Δ(1232) axial-vector and pseudoscalar form factors from the continuum Schwinger function methods

Chen Chen

University of Science and Technology of China

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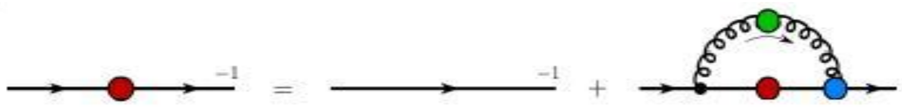
Non-Perturbative QCD:

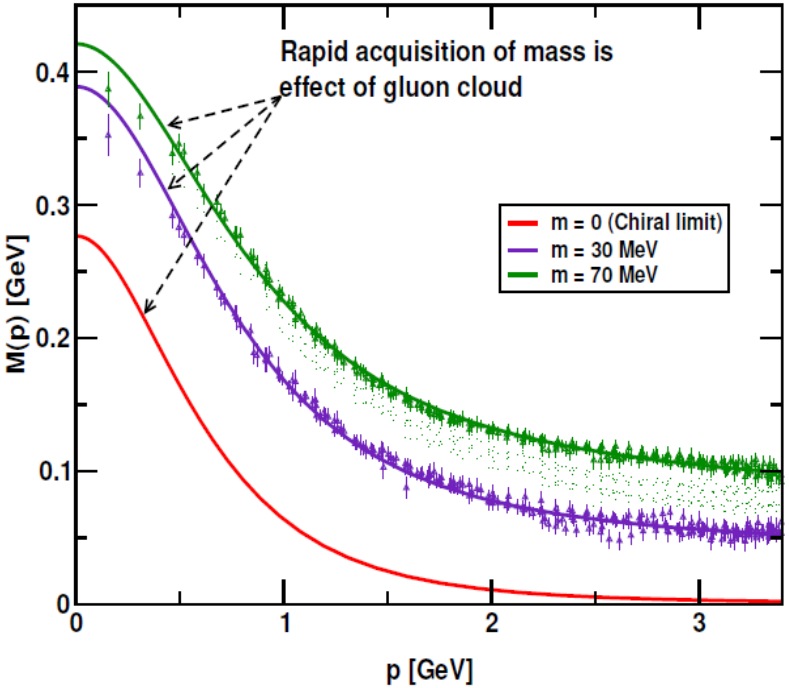
- **Hadrons, as bound states, are dominated by non-perturbative QCD dynamics –** *Two emergent phenomena*
  - **Confinement**: Colored particles have never been seen isolated
    - ✓ Explain how quarks and gluons bind together
  - **Dynamical Chiral Symmetry Breaking (DCSB)**: Hadrons do not follow the chiral symmetry pattern
    - ✓ Explain the most important mass generating mechanism for visible matter in the Universe
  - Neither of these phenomena is apparent in QCD 's Lagrangian, HOWEVER, They play a dominant role in determining the characteristics of real-world QCD!

Non-Perturbative QCD:

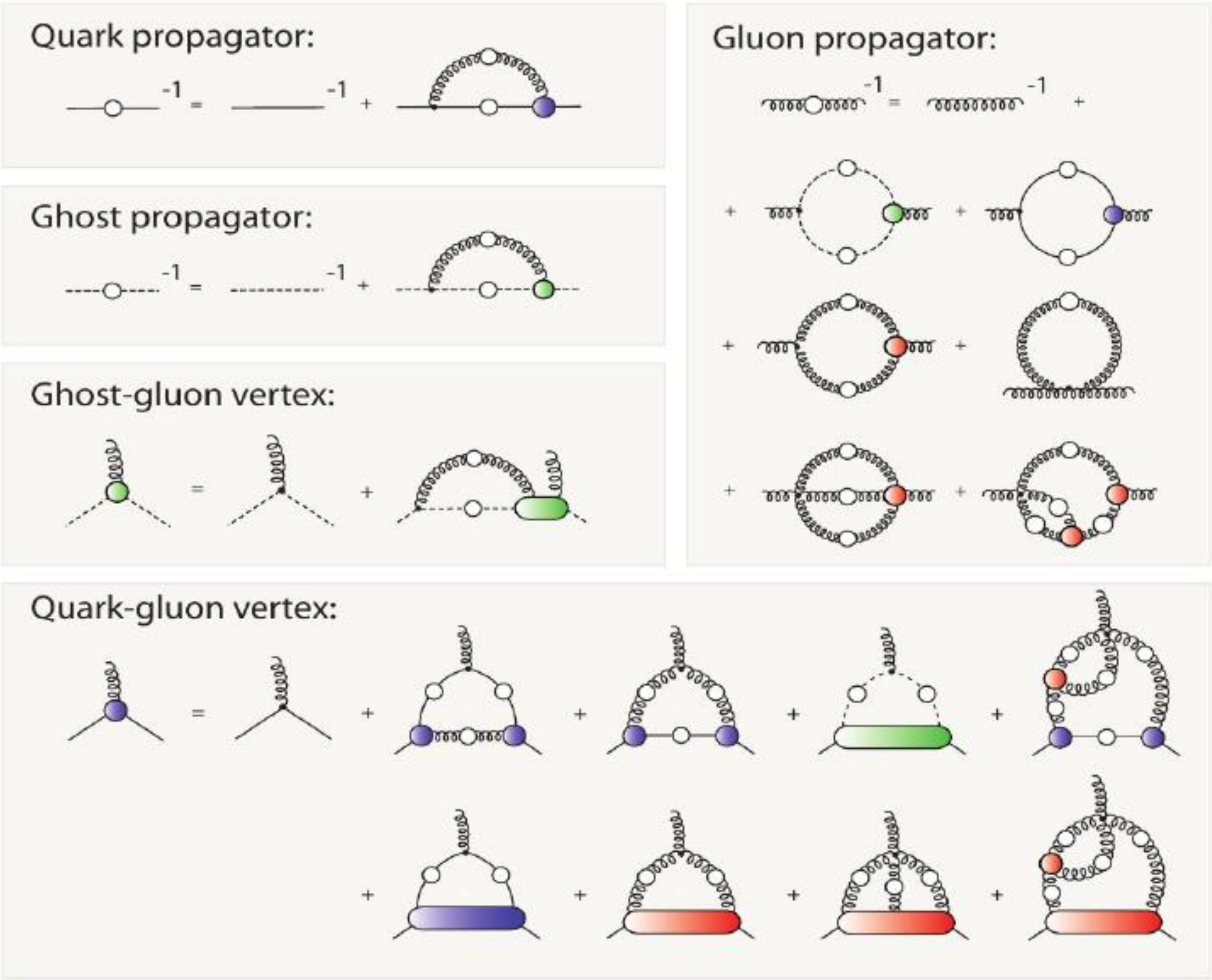
- **From a quantum field theoretical point of view**, these emergent phenomena could be associated with dramatic, dynamically driven changes in the analytic structure of QCD 's Schwinger functions (propagators and vertices). The Schwinger functions are solutions of the quantum equations of motion (**Dyson-Schwinger equations**).
- **Dressed-quark propagator:**



  - Mass generated from the interaction of quarks with the gluon.
  - Light quarks acquire a **HUGE** constituent mass.
  - Responsible of the 98% of the mass of the proton and the large splitting between parity partners.

