Continuum Schwinger function methods for the nucleon's low-lying excitations

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Interdisciplinary Center for Theoretical Study
&
Peng Huanwu Center for Fundamental Theory

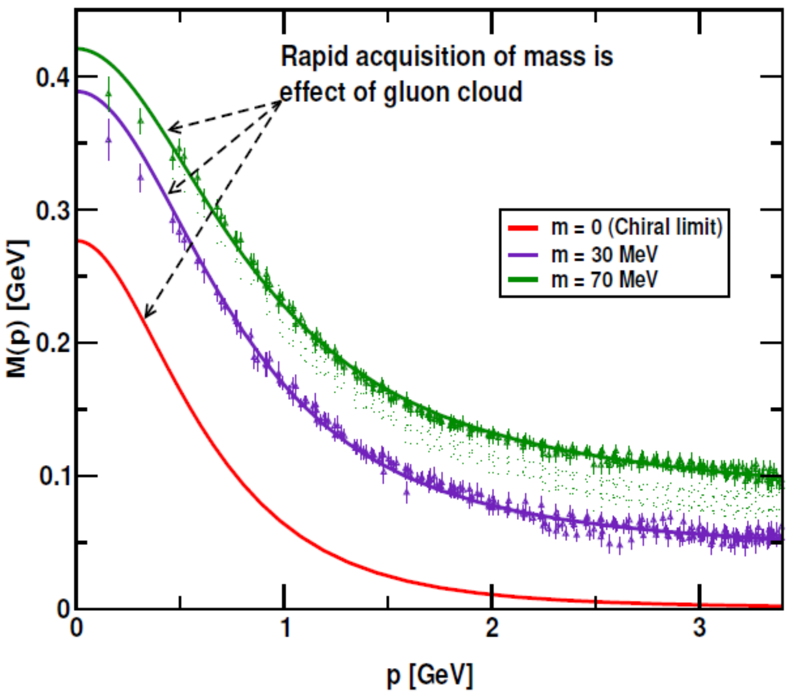
强子质量的非微扰起源
April 21st, 2023

Non-Perturbative QCD:

- **Hadrons, as bound states, are dominated by non-perturbative QCD dynamics – *Two emergent phenomena***
  - **Confinement**: Colored particles have never been seen isolated
    - ✓ Explain how quarks and gluons bind together
  - **Dynamical Chiral Symmetry Breaking (DCSB)**: Hadrons do not follow the chiral symmetry pattern
    - ✓ Explain the most important mass generating mechanism for visible matter in the Universe
  - Neither of these phenomena is apparent in QCD 's Lagrangian, HOWEVER, They play a dominant role in determining the characteristics of real-world QCD!

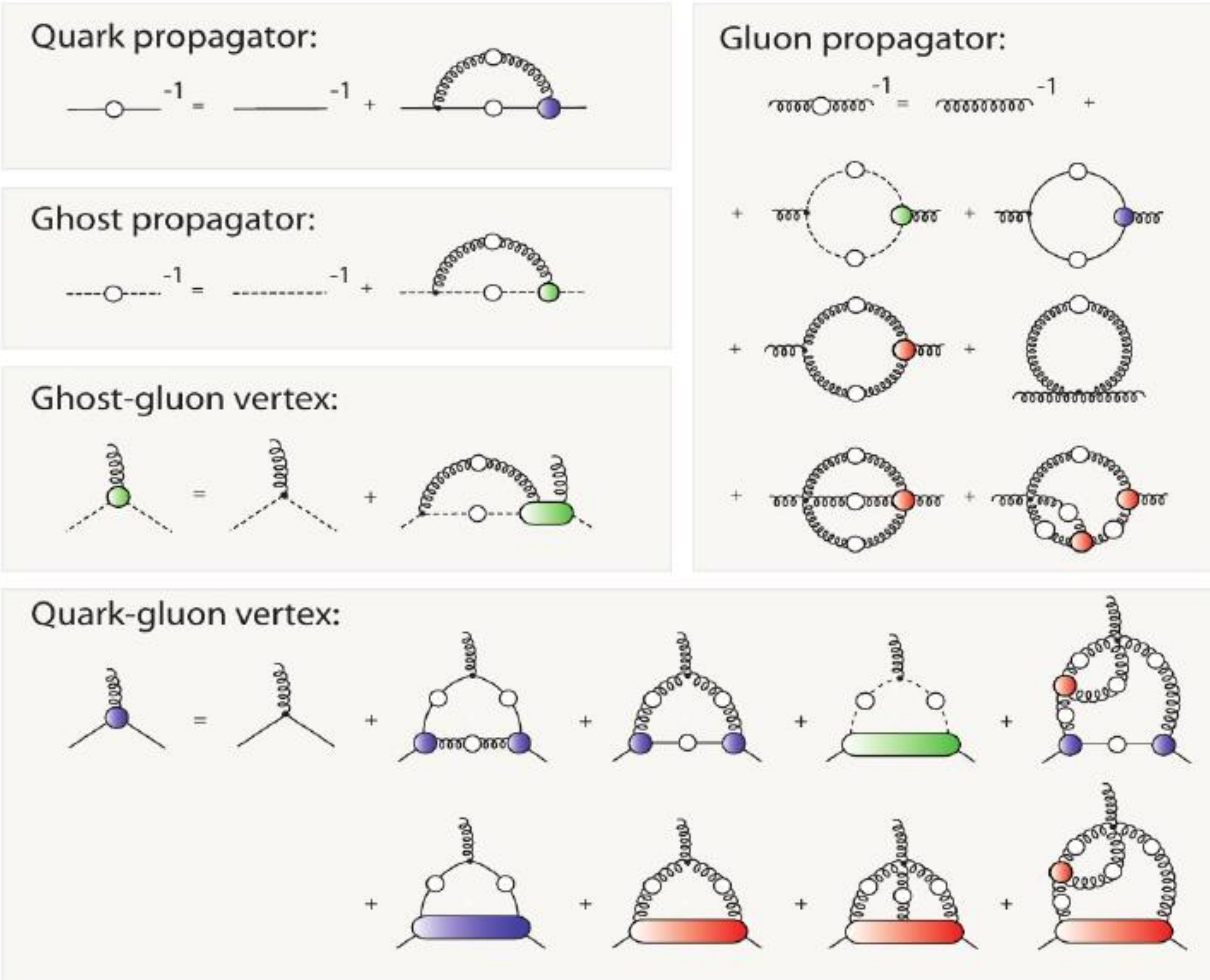
Non-Perturbative QCD:

- **From a quantum field theoretical point of view**, these emergent phenomena could be associated with dramatic, dynamically driven changes in the analytic structure of QCD 's Schwinger functions (propagators and vertices). The Schwinger functions are solutions of the quantum equations of motion (**Dyson-Schwinger equations**).
- **Dressed-quark propagator:**



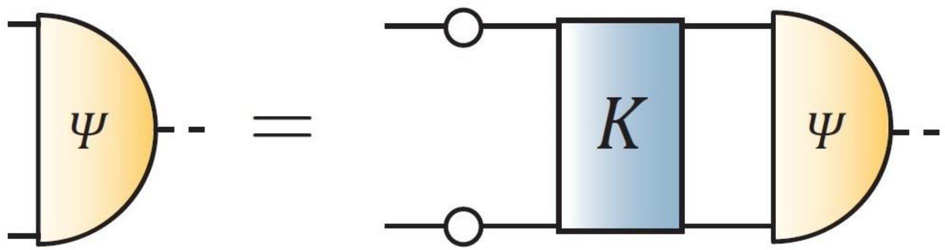
  - Mass generated from the interaction of quarks with the gluon.
  - Light quarks acquire a **HUGE** constituent mass.
  - Responsible of the 98% of the mass of the proton and the large splitting between parity partners.

Continuum Schwinger function methods (CSMs)

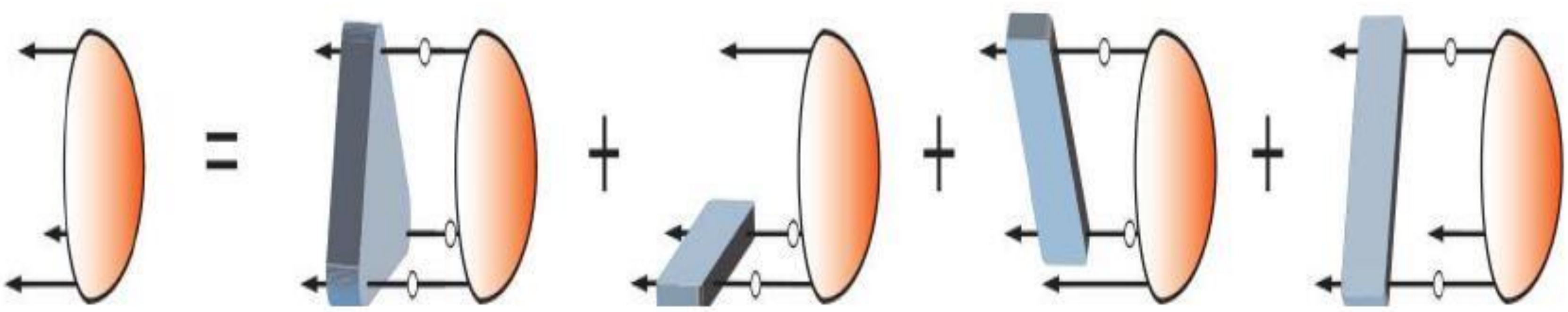


Hadrons: Bound-states in QFT

- **Mesons: a 2-body bound state problem in QFT**
  - ***Bethe-Salpeter Equation***
  - ***K*** **- fully amputated, two-particle irreducible, quark-antiquark scattering kernel**



- **Baryons: a 3-body bound state problem in QFT**
- ***Faddeev equation*: sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.**



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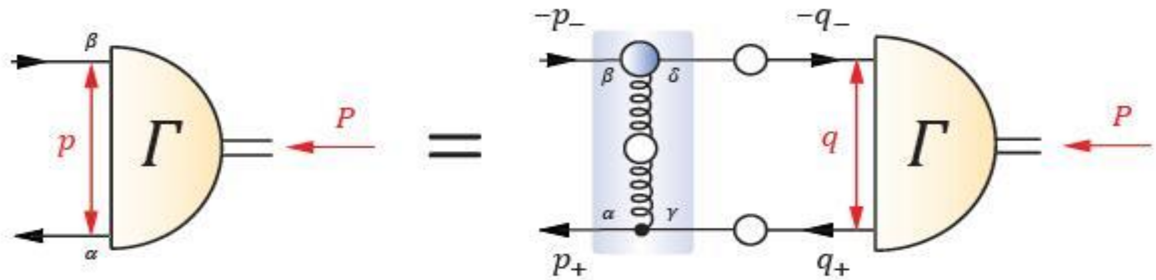

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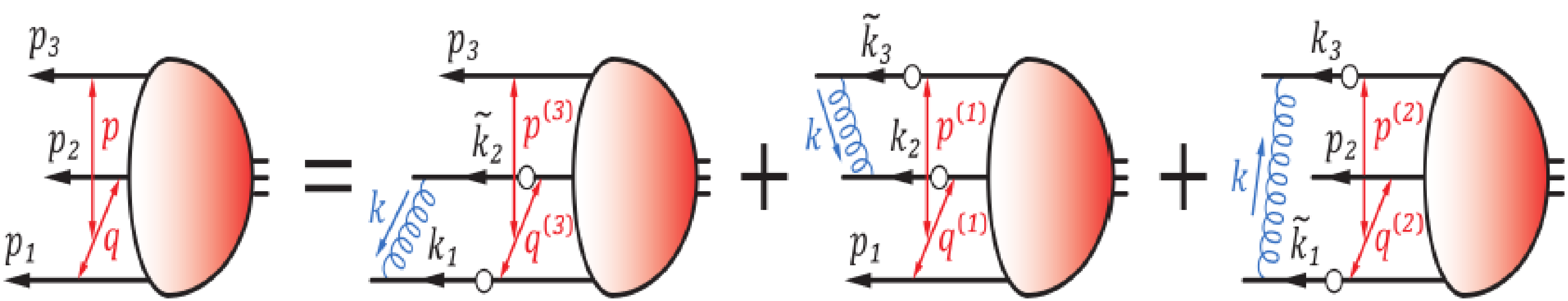
Faddeev equation in rainbow-ladder truncation



Three-body Faddeev equation

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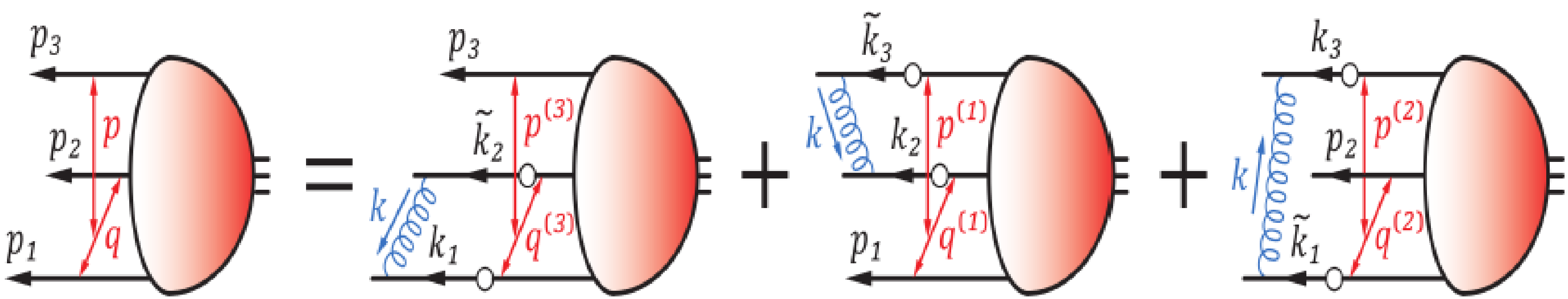

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- G. Eichmann, R. Alkofer, A. Krassnigg and D. Nicmorus, Phys. Rev. Lett. 104, 201601 (2010).

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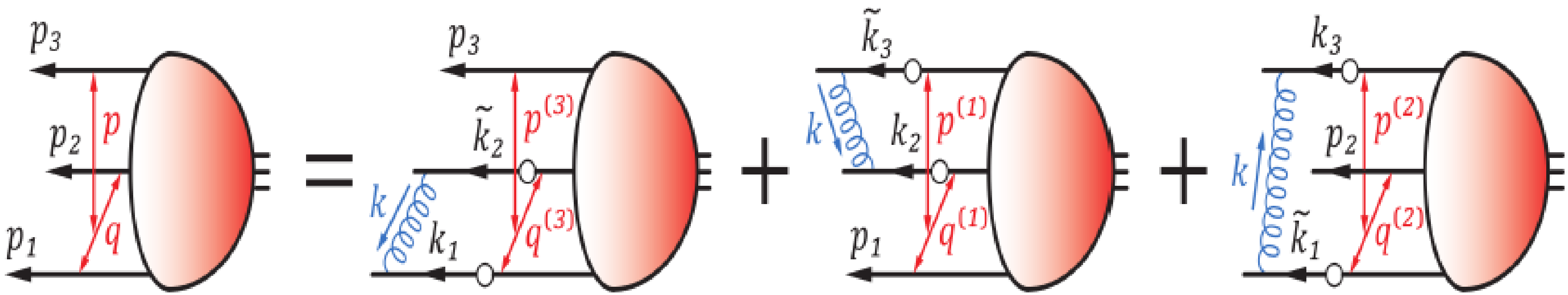

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- **S. X. Qin**, C. D. Roberts and S. M. Schmidt, Phys. Rev. D 97, no.11, 114017 (2018).
- **S. x. Qin**, C. D. Roberts and S. M. Schmidt, Few Body Syst. 60, no.2, 26 (2019).

