’t Hooft 模型中的隐粲夸克部分子分布函数

’t Hooft 模型算符方法

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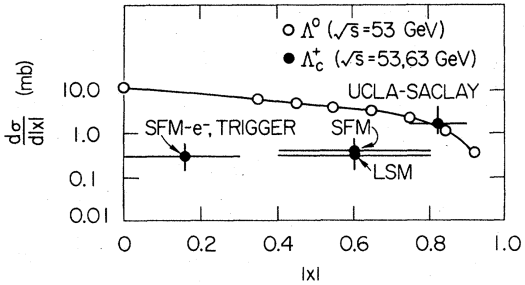
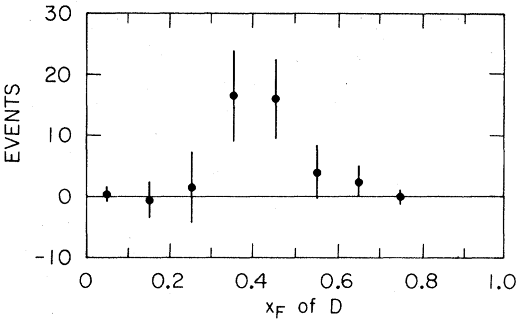
2023 年 4 月 20 日

1.pp 对撞中，Λ_c^+ 的产生并没有被碎裂函数显著压低



2.$\pi^- p \to D\overline{D}pX$ 实验中，观察到 D 的产生集中在大 x 区域



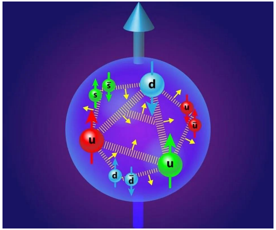
3. 在 pp 对撞中，看见了超过预期的 $D^+(c\bar{d})$ 产生

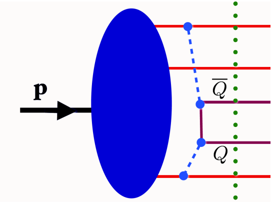
The Intrinsic Charm of the Proton

S.J. Brodsky (SLAC), P. Hoyer (Nordita), C. Peterson (Nordita), N. Sakai (Nordita)
Apr, 1980

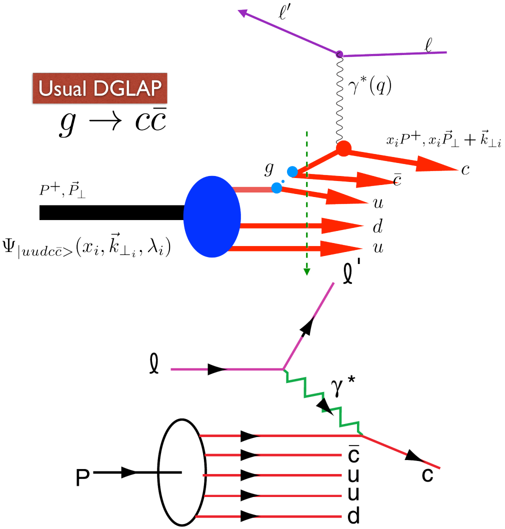
Intrinsic Heavy Quark States

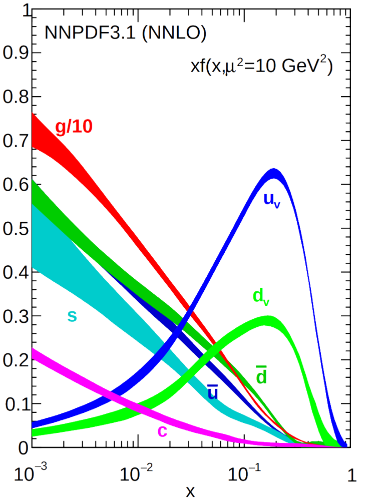
Stanley J. Brodsky (SLAC), C. Peterson (SLAC), N. Sakai (Fermilab)
Jan, 1981





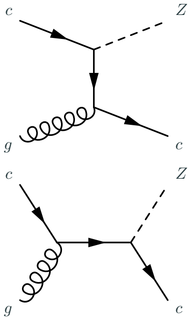
- 夸克模型 $|p\rangle = |uud\rangle$
- 次领头阶 $|uudg\rangle$、$|uudQ\bar{Q}\rangle$ …
  $|uud\rangle \overset{boost}{\longrightarrow} |uudg\rangle$、$|uudq\bar{q}\rangle$ …