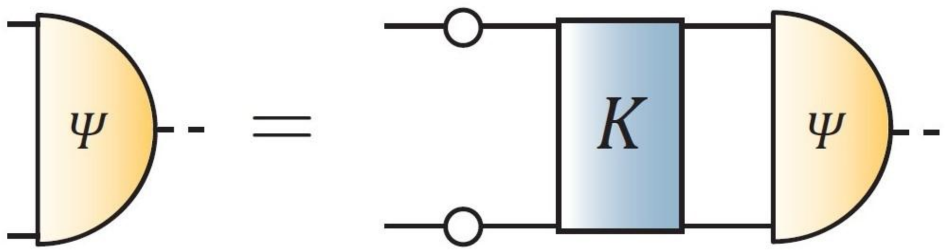


Continuum Schwinger function methods (CSMs)

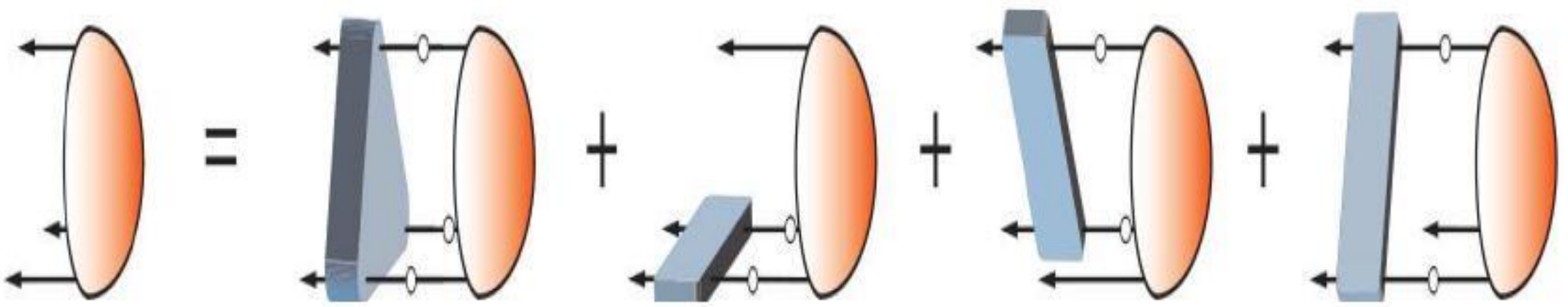


Hadrons: Bound-states in QFT

- **Mesons: a 2-body bound state problem in QFT**
  - ***Bethe-Salpeter Equation***
  - ***K*** **- fully amputated, two-particle irreducible, quark-antiquark scattering kernel**



- **Baryons: a 3-body bound state problem in QFT**
- ***Faddeev equation*: sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.**



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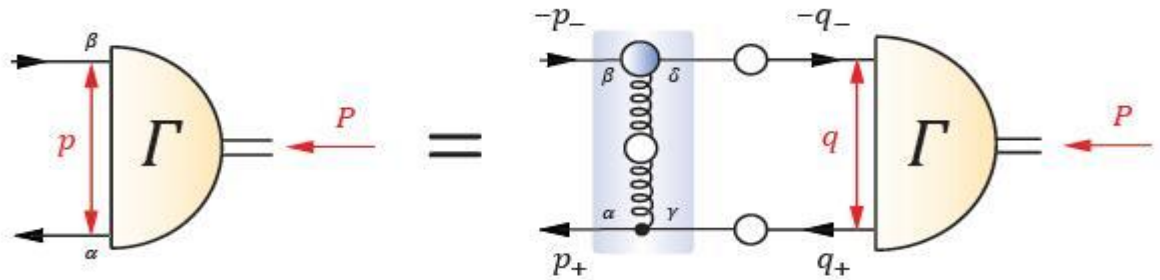

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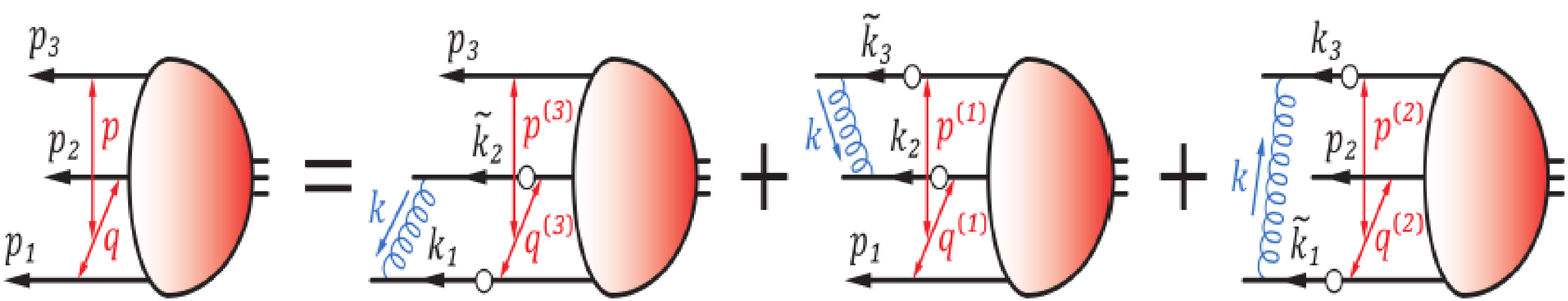
Faddeev equation in rainbow-ladder truncation



2-body correlations

- **Mesons: quark-antiquark correlations -- color-singlet**
- **Diquarks: quark-quark correlations within a color-singlet baryon.**

- **Diquark correlations:**
  - In our approach: non-pointlike color-antitriplet and fully interacting.
  - Diquark correlations are soft, they possess an electromagnetic size.
  - Owing to properties of charge-conjugation, a diquark with spin-parity J^P may be viewed as a partner to the analogous J^{-P} meson.

$$\Gamma_{q\bar{q}}(p;P) = -\int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{q\bar{q}}(q;P)S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

$$\Gamma_{qq}(p;P)C^\dagger = -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{qq}(q;P)C^\dagger S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