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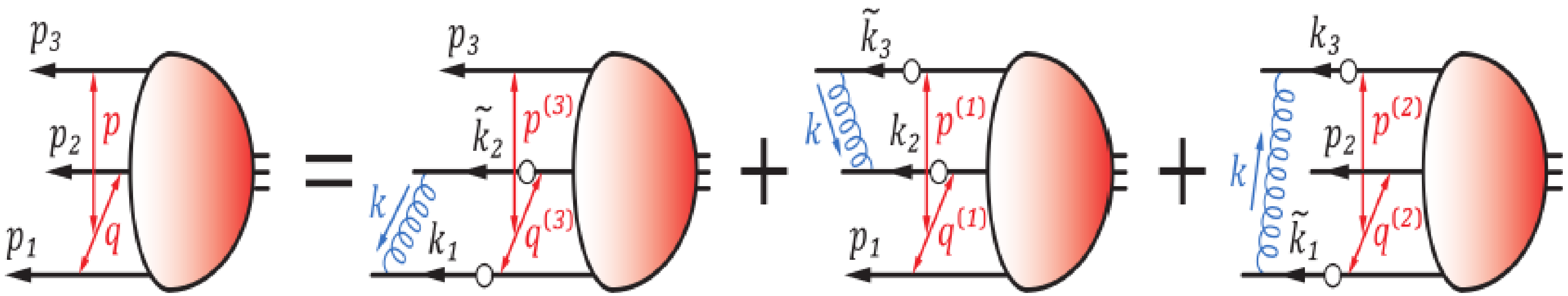

2-body correlations

- **Mesons: quark-antiquark correlations -- color-singlet**
- **Diquarks: quark-quark correlations within a color-singlet baryon.**

- **Diquark correlations:**
  - In our approach: non-pointlike color-antitriplet and fully interacting.
  - Diquark correlations are soft, they possess an electromagnetic size.
  - Owing to properties of charge-conjugation, a diquark with spin-parity J^P may be viewed as a partner to the analogous J^{-P} meson.

$$\Gamma_{q\bar{q}}(p;P) = - \int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{q\bar{q}}(q;P)S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

$$\Gamma_{qq}(p;P)C^\dagger = -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{qq}(q;P)C^\dagger S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

2-body correlations

- Quantum numbers:
  - (I=0, J^P=0^+): isoscalar-scalar diquark
  - (I=1, J^P=1^+): isovector-pseudovector diquark
  - (I=0, J^P=0^-): isoscalar-pesudoscalar diquark
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- ✓ G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer, Prog.Part.Nucl.Phys. 91 (2016) 1-100
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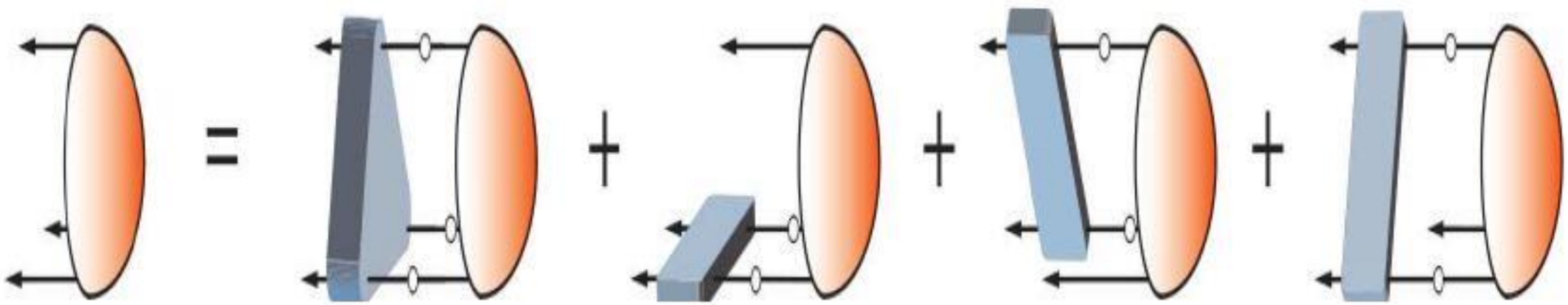

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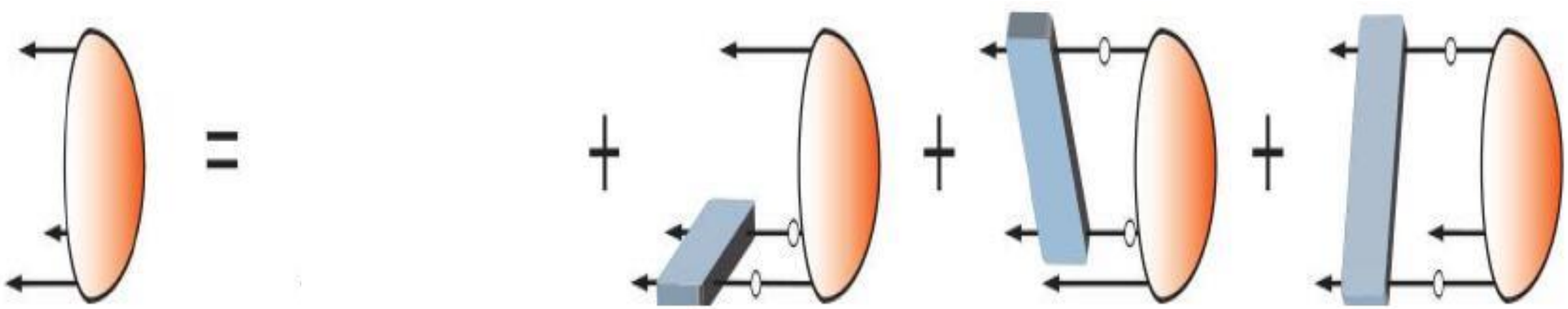

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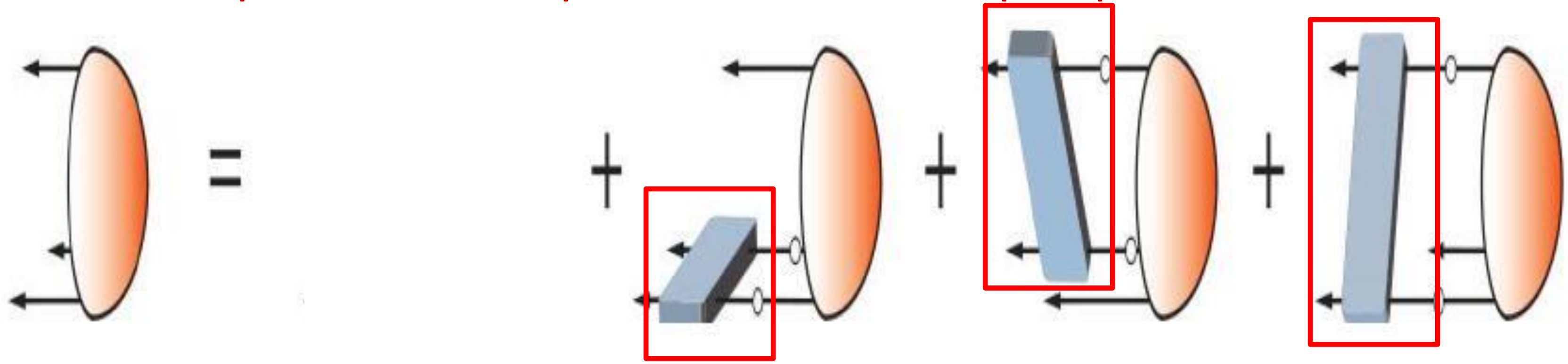

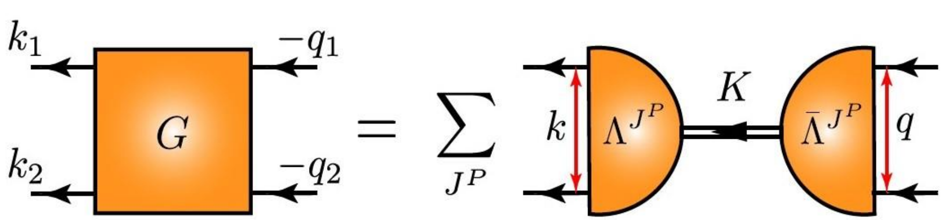

quark-diquark Faddeev equation

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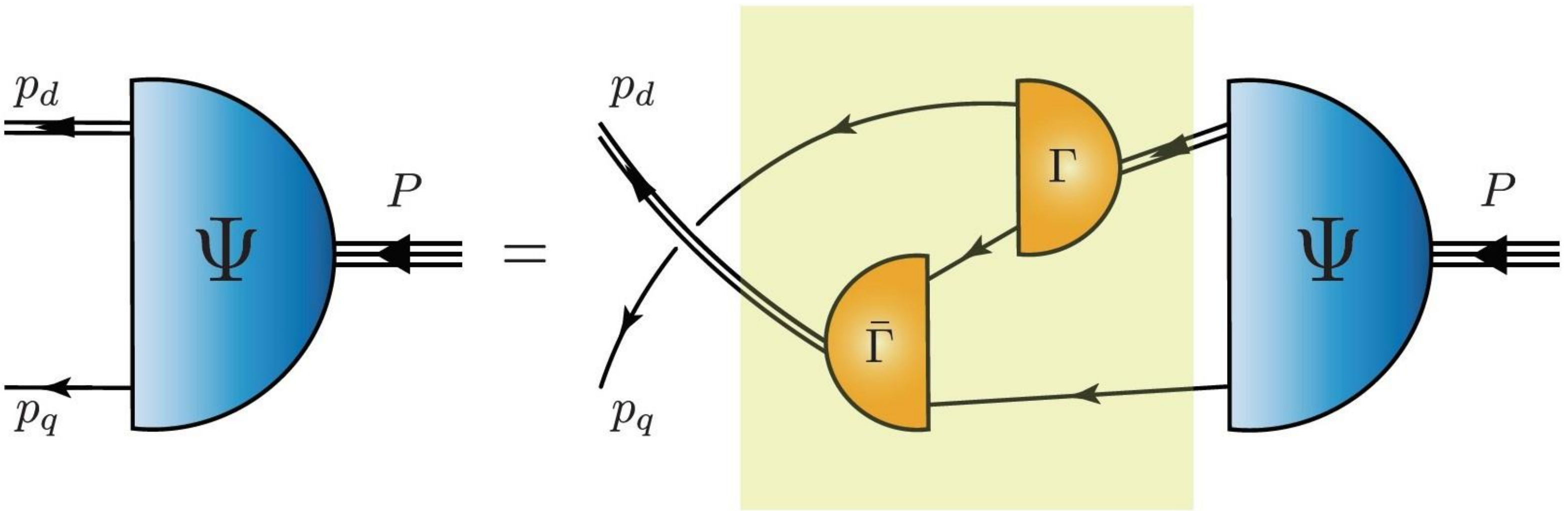
- Three-body bound states ➡
  Quark-diquark two-body bound states

✓ R.T. Cahill, Craig D. Roberts, J. Praschifka, Phys. Rev. D 36 (1987) 2804

✓ R.T. Cahill, Craig D. Roberts, J. Praschifka, Austral.J.Phys. 42 (1989) 129-145



quark-diquark Faddeev equation

Progress in Particle and Nuclear Physics
Volume 116, January 2021, 103835

Review

Diquark correlations in hadron physics: Origin, impact and evidence

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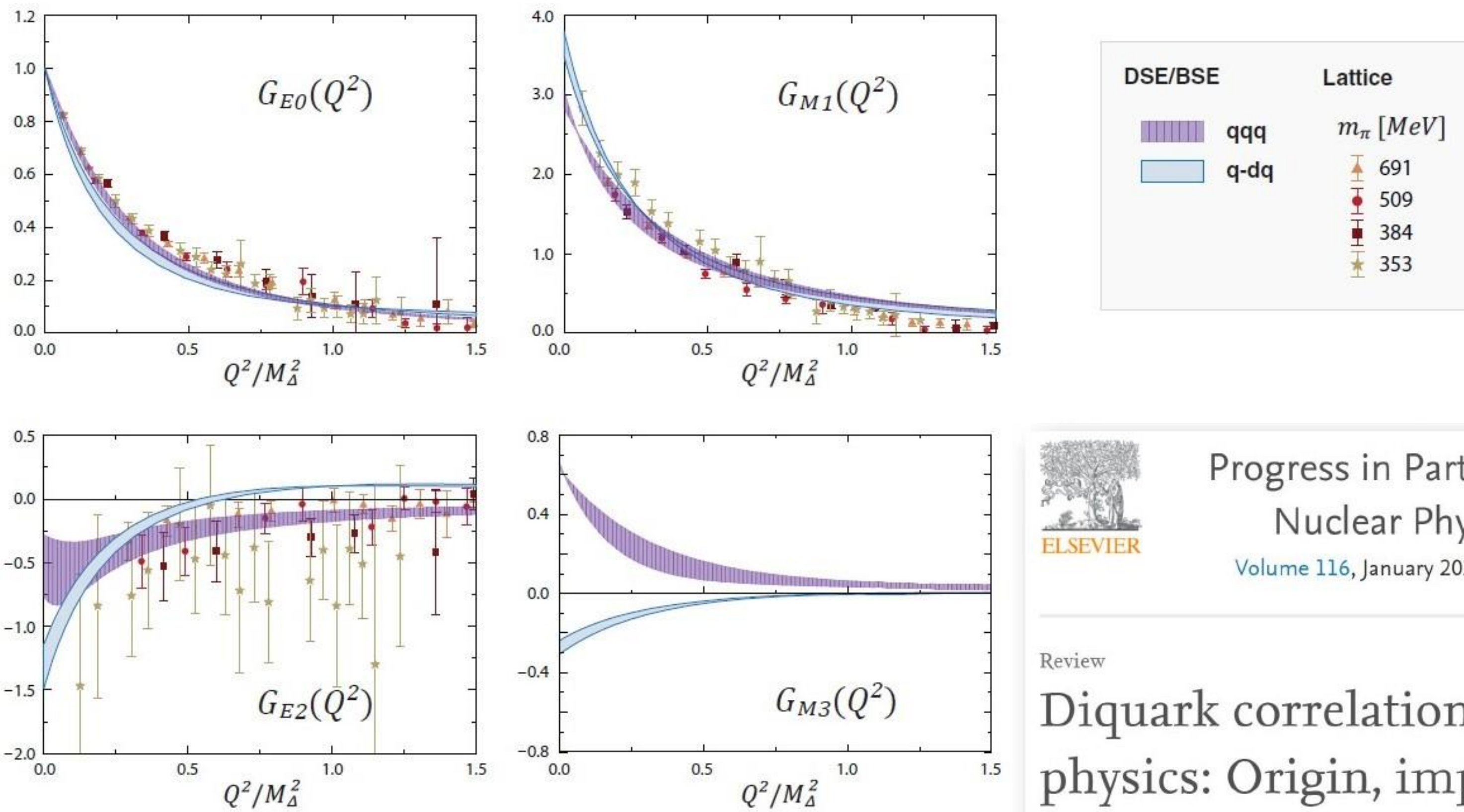

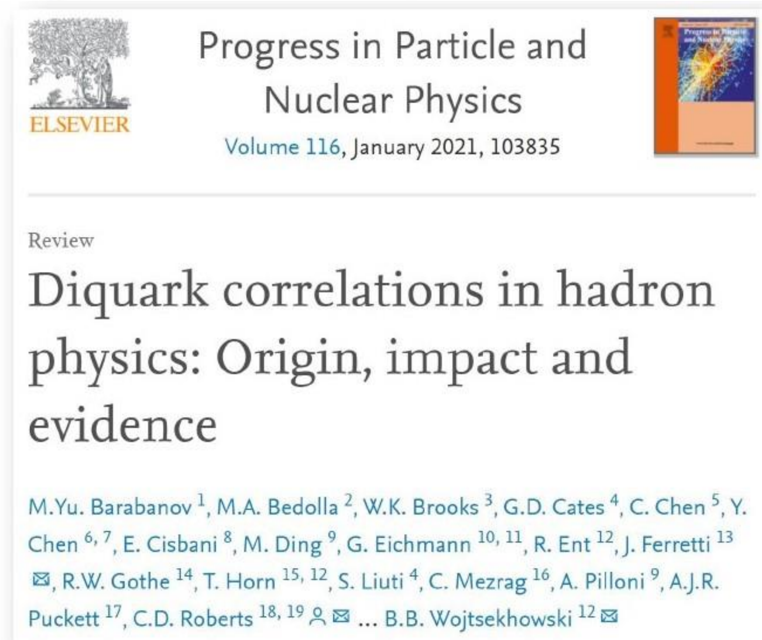
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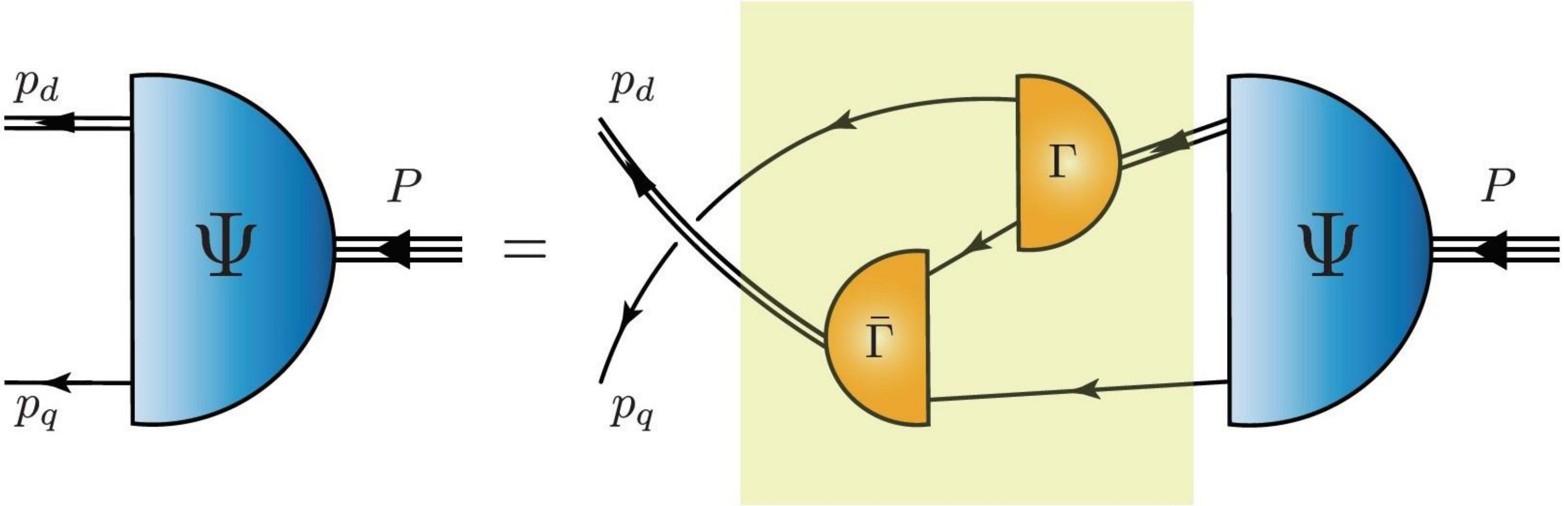

How to solve?