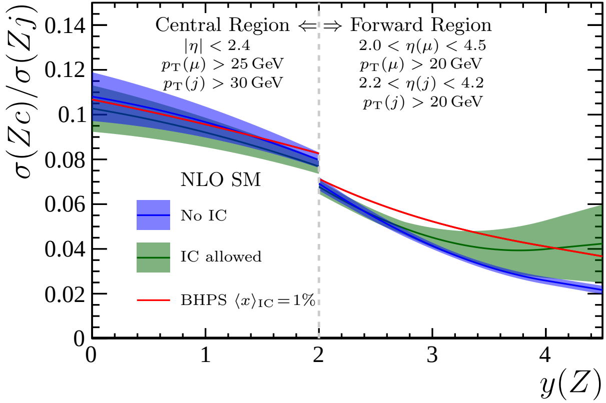


NNLO PDFs at scale $\mu^2 = 10\,\text{GeV}^2$

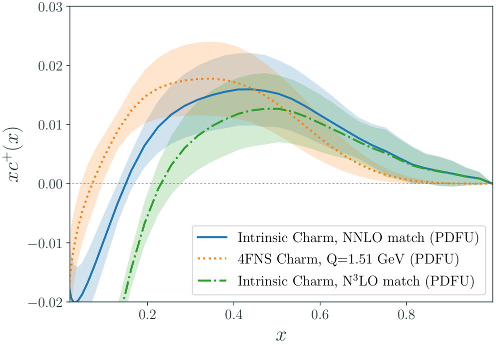
LHCb 上的质子质子对撞，测量 $\mathcal{R} = \frac{\sigma(Zc)}{\sigma(Zj)}$

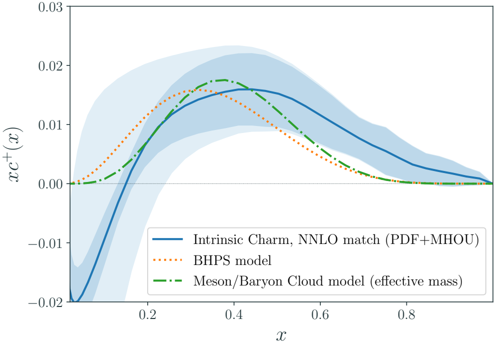


Zc 产生的领头阶贡献



加入 IC 对 SM 预言的修改





NNPDF: 3σ 的意义下确定存在 IC，携带动量约 0.62%

质子中发现 $|uudc\bar{c}\rangle$ 的概率

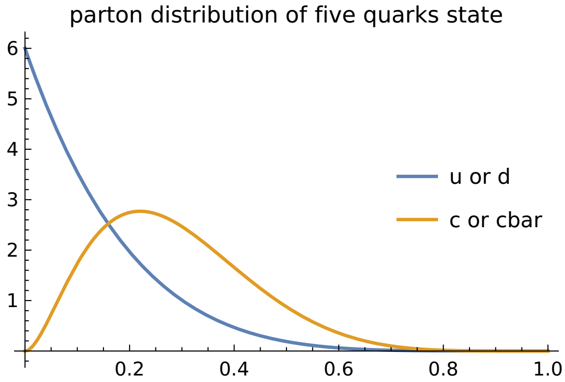
$$P(p \to uudc\bar{c}) \sim \frac{\langle ...|V|...\rangle^2}{\left(m_p^2 - \sum_{i=1}^{5} \frac{m_{\perp,i}^2}{x_i}\right)^2} \delta\left(1 - \sum_{i=1}^{5} x_i\right)$$

$$\propto \frac{x_c^2 x_{\bar{c}}^2}{(x_c + x_{\bar{c}})^2} \delta\left(1 - \sum_i x_i\right), m_{c\bar{c}} \to \infty$$

积分得到

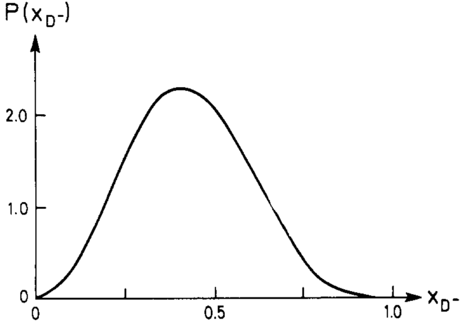
$$f_c(x) = f_{\bar{c}}(x) = Nx^2\left[\frac{1}{3}(1-x)\left(1+10x+x^2\right) + 2x(1+x)\ln x\right]$$