2-body correlations

- Quantum numbers:
  - (I=0, J^P=0^+): isoscalar-scalar diquark
  - (I=1, J^P=1^+): isovector-pseudovector diquark
  - (I=0, J^P=0^-): isoscalar-pesudoscalar diquark
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- ✓ G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer, Prog.Part.Nucl.Phys. 91 (2016) 1-100
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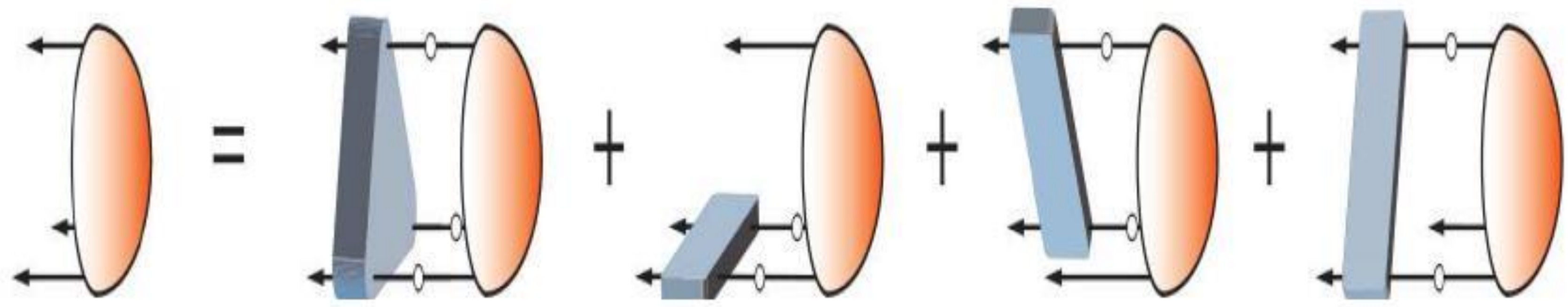

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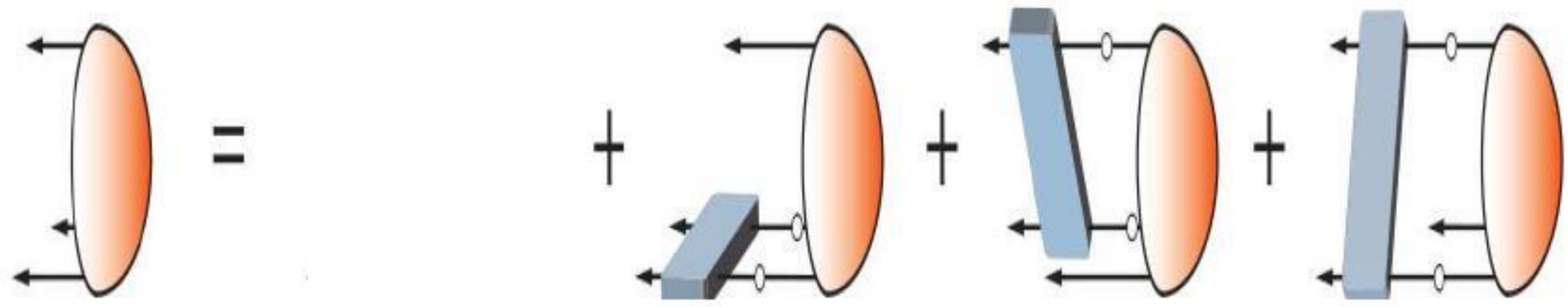

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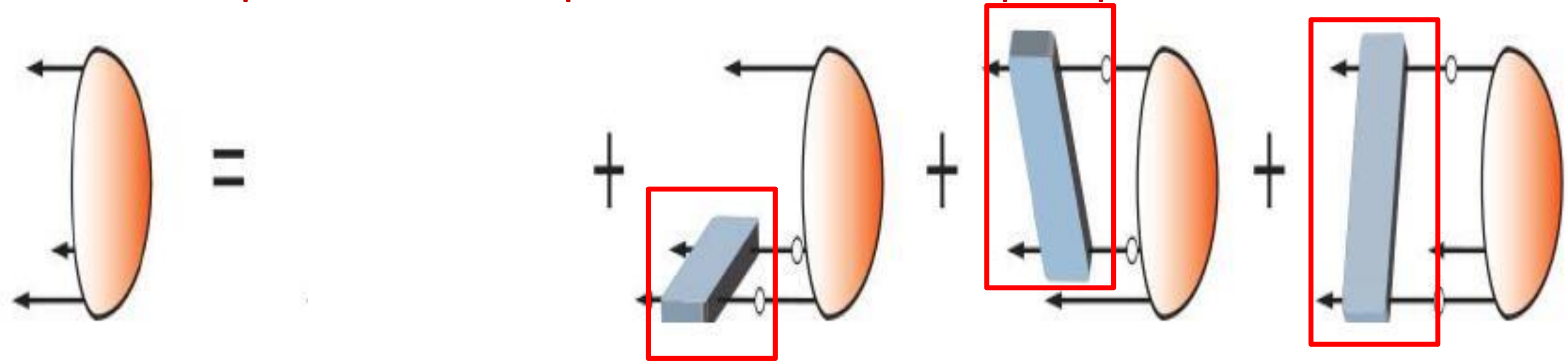

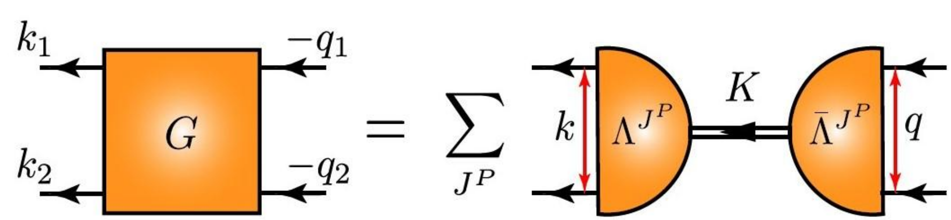

quark-diquark Faddeev equation

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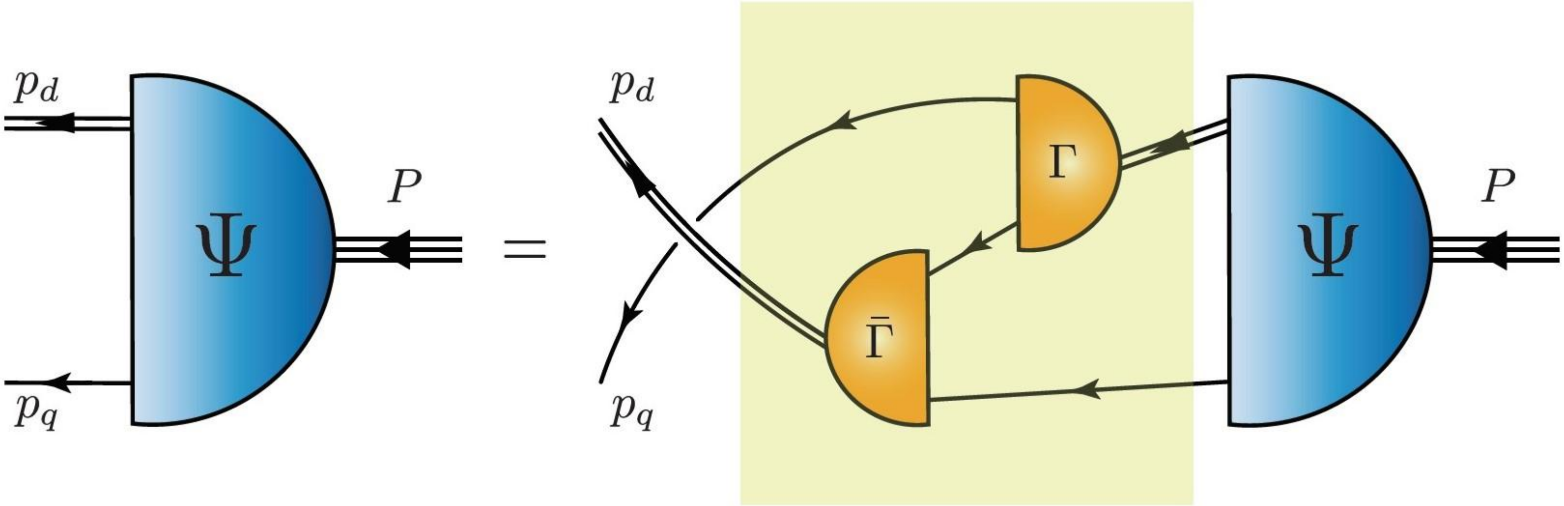