- The dressed-quark propagator
- Diquark amplitudes
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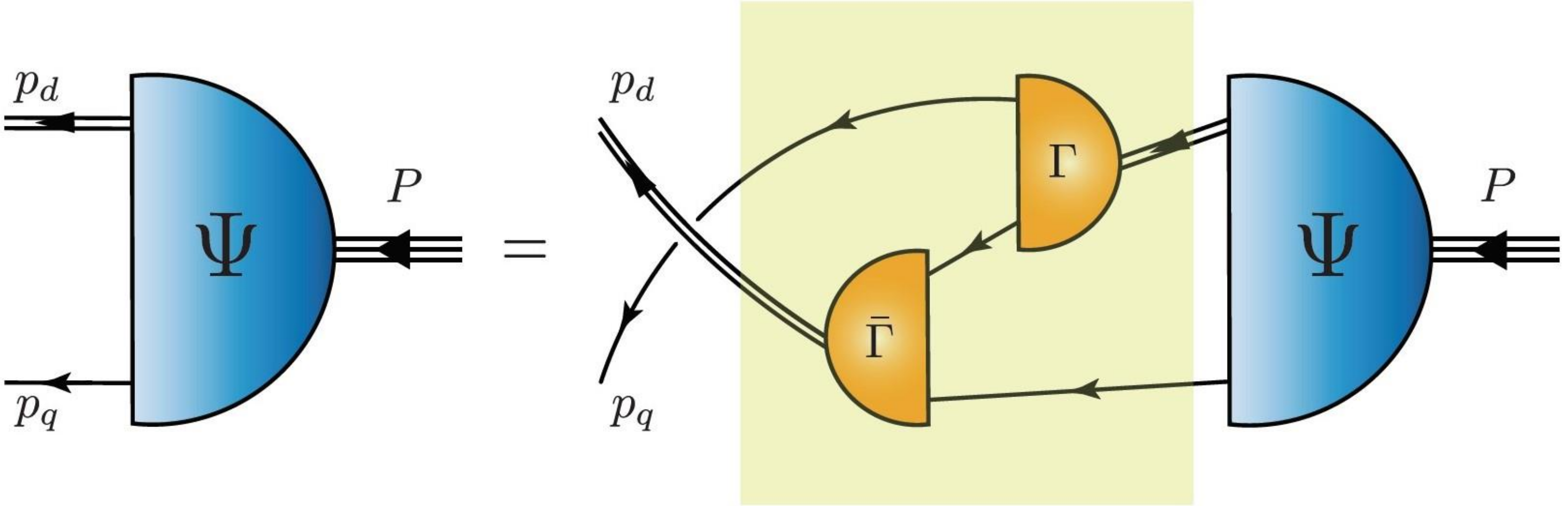

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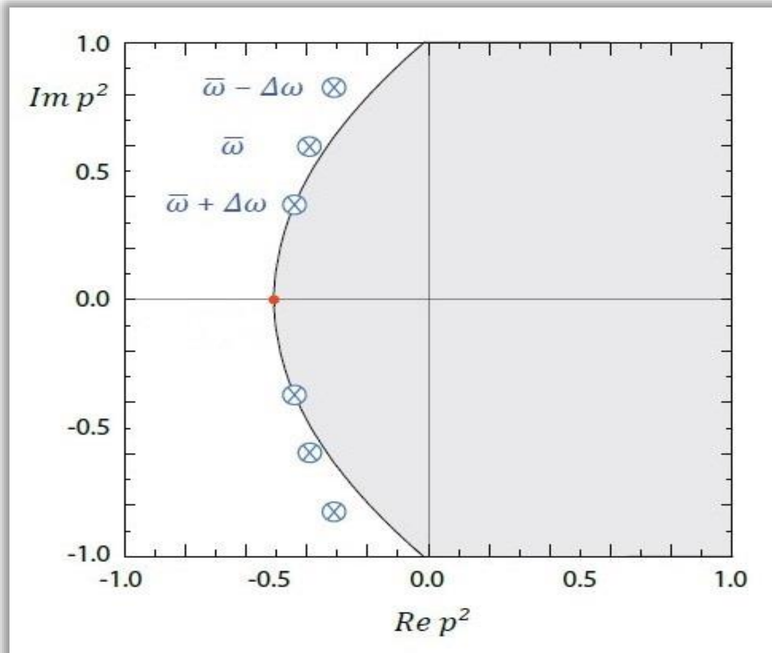

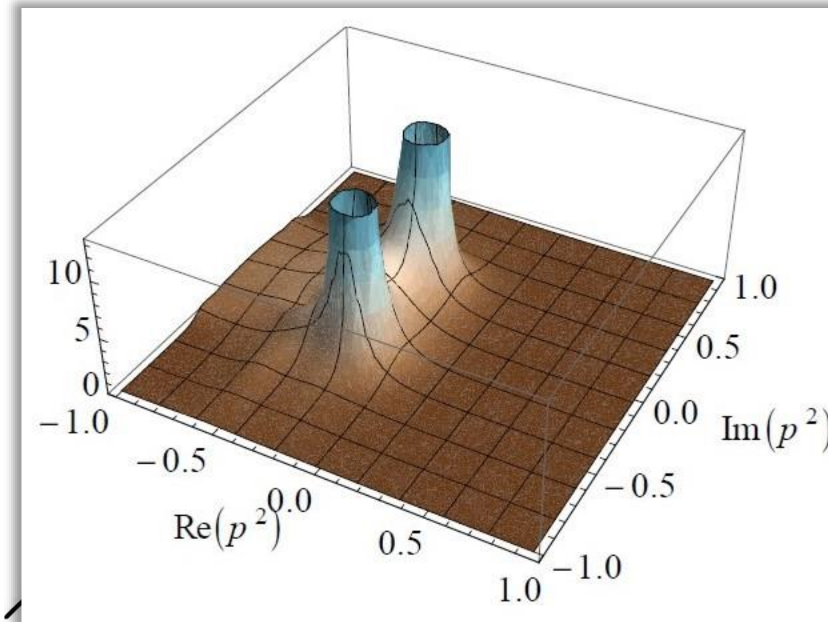

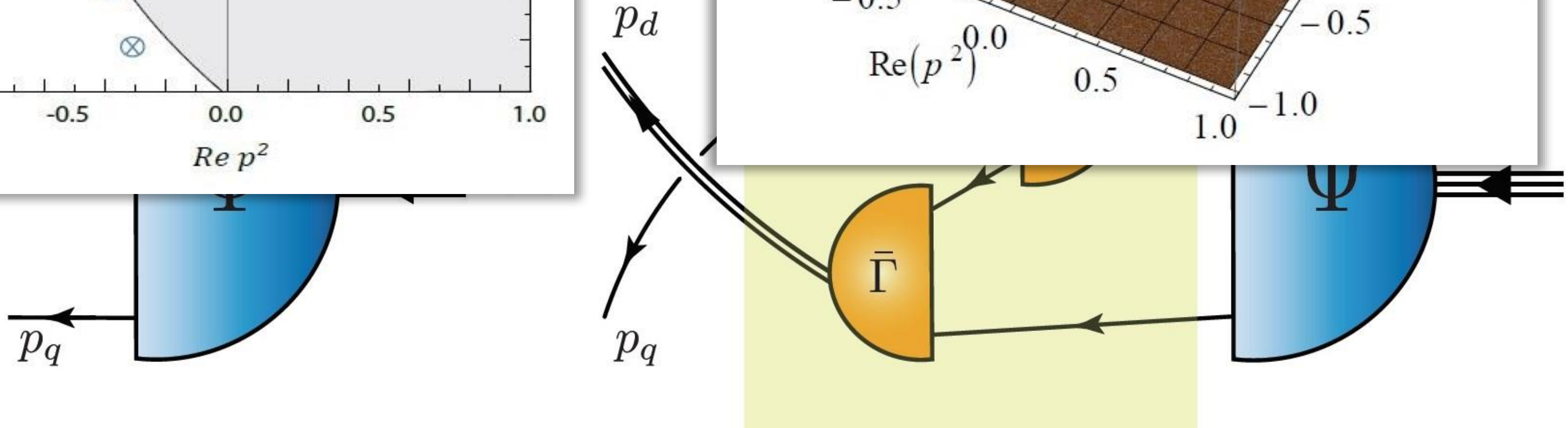

QCD-kindred model

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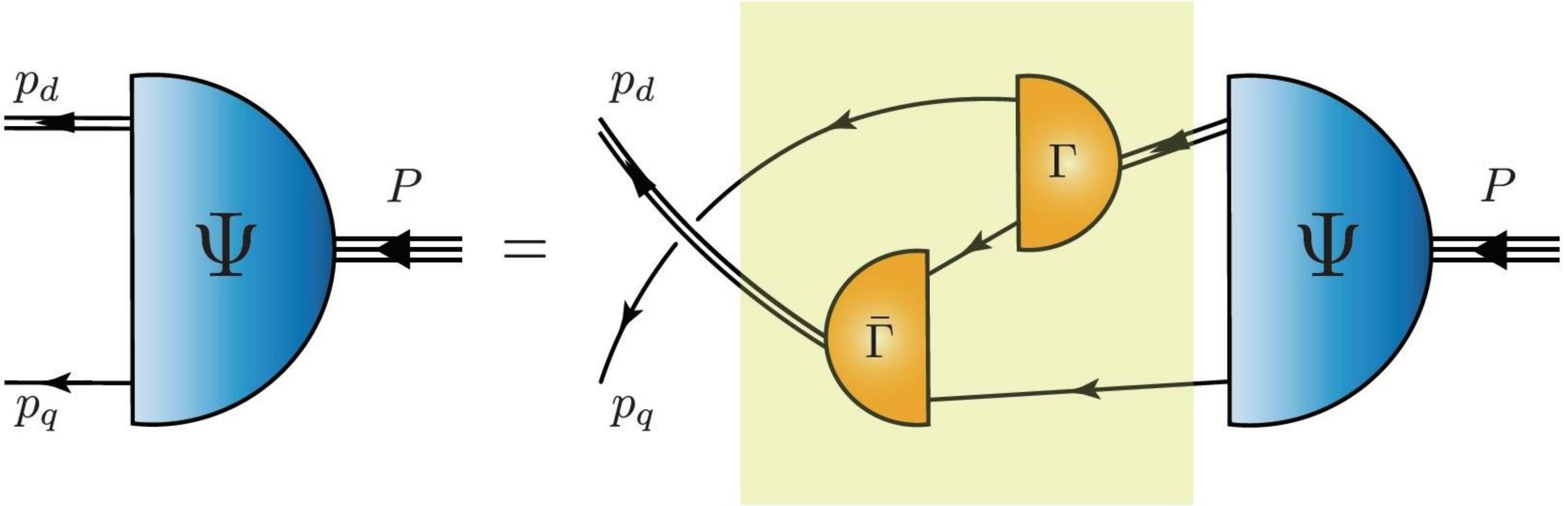

QCD-kindred model

- **Solution to the 60 year puzzle -- Roper resonance: Discovered in 1963, the Roper resonance appears to be an exact copy of the proton except that its mass is *50%* greater and it is unstable…**

PRL **115,** 171801 (2015) PHYSICAL REVIEW LETTERS week ending 23 OCTOBER 2015

Completing the Picture of the Roper Resonance

Jorge Segovia,[1] Bruno El-Bennich,[2,3] Eduardo Rojas,[2,4] Ian C. Cloët,[5] Craig D. Roberts,[5] Shu-Sheng Xu,[6] and Hong-Shi Zong[6]

[1]*Grupo de Física Nuclear and Instituto Universitario de Física Fundamental y Matemáticas (IUFFyM), Universidad de Salamanca, E-37008 Salamanca, Spain*
[2]*Laboratório de Física Teórica e Computacional, Universidade Cruzeiro do Sul, 01506-000 São Paulo, SP, Brazil*
[3]*Instituto de Física Teórica, Universidade Estadual Paulista, 01140-070 São Paulo, SP, Brazil*
[4]*Instituto de Física, Universidad de Antioquia, Calle 70 No. 52-21, Medellín, Colombia*
[5]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[6]*Department of Physics, Nanjing University, Nanjing 210093, China*

(Received 16 April 2015; revised manuscript received 29 July 2015; published 21 October 2015)

We employ a continuum approach to the three valence-quark bound-state problem in relativistic quantum field theory to predict a range of properties of the proton's radial excitation and thereby unify them with those of numerous other hadrons. Our analysis indicates that the nucleon's first radial excitation is the Roper resonance. It consists of a core of three dressed quarks, which expresses its valence-quark content and whose charge radius is 80% larger than the proton analogue. That core is complemented by a meson cloud, which reduces the observed Roper mass by roughly 20%. The meson cloud materially affects long-wavelength characteristics of the Roper electroproduction amplitudes but the quark core is revealed to probes with $Q^2 \gtrsim 3m_N^2$.

DOI: 10.1103/PhysRevLett.115.171801 PACS numbers: 13.40.Gp, 14.20.Dh, 14.20.Gk, 11.15.Tk

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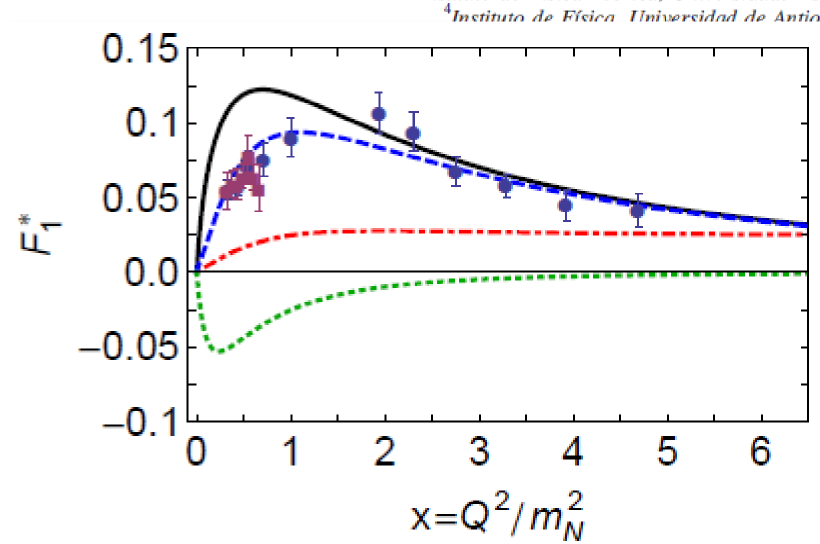
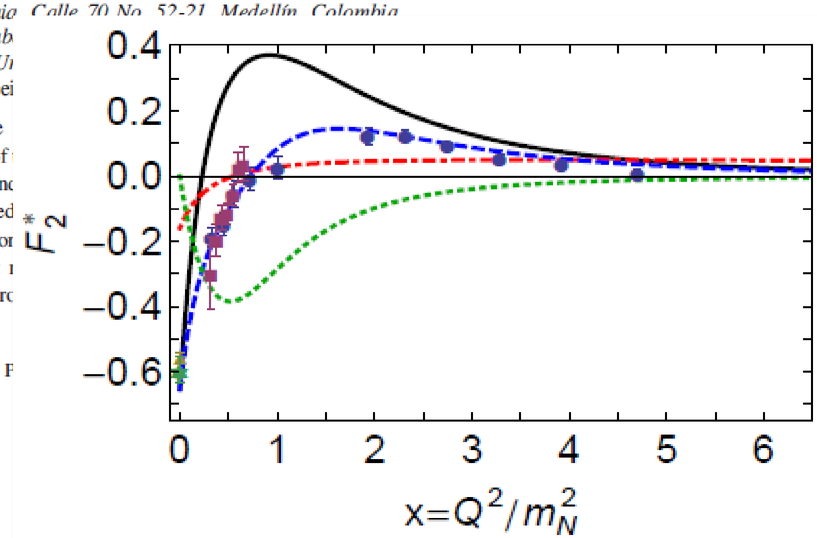




QCD-kindred model

- Roper resonance -- solution to the 60 year puzzle

REVIEWS OF MODERN PHYSICS

REVIEWS OF MODERN PHYSICS, VOLUME 91, JANUARY–MARCH 2019

Colloquium: Roper resonance: Toward a solution to the fifty year puzzle

Volker D. Burkert[*]
Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA

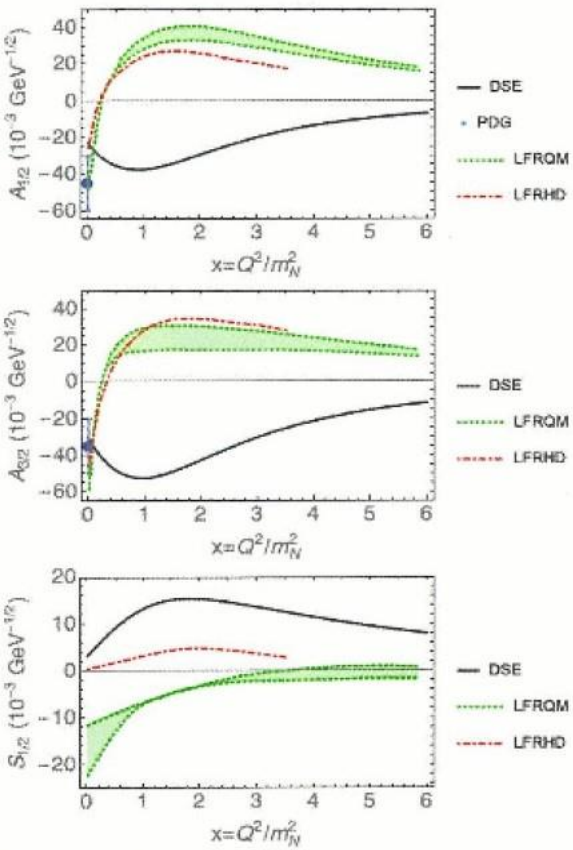
Craig D. Roberts[†]
Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

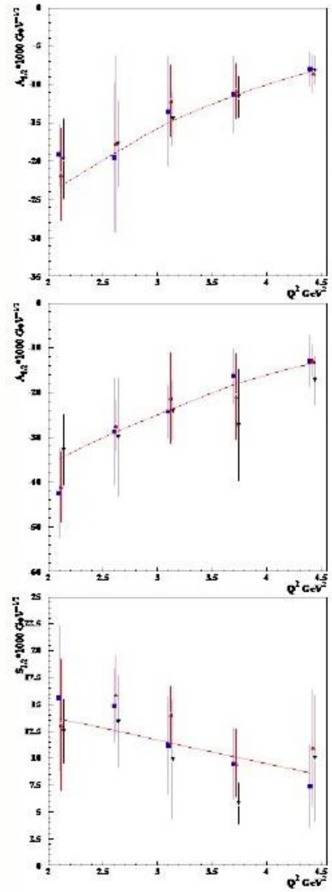
(published 14 March 2019)

