Hadron and parton masses from Lattice QCD

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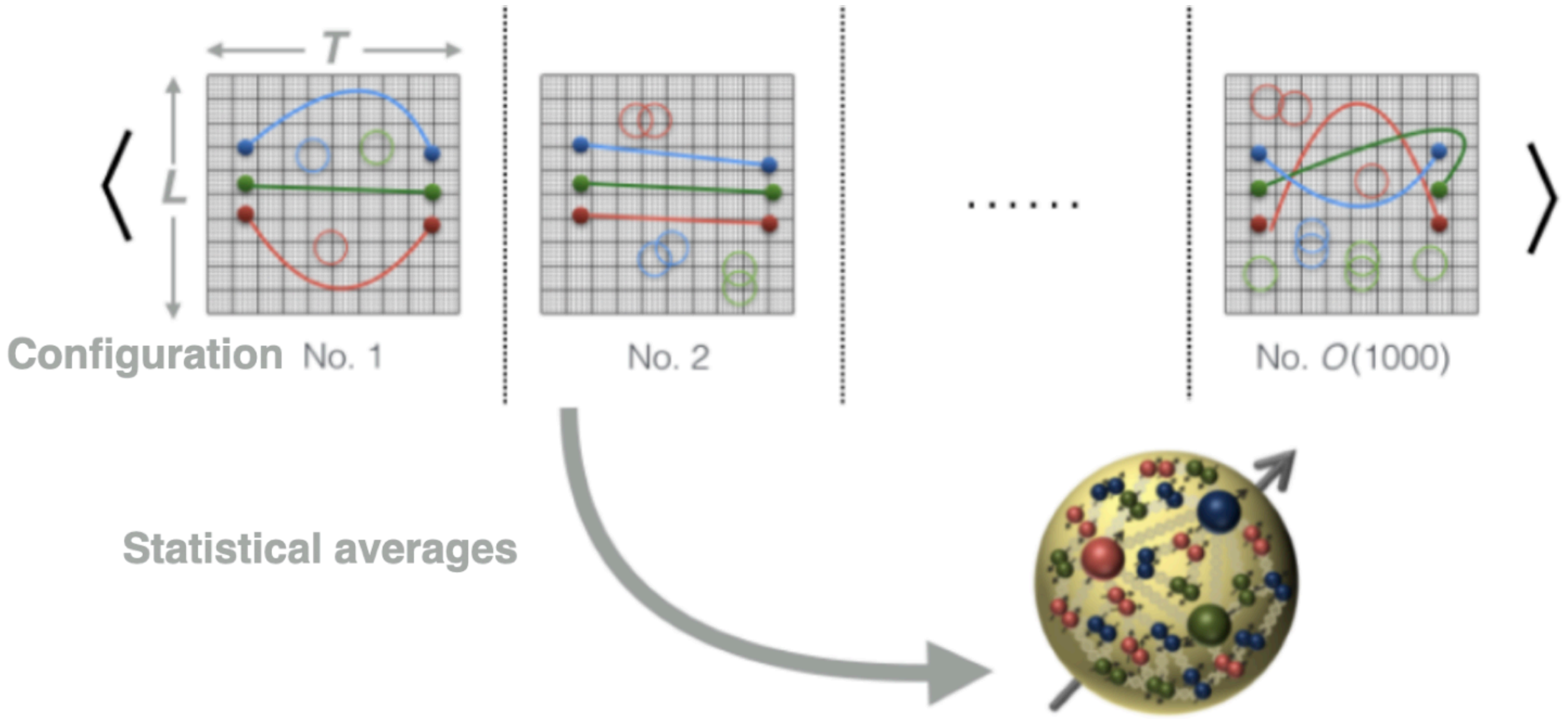
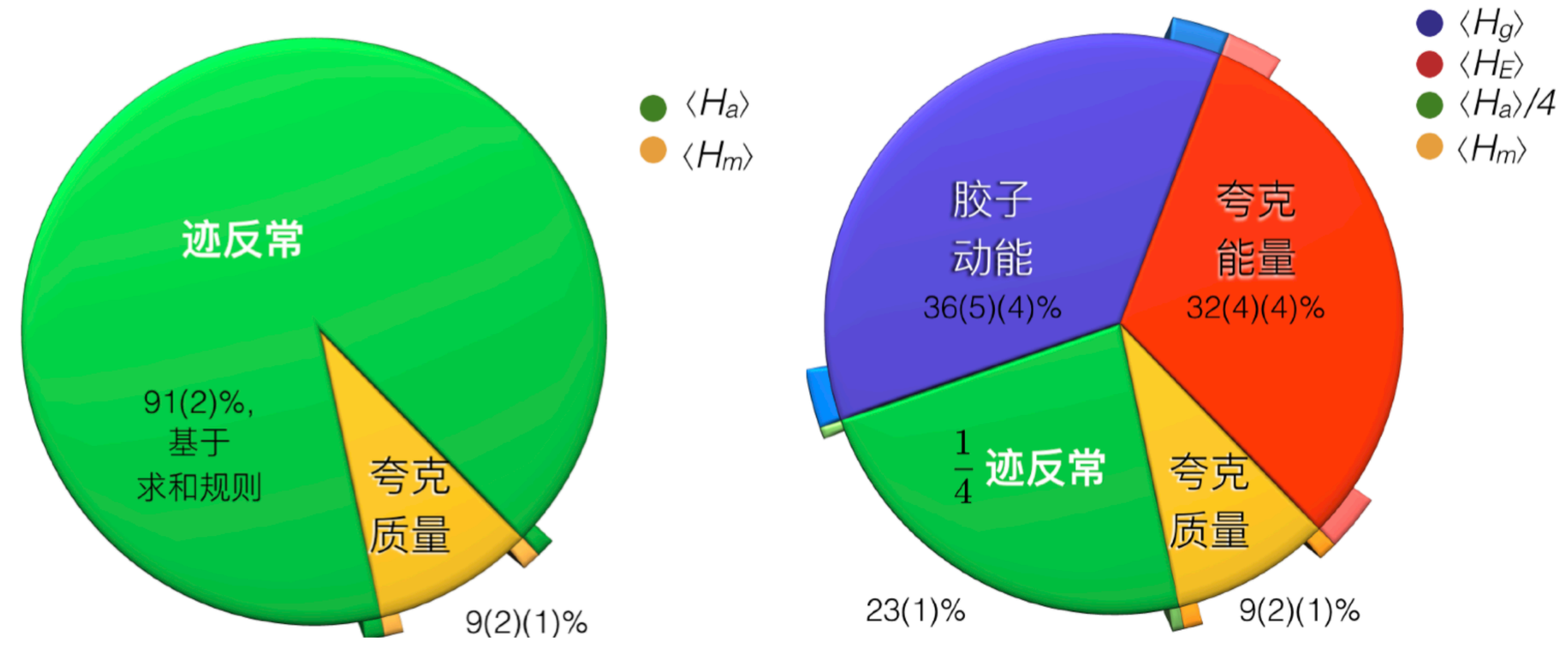
Outline

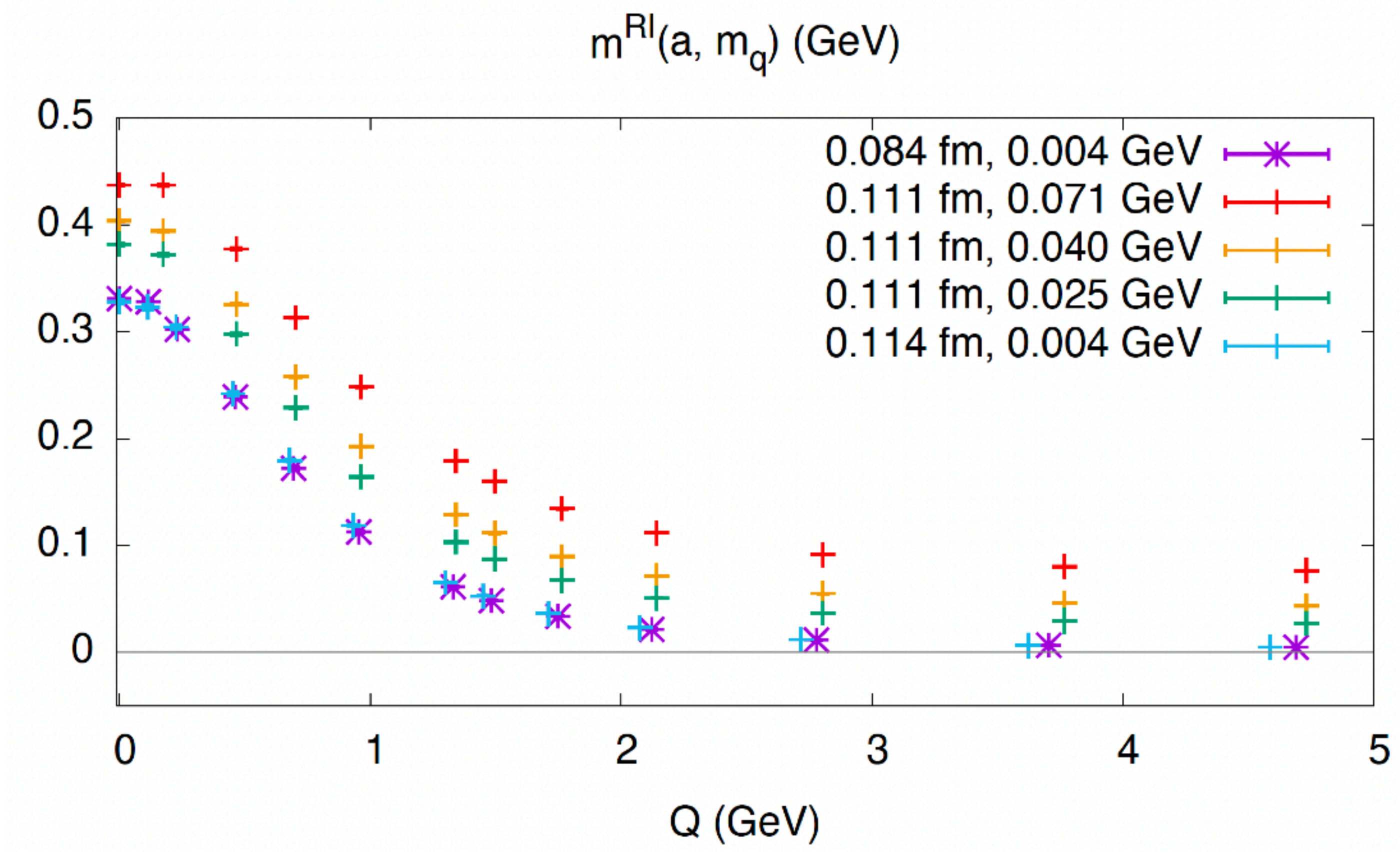
- **Brief introduction on Lattice QCD;**



- 

  Hadron mass and its components;

- Parton masses and their low energy limits.



Discretized quantum field theory

	Continuum	Lattice
EOM	$(\partial_\mu \partial^\mu + m^2)\phi = 0$	$\sum_\mu \left(\phi(x+\hat{n}_\mu) + \phi(x-\hat{n}_\mu)\right) + (m^2a^2 - 2)\phi(x) = 0$
Propagator	$S = \dfrac{1}{p^2 + m^2}$	$S_L = \dfrac{1}{4\sum_\mu \mathrm{Sin}^2(\frac{ap_\mu}{2})/a^2 + m^2}$
Loop Integral	$\int_{-\infty}^{\infty} \mathrm{d}^4 p$	$\int_{-\pi/a}^{\pi/a} \mathrm{d}^4 p$
	Require additional regulation to be finite	The divergence has been regularized into the $1/a^n$ and $\log(a)$ terms

Discretized QCD

Wilson link

The QCD Lagrangian is the following：

$$\bar{\psi}(\gamma_\mu(\partial_\mu - igA_\mu) - m)\psi - \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

with basic variable ψ and A_μ

The lattice gauge theory replaces the basic variable A_μ into the gauge link (or Wilson link) U_μ:

$$A_\mu(x + \frac{1}{2}\hat{n}_\mu) = a^4\frac{U_\mu(x) - U^\dagger_\mu(x)}{2ig_0 a} + \mathcal{O}(a^2g^2), U_\mu(x) \equiv e^{ig_0\int_x^{x+a\hat{\mu}} dyA_\mu(y)};$$

$$U_\mu(x) \equiv e^{ig_0\int_x^{x+a\hat{\mu}} dyA_\mu(y)},\ U^\dagger_\mu(x) \equiv e^{-ig_0\int_{x+a\hat{\mu}}^{x} dyA_\mu(y)} = e^{ig_0\int_{x+a\hat{\mu}}^{x} dyA_\mu(y)}$$

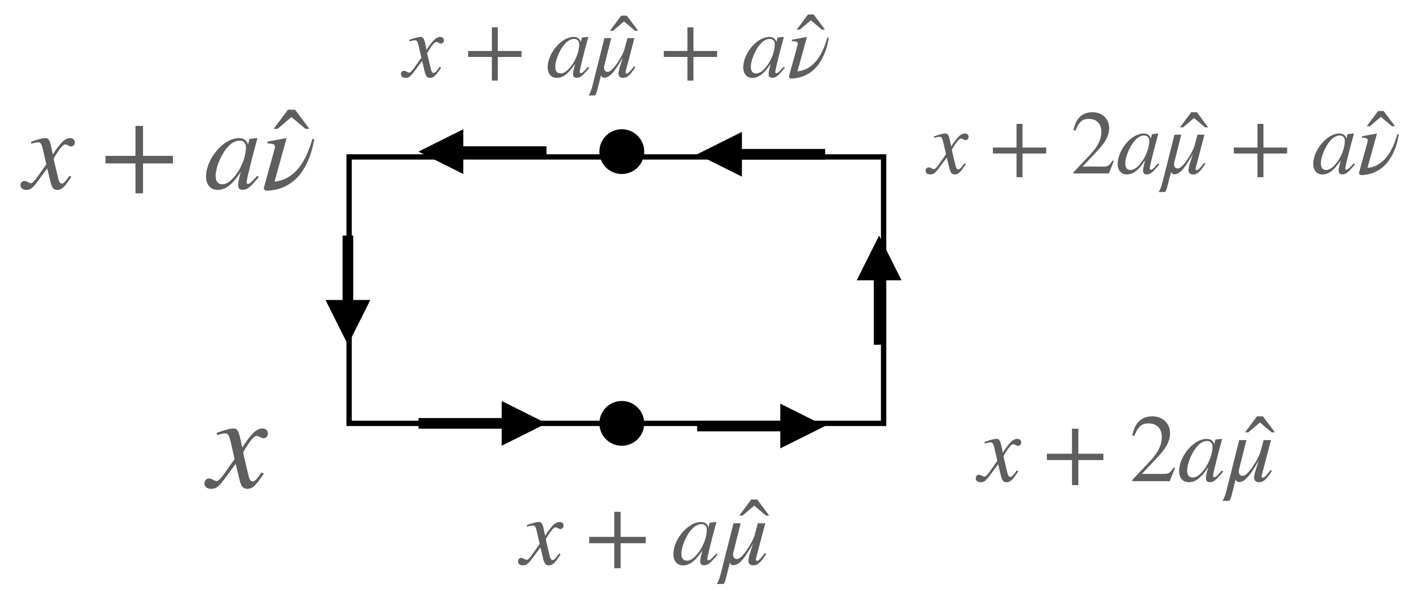
$$x \longrightarrow x + a\hat{\mu}$$

Discretized QCD

gauge action

- $\mathcal{P}_{\mu\nu}(x) = U_\mu(x)U_\nu(x+a\hat{\mu})U^\dagger_\mu(x+a\hat{\nu})U^\dagger_\nu(x) = 1 + ig_0a^2F_{\mu\nu}(x+\frac{a}{2}(\hat{\mu}+\hat{\nu})) - \frac{1}{2}a^4g_0^2F_{\mu\nu}(x+\frac{a}{2}(\hat{\mu}+\hat{\nu}))F_{\mu\nu}(x+\frac{a}{2}(\hat{\mu}+\hat{\nu})) + i\mathcal{O}(a^4)$

- it can also be used to define the gauge field tensor $F_{\mu\nu}$, and also gauge action：
$$\mathcal{S}_g = \frac{1}{2g_0^2}\sum_{x,\mu\nu}\mathrm{Re}\left[1-\mathrm{Tr}[\mathcal{P}_{\mu\nu}(x)]\right] = \frac{1}{2}\mathrm{Tr}[\int \mathrm{d}^4xF_{\mu\nu}F_{\mu\nu}] + \mathcal{O}(a^2)$$
    - Such an action has the $\mathcal{O}(a^2)$ discretization error.
    - It can combine with the 1x2 loop
    $\mathcal{P}^{Rect}_{\mu\nu}(x) = U_\mu(x)U_\mu(x+a\hat{\mu})U_\nu(x+2a\hat{\mu})U^\dagger_\mu(x+a\hat{\mu}+a\hat{\nu})U^\dagger_\mu(x+a\hat{\nu})U^\dagger_\nu(x)$
    to construct the Symanzik or Iwasaki action, to suppress the discretization error to $\mathcal{O}(a^4)$。



$$S_g^{\mathrm{Symanzik}} = \frac{5}{3}S_g^{1x1} - \frac{1}{12}S_g^{1x2}$$

$$S_g^{\mathrm{Iwasaki}} = (1+8\times 0.331)S_g^{1x1} - 0.331S_g^{1x2}$$

Discretized QCD

Naive and Staggered actions

- The naive discretization suffers from the doubling problem：
  - $\mathcal{S}_q^{Naive}(m) = \sum_{x,y} \bar{\psi}(x) D_{Naive}(m; x, y) \psi(y),\ D_{Naive}(m; x, y) = \frac{1}{2a} \sum_{\mu} \gamma_\mu \left(U_\mu(x) \delta_{y,x+a\hat{\mu}} - U_\mu^\dagger(x - a\hat{\mu}) \delta_{y,x-a\hat{\mu}} \right) + m\delta_{y,x}$
  - The propagator has $1/m$ IR poles at $pa = (0/\pi, 0/\pi, 0/\pi, 0/\pi)$, which is different from the continuum theory.
- Staggered fermion:
  - $\psi^{\mathrm{st}}(x) = \gamma_4^{x_4} \gamma_1^{x_1} \gamma_2^{x_2} \gamma_3^{x_3} \psi(x),\ \{\gamma_1^{\mathrm{st}}, \gamma_2^{\mathrm{st}}, \gamma_3^{\mathrm{st}}, \gamma_4^{\mathrm{st}}\} = \{(-1)^{x_4}, (-1)^{x_1+x_4}, (-1)^{x_1+x_2+x_4}, 1\};$
  - 16 IR poles → 4 IR poles.
  - Mixing between IR poles can be suppressed with kinds of the improvement, likes the so-call highly-improved staggered quark (HISQ).

$\boxed{\text{Staggered/HISQ}} \xrightarrow{\text{Cost x10}} \boxed{\text{Wilson/Clover}} \xrightarrow{\text{Cost x10}} \boxed{\text{Domain wall}} \xrightarrow{\text{Cost x10}} \boxed{\text{Overlap}}$

Discretized QCD

Wilson and clover actions

- Wilson fermion action：
  - $D + m \rightarrow D + aD^2 + m$
  - It removes the unphysical IR pole at $p_i = \pi/a$, while introduce the additional chiral symmetry breaking at $\mathcal{O}(\alpha_s/a)$.
- Clover fermion action:
  - $D + m \rightarrow D + aD^2 + m + ac_{\mathrm{sw}}\sigma_{\mu\nu}F^{\mu\nu}$
  - Suppress the additional chiral symmetry breaking at $\mathcal{O}(\alpha_s^2/a)$.
- The cost of either Wilson or Clover action is $\mathcal{O}(10)$ of the Staggered fermion.

Staggered/HISQ $\xrightarrow{\text{Cost x10}}$ Wilson/Clover $\xrightarrow{\text{Cost x10}}$ Domain wall $\xrightarrow{\text{Cost x10}}$ Overlap

Discretized QCD

Ginsparg-Wilson action

- Ginsparg-Wilson relation: $\gamma_5 D_{\rm GW} + D_{\rm GW}\gamma_5 = \frac{1}{\rho} D_{\rm GW}\gamma_5 D_{\rm GW}$。
- Overlap fermion as a possible solution:

$$\mathcal{S}_q^{ov}(m) = \sum_{x,y} \bar{\psi}(x)\Big(\delta_{xy} m + \sum_z D_{ov}(\rho; x, z)\frac{\rho/a}{\delta_{zy} - D_{ov}(\rho; z, y)/2}\Big)\psi(y),\ D_{ov}(\rho) = 1 + \frac{D_w(-\rho)}{\sqrt{D_w(-\rho)D_W^\dagger(-\rho)}}$$

- In $p \to 0$ region, $D_{ov} \to a\gamma_\mu p_\mu$;
- In $p \to \pi/a$ region, $D_{ov} \to \mathcal{O}(1)$.
- But approximate the sign function $\frac{\gamma_5 D_w(-\rho)}{\sqrt{D_w(-\rho)D_W^\dagger(-\rho)}} = \frac{\gamma_5 D_w(-\rho)}{|\gamma_5 D_w(-\rho)|}$ need $\mathcal{O}(100)$ cost of the Wilson/Clover action.
- Domain wall fermion action is an approximation of overlap fermion with $\mathcal{O}(10)$ cost of the Wilson/Clover action.

Staggered/HISQ $\xrightarrow{\text{Cost x10}}$ Wilson/Clover $\xrightarrow{\text{Cost x10}}$ Domain wall $\xrightarrow{\text{Cost x10}}$ Overlap

Lattice regularization

Feynman rules

- The Feynman rule under **the lattice regularization** can be extracted in the weak coupling limit.
- It approaches to the continuum form in the $a \to 0$ limit.
- But the Feynman rule of the multi-gluon vertex is very complicated, especially for the improved discretized actions.
- For example, the 4-gluon vertex of the simplest Wilson gauge actions

$$S_G = \frac{1}{2g_0^2} \sum_{x,\mu\nu} \mathrm{Re}\left[1 - \mathrm{Tr}[\mathscr{P}_{\mu\nu}(x)]\right] = \frac{1}{2} \mathrm{Tr} \int \mathrm{d}^4 x F_{\mu\nu} F_{\mu\nu} + \mathcal{O}(a^2)$$

With $\mathscr{P}_{\mu\nu}(x) = U_\mu(x) U_\nu(x + a\hat{\mu}) U_\mu^\dagger(x + a\hat{\nu}) U_\nu^\dagger(x)$:

$$(S_G)_{A^4} = -\frac{1}{4!} \int_{BZ} \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{d^4r}{(2\pi)^4} \frac{d^4s}{(2\pi)^4} \times \sum_{\mu\nu\lambda\rho} \sum_{ABCD} \Gamma^{ABCD}_{\mu\nu\lambda\rho}(k,q,r,s) \tilde{A}^A_\mu(k) \tilde{A}^B_\nu(q) \tilde{A}^C_\lambda(r) \tilde{A}^D_\rho(s) \ .$$

$$\begin{aligned}
&\Gamma^{ABCD}_{\mu\nu\lambda\rho}(k,q,r,s) = \\
&-g_0^2 \Big[\sum_E f_{ABE} f_{CDE} \Big\{ \delta_{\mu\lambda}\delta_{\nu\rho} [\cos\tfrac{1}{2}a(q-s)_\mu \cos\tfrac{1}{2}a(k-r)_\nu - \tfrac{a^4}{12}\tilde{k}_\nu \tilde{q}_\mu \tilde{r}_\nu \tilde{s}_\mu] \\
&\quad - \delta_{\mu\rho}\delta_{\nu\lambda} [\cos\tfrac{1}{2}a(q-r)_\mu \cos\tfrac{1}{2}a(k-s)_\nu - \tfrac{a^4}{12}\tilde{k}_\nu \tilde{q}_\mu \tilde{r}_\mu \tilde{s}_\nu] \\
&\quad + \tfrac{1}{6}\delta_{\nu\lambda}\delta_{\nu\rho} a^2 (\widetilde{s-r})_\mu \tilde{k}_\nu \cos(\tfrac{1}{2}aq_\mu) \\
&\quad - \tfrac{1}{6}\delta_{\mu\lambda}\delta_{\mu\rho} a^2 (\widetilde{s-r})_\nu \tilde{q}_\mu \cos(\tfrac{1}{2}ak_\nu) \\
&\quad + \tfrac{1}{6}\delta_{\mu\nu}\delta_{\mu\rho} a^2 (\widetilde{q-k})_\lambda \tilde{r}_\rho \cos(\tfrac{1}{2}as_\lambda) \\
&\quad - \tfrac{1}{6}\delta_{\mu\nu}\delta_{\mu\lambda} a^2 (\widetilde{q-k})_\rho \tilde{s}_\lambda \cos(\tfrac{1}{2}ar_\rho) \\
&\quad + \tfrac{1}{12}\delta_{\mu\nu}\delta_{\mu\lambda}\delta_{\mu\rho} a^2 \sum_\sigma (\widetilde{q-k})_\sigma (\widetilde{s-r})_\sigma \Big\} \\
&\quad + (B \leftrightarrow C, \nu \leftrightarrow \lambda, q \leftrightarrow r) + (B \leftrightarrow D, \nu \leftrightarrow \rho, q \leftrightarrow s) \Big] \\
&\quad + \tfrac{g_0^2}{12} a^4 \Big\{ \tfrac{2}{3}(\delta_{AB}\delta_{CD} + \delta_{AC}\delta_{BD} + \delta_{AD}\delta_{BC}) \\
&\quad + \sum_E (d_{ABE}d_{CDE} + d_{ACE}d_{BDE} + d_{ADE}d_{BCE}) \Big\} \\
&\quad \times \Big\{ \delta_{\mu\nu}\delta_{\mu\lambda}\delta_{\mu\rho} \sum_\sigma \tilde{k}_\sigma \ \tilde{q}_\sigma \tilde{r}_\sigma \tilde{s}_\sigma - \delta_{\mu\nu}\delta_{\mu\lambda}\tilde{k}_\rho \tilde{q}_\rho \tilde{r}_\rho \tilde{s}_\mu \\
&\quad - \delta_{\mu\nu}\delta_{\mu\rho}\tilde{k}_\lambda \tilde{q}_\lambda \tilde{s}_\lambda \tilde{r}_\mu - \delta_{\mu\lambda}\delta_{\mu\rho}\tilde{k}_\nu \tilde{r}_\nu \tilde{s}_\nu \tilde{q}_\mu - \delta_{\nu\lambda}\delta_{\nu\rho}\tilde{q}_\mu \tilde{r}_\mu \tilde{s}_\mu \tilde{k}_\nu \\
&\quad + \delta_{\mu\nu}\delta_{\lambda\rho}\tilde{k}_\lambda \tilde{q}_\lambda \tilde{r}_\mu \tilde{s}_\mu + \delta_{\mu\lambda}\delta_{\nu\rho}\tilde{k}_\nu \tilde{r}_\nu \tilde{q}_\mu \tilde{s}_\mu + \delta_{\mu\rho}\delta_{\nu\lambda}\tilde{k}_\nu \tilde{s}_\nu \tilde{q}_\mu \tilde{r}_\mu \Big\}
\end{aligned}$$

H. Rothe, 《Lattice Gauge Theories, An Introduction》, Eq.15.53

Lattice regularization

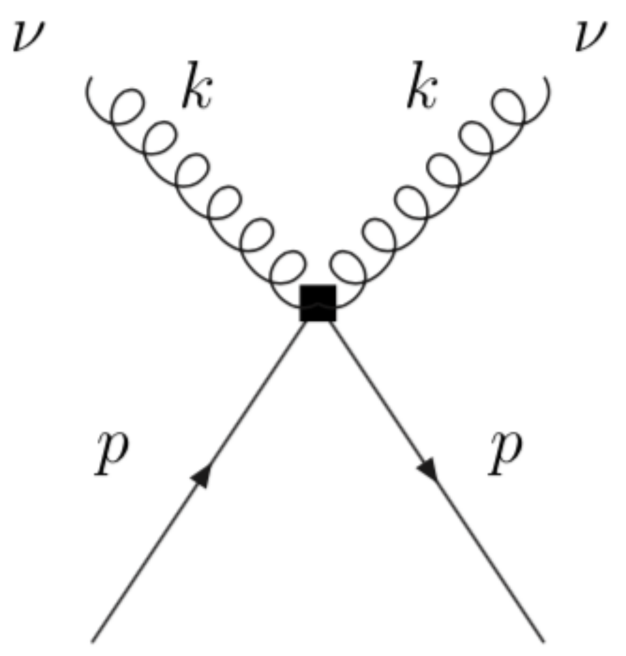
Additional vertex

Taking the simplest Wilson fermion as example:

$$\mathcal{S}_q^W(m) = \sum_{x,y} \bar{\psi}(x) D_w(m;x,y)\psi(y),\ D_w(m;x,y) = \frac{1}{2a}\sum_{\mu}\Big((1+\gamma_\mu)U_\mu(x)\delta_{y,x+a\hat{\mu}} + (1-\gamma_\mu)U_\mu^\dagger(x-a\hat{\mu})\delta_{y,x-a\hat{\mu}}\Big) - (m+\frac{4}{a})\delta_{y,x}$$

where $U(x) \equiv e^{ig_0\int_x^{x+a\hat{\mu}} dy A_\mu(y)}$:

There is a g-g-q-q vertex at $\mathcal{O}(a)$: $\cdot g_0^2 \cdot$



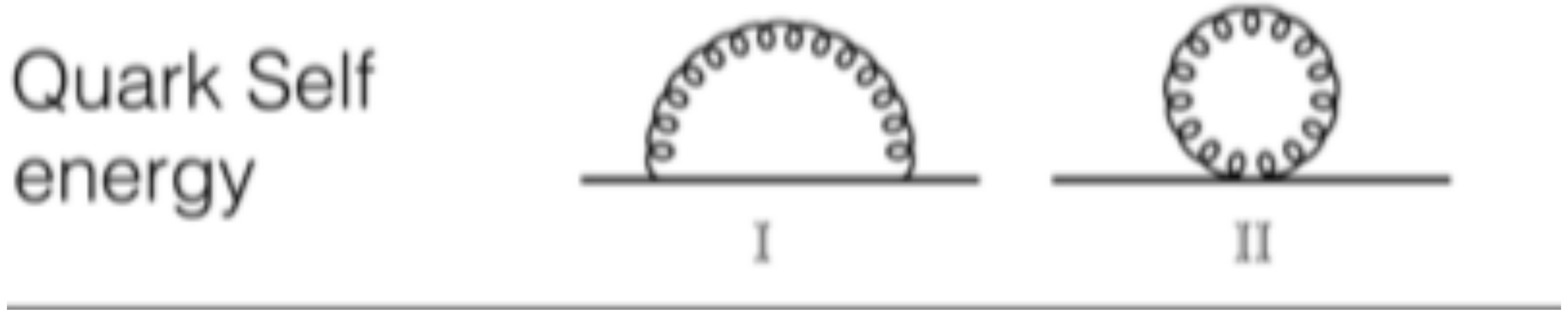
$-\frac{a}{2}\, C_F\, \mathrm{i}\gamma_\mu \sin ap_\nu$

Such a vertex is $\mathcal{O}(a)$ at tree level, but it can introduce $\mathcal{O}(\alpha_s)$ correction at quantum level!

