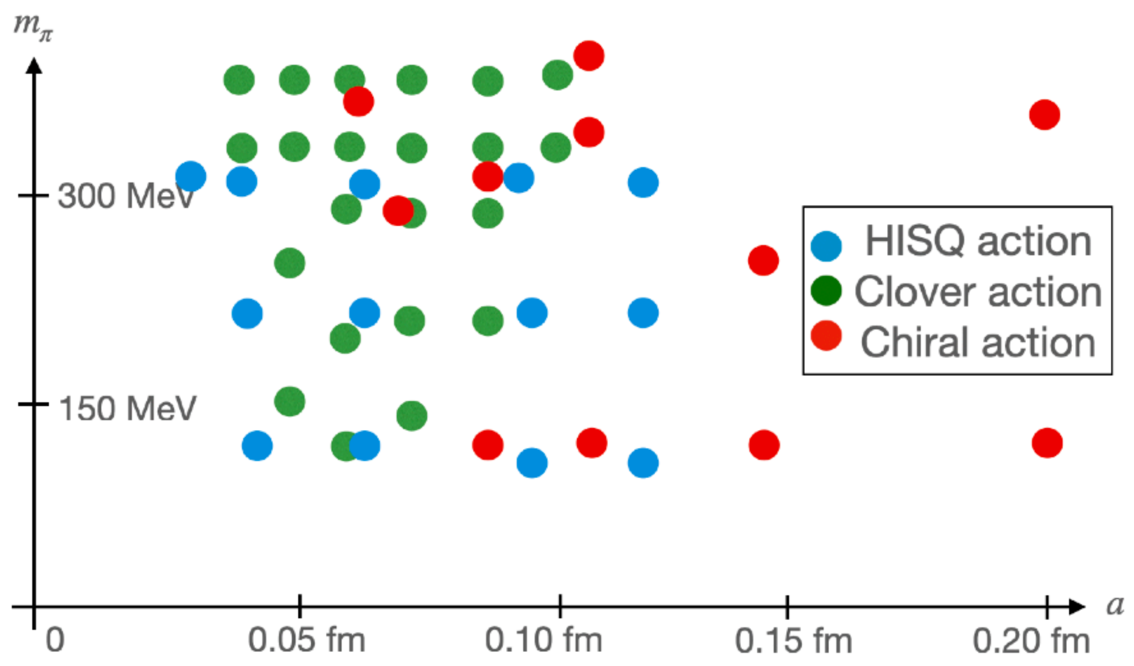



$$f_+^{f_0}(0)|V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

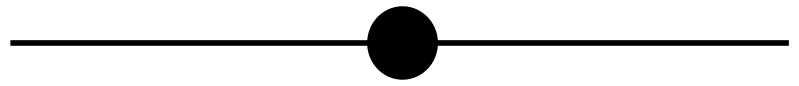
- $D_s^+ \to f_0(980)e^+\nu_e$ form factor has been available (arXiv 2303.12927);
- Form factor of $a_0(980)$ and $f_0(500)$ will be available in a few years based on $20\ \mathrm{fb}^{-1}$@$3.773$ GeV data of BESIII.
- Any more information on these Tetraquark-like states can be extracted?

# Non-perturbative renormalization of Lattice QCD

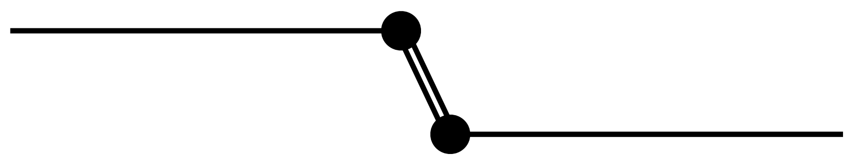



★多种离散化方案的全标度研究

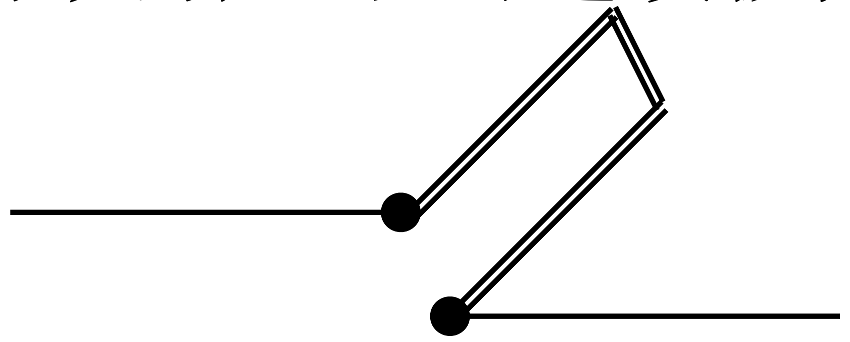
- 在0.03 fm到0.2 fm之间，三种离散化方案共19种格距上对紫外发散的系统研究；
- 确保重整化完整消除紫外发散后，各种离散化方案具有一致的连续极限；