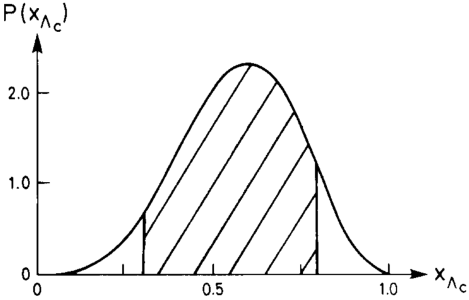
$$f_u = f_d = 6(1-x)^5$$



$$< x_c >=< x_{\bar{c}} >= 2/7 \quad < x_u >=< x_d >= 1/7$$

'rare' but not wee



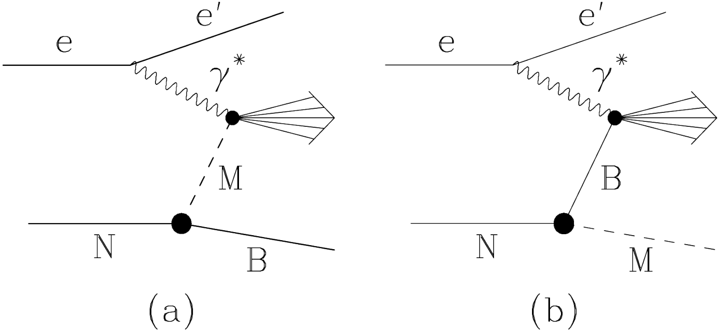


$< x_{D-} >= \frac{1}{7} + \frac{2}{7} = \frac{3}{7}$

$< x_{\Lambda_c} >= \frac{1}{7} + \frac{1}{7} + \frac{2}{7} = \frac{4}{7}$

能够描述在大 x 处的行为，是实验拟合的重要模型（BHPS 1/2/3）

介子云模型 Meson Cloud Model (MCM)



$$f_{c\,\text{in}\,p}(x) = \sum_{B,M} \int_x^1 \frac{dy}{y} \mathcal{F}_{p\to B+M}(y) f_{c\,\text{in}\,M}\left(\frac{x}{y}\right) + \int \ldots f_{c\,\text{in}\,B}$$

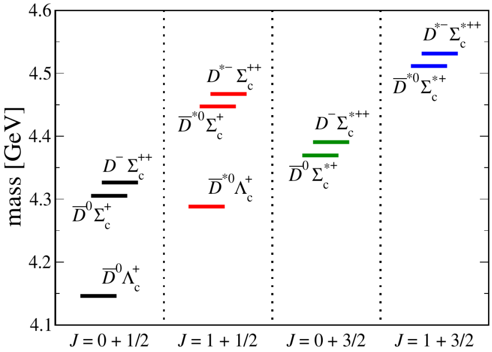
介子云模型 hadronic effective lagrangian

spliting $\mathcal{F}(y)$

$$\mathcal{L}_{pD\Lambda_c} = ig_{pD\Lambda_c}\bar{\psi}_p\gamma_5\psi_B\phi_D + \text{h.c.}$$

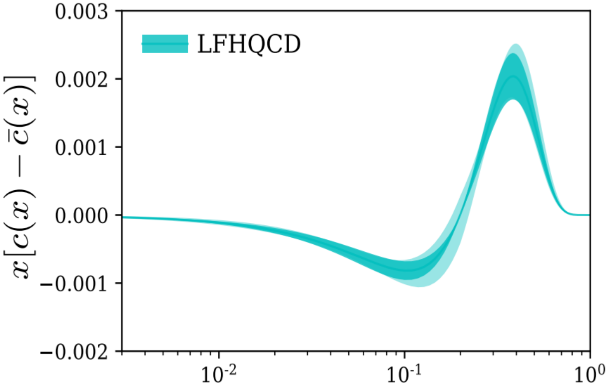
PDF $f_{c\,\text{in}\,\Lambda_c}$

$$\mathcal{L}_{\Lambda_c c[qq]} = g_{\lambda_c c[qq]}\bar{\psi}_{\lambda_c}\psi_c\phi_{[qq]} + \text{h.c.}$$