Lattice regularization

Loop correction

Taking the quark self energy as an example：



The result can be quite different finite $\mathcal{O}(\alpha_s)$ corrections with different discretization：

$$Z_Q^{RI}(p^2) = 1 + \frac{g^2 C_F}{16\pi^2}[(1-\xi)\log(a^2p^2) + B_Q + 4.79\xi] + O(a^2p^2) + O(g^4)$$

Gauge actions

Fermion actions

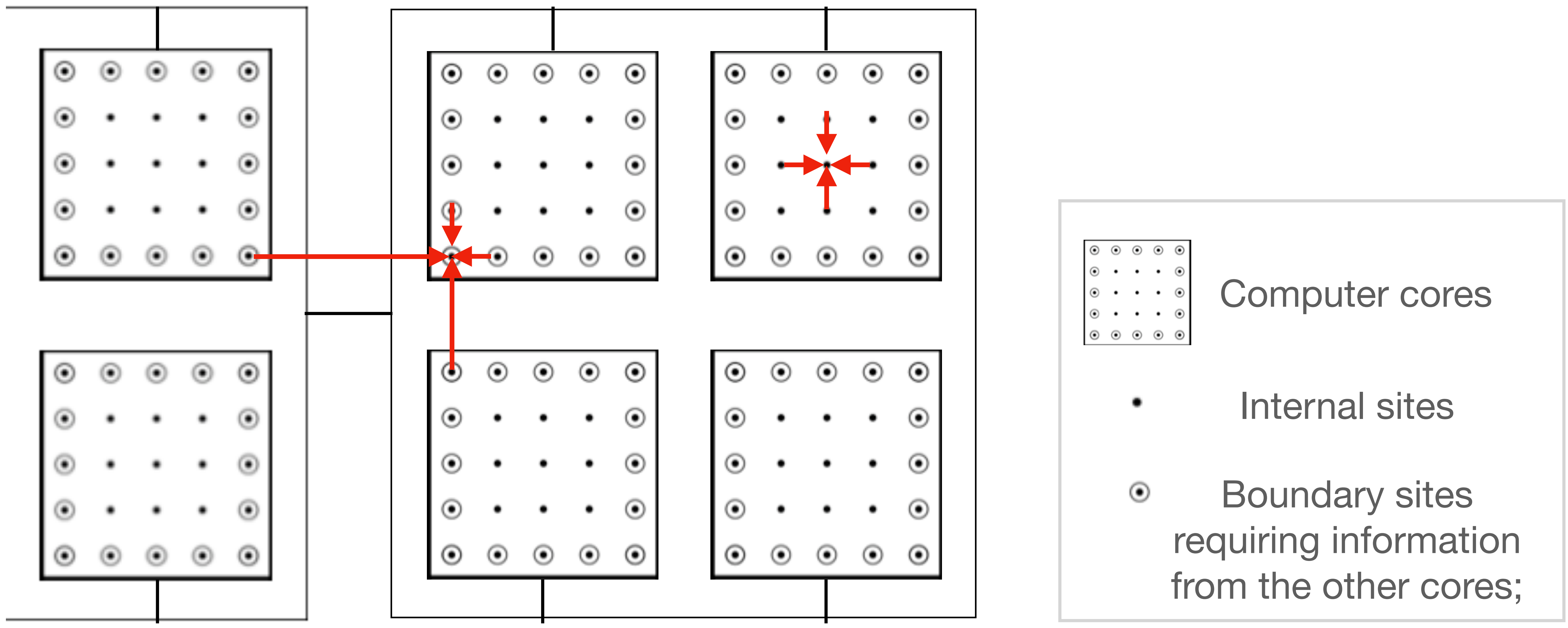
	B_Q Wilson	Iwasaki	Iwasaki^{HYP}
Wilson	11.85	3.32	-4.22
overlap	-21.50	-13.58	-7.56

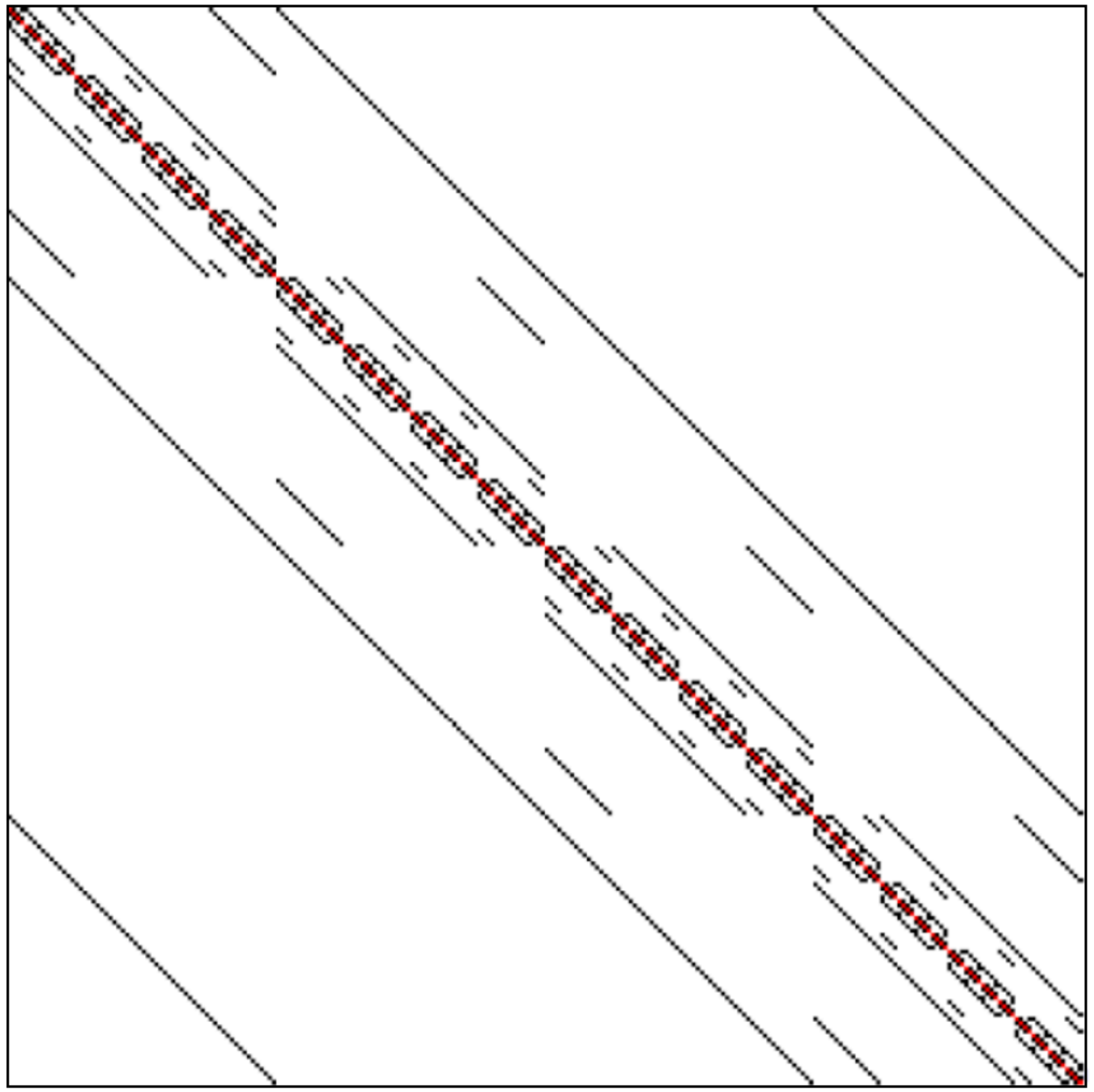
Basic unit of Lattice QCD

$$\left(\gamma_4(\partial_\tau - igA_4)\psi + \sum (\partial_i - igA_i)\gamma_i - m\right)\psi = 0$$

The discretized Dirac equation with the coupling with the non-abelian SU(3) gauge field:

- $\gamma_{1,2,3,4}$ are 4×4 complex matrices, $A_{1,2,3,4}$ are space-time dependent 3x3 complex matrices;
- Can be converted to a problem of sparse matrix inversion.





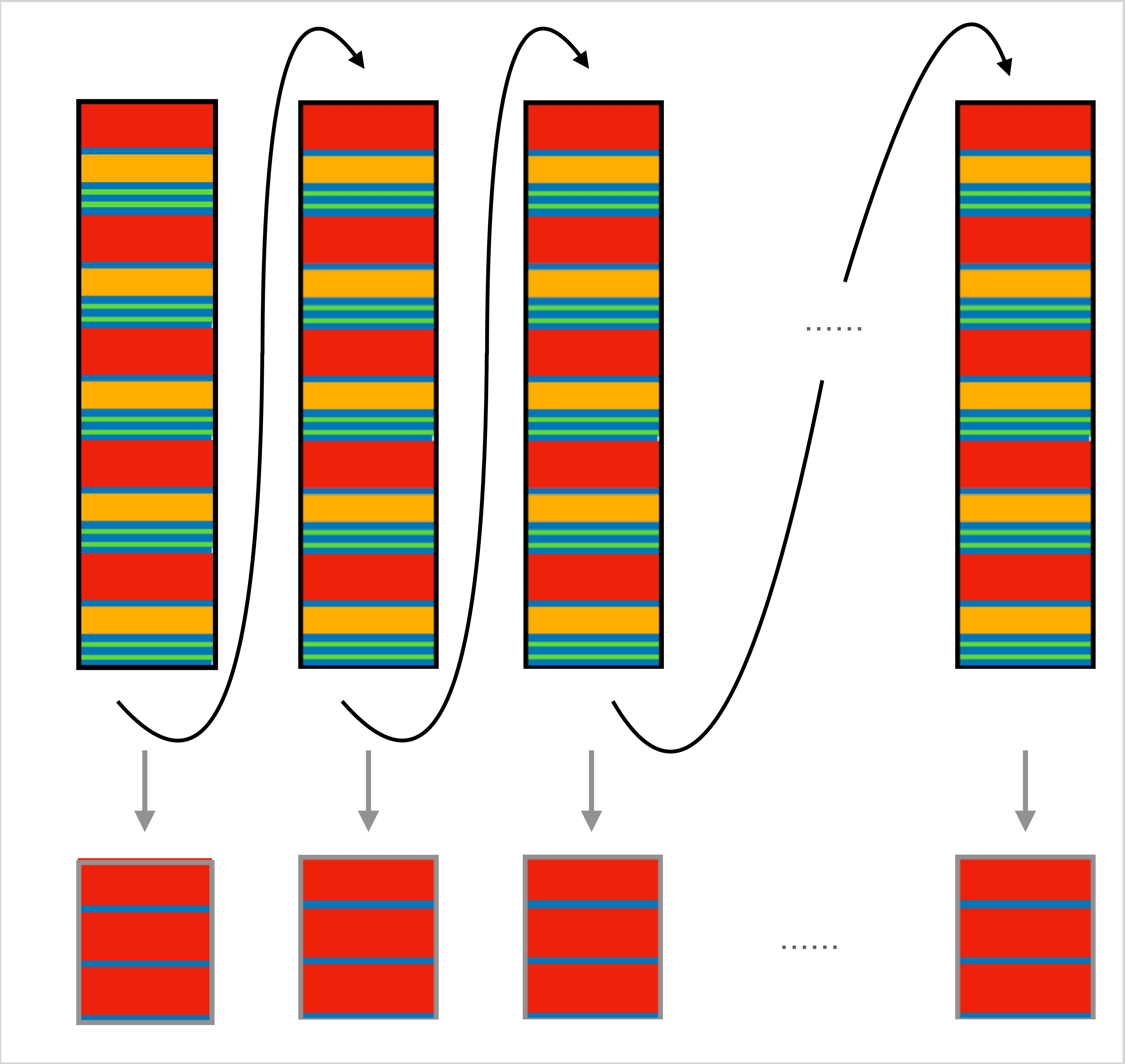
$L^3 \times T = 4^3 \times 4$ lattice:

- Red point: 12×12 diagonal matrix
- Black point: 12×12 sparse matrix

Basic flow of Lattice QCD

hotspot： sparse linear operation,

$$\xi = D\eta$$



Generate **configurations** using important sampling

Analysis the **configurations** to get the physical results

Linear system solver

$$\xi = D^{-1}\eta = \sum_i c_i D^i \eta + \mathcal{O}(10^{-12})$$

Linear algebra operation,

$$\xi = c_1\eta_1 + (\eta_2^\dagger \cdot \eta_3)\eta_4$$

Square root of sparse matrix,

$$\xi = (D+m_0)^{-1/2}\eta = \sum_i \frac{d_i}{D+m_0+e_i}\eta + \mathcal{O}(10^{-12})$$

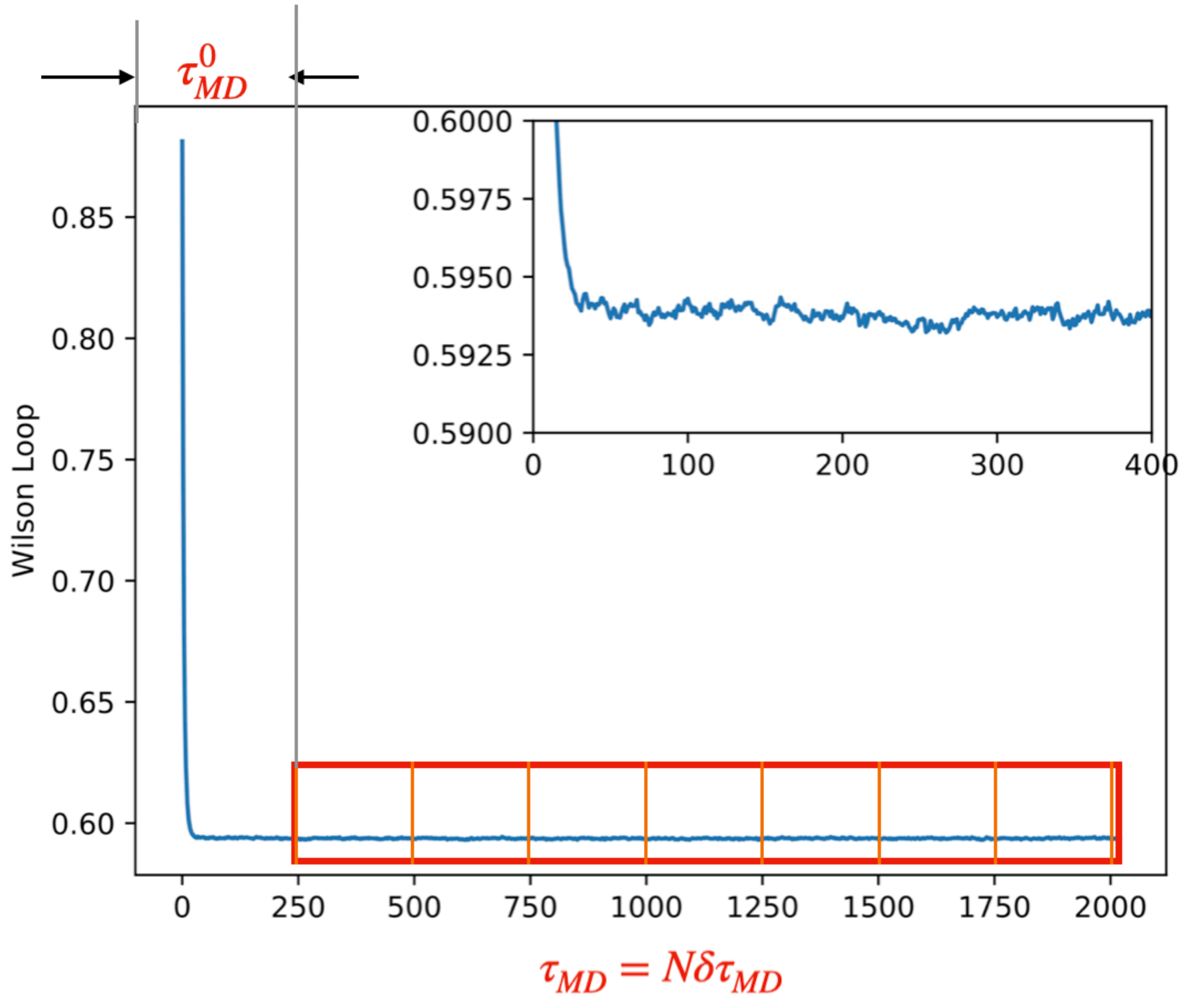
Derivative of sparse matrix,

$$\xi = \frac{\partial D}{\partial U}\eta = D_1 D_2 D_3 \eta$$

- The major hotspot is linear system solver；
- But after the acceleration of this hotspot, linear algebra operation, square root and derivative of sparse matrix will be the bottlenecks of the performance.
- Configurations are the foundation of all the physical analysis!

Configuration：

Markov chain Monte Carlo



- Specify the discretized quark and gluon actions
- Specify the flavors of quarks and their bare masses
- Specify the bare gauge coupling

→Determine the lattice spacing through Λ_{QCD}

- Specify the lattice size $L^3 \times T$

Case 1: Clover fermion, 24^3x72, a=0.108 fm, m_π=300 MeV, 8 V100 GPUs:

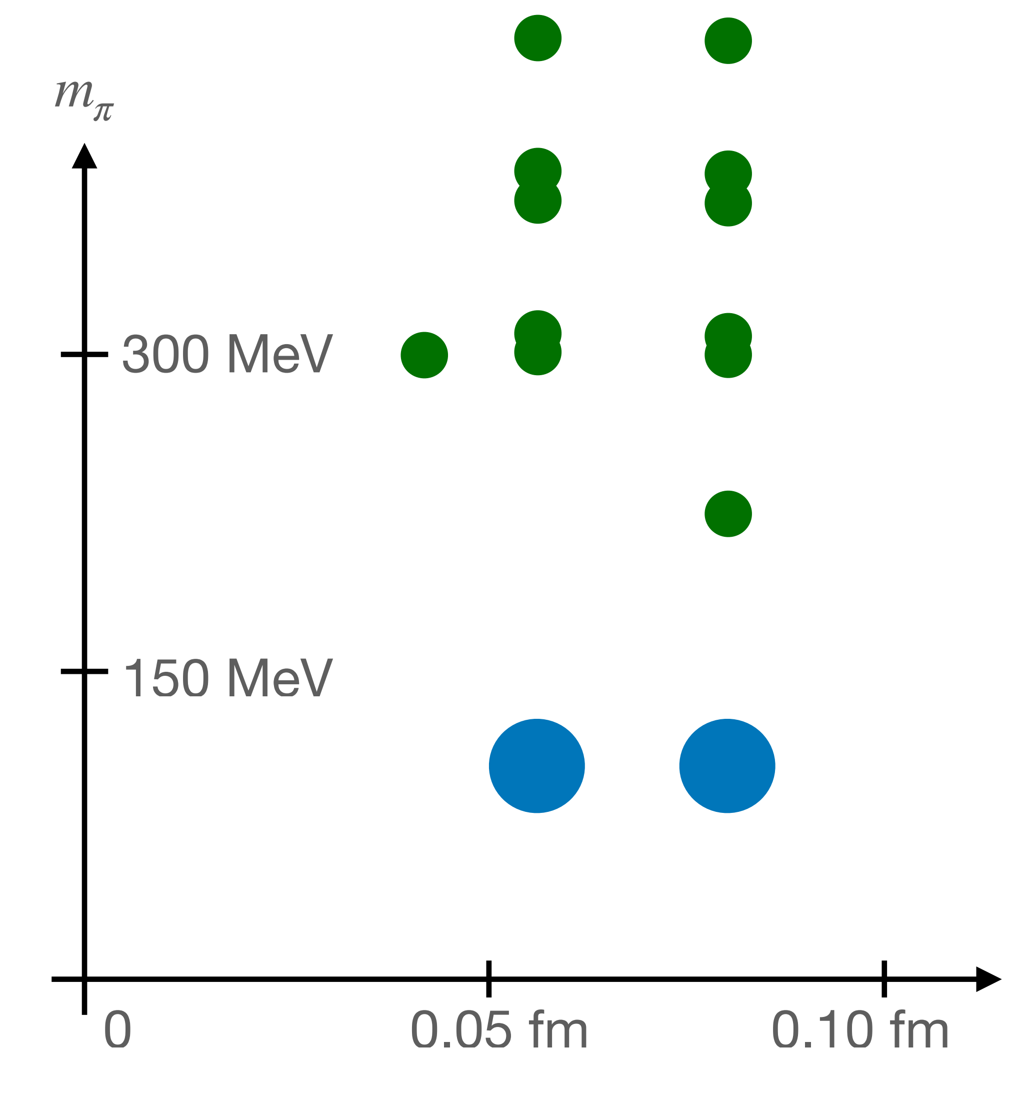
- One week for warn-up;
- Another week for 200 configurations (5 traj. per conf.)

Case 2: Mobius DW (chiral fermion), 96^3x192, a =0.071 fm, m_π=140 MeV, 512 V100 GPUs:

- One year for warn-up;
- Another year for 200 configurations (5 traj. per conf.)

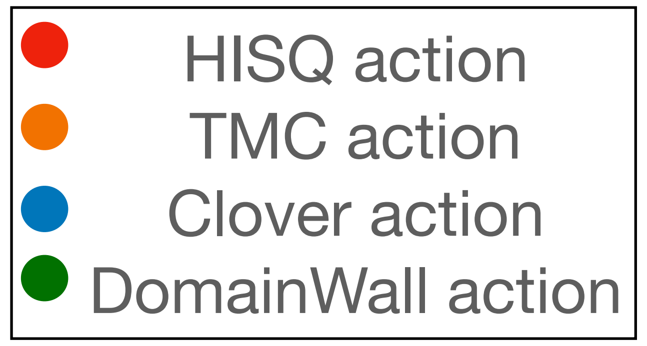
Configurations:

- Three different m_π with the lightest m_π smaller than 200 MeV, or one $m_\pi \sim 135$ MeV and another $m_\pi < 200$ MeV;
- Three lattice spacings a, with two of $a < 0.1$ fm, and $(a_{max}/a_{min})^2 \geq 2$;
- $(m_{\pi,\mathrm{min}}/M_{\pi,\mathrm{phys}})^2 \exp\{4 - m_{\pi,\mathrm{min}} L\} < 2$, or at least three volumes.

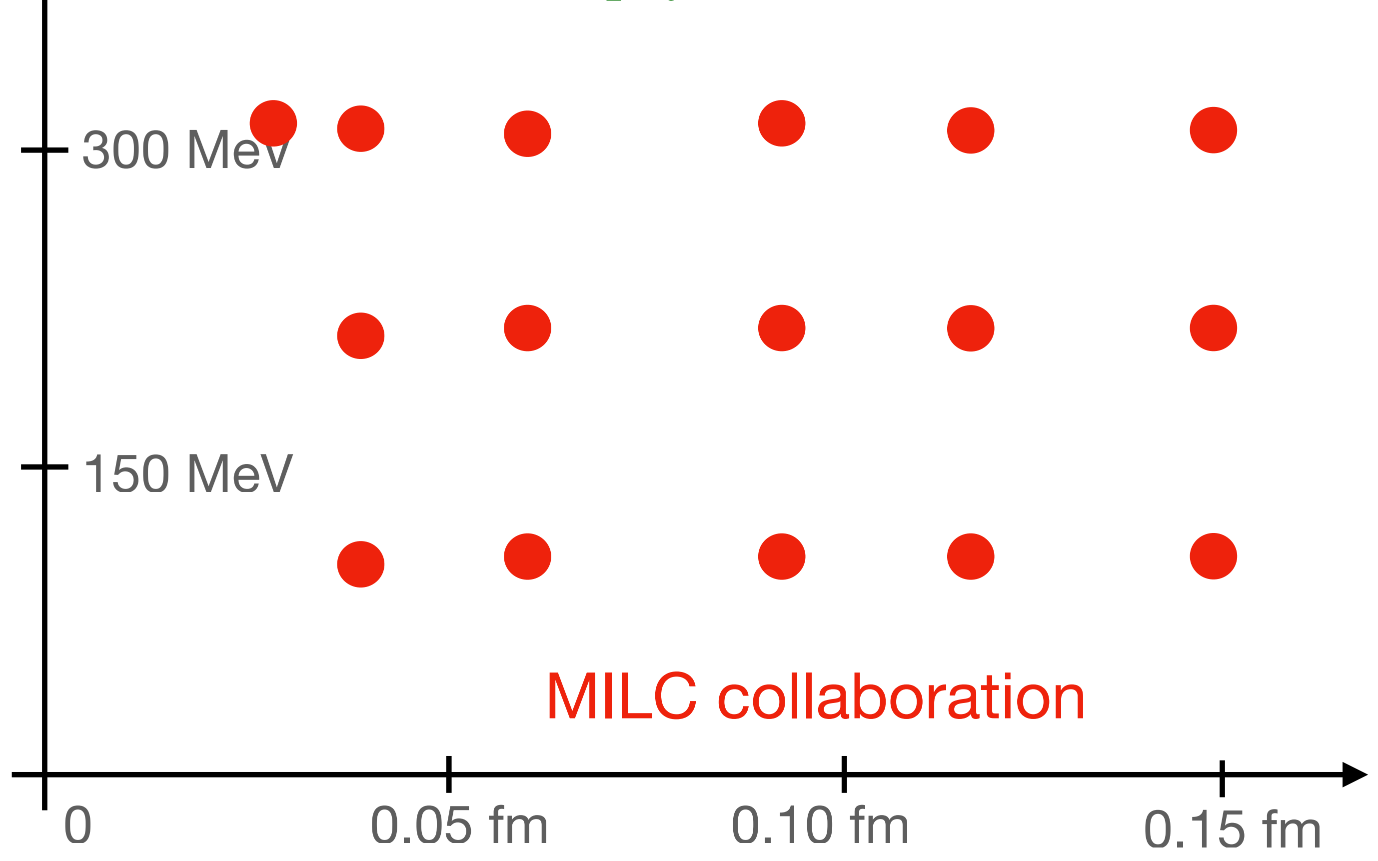


JLQCD collaboration

PACS collaboration



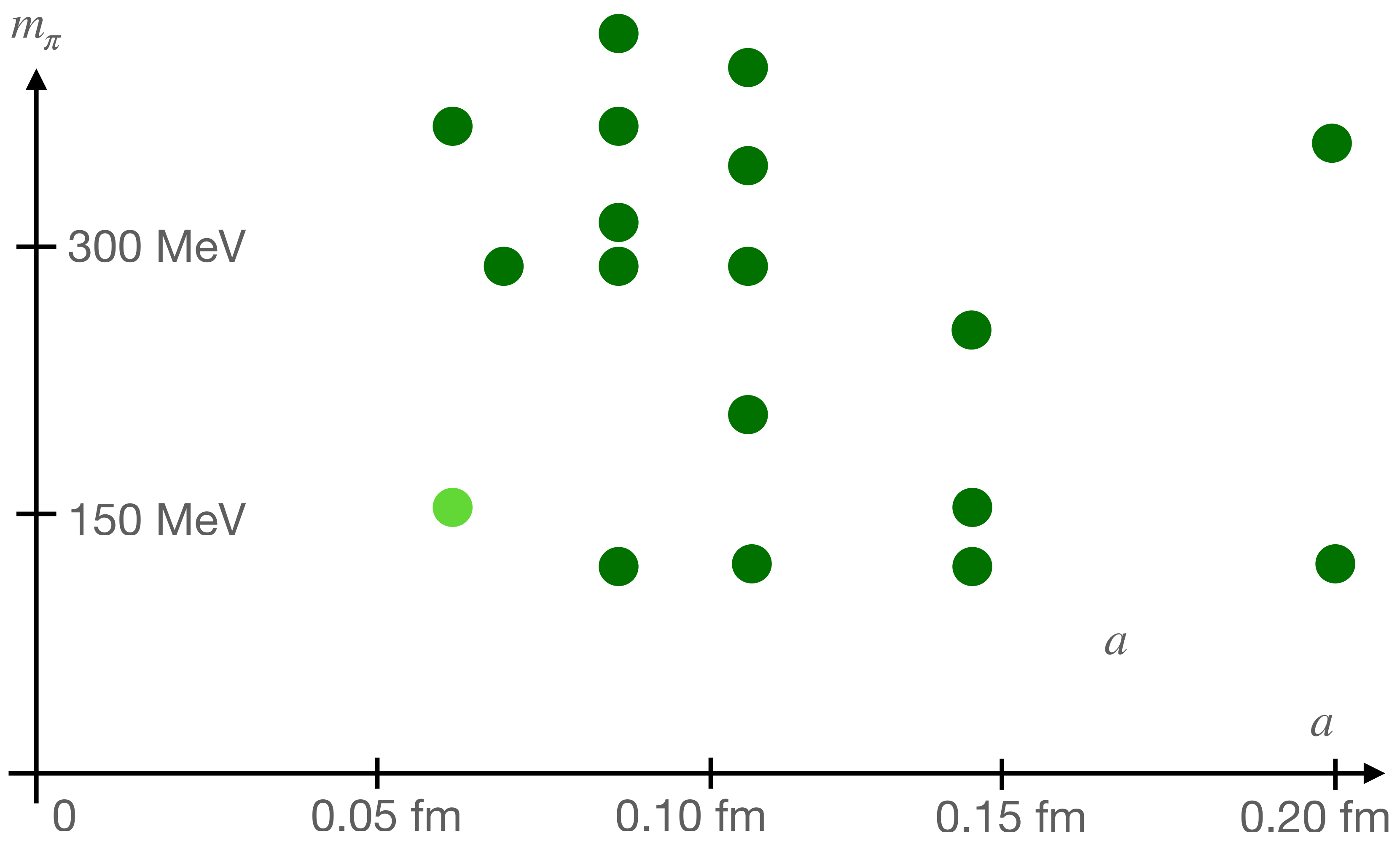
m_π



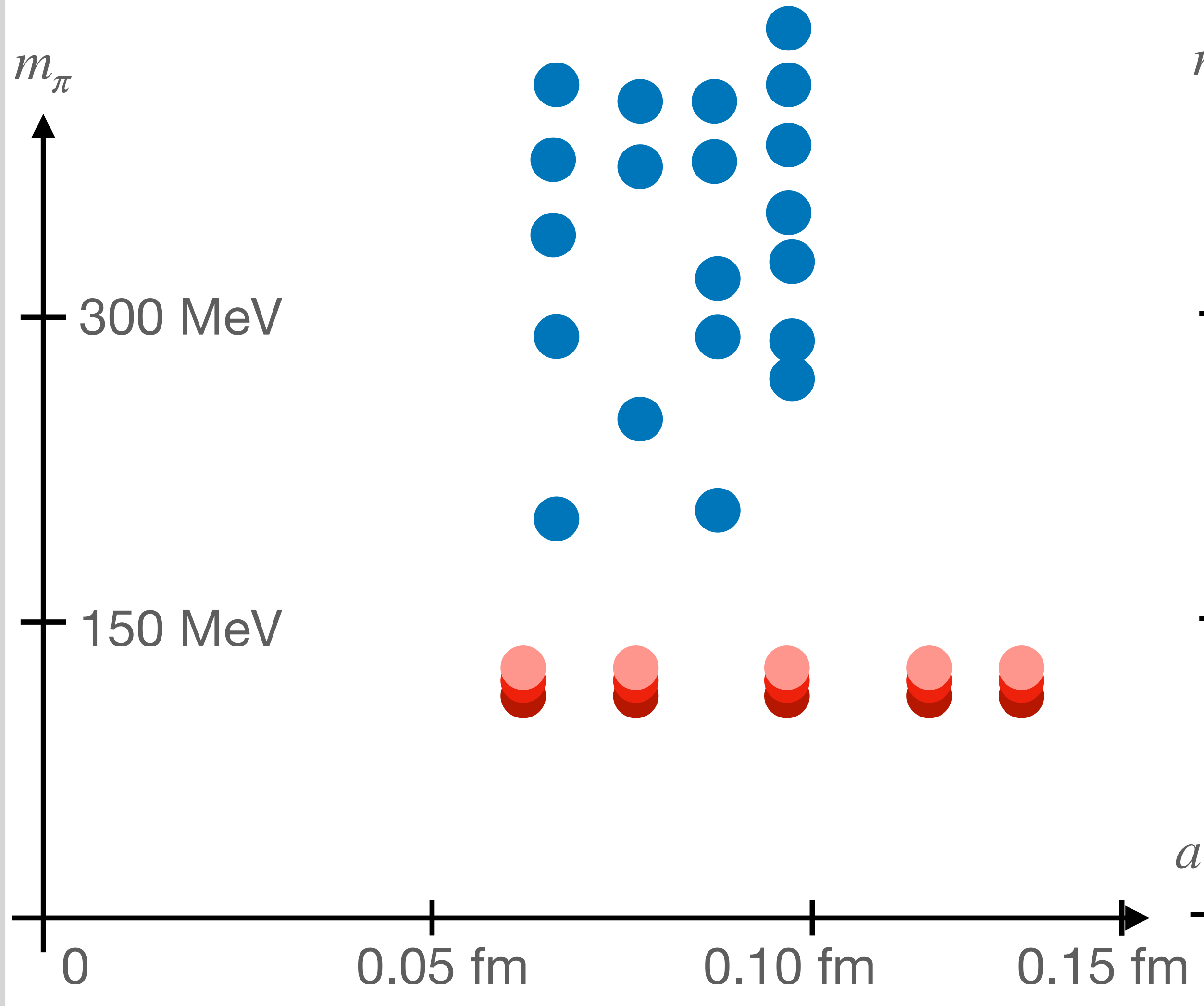
US

JP

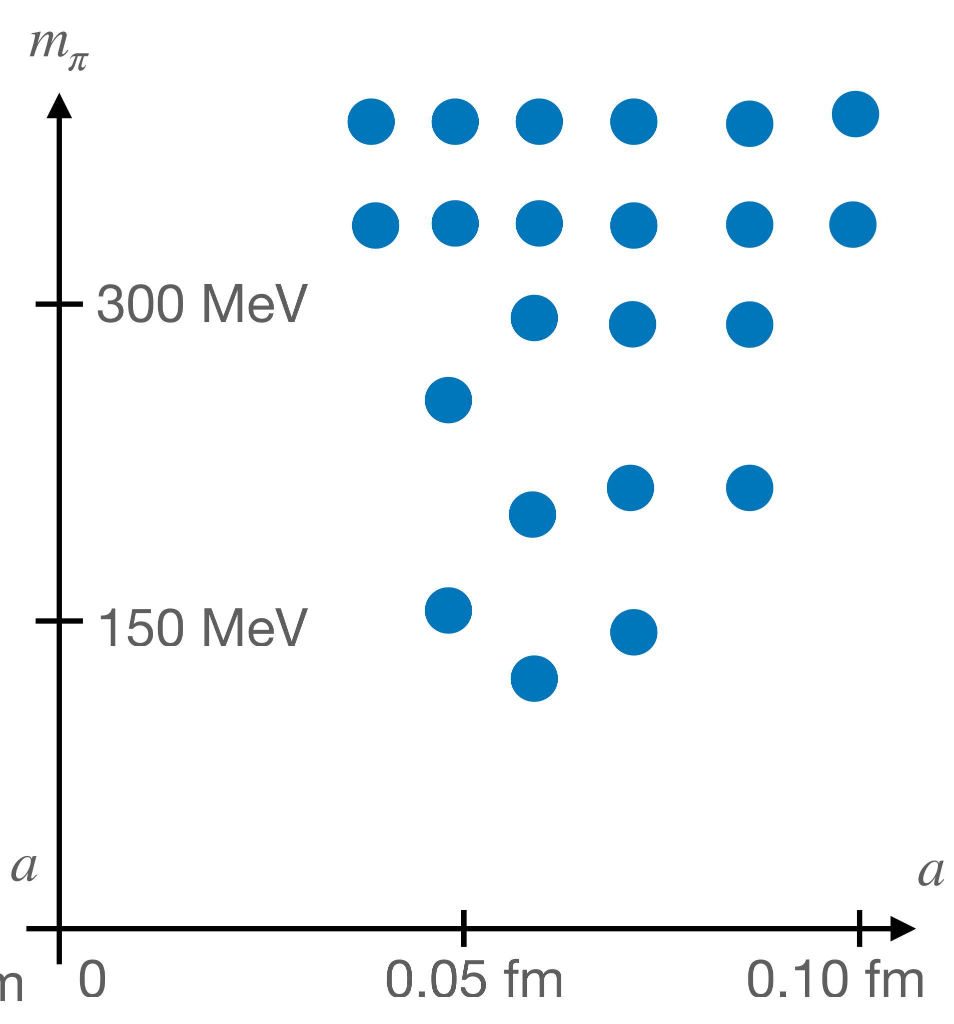
EU



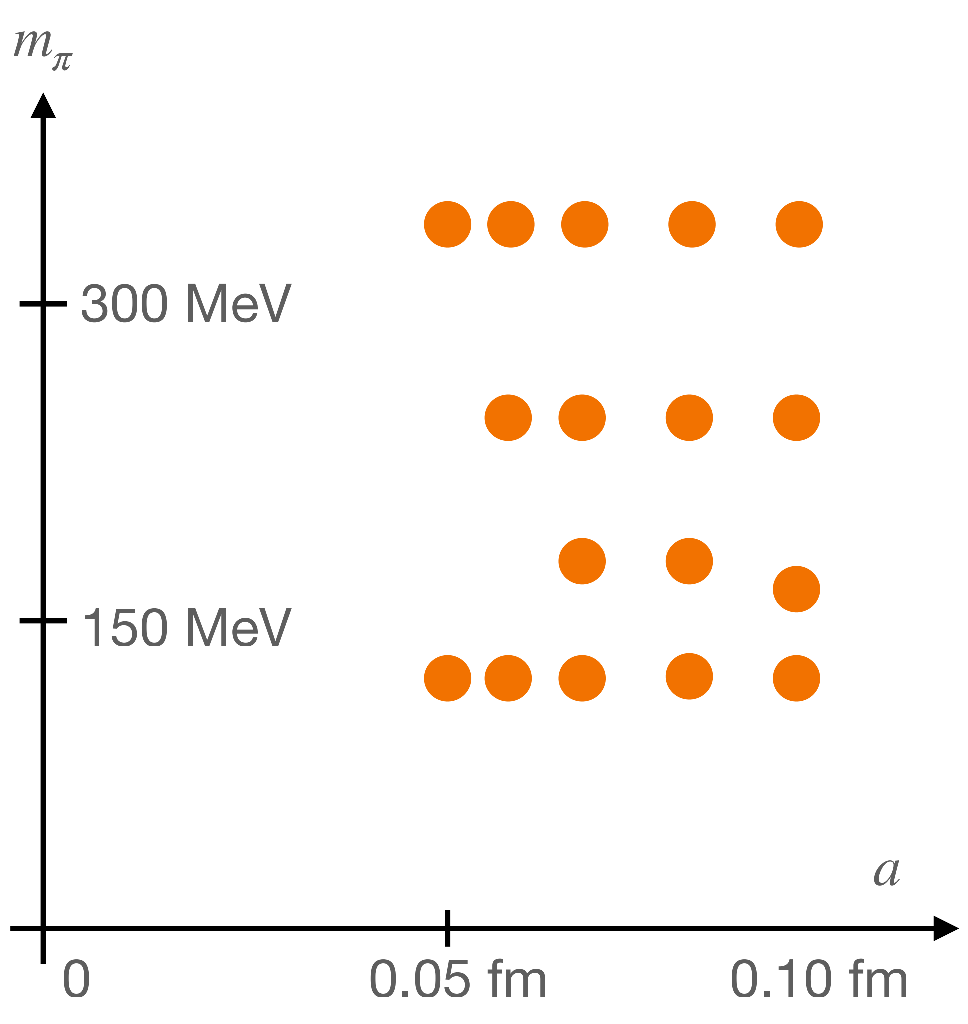
RBC/UKQCD collaboration



BMW collaboration



CLS collaboration



ETM collaboration

Configurations in China：Towards the FLAG criteria

- Generate the configurations using the domestic computers is the foundation of any high-precision lattice QCD study.
- FLAG criteria is the current status-of-the-arts in the lattice community.
- Major contributors：P. Sun, L. Liu, YBY, W. Sun,…

