Revealing Mysteries of QCD

via Hadron Spectroscopy

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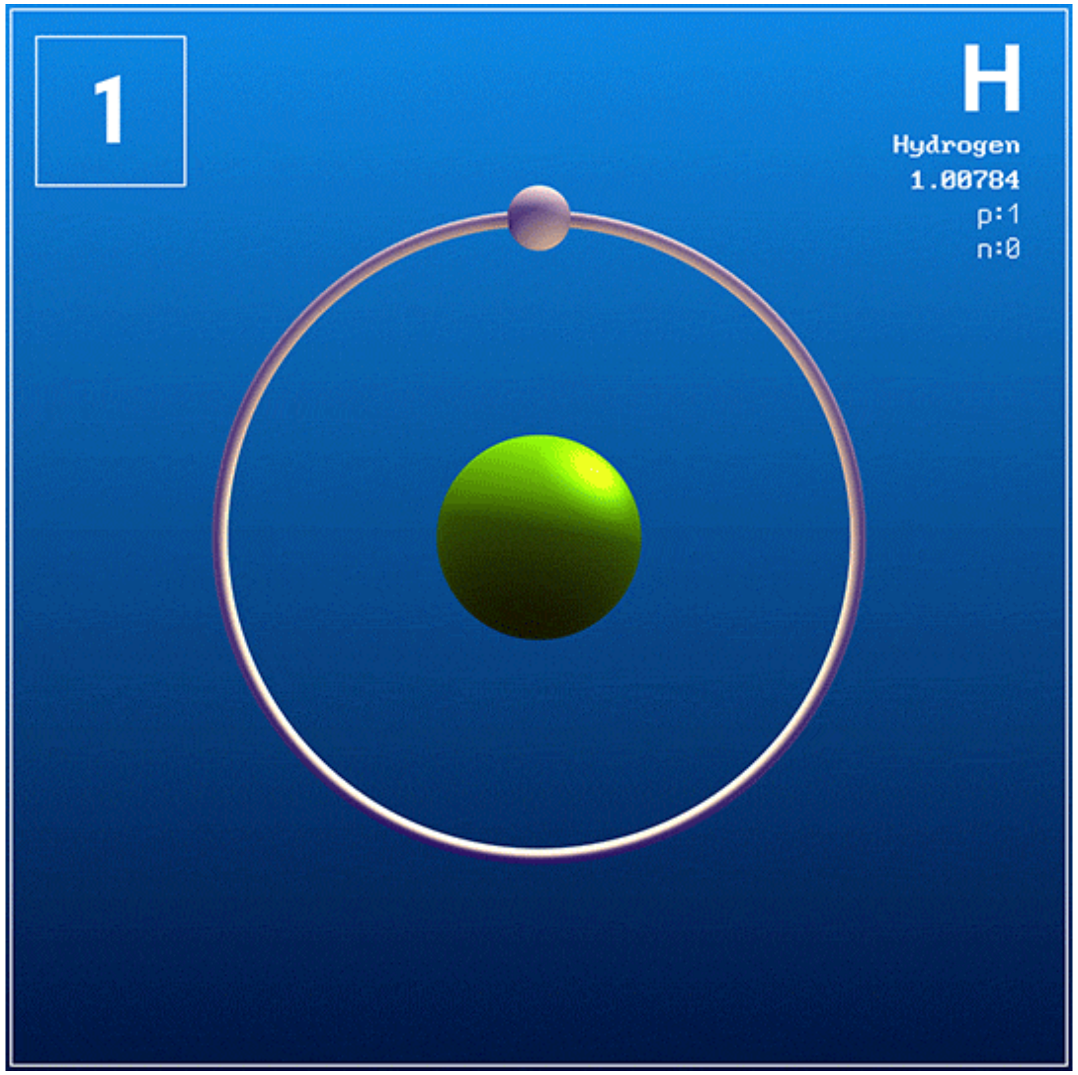
History: Hydrogen atom — The simplest element



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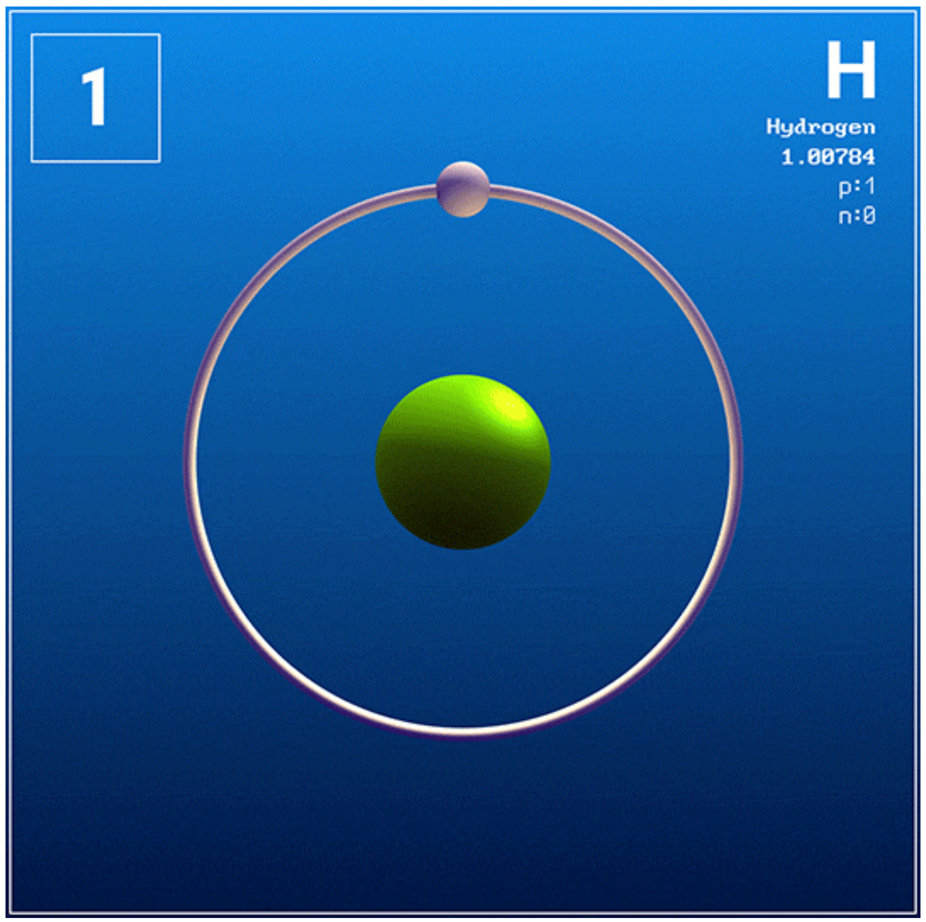

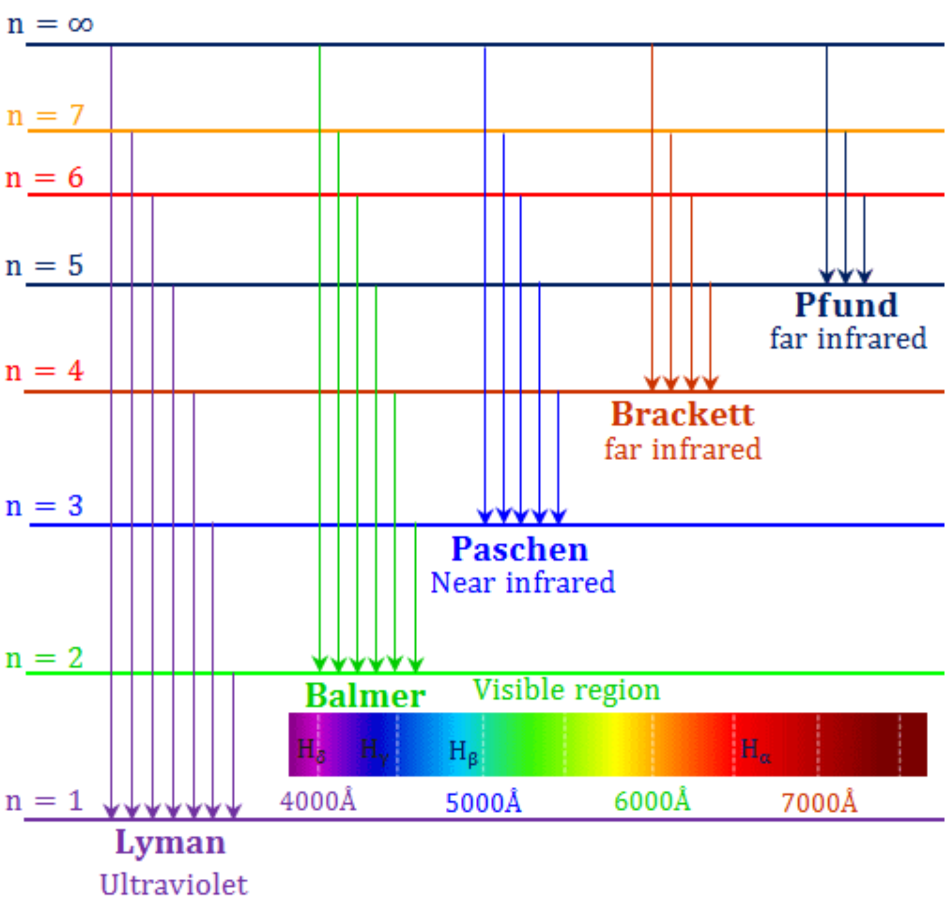

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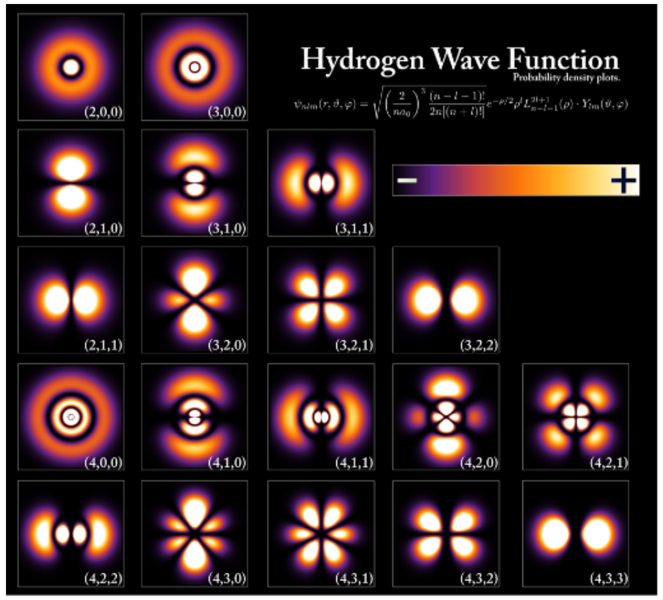