Lattice

粲夸克电磁形状因子



提供了 IC 存在的格点证据

QCD 拉氏量为（光锥规范 $A^+ = 0$，手征 Wely 表象 $\psi^T = 2^{-1/4}(\psi_R, \psi_L)$）

$$
\begin{aligned}
\mathcal{L} =& -\frac{1}{4} F^{\mu\nu,a} F^a_{\mu\nu} + \overline{\psi}\left(i\not{D} - m\right)\psi \\
=& \frac{1}{2}\left(\partial_- A^{-,a}\right)^2 + g_s \psi_R^\dagger A^{-,a} T^a \psi_R + \psi_R^\dagger i\partial_+ \psi_R + \psi_L^\dagger i\partial_- \psi_L - \frac{m_f}{\sqrt{2}}\left(\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L\right)
\end{aligned}
$$

运动方程

$$
\begin{aligned}
&\partial_-^2 A^{-,a} - g_s \psi_R^\dagger T^a \psi_R = 0, \\
&i\partial_- \psi_L - \frac{m}{\sqrt{2}} \psi_R = 0.
\end{aligned}
$$

- **运动学自由度只有** ψ_R，**胶子只提供库伦势**
- **没有三胶子四胶子顶角**，$F_{--}F^{--} = 0$

$V \sim |x-y|$ 规范场自然地出现色禁闭。引入玻色化

$$M\left(k^{+}, p^{+}\right) \equiv \frac{1}{\sqrt{N_c}} \sum_i d^i(k^{+}) b^i(p^{+})$$

$$M^{\dagger}\left(k^{+}, p^{+}\right) \equiv \frac{1}{\sqrt{N_c}} \sum_i b^{i\dagger}(p^{+}) d^{i\dagger}(k^{+})$$

$$\left[M\left(k_1^{+}, p_1^{+}\right), M^{\dagger}\left(k_2^{+}, p_2^{+}\right)\right] = (2\pi)^2\, \delta(k_1^{+} - k_2^{+}) \delta(p_1^{+} - p_2^{+}) + \mathcal{O}\left(\frac{1}{\sqrt{N_c}}\right)$$

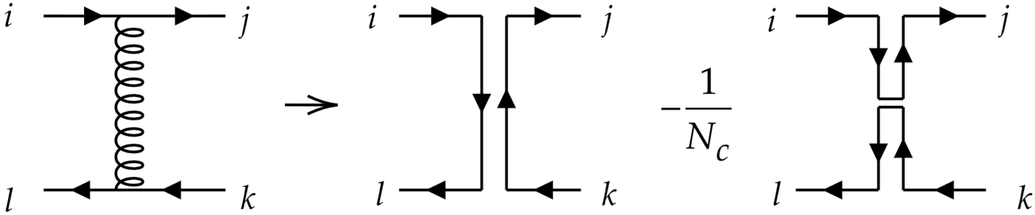
自然地引入大 N_c 极限

$$N_c \to \infty, \qquad \lambda \equiv \frac{g_s^2 N_c}{4\pi} \text{ fixed}, \qquad g_s \sim \frac{1}{\sqrt{N_c}}$$

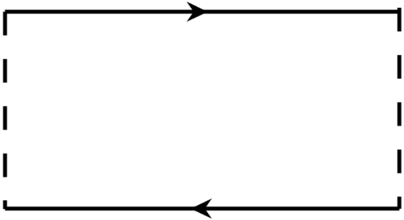
1+1QCD 的相互作用

$$g_s^2 A^{-i,a}\psi_R^\dagger T^a \psi_R \sim g_s^2 \left(\psi_R^{i,\dagger}\psi^j\right)_x \left(\psi_R^{k,\dagger}\psi^l\right)_y \sum_a T^a_{i,j}T^a_{k,l}$$

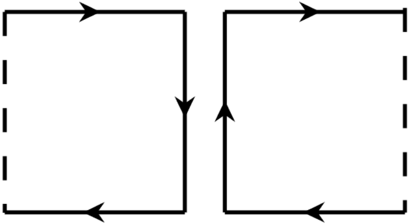
$$\sum_a T^a_{i,j}T^a_{k,l} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}\right)$$