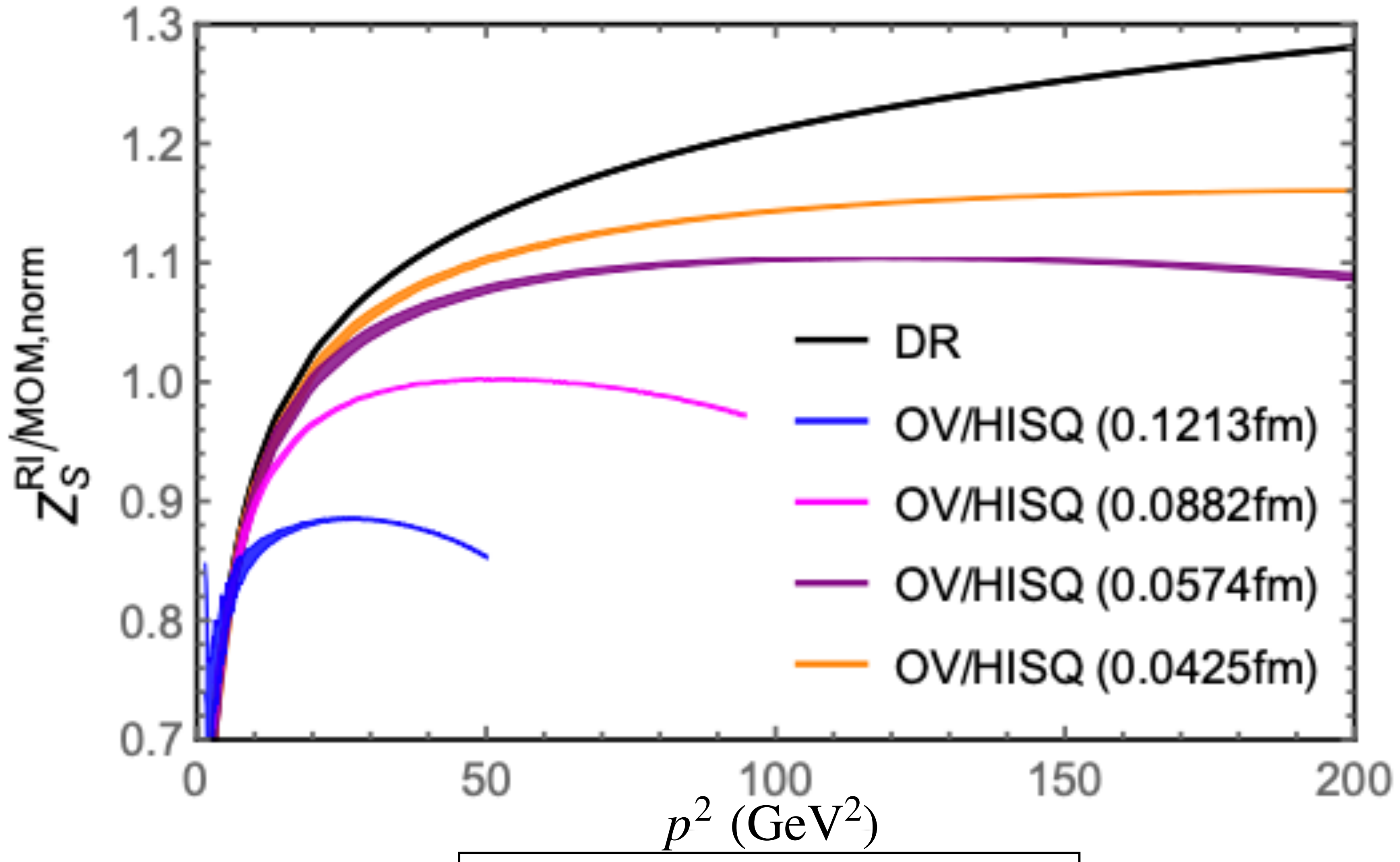
定域算符



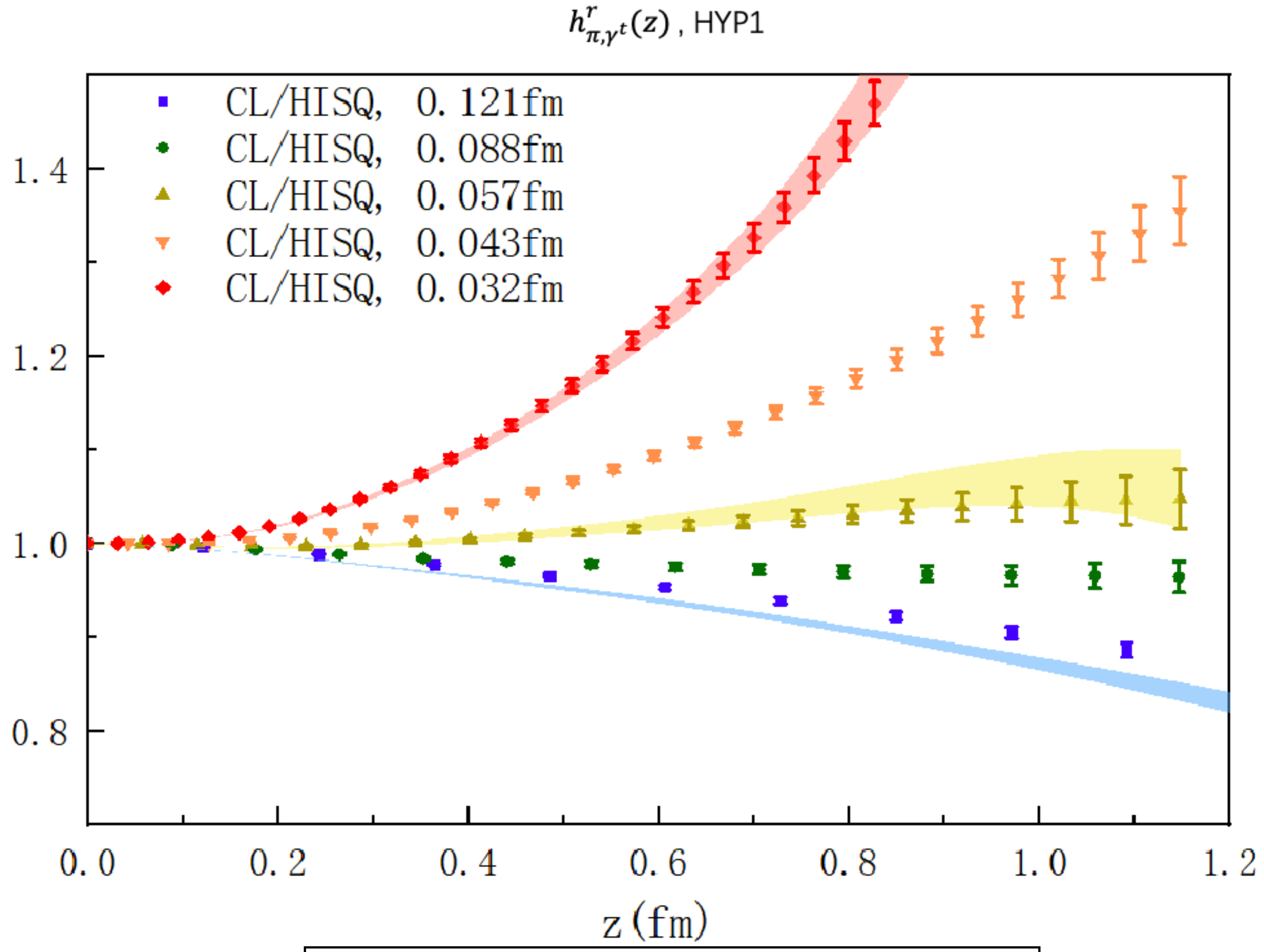
带直线链接的非定域算符



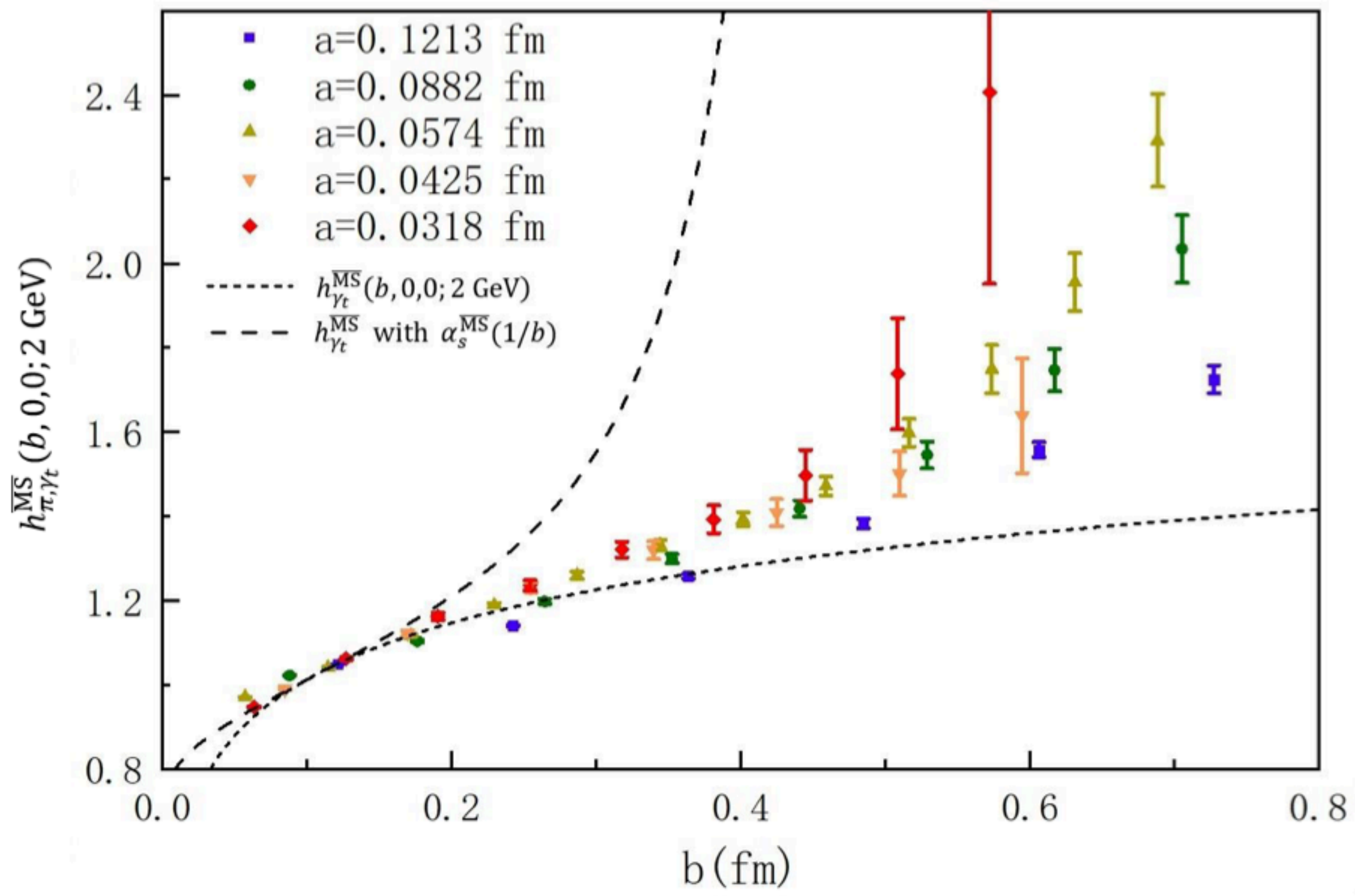
带折线链接的非定域算符




F. He. et.al., $\chi$QCD, PRD106(2022)114506



