强子质量的非微扰起源

2023 年4 月 17-28 日

Hadronic Molecules

郭奉坤

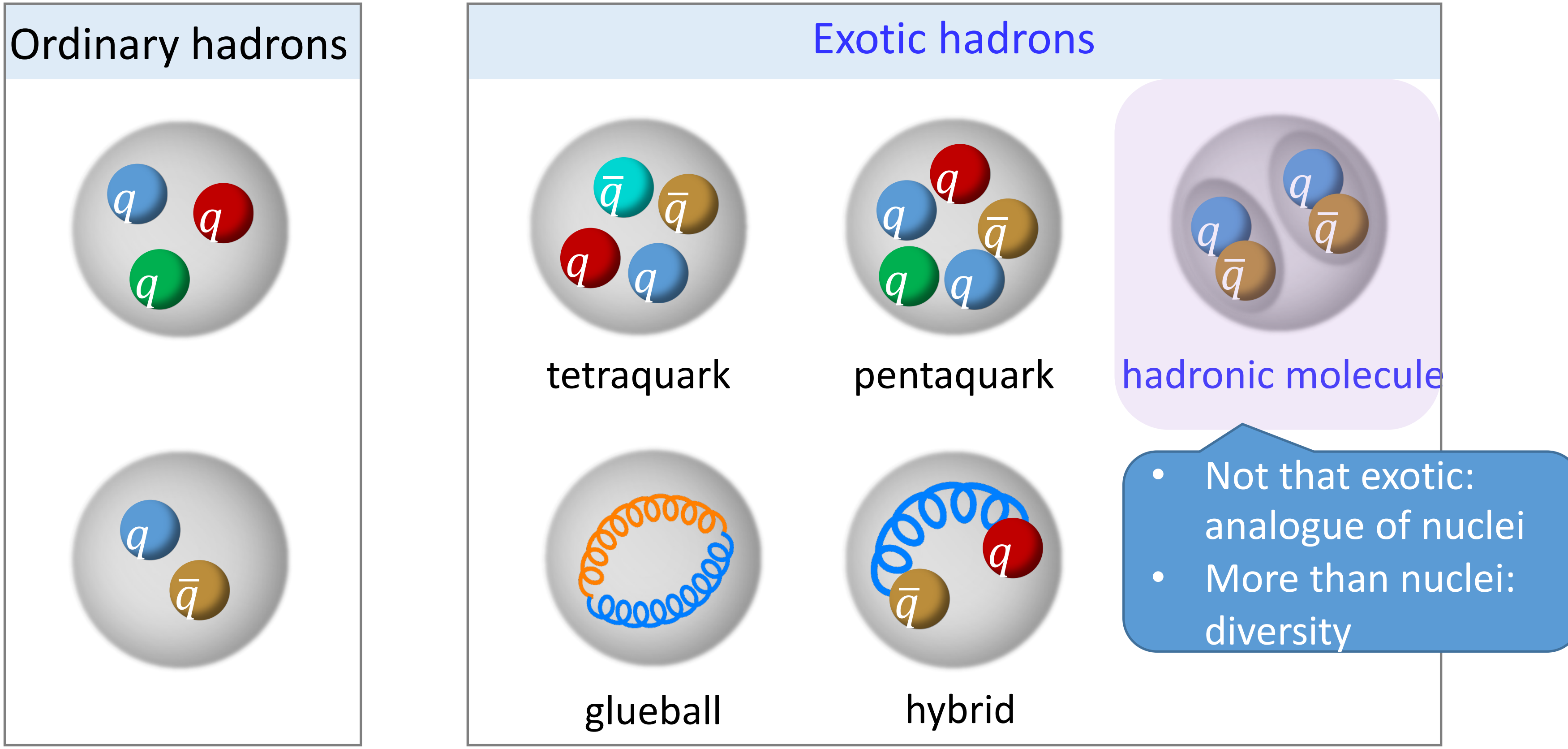
中国科学院理论物理研究所

For a review focusing on hadronic molecules, see:

FKG, C.Hanhart, U.-G.Meißner, Q.Wang, Q.Zhao, B.-S.Zou, Rev. Mod. Phys. **90** (2018) 015004

2023.04.26

Ordinary and exotic hadrons

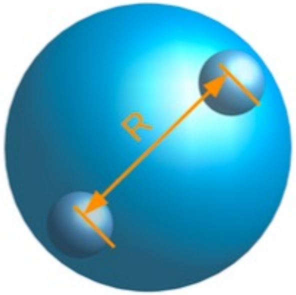


Components with the same quantum numbers always mix, what is the dominant one?

Hadronic molecules

- Hadronic molecule:
  dominant component is a composite state of 2 or more hadrons
- Concept at large distances, so that can be approximated by system of multi-hadrons at low energies
  Consider a 2-body bound state with a mass $M = m_1 + m_2 - E_B$

  size: $\sim \dfrac{1}{\sqrt{2\mu E_B}} \gg r_{\text{hadron}}$



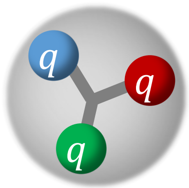
- scale separation $\Rightarrow$ (nonrelativistic) EFT applicable!
- Only narrow hadrons can be considered as components of hadronic molecules, $\Gamma_h \ll 1/r$, r: range of forces

Filin *et al.*, PRL105(2010)019101; FKG, Meißner, PRD84(2011)014013

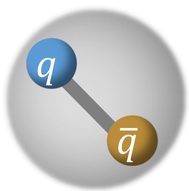
Relation with confinement mechanism?

- Different flux tube configurations: compact multiquarks and hadronic molecules

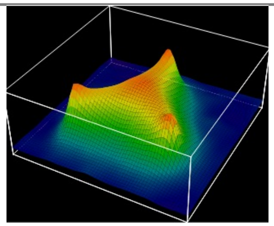
Ordinary hadrons



baryon

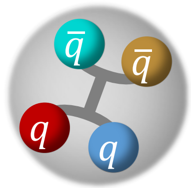


meson

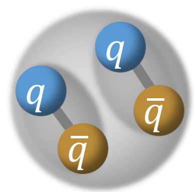


V.G. Bornyakov et al., PRD70(2004)054506

Multiquarks vs hadronic molecules



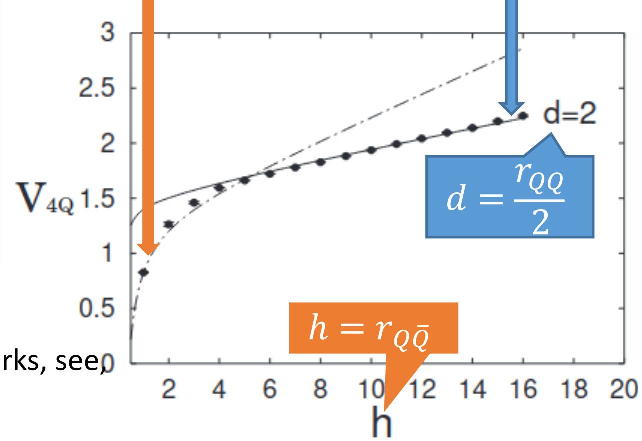
compact tetraquark



hadronic molecule

For large distance between Q and $\bar{Q}$

For small distance between Q and $\bar{Q}$



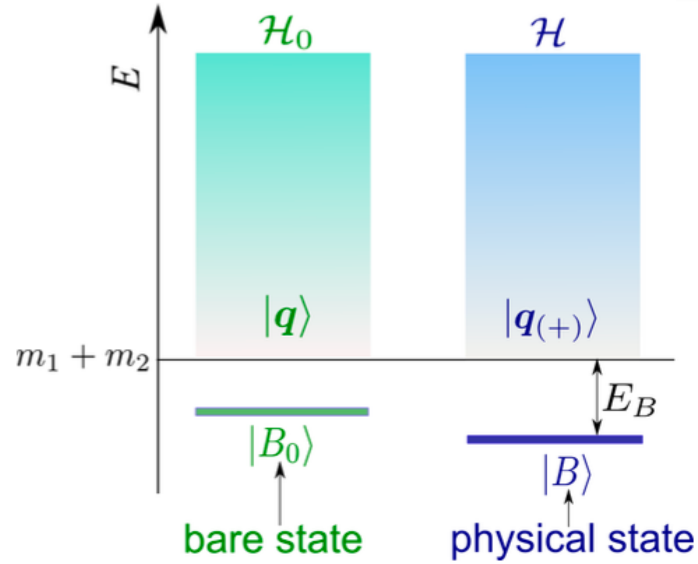
F. Okiharu et al., PRD72(2005)014505

For lattice studies of flux tube picture of multiquarks, see, e.g., F. Okiharu, H. Suganuma, T. T. Takahashi, PRD72(2005)014505; PRL94(2005)192001

An overview: H. Suganuma et al., arXiv:1103.4015

Compositeness

Model-independent result for S-wave loosely bound composite states:



Consider a system with Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + V$$

$\mathcal{H}_0$: free Hamiltonian, V: interaction potential

- **Compositeness**:
  the probability of finding the physical state $|B\rangle$ in the 2-body continuum $|\boldsymbol{q}\rangle$

$$1 - Z = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} |\langle \boldsymbol{q}|B\rangle|^2$$

- $Z = |\langle B_0|B\rangle|^2, \qquad 0 \le (1-Z) \le 1$
  - ☞ $Z = 0$: pure bound (composite) state
  - ☞ $Z = 1$: pure elementary state