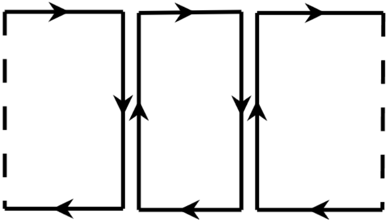
夸克对的相互作用



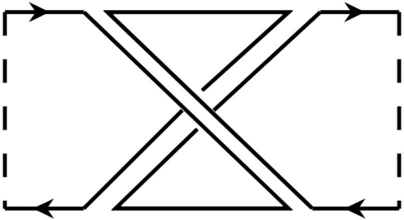
$N_c^{-1} Nc$



$N_c^{-1} g_s^2 Nc^2$



$N_c^{-1} g_s^4 Nc^3$



$N_c^{-1} g_s^4 Nc$

构造介子态算符

$$M^\dagger((1-x)P^+, xP^+) = \sqrt{\frac{2\pi}{P^+}} \sum_{n=0}^{\infty} \varphi_n(x) m_n^\dagger(P^+),$$

$$m_n^\dagger(P^+) = \sqrt{\frac{P^+}{2\pi}} \int_0^1 dx \varphi_n(x) M^\dagger((1-x)P^+, xP^+),$$

我们期待对易关系

$$\left[m_n(P_1^+), m_r^\dagger(P_2^+)\right] = 2\pi\delta_{nr}\delta(P_1^+ - P_2^+)$$

要求

$$\int_0^1 dx\, \varphi_n(x)\varphi_m(x) = \delta_{nm}$$

$$\sum_n \varphi_n(x)\varphi_n(y) = \delta(x-y)$$

夸克对“表象”下的 Hamitonian

$$H_{\mathrm{LF};0} = N\int \frac{dx^-}{2\pi}\left[\frac{\lambda}{2} + \frac{\lambda - m^2}{2}\int_\rho^\infty \frac{dk^+}{k^+}\right]$$

$$:H_{\mathrm{LF};2}: = \frac{1}{(2\pi)^2}\int_\rho^\infty dP^+ \int_0^1 dx M^\dagger((1-x)P^+, xP^+)M((1-x)P^+, xP^+)$$

$$\times\left\{\left[\left(\frac{m^2}{2}-\lambda\right)\frac{1}{x}+\frac{P^+\lambda}{\rho}\right]\Theta\left(x-\frac{\rho}{P^+}\right)+\left[\left(\frac{m^2}{2}-\lambda\right)\frac{1}{1-x}+\frac{P^+\lambda}{\rho}\right]\Theta\left(1-\frac{\rho}{P^+}-x\right)\right\}$$

$$:H_{\mathrm{LF};4}: = -\frac{\lambda}{(2\pi)^2}\int_\rho^\infty dP^+\int_0^1\int_0^1 dxdy\Theta\left(|x-y|-\frac{\rho}{P^+}\right)\frac{1}{(x-y)^2}\times M^\dagger((1-x)P^+, xP^+)M((1-y)P^+, yP^+)$$

介子“表象”

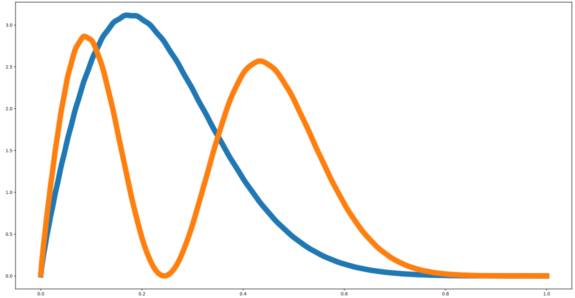
$$H_{\mathrm{LF}} = H_{\mathrm{LF};0} + \int\frac{dP^+}{2\pi}P_n^- m_n^\dagger(P^+)m_n(P^+)$$

对角化给出 ‘t Hooft 方程

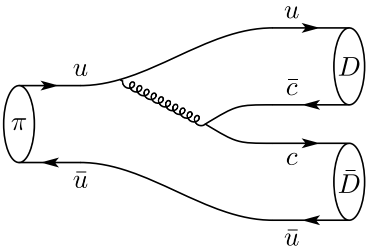
$$\left(\frac{m^2}{x}+\frac{m^2}{1-x}\right)\varphi_n(x) - 2\lambda ⨍ dy\frac{\varphi_n(y)-\varphi_n(x)}{(x-y)^2} = M_n^2\varphi_n(x)$$