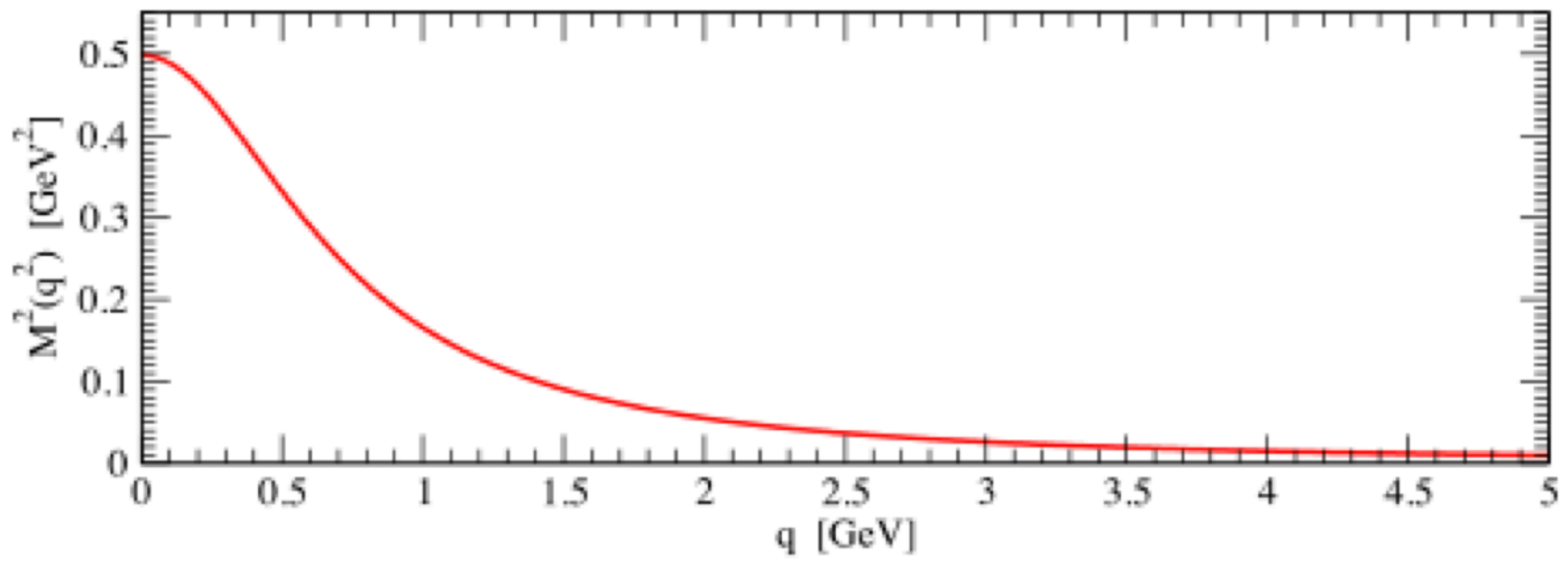

K. Zhang. et.al., $\chi$QCD, PRD104(2021)074501



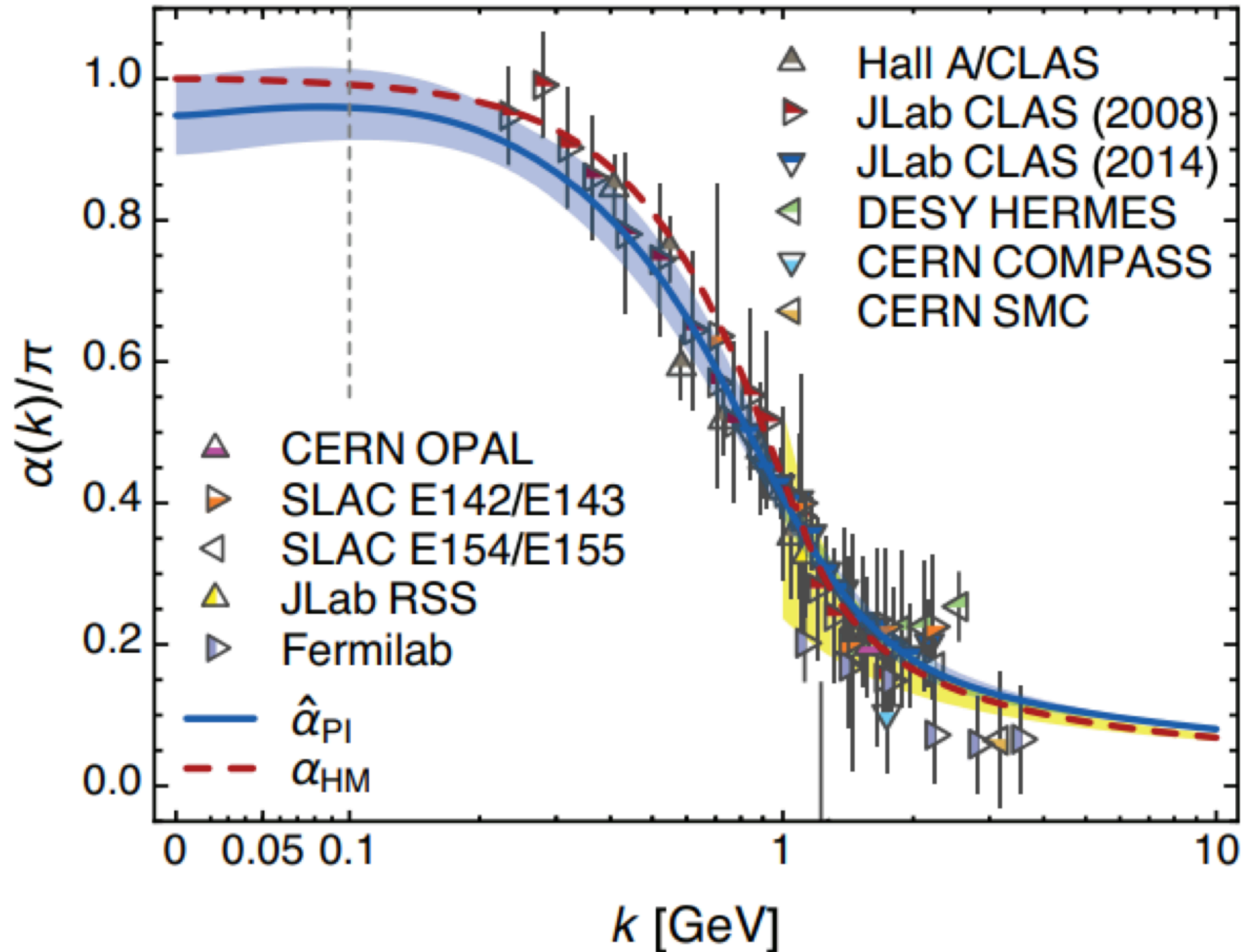

K. Zhang. et.al., LPC, PRL129(2022)082002

# Low energy QCD under Landau gauge

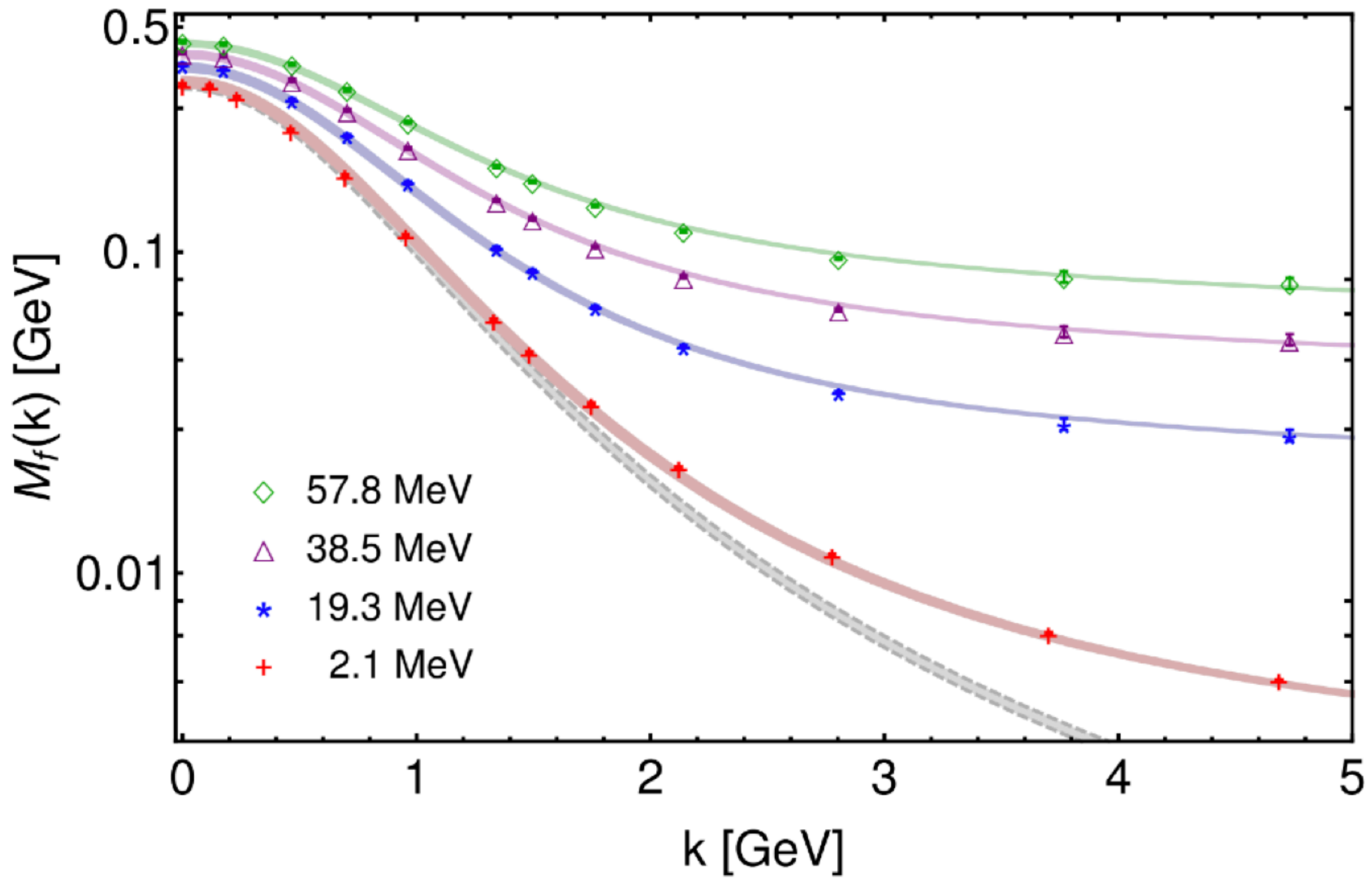
**Gluon mass function:** O. Oliveira et. al., J.Phys. G38, 045003 (2011)




**Running coupling:** Binosi, Mezrag, Papavassiliou, Roberts and Rodriguez-Quintero




Chang et. al., PRD 104, 094509 (2021)