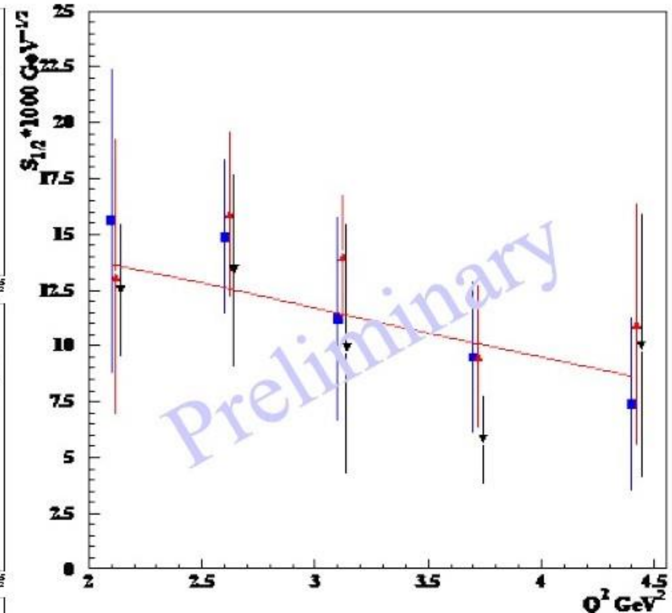
Δ(1600)3/2⁺ Form Factors in CSM Approach

Viktor Mokeev

CSM predictions of the
$\Delta(1600)3/2^+$ electrocouplings







Ya Lu et al., PRD 100, 034001 (2019)

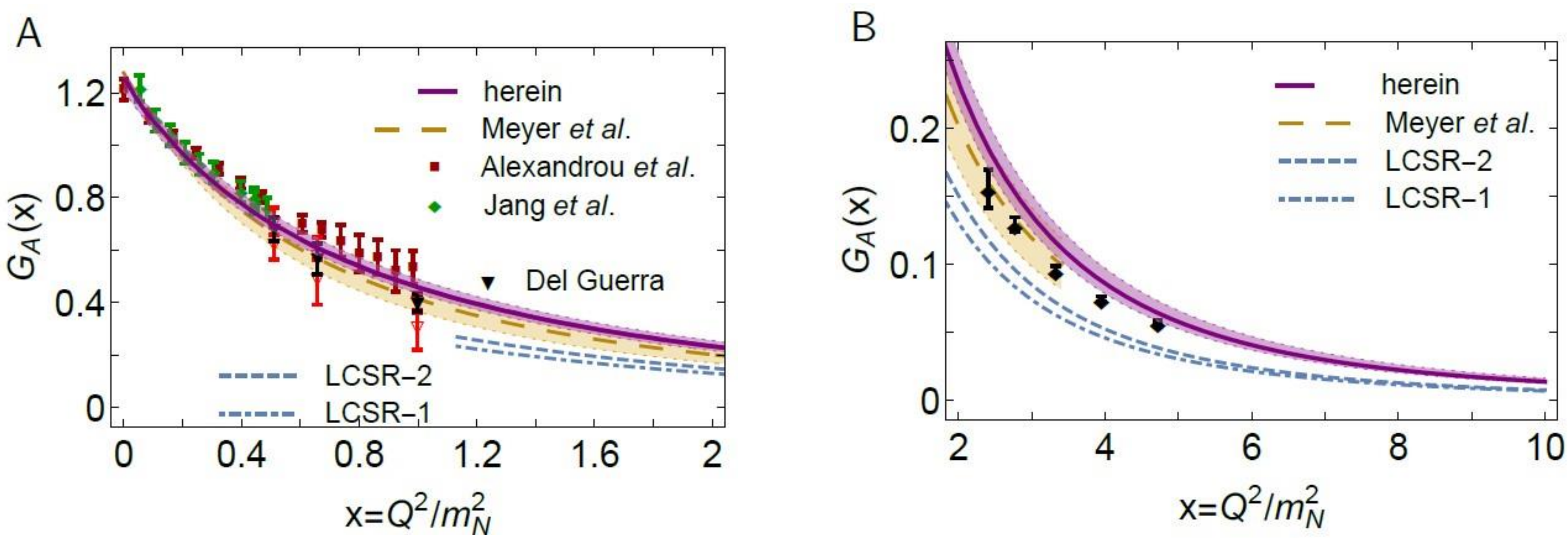
Spring 2022 analysis of Arjun's $\pi^+\pi^- p$ differential cross sections for $2.0\text{GeV}^2<Q^2<5.0\text{GeV}^2$ within three W-intervals, 1.46GeV<W<1.56GeV, 1.51GeV<W<1.61GeV, and 1.56GeV<W<1.66GeV.

Axial Form Factors

- Hellstern, G. and Alkofer, Reinhard and Oettel, M. and Reinhardt, H., *Nucl.Phys.A* 627 (**1997**) 679-709.
- Bloch, Jacques C. R. and Roberts, Craig D. and Schmidt, S. M., *Phys.Rev.C* 61 (**2000**) 065207.
- Oettel, Martin and Pichowsky, Mike and von Smekal, Lorenz, *Eur.Phys.J.A* 8 (**2000**) 251-281.

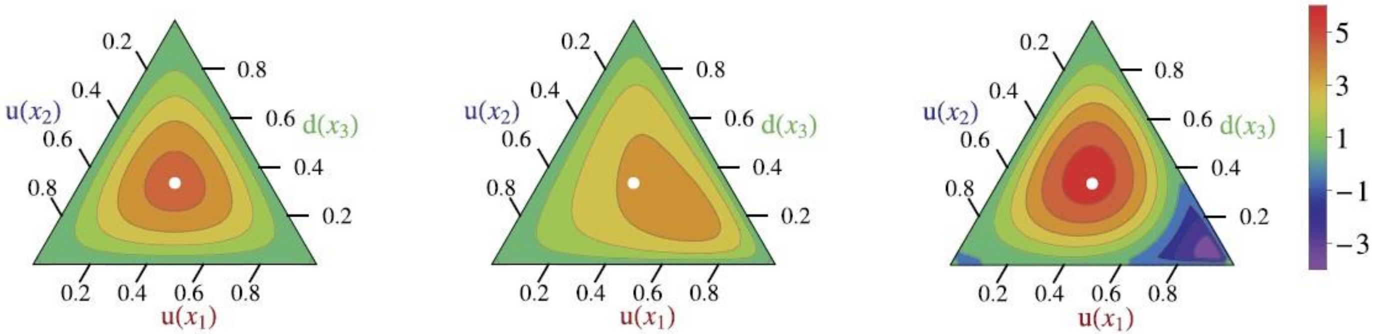
- Eichmann, G. and Fischer, C. S., *Eur.Phys.J.A* 48 (**2012**) 9.

- **Chen Chen**, C. S. Fischer, C. D. Roberts and J. Segovia, Form Factors of the Nucleon Axial Current, *Phys.Lett.B* 815 (**2021**), 136150.
- **Chen Chen**, C. S. Fischer, C. D. Roberts and J. Segovia, Form Factors of the Nucleon Axial and Pseudoscalar Currents, Phys.Rev.D 105 (**2022**) 9, 094022.
- **Chen Chen** and C. D. Roberts, Nucleon axial form factor at large momentum transfers, *Eur.Phys.J.A* (**2022**) 58:206.
- Peng Cheng, Fernando E. Serna, Zhao-Qian Yao, **Chen Chen**, Zhu-Fang Cui, and C. D. Roberts, Contact interaction analysis of octet baryon axial-vector and pseudoscalar form factors, *Phys.Rev.D* 106 (**2022**) 5, 054031.

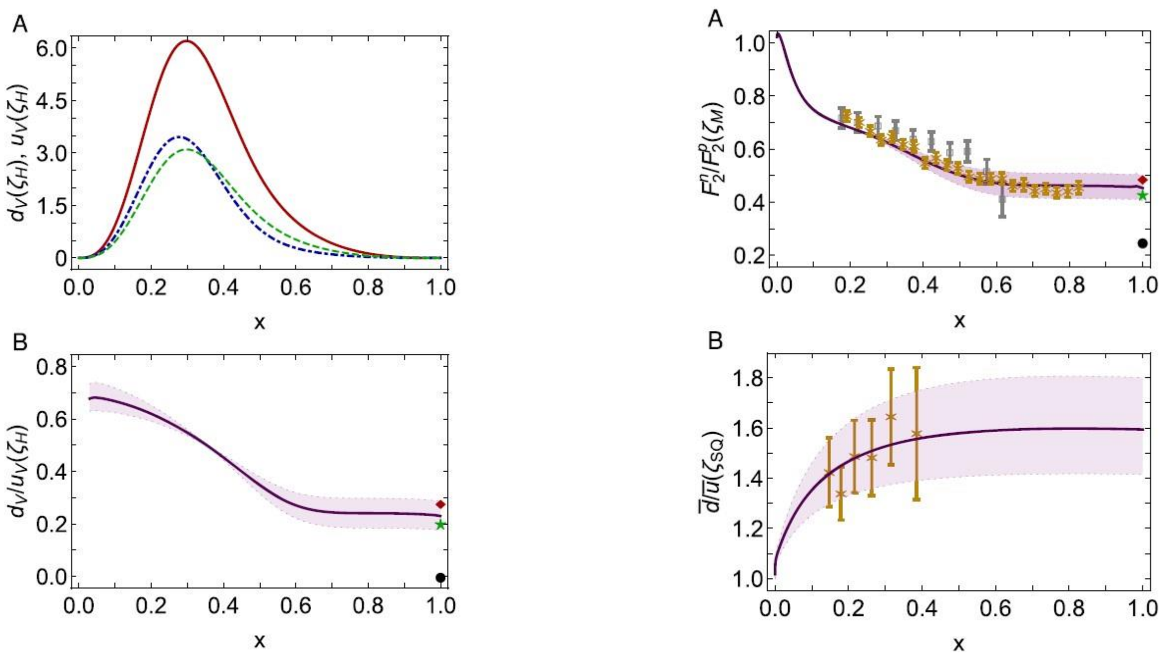


PDAs & PDFs

- C. Mezrag, J. Segovia, **L. Chang** and C. D. Roberts, Phys. Lett. B 783, 263-267 (2018).



- **L. Chang**, F. Gao and C. D. Roberts, Phys. Lett. B 829, 137078 (2022).



Data

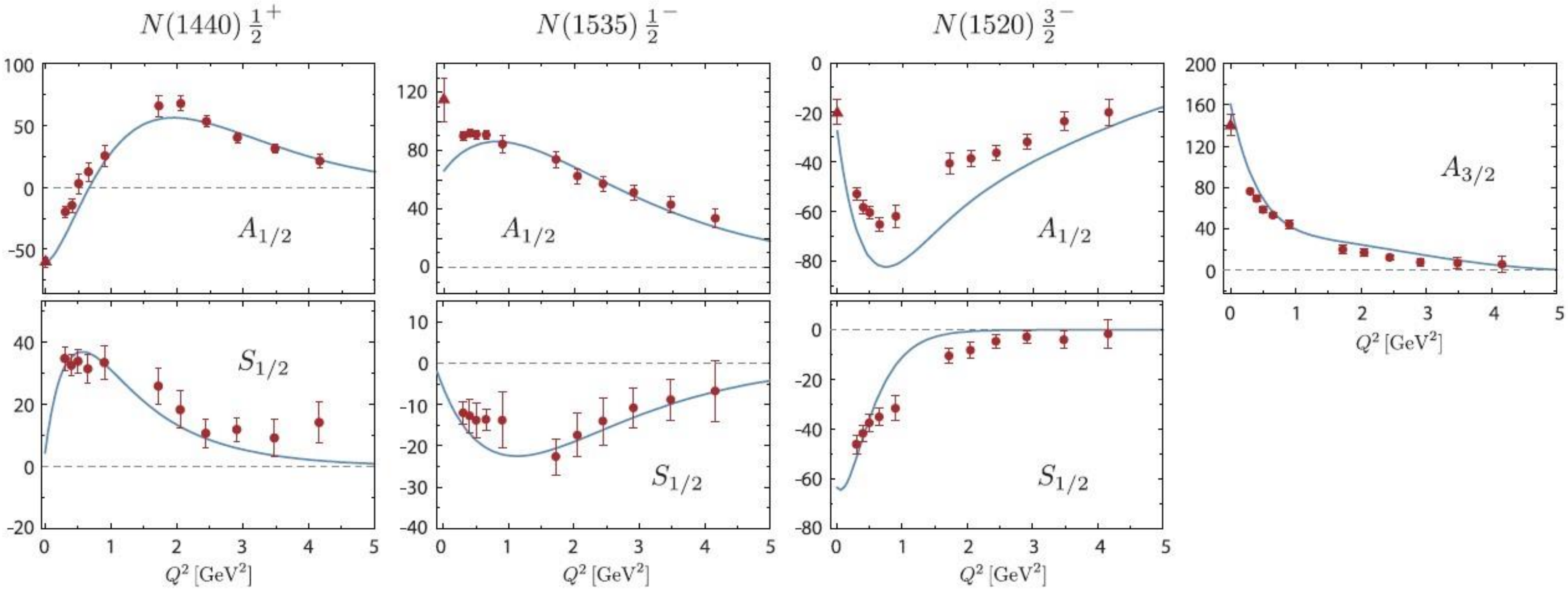
- Two- to four-star baryon resonances below 2 GeV and up to J^P=5/2^{+-} from the PDG. The four-star resonances are shown in bold font and the two-star resonances in grey.

I	S	$J^P = \frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^-$	$\frac{5}{2}^-$
$\frac{1}{2}$	0	**N(940)** **N(1440)** $N(1710)$ $N(1880)$	**N(1720)** $N(1900)$	**N(1680)** $N(1860)$	**N(1535)** **N(1650)** $N(1895)$	**N(1520)** $N(1700)$ $N(1875)$	**N(1675)**
$\frac{3}{2}$	0	**Δ(1910)**	**Δ(1232)** Δ(1600) Δ(1920)	**Δ(1905)**	**Δ(1620)** Δ(1900)	**Δ(1700)** Δ(1940)	Δ(1930)

- The mass range around 1.5 GeV is the so-called *second resonance region* and features a cluster of three nucleon resonances: the Roper N(1440), the nucleon's putative parity partner N(1535) with J^P = 1/2−, and the N(1520) resonance with J^P = 3/2−.

Data

- The mass range around **1.5 GeV** is the so-called ***second resonance region*** and features a cluster of three nucleon resonances: the Roper **N(1440)**, the nucleon's putative parity partner **N(1535)** with **J^P = 1/2−**, and the **N(1520)** resonance with **J^P = 3/2−**.



- The **N(1535)**: its helicity amplitude **S1/2** shows an unusually slow falloff with **Q2**, which translates into a cancellation of the corresponding Pauli form factor **F2(Q2)** that is consistent with zero above **Q2 ∼ 2 GeV2** and rapidly rises below that value.

- The **N(1520)**: its helicity amplitudes look rather ordinary.

CSMs: Present situation & Future plan

	质量谱		电磁形状因子		轴矢形状因子	
	双夸克	三体	双夸克	三体	双夸克	三体
$N(940)1/2^+$	✓	✓	✓	✓	✓	✓
$\Delta(1232)3/2^+$	✓	✓	✓	✓		
$N(1440)1/2^+$	✓		✓			
$N(1535)1/2^-$	✓					
$N(1520)3/2^-$	✓					
超子（正宇称）	✓	✓		✓		
超子（负宇称）						

1. **Computing the electromagnetic, axial-vector, and pseudoscalar form factors for the Δ(1232) and three nucleon resonances in the second resonance region using the diquark method.**
2. **Studying the axial-vector and pseudoscalar form factors of the nucleon and the Δ(1232) at a deeper level using the three-body method.**
3. **Calculating the spectrum of the negative-parity hyperons, and performing the preliminary study on the hyperons' form factors.**

The expected outcomes: making a unified analysis of the spectrum and form factors for the nucleon and its resonances and developing the CSMs furthermore.

QCD-kindred model: diquark masses

- **Diquark masses (in GeV):**

  $m_{0^+} = 0.8, \quad m_{1^+} = 0.9, \quad m_{0^-} = 1.2, \quad m_{1^-} = 1.3,$

- **The first two values (positive-parity) provide for a good description of numerous dynamical properties of the nucleon, Δ-baryon and Roper resonance.**
- **Masses of the odd-parity correlations are based on those computed from a contact interaction.**
- **Such values are typical; and in truncations of the two-body scattering problem that are most widely used (RL), isoscalar- and isovector-vector correlations are degenerate.**
- **Normalization condition → couplings:**

$$g_{0^+} = 14.8, \qquad g_{1^+} = 12.7,$$
$$g_{0^-} = 12.8, \qquad g_{1^-} = 5.4, \qquad g_{\bar{1}^-} = 2.5.$$

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  $$g_{0^-} = 12.8, \qquad g_{1^-} = 5.4, \qquad \cancel{g_{\bar{1}^-} = 2.5}.$$

- **Faddeev kernels:**
  - (I, J^P) = (1/2, 1/2^{+-}): 22 × 22 matrices are reduced to 16 × 16
  - (I, J^P) = (1/2, 3/2^{+-}): 28 × 28 matrices are reduced to 20 × 20

Spectrum publications

- **Chen Chen**, B. El-Bennich, C. D. Roberts, S. M. Schmidt, J. Segovia and S. Wan, “Structure of the nucleon’s low-lying excitations,” Phys. Rev. D **97**, no.3, 034016 (2018).
- **Chen Chen**, G. I. Krein, C. D. Roberts, S. M. Schmidt and J. Segovia, “Spectrum and structure of octet and decuplet baryons and their positive-parity excitations,” Phys. Rev. D **100**, no.5, 054009 (2019).
- L. Liu, **Chen Chen**, Y. Lu, C. D. Roberts and J. Segovia, “Composition of low-lying J=32± Δ-baryons,” Phys. Rev. D **105**, no.11, 114047 (2022).
- L. Liu, **Chen Chen** and C. D. Roberts, “Wave functions of (I,JP)=(12,32∓) baryons,” Phys. Rev. D **107**, no.1, 014002 (2023).