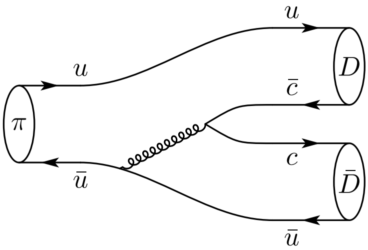
Collins-Soper definition:

$$f(x) = \int \frac{dz^-}{4\pi} e^{-ixP^+ z^-} \langle P^+ | \overline{\psi}(z^-)\gamma^+ \mathcal{P}\left[\exp\left(-ig_s \int_0^{z^-} d\eta^- A^+(\eta^-)\right)\right] \psi(0) | P^+ \rangle_{\text{connected}}$$



cs 介子中 s 夸克的 PDF

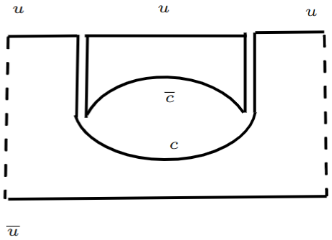




N_c 意义上的微扰

$$|\pi'\rangle \approx |\pi\rangle + \frac{1}{P^- - H_{\mathrm{LF},0} + i\epsilon} V|\pi\rangle$$

$$P^- = P_D^- + P_{\bar{D}}^-, \quad H_{\mathrm{LF},0} = \frac{M_\pi^2}{2P^+}$$



't Hooft model

$$f_{c/\pi}(x) = \sum\!\!\!\!\!\!\!\int_{D\bar{D}} \langle \pi(P^+)|D_{n_3}\overline{D}_{n_4}\rangle \, \langle D_{n_1}\overline{D}_{n_2}|\pi(P^+)\rangle \int \frac{dz^-}{4\pi}\, e^{-ixP^+ z^-} \langle D_{n_3}\overline{D}_{n_4}|\, \bar{c}(z^-)\gamma^+ c(0)\,|D_{n_1}\overline{D}_{n_2}\rangle$$

't Hooft model

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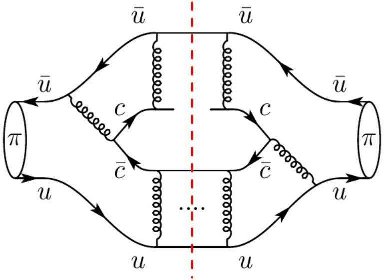
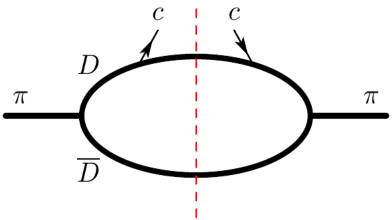
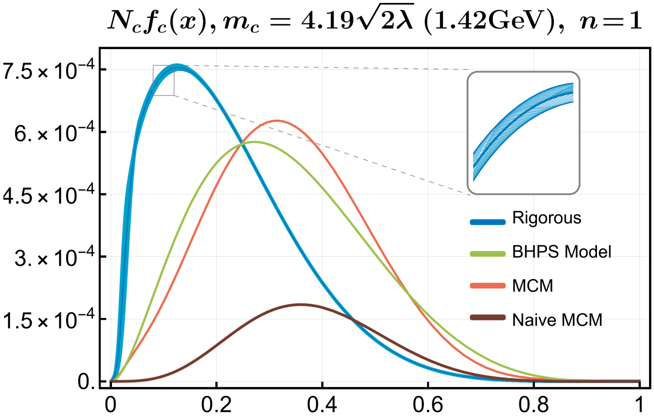
MCM

$$f_{c/\pi}(x) = \sum_{D\bar{D}} \left| \langle \pi(P^+) | D_{n_1} \overline{D}_{n_2} \rangle \right|^2 \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle D_{n_1} \overline{D}_{n_2} | \bar{c}(z^-) \gamma^+ c(0) | D_{n_1} \overline{D}_{n_2} \rangle$$