**Equation of Motion (DSEs)**

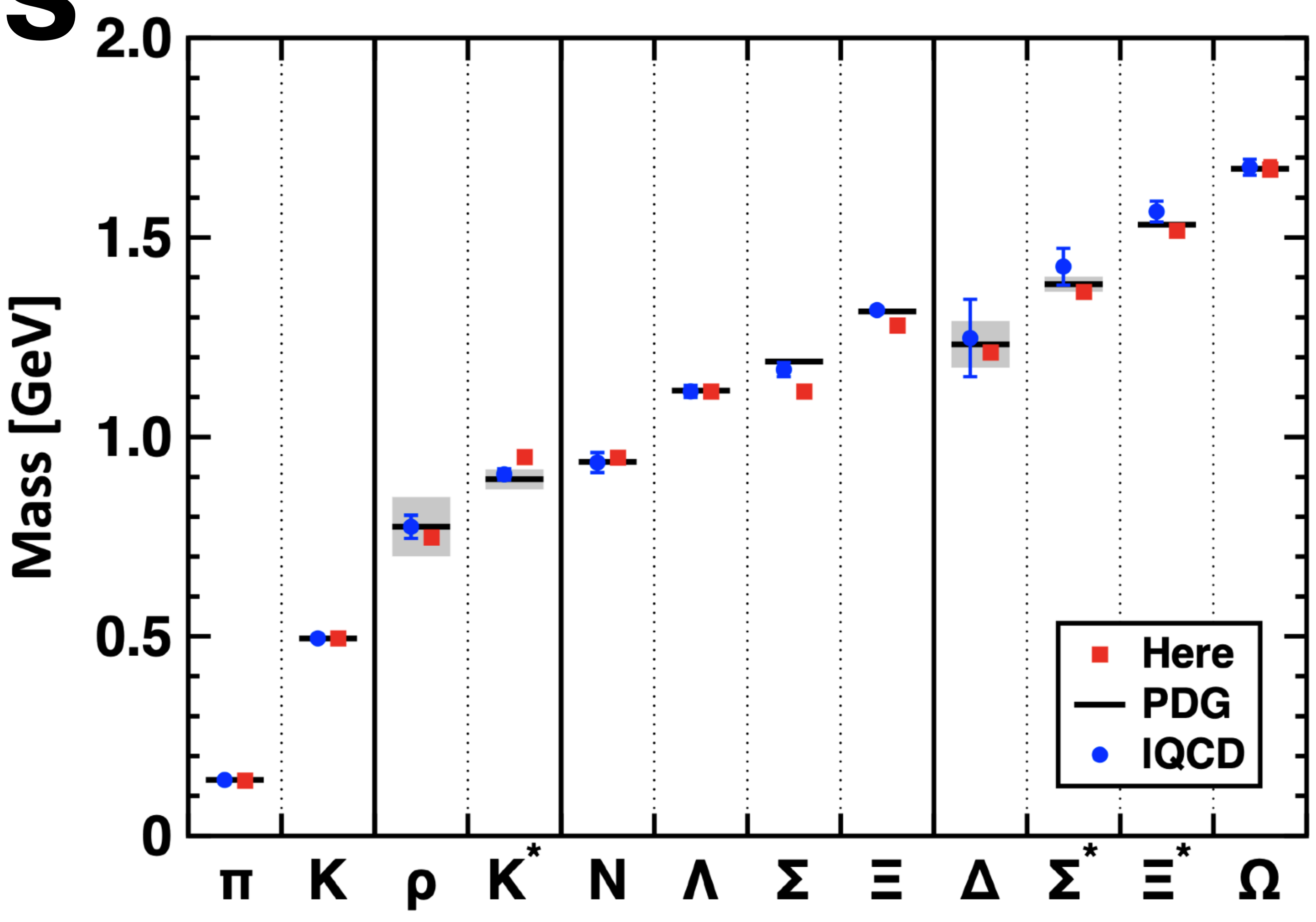
$$S = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

$$\gamma^\mu \Sigma_A + i\sigma_{\mu\nu} Q^\nu \Delta_B + \text{etc.} \sim \Gamma^\mu$$

**Ward identities (Symmetries)**

# Dyson-Schwinger Equations

- Could predict kinds of the hadron spectrum and structure with a few additional parameters;
- Kinds of systematic uncertainties require further efforts.

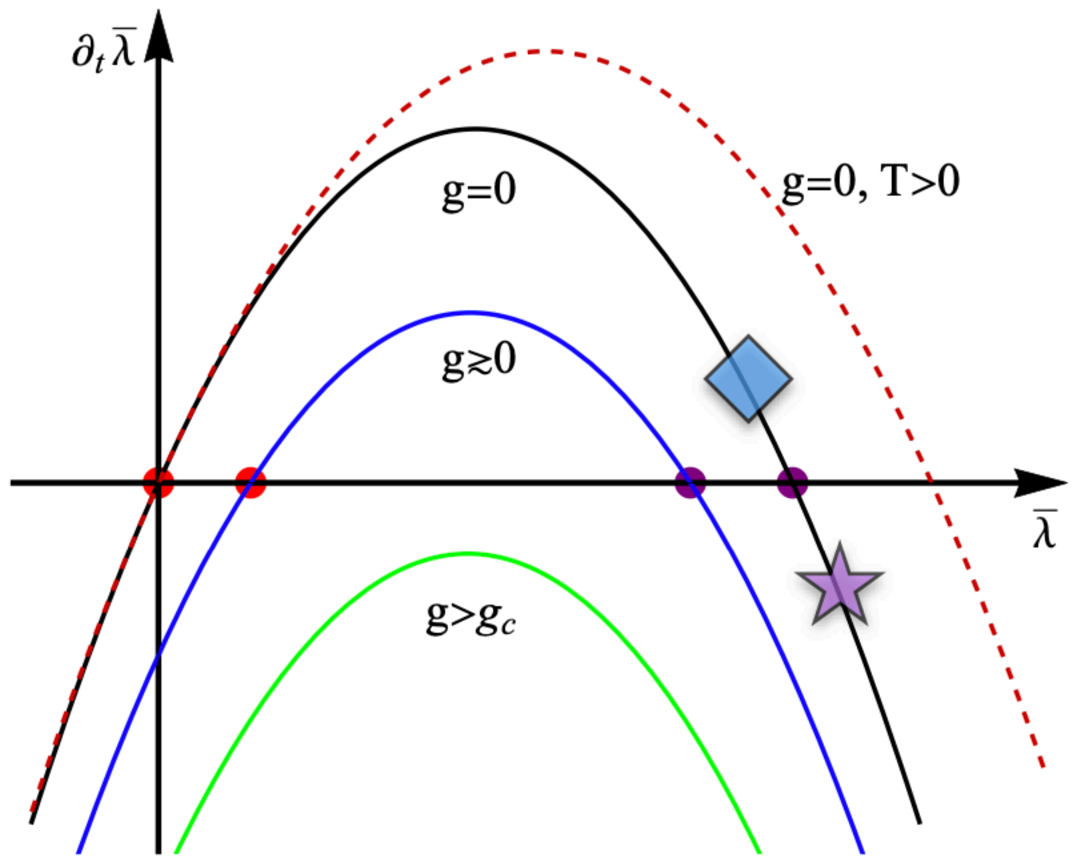



| | 质量谱 | | 电磁形状因子 | | 轴矢形状因子 | |
|---|---|---|---|---|---|---|
| | 双夸克 | 三体 | 双夸克 | 三体 | 双夸克 | 三体 |
| $N(940)1/2^+$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\Delta(1232)3/2^+$ | ✓ | ✓ | ✓ | ✓ | | |
| $N(1440)1/2^+$ | ✓ | | ✓ | | | |
| $N(1535)1/2^-$ | ✓ | | | | | |
| $N(1520)3/2^-$ | ✓ | | | | | |
| 超子（正宇称） | ✓ | ✓ | | ✓ | | |
| 超子（负宇称） | | | | | | |