(I, J^P) = (1/2, 1/2^{+-})

PHYSICAL REVIEW D **97,** 034016 (2018)

Structure of the nucleon's low-lying excitations

Chen Chen,[1,*] Bruno El-Bennich,[2,†] Craig D. Roberts,[3,‡] Sebastian M. Schmidt,[4,§]
Jorge Segovia,[5,∥] and Shaolong Wan[6,¶]

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[2]*Universidade Cruzeiro do Sul, Rua Galvão Bueno, 868, 01506-000 São Paulo, São Paulo, Brazil*
[3]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[4]*Institute for Advanced Simulation, Forschungszentrum Jülich and JARA, D-52425 Jülich, Germany*
[5]*Institut de Física d'Altes Energies (IFAE) and Barcelona Institute of Science and Technology (BIST), Universitat Autònoma de Barcelona, E-08193 Bellaterra (Barcelona), Spain*
[6]*Institute for Theoretical Physics and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China*

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(I, J^P) = (1/2, 3/2^{+-})

Wave functions of $(I,J^P)=(\frac{1}{2},\frac{3}{2}^{\mp})$ baryons

Langtian Liu (刘浪天)[1,2] Chen Chen (陈晨)[3,4,*] and Craig D. Roberts[1,2,†]

[1]*School of Physics, Nanjing University, Nanjing, Jiangsu 210093, China*

[2]*Institute for Nonperturbative Physics, Nanjing University, Nanjing, Jiangsu 210093, China*

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[4]*Peng Huanwu Center for Fundamental Theory, Hefei, Anhui 230026, China*

- **Chen Chen**, B. El-Bennich, C. D. Roberts, S. M. Schmidt, J. Segovia and S. Wan, “Structure of the nucleon’s low-lying excitations,” Phys. Rev. D **97**, no.3, 034016 (2018).
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SOLUTIONS & THEIR PROPERTIES:
Masses

- Our computed values for the masses of the four lightest (I, J^P) = (1/2, 3/2^{+-}) baryon doublets are listed here, in GeV:

$N(1520)\frac{3}{2}^-$	$N(1700)\frac{3}{2}^-$	$N(1720)\frac{3}{2}^+$	$N(1900)\frac{3}{2}^+$
1.68(8)	1.82(9)	1.78(8)	2.05(7)

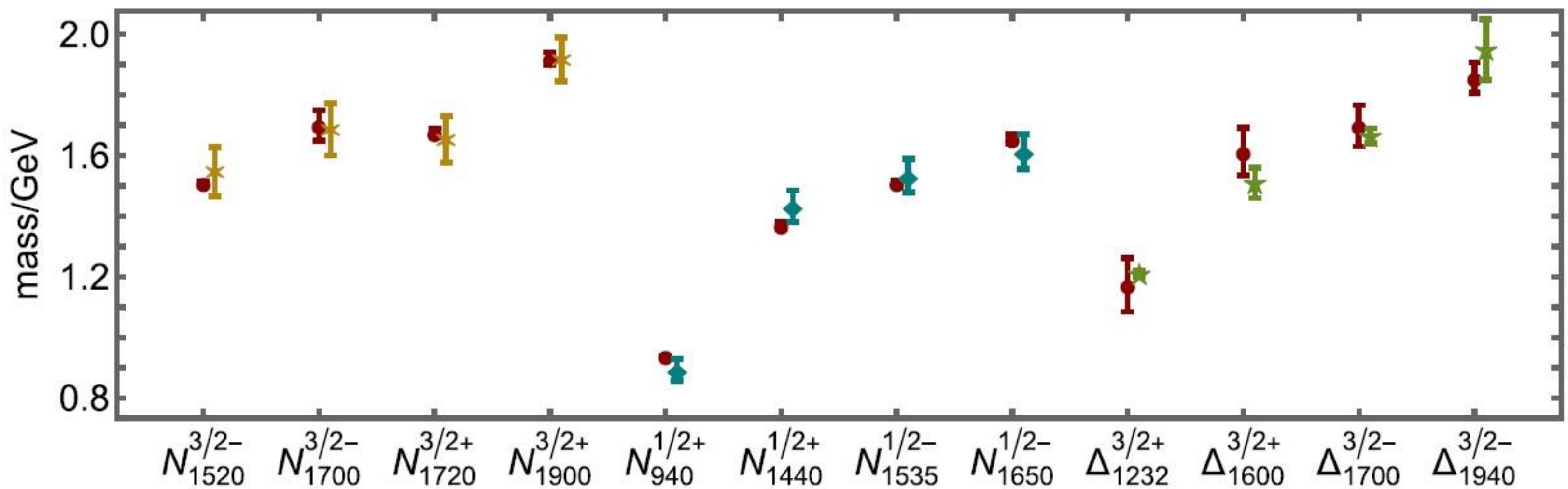
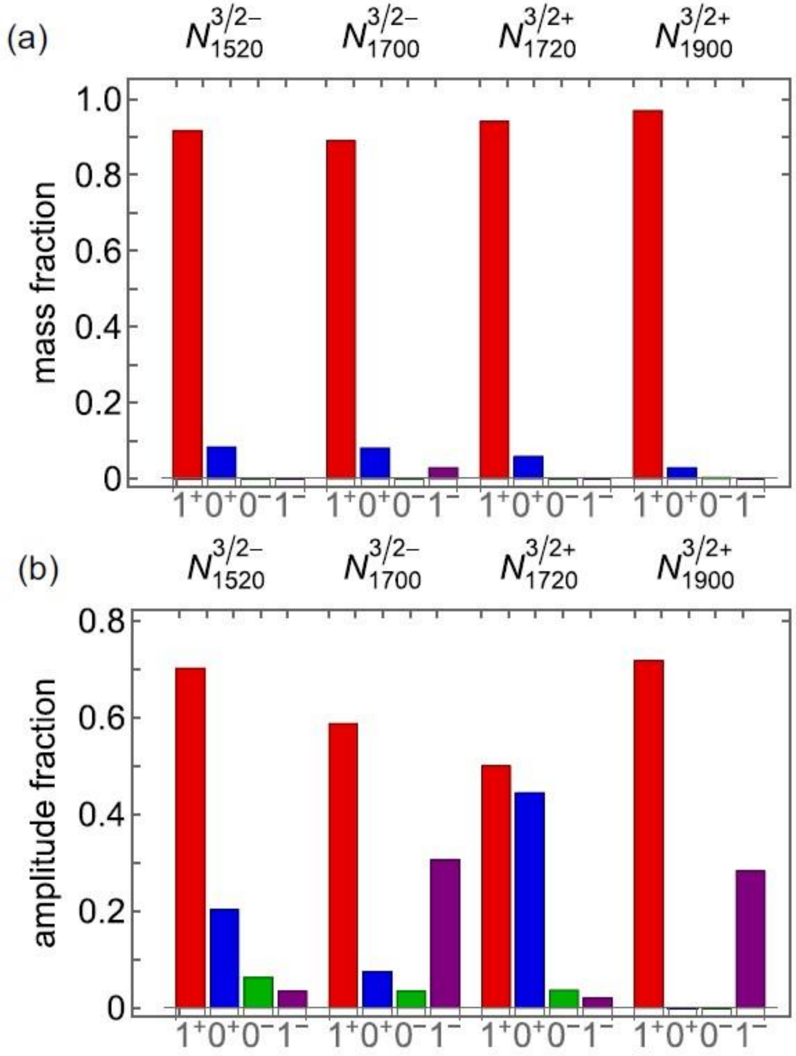


FIG. 2. Real part of empirical pole position for each identified baryon [48] (red circle) compared with: calculated masses in Eq. (17) (gold asterisks) after subtracting $\delta_{\rm MB}^{N_{3/2}} = 0.13$ GeV from each; calculated masses in Ref. [34] [Eq. (15)] (teal diamonds) after subtracting $\delta_{\rm MB}^{N_{1/2}} = 0.30$ GeV; and calculated masses in Ref. [47] (Table II) (green five-pointed stars) after subtracting $\delta_{\rm MB}^{\Delta_{3/2}} = 0.17$ GeV. All theory values are drawn with an uncertainty that reflects a $\pm 5\%$ change in diquark masses, Eq. (1).

SOLUTIONS & THEIR PROPERTIES:
Diquark content

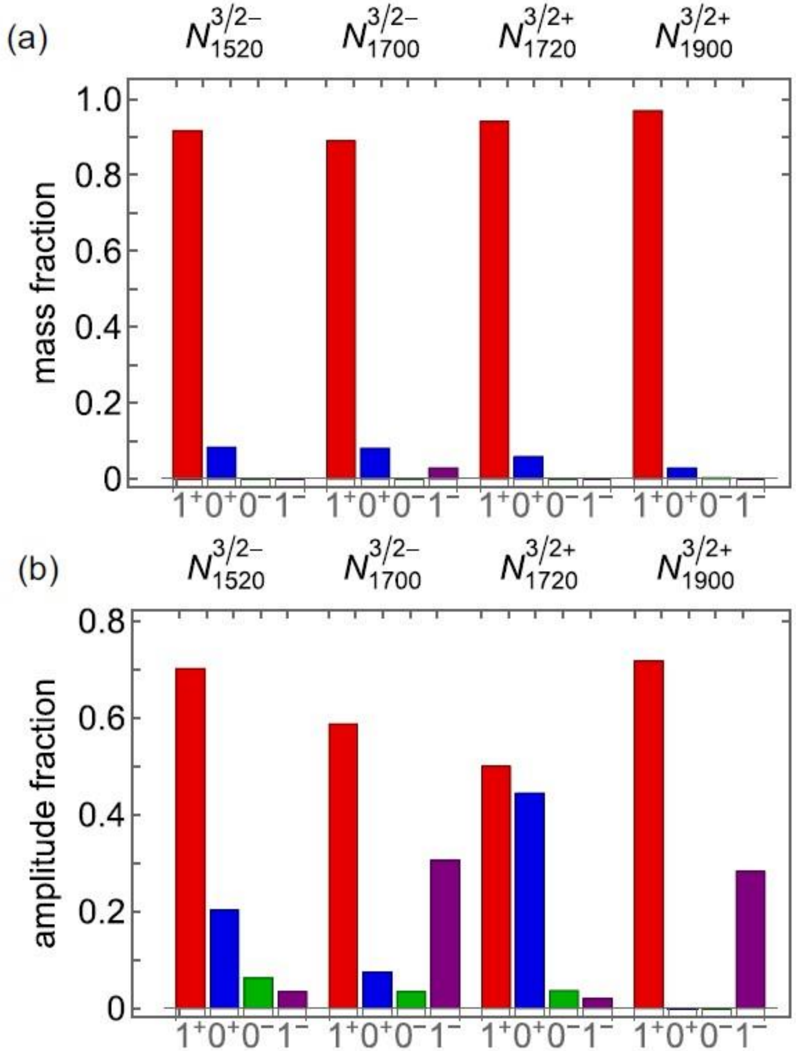


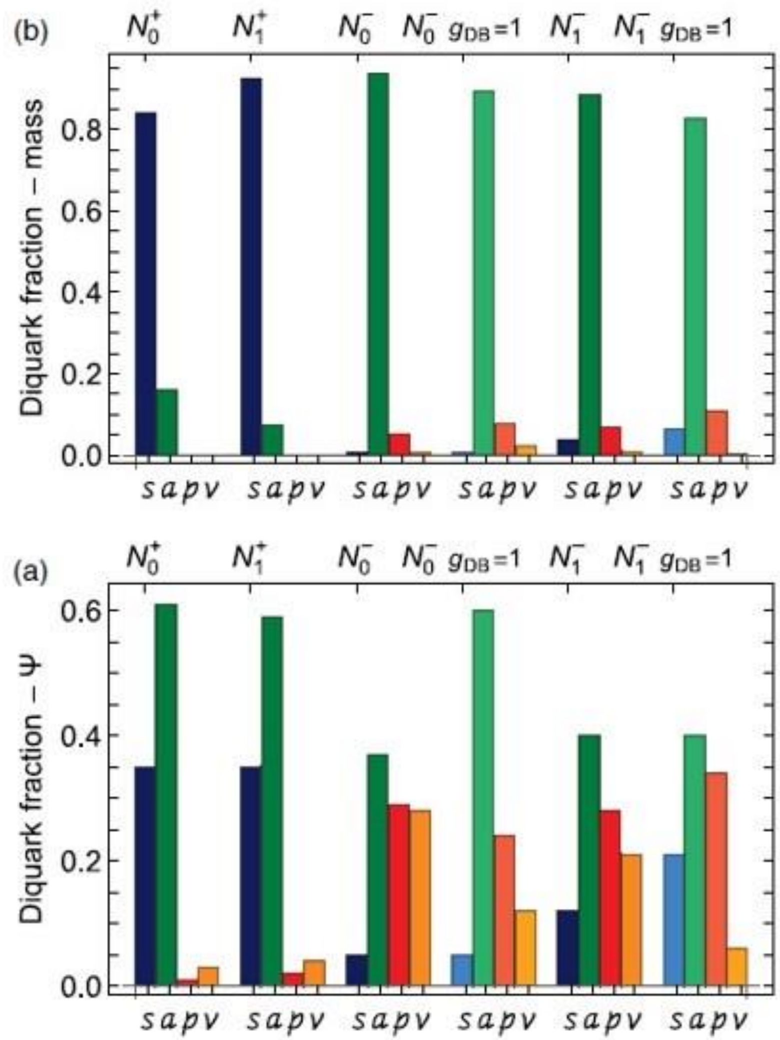
(a) Computed from the relative contributions to the masses.

(b) Computed from the amplitudes directly.

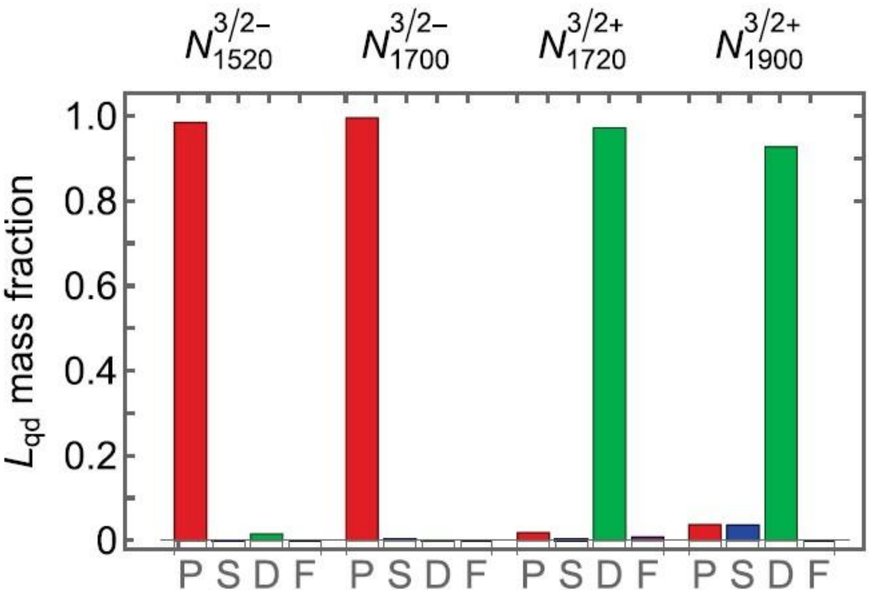
SOLUTIONS & THEIR PROPERTIES:

Diquark content





SOLUTIONS & THEIR PROPERTIES:
Rest-frame orbital angular momentum



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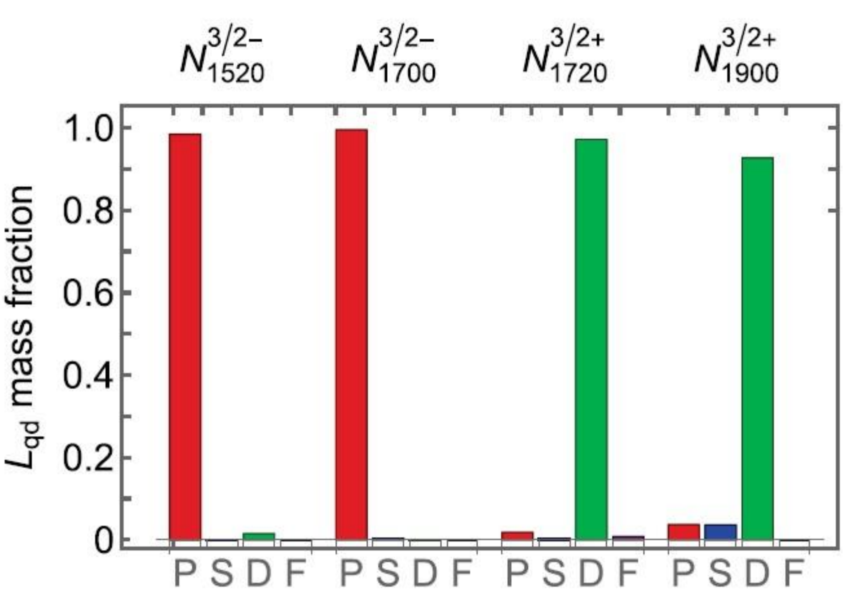
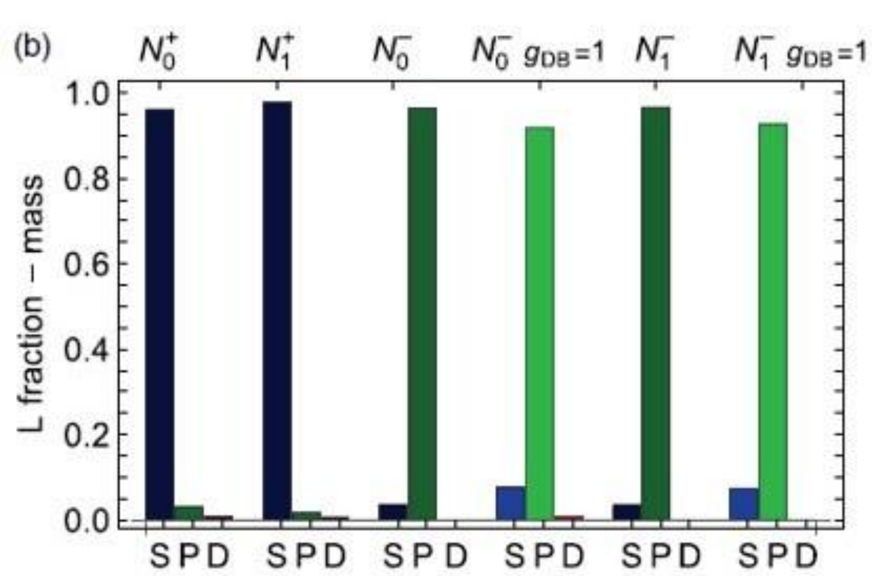
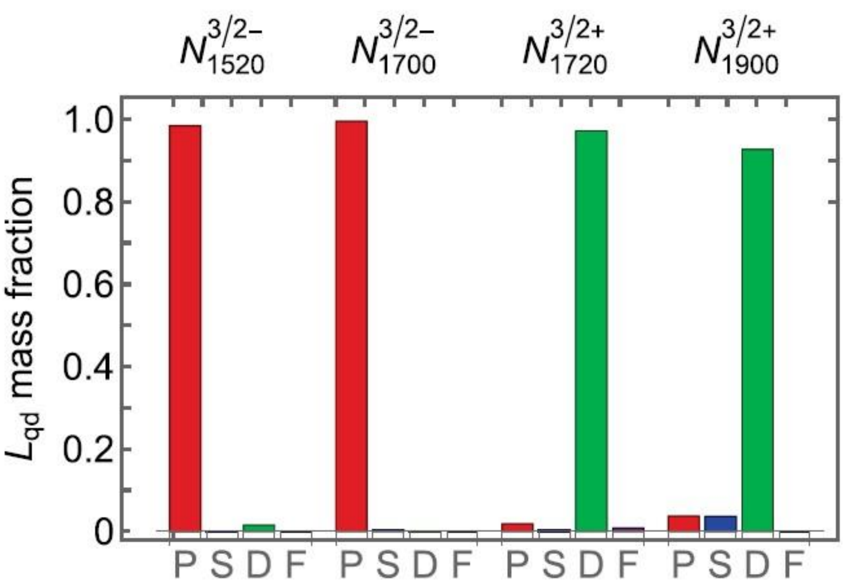
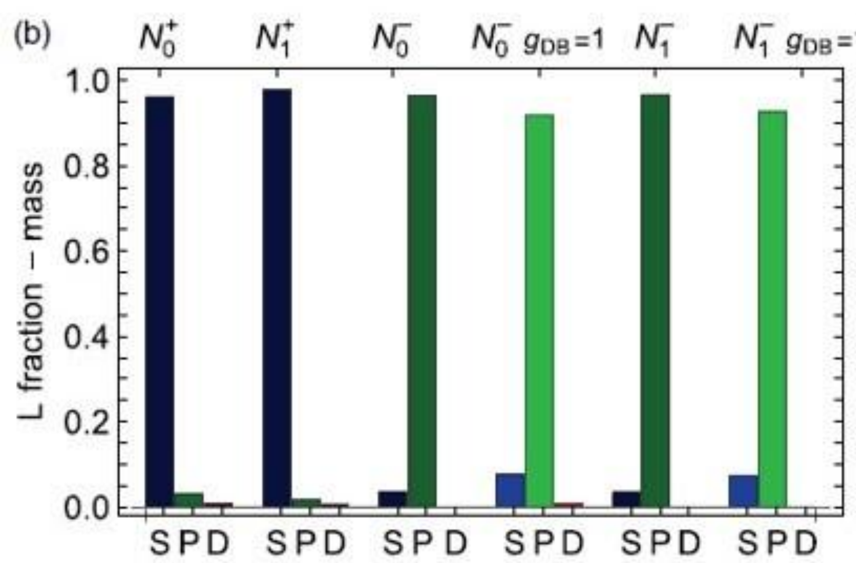
SOLUTIONS & THEIR PROPERTIES:
Rest-frame orbital angular momentum

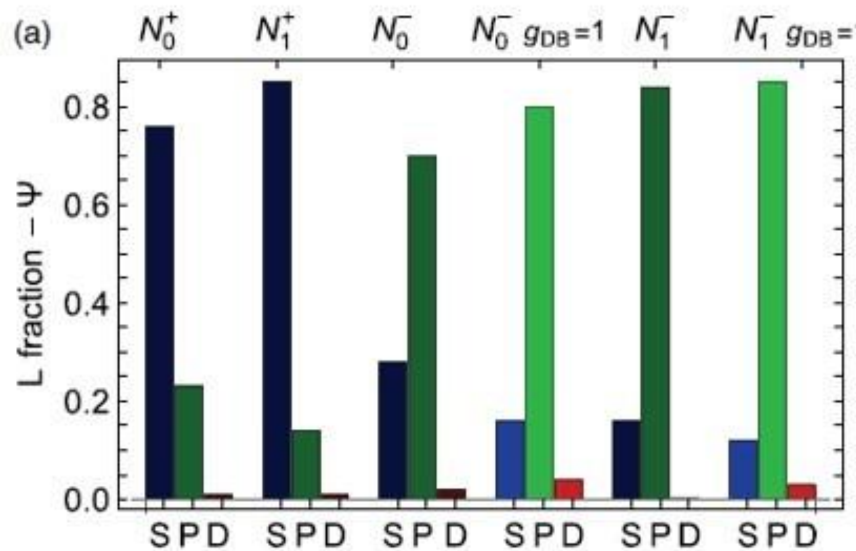




SOLUTIONS & THEIR PROPERTIES:
Rest-frame orbital angular momentum

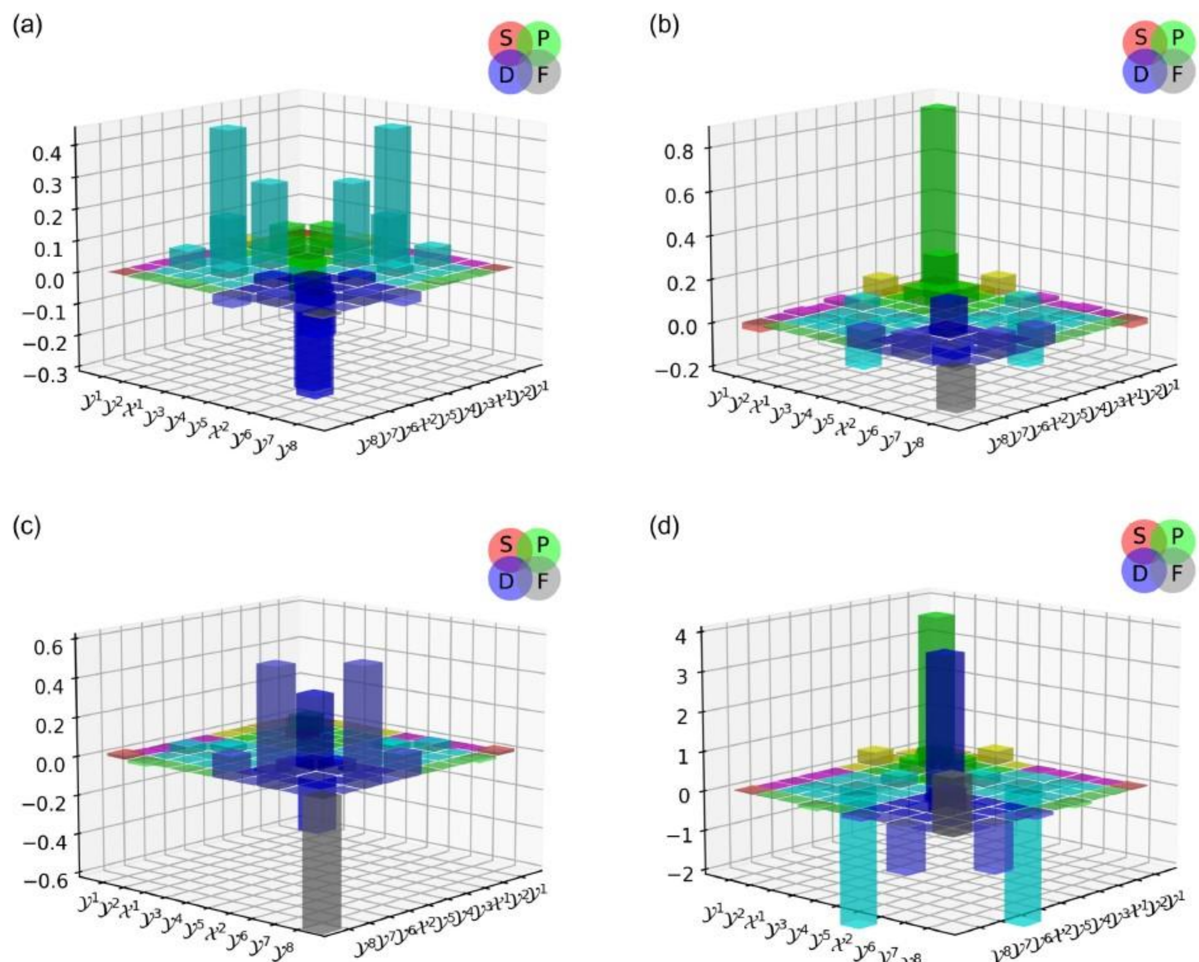






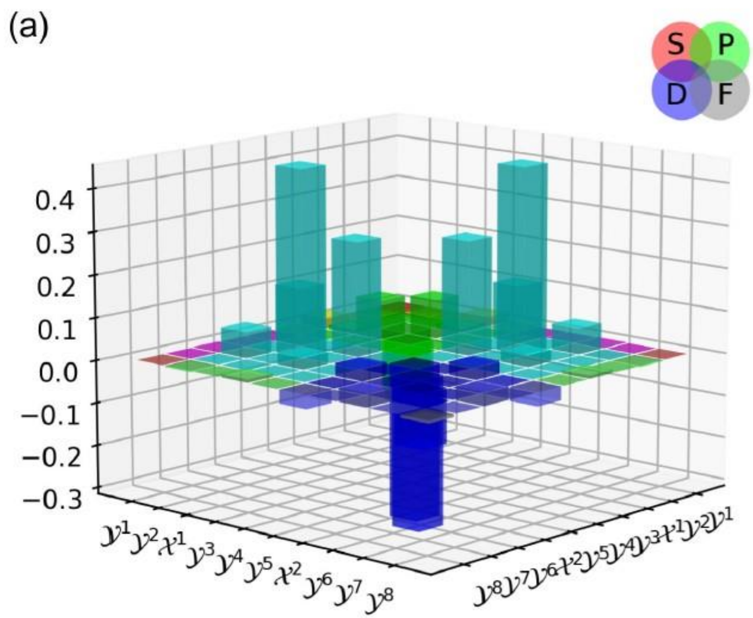
SOLUTIONS & THEIR PROPERTIES:
Canonical normalization - decomposition

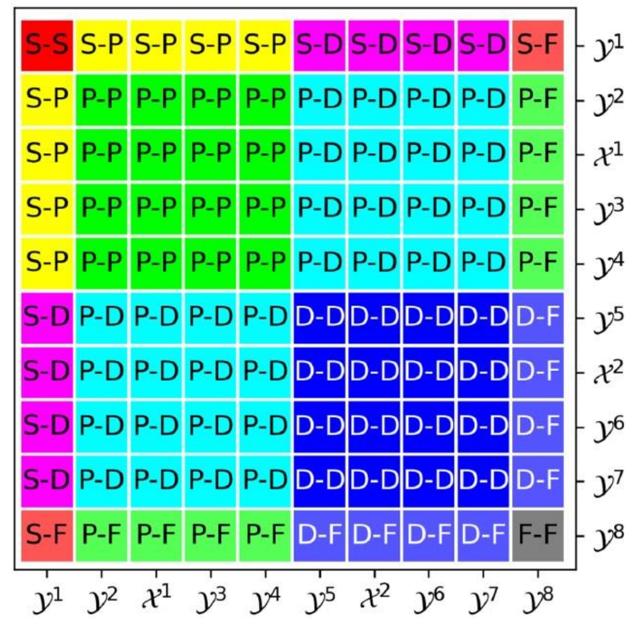
- A baryon's decomposition: a quantity that is related to the zero momentum transfer value of the electric form factor of the valence quarks within the state; hence, observable.



SOLUTIONS & THEIR PROPERTIES:
Canonical normalization - decomposition

- N(1520) 3/2-

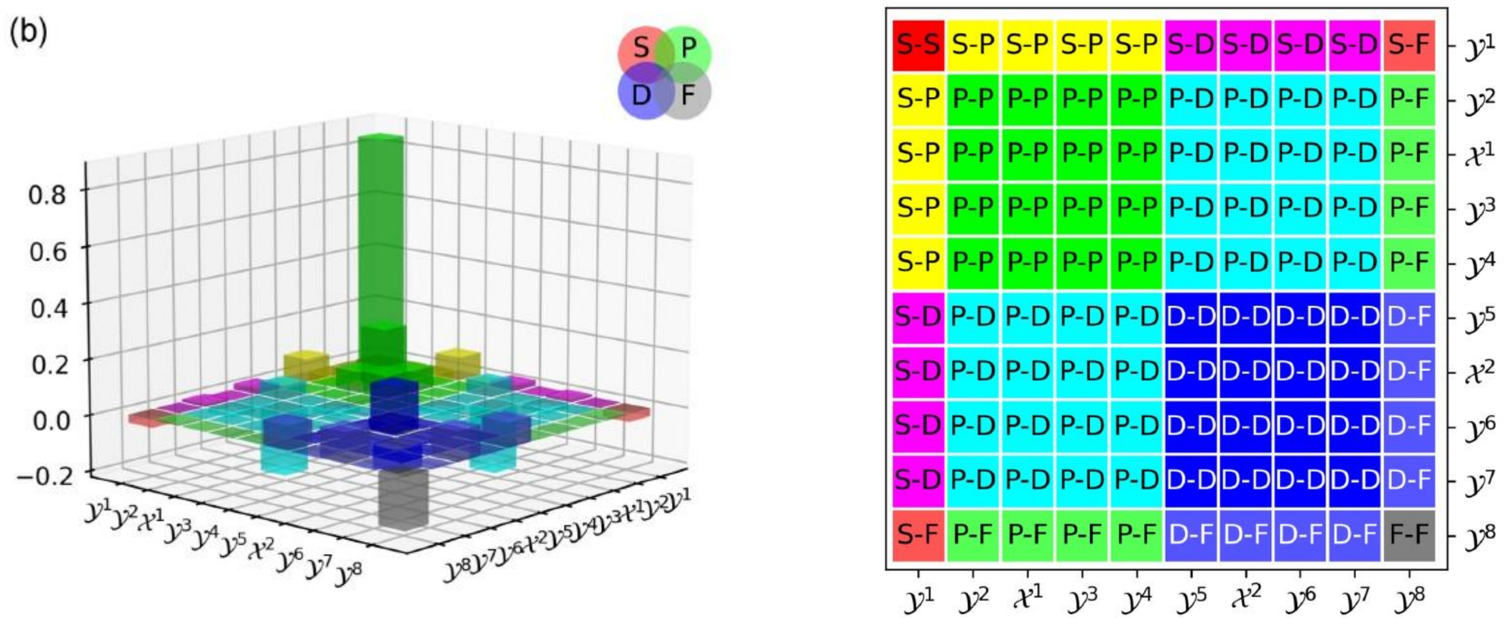




- The most prominent positive contributions are provided by constructive P ⊗ D-wave interference terms; contributions from purely P-wave components are visible, but interfere destructively; and pure D-wave terms are responsible for largely destructive interference.

SOLUTIONS & THEIR PROPERTIES:
Canonical normalization - decomposition

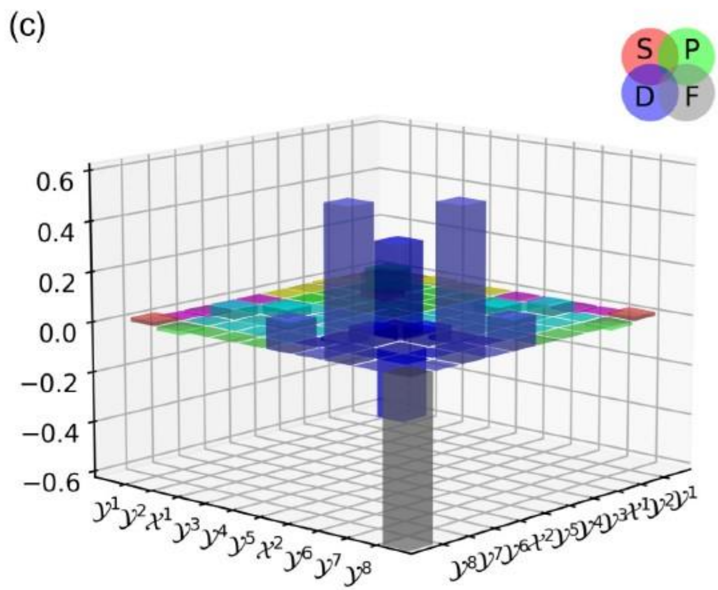
- N(1700) 3/2-

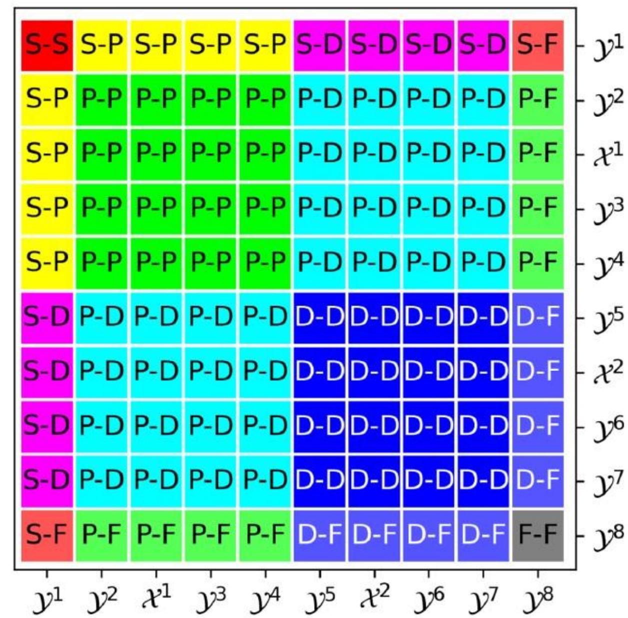


- Pure P-wave contributions to the canonical normalization are dominant; there is some destructive P $\otimes$ D-wave interference; simple D-wave contributions largely cancel among themselves; and D $\otimes$ F-wave constructive interference offsets a destructive F-wave contribution.