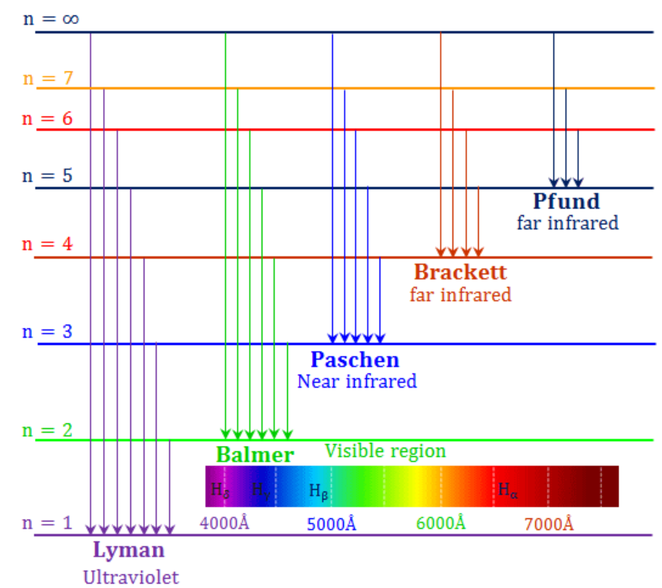

$$H\,|\psi_n\rangle = E_n\,|\psi_n\rangle$$

$$H = H_{\text{kinetic}} + H_{\text{Coulomb}}$$

History: Hydrogen atom — The simplest element

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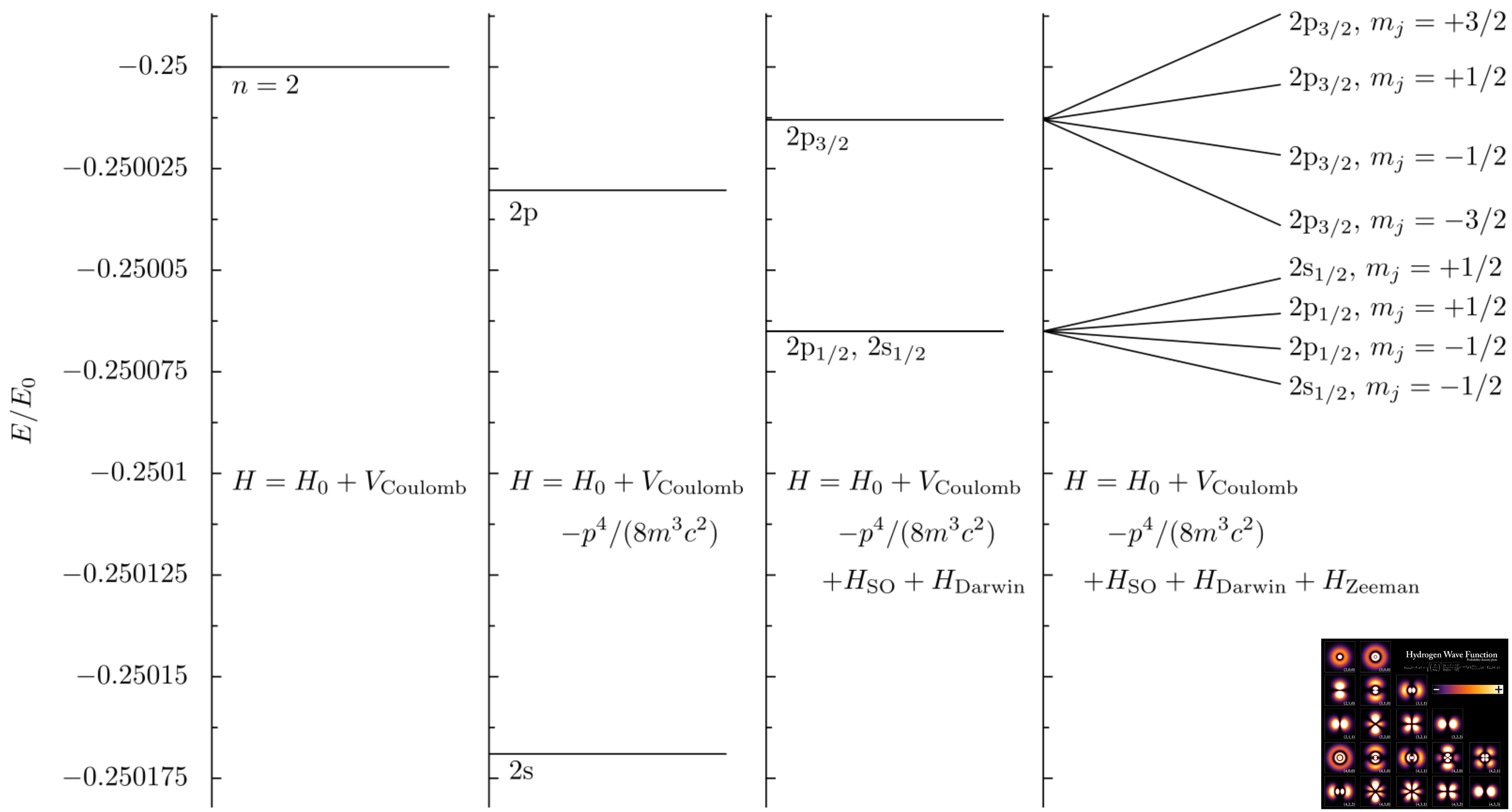
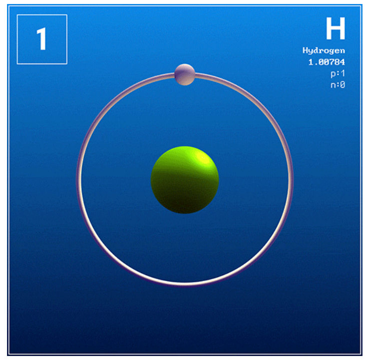
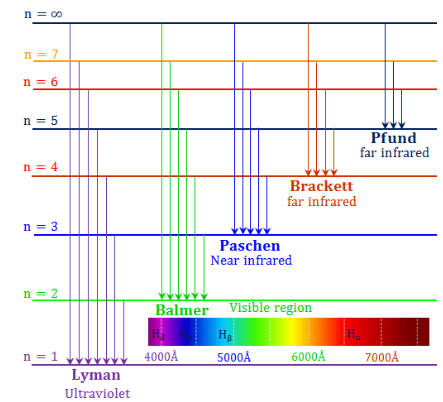
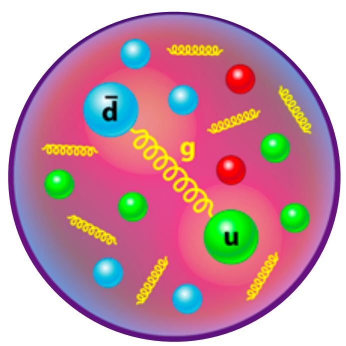


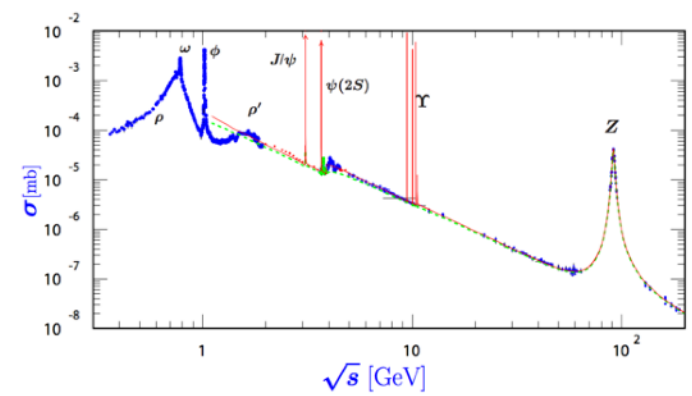
$$H = H_{\text{kinetic}} + H_{\text{Coulomb}} + H_{\text{spin-orbit}} + H_{\text{relativistic}} + H_{\text{QED}}$$

Background: Hadron — The QCD “atom”









✦ **Theories**: Simple (few-body) objects could involve surprisingly rich physics.

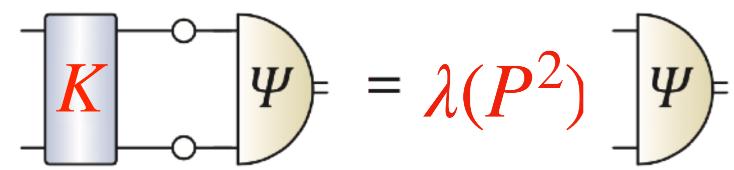
✦ **Experiments**: High-precision measurements could make the story very different.

Background: Hadron — The QCD “atom”

Quantum Mechanics → **Quantum Field Theory**

Schroedinger equation → Bound-state equation

$$H\,|\,\psi\rangle = E\,|\,\psi\rangle$$



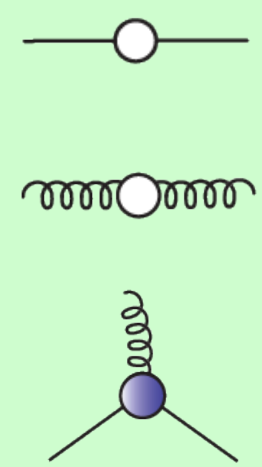
H_{kinetic}

H_{Coulomb}

$H_{\mathrm{spin-orbit}}$

$H_{\mathrm{relativistic}}$

H_{QED}



Quark

Gluon

Vertex

Architecture

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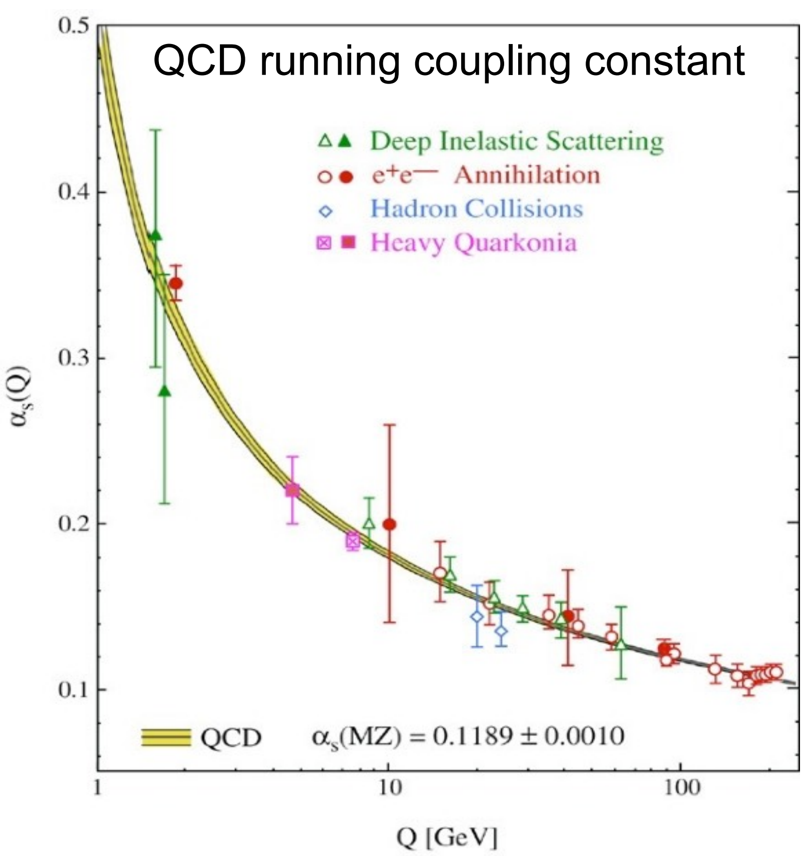
• Relativistic bound states

"These problems are those involving bound states [...] such problems necessarily involve a breakdown of ordinary perturbation theory. [...] The pole therefore can only arise from a divergence of the sum of all diagrams [...]"