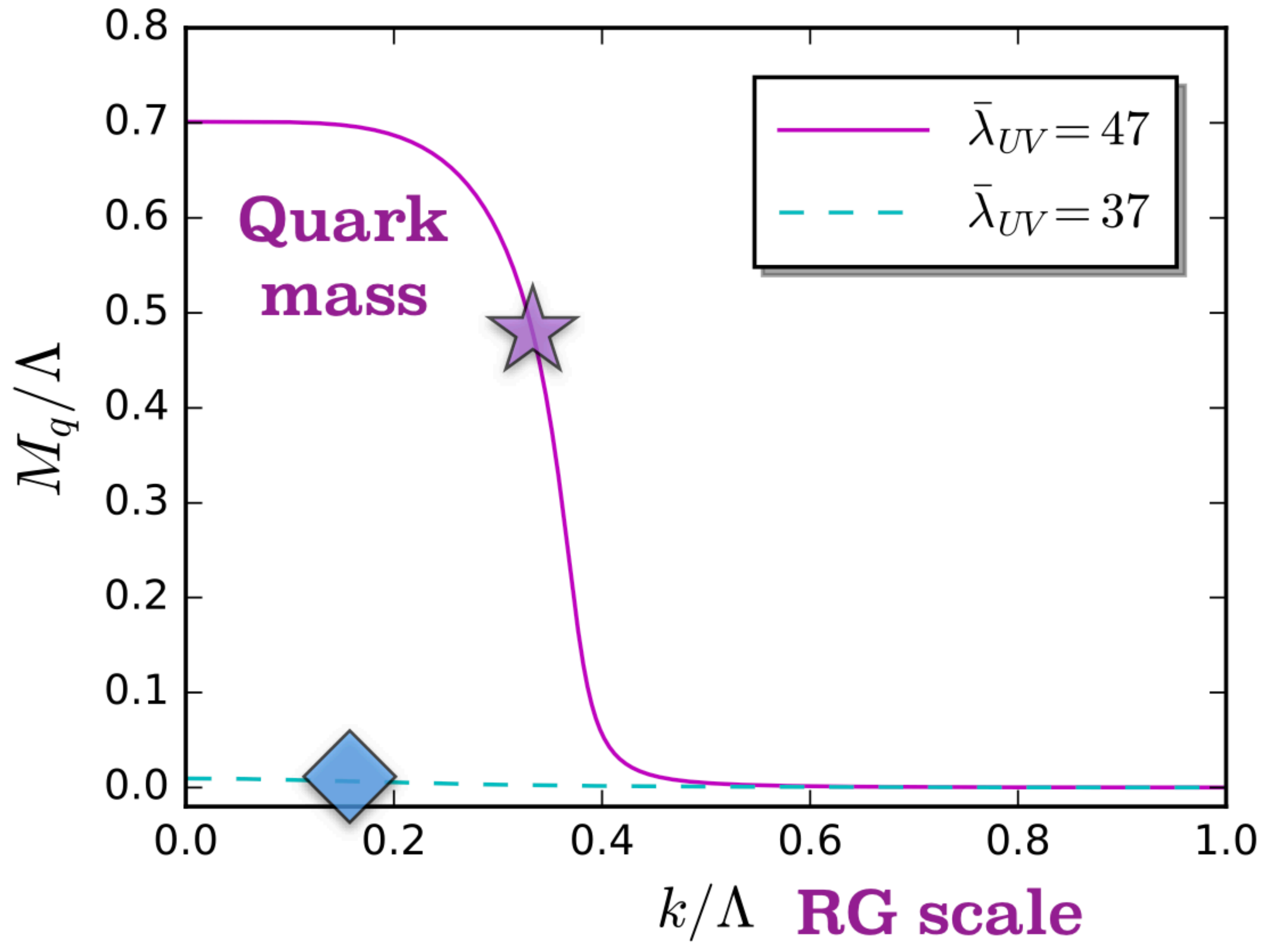
- Gauge invariant check through the $\xi$ dependence;
- Verification from Lattice on the quark-gluon magnetic vertex

# Functional renormalization group with four quark vertices

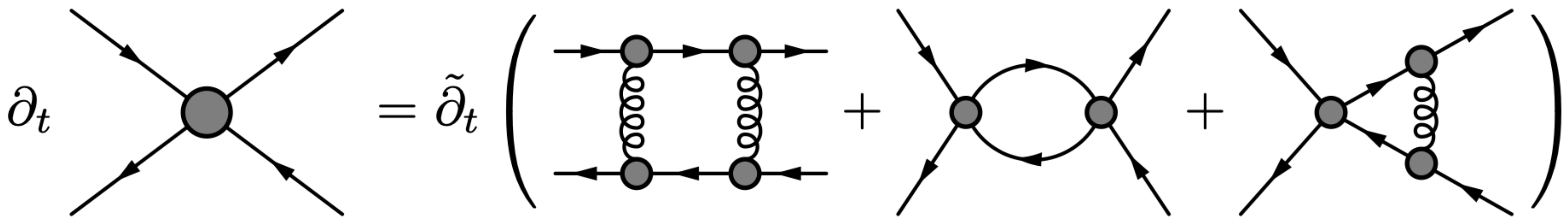
- **$\beta$ function of 4-quark coupling:**




Braun, Gies, *JHEP* 06 (2006) 024.