Compositeness

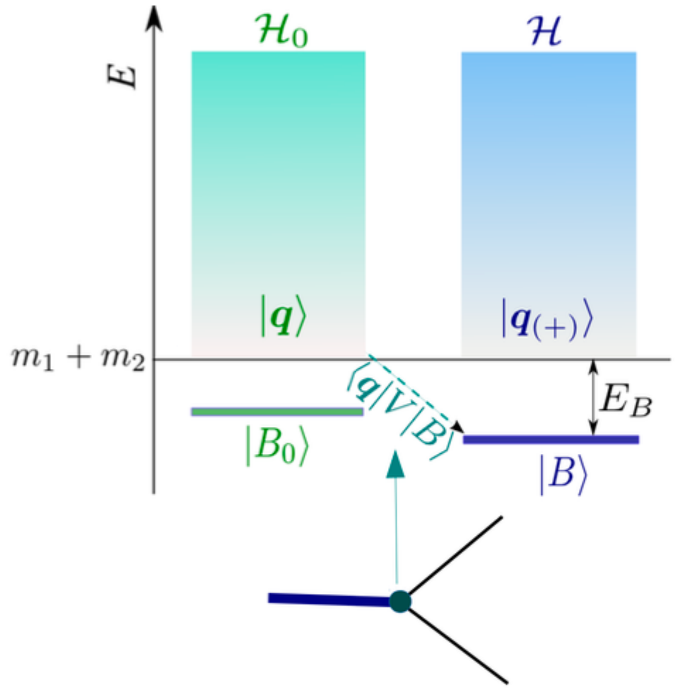
Compositeness : $1 - Z = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} |\langle \boldsymbol{q}|B\rangle|^2$

- Schrödinger equation

$$(\mathcal{H}_0 + V)|B\rangle = -E_B|B\rangle$$

multiplying by $\langle \boldsymbol{q}|$ and using $\mathcal{H}_0|\boldsymbol{q}\rangle = \frac{\boldsymbol{q}^2}{2\mu}|\boldsymbol{q}\rangle$:
$\Rightarrow$ momentum-space wave function:

$$\langle \boldsymbol{q}|B\rangle = -\frac{\langle \boldsymbol{q}|V|B\rangle}{E_B + \boldsymbol{q}^2/(2\mu)}$$



- S-wave, small binding energy so that $R = 1/\sqrt{2\mu E_B} \gg r$, r: range of forces

$$\langle \boldsymbol{q}|V|B\rangle = g_{\rm NR}\left[1 + \mathcal{O}(r/R)\right]$$

- Compositeness:

$$1 - Z = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} \frac{g_{\rm NR}^2}{[E_B + \boldsymbol{q}^2/(2\mu)]^2}\left[1 + \mathcal{O}\left(\frac{r}{R}\right)\right] = \frac{\mu^2 g_{\rm NR}^2}{2\pi\sqrt{2\mu E_B}}\left[1 + \mathcal{O}\left(\frac{r}{R}\right)\right]$$

Compositeness

- Coupling constant measures the compositeness for an S-wave shallow bound state
$$g_{\rm NR}^2 \approx (1-Z)\frac{2\pi}{\mu^2}\sqrt{2\mu E_B} \le \frac{2\pi}{\mu^2}\sqrt{2\mu E_B}$$
bounded from the above

➢ $g_{\rm NR}^2$ is the residue of the T-matrix element at the pole $E=-E_B(E\equiv\sqrt{s}-m_1-m_2)$:

$$g_{\rm NR}^2 = \lim_{E\to -E_B}(E+E_B)\langle \boldsymbol{k}|T_{\rm NR}|\boldsymbol{k}\rangle$$

here nonrelativistic normalization is used: $T_{\rm NR} = -\frac{T}{4\mu\sqrt{s}} \simeq -\frac{T}{4m_1m_2}$

➢ use the LSE $T_{\rm NR} = V + V\frac{1}{E-\mathcal{H}_0+i\epsilon}T_{\rm NR}$ and $|B\rangle\left\langle B\right| + \int\frac{d^3q}{(2\pi)^3}\left|\boldsymbol{q}_{(+)}\right\rangle\left\langle\boldsymbol{q}_{(+)}\right| = 1$ to derive the Low equation (noticing $T_{\rm NR}|\boldsymbol{q}\rangle = V\left|\boldsymbol{q}_{(+)}\right\rangle$):

$$\langle \boldsymbol{k}'|T_{\rm NR}|\boldsymbol{k}\rangle = \langle \boldsymbol{k}'|V|\boldsymbol{k}\rangle + \frac{\langle \boldsymbol{k}'|V|B\rangle\langle B|V|\boldsymbol{k}\rangle}{E+E_B+i\epsilon} + \int\frac{d^3q}{(2\pi)^3}\frac{\langle \boldsymbol{k}'|T_{\rm NR}|\boldsymbol{q}\rangle\langle \boldsymbol{q}|T_{\rm NR}^\dagger|\boldsymbol{k}\rangle}{E-\boldsymbol{q}^2/(2\mu)+i\epsilon}$$

Compositeness

- Z can be related to scattering length a and effective range r_e — Weinberg (1965)

$$a_0 = -\frac{2R(1-Z)}{2-Z}\left[1+\mathcal{O}\left(\frac{r}{R}\right)\right], r_{e0} = -\frac{RZ}{1-Z}\left[1+\mathcal{O}\left(\frac{r}{R}\right)\right]$$

Effective range expansion (S-wave): $f_0^{-1}(k) = 1/a_0 + r_{e0}\,k^2/2 - ik + \mathcal{O}(k^4)$

Derivation:

$$T_{\rm NR}(E) \equiv \langle k|T_{\rm NR}|k\rangle = -\frac{2\pi}{\mu} f_0(k) \;\Rightarrow\; {\rm Im}\, T_{\rm NR}^{-1}(E) = \frac{\mu}{2\pi}\sqrt{2\mu E}\,\theta(E)$$

Twice-subtracted dispersion relation for $t^{-1}(E)$

$$T_{\rm NR}^{-1}(E) = \frac{E+E_B}{g_{\rm NR}^2} + \frac{(E+E_B)^2}{\pi}\int_0^{+\infty} dw \frac{{\rm Im}\, T_{\rm NR}^{-1}(w)}{(w-E-i\epsilon)(w+E_B)^2}$$

$$= \frac{E+E_B}{g_{\rm NR}^2} + \frac{\mu R}{4\pi}\left(\frac{1}{R} - \sqrt{-2\mu E - i\epsilon}\right)^2$$

- Example: deuteron as pn bound state. Exp.: $E_B = 2.2$ MeV, $a(^3S_1) = -5.4$ fm

$$a_{Z=1} = 0 \text{ fm},\ a_{Z=0} = -(4.3 \pm 1.4)\text{fm}$$

Note:

- Only for S-wave loosely bound state
- Problematic for $r_{e0} > 0 \Rightarrow Z < 0$ $\quad 1 - Z = \sqrt{\frac{a_0}{a_0 + 2r_{e0}}}$

I. Matuschek, V. Baru, FKG, C. Hanhart, EPJA 57 (2021) 101;
Y. Li, FKG, J.-Y. Pang, J.-J. Wu, PRD 105 (2022) L071502

$$X = 1 - \exp\left(\frac{1}{\pi}\int_0^\infty dE\, \frac{\delta_0(E)}{E - E_B}\right)$$

NREFT at LO

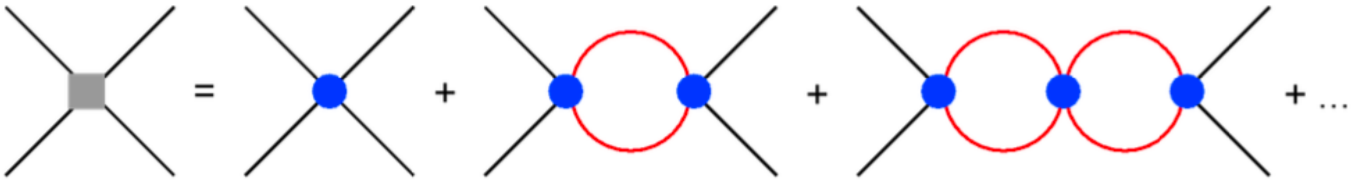
We consider a system of two particles of masses m_1, m_2

- in the near-threshold region, a momentum expansion for the interactions with the LO being a constant

$$\mathcal{L} = \sum_{i=1,2} \phi_i^\dagger \left(i\partial_0 - m_i + \frac{\nabla^2}{2m_i} \right) \phi_i - C_0 \, \phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2 + \ldots$$

nonrelativistic propagator: $\dfrac{i}{p^0 - m_i - \boldsymbol{p}^2/(2m_i) + i\epsilon}$

- to have a near-threshold bound state (hadronic molecule)

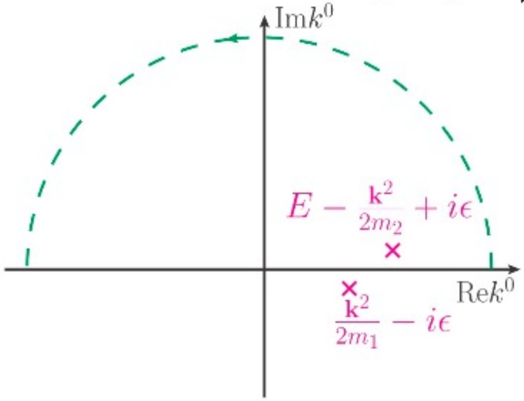


$$T_{\rm NR}(E) = C_0 + C_0 \, G_{\rm NR}(E) \, C_0 + C_0 \, G_{\rm NR}(E) \, C_0 \, G_{\rm NR}(E) \, C_0 + \ldots$$
$$= \frac{1}{C_0{}^{-1} - G_{\rm NR}(E)}$$