The QFT book vol1 p564 Weinberg

• Strongly coupled systems



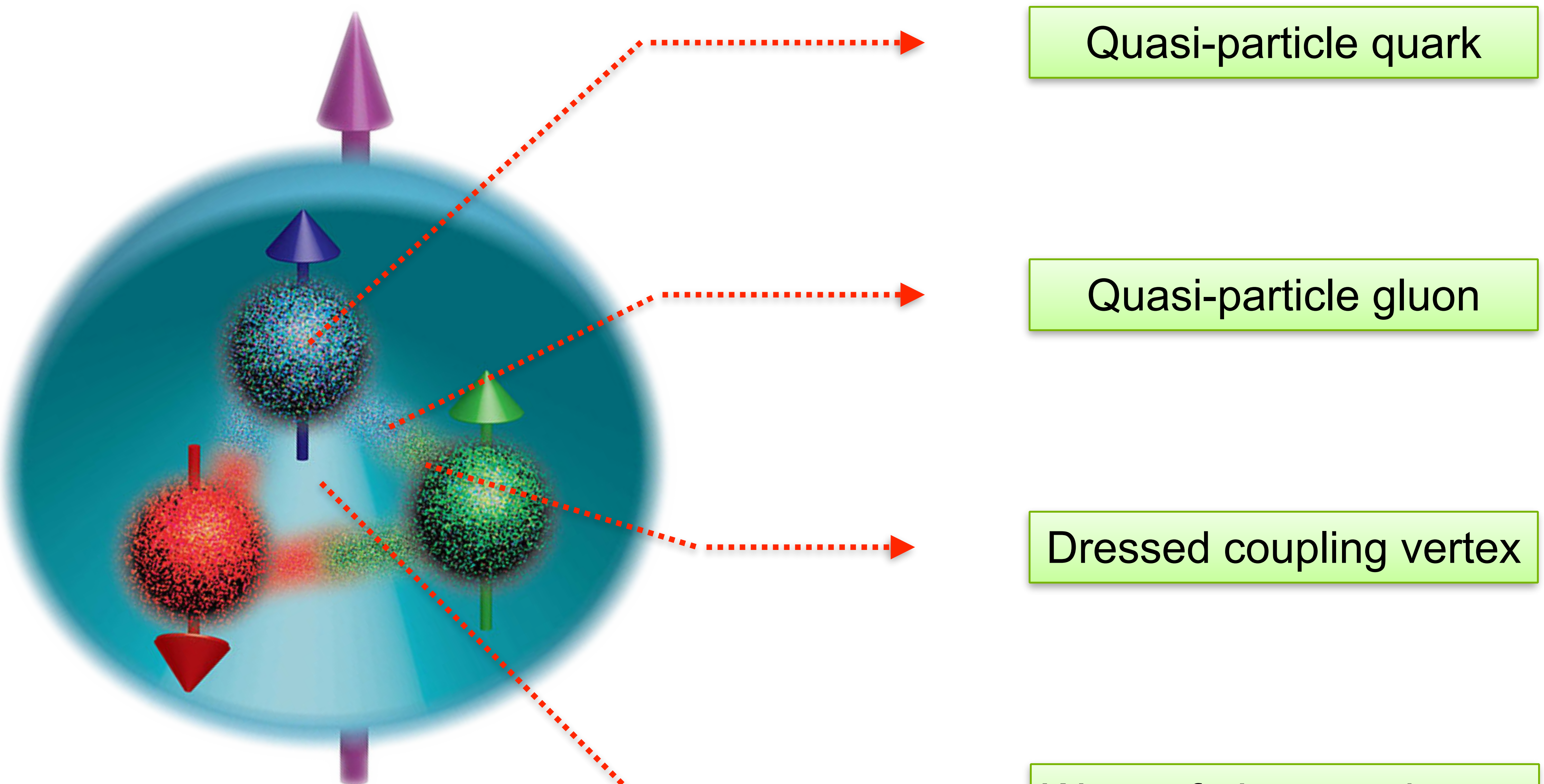
- **Color Confinement**: No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon (Millennium Problems).

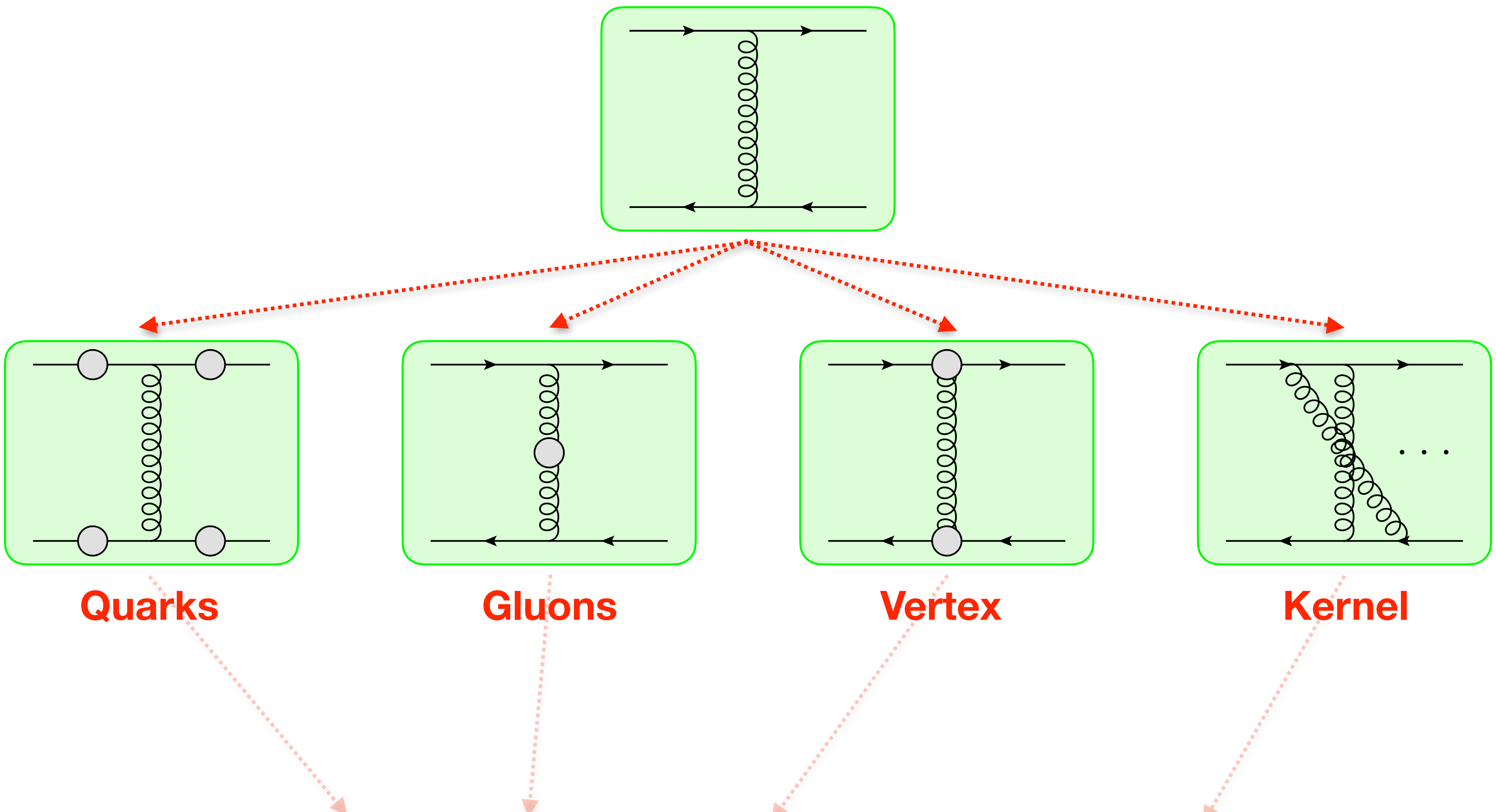
- **Dynamical Chiral Symmetry Breaking**: Mystery of bound state masses, e.g., current quark mass (Higgs) is small, and no degeneracy between *parity partners.*

Chapter I: Theory

Physics of quark, gluon, vertex, and kernel

Continuum QCD: Interaction between quarks



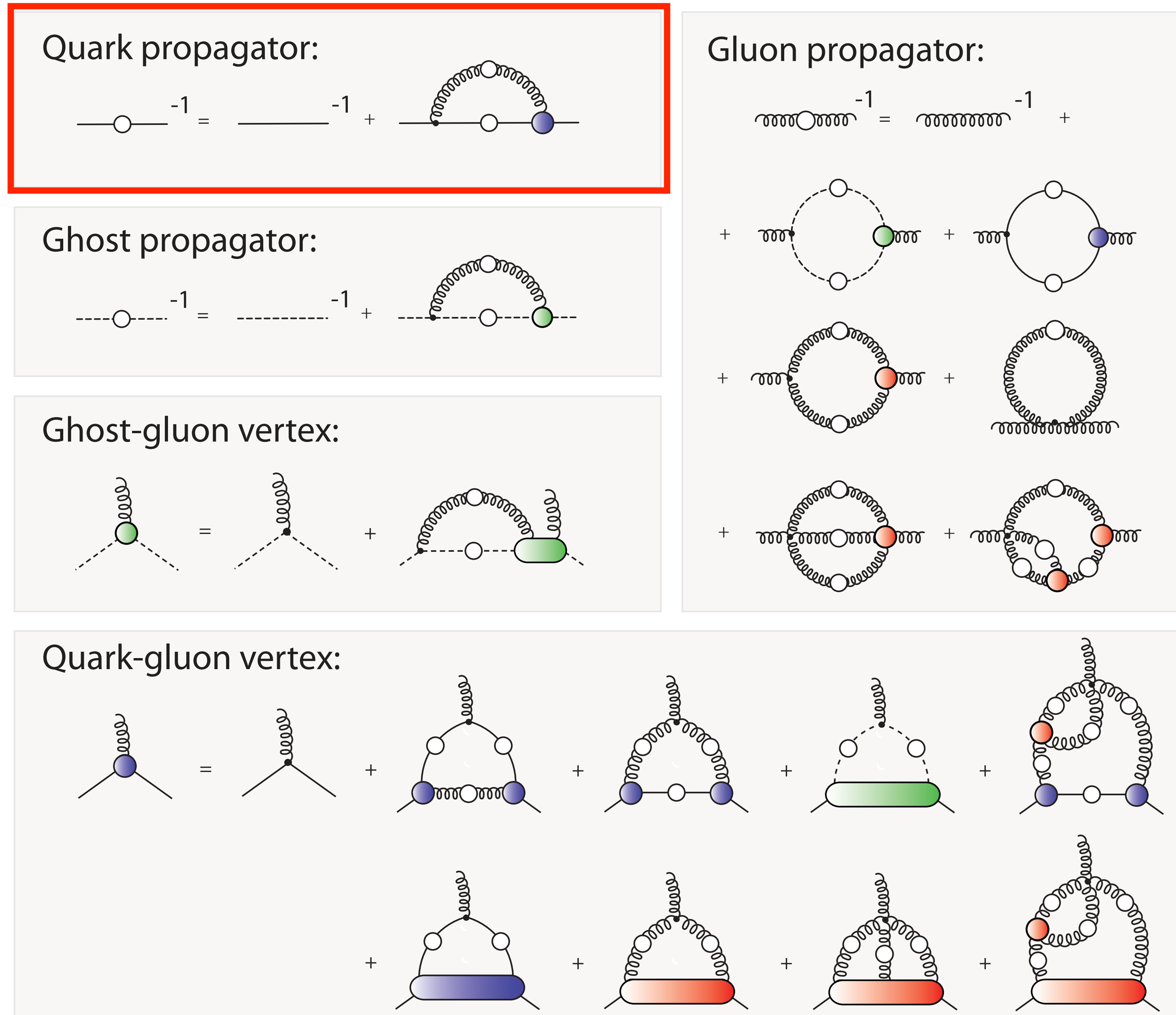


$$H = H_{\text{kinetic}} + H_{\text{Coulomb}} + H_{\text{spin-orbit}} + H_{\text{relativistic}} + H_{\text{QED}}$$