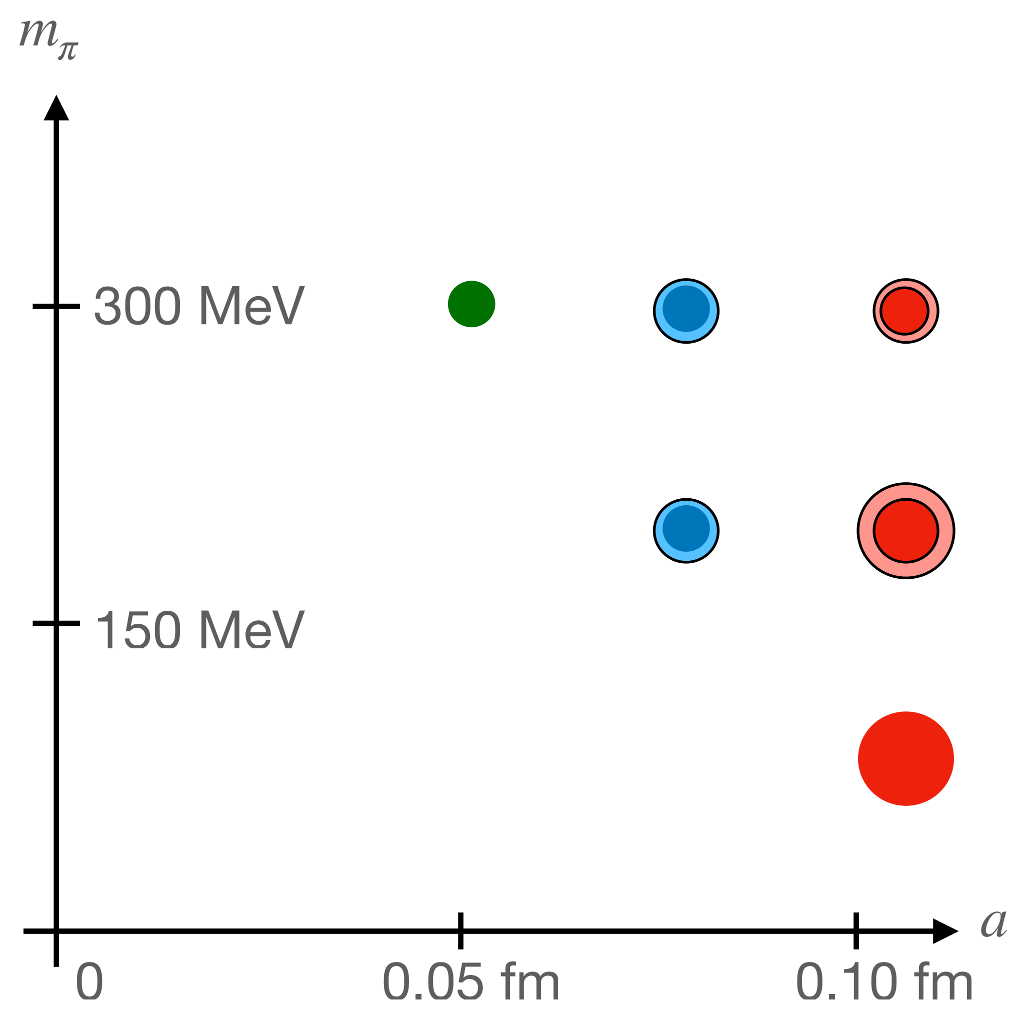
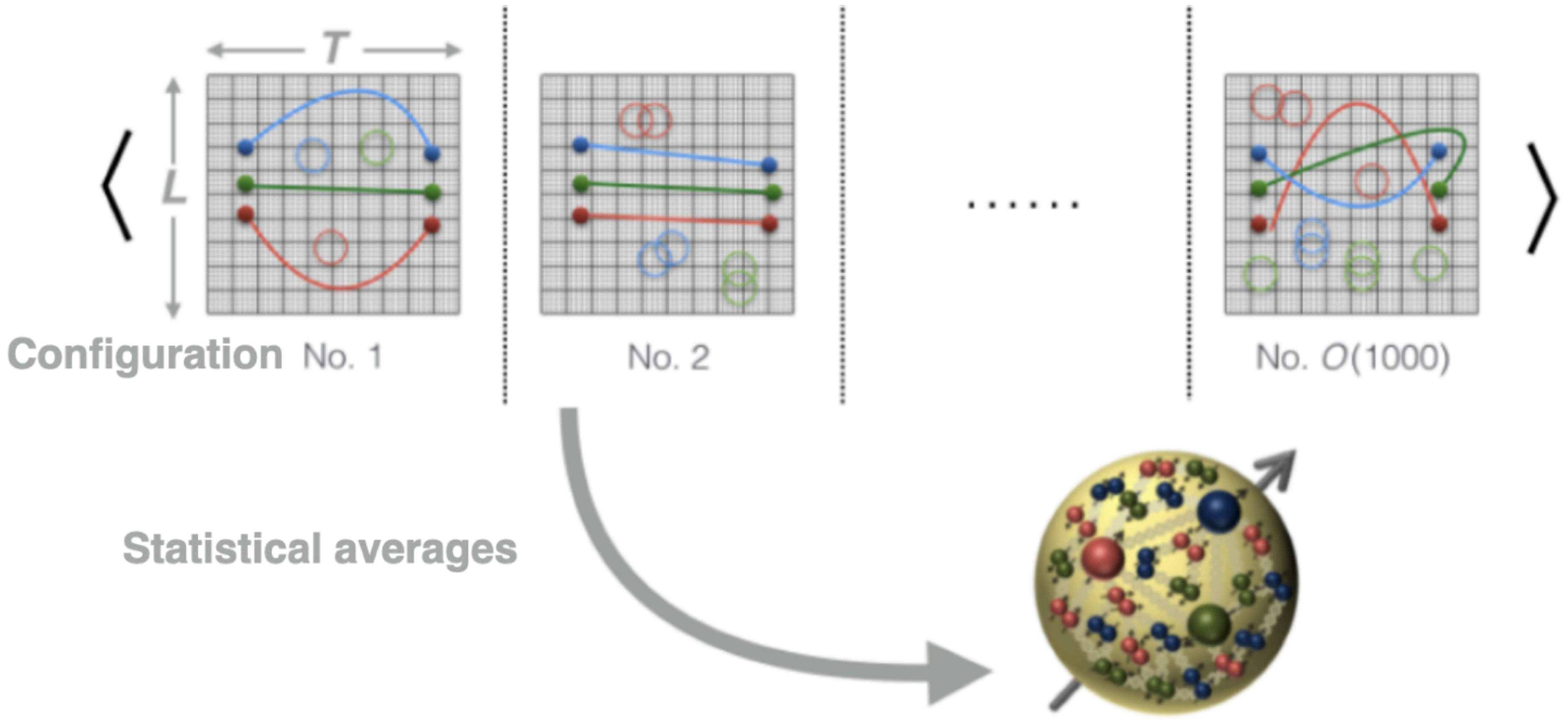


TABLE I. Lattice size $L^3 \times T$, gauge coupling $10/g^2$, bare quark masses $m^{\mathrm{b}}_{l,s,c}$, tadpole improvement factors u_0/v_0 and scale parameter w_0 of the ensembles used in this work. The bare light and starnge quark masses $m^{\mathrm{b}}_{l,s}$ with the bold font on each ensemble are the unitary quark masses, and the other values of $m^{\mathrm{b}}_{l,s,c}$ are those used for the valence quark propagators. The values u^{I}_0 and v^{I}_0 are the tadpole improvement factors used in the gauge and fermion actions, respectively; and u_0, v_0, $w_0 a$ are those measured from the realistic configurations generated using the Paramters here.

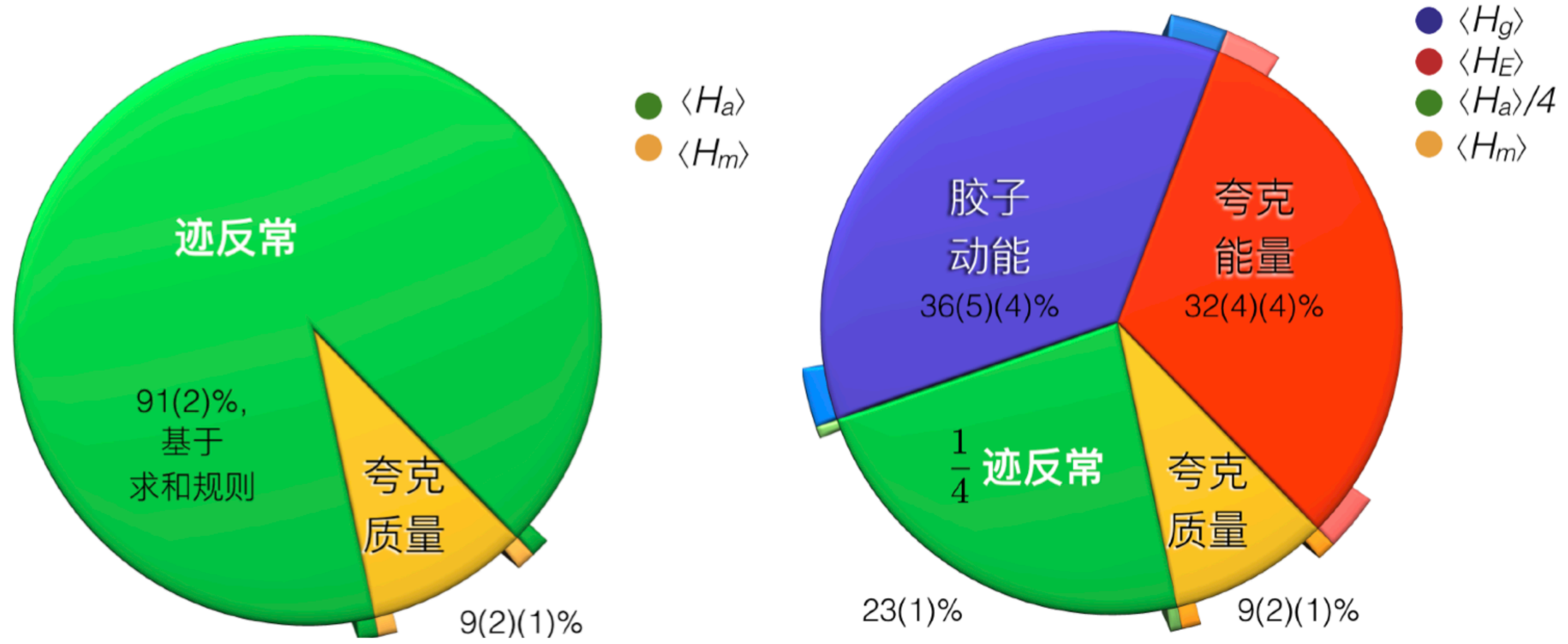
	C11P29Ss	C11P29S	C11P29M	C11P22M	C11P22L	C11P14L	C08P30S	C08P30M	C08P22S	C08P22M	C06P30S
$L^3 \times T$	$24^3 \times 64$	$24^3 \times 72$	$32^3 \times 64$	$32^3 \times 64$	$48^3 \times 96$	$48^3 \times 96$	$32^3 \times 96$	$48^3 \times 96$	$32^3 \times 64$	$48^3 \times 96$	$48^3 \times 144$
$10/g^2$	6.20						6.41				6.72
m^{b}_l	**-0.2770** -0.2760 -0.2750	**-0.2770** -0.2760 -0.2750	-0.2780 **-0.2770** -0.2760	**-0.2790** -0.2780 -0.2770	**-0.2790** -0.2780 -0.2770	**-0.2825** -0.2820 -0.2815	**-0.2295** -0.2288 -0.2275	-0.2307 **-0.2295** -0.2288	**-0.2320** -0.2307 -0.2295	**-0.2320** -0.2307 -0.2295	**-0.1850** -0.1845 -0.1840
m^{b}_s	-0.2400 -0.2355 **-0.2310**	**-0.2400** -0.2355 -0.2310	**-0.2400** -0.2355 -0.2310	**-0.2400** -0.2355 -0.2310	**-0.2400** -0.2355 -0.2310	-0.2400 -0.2355 **-0.2310**	**-0.2050** -0.2030 -0.2010	**-0.2050** -0.2030 -0.2010	**-0.2050** -0.2030 -0.2010	**-0.2050** -0.2030 -0.2010	**-0.1700** -0.1694 -0.1687
m^{b}_c	0.4780 0.4800 0.4820	0.4780 0.4800 0.4820	0.4780 0.4800 0.4820	0.4780 0.4800 0.4820	0.4780 0.4800 0.4820	0.4780 0.4800 0.4820	0.2326 0.2340 0.2354	0.2326 0.2340 0.2354	0.2326 0.2340 0.2354	0.2326 0.2340 0.2354	0.0770 0.0780 0.0790
δ_τ	1.0	0.7	0.7	0.7	0.7	1.0	0.5	0.5	0.5	0.5	1.0
n_{min}		4050	11000	4100	1000	1600	1000	2690	13500	1600	1000
n_{max}		48000	35050	26600	5050	2200	26200	6700	36400	6060	4070
u^{I}_0	0.855453	0.855453	0.855453	0.855520	0.855520	0.855548	0.863437	0.863473	0.863488	0.863499	0.873378
v^{I}_0	0.951479	0.951479	0.951479	0.951545	0.951545	0.951570	0.956942	0.956984	0.957017	0.957006	0.963137
u_0		0.855440	0.855422			0.855539	0.863463				0.873373
v_0		0.951463	0.951444			0.951561	0.956971				0.963135
$w_0 a$											

Outline

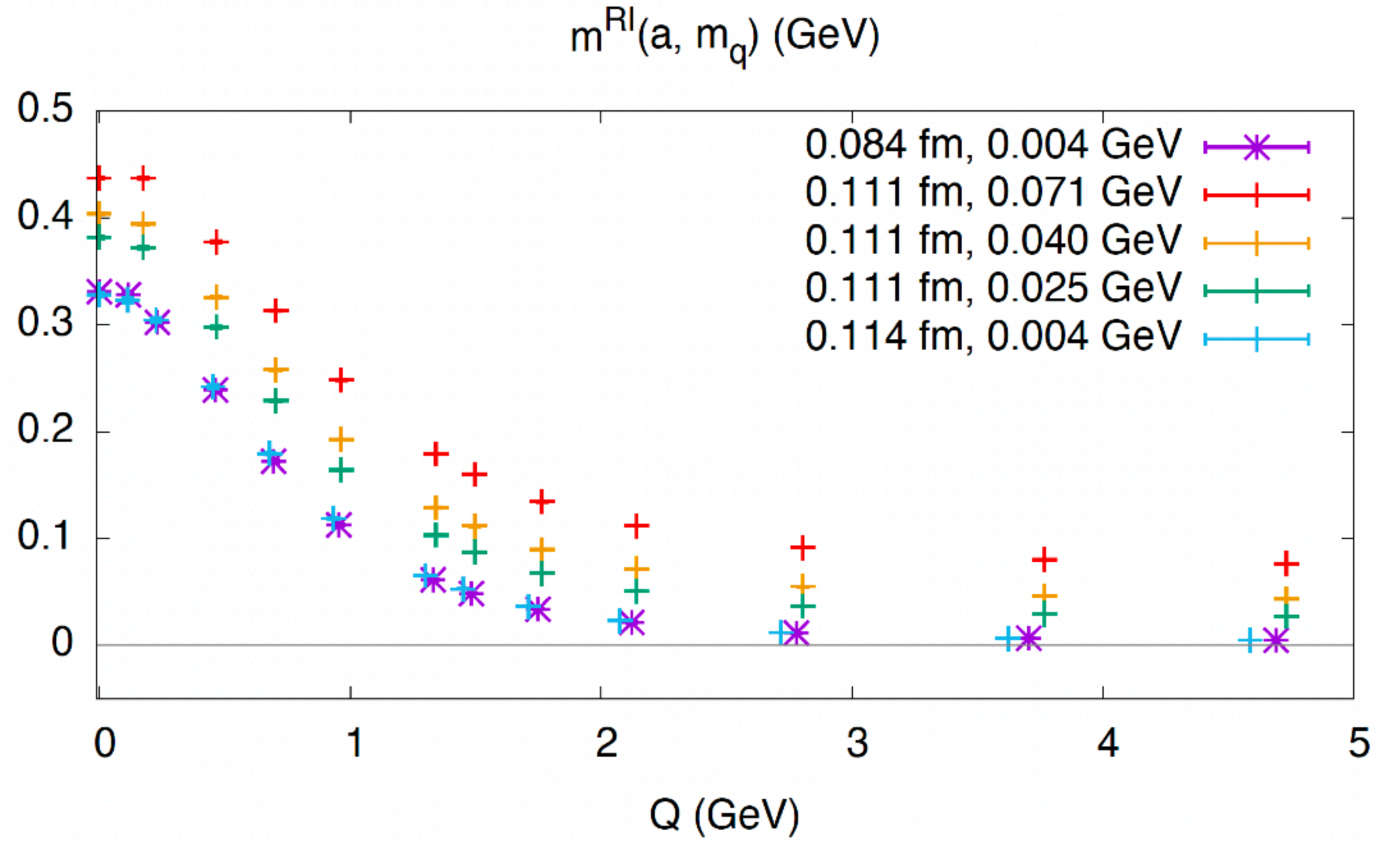
- Brief introduction on Lattice QCD



- Hadron mass and its components;



- Parton masses and their low energy limits.



Hadron mass from Lattice QCD

- From the time order product ($\mathcal{O} = \epsilon_{abc}(u^{a,T}Cd^b)u^c$):

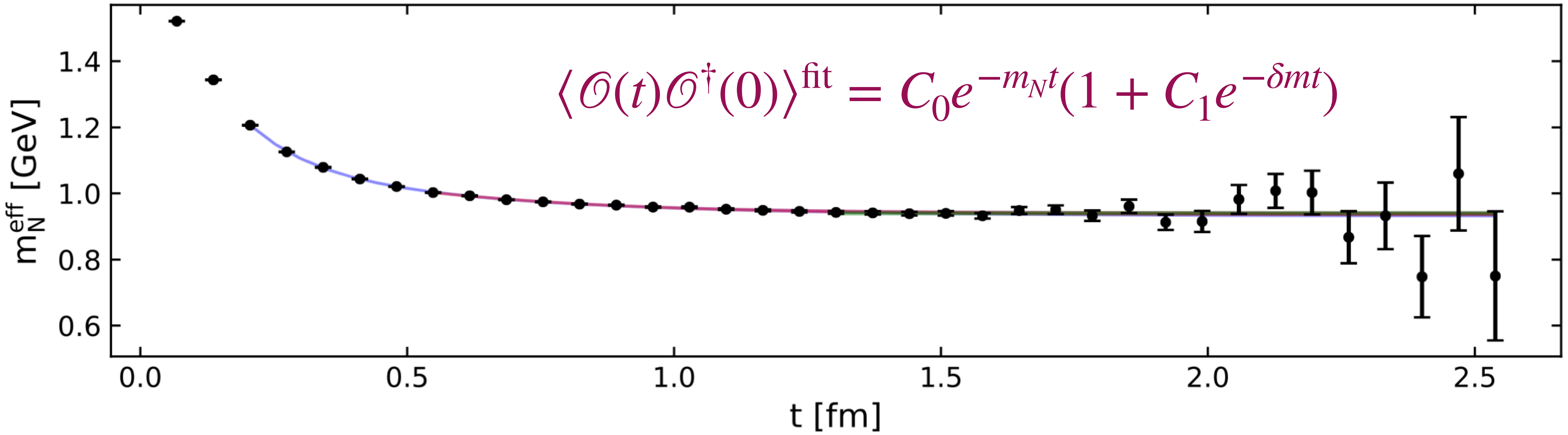
$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \sum_n \langle \mathcal{O}(t)|n\rangle \frac{e^{-E_n t}}{2E_n}\langle n|\mathcal{O}^\dagger(0)\rangle \xrightarrow[t\to\infty]{} \frac{|\langle \mathcal{O}(t)|0\rangle|^2}{2E_0}e^{-E_0 t}$$

- From the path integral ($S(x;y) = (D(x;y)+m)^{-1}$):

$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \sum_{\vec{x}} \langle \epsilon_{abc}\big((u^{a,T}Cd^b)u^c\big)(\vec{x},t)\Gamma_e \epsilon_{a'b'c'}\big((u^{a',T}Cd^{b'})u^{c'}\big)^\dagger(\vec{0},0)\rangle$$

$$= \sum_x \epsilon_{abc}\epsilon_{a'b'c'}\langle \mathrm{Tr}[S_u^{aa',T}(\vec{x},\tau;\vec{0},0)CS_d^{bb'}(\vec{x},\tau;\vec{0},0)C^\dagger]\mathrm{Tr}[\Gamma^e S_u^{cc'}(\vec{x},\tau;\vec{0},0)] + \mathrm{Tr}[S_u^{aa',T}(\vec{x},\tau;\vec{0},0)CS_d^{bb'}(\vec{x},\tau;\vec{0},0)\Gamma^e S_u^{cc'}(\vec{x},\tau;\vec{0},0)C^\dagger]\rangle$$

- All the ground state hadron masses can be obtained with different $\mathcal{O}$ and m.



$$m_N^{\mathrm{eff}} = \frac{1}{a}\log\frac{\langle \mathcal{O}(t-a)\mathcal{O}^\dagger(0)\rangle}{\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle}$$

C. Alexandrou, et,al. ETMC, PRD104(2021)074515

The light quark masses

From lattice QCD

P.Zyla et,al, PTEP(2020)083C01 (PDG2020):

- $m_p = 938.27\ \mathrm{MeV} = m_{p,\mathrm{QCD}} + 1.00(16)\ \mathrm{MeV} + \ldots;$

Sz. Borsanyi, et,al. Science347(2015)1452

- $m_n = 939.57\ \mathrm{MeV};$
- $m_\pi^0 = 134.98\ \mathrm{MeV};$

$(m_{p,\mathrm{QCD}} + m_n)/2 = 938.4(1)\ \mathrm{MeV}$

- $m_\pi^+ = 139.57\ \mathrm{MeV} = m_\pi^0 + 4.53(6)\ \mathrm{MeV} + \ldots;$

X. Feng, et,al. PRL128(2022)062003

- $m_K^0 = 497.61(1)\ \mathrm{MeV} = m_{K,\mathrm{QCD}}^0 + 0.17(02)\ \mathrm{MeV} + \ldots;$
- $m_K^+ = 493.68(2)\ \mathrm{MeV} = m_{K,\mathrm{QCD}}^+ + 2.07(15)\ \mathrm{MeV} + \ldots$

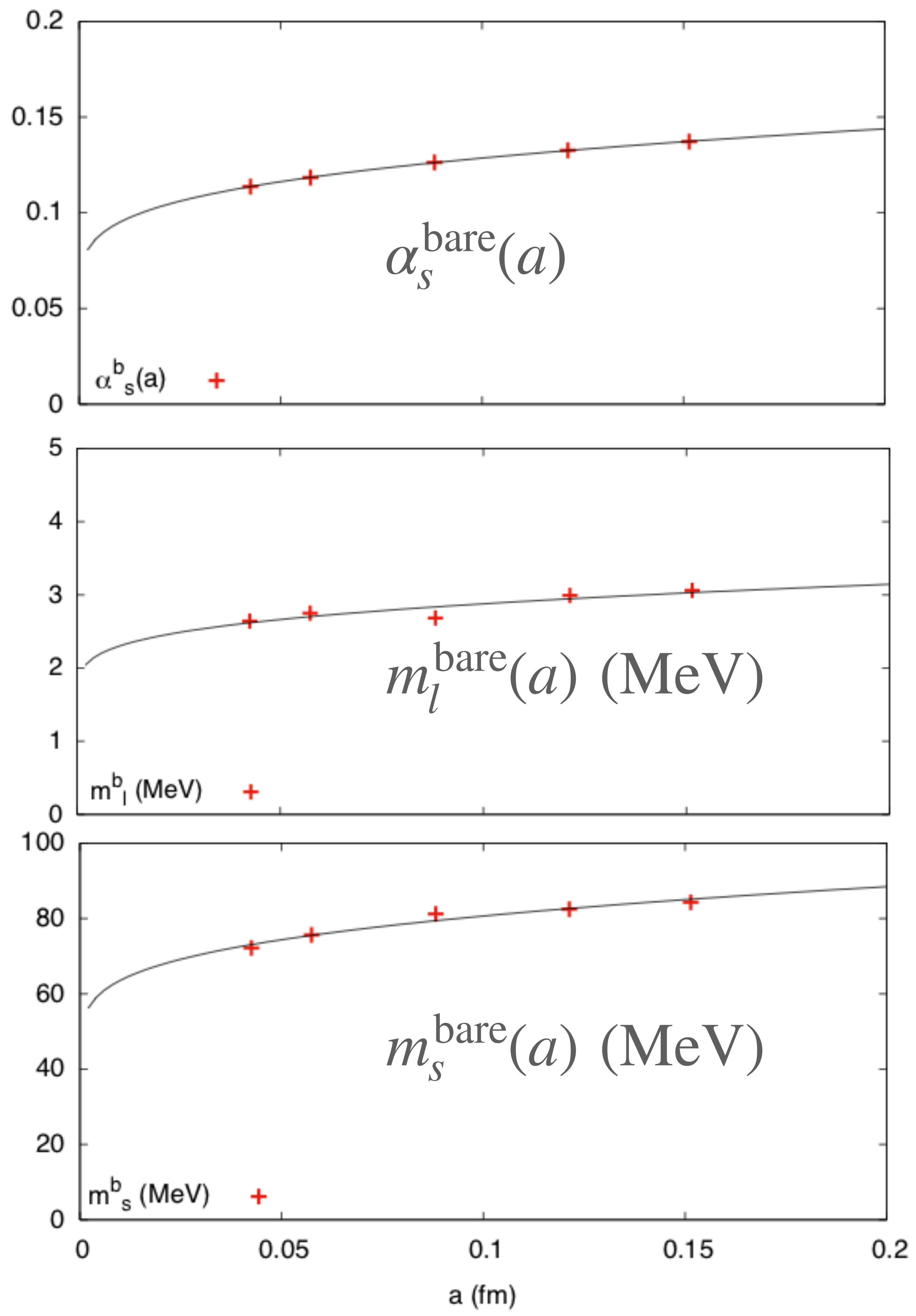
D. Giusti, et,al. PRD95(2017)114504

α_s^{bare}

$(m_u^{\mathrm{bare}} + m_d^{\mathrm{bare}})/2$ and a (lattice spacing)

m_u^{bare}, m_d^{bare} and m_s^{bare}

The light quark masses



A. Bazavov, et,al., MILC, PRD87(2013)054505

Lattice spacing dependence

- The lattice spacing a is very sensitive to the bare coupling;
- The light quark mass to satisfy the condition is very small;
- Renormalization is needed to convert the result to MS-bar.

$$\boxed{\alpha_s^{\mathrm{bare}}}$$

$$\downarrow$$

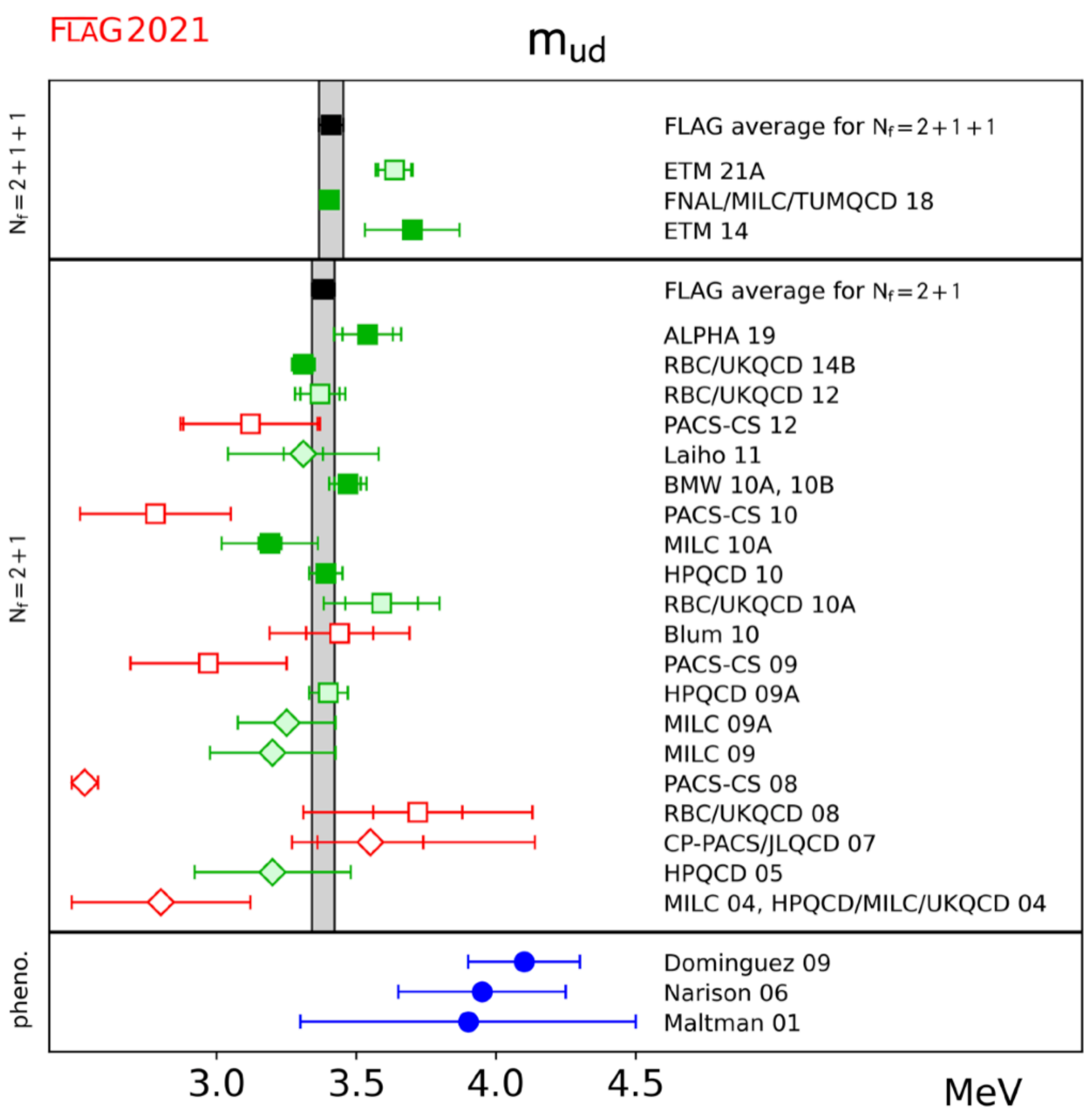
$$\boxed{(m_u^{\mathrm{bare}} + m_d^{\mathrm{bare}})/2 \text{ and } a \text{ (lattice spacing)}}$$