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- Three-body Faddeev equation → Quark-diquark two-body Faddeev equation

✓ R.T. Cahill, Craig D. Roberts, J. Praschifka, Phys. Rev. D 36 (1987) 2804

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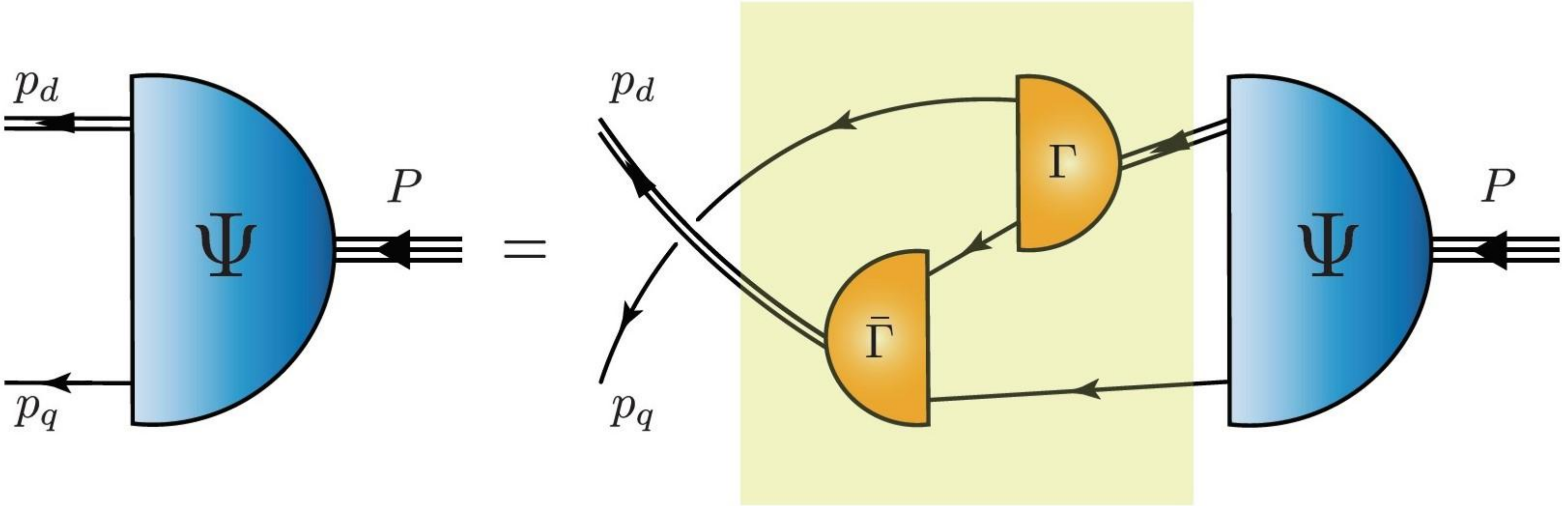

quark-diquark Faddeev equation

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Progress in Particle and Nuclear Physics