Principle of Least Action

$$\left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$$

Dyson-Schwinger Equations

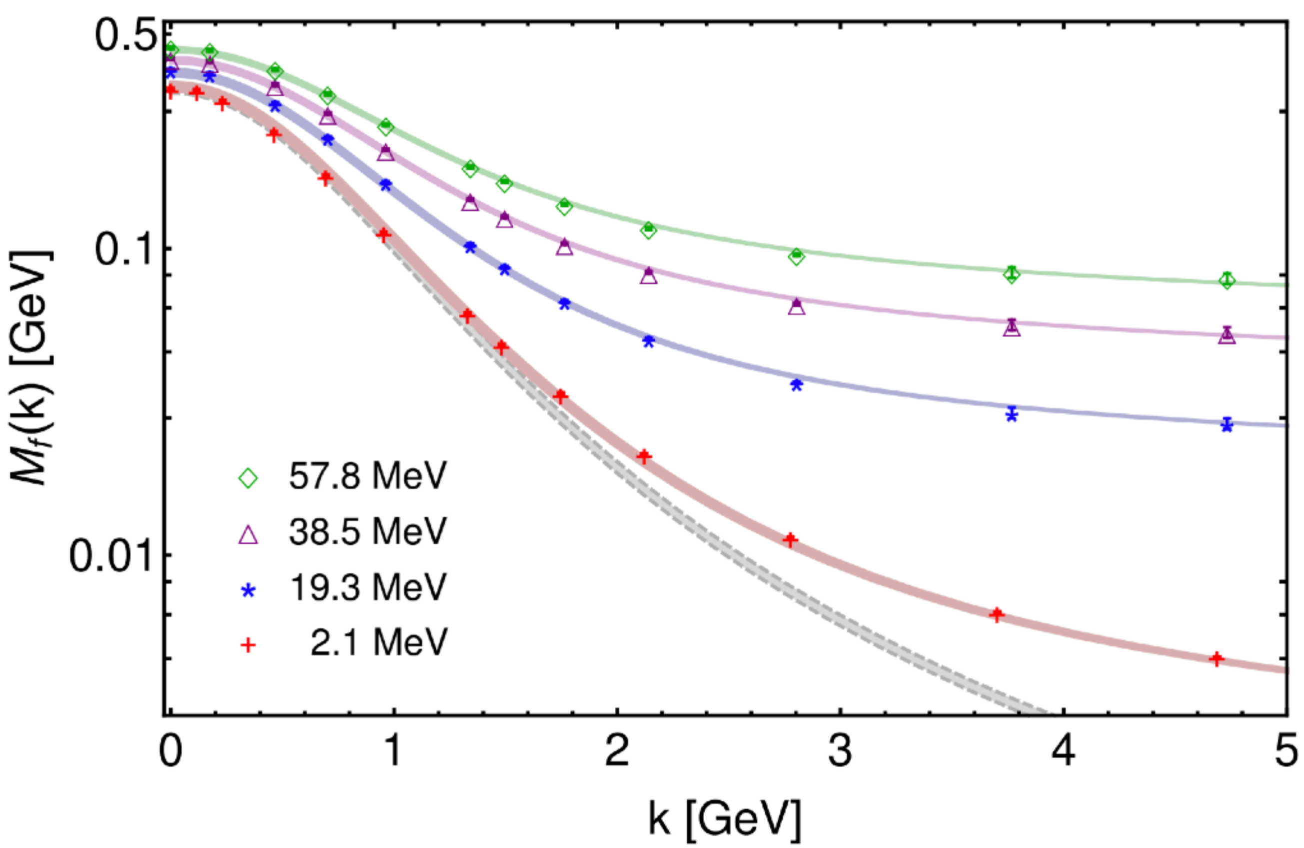


G. Eichmann, arXiv:0909.0703

Quark: Running mass function

$$S(p) = \frac{1}{i\gamma \cdot pA(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

Chang et. al., PRD 104, 094509 (2021)



✦ **Now:**

1. The quark's **effective mass** runs with its momentum.
2. The most **constituent mass** of a light quark comes from a cloud of gluons.

✦ **Next:**

1. What is the **infrared scale** of quark mass function?
2. How does the **transition** connect the non-perturbative and perturbative regions?

Gluon: Dynamical mass scale

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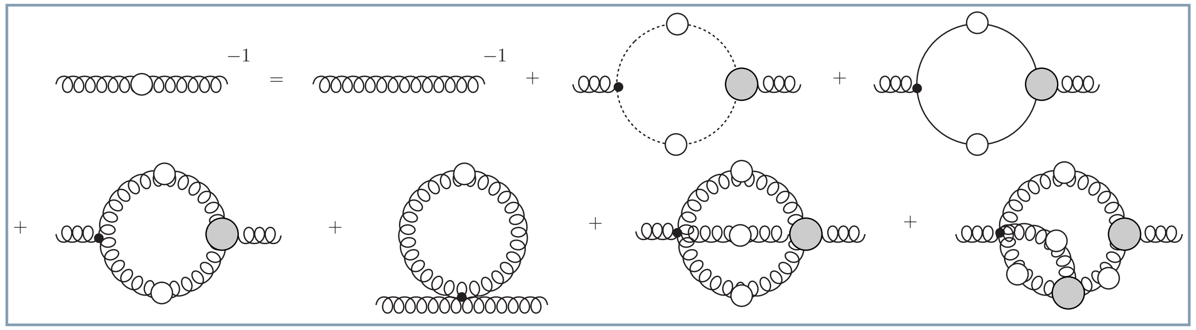
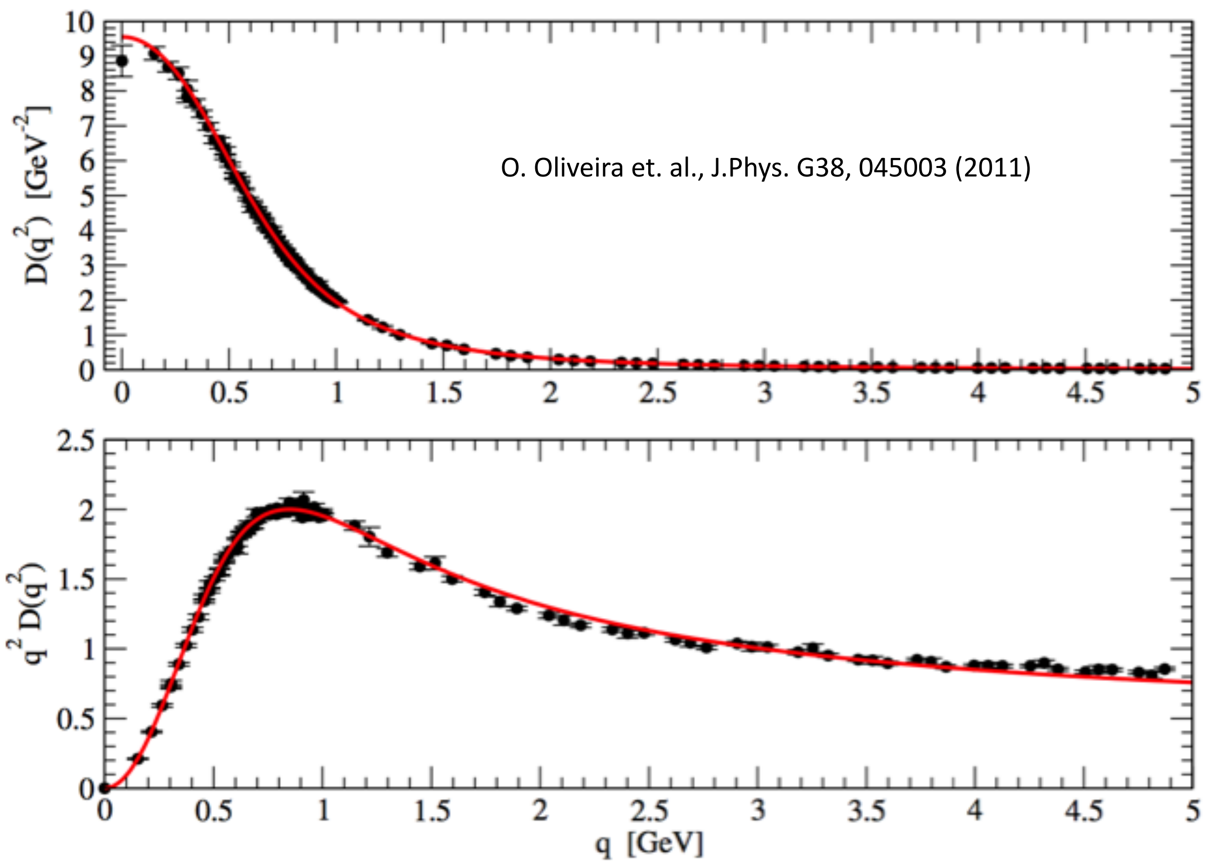
Gluon gap equation:

Aguilar, Binosi, Papavassiliou and Rodriguez-Quintero



Lattice QCD simulations:



- The interaction can be decomposed: ***gluon running mass*** + ***effective running coupling***

$$g^2 D_{\mu\nu}(k) = \mathcal{G}(k^2)\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

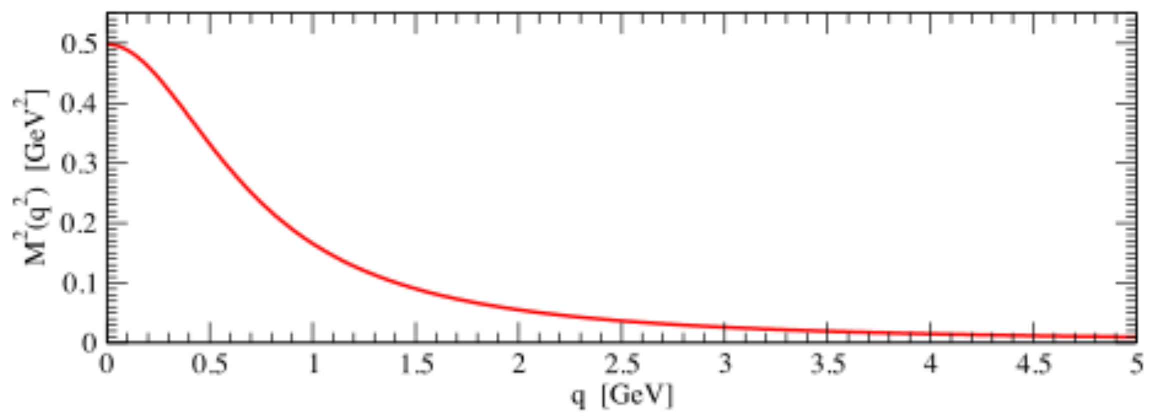
$$\mathcal{G}(k^2) \approx \frac{4\pi\alpha_{RL}(k^2)}{k^2 + m_g^2(k^2)}$$

- In QCD: Gluons are ***cannibals*** – a particle species whose members become **massive** by eating each other!