SOLUTIONS & THEIR PROPERTIES:
Canonical normalization - decomposition

- N(1720) 3/2+

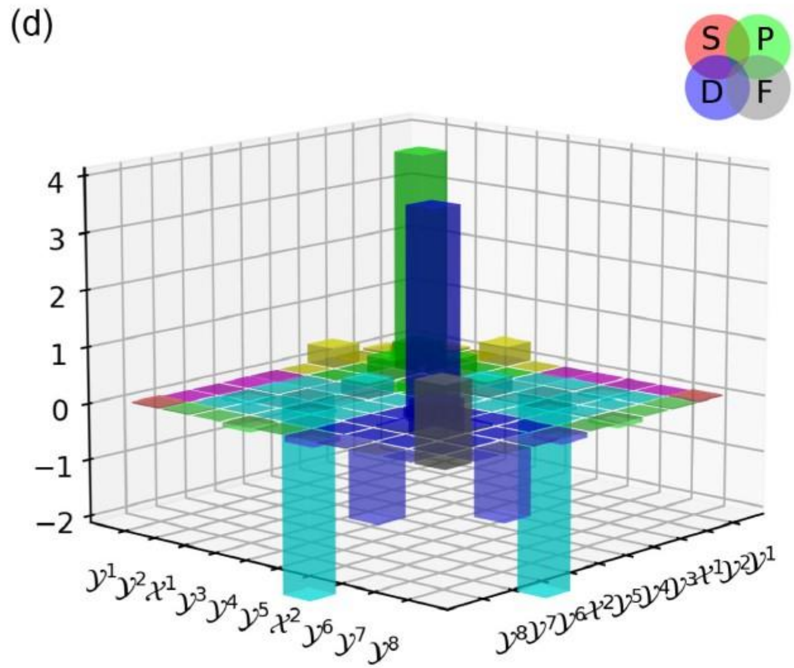


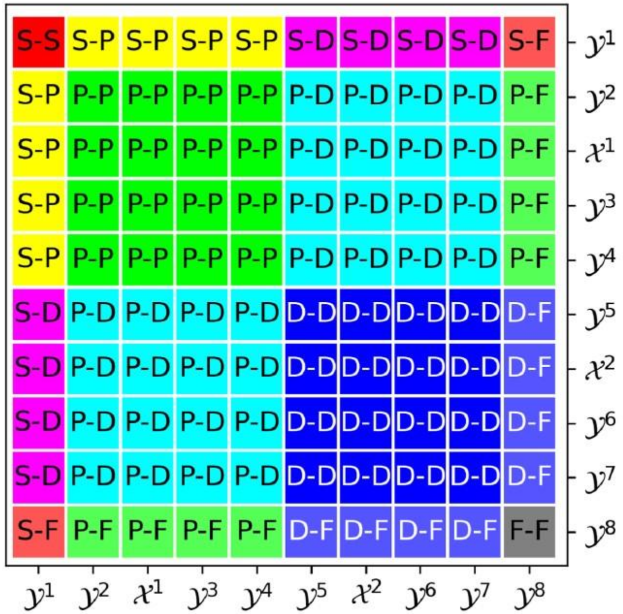


- Normalization contributions related to D-waves are most prominent: the largest positive terms are generated by constructive D $\otimes$ F-wave interference. Pure D-wave contributions largely cancel among themselves; and there is a sizeable destructive F-wave contribution.

SOLUTIONS & THEIR PROPERTIES:
Canonical normalization - decomposition

- N(1900) 3/2+





- Strong pure P- and D-wave contributions. There is also a prominent constructive F-wave contribution; and P $\otimes$ D-wave and D $\otimes$ F-wave interference are strongly destructive.

Pointwise structure

- We consider the zeroth Chebyshev moment of P- and D-wave components in a given baryon's Faddeev wave function.

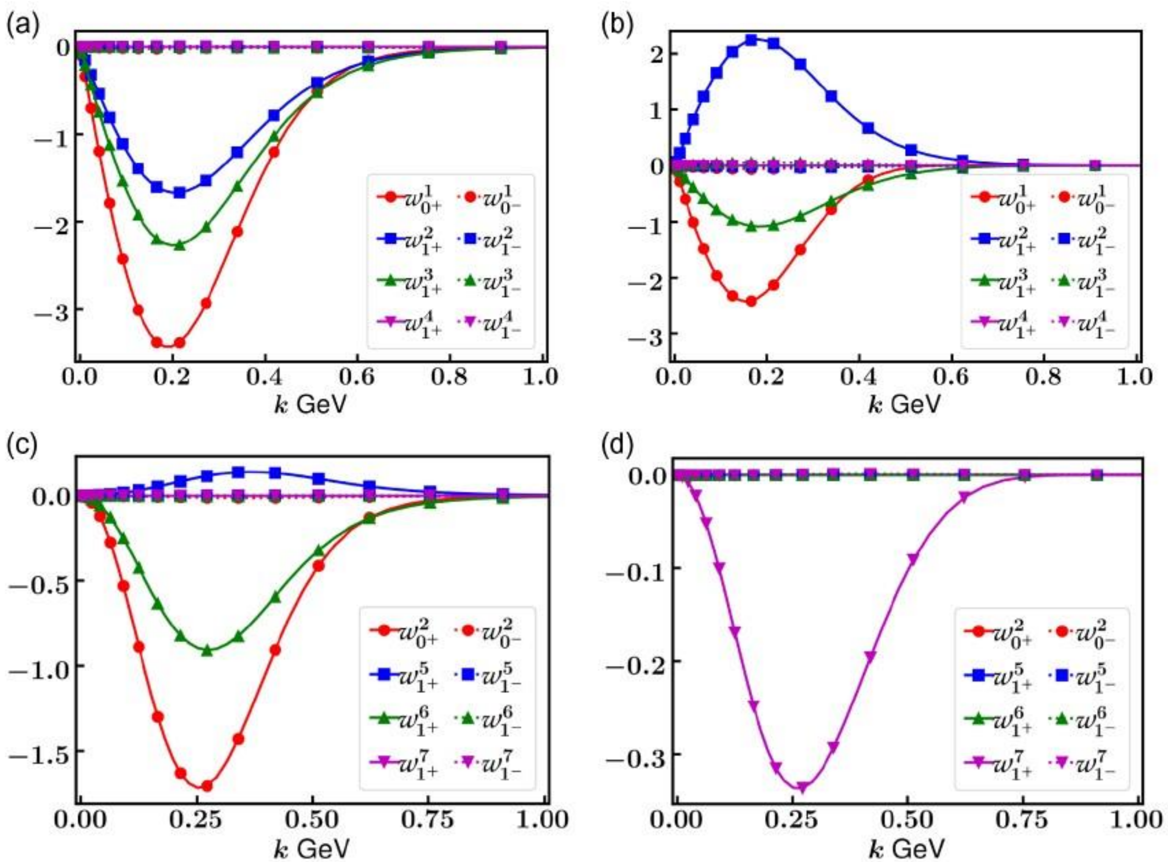
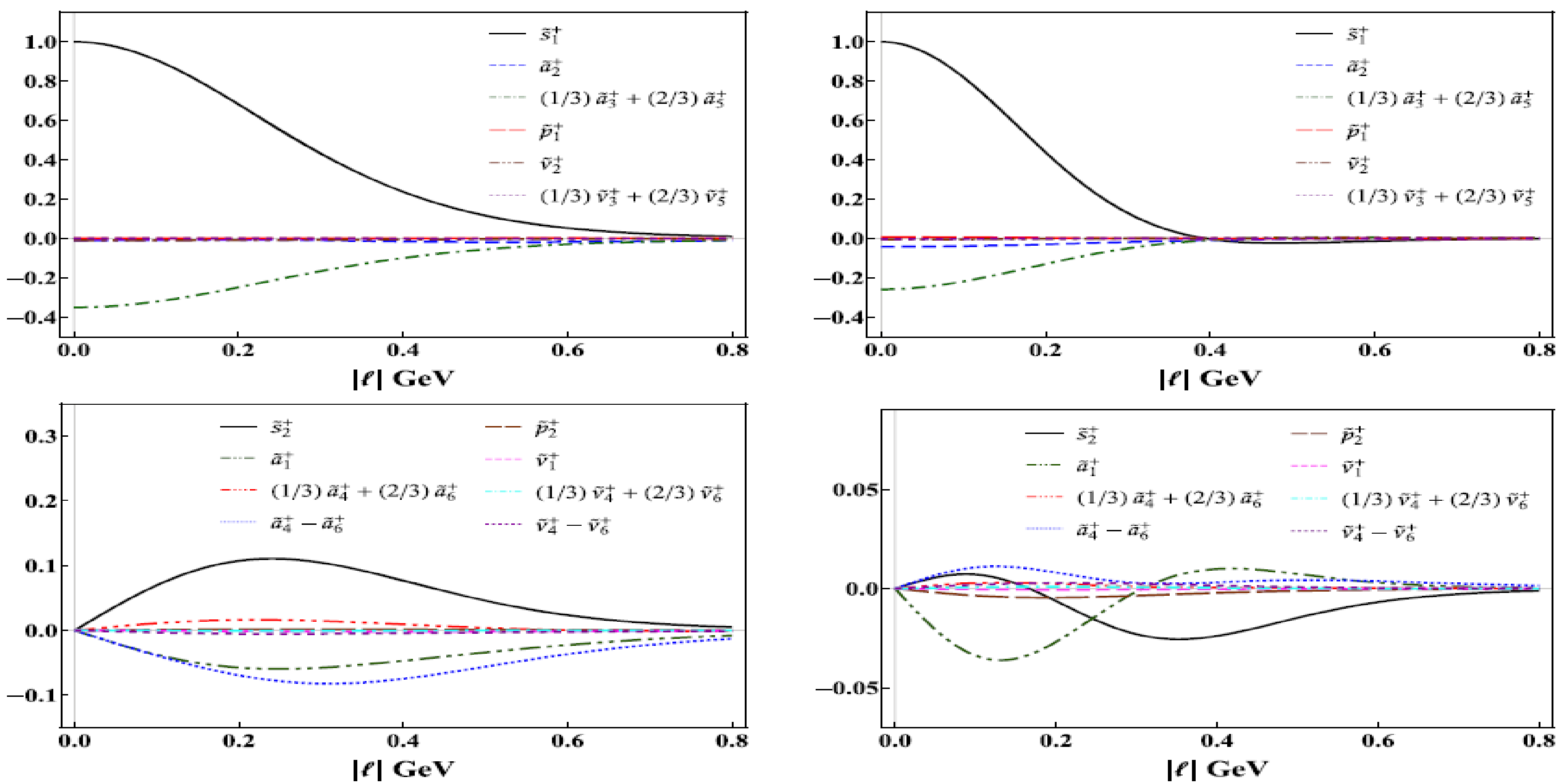


FIG. 5. Zeroth Chebyshev moments—Eq. (20). Upper panels: rest-frame P-wave components in wave functions of the negative parity baryons: (a), $N(1520)\frac{3}{2}^-$; and (b), $N(1700)\frac{3}{2}^-$. Lower panels: rest-frame D-wave components in wave functions of the positive parity baryons: (c), $N(1720)\frac{3}{2}^+$; and (d), $N(1900)\frac{3}{2}^+$.

SOLUTIONS & THEIR PROPERTIES:

Pointwise structure

- We consider the zeroth Chebyshev moment of all S- and P-wave components in a given baryon's Faddeev wave function.
- Nucleon's first positive-parity excitation: all S-wave components exhibit a single zero; and four of the P-wave projections also possess a zero. This pattern of behavior for the first excited state indicates that it may be interpreted as a radial excitation.

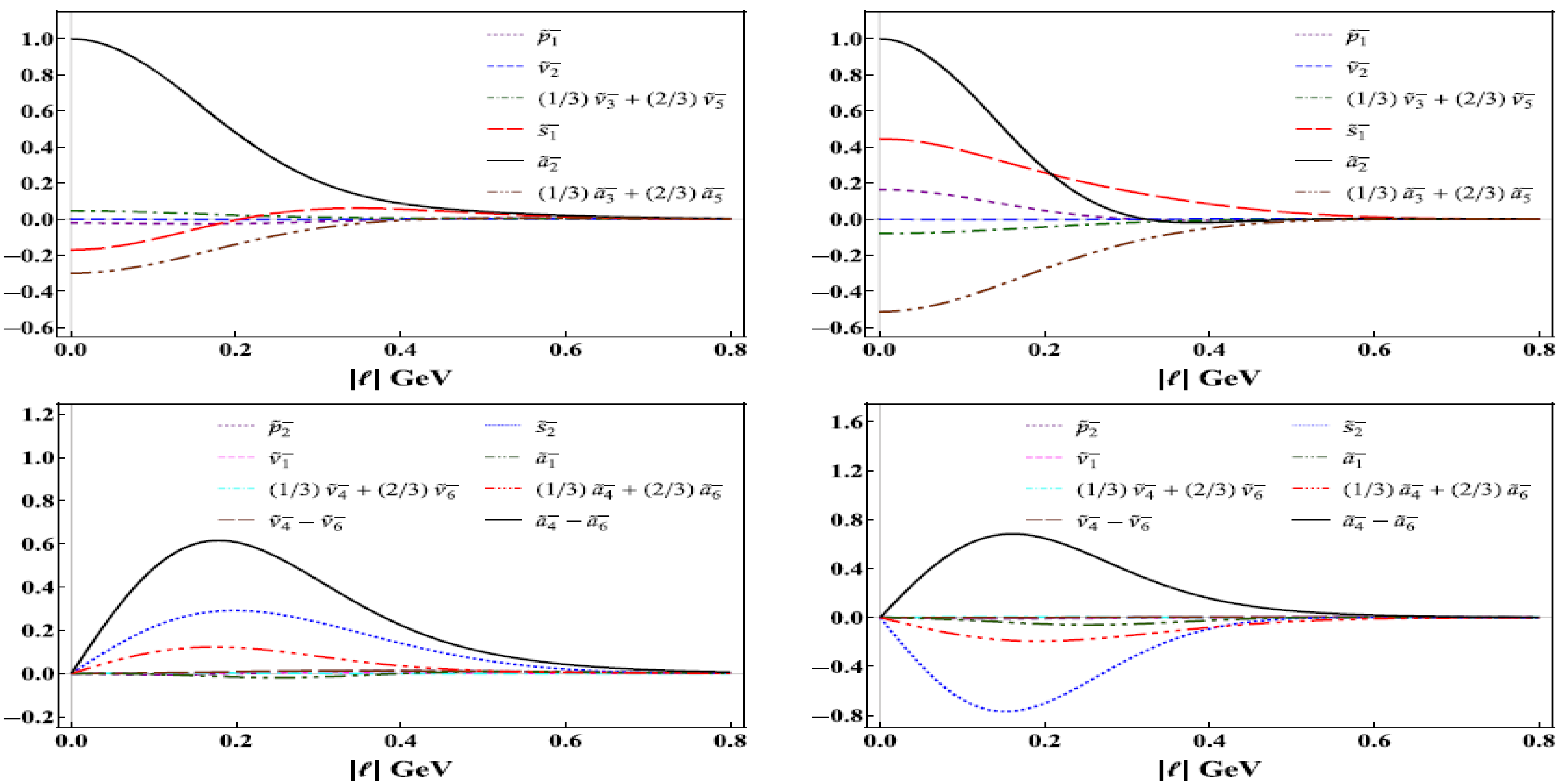


SOLUTIONS & THEIR PROPERTIES:
Pointwise structure

- For N(1535)1/2^-,N(1650)1/2^- : the contrast with the positive-parity states is stark.

  In particular, there is no simple pattern of zeros, with all panels containing at least one function that possesses a zero.

- In their rest frames, these systems are predominantly P-wave in nature, but possess material S-wave components; and the first excited state in this negative parity channel—*N(1650)1/2^−*—has little of the appearance of a radial excitation, since most of the functions depicted in the right panels of the figure do not possess a zero.



Summary & Perspective

- **Continuum Schwinger function methods: a introduction**
- **Results for the nucleon's low-lying excitations**
  - **(I, J^P) = (1/2, 1/2^{+-})**
  - **(I, J^P) = (1/2, 3/2^{+-})**

Summary & Perspective

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- Next: Filling the TABLE

	质量谱		电磁形状因子		轴矢形状因子	
	双夸克	三体	双夸克	三体	双夸克	三体
$N(940)1/2^+$	✓	✓	✓	✓	✓	✓
$\Delta(1232)3/2^+$	✓	✓	✓	✓		
$N(1440)1/2^+$	✓		✓			
$N(1535)1/2^-$	✓					
$N(1520)3/2^-$	✓					
超子（正宇称）	✓	✓		✓		
超子（负宇称）						

Summary & Perspective

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$\Delta(1232)3/2^+$	✓	✓	✓	✓		
$N(1440)1/2^+$	✓		✓			
$N(1535)1/2^-$	✓					
$N(1520)3/2^-$	✓					
超子（正宇称）	✓	✓		✓		
超子（负宇称）						

Thank you!