NREFT at LO

- The loop integral is linearly divergent (E defined relative to $m_1 + m_2$), regularized with, e.g., a sharp cut

$$G_{\rm NR}(E) = i\int\frac{d^3\boldsymbol{k}dk^0}{(2\pi)^4}\left[\left(k^0 - \frac{\boldsymbol{k}^2}{2m_1} + i\epsilon\right)\left(E - k^0 - \frac{\boldsymbol{k}^2}{2m_2} + i\epsilon\right)\right]^{-1}$$



$$= -i2\mu(2\pi i)\int^{\Lambda}\frac{d^3\boldsymbol{k}}{(2\pi)^4}\frac{1}{2\mu E - \boldsymbol{k}^2 + i\epsilon}$$

$$= -\frac{\mu}{\pi^2}\left(\Lambda - \sqrt{-2\mu E - i\epsilon}\,\arctan\frac{\Lambda}{\sqrt{-2\mu E - i\epsilon}}\right)$$

$$= -\frac{\mu}{\pi^2}\Lambda + \frac{\mu}{2\pi}\sqrt{-2\mu E - i\epsilon} + \mathcal{O}\left(\Lambda^{-1}\right)$$

for real E, $\sqrt{-2\mu E - i\epsilon} = \sqrt{-2\mu E}\,\theta(-E) - i\sqrt{2\mu E}\,\theta(E)$

NREFT at LO

- Renormalization: $T_{\rm NR}$ is Λ-independent,

$$T_{\rm NR}(E) = \frac{1}{C_0^{-1} - G_{\rm NR}}$$

$$= \Bigg(\underbrace{\frac{1}{C_0} + \frac{\mu}{\pi^2}\Lambda}_{\equiv 1/C_0^r} - \frac{\mu}{2\pi}\sqrt{-2\mu E - i\epsilon}\Bigg)^{-1}$$

$$= \frac{2\pi/\mu}{2\pi/(\mu\, C_0^r) - \sqrt{-2\mu E - i\epsilon}}$$

- Other regularization can be used as well, equiavalent to the sharp cutoff up to $1/\Lambda$ suppressed terms, e.g.
  - ☞ with a Gaussian regulator $\exp\left(-\boldsymbol{k}^2/\Lambda_{\rm G}^2\right)$, $\Lambda_{\rm G} = \sqrt{2/\pi}\Lambda$
  - ☞ with the power divergence subtraction (PDS) scheme in dimensional regularization by letting, $\Lambda_{\rm PDS} = 2\Lambda/\pi$ Kaplan, Savage, Wise (1998)

NREFT at LO

$$T_{\rm NR}(E) = \frac{2\pi/\mu}{2\pi/(\mu\, C_0^r) - \sqrt{-2\mu E - i\epsilon}} = \frac{2\pi/\mu}{2\pi/(\mu\, C_0^r) + i\, k}$$

- from matching to effective range expansion,

$$f_0^{-1}(k) = -\frac{2\pi}{\mu} T_{\rm NR}^{-1} = -\frac{1}{a_0} + \frac{1}{2} r_{e0} k^2 - i\, k + \mathcal{O}\left(k^4\right)$$

$2\pi/(\mu C_0^r) = 1/a_0$; higher terms are necessary to match both a and r_e

- pole below threshold at $E = -E_B$ with $E_B > 0$

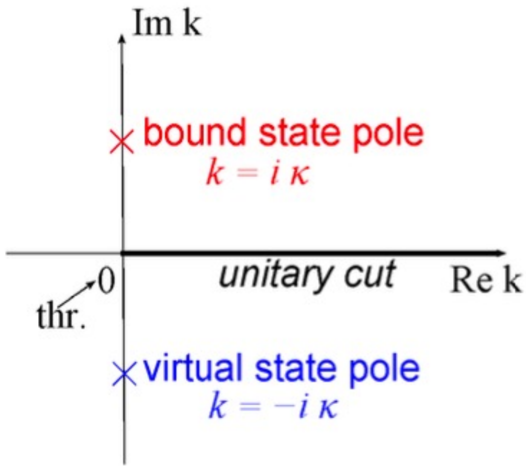
$\kappa \equiv |\sqrt{2\mu E_B}|$

  - bound state pole, in the 1st Riemann sheet

    $\Rightarrow 2\pi/(\mu C_0^r) = \kappa$

  - virtual state pole, in the 2nd Riemann sheet

    $\Rightarrow 2\pi/(\mu C_0^r) = -\kappa$



  - unable to get a resonance pole at LO with a single channel

Bound state and virtual state

- If the same binding energy, bound and virtual states cannot be distinguished above threshold ($E > 0$):

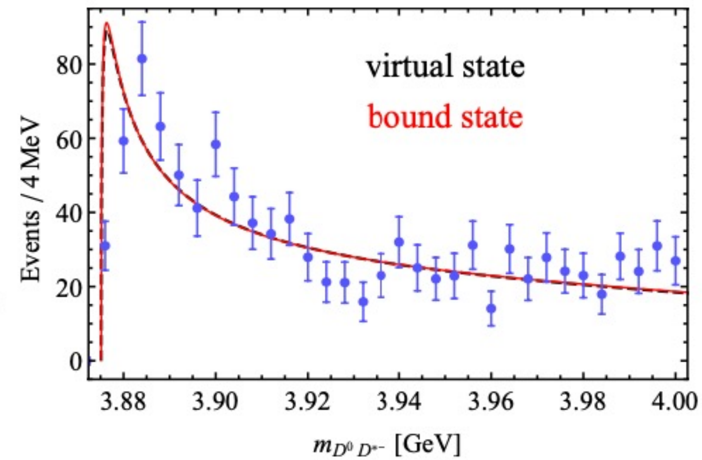
$$|T_{\rm NR}(E)|^2 \propto \left|\frac{1}{\pm\kappa + i\sqrt{2\mu E}}\right|^2 = \frac{1}{\kappa^2 + 2\mu E}$$

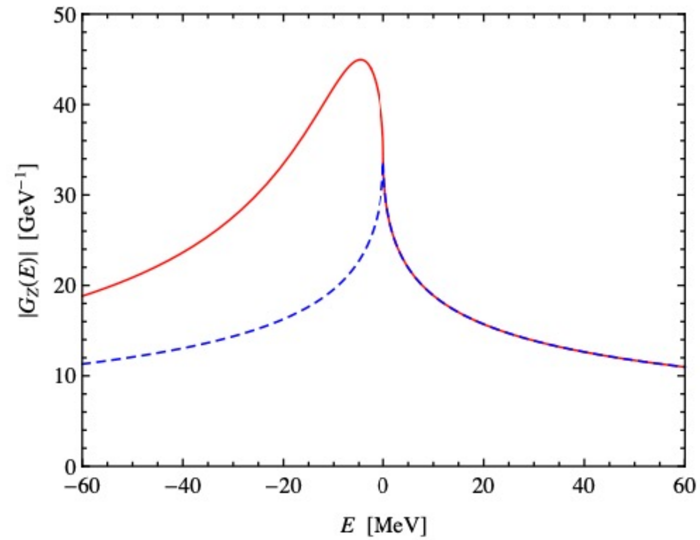
- Bound state and virtual state are different below threshold ($E < 0$):
  - ☞ bound state: peaked below threshold

$$|T_{\rm NR}(E)|^2 \propto \frac{1}{(\kappa - \sqrt{-2\mu E})^2}$$

  - ☞ virtual state: a sharp cusp at threshold

$$|T_{\rm NR}(E)|^2 \propto \frac{1}{(\kappa + \sqrt{-2\mu E})^2}$$





Lower Fig.: bound state and virtual state with $E_B = 5$ MeV and a small width to the inelastic channel

Cleven et al., EPJA47(2011)120

Bound state and virtual state

$$T_{\rm NR}(E) = \frac{2\pi/\mu}{2\pi/(\mu\, C_0^r) - \sqrt{-2\mu E - i\epsilon}}$$

At LO, effective coupling strength for bound state

$$g_{\rm NR}^2 = \lim_{E\to -E_B}(E+E_B)\,T_{\rm NR}(E) = -\frac{2\pi}{\mu}\left(\frac{d}{dE}\sqrt{-2\mu E - i\epsilon}\right)^{-1}_{E=-E_B}$$
$$= \frac{2\pi}{\mu^2}\sqrt{2\mu E_B}$$

Recall the compositeness formula:

$$g_{\rm NR}^2 = (1-Z)\frac{2\pi}{\mu^2}\sqrt{2\mu E_B}$$

This means that the pole obtained at LO NREFT with only a constant contact term corresponds to a purely composite state $(Z=0)$

- Range corrections: other components at shorter distances
  - coupling to additional states/channels
  - energy/momentum-dependent interactions: higher order

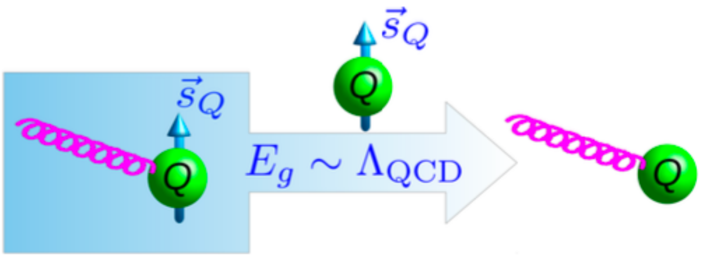
Heavy quark spin symmetry

- For heavy quarks (charm, bottom) in a hadron, typical momentum transfer Λ_{QCD}

  ☞ heavy quark spin symmetry (HQSS):

  chromomag. interaction $\propto \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{m_Q}$

  spin of the heavy quark decouples



Let total angular momentum $\boldsymbol{J} = \boldsymbol{s}_Q + \boldsymbol{s}_\ell$,

$\boldsymbol{s}_Q$: heavy quark spin,

$\boldsymbol{s}_\ell$: spin of the light degrees of freedom (including orbital angular momentum)

✓ HQSS:

s_ℓ and s_Q are conserved separately in the heavy quark limit!