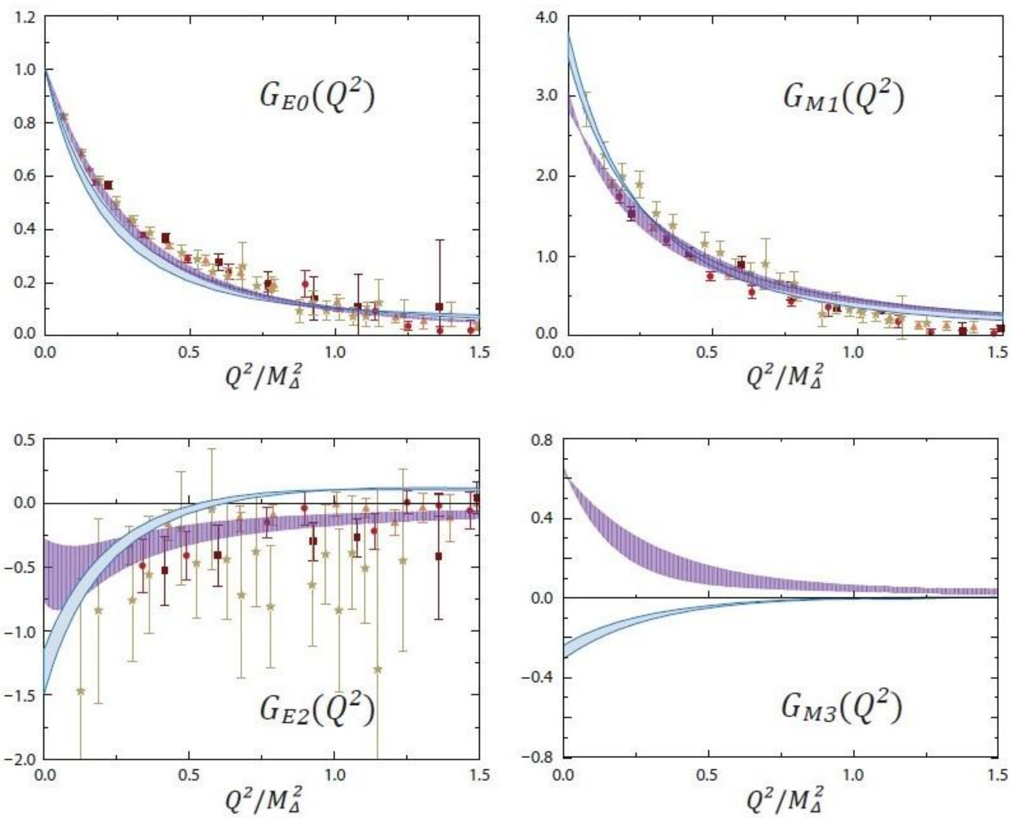
Volume 116, January 2021, 103835

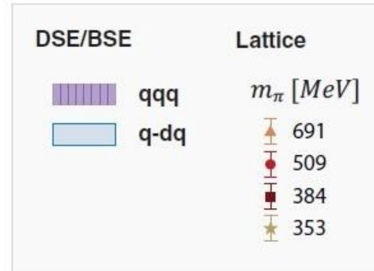
Review

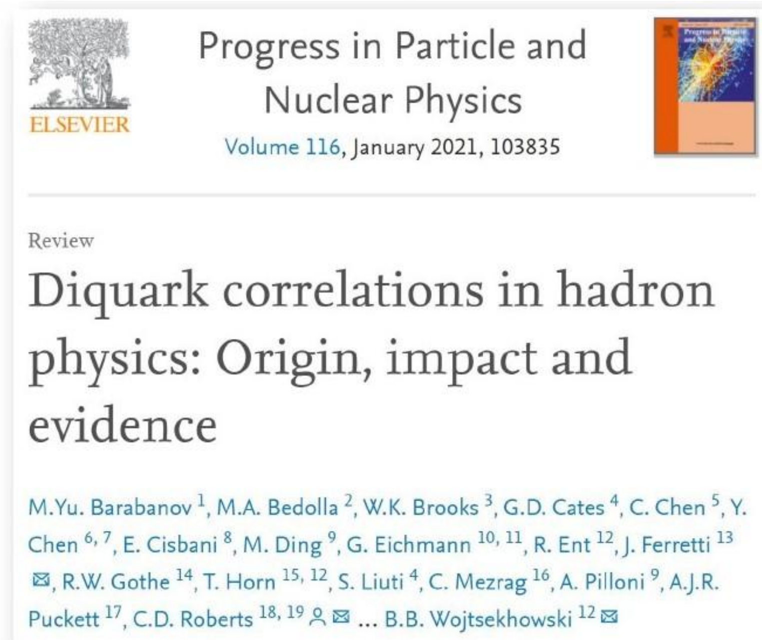
Diquark correlations in hadron physics: Origin, impact and evidence

M.Yu. Barabanov [1], M.A. Bedolla [2], W.K. Brooks [3], G.D. Cates [4], C. Chen [5], Y. Chen [6, 7], E. Cisbani [8], M. Ding [9], G. Eichmann [10, 11], R. Ent [12], J. Ferretti [13] ✉, R.W. Gothe [14], T. Horn [15, 12], S. Liuti [4], C. Mezrag [16], A. Pilloni [9], A.J.R. Puckett [17], C.D. Roberts [18, 19] ✉ … B.B. Wojtsekhowski [12] ✉

quark-diquark Faddeev equation







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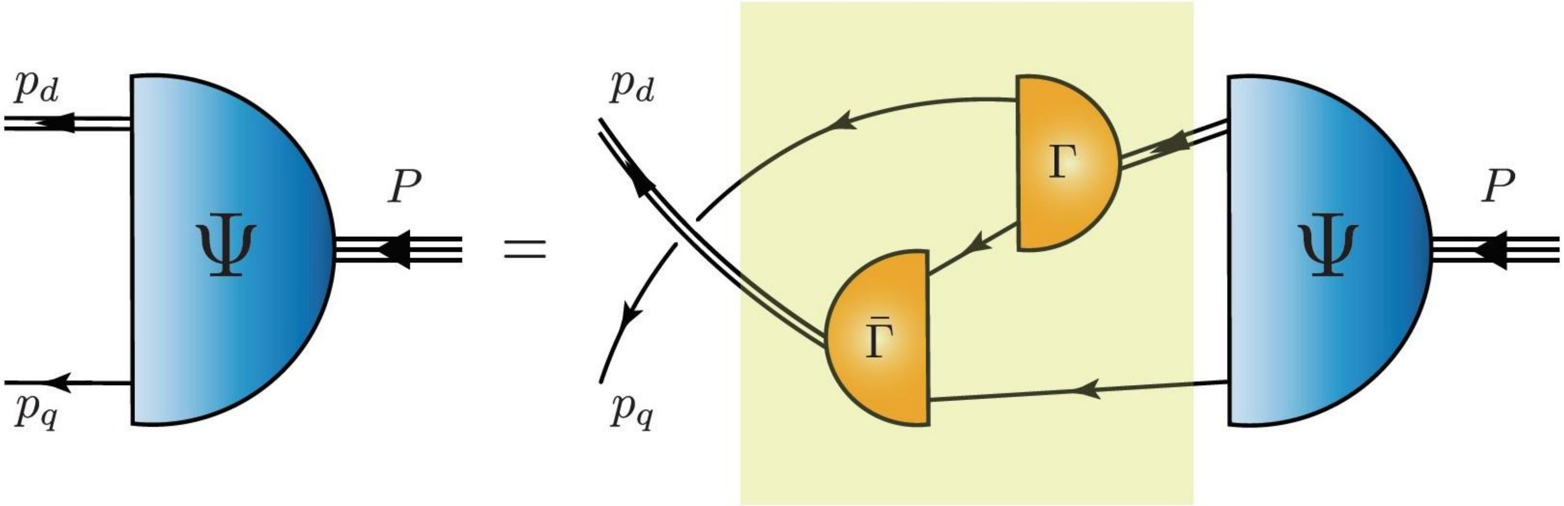

How to solve?

- The dressed-quark propagator
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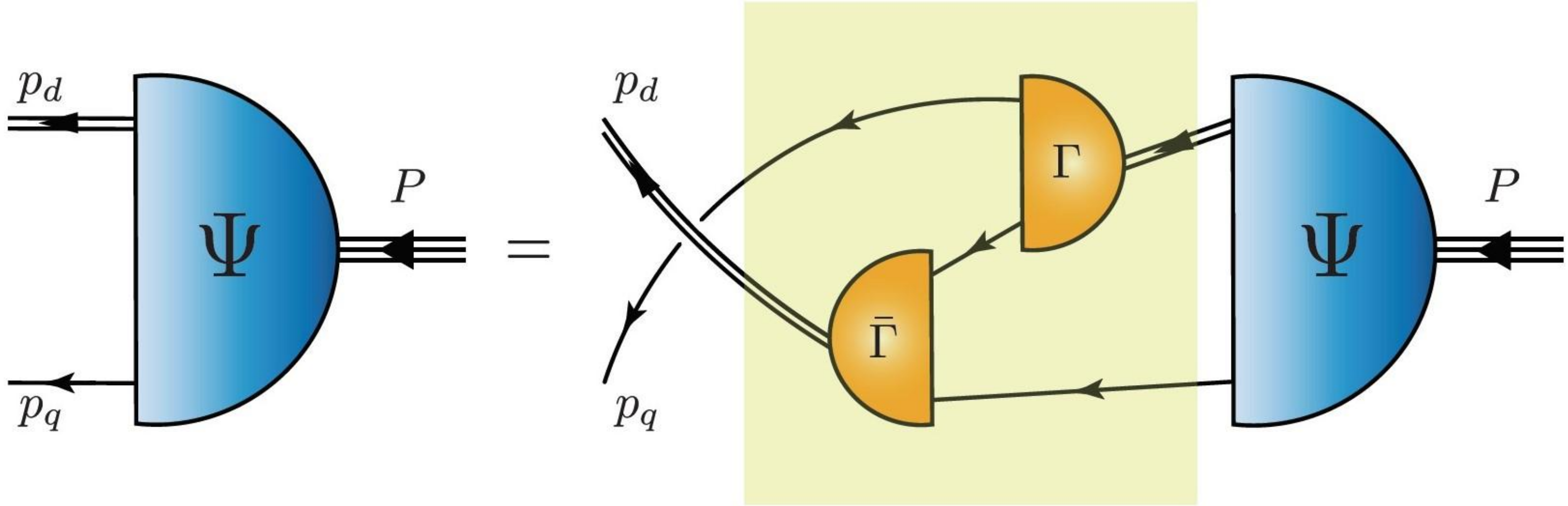