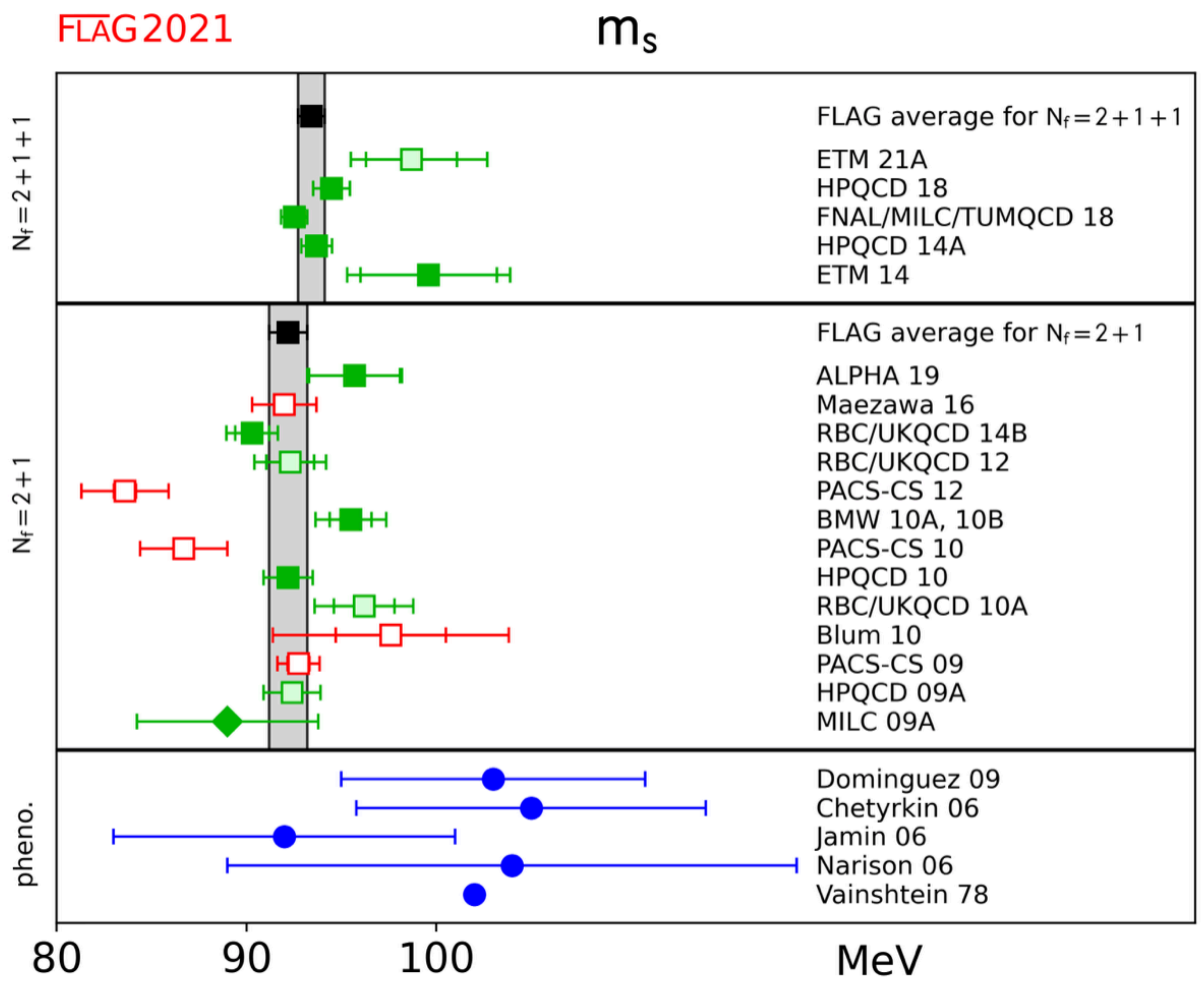
$$\downarrow$$

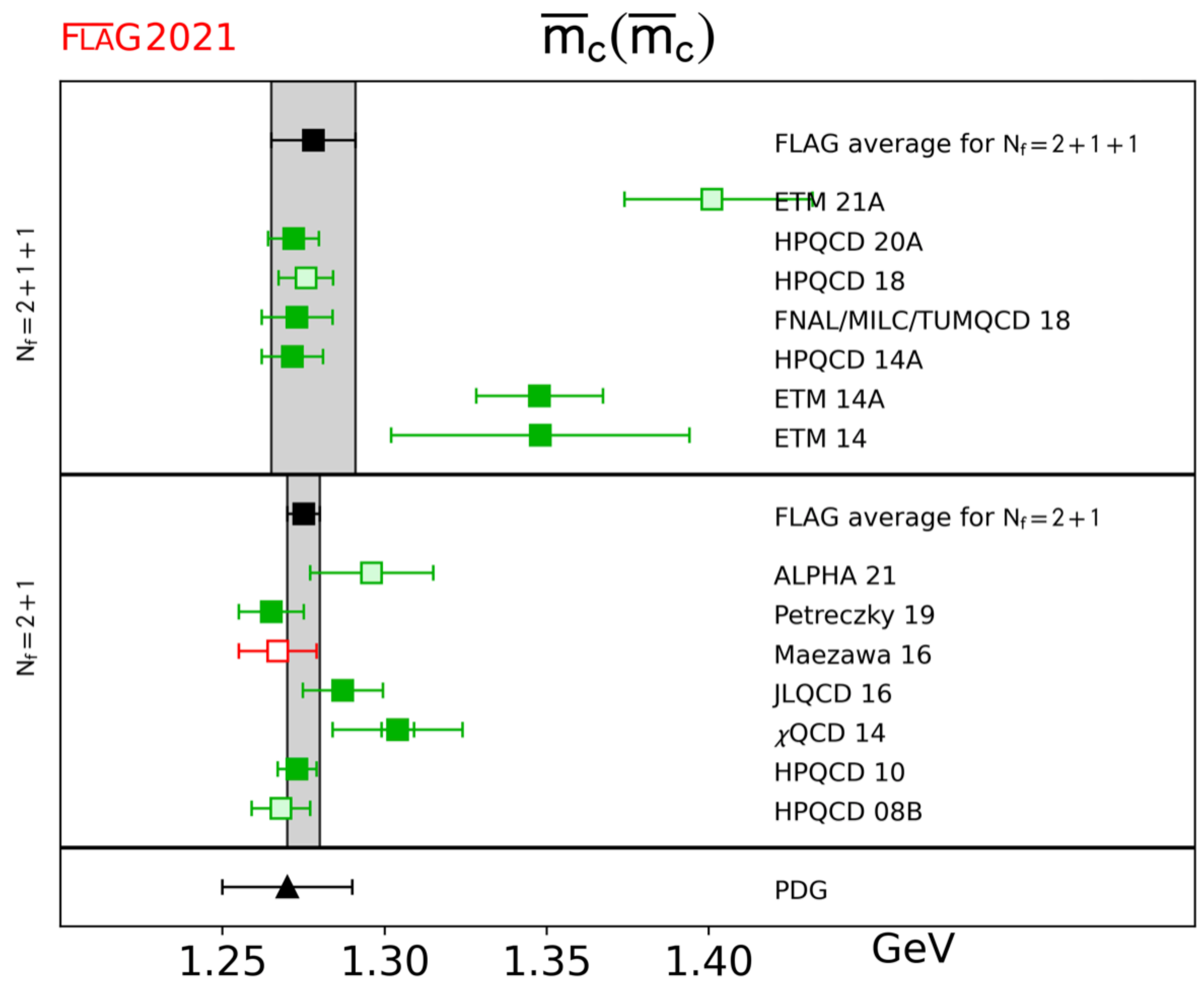
$$\boxed{m_u^{\mathrm{bare}},\ m_d^{\mathrm{bare}} \text{ and } m_s^{\mathrm{bare}}}$$

Quark masses

From lattice QCD



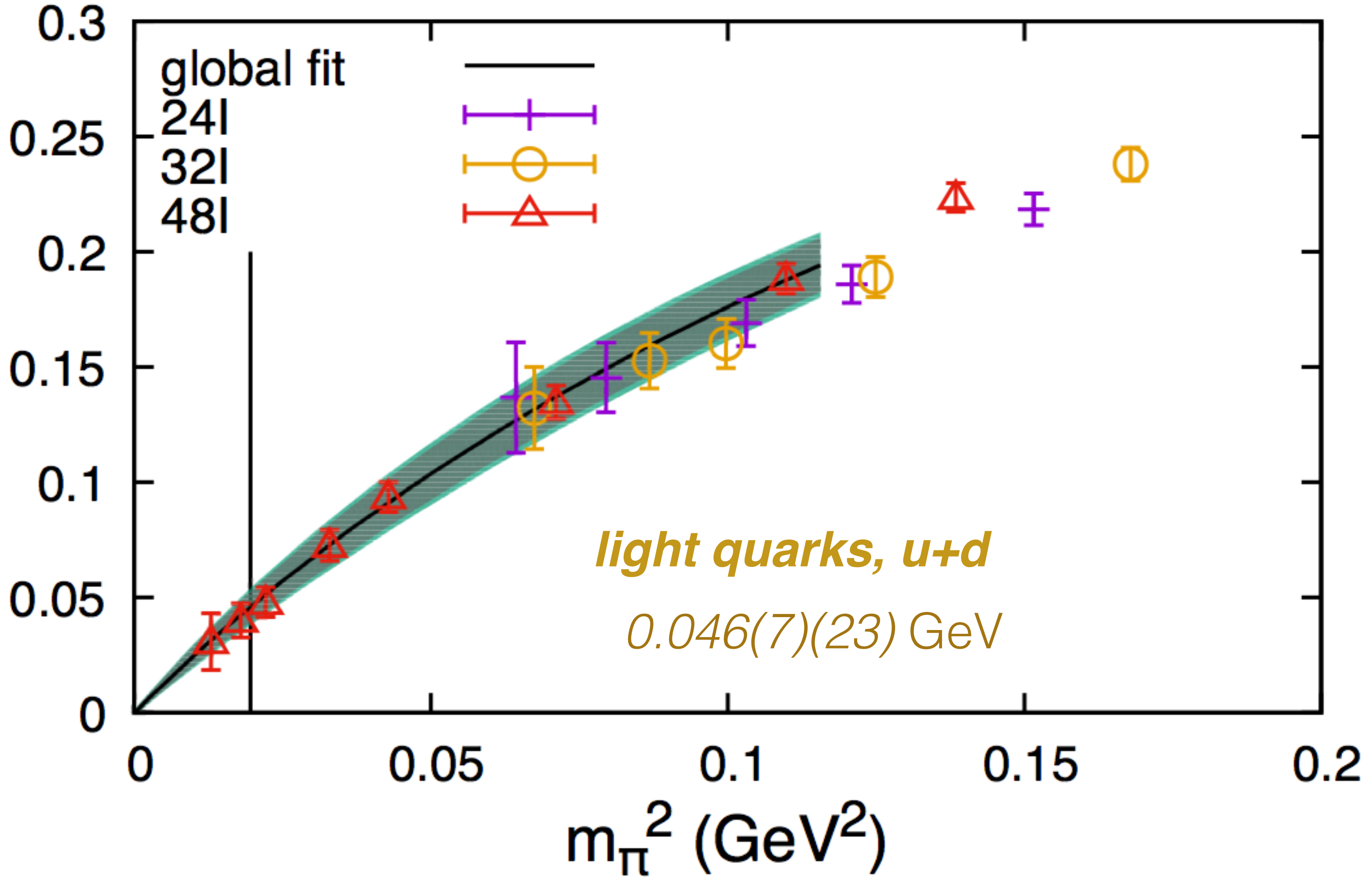




FLAG, EPJC80(2020)113

The quark masses are small…

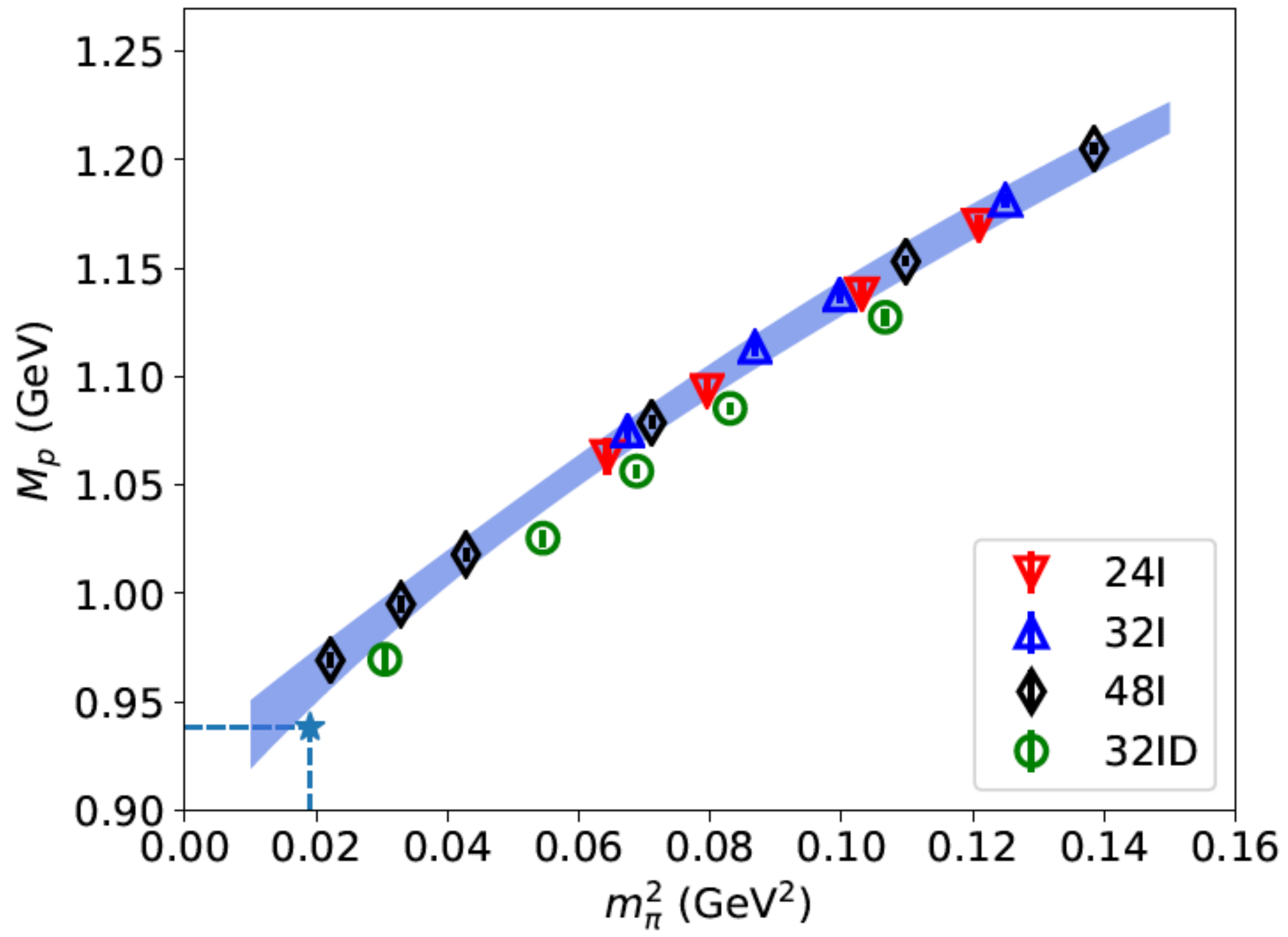
but the nucleon mass is much heavier



YBY, et.al. χQCD Collaboration, PRD94(2016)054503

- Lattice QCD determines the u/d averaged quark mass to be 3.4(1) MeV at MS-bar 2 GeV.
- But the proton mass is 938 MeV…
- and it not very sensitive to the quark mass:

1. Constant + ~15 $m_q^{\overline{\rm MS}(2\rm GeV)}$ around the chiral limit;
2. ~3 $m_q^{\overline{\rm MS}(m_q)}$ with the quark mass is very heavy.



YBY, J. Liang, et. al., χQCD Collaboration, PRL121(2018)212001
ViewPoint and Editor's suggestion

Ji's sum rule for the hadron energy

Xiangdong Ji, PRL 74(1995)1071

$$M = -\langle T_{44}\rangle = \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \frac{1}{4}\langle H_a\rangle,$$

$$M = -\langle \hat{T}_{44}\rangle = \langle H_m\rangle + \langle H_a\rangle$$

With

$$H_m = \sum_{u,d,s\cdots}\int d^3x\, m\overline{\psi}\psi,$$

The quark mass

Gauge Invariant and scale independent combinations.

The QCD anomaly

$$H_a = H_g^a + H_m^\gamma,$$

The glue anomaly

$$H_g^a = \int d^3x\, \frac{-\beta(g)}{g}(E^2+B^2),$$

$$H_m^\gamma = \sum_{u,d,s\cdots}\int d^3x\, \gamma_m m\overline{\psi}\psi.$$

The quark mass anomaly

The total energy

$$H_E = \sum_{u,d,s\ldots}\int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

The quark energy

$$H_g = \int d^3x\, \frac{1}{2}(B^2-E^2),$$

The glue field energy

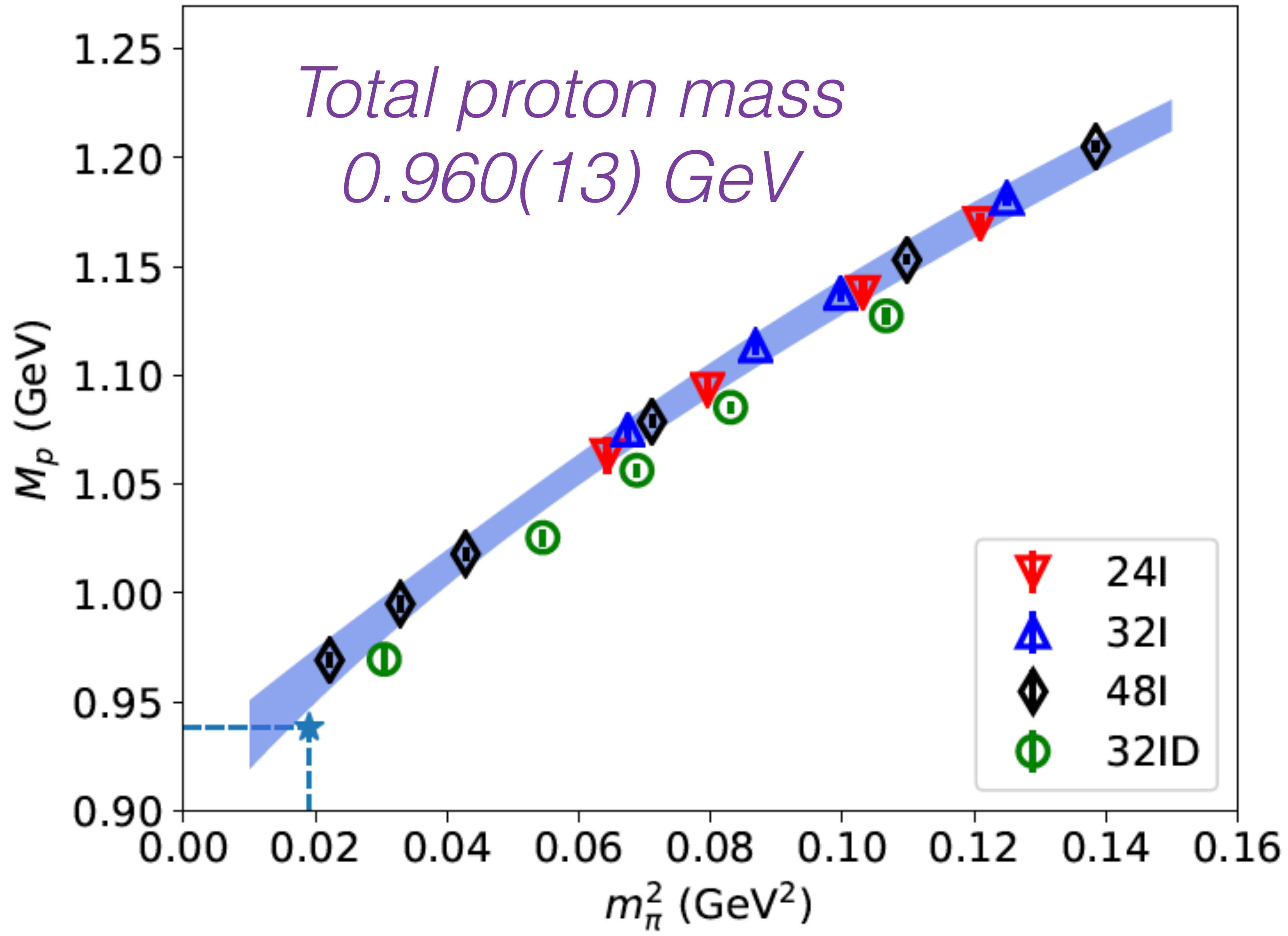
Ji's sum rule

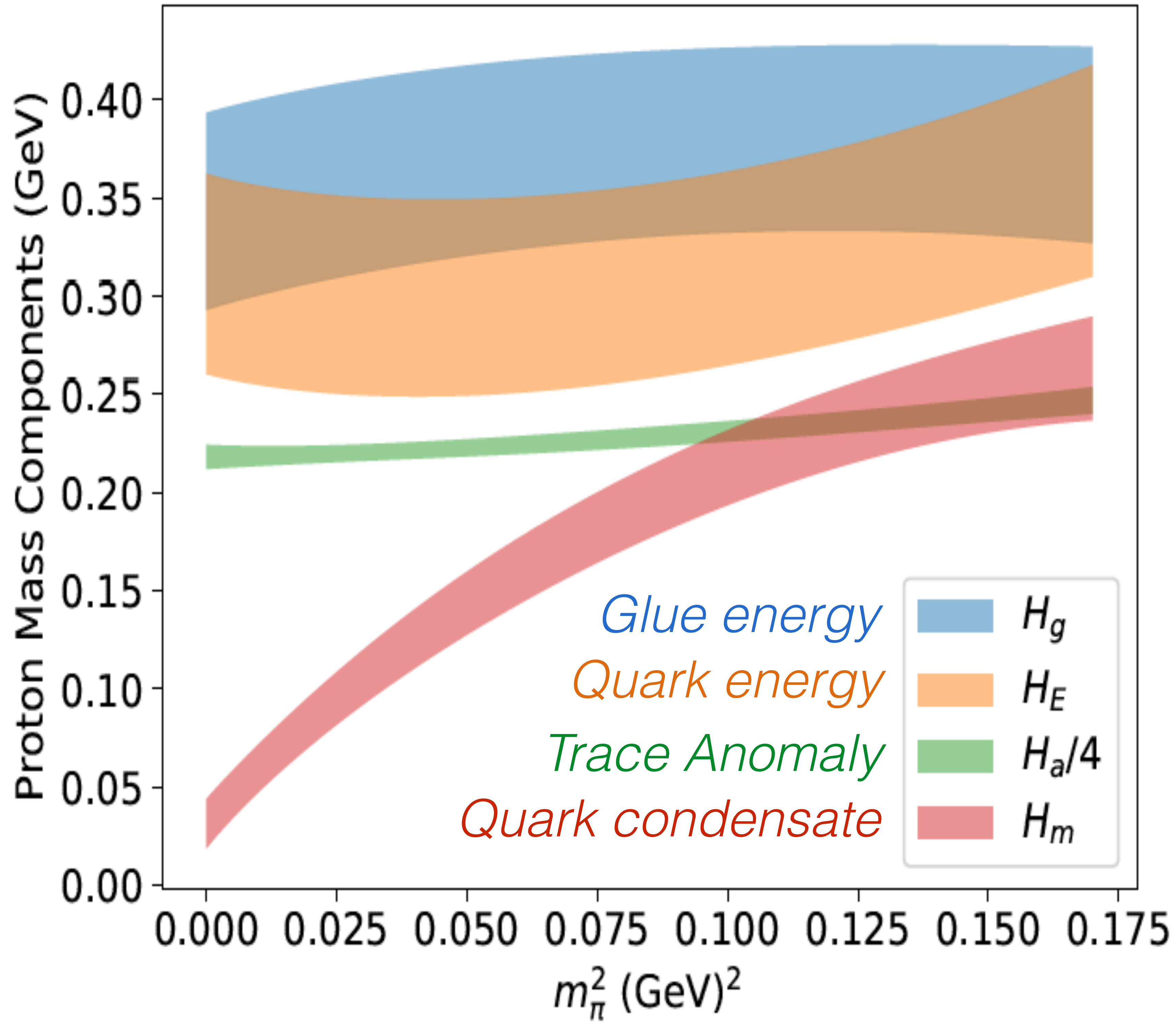
for the nucleon

- Direct calculation of the quark/glue momentum fraction with non-perturbative renormalization and normalization.

- Trace anomaly contribution deduced by the direct calculation of the quark scalar condensate in nucleon, based on the sum rule.

$$\frac{1}{4}M = -\langle \hat{T}_{44} \rangle = \frac{1}{4}\langle H_m \rangle + \frac{1}{4}\langle H_a \rangle$$

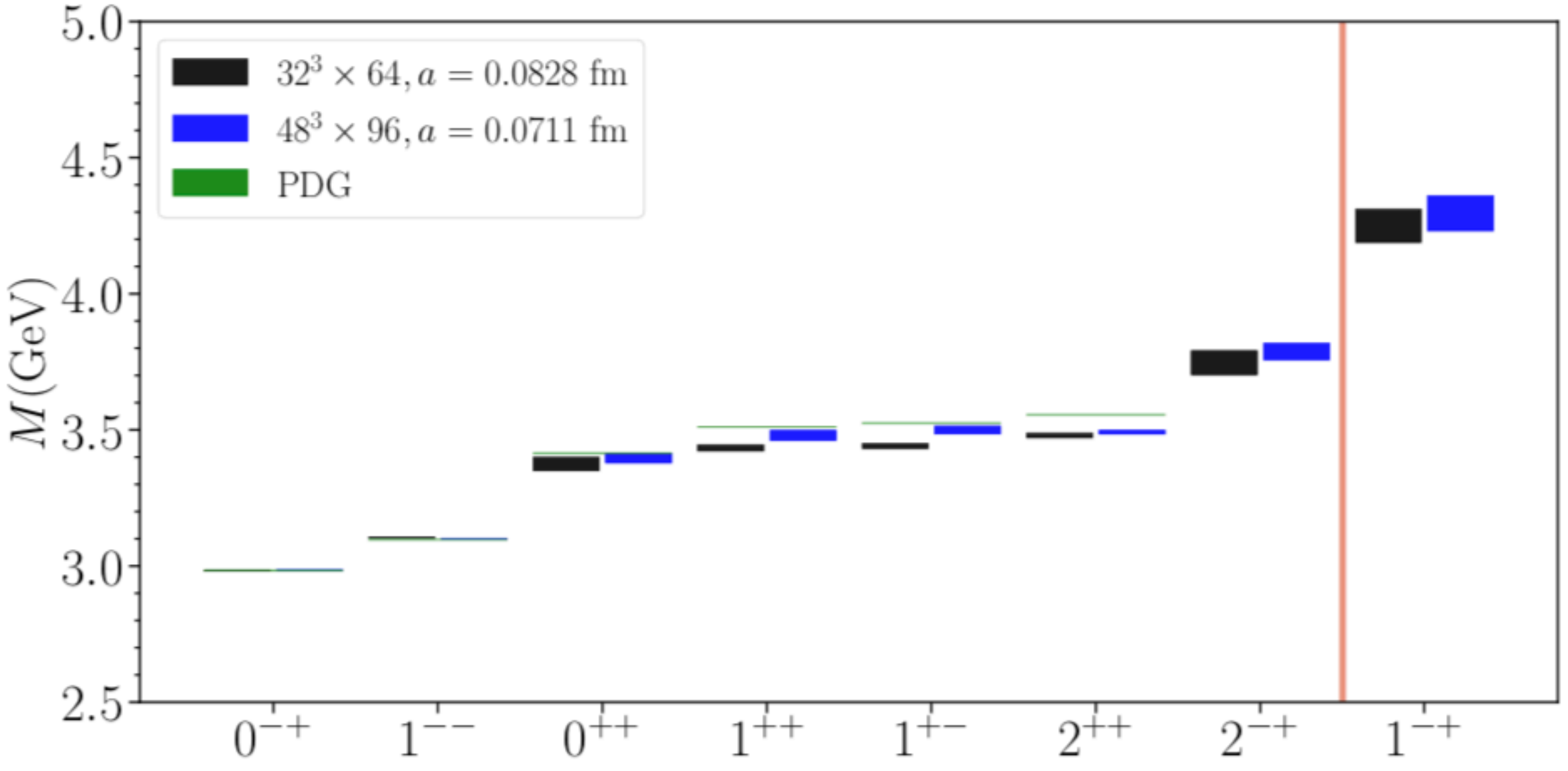




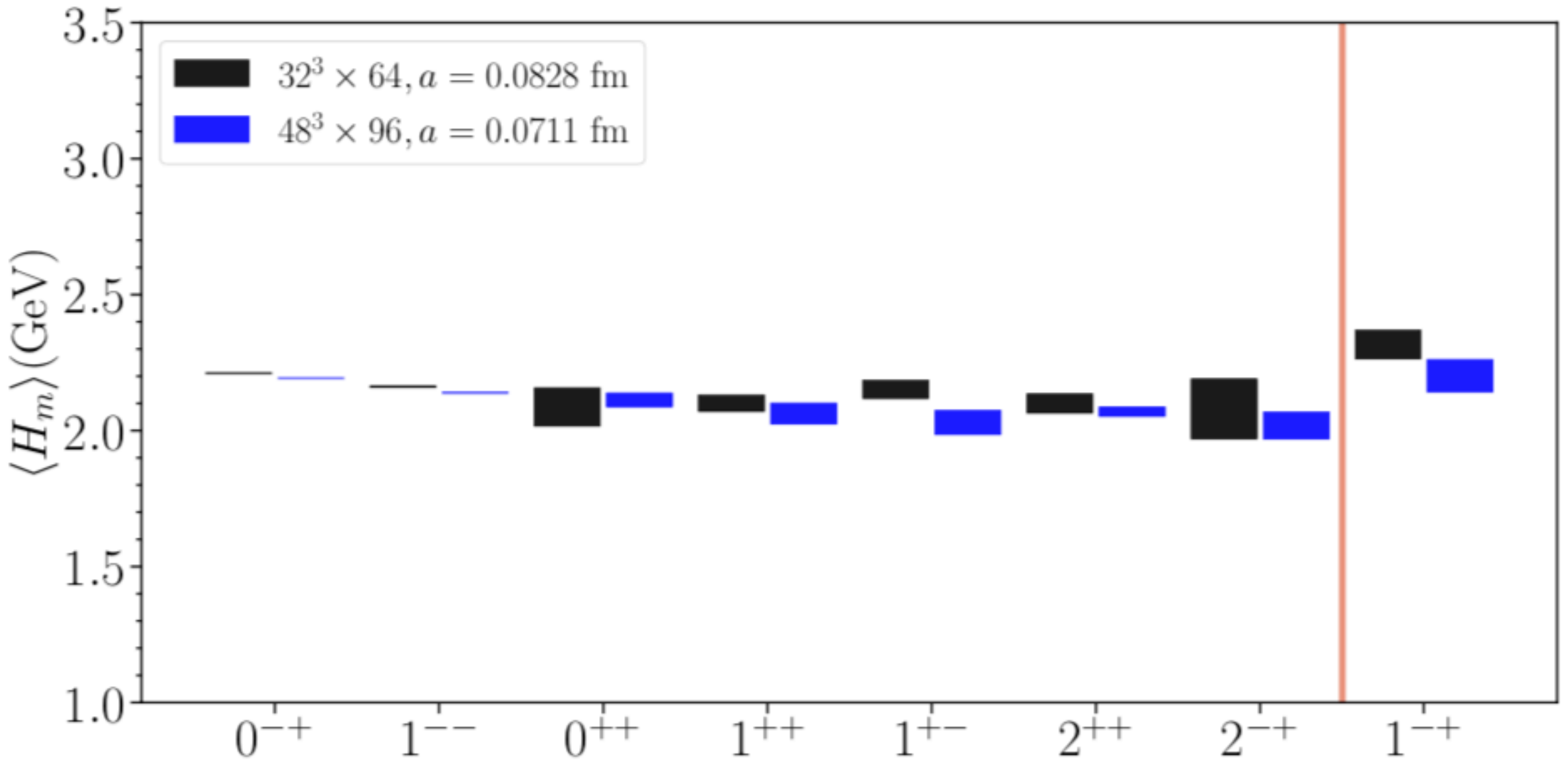
YBY, J. Liang, et. al., χQCD Collaboration, PRL121(2018)212001, **ViewPoint and Editor's suggestion**