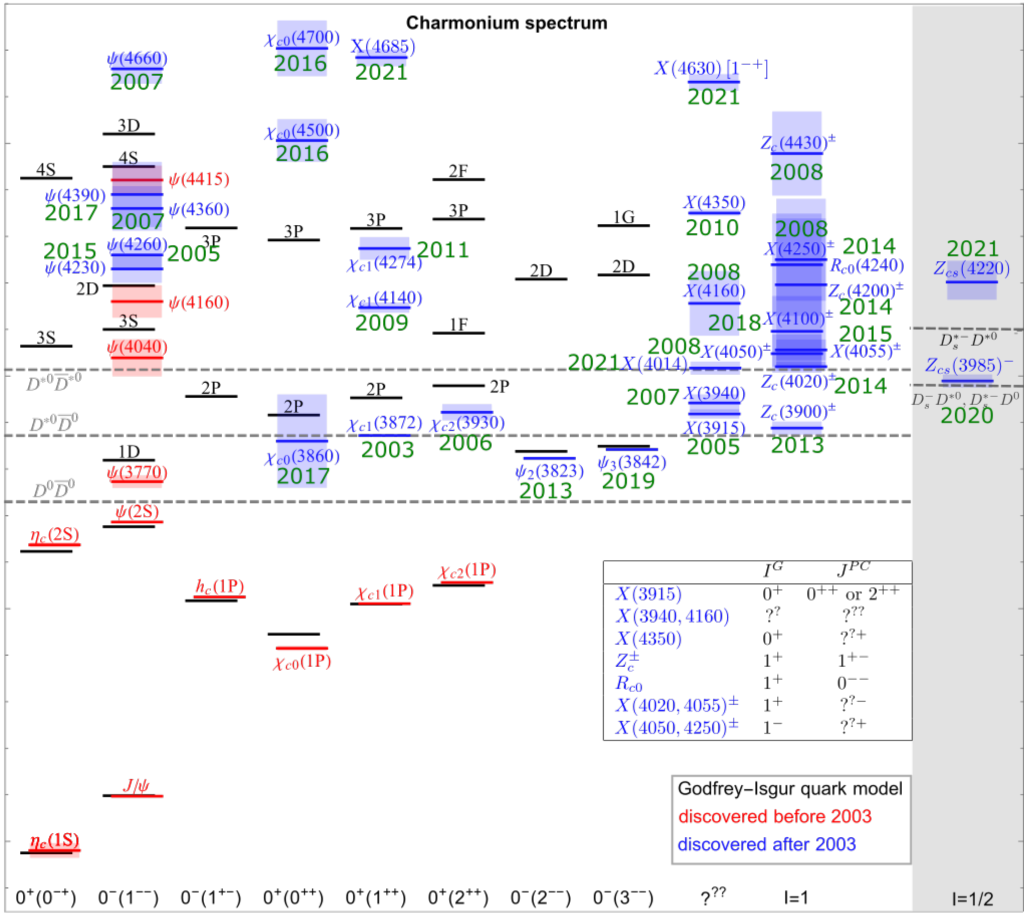
✓ spin multiplets:

for singly heavy mesons, e.g. $\{D, D^*\}$, $\{B, B^*\}$ with $s_\ell^P = \frac{1}{2}^-$;

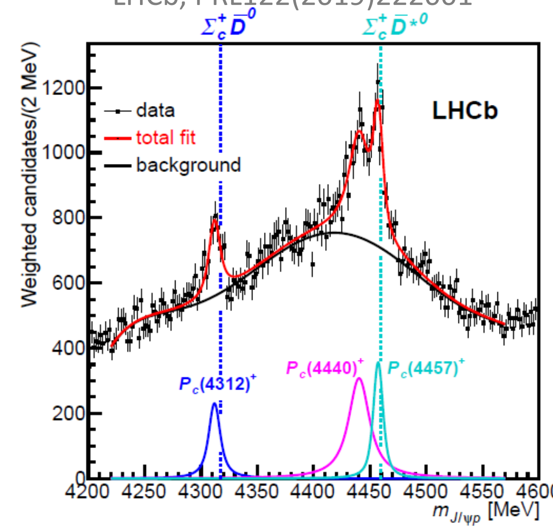
for heavy quarkonia, e.g. S-wave: $\{\eta_c, J/\psi\}$, $\{\eta_b, \Upsilon\}$;

P-wave: $\{h_c, \chi_{c0,c1,c2}\}$, $\{h_b, \chi_{b0,b1,b2}\}$

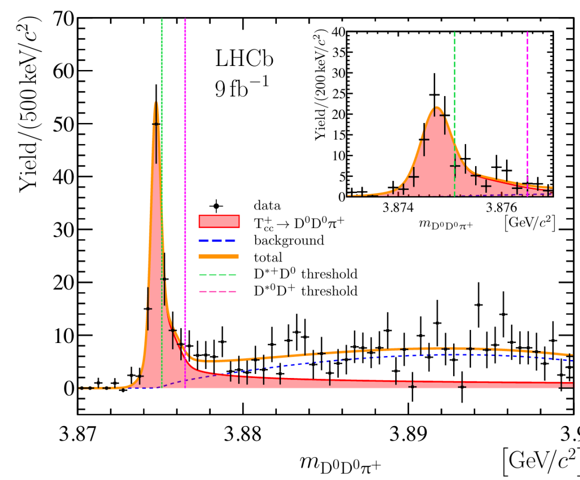
XYZ states and more



LHCb, PRL122(2019)222001



LHCb, Nature Phys. 18 (2021) 751



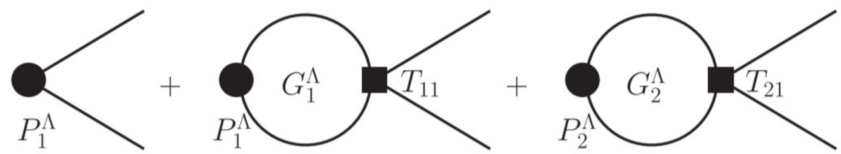
- Milestones:
  - ✓ $X(3872)$ Belle (2003)
  - ✓ $Z_b(10610,10650)^\pm$ Belle (2011)
  - ✓ $Z_c(3900)^\pm$ BESIII, Belle (2013)
  - ✓ P_c LHCb (2015, 2019)
  - ✓ T_{cc}^+ LHCb (2021)

- Prominence of near-threshold structures

Near-threshold structures

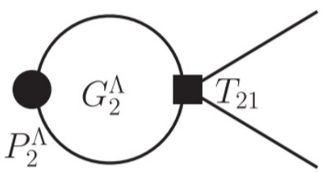
X.-K. Dong, FKG, B.-S. Zou, PRL126,152001(2021)

- Consider a production process, must go through final-state interaction (unitarity)



$$
\begin{aligned}
&P_1^\Lambda[1+G_1^\Lambda T_{11}(E)] + P_2^\Lambda G_2^\Lambda(E) T_{21}(E) \\
&= P_1^\Lambda (V_{11}^\Lambda)^{-1} T_{11}(E) + \left[P_1^\Lambda (V_{11}^\Lambda)^{-1} V_{12}^\Lambda + P_2^\Lambda\right] G_2^\Lambda T_{21}(E) \\
&\equiv P_1 T_{11}(E) + P_2 T_{21}(E)
\end{aligned}
$$

- All nontrivial energy dependence are contained in $T_{11}(E)$ and $T_{21}(E)$
- Case-1: dominated by $T_{21}(E)$,

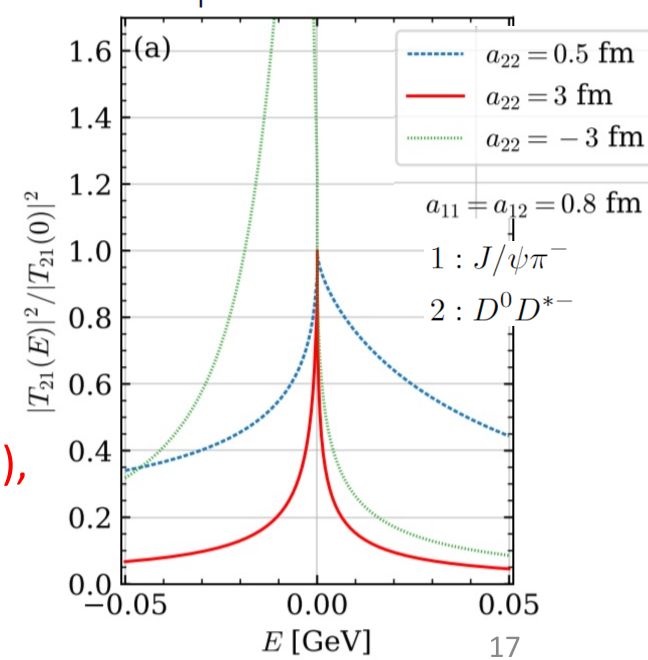


$$
T_{21}(E) = \frac{-8\pi\Sigma_2}{a_{12}(1/a_{11} - i k_1)} \left[\frac{1}{a_{22,\text{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]^{-1}.
$$

$$
|T_{21}(E)|^2 \propto |T_{22}(E)|^2 \propto
$$

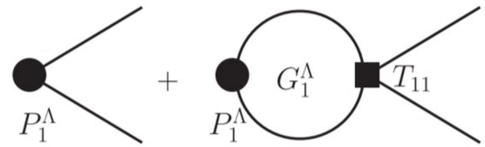
$$
\begin{cases}
\left[\left(\text{Re}\,\frac{1}{a_{22,\text{eff}}}\right)^2 + \left(\text{Im}\,\frac{1}{a_{22,\text{eff}}} - \sqrt{2\mu E}\right)^2\right]^{-1} & \text{for } E \geq 0 \\
\left[\left(\text{Im}\,\frac{1}{a_{22,\text{eff}}}\right)^2 + \left(\text{Re}\,\frac{1}{a_{22,\text{eff}}} + \sqrt{-2\mu E}\right)^2\right]^{-1} & \text{for } E < 0
\end{cases}
$$

- Maximal at threshold for positive $\text{Re}(a_{22,\text{eff}})$ (attraction), FWHM $\propto 1/\mu$
- Peaking at pole for negative $\text{Re}(a_{22,\text{eff}})$