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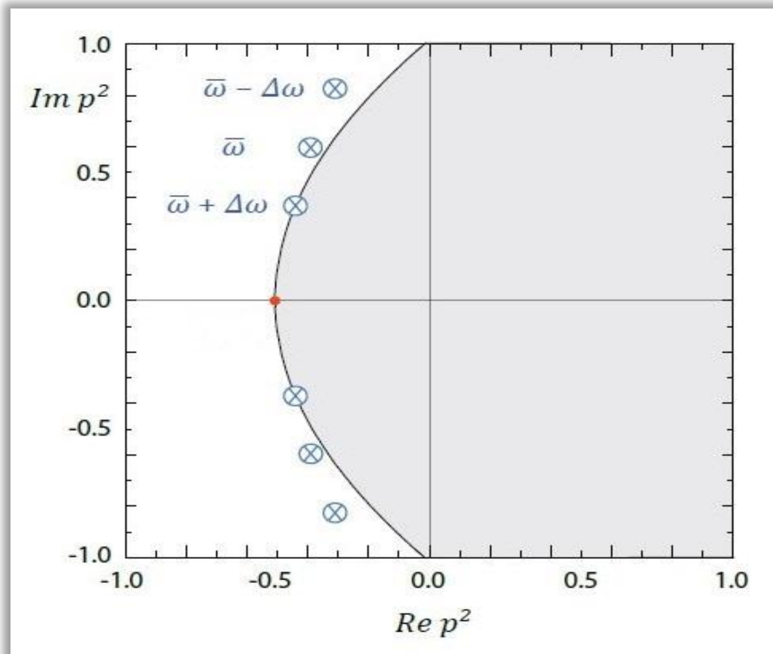