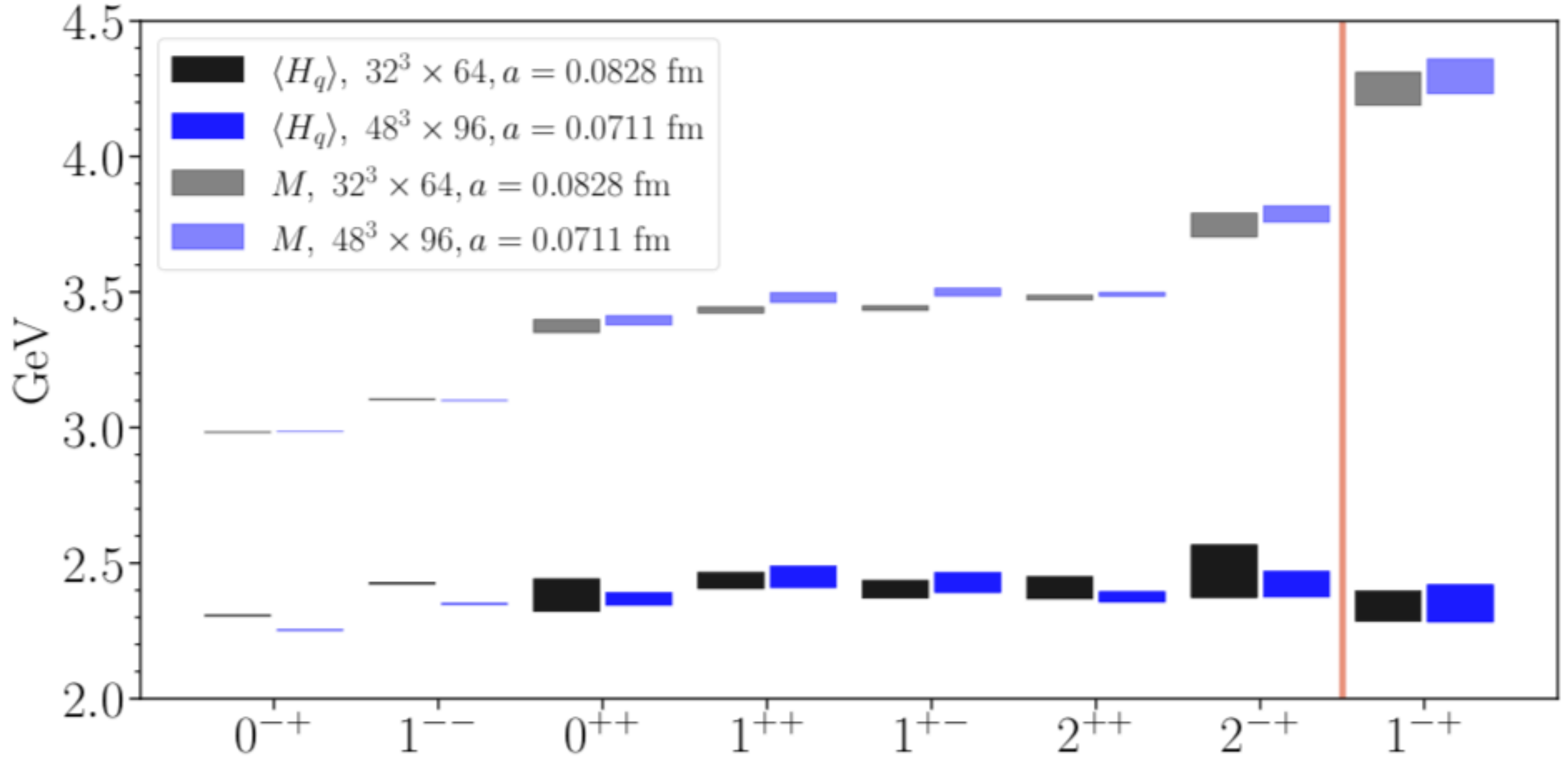
Ji's sum rule

for the charmonium



- The charm quark mass contribution in all the different charmonia are around 2.2 GeV.
- Most of the mass difference between different charmonia comes from the dynamics.





W. Sun, et.al, χQCD Collaboration, PRD103(2021)094503

Chinese Academy of Sciences, Grant No. XDC01040100

Trace anomaly

Defintion

- The trace of QCD energy momentum tensor (EMT),

$$T_{\mu\nu} = \frac{i}{2}\bar{\psi}\overleftrightarrow{D}_{(\mu}\gamma_{\nu)}\psi + \frac{1}{4}g_{\mu\nu}F^2 - F_{\mu\rho}F_{\rho\nu}$$

is just the quark mass term $m\bar{\psi}\psi$ at the classical level.

- But with the quantum corrections, the quantum correction changes the trace term into:

J.Collins et,al. PRD16(1977) 438

$$T^{\mu}_{\mu} = \left[1 + \frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)\right]m\bar{\psi}\psi + \left[(-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)\right]F^2,$$

where the terms proportional to α_s are the QCD quantum corrections.

- Then the hadron mass can be decomposed into three pieces:

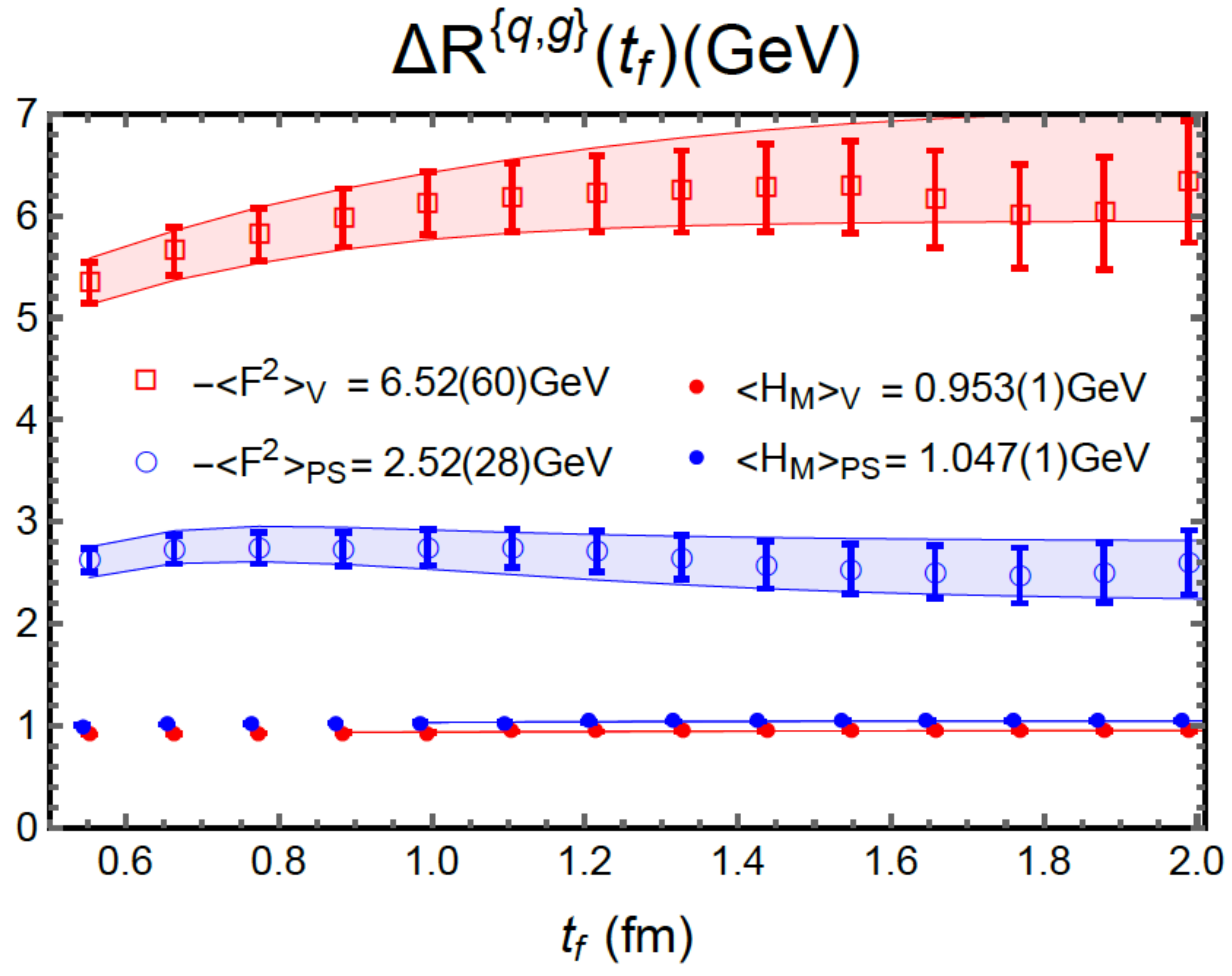
M.A. SHIFMAN et,al. PLB78(1978)

$$m_N = m\langle\bar{\psi}\psi\rangle_N + \left[\frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)\right]m\langle\bar{\psi}\psi\rangle_N + \left[(-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)\right]\langle F^2\rangle_N.$$

YBY, et. al., χQCD Collaboration, PRD94(2016)054503

Trace anomaly

Hadron matrix elements



$$\langle H_m \rangle_H = \Delta R^q(t_f; H) + \mathcal{O}(e^{-\delta m t_f}),$$
$$\langle F^2 \rangle_H = \Delta R^g(t_f; H) + \mathcal{O}(e^{-\delta m t_f}),$$

F. He, P. Sun, **YBY**, χQCD Collaboration, PRD104 (2021) 074507

- Valence quark mass ~ 500 MeV.
- Considering pseudo-scalar (PS) and vector (V) mesons
- Their quark matrix elements are similar within ~10% difference;
- But the gluon matrix elements differ by a factor of 2 in those two hadrons !
- ……and the uncertainty is much larger even with 5 steps of HYP smearing on the gluon operator.

Supported by Strategic Priority Research Program of Chinese Academy of Sciences, Grant No. XDC01040100

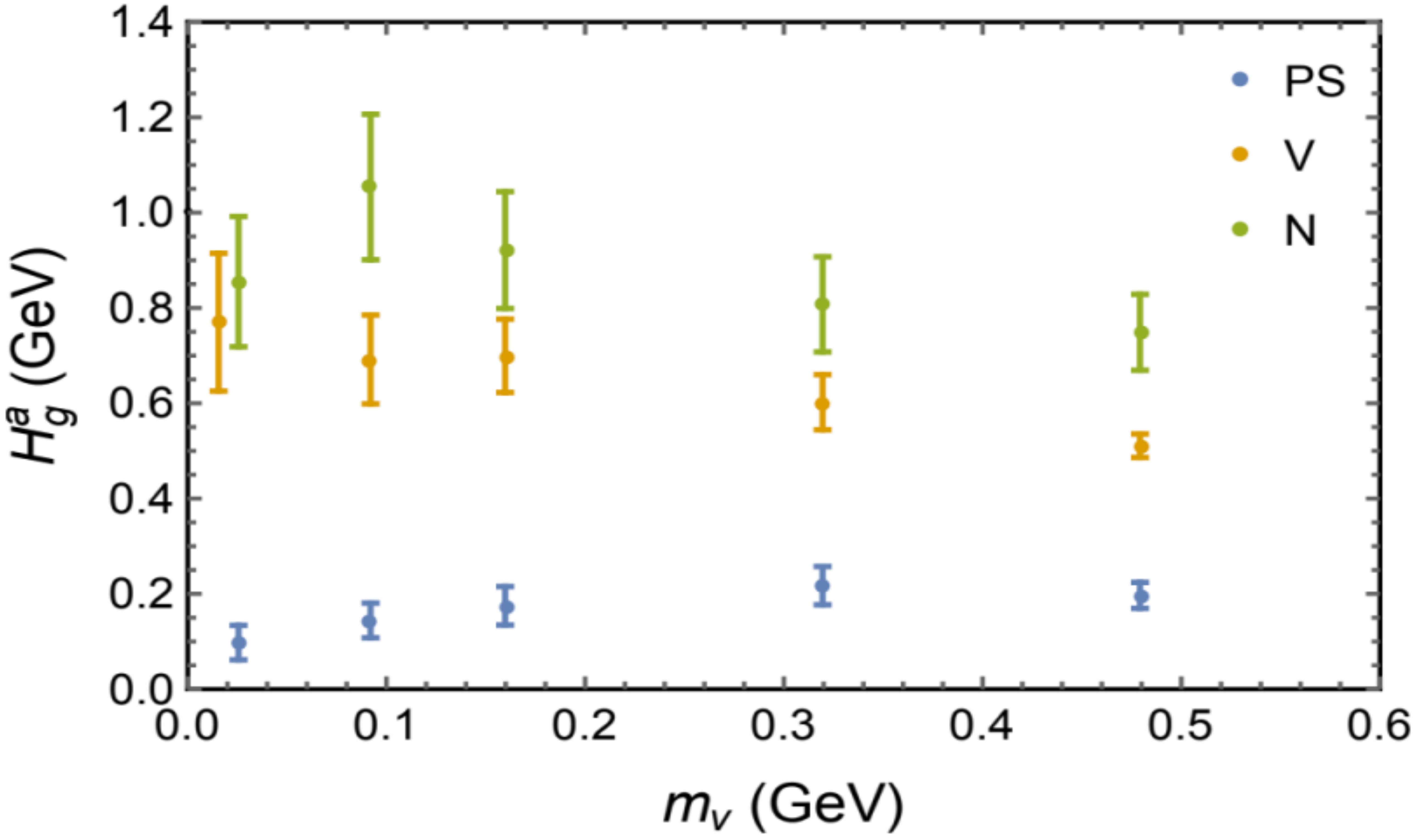
Trace anomaly

Gluon contribution

- The gluon contribution $\frac{\beta}{2g}\langle F^2\rangle_H$ to the hadron mass m_H.

- **$\frac{\beta}{2g}\langle F^2\rangle_N$ is around 800 MeV in the chiral limit $m_q \to 0$.**
- **$\frac{\beta}{2g}\langle F^2\rangle_\pi$ will be less than 100 MeV in the chiral limit $m_q \to 0$.**

- They are exact what QCD predict.



F. He, P. Sun, **YBY**, χQCD Collaboration, PRD104 (2021) 074507

Supported by Strategic Priority Research Program of Chinese Academy of Sciences, Grant No. XDC01040100

Trace anomaly

Density inside hadron

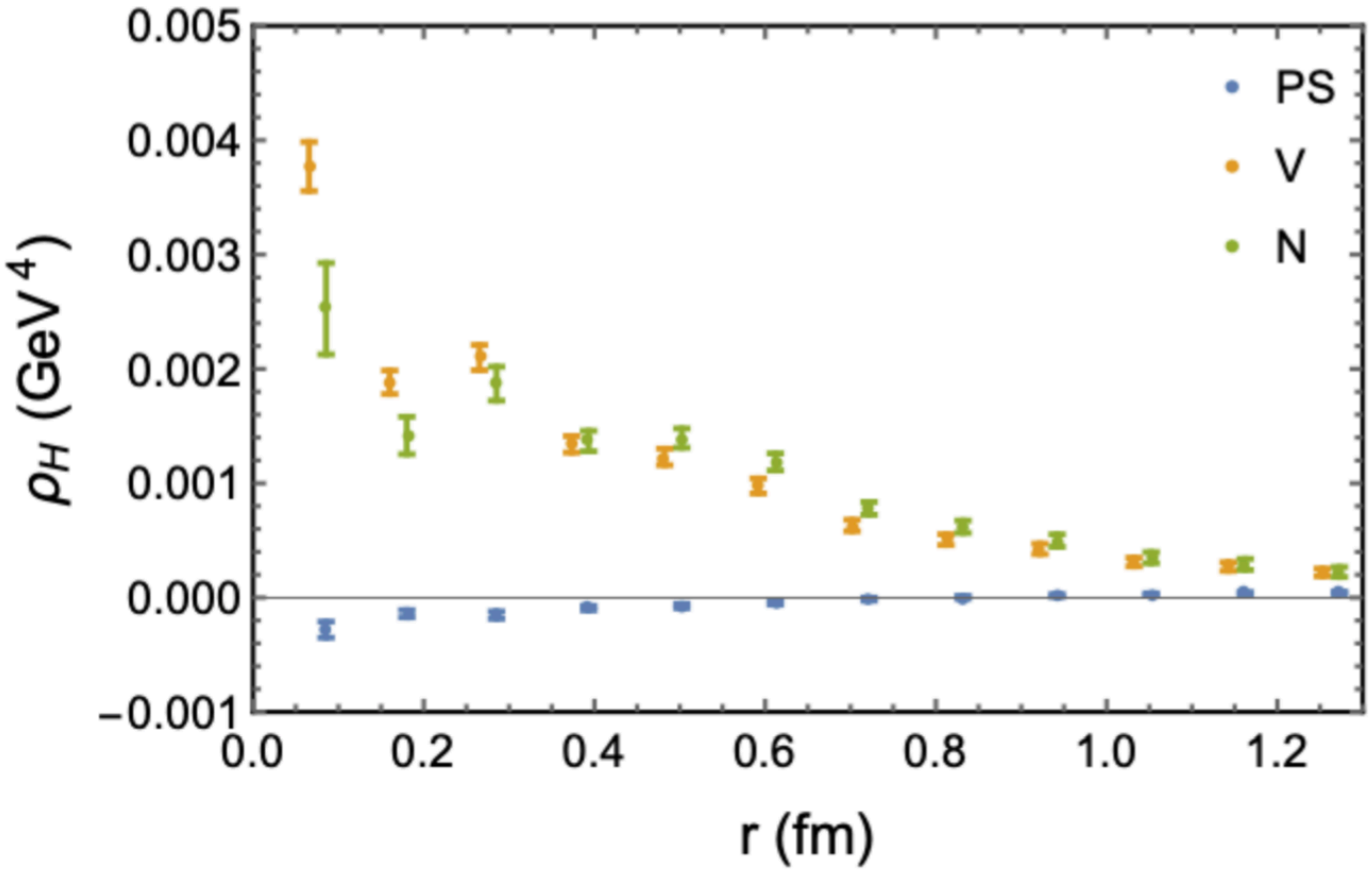
$$\rho_H(|r|) = \frac{\langle \sum_{\vec{y}} \mathcal{H}(t_f,\vec{y}) H_a^g(t,\vec{y}+\vec{r}) \sum_{\vec{x}} \mathcal{H}^\dagger(0,\vec{x})\rangle}{\langle \sum_{\vec{y}} \mathcal{H}(t_f,\vec{y}) \sum_{\vec{x}} \mathcal{H}^\dagger(0,\vec{x})\rangle}\Big|_{t,t_f-t\to\infty},$$

- Similar strategy as that proposed for the pion charge radius;

X. Feng. et.al, PRD101(2020)051502, arXiv:1911.04064

- The density is an order of magnitude smaller than the usual estimate of $\langle F^2 \rangle \sim 0.01 - 0.06$

M. Shifman. et.al, NPB147(1979)448
G. Bali. et.al, PRL113(2014)092001, arXiv:1403.6477

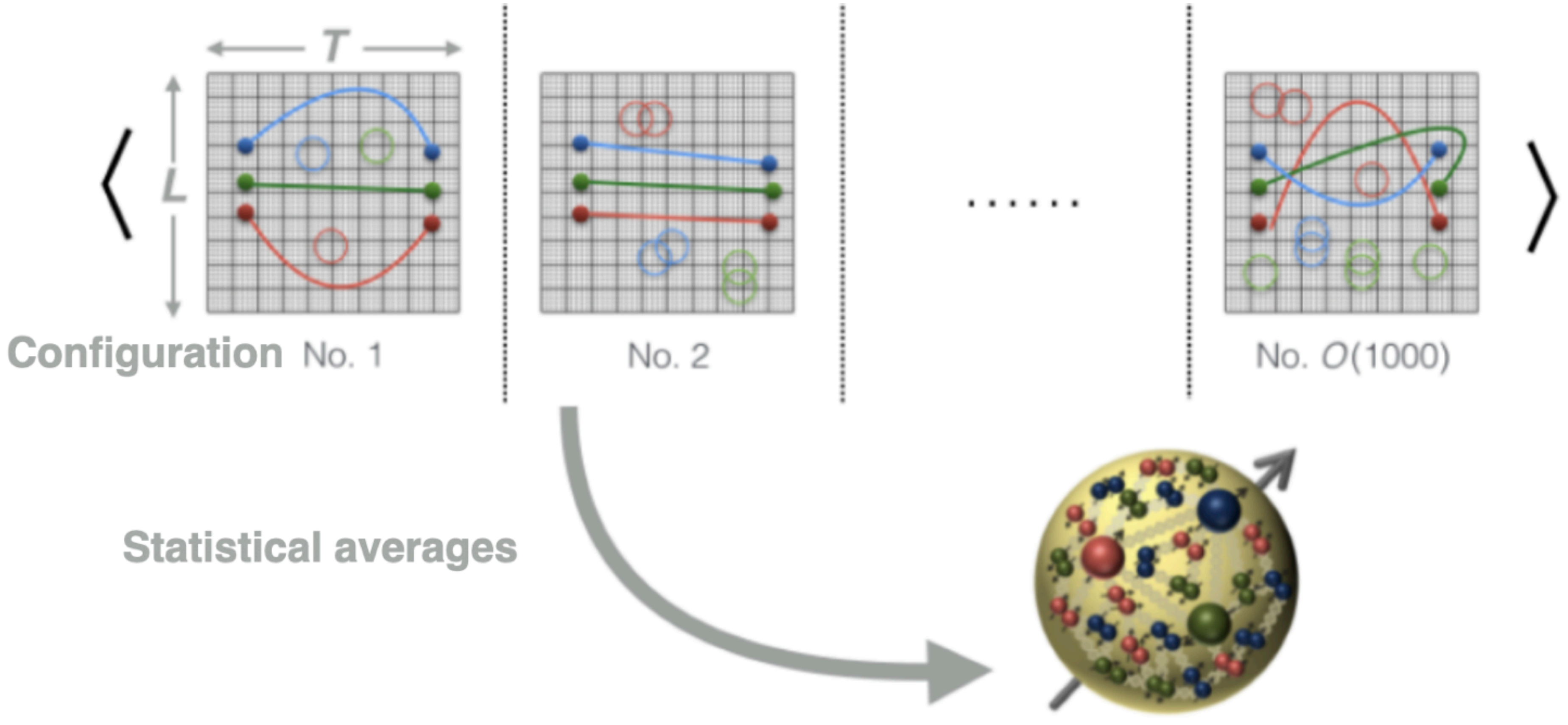


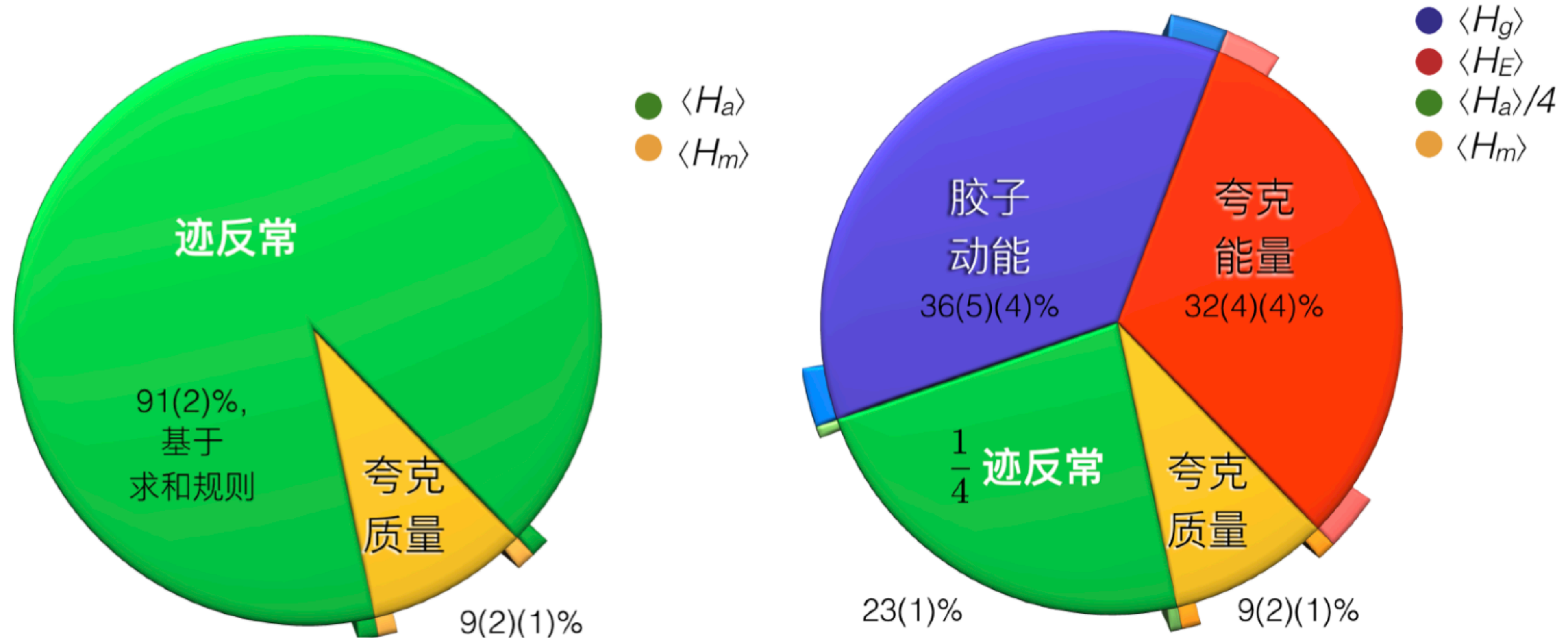
F. He, P. Sun, **YBY**, χQCD Collaboration, arXiv:2101.04942

Supported by Strategic Priority Research Program of Chinese Academy of Sciences, Grant No. XDC01040100

Outline

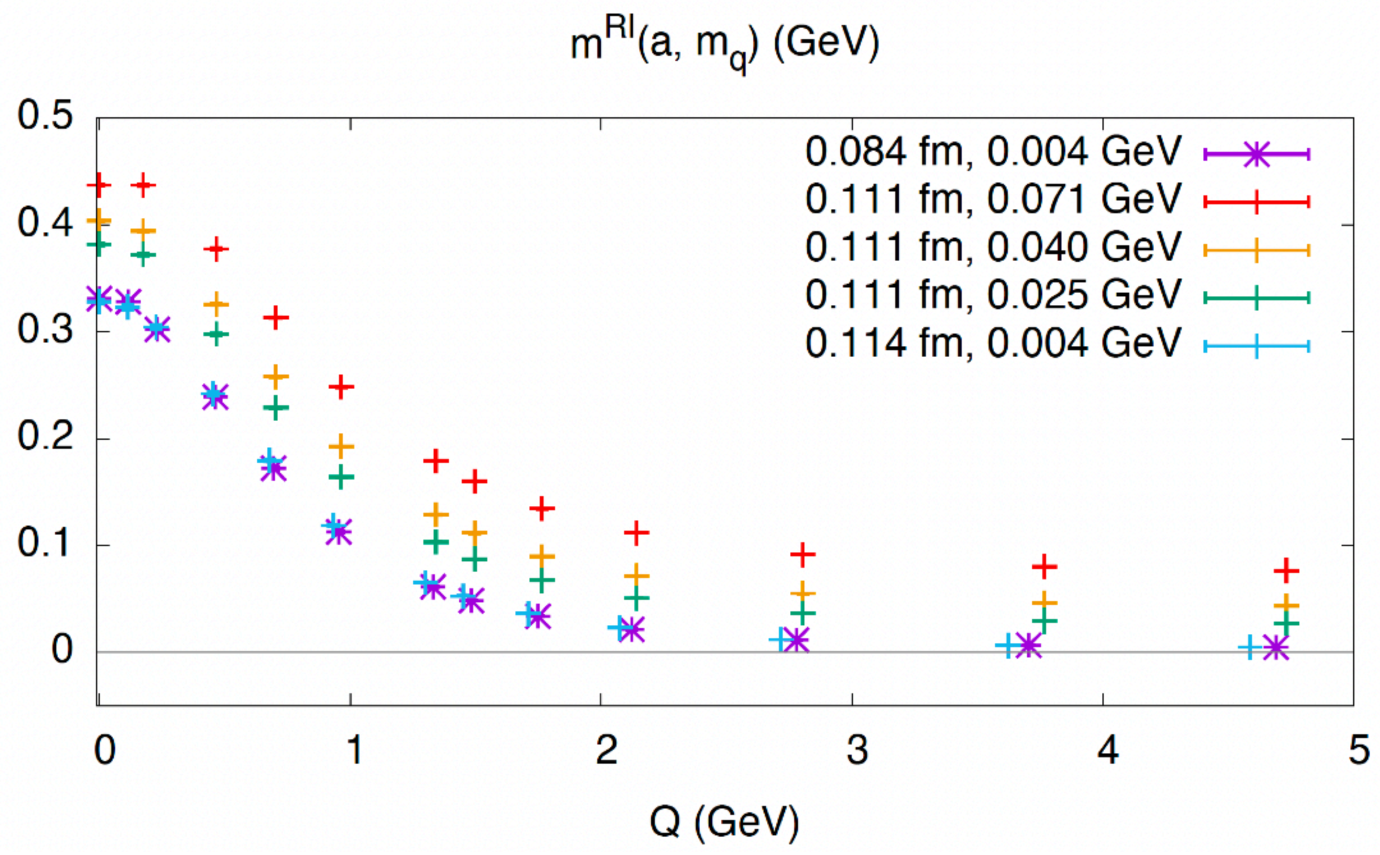
- Brief introduction on Lattice QCD;



- 

Hadron mass and its components;

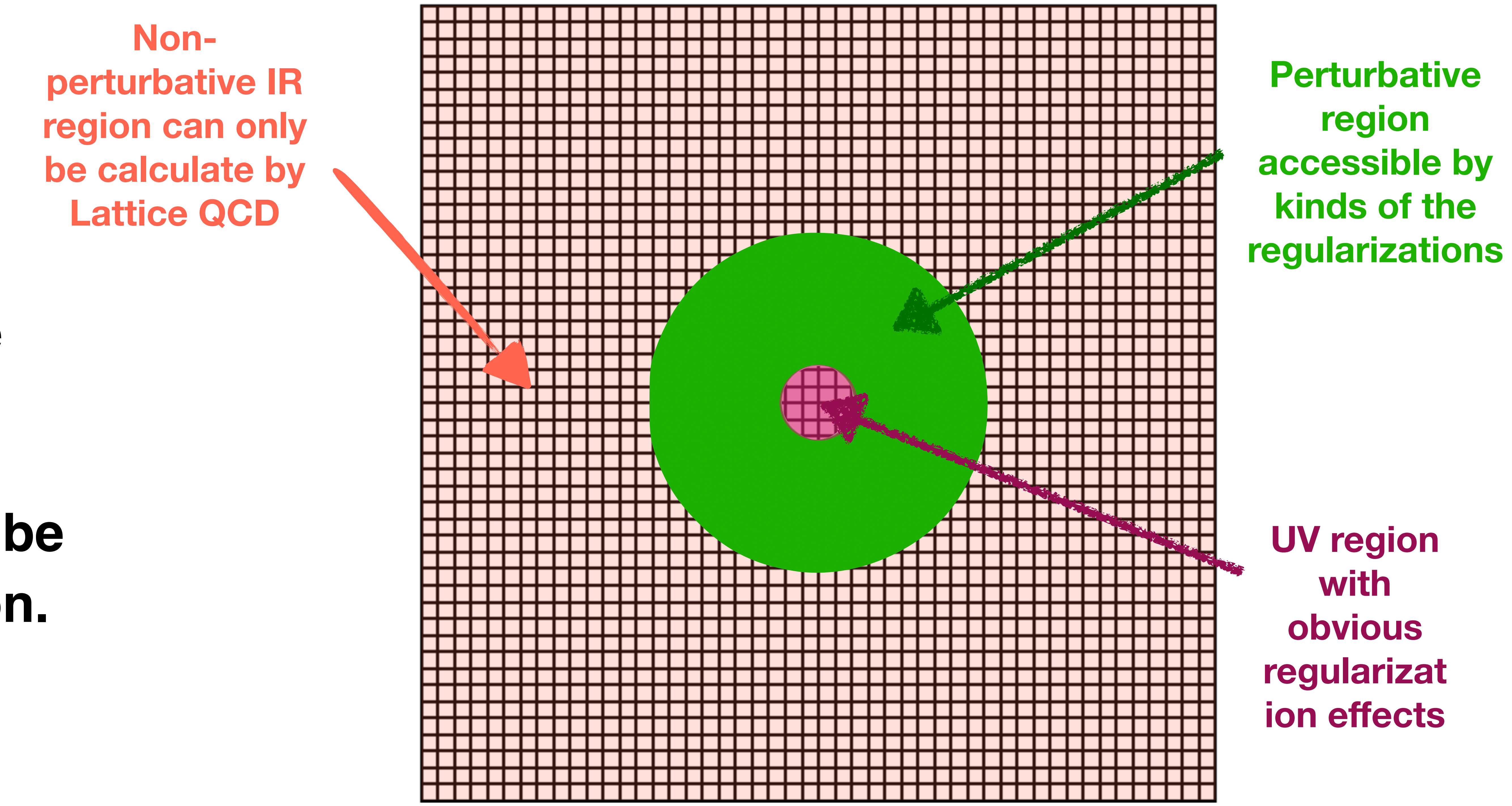
- **Parton masses and their low energy limits.**