Near-threshold structures

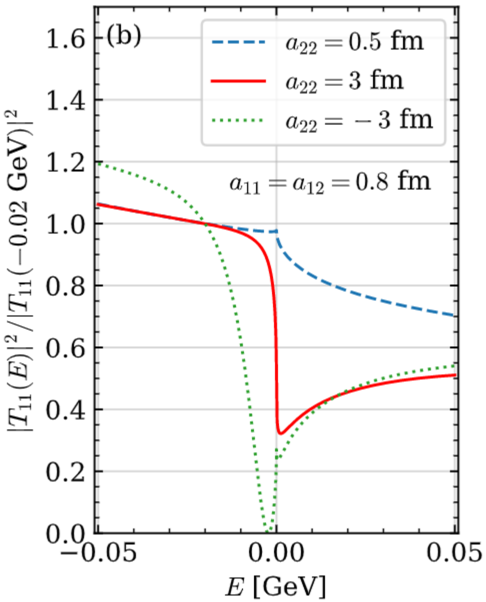
- Case-2: dominated by $T_{11}(E)$



$$T_{11}(E) = \frac{-8\pi\Sigma_2\left(\frac{1}{a_{22}} - i\sqrt{2\mu_2 E}\right)}{\left(\frac{1}{a_{11}} - i\,k_1\right)\left[\frac{1}{a_{22,\mathrm{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]}$$

- One pole and one zero
- For strongly interacting channel-2 (large a_{22}), there must be a dip around threshold (zero close to threshold)

$$\frac{1}{a_{22,\mathrm{eff}}} = \frac{1}{a_{22}} - \frac{a_{11}}{a_{12}^2(1+a_{11}^2k_1^2)} - i\frac{a_{11}^2k_1}{a_{12}^2(1+a_{11}^2k_1^2)}.$$



Poles in complex momentum plane:

$(-0.08 - i0.37)$GeV

$(-0.08 - i0.04)$GeV

$(-0.08 - i0.09)$GeV

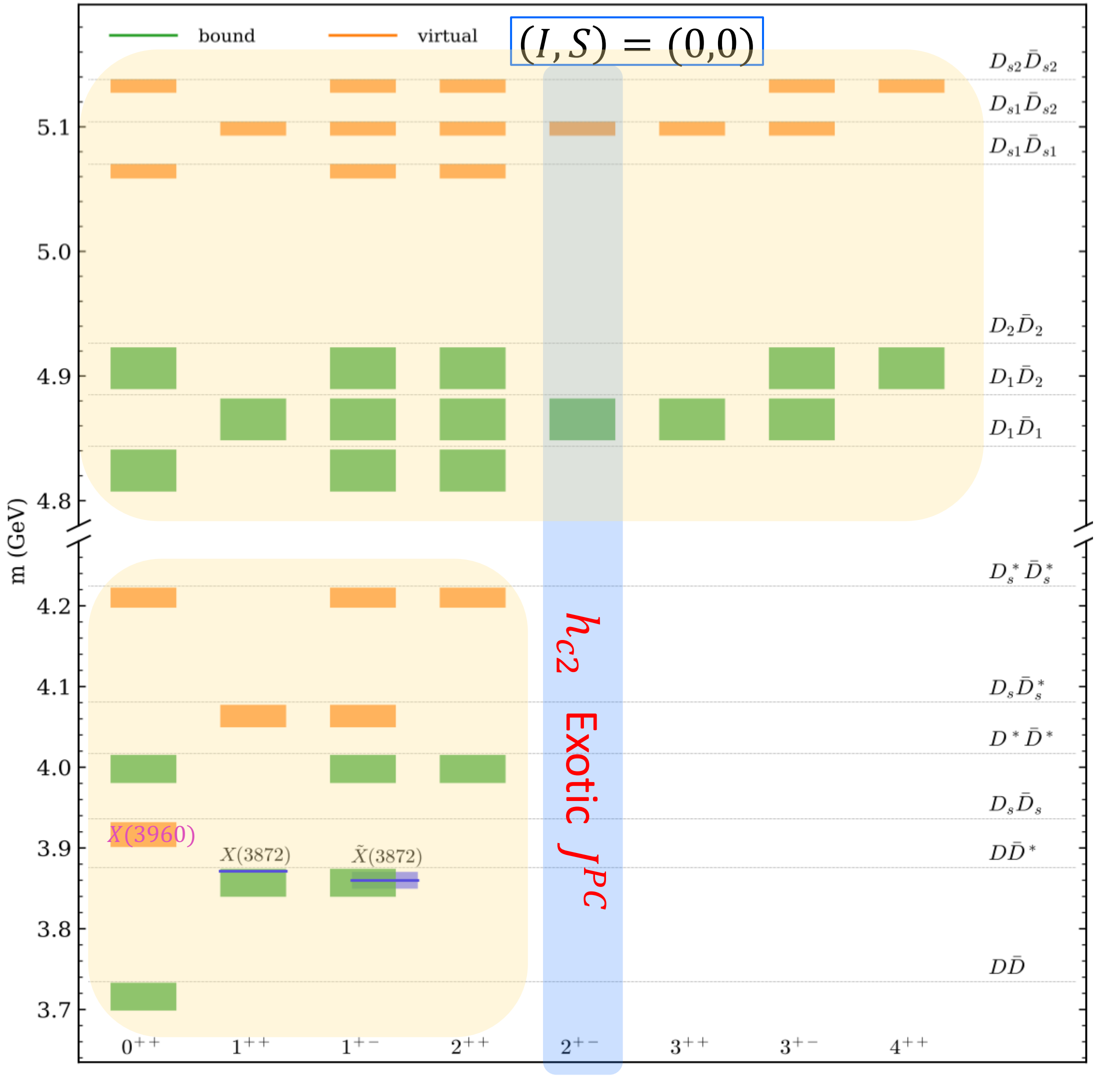
- S-wave attraction => Nontrivial structure
- Pole produced by LO interaction: a hadronic molecular state
- More complicated line shape if both channels are important for the production

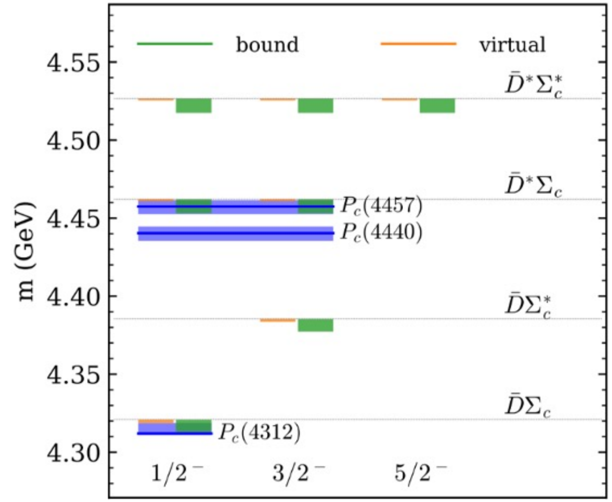
Spectrum of hadronic molecules

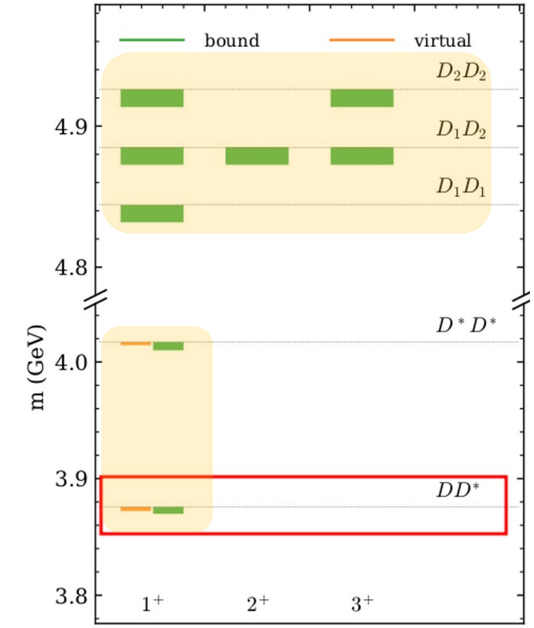
X.-K. Dong, FKG, B.-S. Zou, 物理学进展 41 (2021) 65; CTP73(2021)125201

- Predictions based on a vector-meson exchange model (HQSS respected):