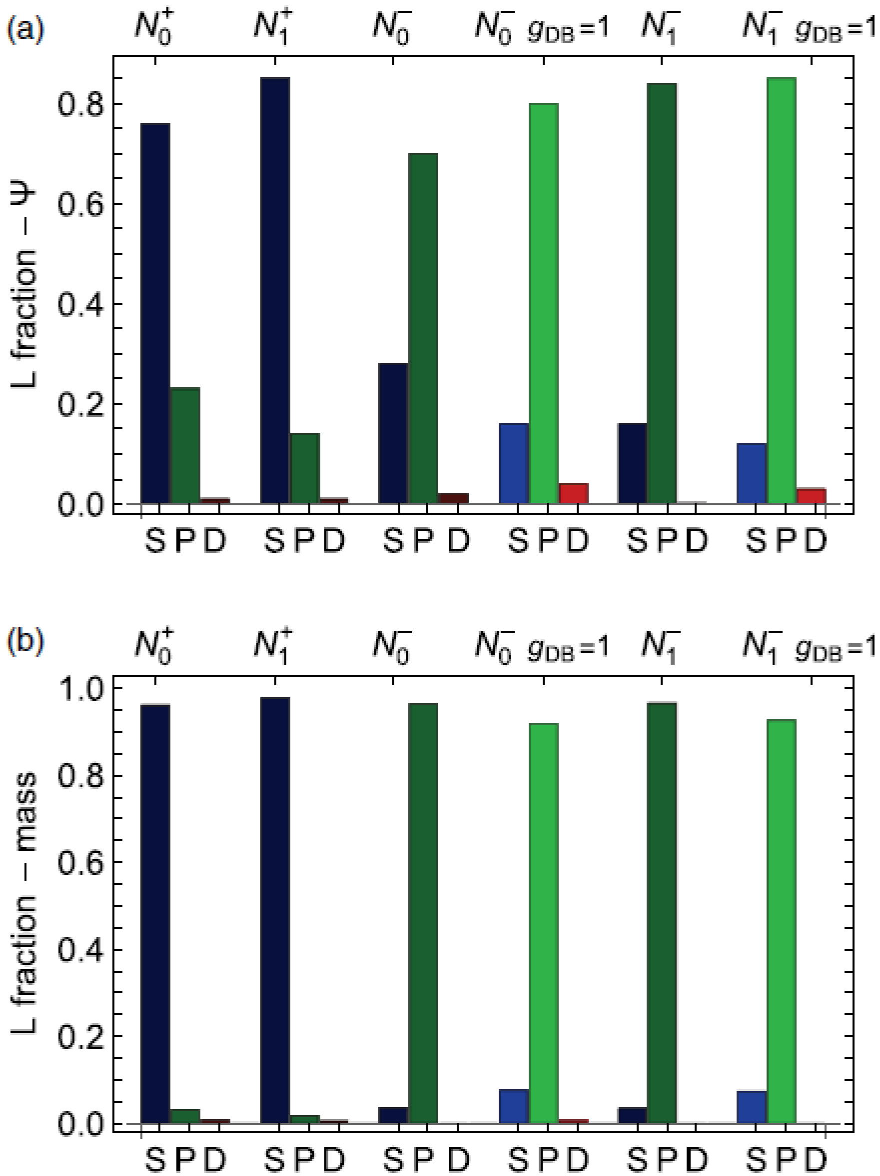
SOLUTIONS & THEIR PROPERTIES:
Masses

- We choose gDB=0.43 so as to produce a mass splitting of 0.1 GeV between the lowest-mass P=- state and the first excited P=+ state, viz. the empirical value.
- Our computed values for the masses of the four lightest 1/2^{+-} baryon doublets are listed here, in GeV:

$g_{\rm DB}$	m_N	$m_{N(1440)}^{1/2^+}$	$m_{N(1535)}^{1/2^-}$	$m_{N(1650)}^{1/2^-}$
0.43	1.19	1.73	1.83	1.91
1.0	1.19	1.73	1.43	1.61

- Pseudoscalar and vector diquarks have no impact on the mass of the two positive-parity baryons, whereas scalar and pseudovector diquarks are important to the negative parity systems.
- Although 1/2^- solutions exist even if one eliminates pseudoscalar and vector diquarks, 1/2^+ solutions do not exist in the absence of scalar and pseudovector diquarks.
- It indicates that, with our Faddeev kernel, the so-called P-wave (negative-parity) baryons can readily be built from positive-parity diquarks.

SOLUTIONS & THEIR PROPERTIES:
Rest-frame orbital angular momentum



- (a) Computed from the wave functions directly.

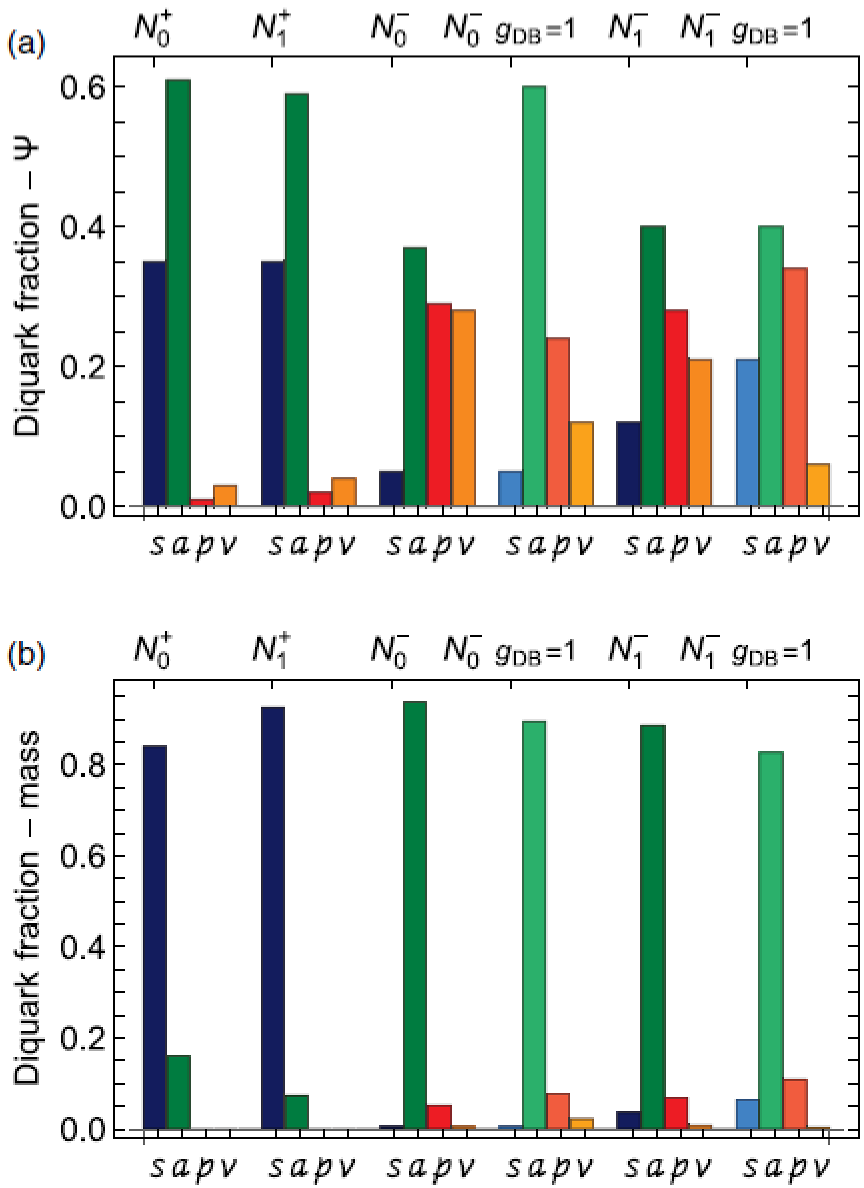
  (b) Computed from the relative contributions to the masses.

- The nucleon and Roper are primarily **S-wave** in nature, since they are not supported by the Faddeev equation unless **S-wave** components are contained in the wave function. On the other hand, the N(1535)1/2^- ,N(1650)1/2^- are essentially **P-wave** in character.

- (b) delivers the same qualitative picture of each baryon's internal structure as that presented in (a).

SOLUTIONS & THEIR PROPERTIES:

Diquark content



- (a) Computed from the amplitudes directly.

  (b) Computed from the relative contributions to the masses.
- From (a): although gDB < 1 has little impact on the nucleon and Roper, it has a significant effect on the structure of the negative parity baryons, serving to enhance the net negative-parity diquark content. The amplitudes associated with these negative-parity states contain roughly ***equal fractions*** of even and odd parity diquarks.
- From (b): In each case depicted in the lower panel, there is a single dominant diquark component. There are **significant interferences** between different diquarks.

QCD-kindred model

- **The dressed-quark propagator**

$$S(p) = -i\gamma \cdot p\sigma_V(p^2) + \sigma_S(p^2)$$

- **algebraic form:**

$$\bar{\sigma}_S(x) = 2\bar{m}\mathcal{F}(2(x+\bar{m}^2)) + \mathcal{F}(b_1 x)\mathcal{F}(b_3 x)[b_0 + b_2\mathcal{F}(\epsilon x)], \tag{A3a}$$

$$\bar{\sigma}_V(x) = \frac{1}{x+\bar{m}^2}[1-\mathcal{F}(2(x+\bar{m}^2))], \tag{A3b}$$

with $x = p^2/\lambda^2$, $\bar{m} = m/\lambda$,

$$\mathcal{F}(x) = \frac{1-e^{-x}}{x}, \tag{A4}$$

$\bar{\sigma}_S(x) = \lambda\sigma_S(p^2)$ and $\bar{\sigma}_V(x) = \lambda^2\sigma_V(p^2)$. The mass scale, $\lambda = 0.566$ GeV, and parameter values,

$\bar{m}$	b_0	b_1	b_2	b_3
0.00897	0.131	2.90	0.603	0.185

(A5)

associated with Eq. (A3) were fixed in a least-squares fit to light-meson observables [79,80]. [$\epsilon = 10^{-4}$ in Eq. (A3a) acts only to decouple the large- and intermediate-p^2 domains.]

QCD-kindred model

The dressed-quark propagator

$$S(p) = -i\gamma \cdot p\sigma_V(p^2) + \sigma_S(p^2)$$

- Based on solutions to the gap equation that were obtained with a dressed gluon-quark vertex.
- Mass function has a real-world value at p^2 = 0, NOT the highly inflated value typical of ~~RL~~ truncation.
- Propagators are entire functions, consistent with sufficient condition for confinement and completely unlike known results from ~~RL~~ truncation.
- Parameters in quark propagators were fitted to a diverse array of meson observables. **ZERO** parameters changed in study of baryons.
- Compare with that computed using the DCSB-improved gap equation kernel (DB). The parametrization is a sound representation numerical results, although simple and introdu long beforehand.

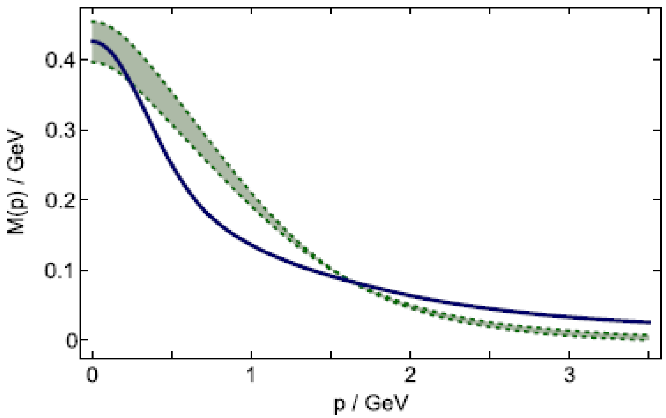


FIG. 6. Solid curve (blue)—quark mass function generated by the parametrization of the dressed-quark propagator specified by Eqs. (A3) and (A4) (A5); and band (green)—exemplary range of numerical results obtained by solving the gap equation with the modern DCSB-improved kernels described and used in Refs. [16,81–83].

QCD-kindred model

- **Diquark amplitudes: five types of correlation are possible in a *J=1/2* bound state: isoscalar scalar(I=0,J^P=0^+), isovector pseudovector, isoscalar pseudoscalar, isoscalar vector, and isovector vector.**
- **The LEADING structures in the correlation amplitudes for each case are, respectively (Dirac-flavor-color),**

$$\Gamma^{0^+}(k;K) = g_{0^+}\gamma_5 C\tau^2 \vec{H}\mathcal{F}(k^2/\omega_{0^+}^2),$$

$$\vec{\Gamma}_\mu^{1^+}(k;K) = ig_{1^+}\gamma_\mu C\vec{\tau}\vec{H}\,\mathcal{F}(k^2/\omega_{1^+}^2),$$

$$\Gamma^{0^-}(k;K) = ig_{0^-}C\tau^2\vec{H}\mathcal{F}(k^2/\omega_{0^-}^2),$$

$$\Gamma_\mu^{1^-}(k;K) = g_{1^-}\gamma_\mu\gamma_5 C\tau^2\vec{H}\mathcal{F}(k^2/\omega_{1^-}^2),$$

$$\vec{\Gamma}_\mu^{\bar{1}^-}(k;K) = ig_{\bar{1}^-}[\gamma_\mu,\gamma\cdot K]\gamma_5 C\vec{\tau}\vec{H}\,\mathcal{F}(k^2/\omega_{\bar{1}^-}^2),$$

- **Simple form. Just one parameter: diquark masses.**
- **Match expectations based on solutions of meson and diquark Bethe-Salpeter amplitudes.**

QCD-kindred model

- **The diquark propagators**

$$\Delta^{0^\pm}(K) = \frac{1}{m_{0^\pm}^2}\mathcal{F}(k^2/\omega_{0^\pm}^2),$$

$$\Delta_{\mu\nu}^{1^\pm}(K) = \left[\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1^\pm}^2}\right]\frac{1}{m_{1^\pm}^2}\mathcal{F}(k^2/\omega_{1^\pm}^2).$$

- **The *$\mathcal{F}$-functions*: Simplest possible form that is consistent with infrared and ultraviolet constraints of confinement (IR) and 1/q^2 evolution (UV) of meson propagators.**

- **Diquarks are confined.**
  - **free-particle-like at spacelike momenta**
  - **pole-free on the timelike axis**
  - **This is NOT true of ~~RL~~ studies. It enables us to reach arbitrarily high values of momentum transfer.**

QCD-kindred model

- **The Faddeev amplitudes:**

$$\psi^{\pm}(p_i,\alpha_i,\sigma_i) = [\Gamma^{0^+}(k;K)]^{\alpha_1\alpha_2}_{\sigma_1\sigma_2}\Delta^{0^+}(K)\boxed{[\varphi^{\pm}_{0^+}(\ell;P}u(P)]^{\alpha_3}_{\sigma_3}$$
$$+ [\Gamma^{1^+j}_{\mu}]\Delta^{1^+}_{\mu\nu}[\varphi^{j\pm}_{1^+\nu}(\ell;P)u(P)]$$
$$+ [\Gamma^{0^-}]\Delta^{0^-}[\varphi^{\pm}_{0^-}(\ell;P)u(P)]$$
$$+ [\Gamma^{1^-}_{\mu}]\Delta^{1^-}_{\mu\nu}[\varphi^{\pm}_{1^-\nu}(\ell;P)u(P)], \qquad (9)$$

- **Quark-diquark vertices:**

$$\varphi^{\pm}_{0^+}(\ell;P) = \sum_{i=1}^{2} \mathscr{s}^{\pm}_i(\ell^2,\ell\cdot P)\mathcal{S}^i(\ell;P)\mathcal{G}^{\pm},$$

$$\varphi^{j\pm}_{1^+\nu}(\ell;P) = \sum_{i=1}^{6} \mathscr{a}^{j\pm}_i(\ell^2,\ell\cdot P)\gamma_5\mathcal{A}^i_\nu(\ell;P)\mathcal{G}^{\pm},$$

$$\varphi^{\pm}_{0^-}(\ell;P) = \sum_{i=1}^{2} \mathscr{p}^{\pm}_i(\ell^2,\ell\cdot P)\mathcal{S}^i(\ell;P)\mathcal{G}^{\mp},$$

$$\varphi^{\pm}_{1^-\nu}(\ell;P) = \sum_{i=1}^{6} \mathscr{v}^{\pm}_i(\ell^2,\ell\cdot P)\gamma_5\mathcal{A}^i_\nu(\ell;P)\mathcal{G}^{\mp},$$

where $\mathcal{G}^{+(-)} = \mathbf{I}_{\rm D}(\gamma_5)$ and

$$\mathcal{S}^1 = \mathbf{I}_{\rm D}, \qquad \mathcal{S}^2 = i\gamma\cdot\hat{\ell} - \hat{\ell}\cdot\hat{P}\mathbf{I}_{\rm D}$$
$$\mathcal{A}^1_\nu = \gamma\cdot\ell^\perp\hat{P}_\nu, \qquad \mathcal{A}^2_\nu = -i\hat{P}_\nu\mathbf{I}_{\rm D}, \qquad \mathcal{A}^3_\nu = \gamma\cdot\hat{\ell}^\perp\hat{\ell}^\perp_\nu$$
$$\mathcal{A}^4_\nu = i\hat{\ell}^\perp_\nu\mathbf{I}_{\rm D}, \quad \mathcal{A}^5_\nu = \gamma^\perp_\nu - \mathcal{A}^3_\nu, \quad \mathcal{A}^6_\nu = i\gamma^\perp_\nu\gamma\cdot\hat{\ell}^\perp - \mathcal{A}^4_\nu,$$

QCD-kindred model

- **Both the Faddeev amplitude and wave function are Poincare covariant, i.e. they are qualitatively identical in all reference frames.**
- **Each of the scalar functions that appears is frame independent, but the frame chosen determines just how the elements should be combined.**
- **In consequence, the manner by which the dressed quarks' spin, *S*, and orbital angular momentum, *L*, add to form the total momentum *J,* is frame dependent: *L*, *S* are not independently Poincare invariant.**
- **The set of baryon rest-frame quark-diquark angular momentum identifications:**

$$^2S:\ \mathcal{S}^1, \mathcal{A}_\nu^2, (\mathcal{A}_\nu^3 + \mathcal{A}_\nu^5),$$

$$^2P:\ \mathcal{S}^2, \mathcal{A}_\nu^1, (\mathcal{A}_\nu^4 + \mathcal{A}_\nu^6),$$

$$^4P:\ (2\mathcal{A}_\nu^4 - \mathcal{A}_\nu^6)/3,$$

$$^4D:\ (2\mathcal{A}_\nu^3 - \mathcal{A}_\nu^5)/3,$$

- **The scalar functions associated with these combinations of Dirac matrices in a Faddeev wave function possess the identified angular momentum correlation between the quark and diquark.**