Gluon: Dynamical mass scale

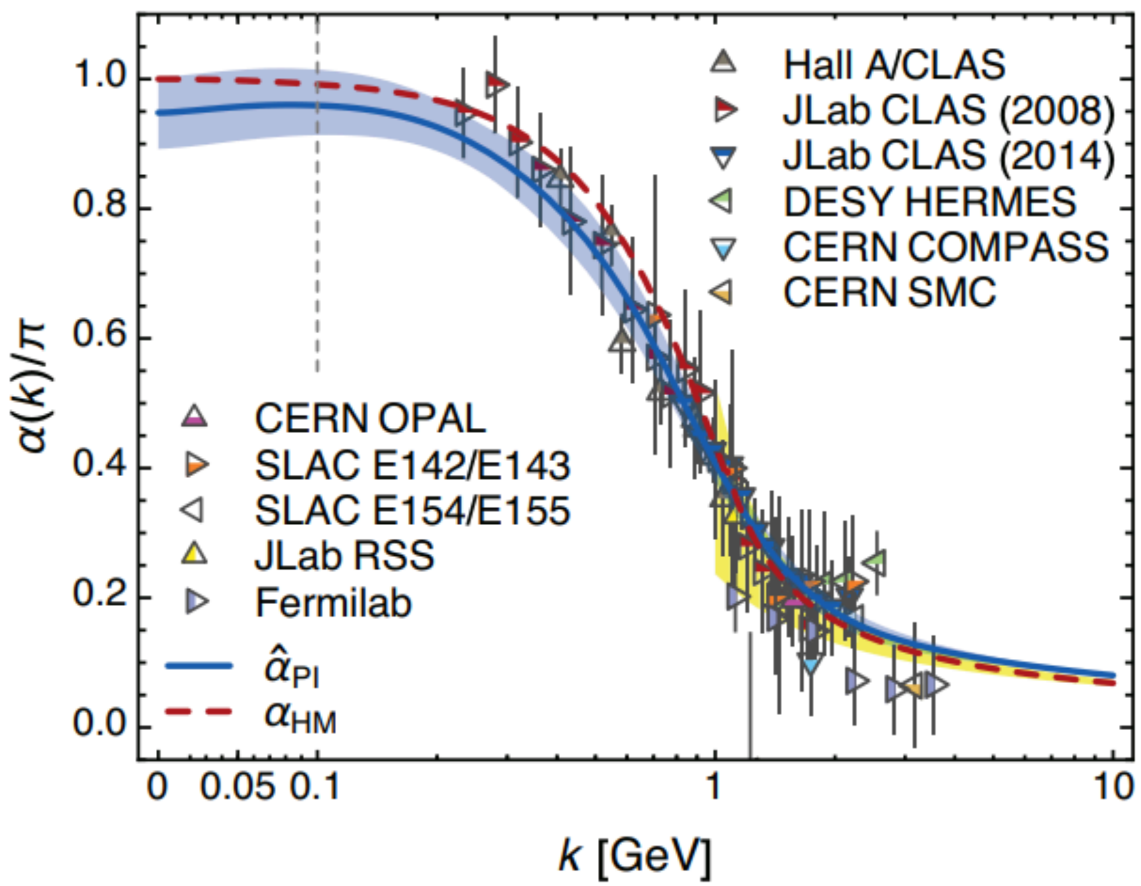
Gluon mass function:

O. Oliveira et. al., J.Phys. G38, 045003 (2011)



Running coupling:

Binosi, Mezrag, Papavassiliou, Roberts and Rodriguez-Quintero



✦ **Now:**

1. The dressed gluon can be well parameterized by a **mass scale**

$$m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2}$$

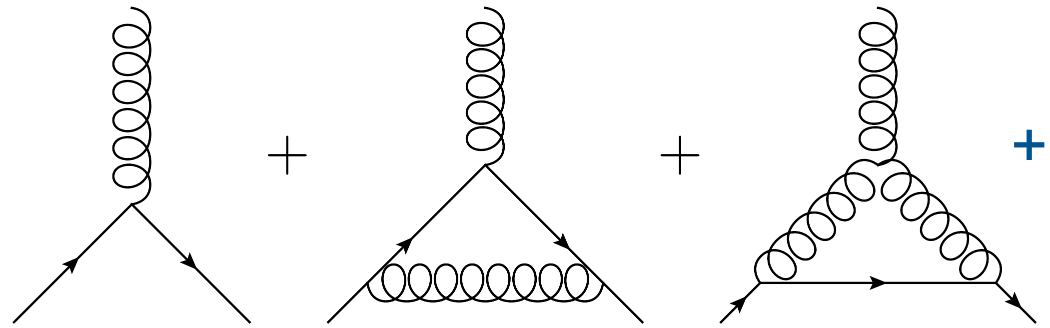
2. The effective running coupling **saturates** in the infrared limit.

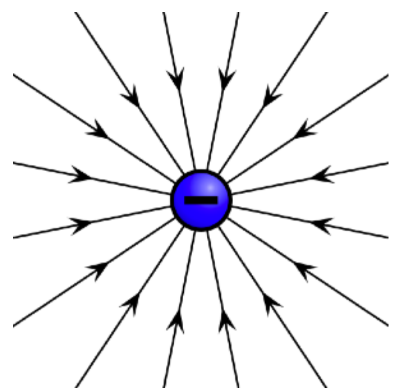
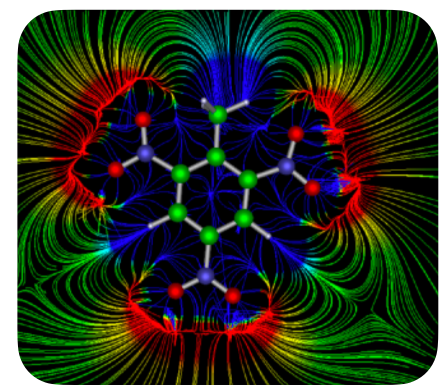
✦ **Next:**

1. What is the **mass scale** of gluon?

2. What is the **infrared magnitude** of running coupling?

Vertex: DCSB feedback

Quark-gluon vertex:  $+ \ \ldots \ =$?

point charge   distributed charge

- ✦ The **Dirac** and **Pauli** terms: for an on-shell fermion, the vertex can be decomposed by two form factors:

$$\Gamma^\mu(P', P) = \gamma^\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M_f} Q^\nu F_2(Q^2)$$

- ✦ The form factors express (color-)**charge** and (color-)**magnetization** densities. And the so-called **anomalous magnetic moment** is proportional to the Pauli term.

See, e.g., PLB722, 384 (2013)

Vertex: DCSB feedback

Spacetime

- **Poincaré symmetry**

Fields

- **Gauge symmetry**
- **Chiral symmetry**

"Symmetry dictates interaction." — CN Yang

- **Gauge symmetry:** Longitudinal WGTI

$$iq_\mu \Gamma_\mu(k,q) = S^{-1}(k) - S^{-1}(p)$$

- **Lorentz symmetry + :** Transverse WGTIs

$$\begin{aligned} q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) &= S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k) \\ &\quad + 2im\Gamma_{\mu\nu}(k,p) + t_\lambda \epsilon_{\lambda\mu\nu\rho}\Gamma^A_\rho(k,p) \\ &\quad + A^V_{\mu\nu}(k,p)\,, \end{aligned}$$

$$\begin{aligned} q_\mu \Gamma^A_\nu(k,p) - q_\nu \Gamma^A_\mu(k,p) &= S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu} S^{-1}(k) \\ &\quad + t_\lambda \epsilon_{\lambda\mu\nu\rho}\Gamma_\rho(k,p) \\ &\quad + V^A_{\mu\nu}(k,p)\,, \end{aligned} \qquad \sigma^5_{\mu\nu} = \sigma_{\mu\nu}\gamma_5$$

The WGTIs of the vertices can be decoupled and (partially) solved.

See, e.g., PLB722, 384 (2013)

Vertex: DCSB feedback

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Equation of Motion (DSEs)

$$S = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

$$\gamma^\mu \Sigma_A + i\sigma_{\mu\nu} Q^\nu \Delta_B + \text{etc.} \sim \Gamma^\mu$$

Ward identities (Symmetries)

✦ **Now:**

1. There is a dynamic chiral symmetry breaking (**DCSB**) **feedback**.

2. The **appearance** of the vertex is dramatically modified by the **dynamics**.

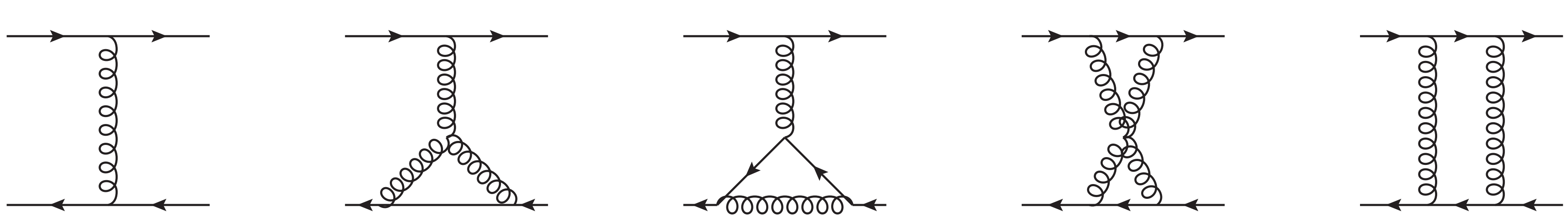
✦ **Next:**

1. What are the exact **strengths** of the terms in the vertex?

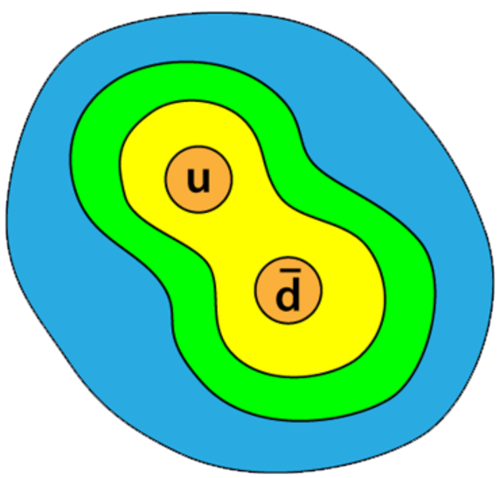
2. What the exact **behaviors** of the form factors in the vertex?