  >200 hidden-charm + >100 double-charm states

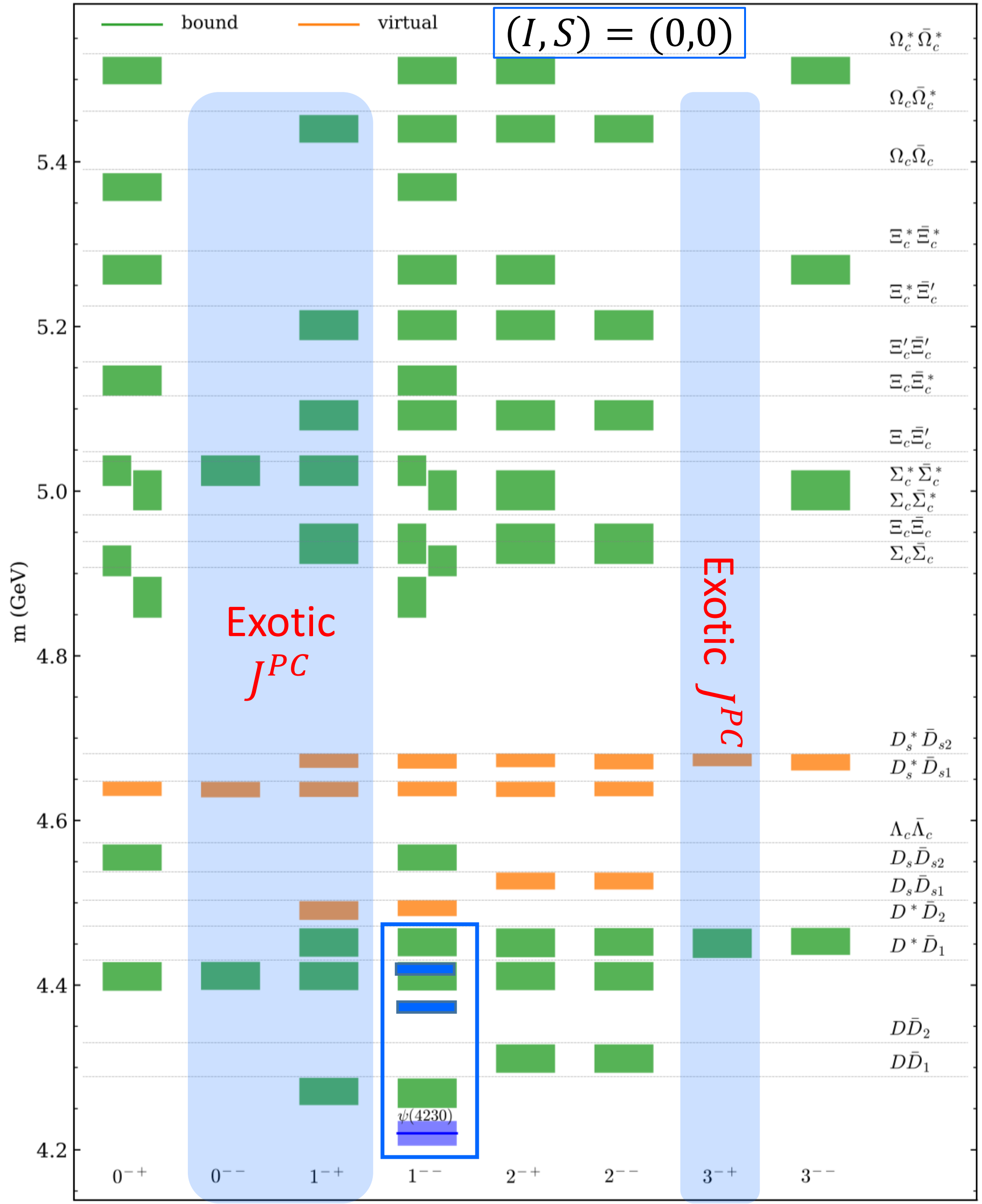






Spectrum of hadronic molecules

X.-K. Dong, FKG, B.-S. Zou, 物理学进展 41 (2021) 65; CTP73(2021)125201



Suggestions for lattice:

- ➢ operators with exotic quantum numbers
  - $D\bar{D}_1$ $[1^{-+}]$
  - $D^*\bar{D}_1$ $[0^{--}]$
  - $D^*\bar{D}_2^*$ $[3^{-+}]$
  - $D_1\bar{D}_1$ $[2^{+-}]$
- with a pion mass larger than the physical value ($\sim$ 400 MeV), so that D^*, D_1, D_2^* are all stable

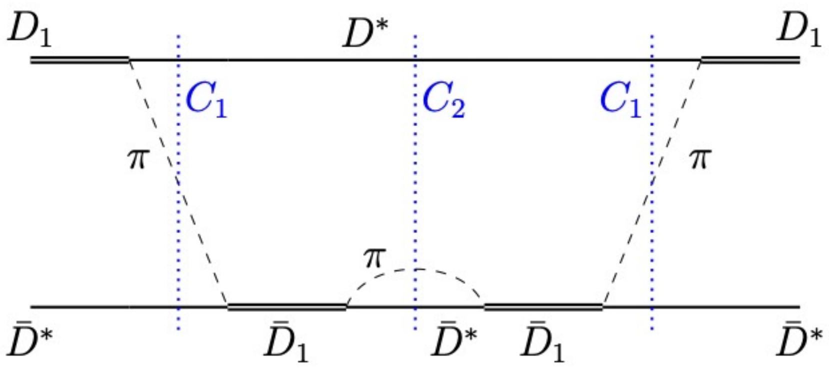
0^{--}

T. Ji, X.-K. Dong, FKG, B.-S. Zou, PRL 129 (2022) 102002

$$M_{\psi(4360)} - M_{\psi(4230)} \approx M_{D^*} - M_D, M_{\psi(4415)} - M_{\psi(4360)} \approx M_{D_2^*} - M_{D_1}$$

- Exotic 0^{--} spin partner $\psi_0(4360)$ $[D^*\bar{D}_1]$ of $\psi(4230), \psi(4360), \psi(4415)$ as $D\bar{D}_1, D^*\bar{D}_1, D^*\bar{D}_2$ hadronic molecules
- Robust against the inclusion of coupled channels and three-body effects

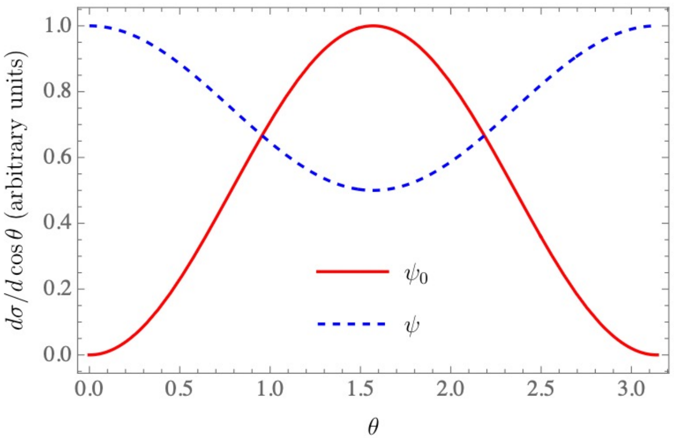
Molecule	Components	J^{PC}	Threshold	E_B
$\psi(4230)$	$\frac{1}{\sqrt{2}}(D\bar{D}_1 - \bar{D}D_1)$	1^{--}	4287	67 ± 15
$\psi(4360)$	$\frac{1}{\sqrt{2}}(D^*\bar{D}_1 - \bar{D}^*D_1)$	1^{--}	4429	62 ± 14
$\psi(4415)$	$\frac{1}{\sqrt{2}}(D^*\bar{D}_2^* - \bar{D}^*D_2^*)$	1^{--}	4472	49 ± 4
ψ_0	$\frac{1}{\sqrt{2}}(D^*\bar{D}_1 + \bar{D}^*D_1)$	0^{--}	4429	**63 ± 18**



- May be searched for using $e^+e^- \to \psi_0\eta$, $\psi_0 \to J/\psi\eta, D\bar{D}^*, D^*\bar{D}^*\pi, \ldots$

  $M = (4366 \pm 18)$ MeV,

  $\Gamma < 10$ MeV



HQSS: P_c pentaquarks

The LHCb P_c states might be $\Sigma_c^{(*)}\bar{D}^{(*)}$ molecules predicted in Wu, Molina, Oset, Zou (2010)

$P_c(4312) \sim \Sigma_c\bar{D}$, $P_c(4440, 4457) \sim \Sigma_c\bar{D}^*$

Consider S-wave pairs of $\Sigma_c^{(*)}\bar{D}^{(*)}$ [$J_{\Sigma_c} = \frac{1}{2}, J_{\Sigma_c^*} = \frac{3}{2}$]:

$$J^P = \frac{1}{2}^- : \ \Sigma_c\bar{D},\ \Sigma_c\bar{D}^*,\ \Sigma_c^*\bar{D}^*$$

$$J^P = \frac{3}{2}^- : \ \Sigma_c^*\bar{D},\ \Sigma_c\bar{D}^*,\ \Sigma_c^*\bar{D}^*$$

$$J^P = \frac{5}{2}^- : \ \Sigma_c^*\bar{D}^*$$

Spin of the light degrees of freedom s_ℓ: $s_\ell(D^{(*)}) = \frac{1}{2}$, $s_\ell(\Sigma_c^{(*)}) = 1$. Thus, $s_L = \frac{1}{2}, \frac{3}{2}$

For each isospin, 2 independent terms

$$\left\langle 1, \frac{1}{2}, \frac{1}{2} \left| \hat{\mathcal{H}} \right| 1, \frac{1}{2}, \frac{1}{2} \right\rangle, \qquad \left\langle 1, \frac{1}{2}, \frac{3}{2} \left| \hat{\mathcal{H}} \right| 1, \frac{1}{2}, \frac{3}{2} \right\rangle$$

Thus, the 7 pairs are in two spin multiplets: 3 with $s_L = \frac{1}{2}$ and 4 with $s_L = \frac{3}{2}$

HQSS: P_c pentaquarks

Seven P_c generally expected in this hadronic molecular model Xiao, Nieves, Oset (2013); Liu et al. (2018, 2019); Sakai et al. (2019); . . .