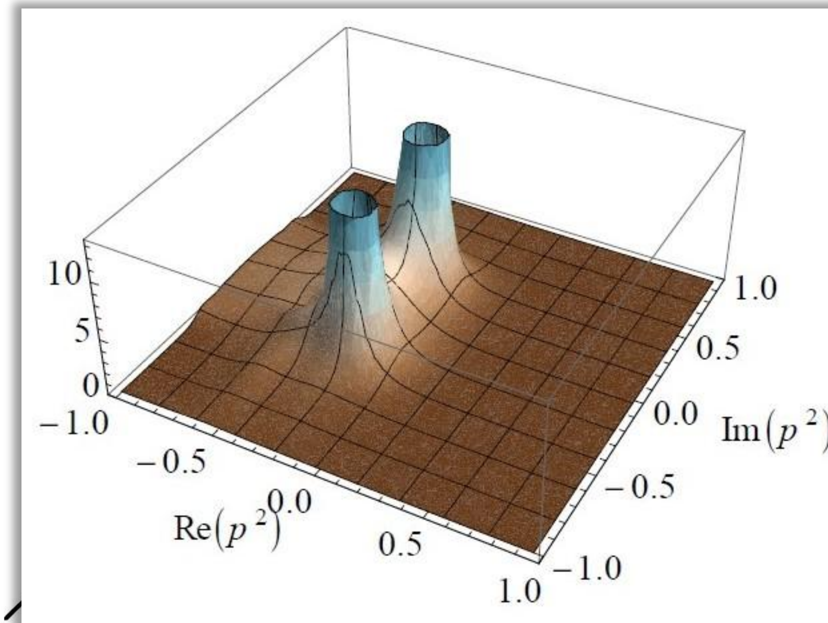

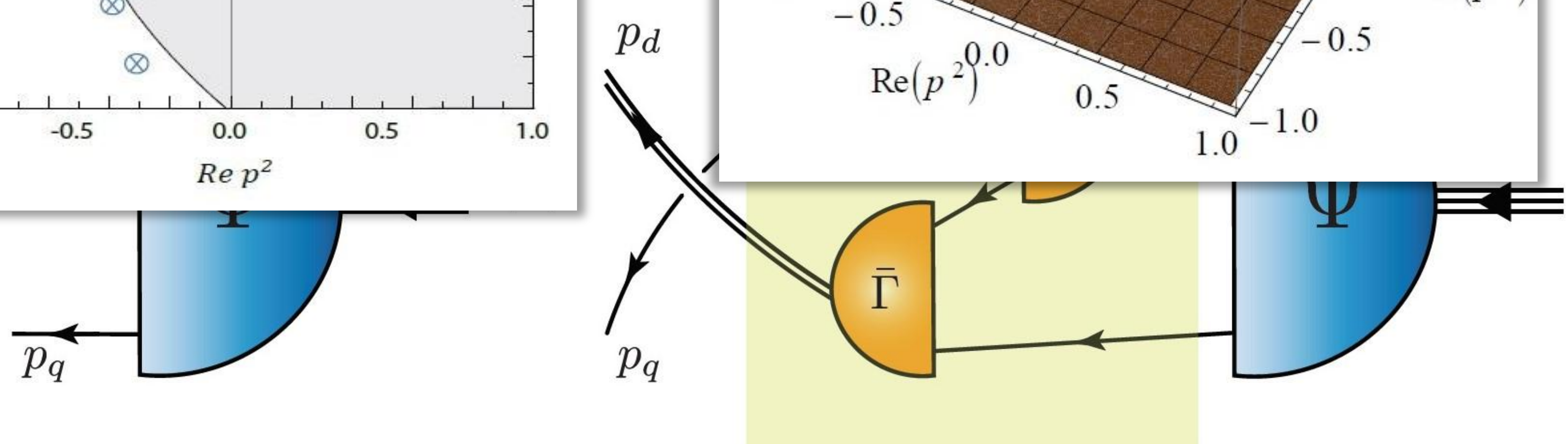

QCD-kindred model

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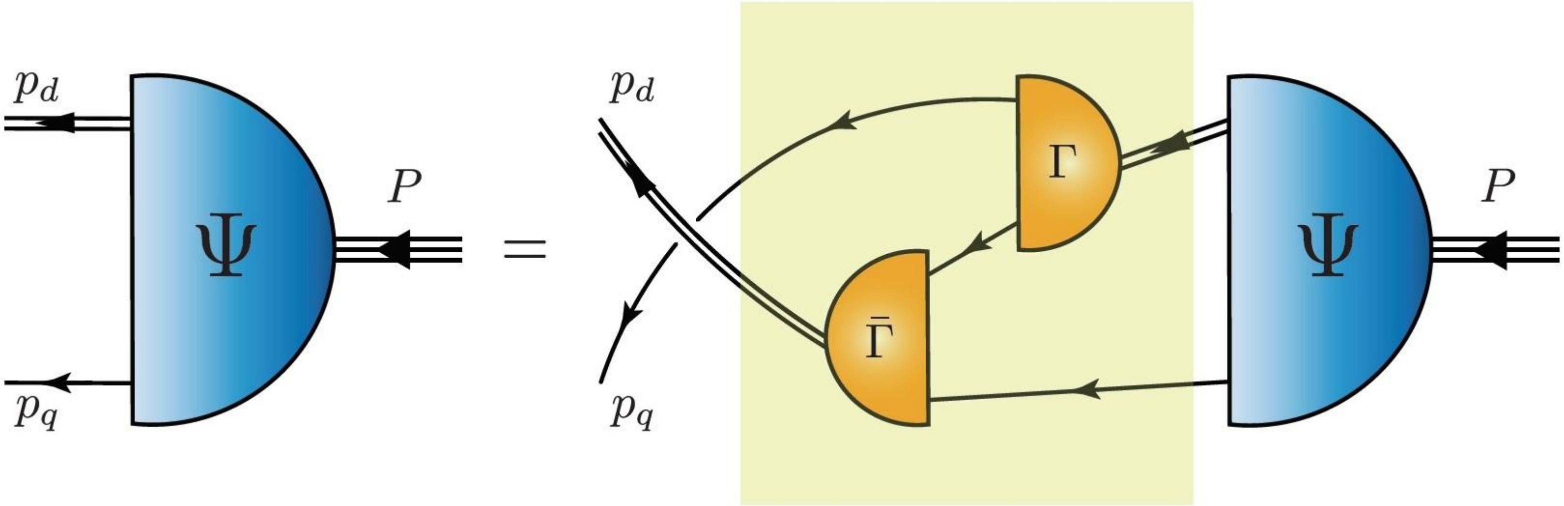

QCD-kindred model

- **Diquark masses (in GeV):**

$$m_{[ud]_{0+}} = 0.80\,\text{GeV}$$

$$m_{\{uu\}_{1+}} = m_{\{ud\}_{1+}} = m_{\{dd\}_{1+}} = 0.89\,\text{GeV}$$

- **These two values provide for a good description of numerous dynamical properties of the nucleon, Δ-baryon and Roper resonance.**
- **Solution to the 50 year puzzle -- Roper resonance: Discovered in 1963, the Roper resonance appears to be an exact copy of the proton except that its mass is *50%* greater and it is unstable…**

PRL 115, 171801 (2015) PHYSICAL REVIEW LETTERS week ending 23 OCTOBER 2015

Completing the Picture of the Roper Resonance

Jorge Segovia,[1] Bruno El-Bennich,[2,3] Eduardo Rojas,[2,4] Ian C. Cloët,[5] Craig D. Roberts,[5] Shu-Sheng Xu,[6] and Hong-Shi Zong[6]

[1]*Grupo de Física Nuclear and Instituto Universitario de Física Fundamental y Matemáticas (IUFFyM), Universidad de Salamanca, E-37008 Salamanca, Spain*
[2]*Laboratório de Física Teórica e Computacional, Universidade Cruzeiro do Sul, 01506-000 São Paulo, SP, Brazil*
[3]*Instituto de Física Teórica, Universidade Estadual Paulista, 01140-070 São Paulo, SP, Brazil*
[4]*Instituto de Física, Universidad de Antioquia, Calle 70 No. 52-21, Medellín, Colombia*
[5]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[6]*Department of Physics, Nanjing University, Nanjing 210093, China*

(Received 16 April 2015; revised manuscript received 29 July 2015; published 21 October 2015)

We employ a continuum approach to the three valence-quark bound-state problem in relativistic quantum field theory to predict a range of properties of the proton's radial excitation and thereby unify them with those of numerous other hadrons. Our analysis indicates that the nucleon's first radial excitation is the Roper resonance. It consists of a core of three dressed quarks, which expresses its valence-quark content and whose charge radius is 80% larger than the proton analogue. That core is complemented by a meson cloud, which reduces the observed Roper mass by roughly 20%. The meson cloud materially affects long-wavelength characteristics of the Roper electroproduction amplitudes but the quark core is revealed to probes with $Q^2 \gtrsim 3m_N^2$.

DOI: 10.1103/PhysRevLett.115.171801 PACS numbers: 13.40.Gp, 14.20.Dh, 14.20.Gk, 11.15.Tk

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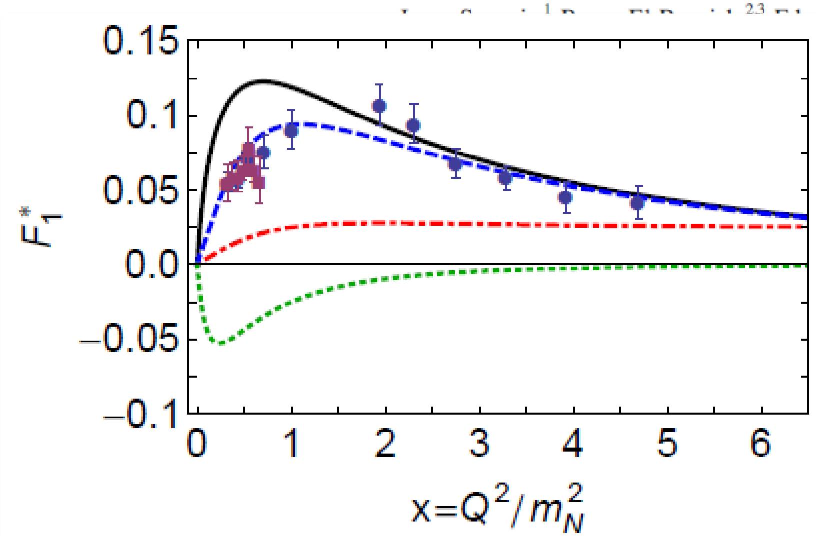
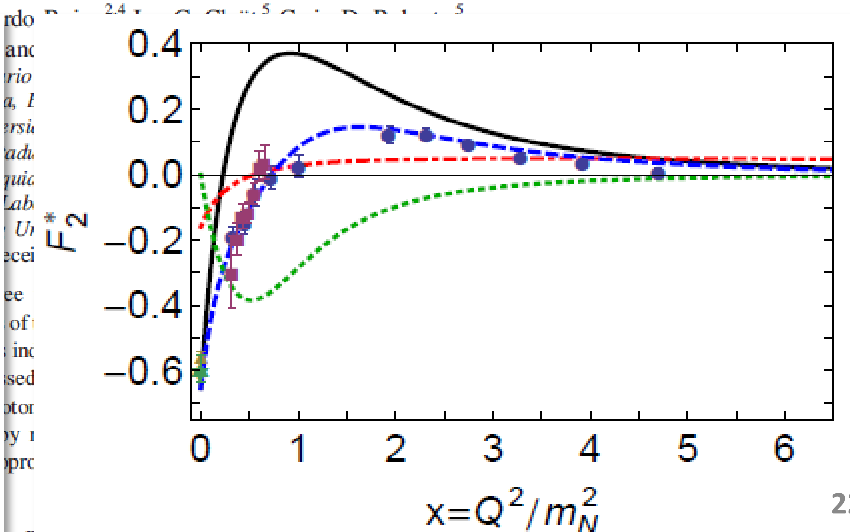