See, e.g., PLB722, 384 (2013)

Kernel: Twofold role of pion



+ … = ?



Pion

- **Bound state** of quark and anti-quark, but abnormally light:

$$M_\pi \ll M_u + M_{\bar{d}}$$

- **Goldstone's theorem:** If a generic continuous symmetry is spontaneously broken, then new massless scalar particles appear in the spectrum of possible excitations.

- The **discrete** and **continuous** **symmetries** strongly constrain the kernel:

Poincaré symmetry **C-, P-, T-symmetry**	**Gauge symmetry** **Chiral symmetry**

Kernel: Twofold role of pion

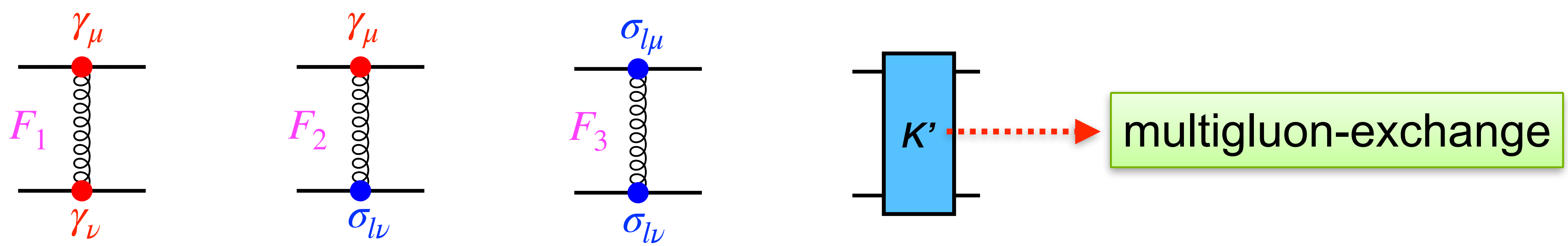
✦ **Now:**

1. A deep connection between one-body and two-body problem:

$$\boxed{f_\pi E_\pi(k^2) = B(k^2)}$$

- **Pion** exists if, and only if, the **quark mass** is dynamically generated.
- **Two-body problem** solved, almost completely, once solution of **one-body** problem is known.

2. A realistic kernel must involves the Dirac and Pauli structures:



✦ **Next:**

1. How to further **pin down structures** of the kernel?

2. How to **simplify the kernel** for more practical applications?

See, e.g., CPL 38 (2021) 7, 071201

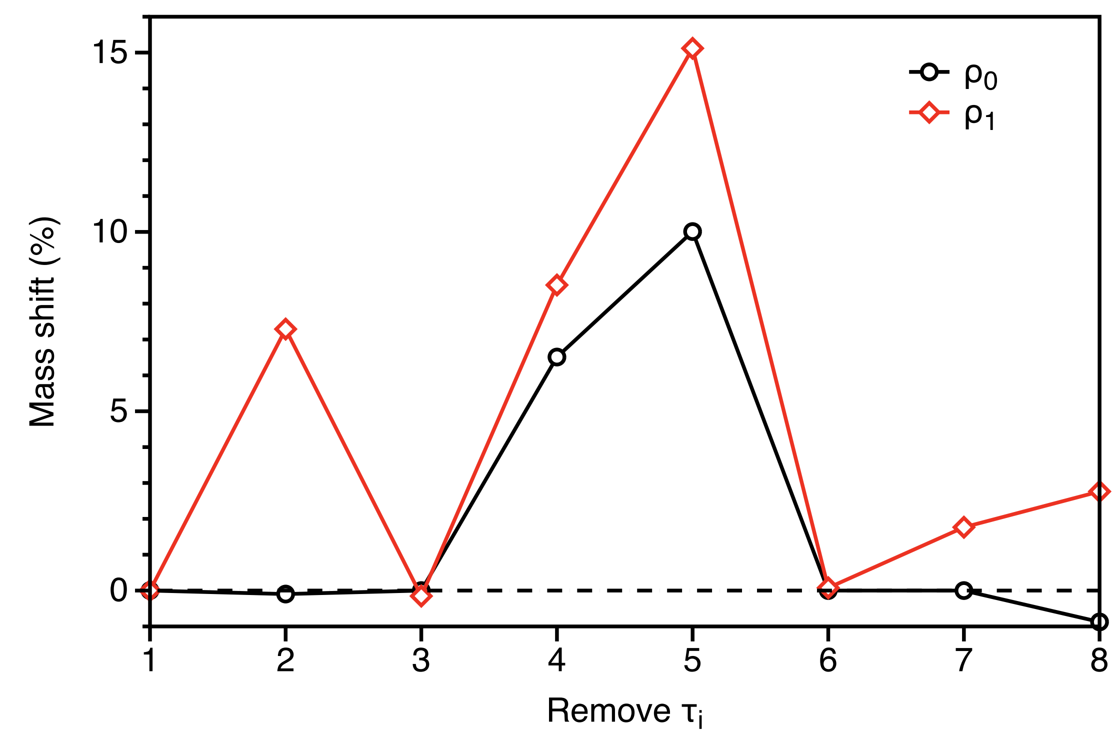
Chapter II: Applications

Spectra of mesons and baryons with light and heavy flavors.

Application: Partial-wave structures of mesons

✦ Structure of wave function, e.g. $\boldsymbol{\rho}$ meson:

$$
\begin{aligned}
\tau^1_{1^-} &= i\gamma^T_\mu, \\
\tau^2_{1^-} &= i\left[3k^T_\mu \gamma\cdot k^T - \gamma^T_\mu k^T\cdot k^T\right], \\
\tau^3_{1^-} &= ik^T_\mu\, k\cdot P\,\gamma\cdot P, \\
\tau^4_{1^-} &= i\left[\gamma^T_\mu \gamma\cdot P\,\gamma\cdot k^T + k^T_\mu \gamma\cdot P\right], \\
\tau^5_{1^-} &= k^T_\mu, \\
\tau^6_{1^-} &= k\cdot P\left[\gamma^T_\mu \gamma^T\cdot k - \gamma\cdot k^T \gamma^T_\mu\right], \\
\tau^7_{1^-} &= (k^T)^2\left(\gamma^T_\mu \gamma\cdot P - \gamma\cdot P\gamma^T_\mu\right) - 2k^T_\mu \gamma\cdot k^T\gamma\cdot P, \\
\tau^8_{1^-} &= k^T_\mu \gamma\cdot k^T\gamma\cdot P.
\end{aligned}
$$



$$
\boxed{\text{J = 1} \begin{cases} \text{S = 0, L = 1} \\ \text{S = 1, L = 0} \\ \text{S = 1, L = 2} \end{cases}}
$$

✦ Total angular momentum **J** is a good quantum number, but **S** and **L** are not. The partial waves mix together.

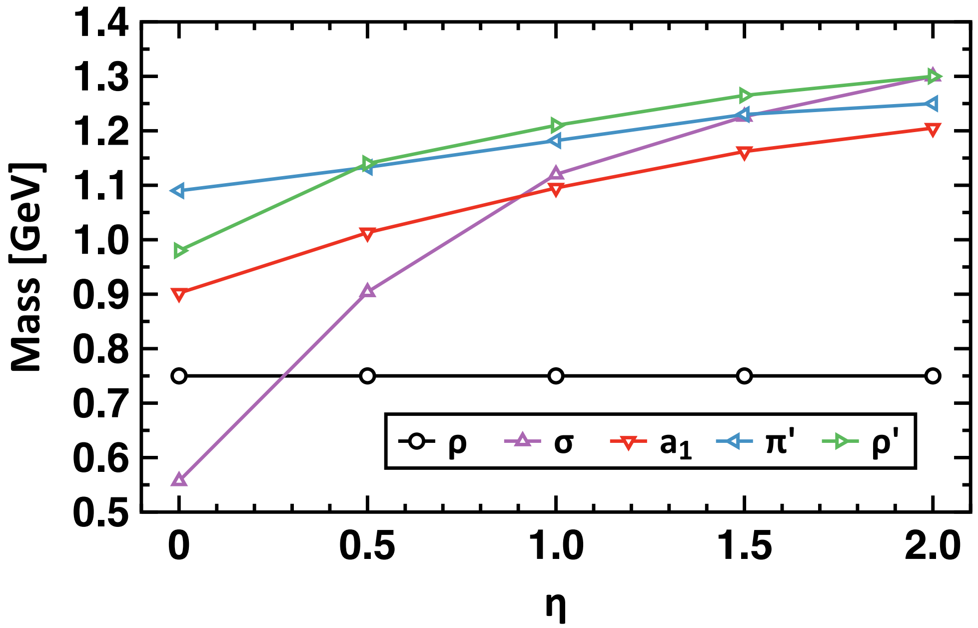
$$P \neq (-1)^{L+1}$$

✦ Missing some partial waves could remarkably affect the mass, especially of radial excitation states.