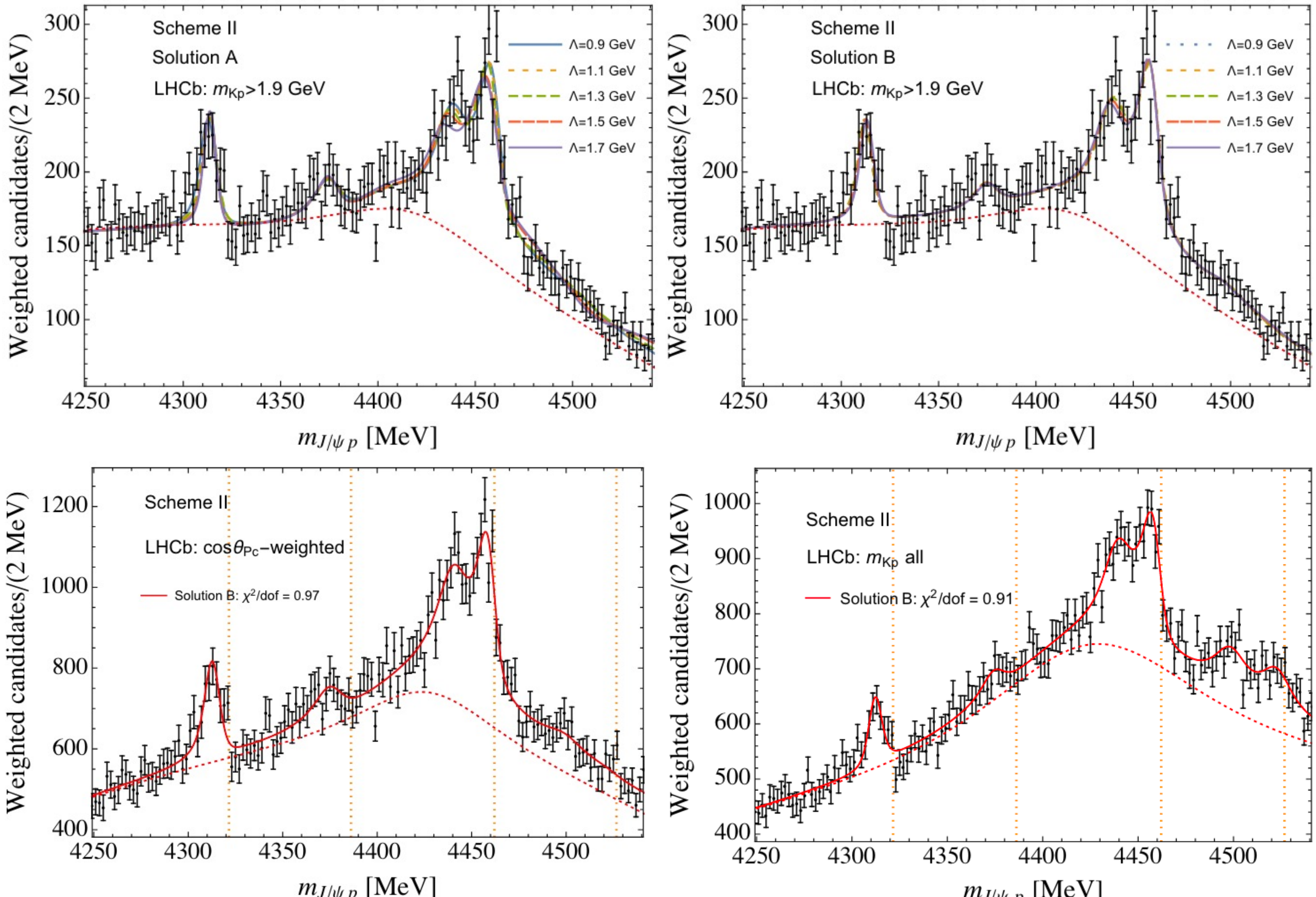
Predictions using the masses of $P_c(4440, 4457)$ as inputs Liu et al., PRL122(2019)242001

Scenario	Molecule	J^P	B (MeV)	M (MeV)
A	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	7.8 – 9.0	4311.8 – 4313.0
A	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	8.3 – 9.2	4376.1 – 4377.0
A	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	Input	4440.3
A	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	Input	4457.3
A	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	25.7 – 26.5	4500.2 – 4501.0
A	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	15.9 – 16.1	4510.6 – 4510.8
A	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	3.2 – 3.5	4523.3 – 4523.6
B	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	13.1 – 14.5	4306.3 – 4307.7
B	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	13.6 – 14.8	4370.5 – 4371.7
B	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	Input	4457.3
B	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	Input	4440.3
B	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	3.1 – 3.5	4523.2 – 4523.6
B	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	10.1 – 10.2	4516.5 – 4516.6
B	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	25.7 – 26.5	4500.2 – 4501.0

HQSS: P_c pentaquarks

Solution B is favored after fitting to the LHCb data with an EFT with coupled channels, including both contact terms + one-pion exchanges

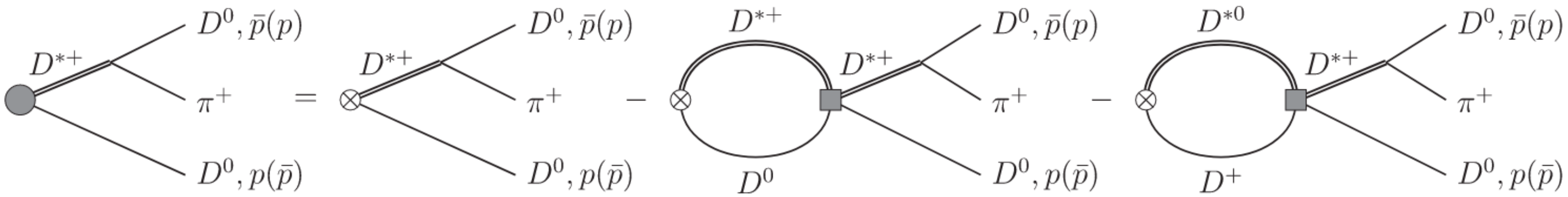
M.-L. Du, V. Baru, F.-K. Guo, C. Hanhart, U.-G. Meißner, J. A. Oller, Q. Wang, PRL124(2020)072001; JHEP08(2021)157

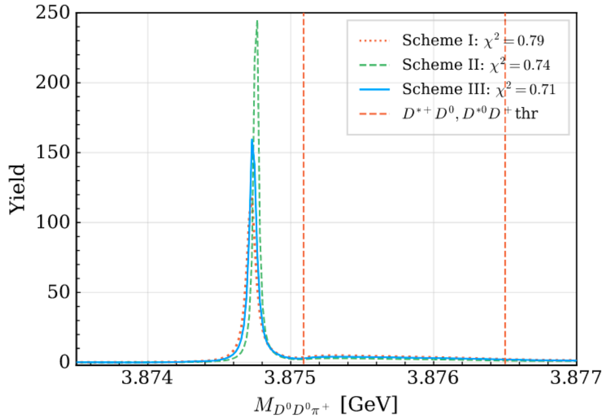


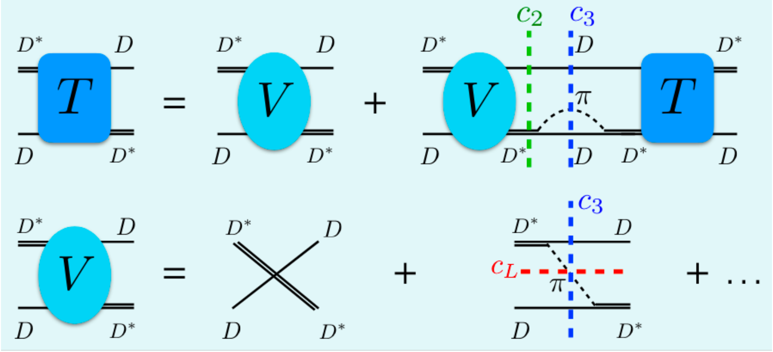
T_{cc}^+

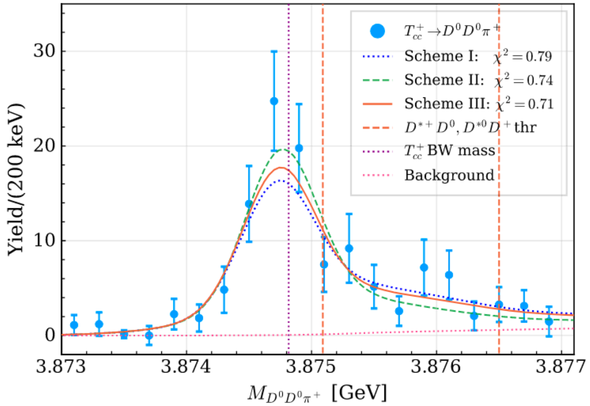
M.-L. Du et al., PRD105(2022)014024; arXiv:2303.09441

- Channels: $D^{*+}D^0, D^{*0}D^+$
- Contact term + one-pion exchange; three-body effects









Pole w.r.t. the $D^{*+}D^0$ th: $-356^{+39}_{-38} - i(28 \pm 1)$ keV

Compositeness

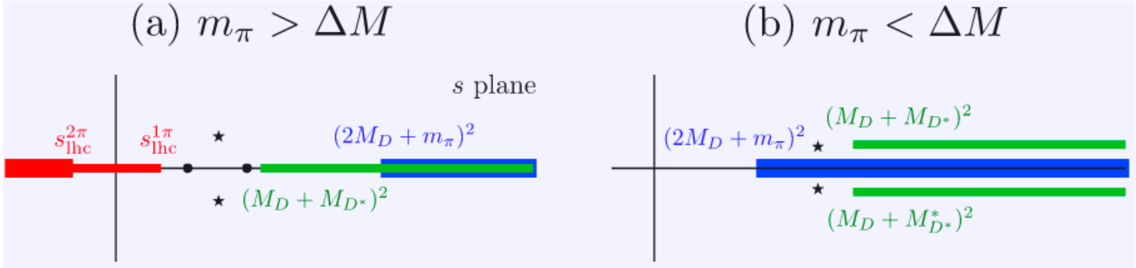
$X\,[D^{*+}D^0]$	$X\,[D^{*0}D^+]$
0.73 ± 0.11	0.29 ± 0.02

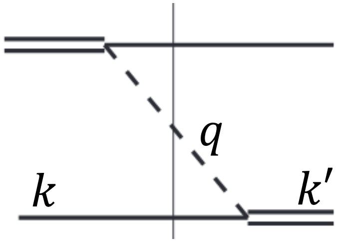
M.-L. Du et al., PRD105(2022)014024; arXiv:2303.09441

- Left-hand cut (lhc) due to the OPE could affect the precise extraction of the T_{cc} pole from lattice calculations
  - Virtual state pole from lattice results with unphysical pion masses:

    $M_\pi = 280$ MeV, M. Padmanath and S. Prelovsek, PRL 129 (2022) 032002;
    $M_\pi = 350$ MeV, S. Chen, C. Shi, Y. Chen, M. Gong, Z. Liu, W. Sun, and R. Zhang, PLB 833 (2022) 137391;
    $M_\pi = 146$ MeV, Y. Lyu, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, and J. Meng, arXiv:2302.04505 [hep-lat]

  - Left-hand cut due to the u-channel pion being on-shell





$$G_\pi^{-1}(E,\boldsymbol{k}',\boldsymbol{k}) = \Delta M + \frac{p^2}{2\mu} - \frac{k^2+k'^2}{2M_D} - \omega_\pi(q^2)$$