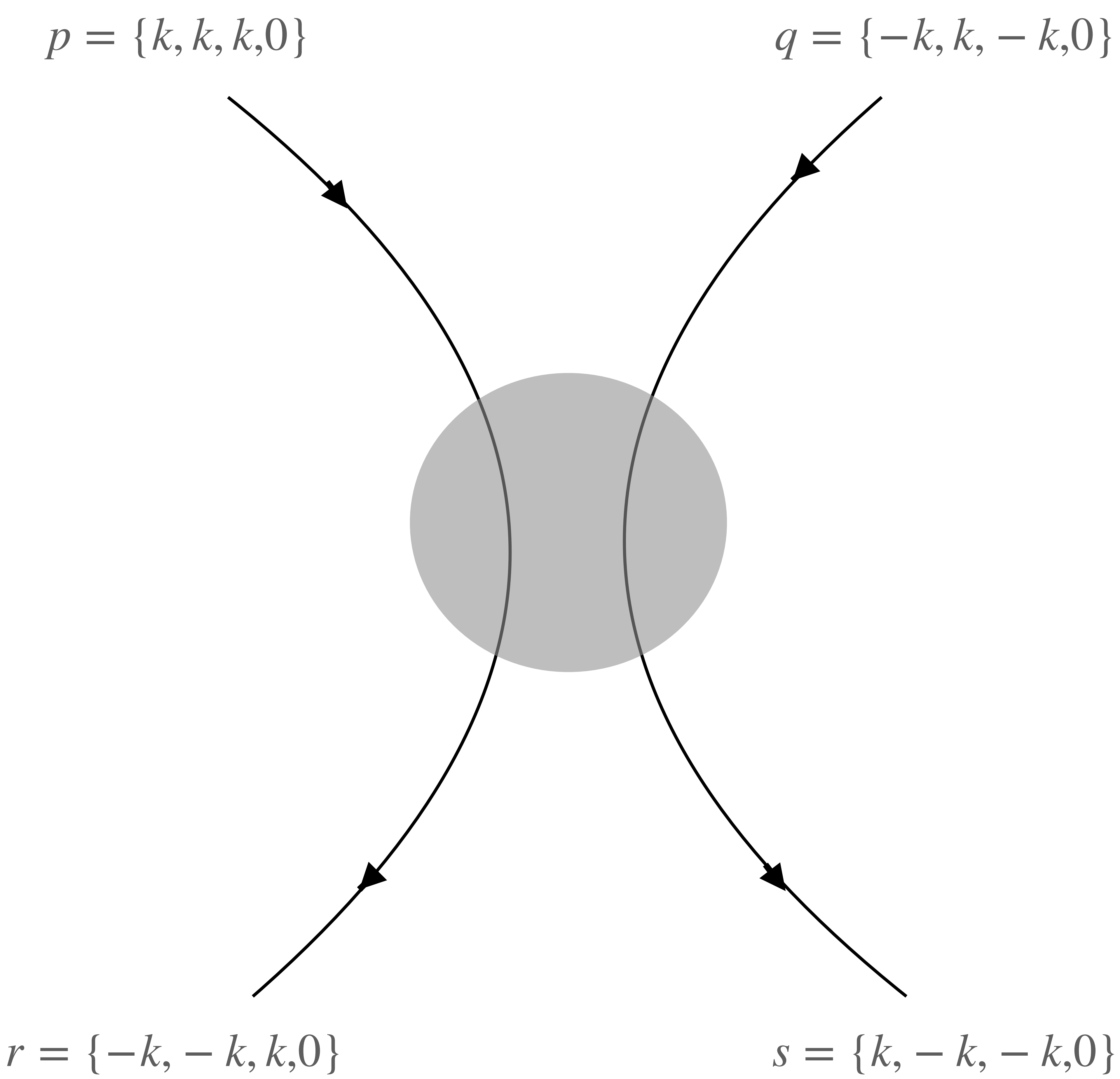

WF, Huang, Pawlowski, Tan, *SciPost Phys.* 14 (2023) 069, arXiv:2209.13120




- A strong 4-quark interaction can also generate dynamical quark mass;
- The low energy prediction is still different from the current lattice result, and then requires further improvements.

# Four quark vertices

# from Lattice QCD



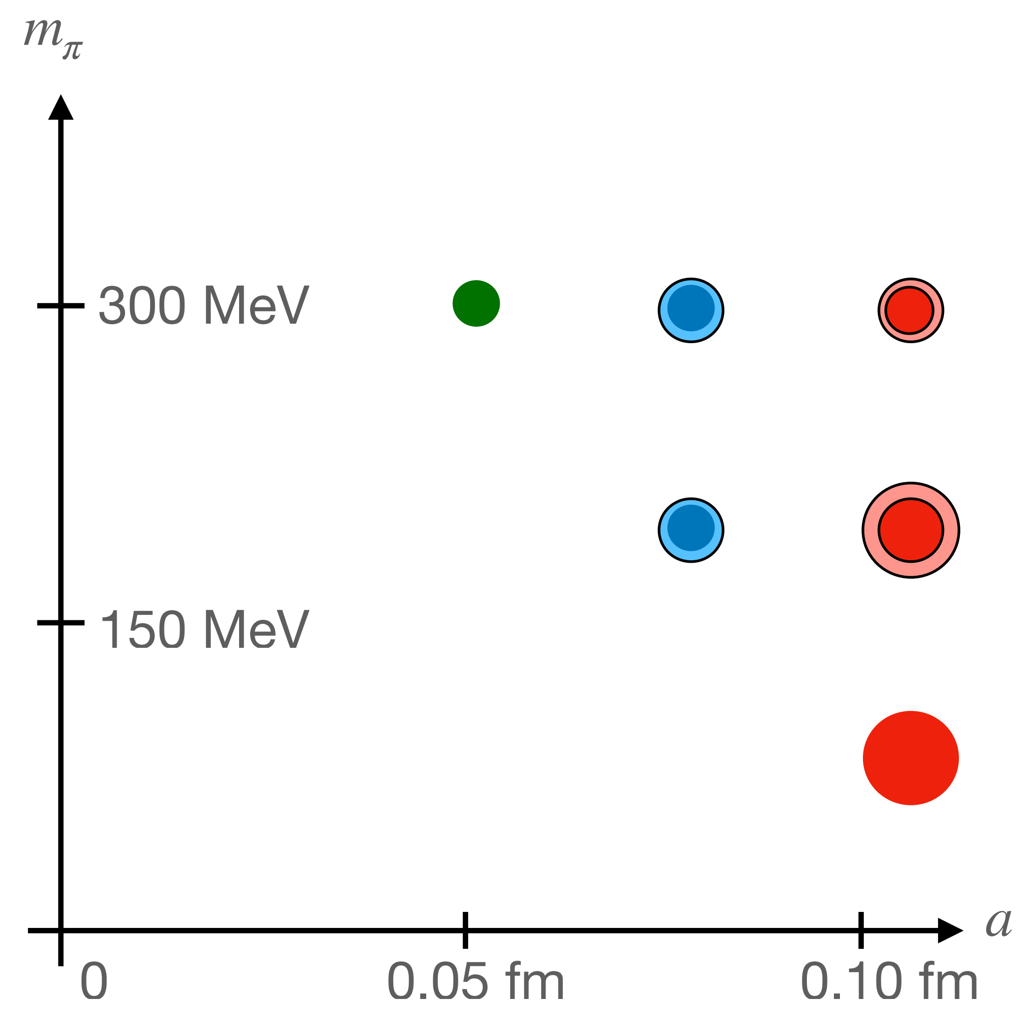

- $p + q + r + s = 0;$
- $p^2 = q^2 = r^2 = s^2 = 3k^2;$
- $(p+q)^2 = (p+r)^2 = (p+s)^2 = 4k^2;$