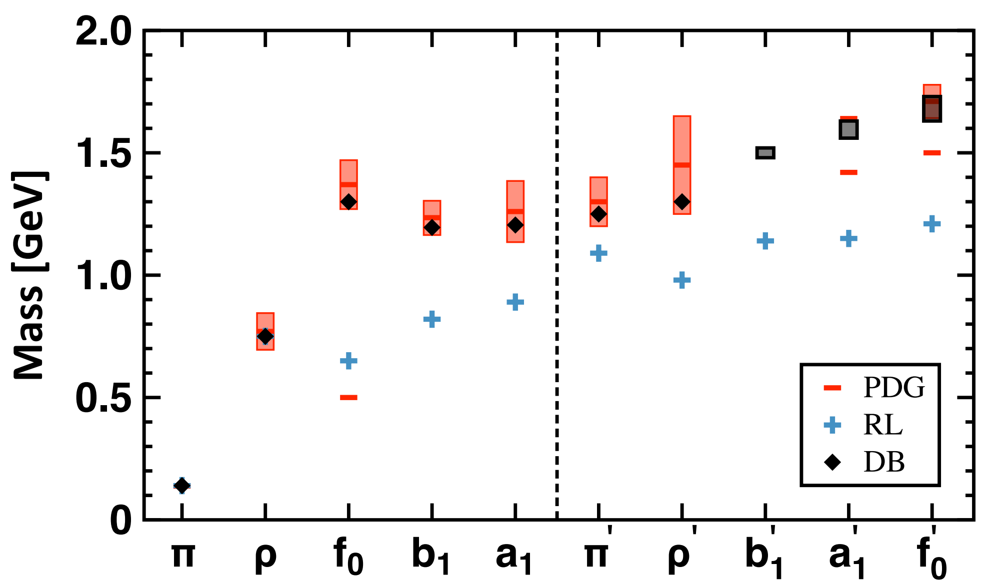
See, e.g., PRC 85, 035202 (2012)

Application: Spin-orbit interaction of light mesons

✦ Impact of the Pauli term (AM):



✦ Light-flavor meson spectrum:



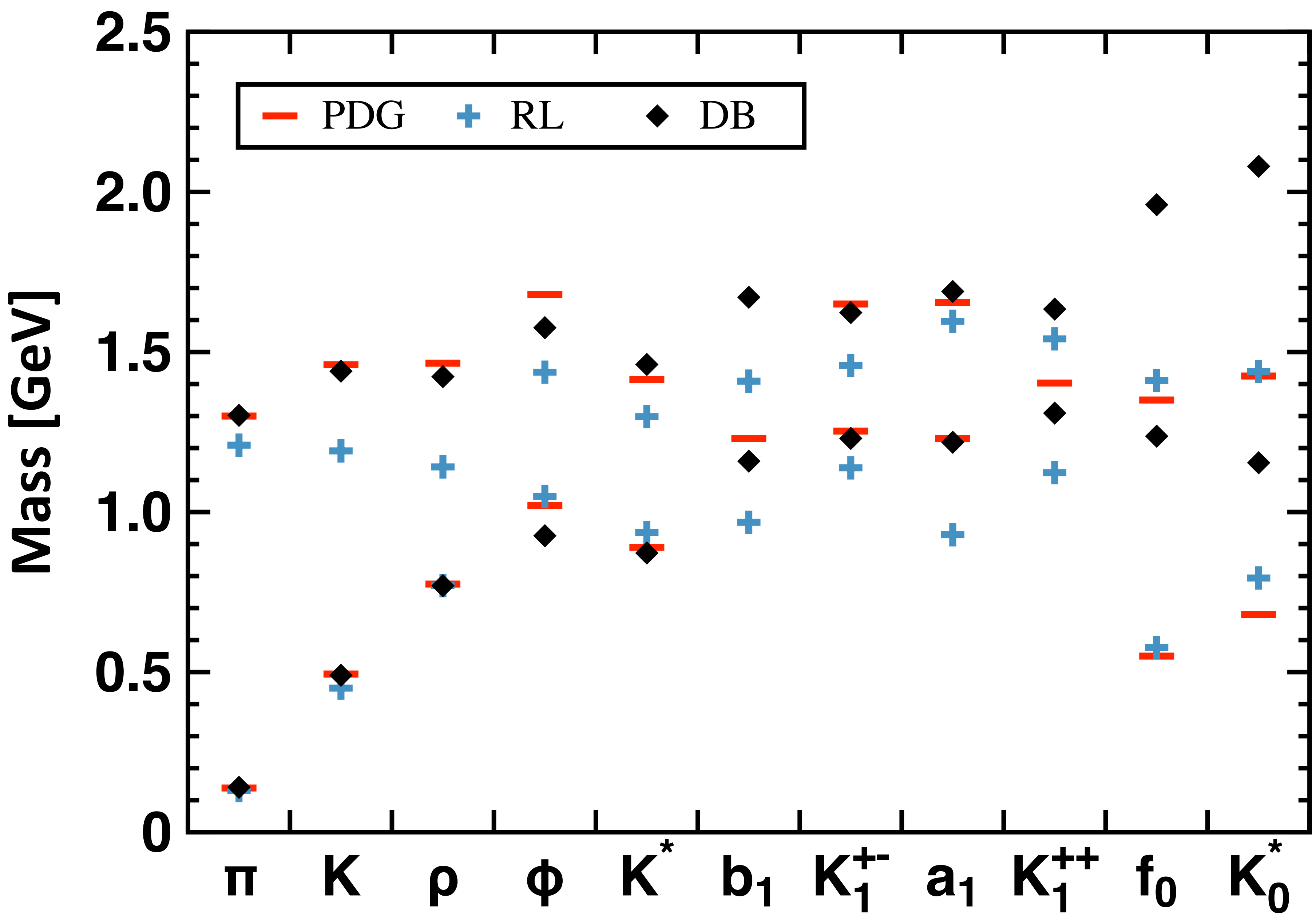
✦ With increasing the AM strength, the a_1−ρ mass-splitting rises very rapidly. From a quark model perspective, the DCSB-enhanced kernel increases spin-orbit repulsion.

✦ The spin-orbit boosted quark-core mass of the f_0 is greater than the empirical value, and matches the estimated result obtained using chiral perturbation theory.

✦ The magnitude and ordering of radial excitation states can be fixed with the DCSB-enhanced kernel.

See, e.g., CPL 38 (2021) 7, 071201

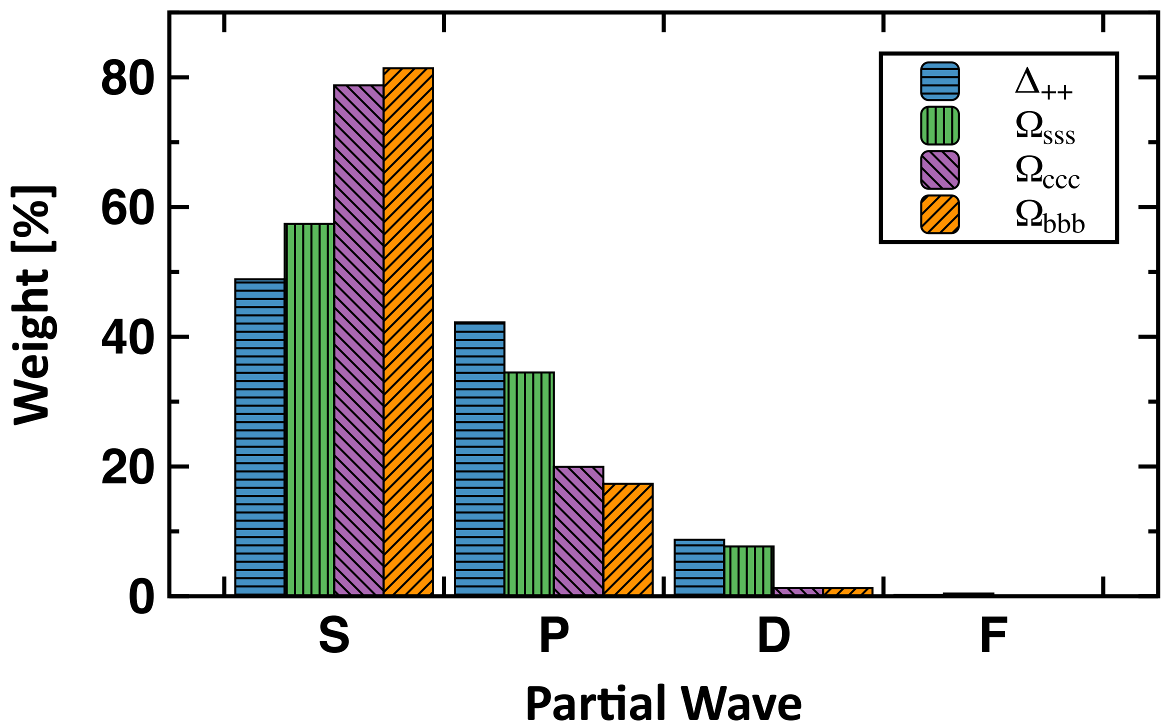
Application: Spectrum of low-lying mesons

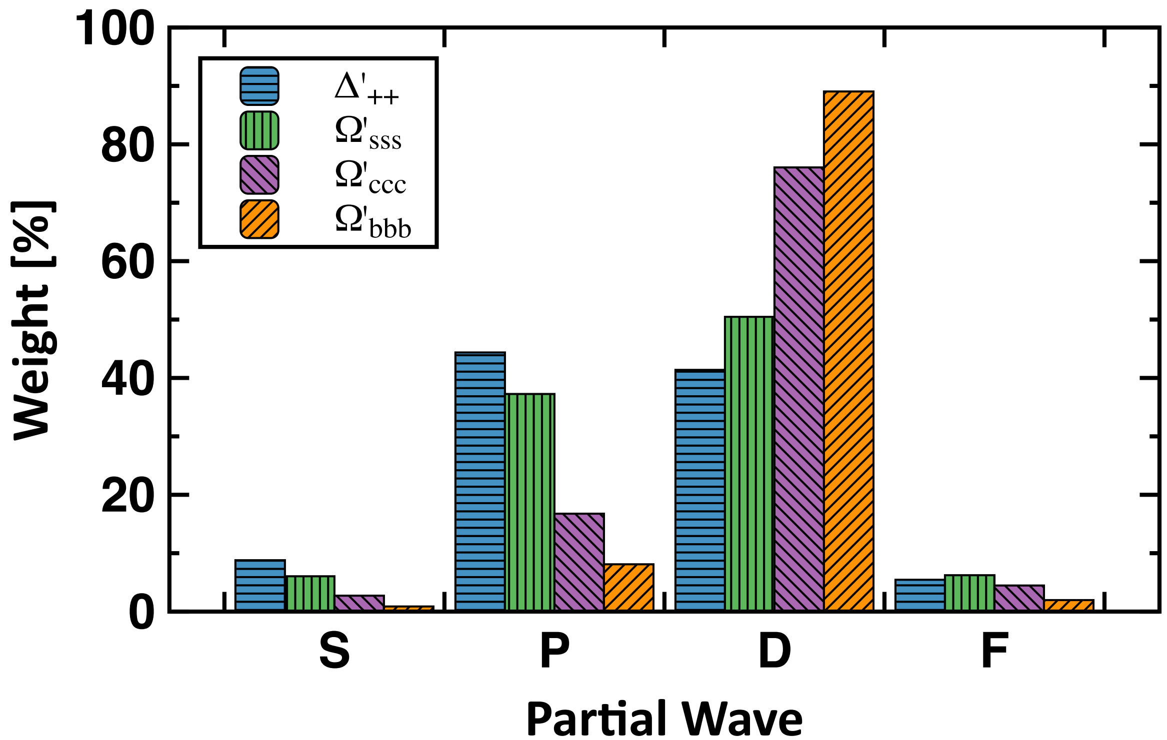


The low-lying mesons with strangeness can be well produced by the DCSB-enhanced kernel, e.g., pion, kaon, and etc.