The light quark masses

Renormalization

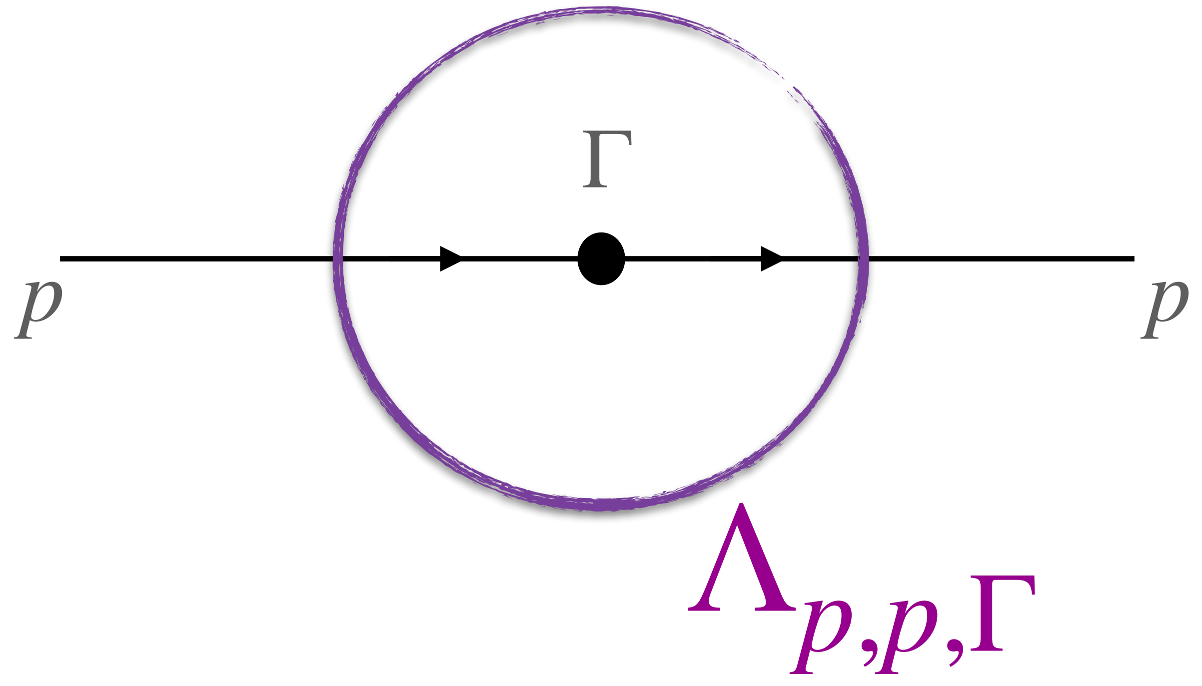
$$m_q^{\overline{\rm MS}}(\mu) = \frac{Z_m^{\rm MOM,Lat}(Q,1/a)}{Z_m^{\rm MOM,Dim}(Q,\mu,\epsilon)} Z_m^{\overline{\rm MS},\rm Dim}(\epsilon) m_q^{\rm Lat}(1/a) + \mathcal{O}(a^m, \alpha_s^n)$$



- The RI/MOM renormalization targets to cancel the $\alpha_s \log(a)$ divergences using the off-shell quark matrix element;
- **Up to the $\mathcal{O}(a^2p^2)$ correction which can be eliminated by the $a^2p^2 \to 0$ extrapolation.**

G. Martinelli, et.al, NPB445(1995)81, arXiv: hep-lat/9411010

The regularization-independent momentum subtraction scheme



First of all, we introduce a perturbative calculable scale $Q^2 = -p^2$,

- Then we can calculate the quark propagator $S(p) = \sum_x e^{-i(p\cdot x)}\langle\psi(x)\bar{\psi}(0)\rangle$,
- And also vertex function $\Lambda(p,p,\Gamma) = S^{-1}(p)\sum_{x,y} e^{-i\,p\cdot(x-y)}\langle\psi(x)\bar{\psi}(0)\Gamma\psi(0)\bar{\psi}(y)\rangle S^{-1}(p)$ under the Landau gauge.
- Eventually we can define the RI/MOM renormalization condition as the following: $\dfrac{12Z_q^{\mathrm{RI}}(Q)}{\mathrm{Tr}[\Gamma^\dagger\Lambda(p,p,\Gamma)]}\Big|_{p^2=-Q^2} = \dfrac{C_0}{m_q^n} + Z_S^{\mathrm{MOM}}(Q) + \mathcal{O}(m_q)$。
- It can be applied to any regularization scheme.

RI/MOM scheme

Perturbative calculation

- The RI/MOM renormalization constant of the quark mass under the lattice regularization is:

$$Z_m^{\mathrm{MOM,Lat}}(Q,1/a,\xi) = (Z_S^{\mathrm{MOM,Lat}}(Q,1/a,\xi))^{-1} = \langle q|\mathcal{O}|q\rangle^{\mathrm{Lat}} = 1 + \frac{\alpha_s C_F}{4\pi}[-3\log(a^2Q^2) - \xi + b_S] + \mathcal{O}(\alpha_s^2, a^2Q^2);$$

- The RI/MOM and $\overline{\mathrm{MS}}$ renormalization constants under the dimensional regularization are:

$$Z_m^{\mathrm{MOM,Dim}}(Q,\mu,\epsilon,\xi) = \langle q|\mathcal{O}|q\rangle^{\mathrm{Dim}} = 1 + \frac{\alpha_s C_F}{4\pi}[\frac{3}{\tilde{\epsilon}} - 3\log(\frac{Q^2}{\mu^2}) - \xi + 5] + \mathcal{O}(\alpha_s^2)$$

$$Z_m^{\overline{\mathrm{MS}},\mathrm{Dim}}(Q,\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\frac{3}{\tilde{\epsilon}} + \mathcal{O}(\alpha_s^2)$$

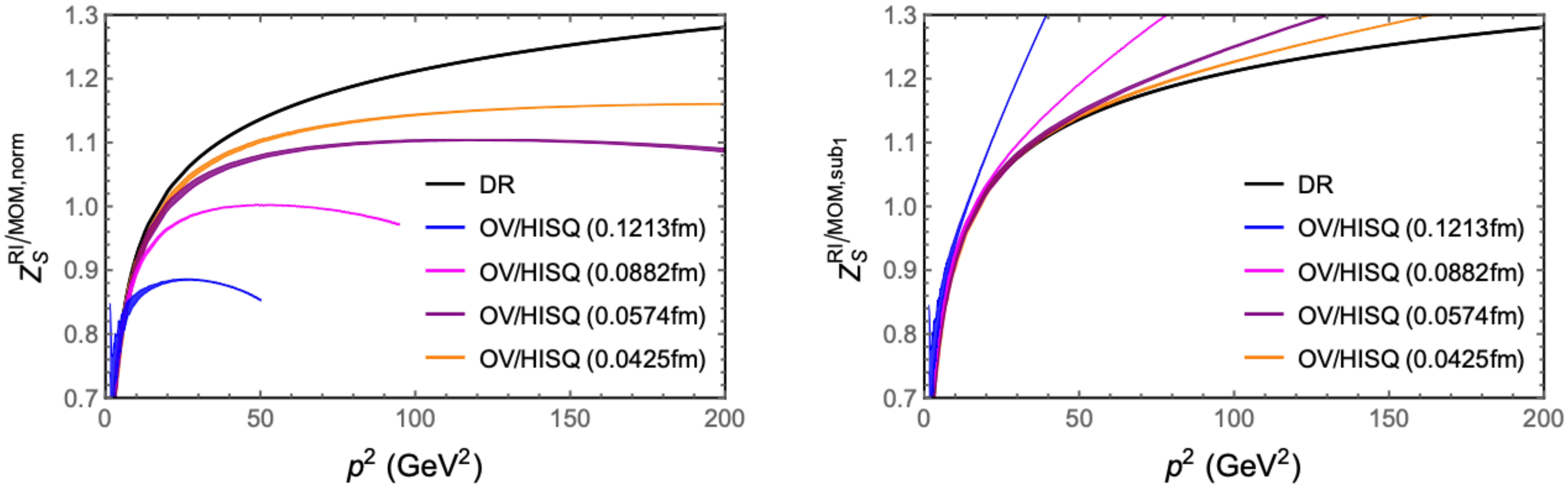
- Thus the renormalized quark mass under the $\overline{\mathrm{MS}}$ scheme can be defined by:

$$m_q^{\overline{\mathrm{MS}}}(\mu) = \frac{Z_m^{\mathrm{MOM,Lat}}(Q,1/a,\xi)}{Z_m^{\mathrm{MOM,Dim}}(Q,\mu,\epsilon,\xi)} Z_m^{\overline{\mathrm{MS}},\mathrm{Dim}}(\epsilon) m_q^{\mathrm{Lat}}(1/a) + \mathcal{O}(a^{2m}Q^{2m}, \alpha_s^n)$$

RI/MOM scheme

Discretization errors

$$Z_m^{\mathrm{MOM,Lat}}(Q, a, 0) = (Z_S^{\mathrm{MOM,Lat}}(Q, a, 0))^{-1}$$



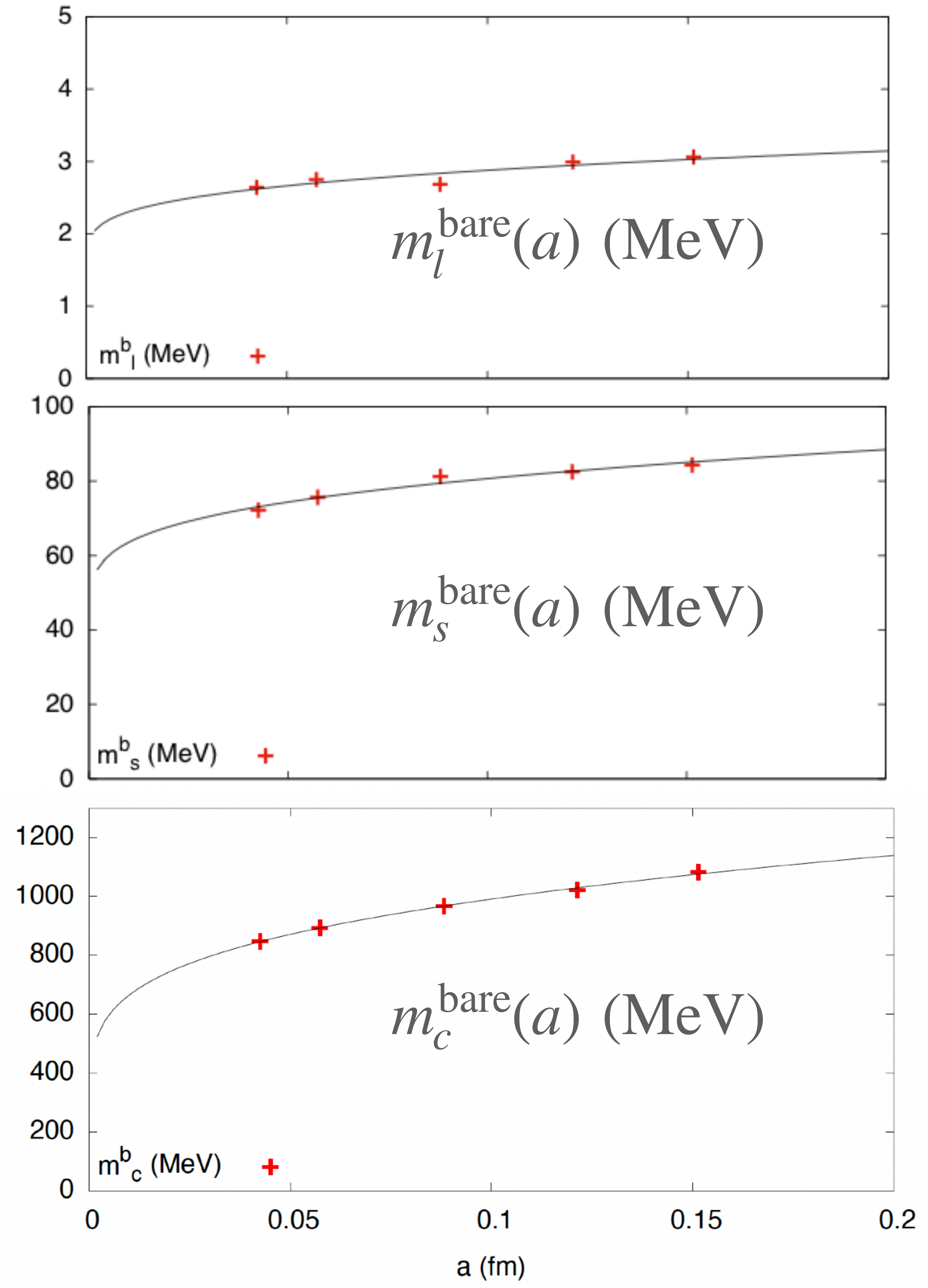
Original lattice results

$\mathcal{O}(a^2p^2)$ term subtracted

F. He, et.al, χQCD, arXiv: 2204.09246

- The discretization is sizable at $a \sim 0.1$ fm;
- Becomes much smaller after the $\mathcal{O}(a^2p^2)$ correction is removed;
- The higher order $a^{2n}p^{2n}$ correction can also be removed in the practical calculation.

The light quark masses

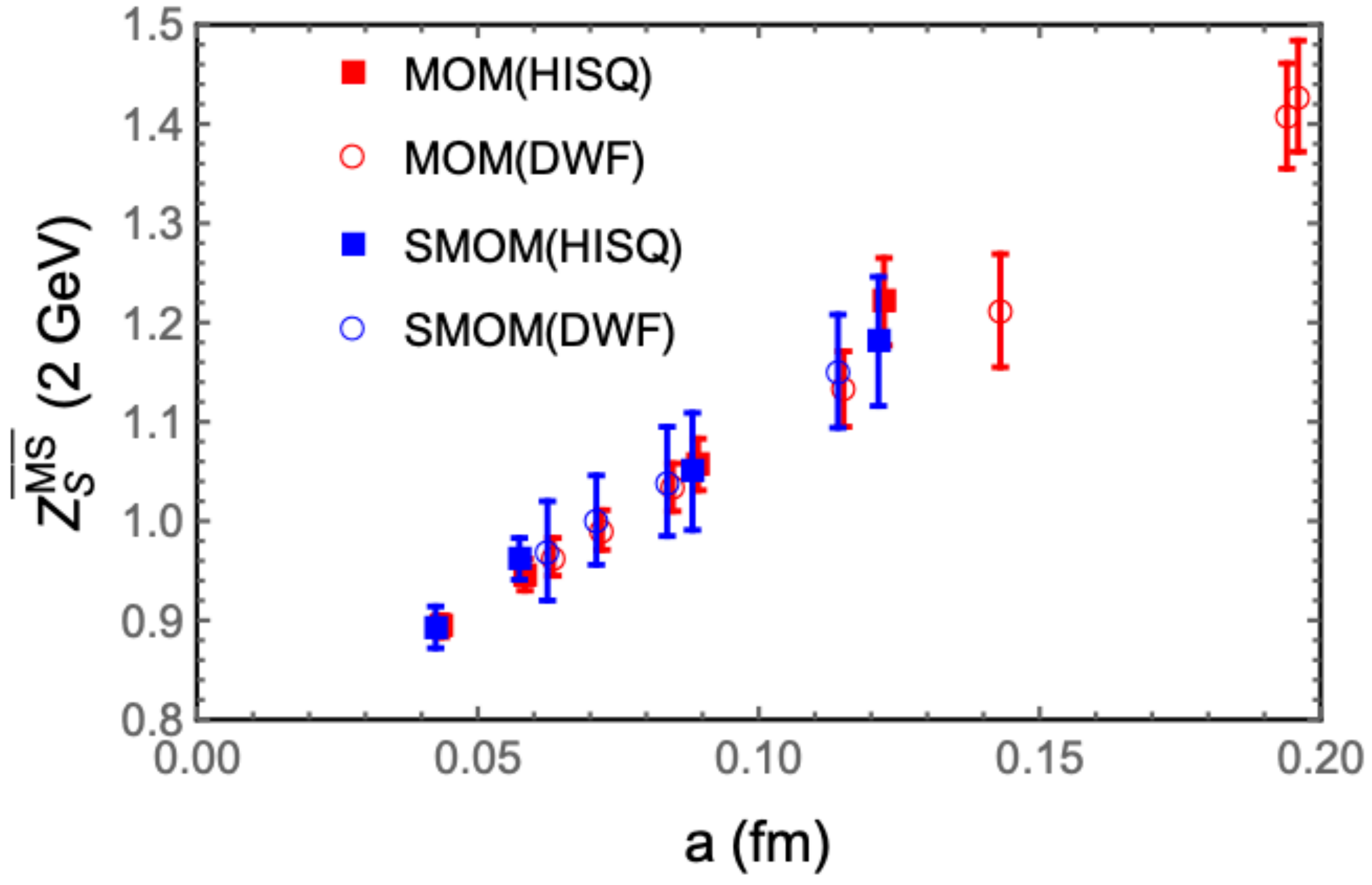


A. Bazavov, et,al., MILC, PRD87(2013)054505

Renormalization

$$m_q^{\overline{\text{MS}}}(\mu) = Z_m^{\overline{\text{MS}},\text{Lat}}(\mu, 1/a) m_q^{\text{Lat}}(1/a) + \mathcal{O}(a^{2m}Q^{2m}, \alpha_s^n)$$

$$Z_m^{\overline{\text{MS}}}(\mu) = (Z_S^{\overline{\text{MS}}}(\mu))^{-1} = \frac{Z_m^{\text{MOM,Lat}}(Q, 1/a, \xi)}{Z_m^{\text{MOM,Dim}}(Q, \mu, \epsilon, \xi)} Z_m^{\overline{\text{MS}},\text{Dim}}(\epsilon)|_{a^2Q^2\to 0} + \mathcal{O}(\alpha_s^4)$$

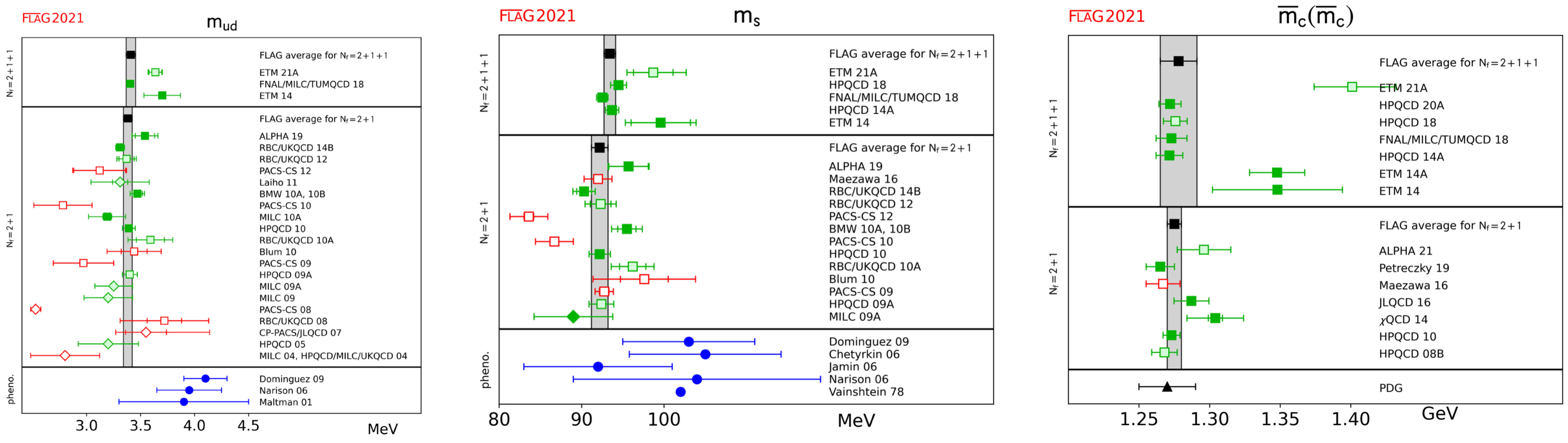


F. He, et.al, χQCD, arXiv: 2204.09246

- The scalar renormalization constant $Z_S^{\overline{\text{MS}}}(\mu)$ shares the similar lattice spacing dependence as the bare quark mass $m_q^{\text{Lat}}(1/a)$;
- The renormalized quark mass $m_q^{\overline{\text{MS}}}(\mu) = \dfrac{m_q^{\text{Lat}}(1/a)}{Z_S^{\overline{\text{MS}},\text{Lat}}(\mu, 1/a)}$ should be free of $1/a$.

Quark masses

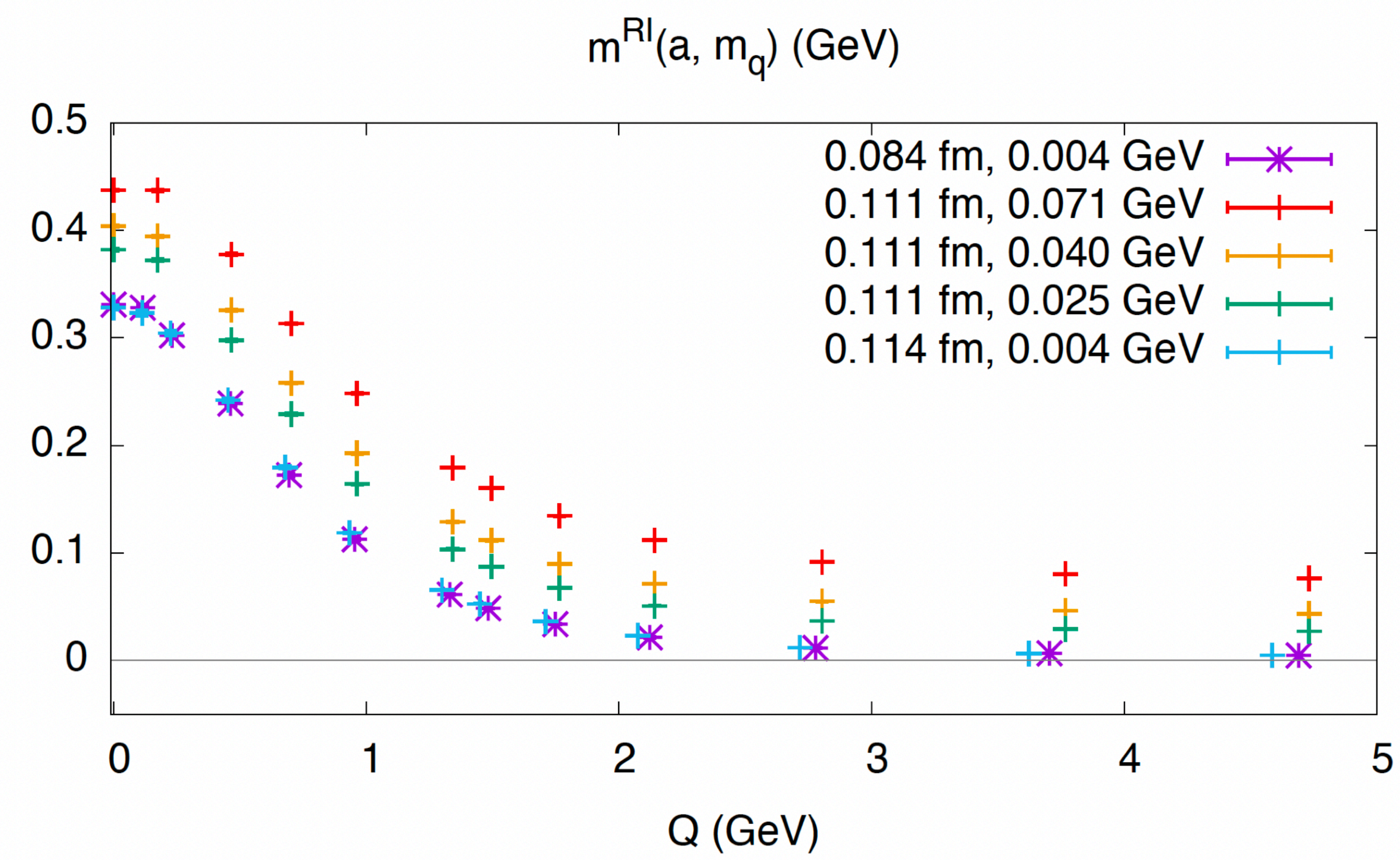
From lattice QCD



FLAG, EPJC80(2020)113

Chiral symmetry breaking

dynamical quark mass under the Landau gauge



YBY, χQCD, Lattice2019 001, 2003.12914

L. Chang et.al., PRD104(2021)094509, 2105.06596

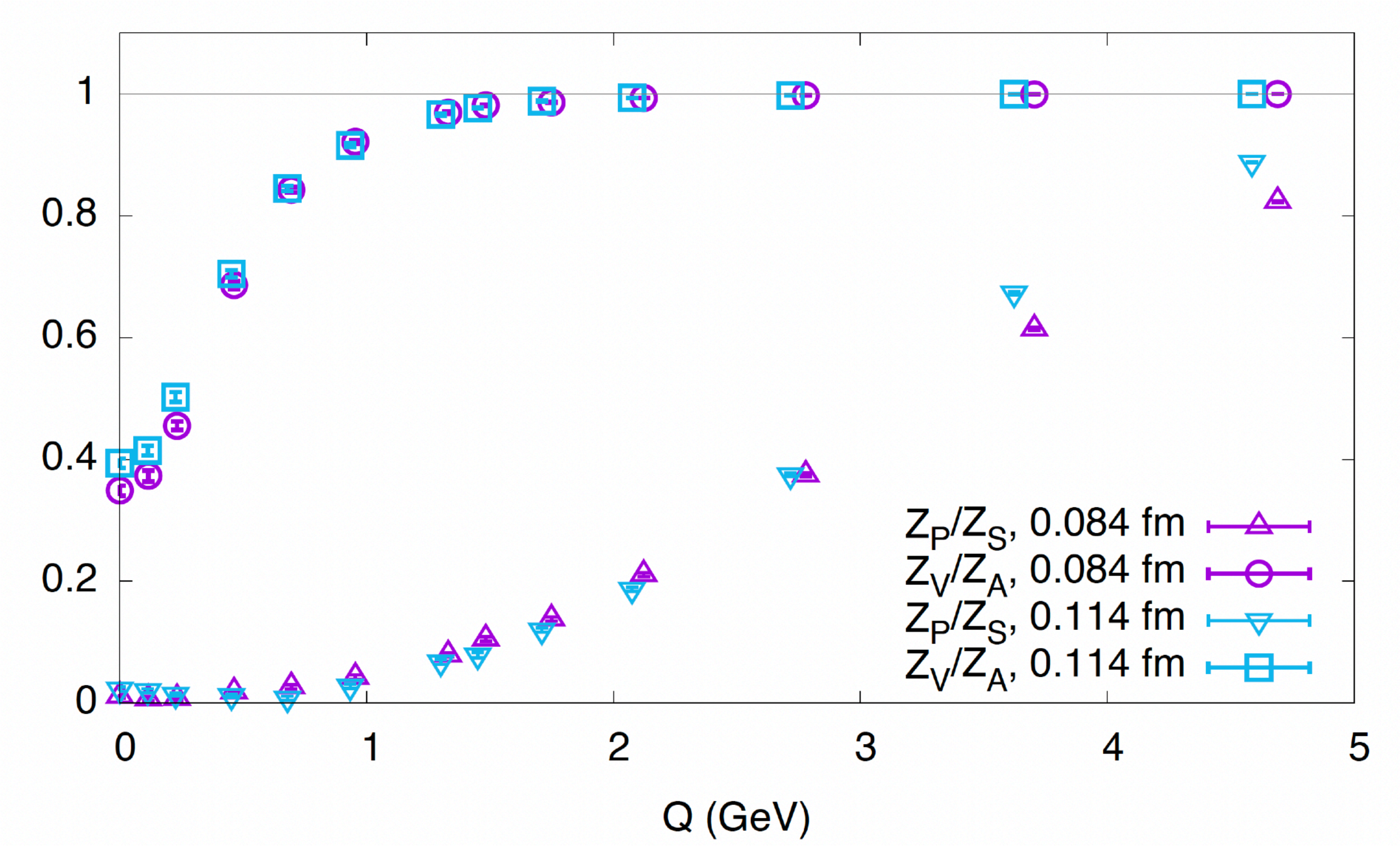
- If we define the mass renormalization constant through $m^{RI}(Q) = \frac{Tr[\langle S(p)\rangle^{-1}]}{Z_q(Q)}|_{Q=\sqrt{-p^2}}$, then we have $Z_p(Q)Z_m(Q) = 1$ for arbitrary quark mass and scale, and then a non-zero "dynamical mass" will appear in the renormalized quark mass

$$m_q^R(a, m_q; Q) = Z_m m_q^{\text{bare}}(a, m_q).$$

- It is an important feature in the DSE approach to understand the IR physics of QCD.
- But where does the feature come from?