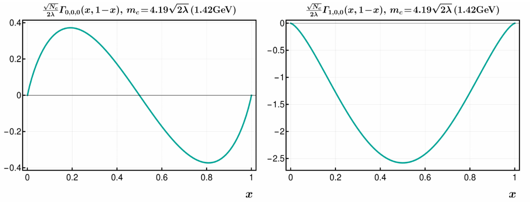
- 在二维下求和是必要的
- 干涉项没有影响 IC 的含量，但是影响了分布

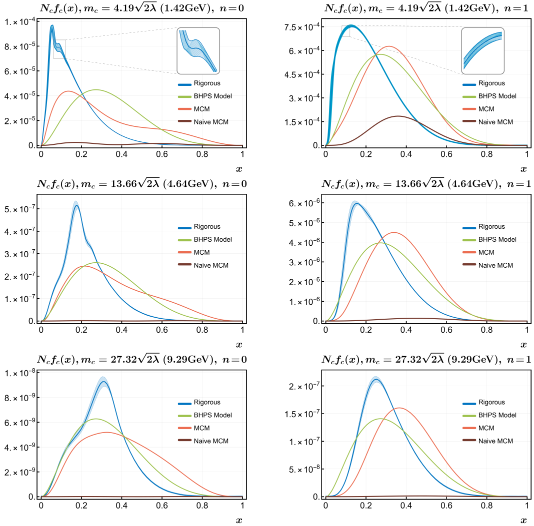
$$\varphi_n^{u\bar{u}}(x) = (-1)^n \varphi_n^{u\bar{u}}(1-x)$$

给出

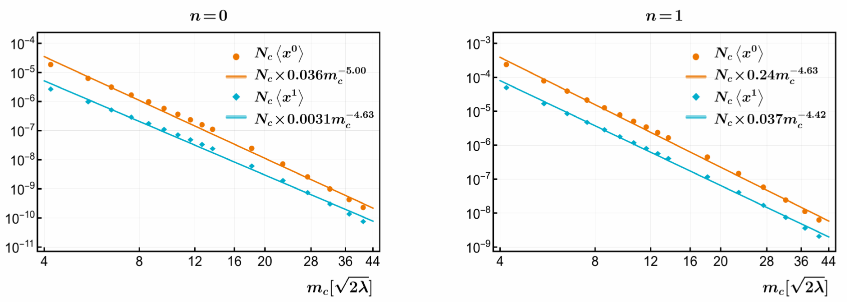
$$\mathcal{C} m_n^{u\bar{u}}(P^+)\mathcal{C}^{-1} = (-1)^{n+1} m_n^{u\bar{u}}(P^+)$$

π_n 有固定宇称，宇称守恒压低





数值结果

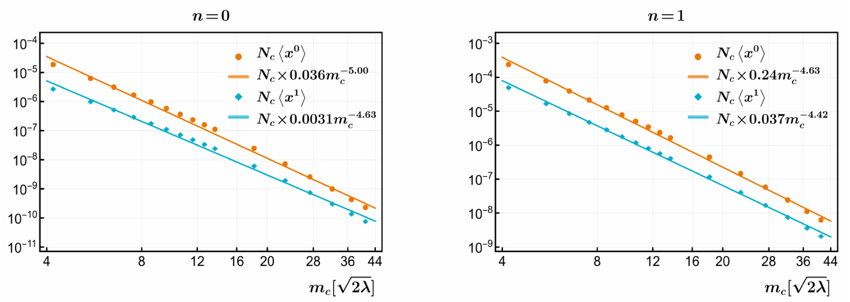


3+1 维 OPE:

QED$\langle p|\frac{F^4_{\mu\nu}}{m_l^4}|p\rangle \sim \frac{\alpha^4}{M_l^4}$

QCD$\langle p|\frac{G^3_{\mu\nu}}{m_Q^2}|p\rangle \sim \frac{\Lambda^2_{\text{QCD}}}{M_Q^2}$

$$\mathcal{L}^{eff}_{QCD} = -\frac{1}{4}G_{\mu\nu a}G^{\mu\nu a} - \frac{g^2 N_C}{120\pi^2 M_Q^2}D_a G_{\mu\nu a}D^a G^{\mu\nu a}$$
$$+C\frac{g^3}{\pi^2 M_Q^2}G^{\nu a}_{\mu}G^{\tau b}_{\nu}G^{\mu c}_{\tau}f_{abc} + \mathcal{O}\left(\frac{1}{M_Q^4}\right)$$



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$$+C\frac{g^3}{\pi^2 M_Q^2} G^{\nu a}_{\mu} G^{\tau b}_{\nu} G^{\mu c}_{\tau} f_{abc} + \mathcal{O}\left(\frac{1}{M_Q^4}\right)$$

THANK YOU