See, e.g., Eur.Phys.J. A59, 39 (2023)

Application: Partial-wave structures of baryons





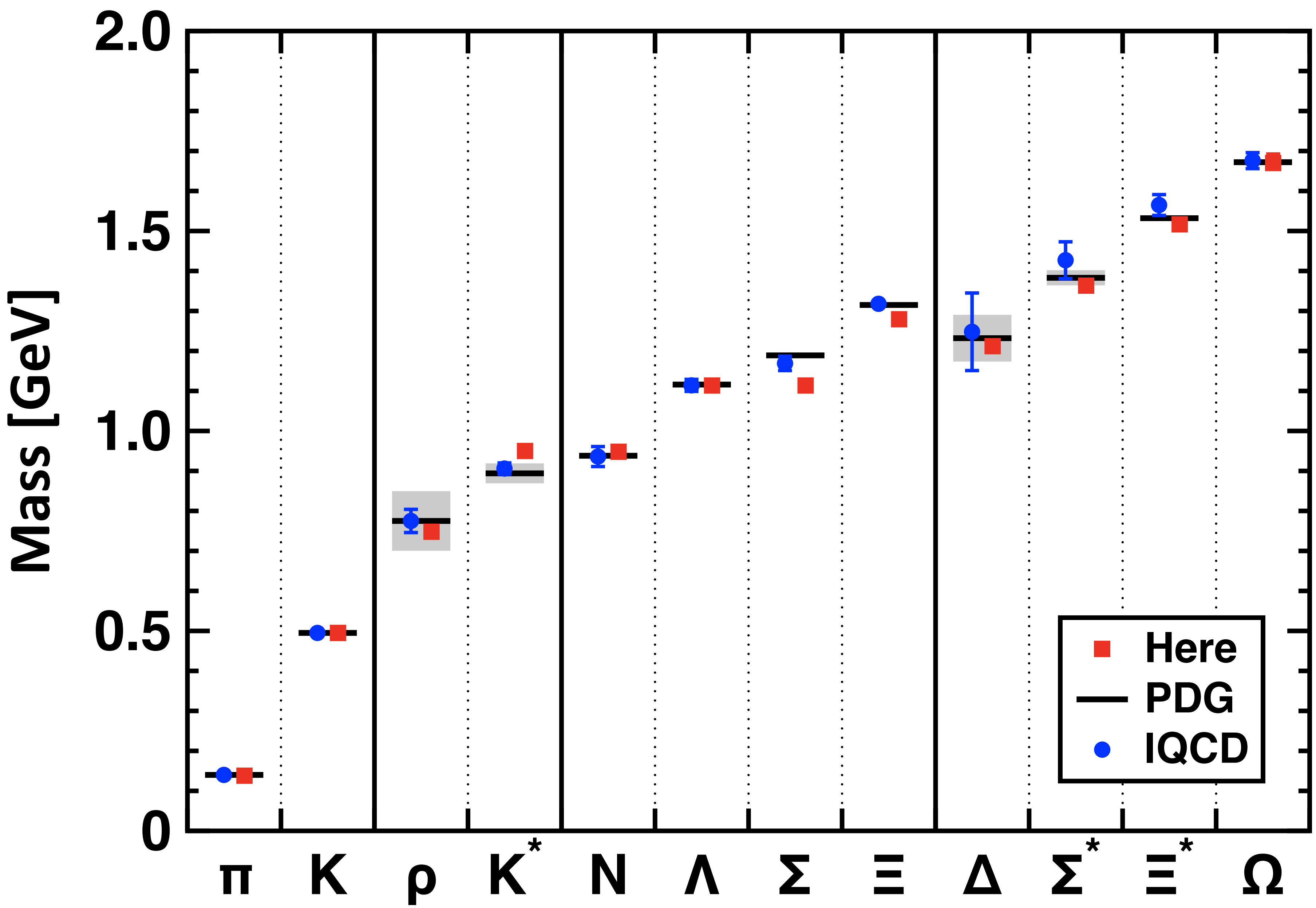
✓ S-waves dominate for ground states, but p-waves grow for light baryons.

How NR quark model works

✓ D-waves dominate for excited states, but p-waves grow for light baryons.

See, e.g., PRD 97, 114017 (2018)

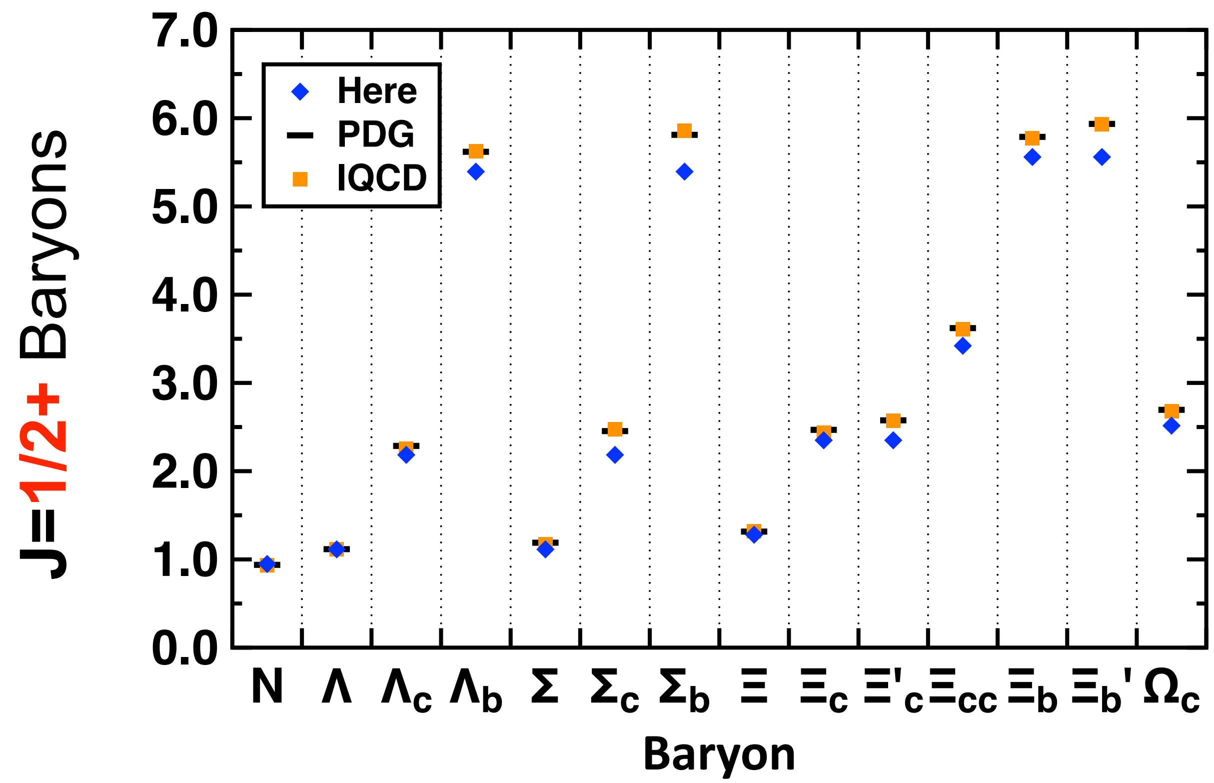
Application: Light & Strange flavors

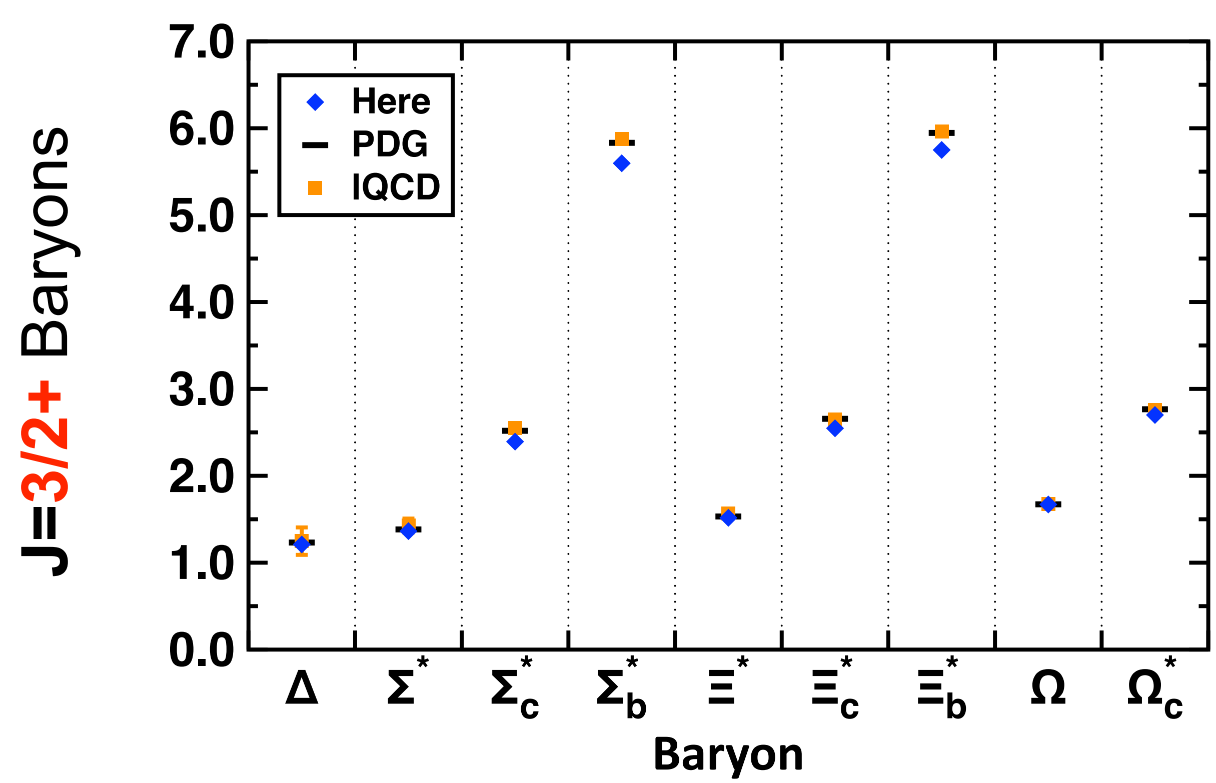


The interaction strength and current quark masses are fixed by properties of pseudo-scalar mesons, e.g., pion, kaon, and etc.

See, e.g., Few-Body Syst 60, 26 (2019)

Application: Charm & Bottom flavors





- The ground states of Nucleon and Delta families can be described by a simple kernel.

- The excited states and the parity partners require a DCSB-enhanced kernel.

More is coming soon

See, e.g., Few-Body Syst 60, 26 (2019)

Summary

- ✦ **Hadrons** play a role for revealing **QCD**'s mysteries. **High-precision measurements** and **simulations** of their spectrum are critical for the next step.

- ✦ The **bound-state equation** for describing hadron properties has made important progress by exposing **QCD**'s **fundamental features** (e.g., in quark, gluon, vertex, and kernel).

Outlook

- ◆ With the **sophisticated** approach, we can further iterate with future experiments and simulations on **light** and **heavy** hadrons, from spectroscopy to structures.

- ◆ Hopefully, based on more and more **successful applications**, we may provide a **faithful path** to understand **QCD** and **mass**.