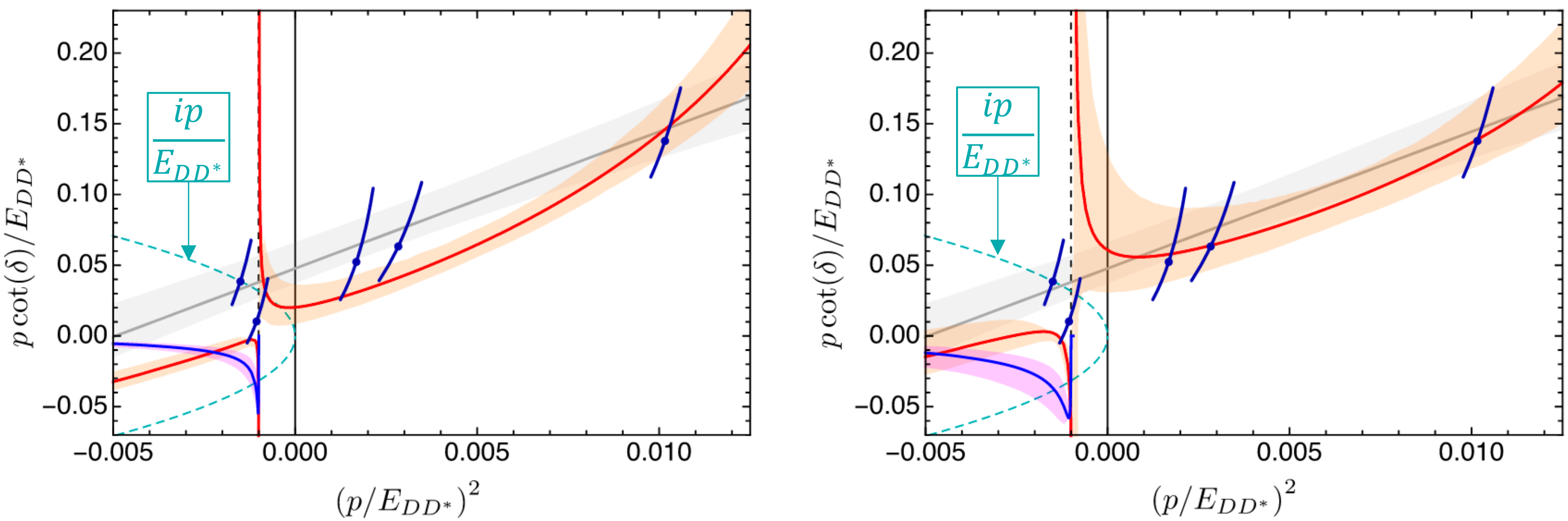
For on-shell momenta $k=k'=p$, $q^2=2p^2(1-\cos\theta)$, we have

$$G_\pi^{-1}(E,\boldsymbol{k}',\boldsymbol{k}) \approx \Delta M - \omega_\pi(q^2)$$

Left-hand cut: $\Delta M - \omega_\pi(q^2) = 0 \Rightarrow$ branch point at (w.r.t. threshold.=): $\approx \frac{(\Delta M)^2 - M_\pi^2}{8\mu}$

M.-L. Du et al., arXiv:2303.09441

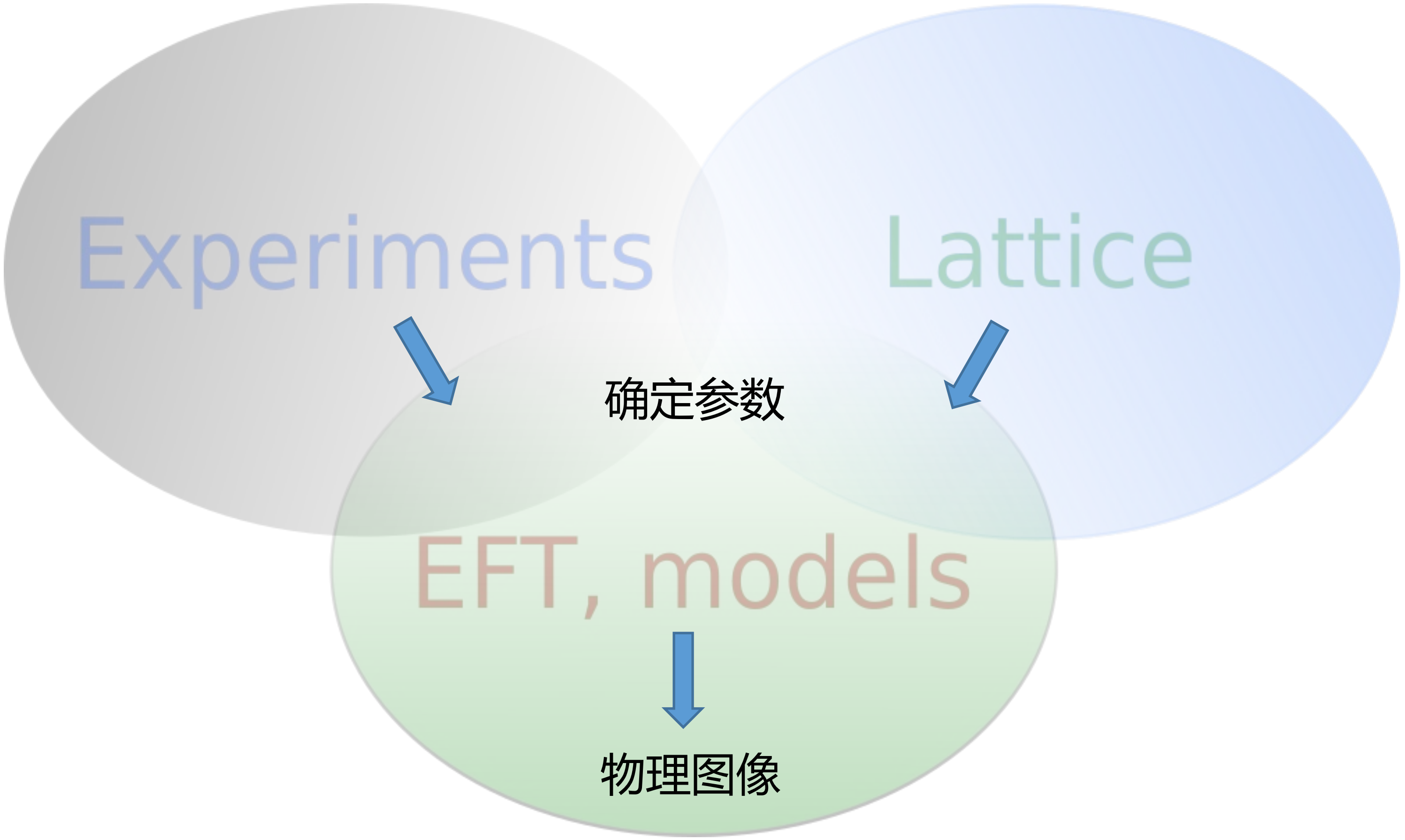
- Fits to the lattice results by Padmanath & Prelovsek right to the lhc



- Could also influence other calculations with nearby lhc
- Luescher formalism with lhc [EFT formalism with 3-body cut put into a finite box] needs to be employed to extract scattering observables

近几年的综述不完全列表

- H.-X. Chen, W. Chen, X. Liu, S.-L. Zhu, *The hidden-charm pentaquark and tetraquark states*, Phys. Rept. 639 (2016) 1 [arXiv:1601.02092]
- A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, S. Yasui, *Exotic hadrons with heavy flavors* $-X, Y, Z$ *and related states*, Prog. Theor. Exp. Phys. 2016, 062C01 [arXiv:1603.09229]
- R. F. Lebed, R. E. Mitchell, E. Swanson, *Heary-quark QCD exotica*, Prog. Part. Nucl. Phys. 93 (2017) 143 [arXiv:1610.04528]
- A. Esposito, A. Pilloni, A. D. Polosa, *Multiquark resonances*, Phys. Rept. 668 (2017) 1 [arXiv:1611.07920]
- F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, B.-S. Zou, *Hadronic molecules*, Rev. Mod. Phys. 90 (2018) 015004 [arXiv:1705.00141]
- S. L. Olsen, T. Skwarnicki, *Nonstandard heavy mesons and baryons: Experimental evidence*, Rev. Mod. Phys. 90 (2018) 015003 [arXiv:1708.04012]
- M. Karliner, J. L. Rosner, T. Skwarnicki, *Multiquark states*, Ann. Rev. Nucl. Part. Sci. 68 (2018) 17 [arXiv:1711.10626]
- C.-Z. Yuan, *The XYZ states revisited*, Int. J. Mod. Phys. A 33 (2018) 1830018 [arXiv:1808.01570]
- Y.-R. Liu, H.-X. Chen, W. Chen, X. Liu, S.-L. Zhu, *Pentaquark and tetraquark states*, Prog.Part.Nucl.Phys. 107 (2019) 237 [arXiv:1903.11976]
- N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C.-P. Shen, C. E. Thomas, A. Vairo, C.-Z. Yuan, *The* XYZ *states: experimental and theoretical status and perspectives*, Phys. Rept. 873 (2020) 154 [arXiv:1907.07583]
- F.-K. Guo, X.-H. Liu, S. Sakai, *Threshold cusps and triangle singularities in hadronic reactions*, Prog. Part. Nucl. Phys. 112 (2020) 103757 [arXiv:1912.07030]
- G. Yang, J. Ping, J. Segovia, *Tetra- and penta-quark structures in the constituent quark model*, Symmetry 12 (2020) 1869 [arXiv:2009.00238]
- H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu, S.-L. Zhu, *An updated review of the new hadron states*, arXiv:2204.02649
- L. Meng, B. Wang, G.-J. Wang, S.-L. Zhu, *Chiral perturbation theory for heavy hadrons and chiral effective field theory for heavy hadronic molecules*, arXiv:2204.08716
- …



谢 谢！