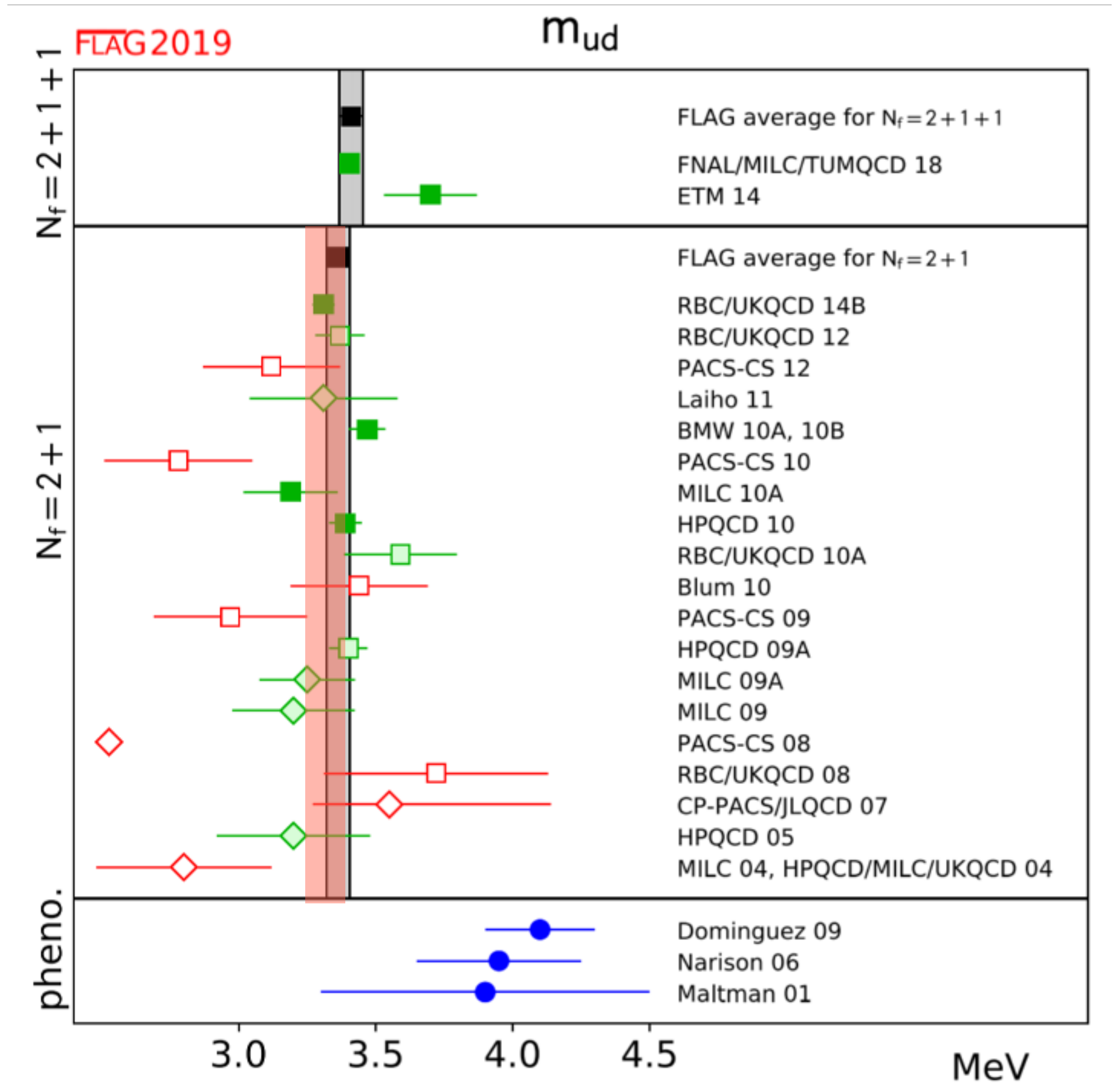
- Direct lattice calculation at low energy scale is doable, and can provide more information on the effective interaction between quarks.

# Configurations in China：

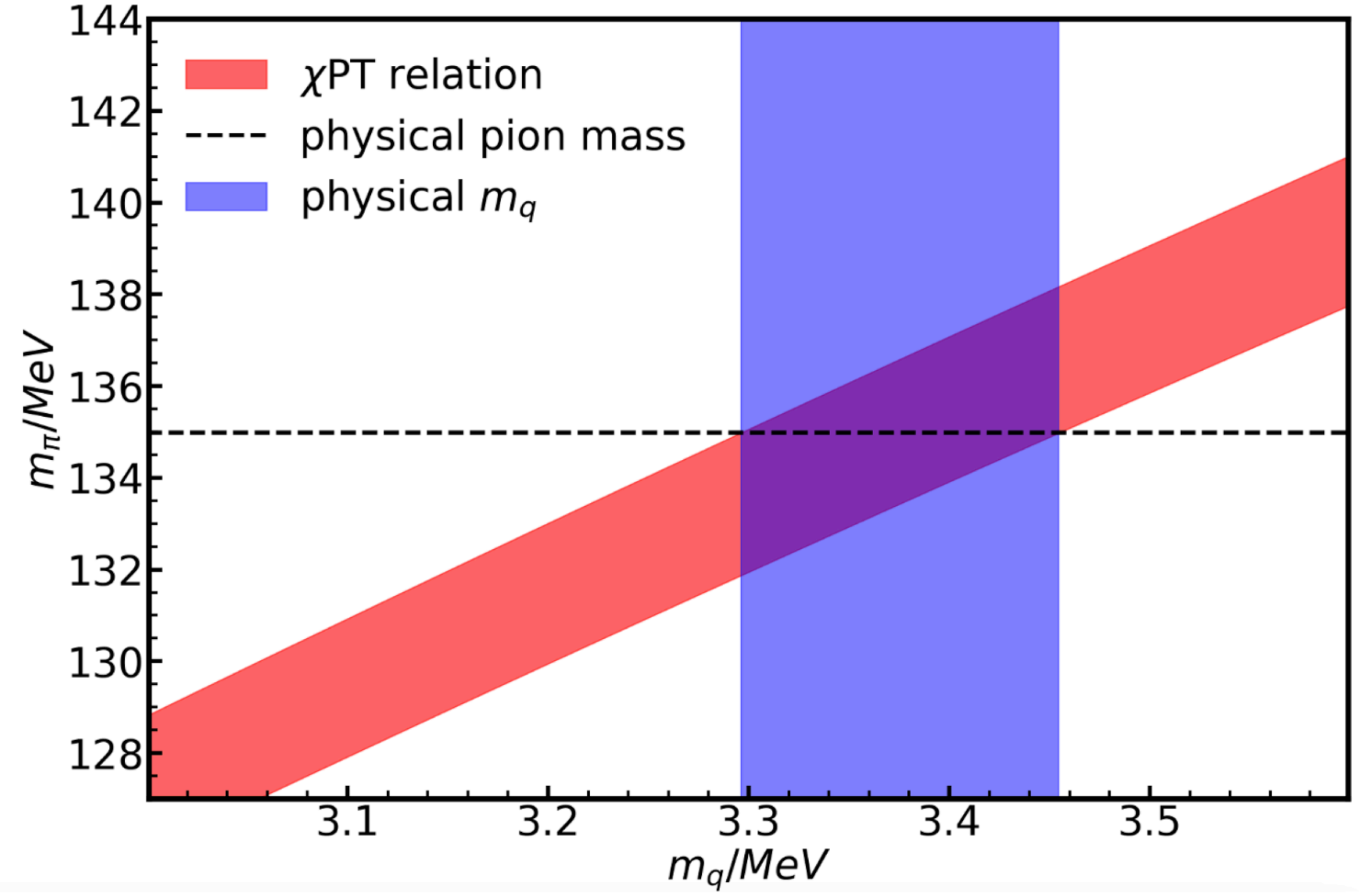
## Towards the FLAG criteria

CLQCD




$$m_{\rm ud}(a \to 0,\, L \to \infty) = 3.38(8)(?)\ \text{MeV}$$



# Configurations in China：

## Application at finite temperature

CLQCD

- $N_f = 2 + 1,\ T = 234\ \mathrm{MeV}$, Physical pion mass ensemble at $a = 0.105\ \mathrm{fm}$;
- Overlap valence fermion and Clover fermion sea.