Chiral symmetry breaking

in the vertex correction under the Landau gauge



- Under the Landau gauge, we can define the vertex correction at given off-shell scale as:

$$Z_\Gamma(Q) \equiv \frac{Z_q(Q)}{\mathrm{Tr}[\langle S(p)\rangle^{-1} . \langle S(p) . \Gamma . S(p)\rangle . \langle S(p)\rangle^{-1}]}\Big|_{Q=\sqrt{-p^2}} .$$

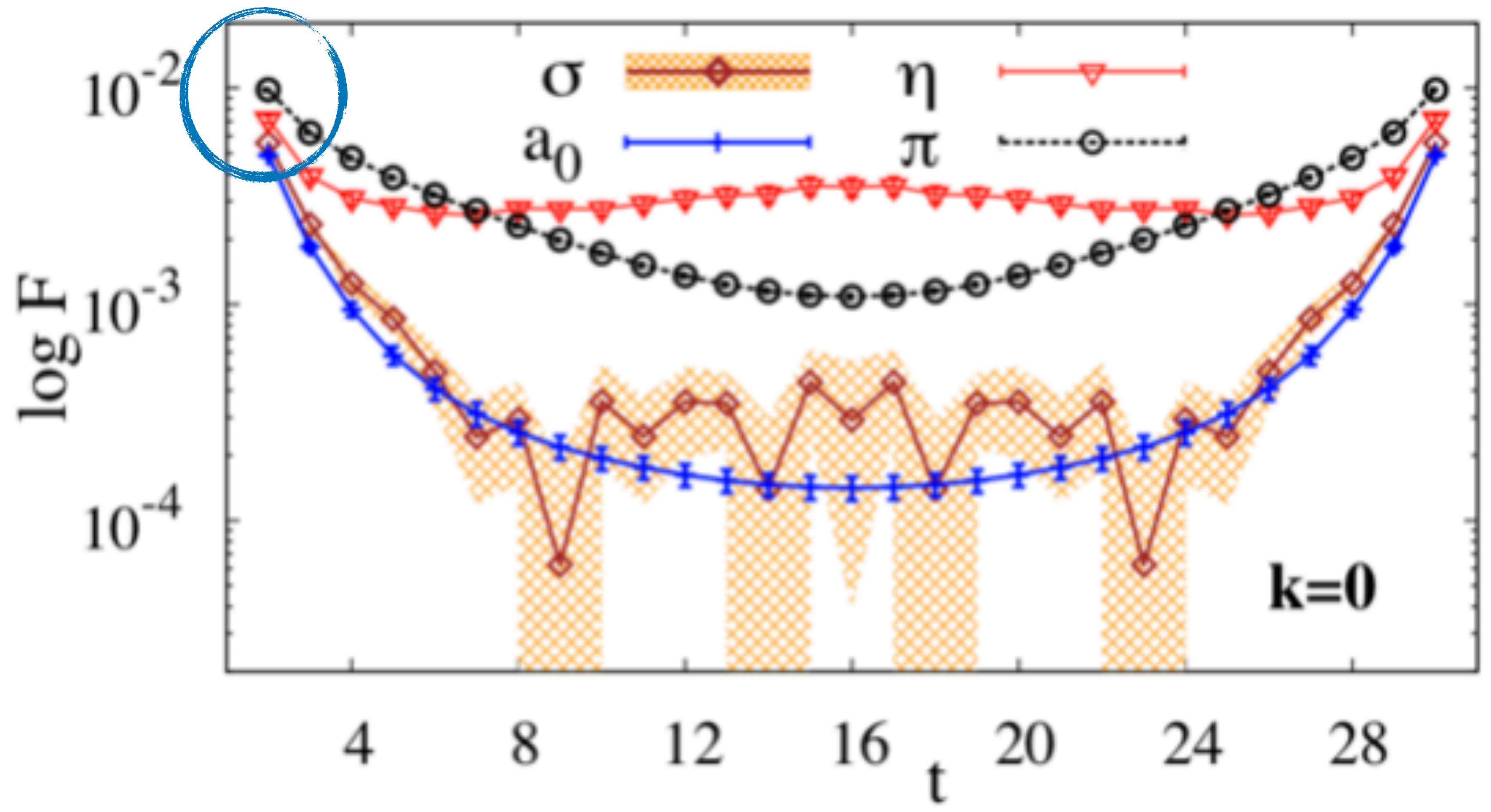
- Then we have

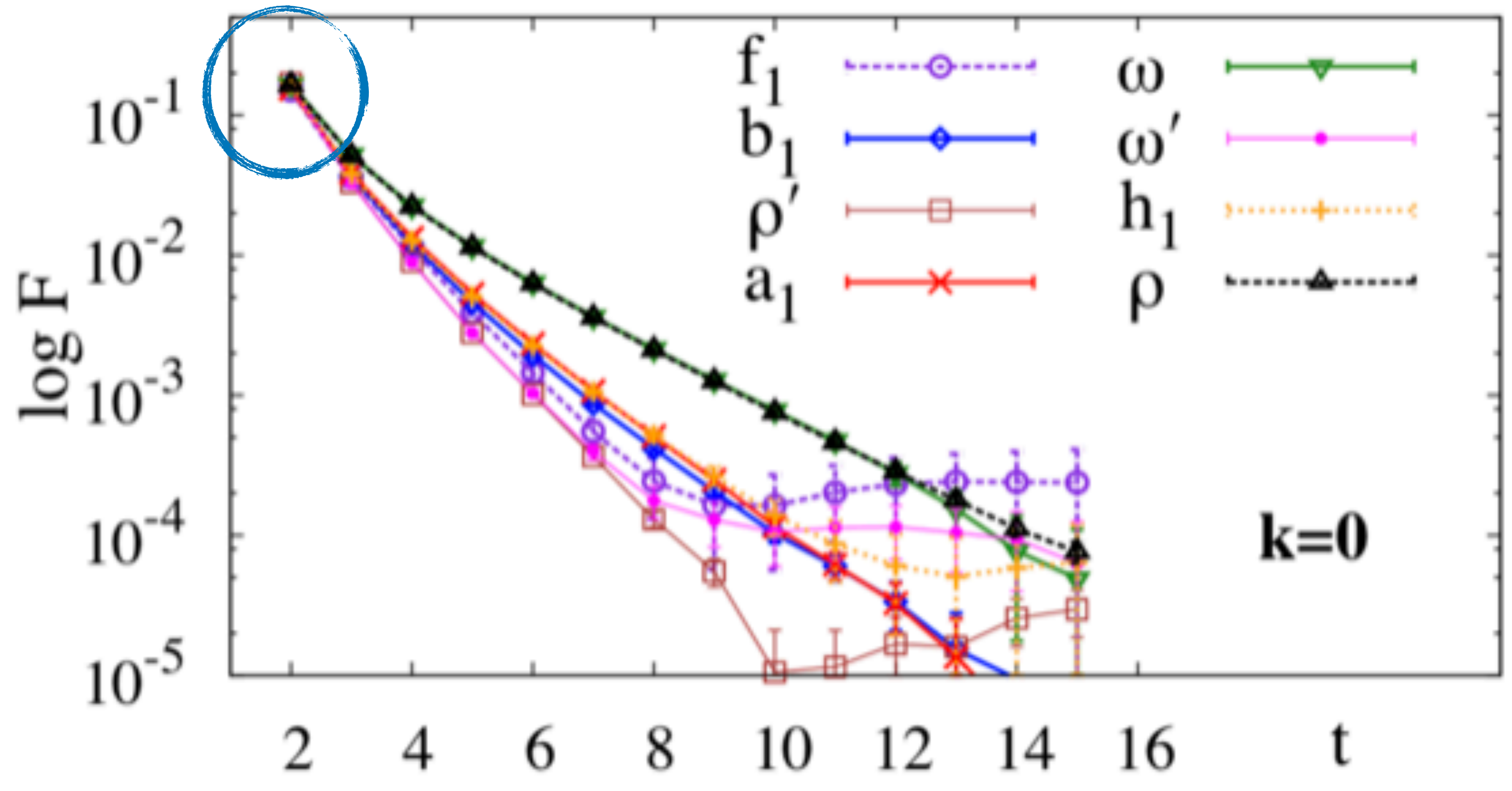
$Z_S \neq Z_P,\ Z_V \neq Z_A$, if Q is small.

- And we can see $Z_P = Z_m^{-1}$ approaches zero when $Q \to 0$.

Chiral symmetry breaking

in the hadron masses



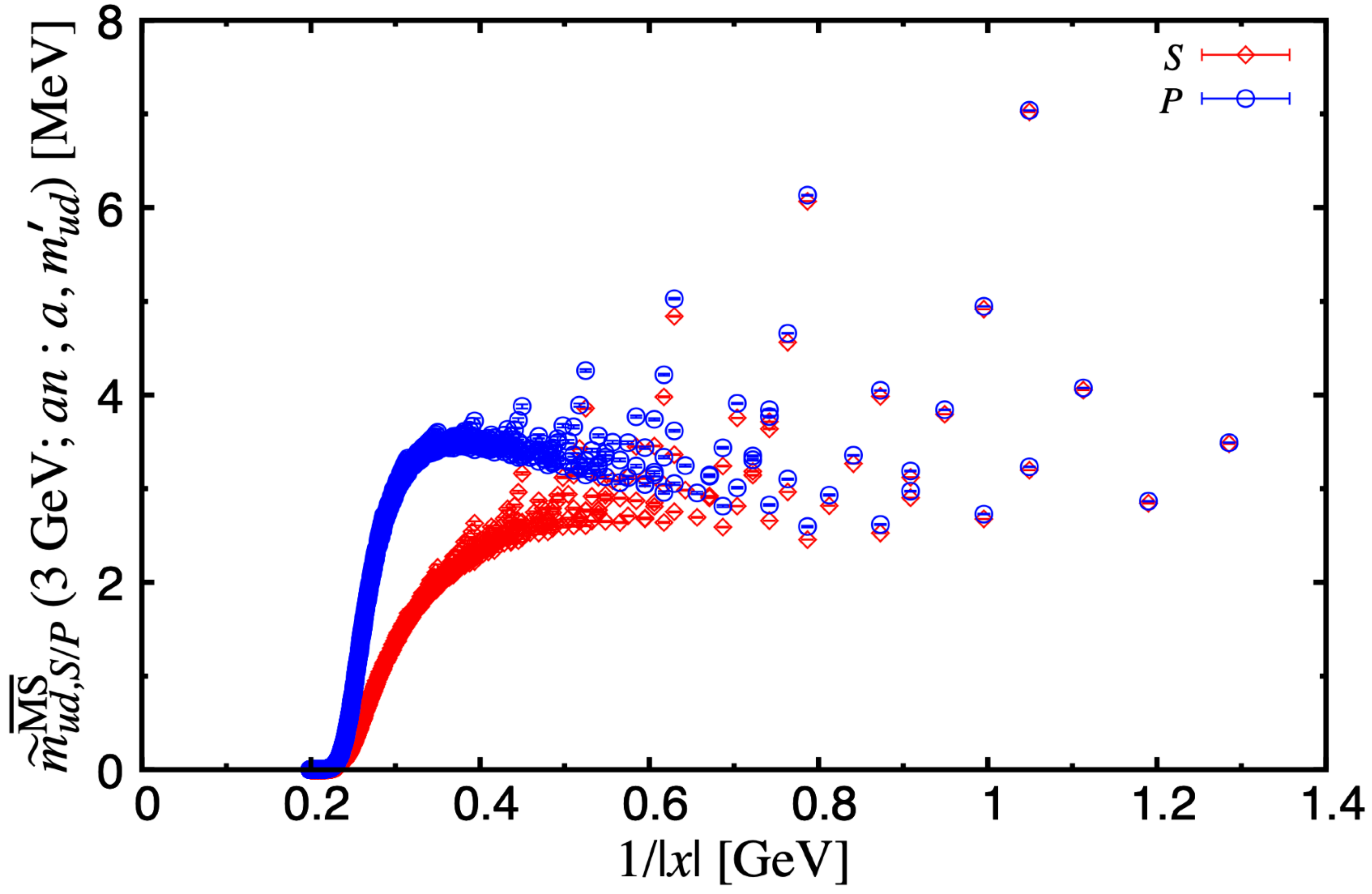


M. Denissenya, et al., PRD91(2015)034505, 1410.8751

- Based on the 2pt correlator
$$C_2(t,\Gamma) = \sum_{\vec{x}} \mathrm{Tr}[\langle \Gamma S(\vec{0},0;\vec{x},t)\Gamma S(\vec{x},t;\vec{0},0)\rangle]$$
in the Euclidean space, the ground state mass with given interpolation field $\bar{q}_1\Gamma q_2$ can be defined by:
$$m_\Gamma \equiv \frac{1}{a}\lim_{t\to\infty}\log\frac{C_2(t,\Gamma)}{C_2(t+a,\Gamma)}.$$
- The spontaneous chiral symmetry breaking makes
$$m_{a_0} \equiv m_I \neq m_{\gamma_5} \equiv m_\pi, m_{a_1} \equiv m_{\gamma_5\gamma_i} \neq m_{\gamma_i} \equiv m_\rho.$$
- But the correlators seem to be closer at smaller t…

Chiral symmetry breaking

in the correlators



M. Tomii, et al., PRD99(2019)014515, 1811.11238

- Based on the 2pt correlator $C_2(|x|,\Gamma) = \mathrm{Tr}[\langle \Gamma S(0;x)\Gamma S(x;0)\rangle]$ in the Euclidean space, the renormalized quark mass can be defined by:

$$m^{\overline{\rm MS}}_{ud,S/P}(1/|x|) = \sqrt{\frac{C_2(|x|,I/\gamma_5)}{C_2^{\overline{\rm MS}}(|x|,I/\gamma_5)}}\, m^{\rm bare}_{ud};$$

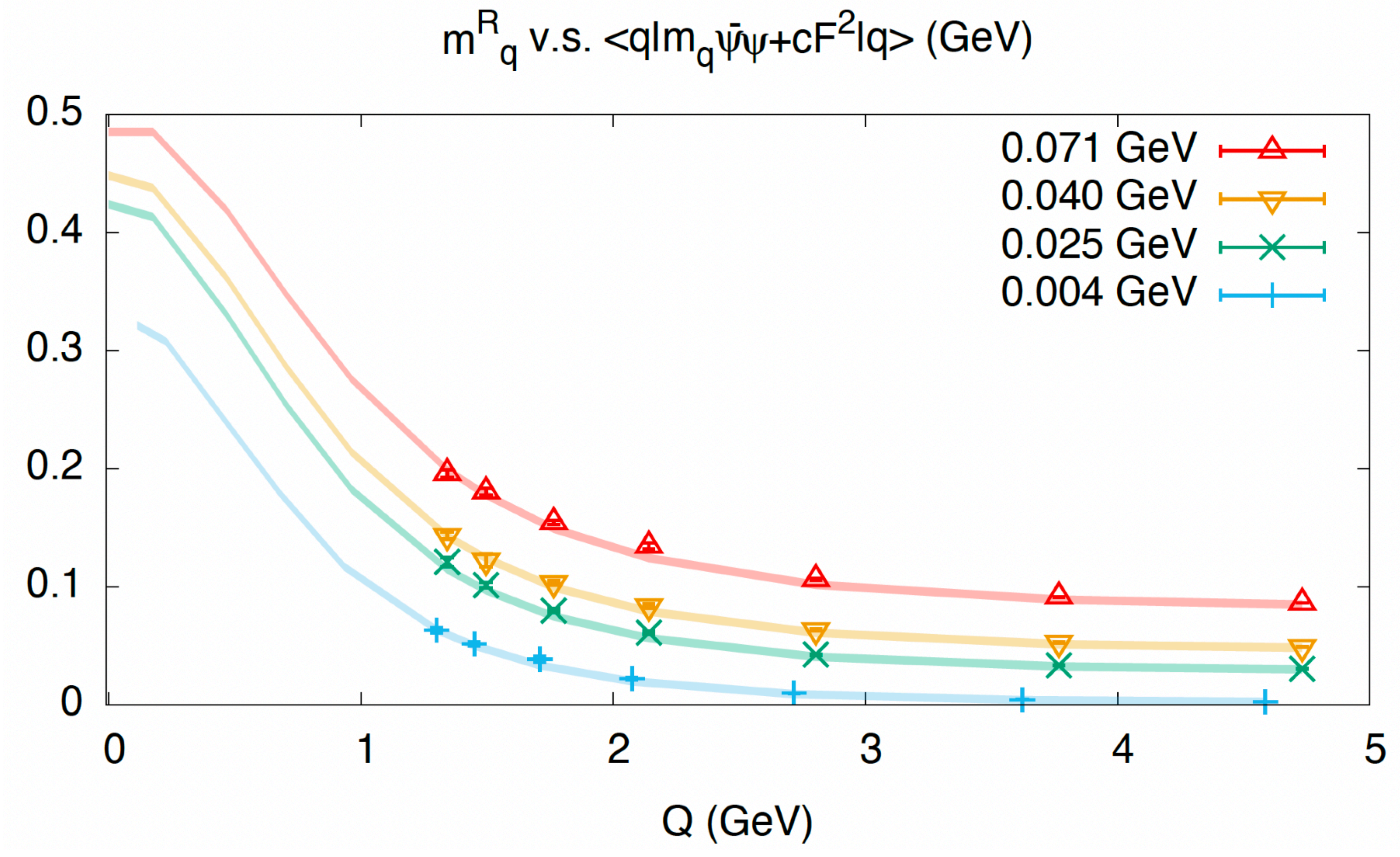
- Then we have

1. $m^{\overline{\rm MS}}_{ud,S}(1/|x|) \simeq m^{\overline{\rm MS}}_{ud,P}(1/|x|), C_2(|x|,I) \simeq C_2(|x|,\gamma_5)$, if $1/|x| \gg 1$ GeV;

2. $m^{\overline{\rm MS}}_{ud,S}(1/|x|) \neq m^{\overline{\rm MS}}_{ud,P}(1/|x|), C_2(|x|,I) \simeq C_2(|x|,\gamma_5)$, if $1/|x| \ll 1$ GeV.

Chiral symmetry breaking

dynamical quark mass and trace anomaly



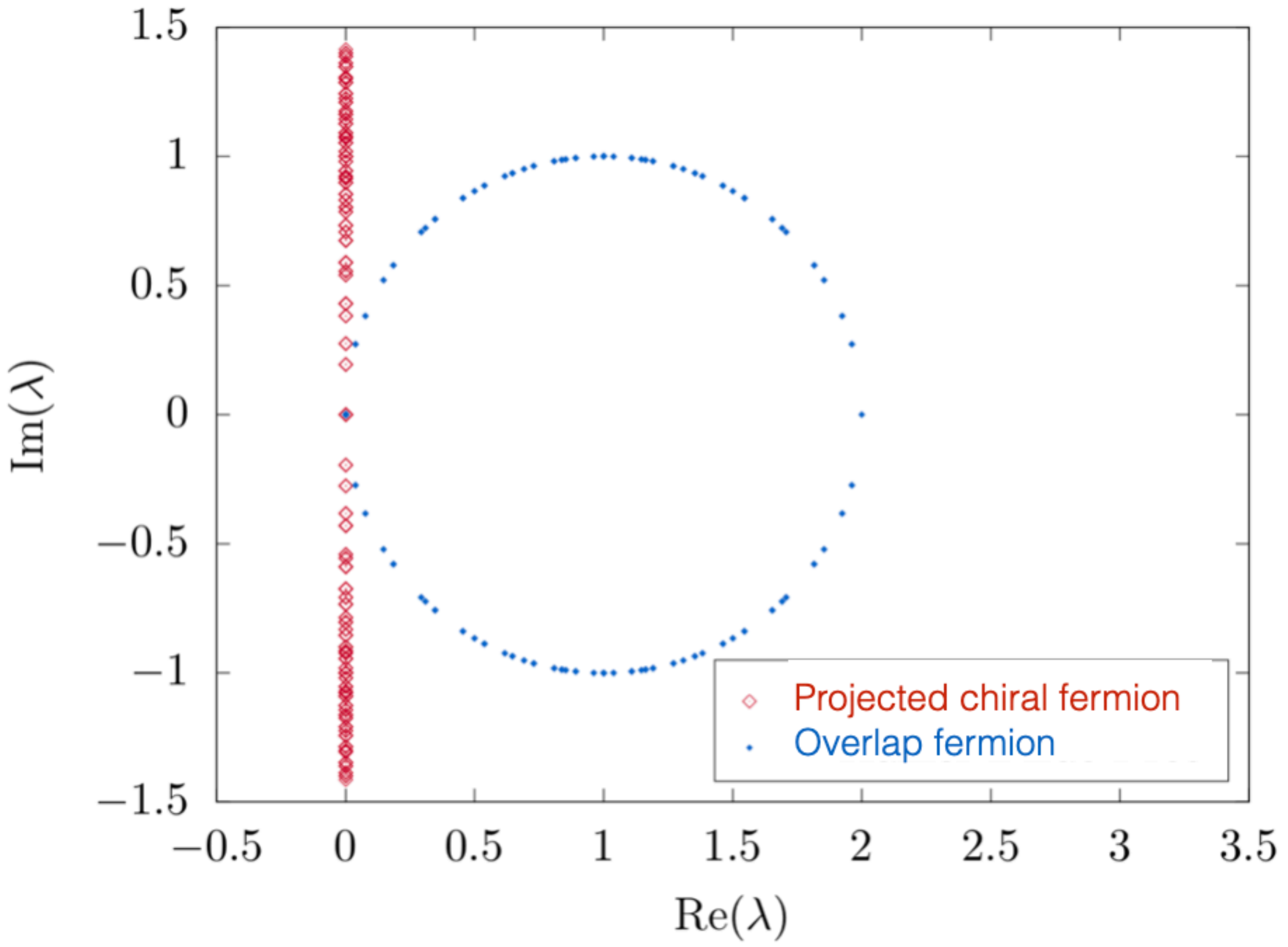
YBY, χQCD, Lattice2019 001, 2003.12914

- If we compare $m_q^R(a, m_q; Q) = Z_m m_q^{\text{bare}}(a, m_q)$ with $\langle q\,|\,m_q\bar{\psi}\psi - \dfrac{\beta}{2g}F^2\,|\,q\rangle$, they are somehow close to each other at large Q.
- But it would not be a well-defined comparison and requires further investigation.
- Let us consider the problem in another way, through the Dirac spectrum…

Dirac spectrum

and chiral symmetry breaking

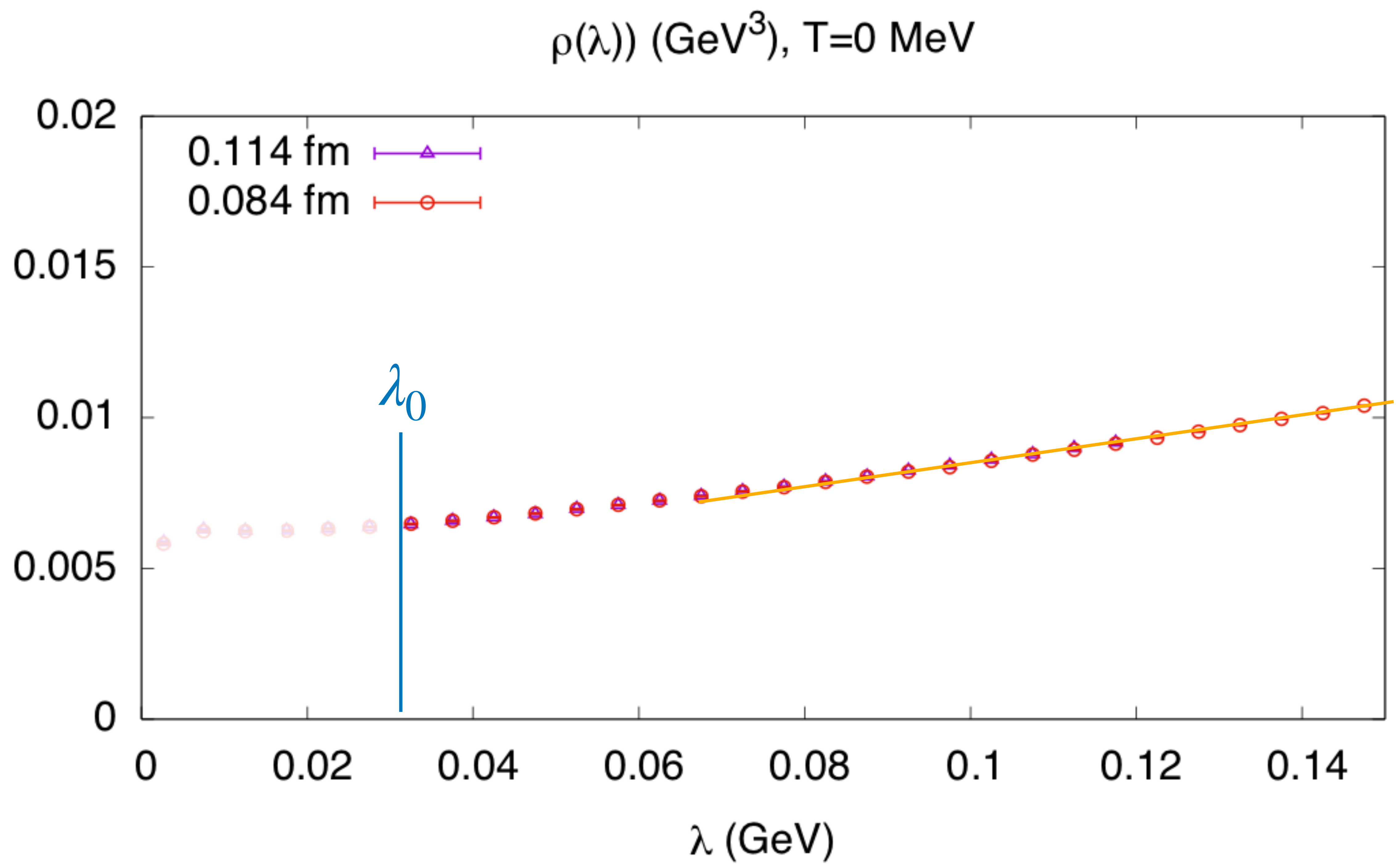
- The non-zero and finite eigenvalues of the overlap fermion are all paired,
  $D_c v = \mathrm{i}\lambda_c v,\ D_c\gamma_5 v = -\mathrm{i}\lambda_c\gamma_5 v$
- Thus if $|\lambda_c|$ has a lower band $\lambda_0 \gg m_q$, then
  $\gamma_5 \frac{1}{D_c + m_q} + \frac{1}{D_c + m_q}\gamma_5 \propto \frac{m_q}{\lambda_0^2} \propto \frac{m_q}{\lambda_0}\frac{\gamma_5}{D_c + m_q}$
- and then the chiral symmetry restores in such a case.



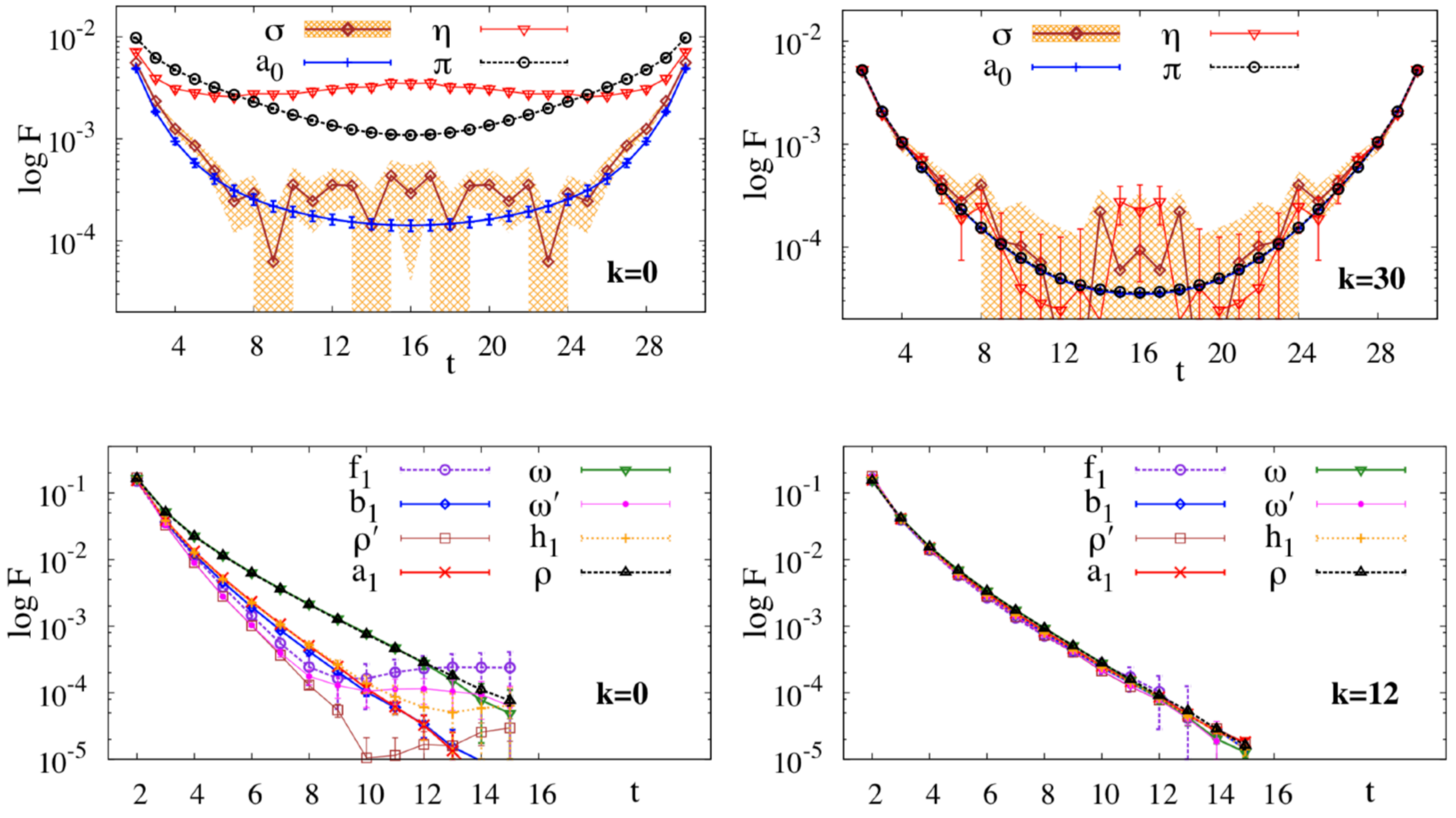
Dirac spectrum

with hard cutoff at small eigenvalues

$$S_h(y,x;\lambda_c) = \langle \bar{\psi}(x)\psi(y)\rangle - \sum_{-\lambda_0<\lambda<\lambda_0} \mathrm{v}_\lambda(x)\frac{1}{i\lambda+m}\mathrm{v}_\lambda^\dagger(y)$$



YBY, χQCD, Lattice2019 001, 2003.12914

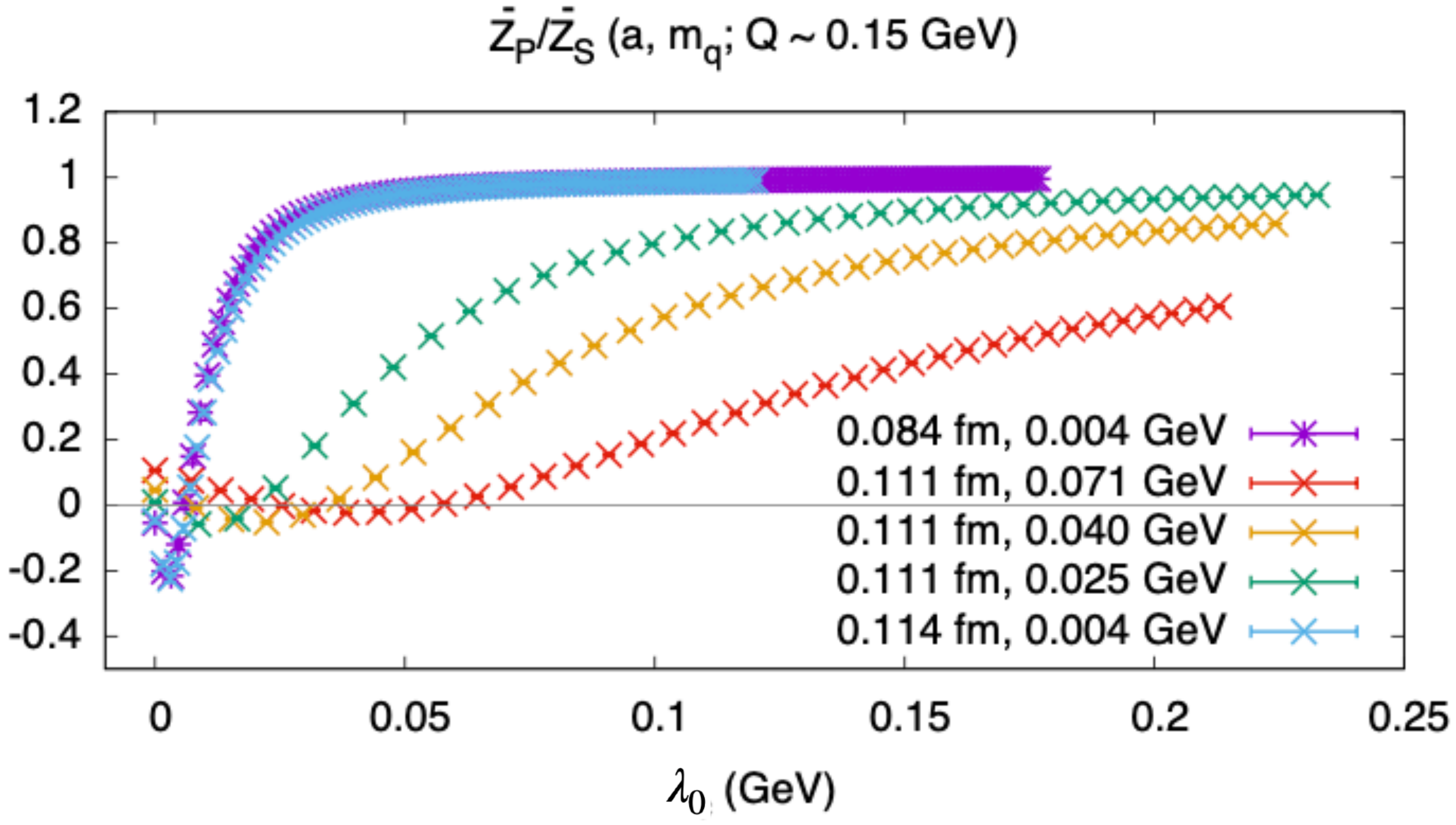


M. Denissenya, et al., PRD91(2015)034505, 1410.8751

If we put a hard cutoff λ_0 at small λ, then one would expect that the chiral condensate "vanishes", and chiral symmetry "restores" with a "spin symmetry".

Dirac spectrum

with hard cutoff at small eigenvalues



YBY, χQCD, Lattice2019 001, 2003.12914

- We define the subtracted propagator as

$$S_h(p,x;\lambda_c) = \sum_y e^{ipy}(\langle\bar{\psi}(x)\psi(y)\rangle - \sum_{-\lambda_0<\lambda<\lambda_0} \mathrm{v}_\lambda(x)\frac{1}{i\lambda+m}\mathrm{v}^\dagger_\lambda(y));$$

- And then the subtracted vertex correction is defined by

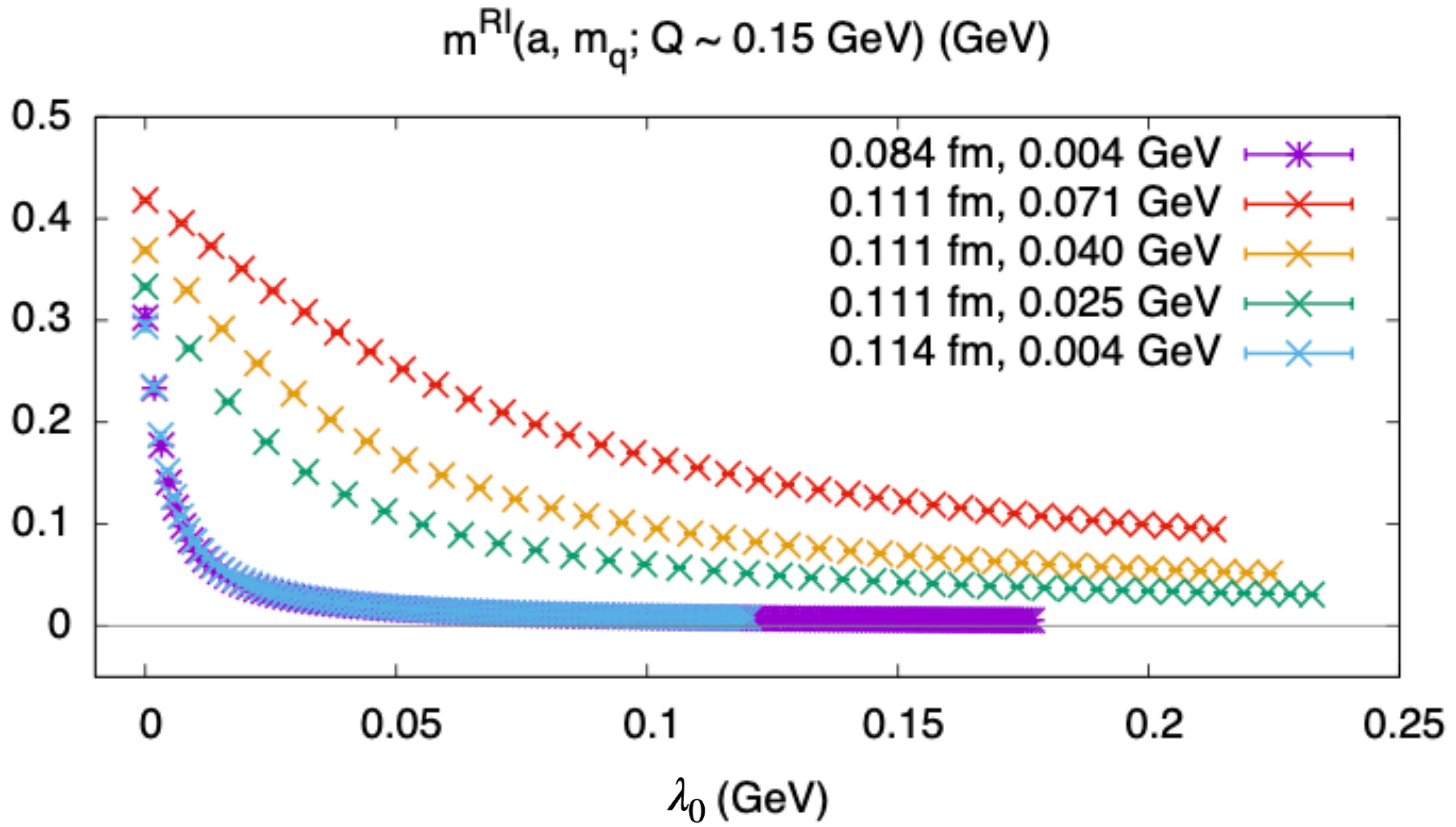
$$\frac{Z_p}{Z_s}(\lambda_0) = \frac{Tr[\langle S_h\rangle^{-1}.\langle S_h.\gamma_5.S_h\rangle.\langle S_h\rangle^{-1}]}{Tr[\langle S_h\rangle^{-1}.\langle S_h.S_h\rangle.\langle S_h\rangle^{-1}]}(\lambda_0).$$

- One can see that the chiral symmetry restores after the low mode with $\lambda < 10m_q$ are subtracted.

$$\gamma_5\frac{1}{D_c+m_q} + \frac{1}{D_c+m_q}\gamma_5 \propto \frac{m_q}{\lambda_0^2} \propto \frac{m_q}{\lambda_0}\frac{\gamma_5}{D_c+m_q}$$

Dirac spectrum

with hard cutoff at small eigenvalues



YBY, χQCD, Lattice2019 001, 2003.12914

- Similar feature appears in the "dynamical quark mass", when we define the subtracted quark mass as
$$m^{RI}(Q;\lambda_0) = \frac{Tr[\langle S_h(p;\lambda_0)\rangle^{-1}]}{Z_q(Q;\lambda_0)}|_{Q=\sqrt{-p^2}}.$$

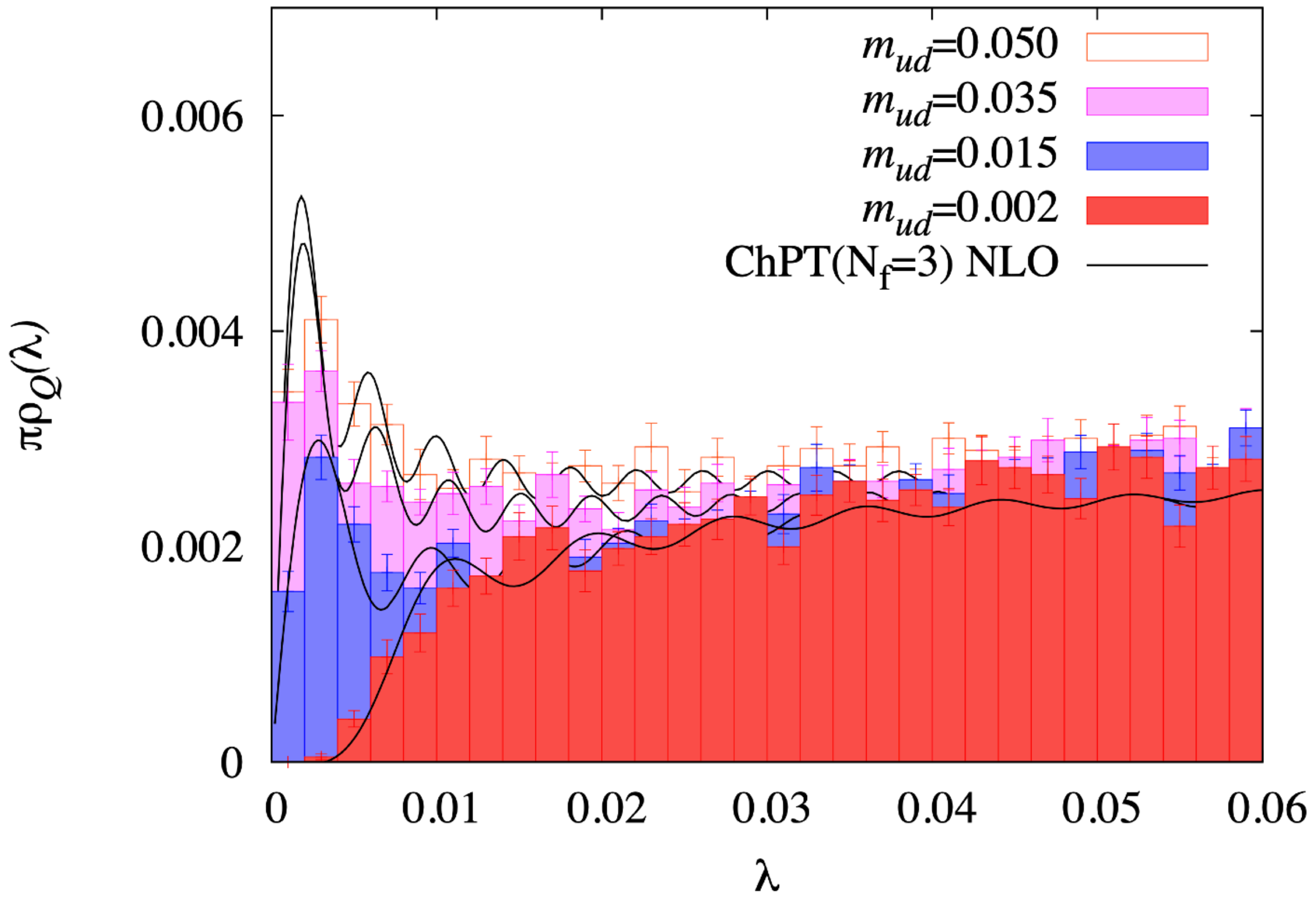
- Subtracted quark mass $m^{RI}(m_q;Q;\lambda_0) \propto m_q$ when the low mode with $\lambda < 10m_q$ are subtracted.

- It is consistent with the chiral symmetry "restoring" picture in the other examples.

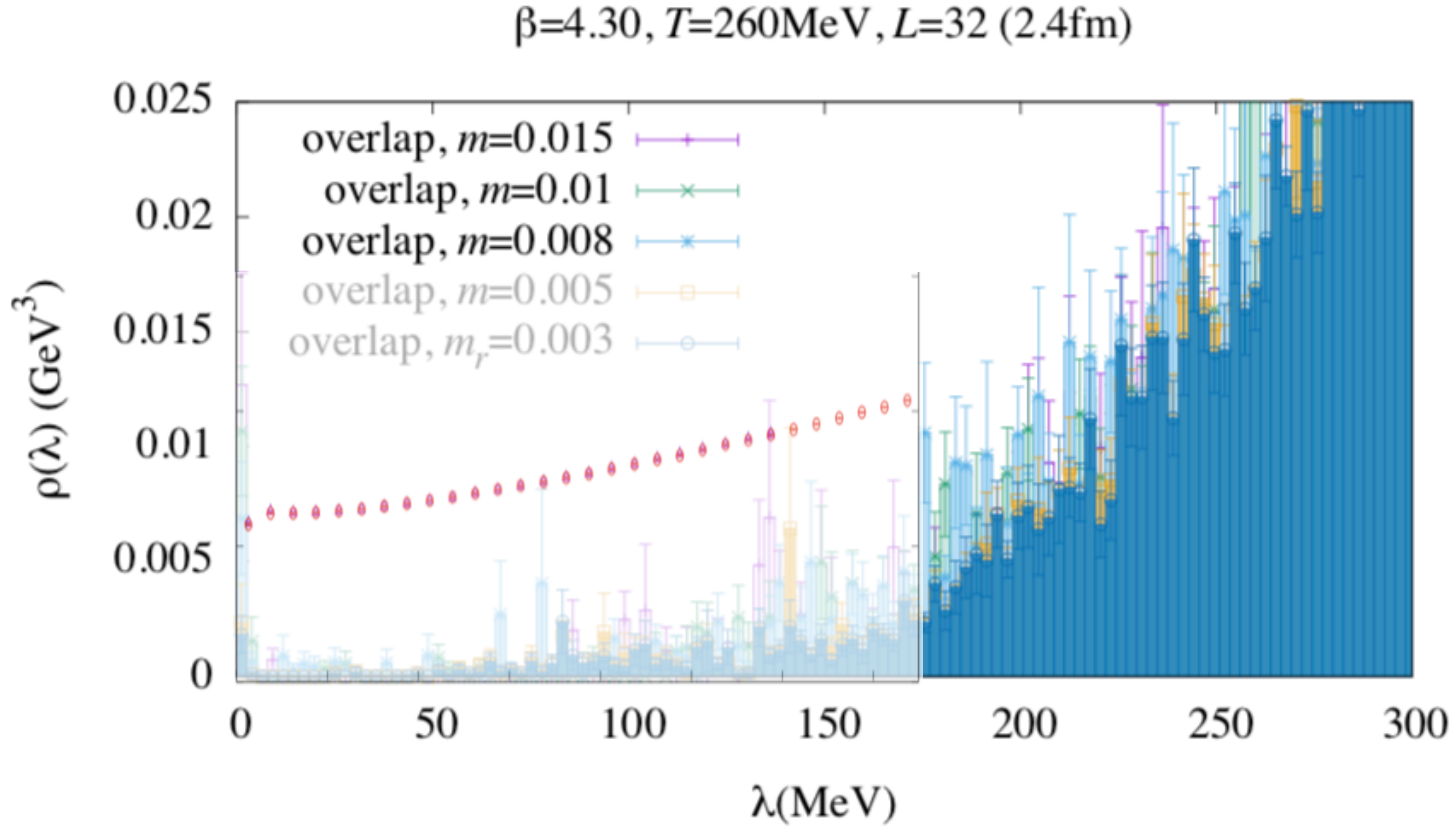
$$\gamma_5 \frac{1}{D_c+m_q} + \frac{1}{D_c+m_q}\gamma_5 \propto \frac{m_q}{\lambda_0^2} \propto \frac{m_q}{\lambda_0}\frac{\gamma_5}{D_c+m_q}$$

Dirac spectrum

on a lattice with small volume $L_S^3 \times L_T$



JLQCD and TWQCD, PRD83(2010), 074501, 1012.4052



S. Aoki, et.al, JLQCD, PRD103(2021), 074506, 2011.01499

- The $\rho(\lambda)$ will be suppressed at small λ when L_S and/or $L_T = 1/T$ is small enough (*T* here corresponds the temperature):

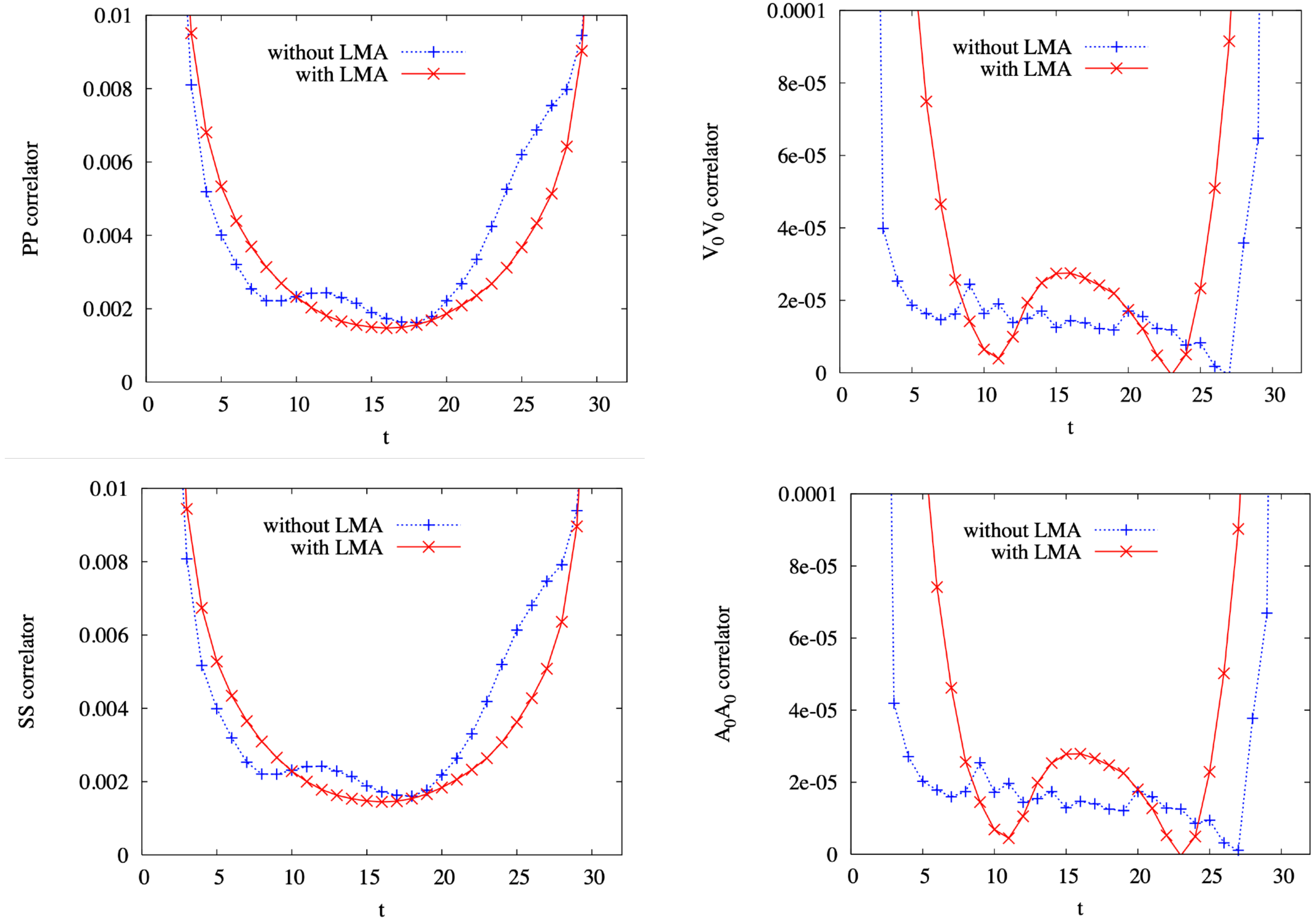
1. $L_S < \dfrac{1}{m_\pi} \ll L_T$, ϵ-regime;

2. $L_T < \dfrac{1}{m_\pi} \ll L_S$, finite temperature regime.

- The chiral symmetry breaking are also suppressed in these two cases.

Dirac spectrum

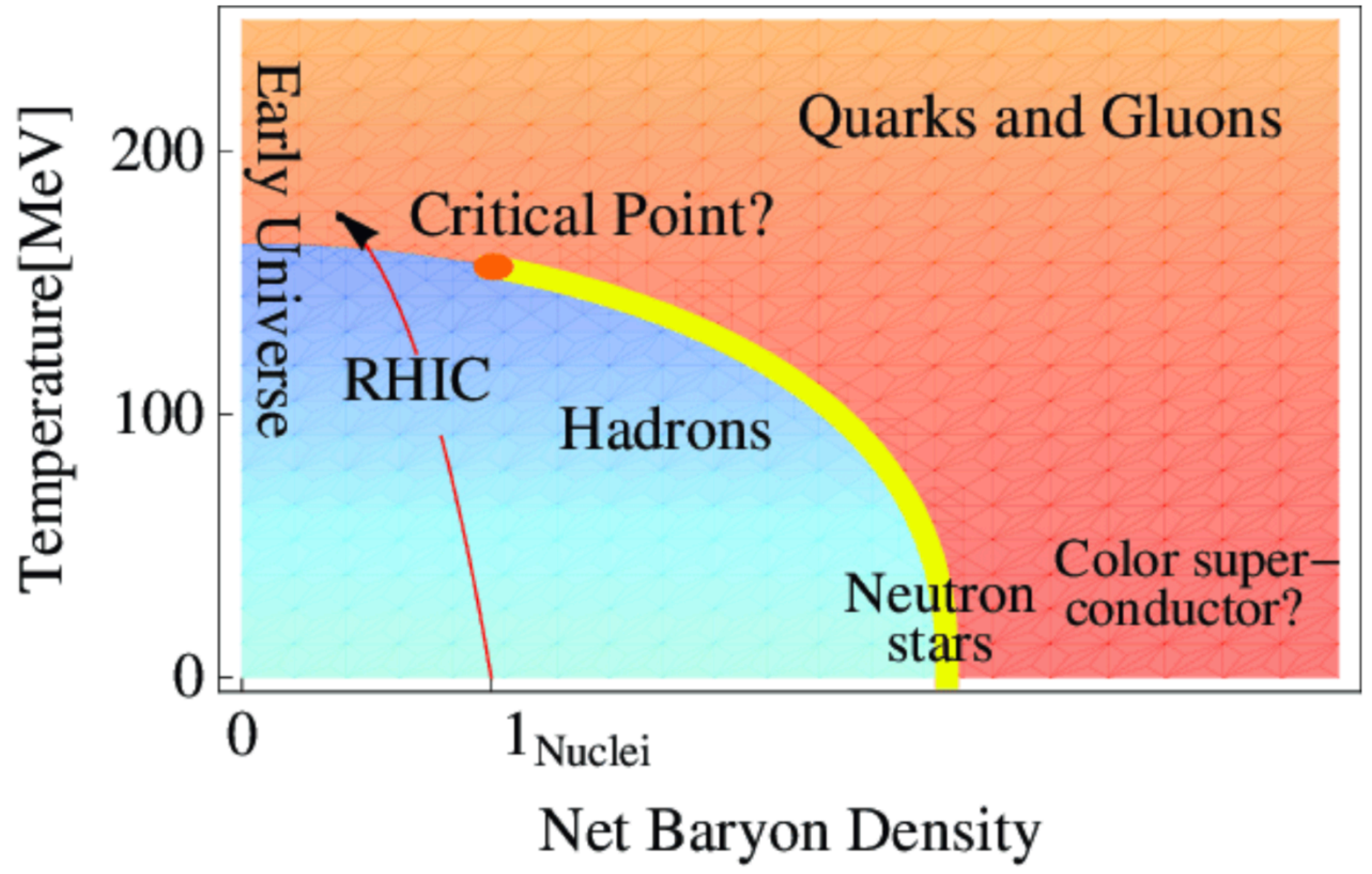
on a lattice with small spacial volume

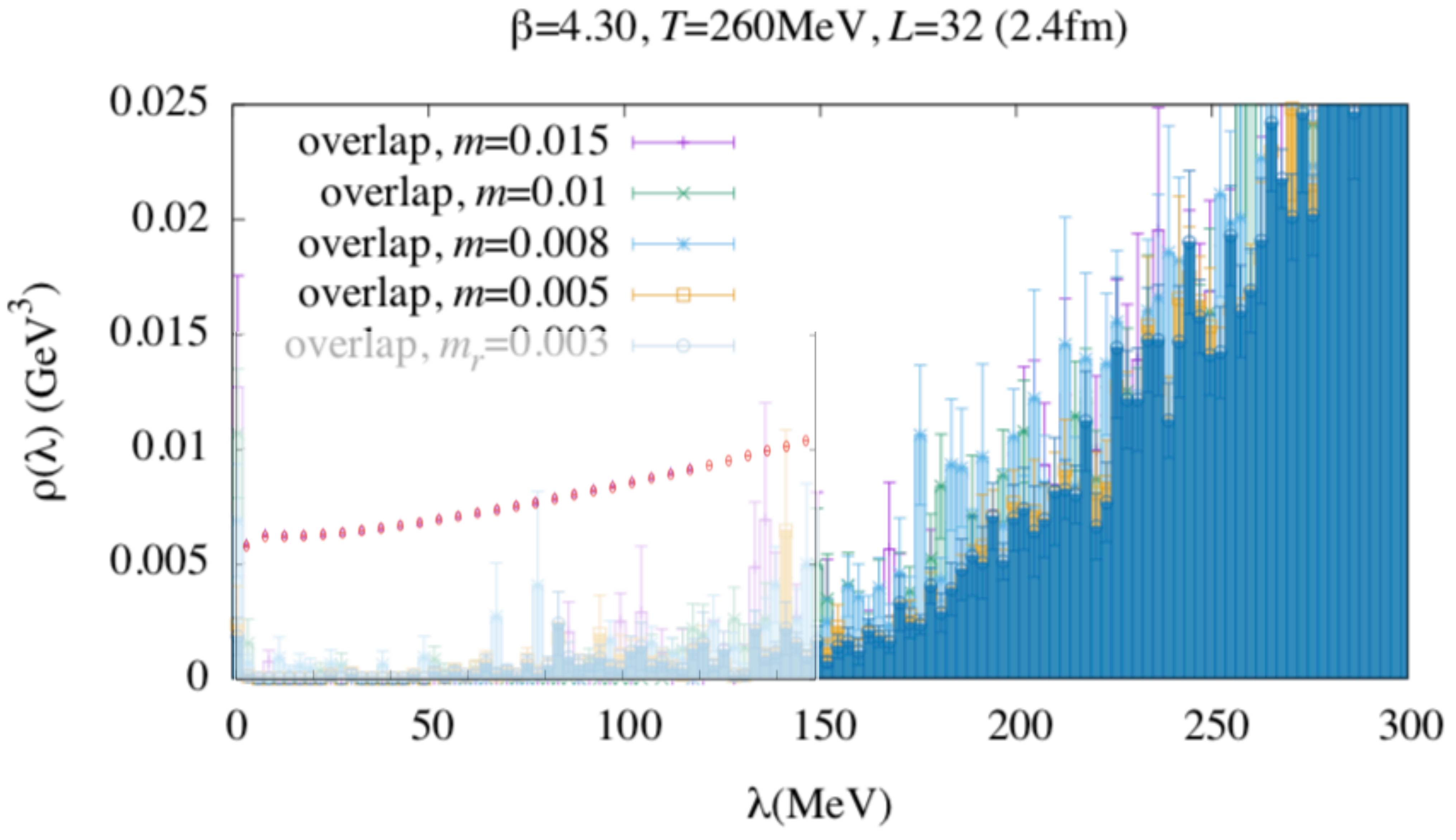


JLQCD, PRD77(2008), 074503, 0711.4965

- The result suggests that the chiral symmetry is restored in the ϵ-regime: $C_{2,P} = C_{2,S}$, $C_{2,V} = C_{2,A}$.
- The effective mass of “the vector meson” would be also small in such a case.

Dirac spectrum





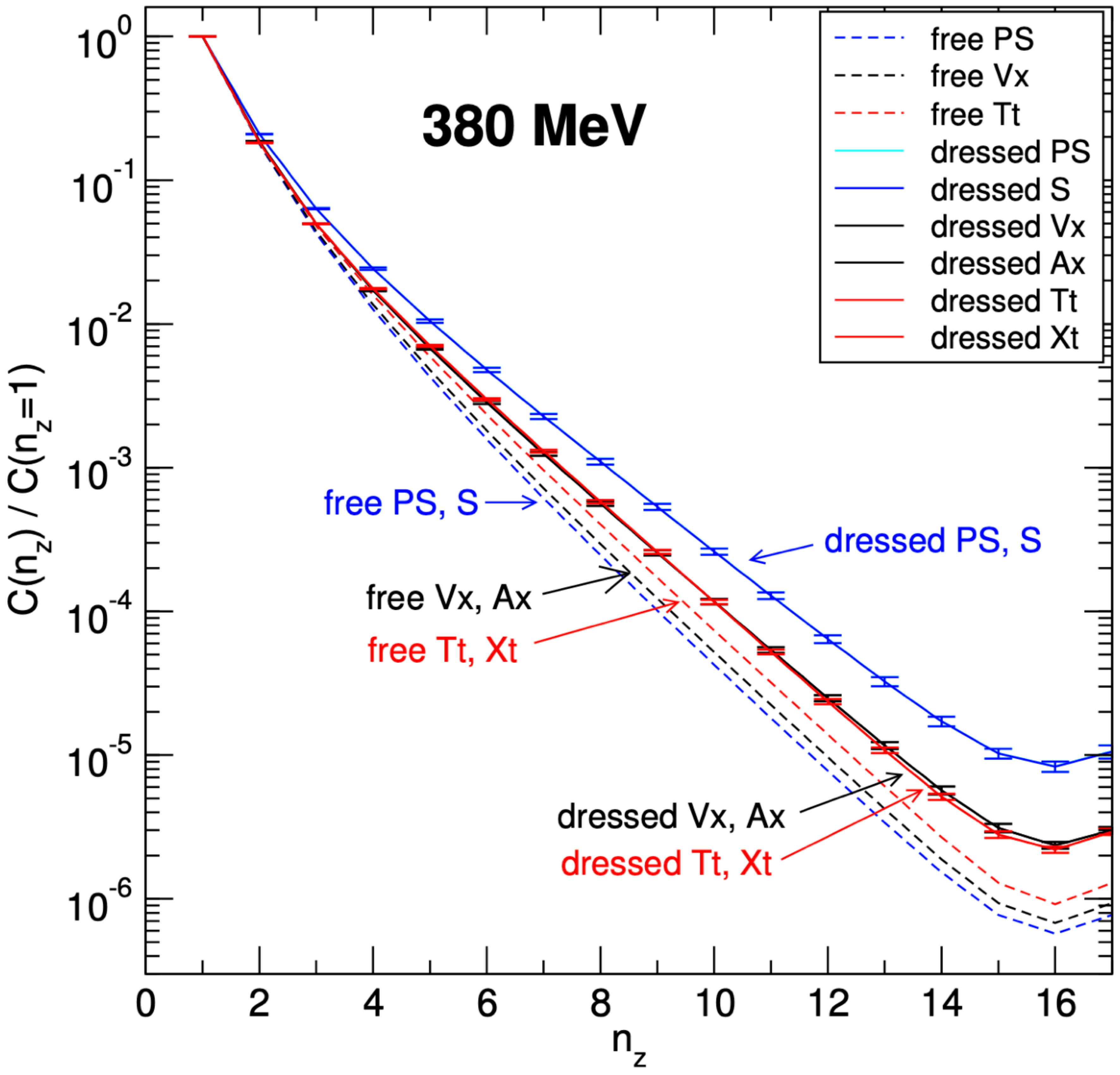
S. Aoki, et.al, JLQCD, PRD103(2021), 074506, 2011.01499

at high temperature

- Similarly, the $\rho(\lambda)$ is suppressed at small λ, when the temperature T is above the critical (cross-over) temperature ~ 150 MeV.
- The chiral condensates $\Sigma = \pi \lim_{\lambda \to 0} \lim_{V \to \infty} \rho(\lambda)$ vanishes;
- The chiral symmetry should also restores in such a case.

Dirac spectrum

at high temperature



L. Ya Glozman, PRD101(2020), 074516, 1912.06505

- $32^3 \times 8$, $T = 380$ MeV $\sim 2.2T_c$, $N_f = 2$
- The chiral symmetry is restored, as $C_{2,P} = C_{2,S}$, $C_{2,V} = C_{2,A}$.
- Even more, $C_{2,T} = C_{2,V} = C_{2,A}$.
- Chiral-spin symmetry within the $T_c - 3T_c$ intervals?

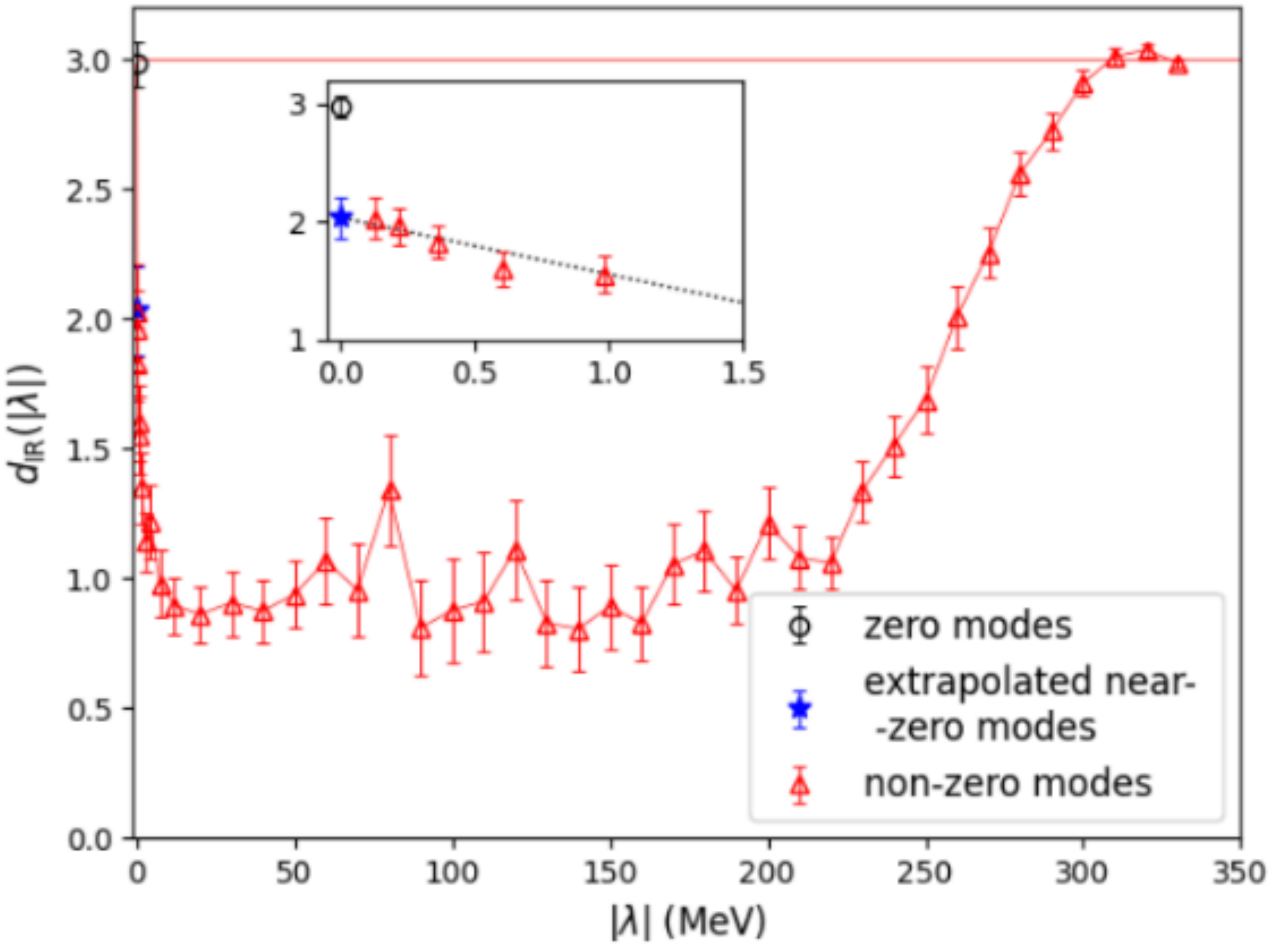
Distribution

at different temperatures

X. Meng, et.al, χQCD &CLQCD, in preparation

Measure-based dimension

in the 2+1 flavor case



X. Meng, et.al, χQCD &CLQCD, in preparation

- $N_f = 2 + 1,\ T = 234$ MeV.
- Overlap valence fermion and Clover fermion sea
- $d_{\mathrm{IR}} = 3$ for the eigenvector with $\lambda = 0$ and $\lambda \geq 300$ MeV.
- $d_{\mathrm{IR}} \to 2$ for the non-zero mode cases with $\lambda \to 0$.
- $d_{\mathrm{IR}} = 1$ for the cases with $\lambda \in [10,200]$ MeV.

Summary

- Lattice QCD provides a systematic solution to describe QCD precisely and accurately;
- Most of the light hadron masses and more than 1/3 charmonium masses come from the QCD dynamics.
- The spontaneous chiral symmetry breaking directly relates to the low lying eigen modes of the Dirac operator spectrum in the Euclidean space.
- Non-perturbative understandings of QCD are essential to understand the Lattice QCD “observations”.