QCD-kindred model

- **Roper resonance -- solution to the 50 year puzzle**

REVIEWS OF MODERN PHYSICS

REVIEWS OF MODERN PHYSICS, VOLUME 91, JANUARY–MARCH 2019

Colloquium: Roper resonance: Toward a solution to the fifty year puzzle

Volker D. Burkert*
Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA

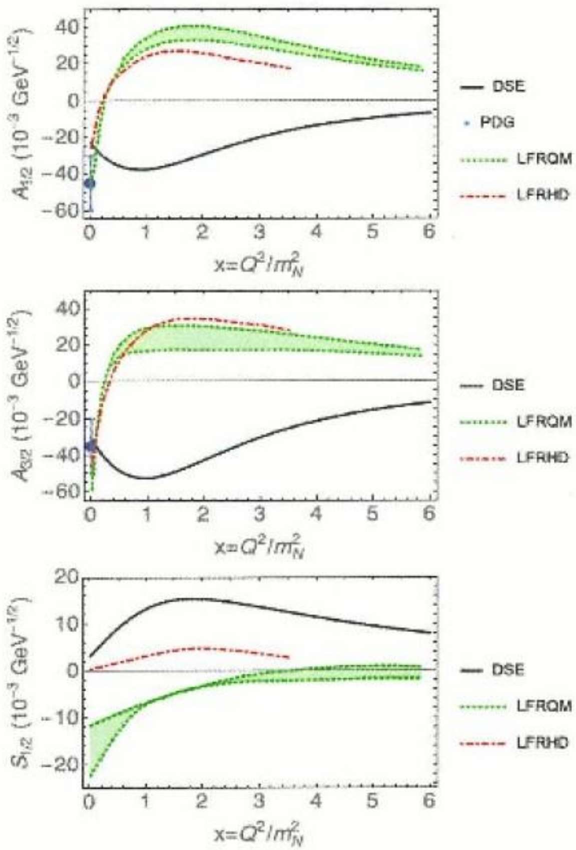
Craig D. Roberts†
Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

(published 14 March 2019)

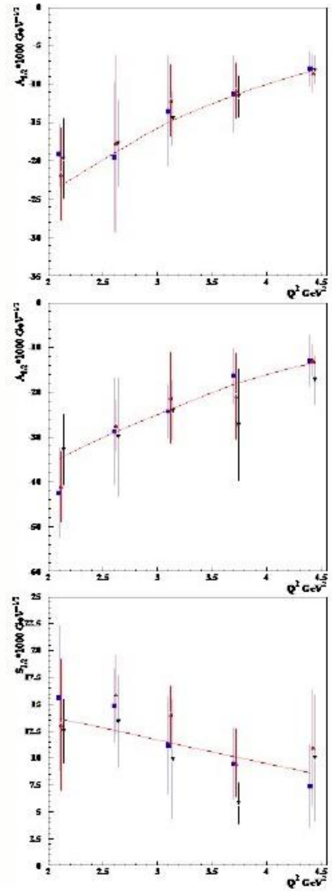
Δ(1600)3/2⁺ Form Factors in CSM Approach

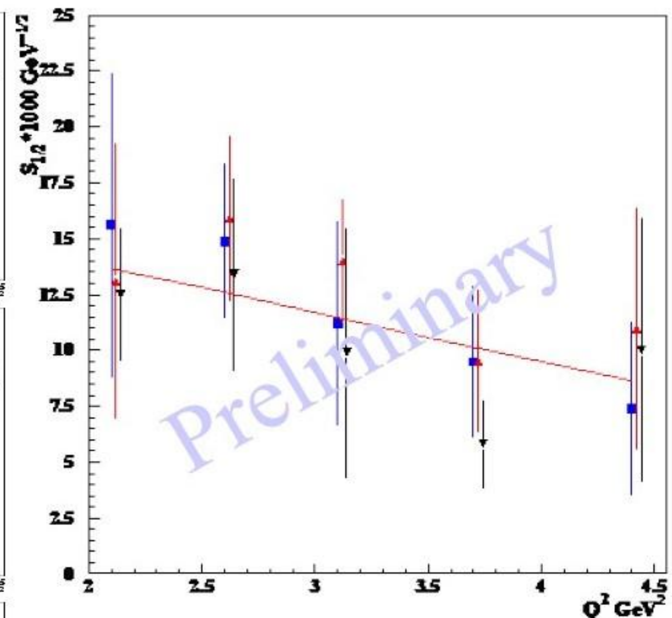
Viktor Mokeev

CSM predictions of the
Δ(1600)3/2⁺ electrocouplings



Ya Lu et al., PRD 100, 034001 (2019)





Spring 2022 analysis of Arjun's $\pi^+\pi^-p$ differential cross sections for $2.0GeV^2<Q^2<5.0GeV^2$ within three W-intervals, 1.46GeV<W<1.56GeV, 1.51GeV<W<1.61GeV, and 1.56GeV<W<1.66GeV.

Nucleon Form Factors

- **Form factors: contain important information about the structure and the properties of hadrons.**
- **Different probes correspond to different form factors.**
- **The nucleon electromagnetic current:**

$$J_\mu^{\mathrm{EM}}(K,Q) = \bar{u}(P_f)\left[\gamma_\mu F_1(Q^2) + \frac{1}{2m_N}\sigma_{\mu\nu}Q_\nu F_2(Q^2)\right]u(P_i)$$

  - **A large number of experimental measurements, with high precision and up to large momentum transfer.**

- **The nucleon axial current:**

$$J_{5\mu}^j(K,Q) = \bar{u}(P_f)\frac{\tau^j}{2}\gamma_5\left[\gamma_\mu G_A(Q^2) + i\frac{Q_\mu}{2m_N}G_P(Q^2)\right]u(P_i)$$

  - **The relative measurements are much more difficult, since they are related to weak processes.**
  - ***G_A* – axial form factor: experimental data are rather sparse and with large uncertainties.**
  - ***G_P* – induced pseudoscalar form factor: ONLY 4 empirical results.**

- **The nucleon pseudoscalar current (pseudoscalar form factor):**

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- **The Partially Conservation of the Axial Current (PCAC) relation:**

$$G_A(Q^2) - \frac{Q^2}{4m_N^2}G_P(Q^2) = \frac{m_q}{m_N}G_5(Q^2)$$

Δ(1232) Form Factors

- **Δ(1232) -> Δ(1232)**

$$J^{\Delta\Delta}_{(5)(\mu),\alpha\beta}(K,Q) = \bar{u}^{\Delta}_{\alpha}(P_f)\Gamma_{(5)(\mu),\alpha\beta}(K,Q)u^{\Delta}_{\beta}(P_i)\,,$$

where

$$\begin{aligned}
\Gamma^{\rm EM}_{\mu,\alpha\beta}(K,Q) &= \left[(F_1^*(Q^2)+F_2^*(Q^2))i\gamma_\mu - \frac{F_2^*(Q^2)}{m_\Delta}K_\mu\right]\delta_{\alpha\beta} - \left[(F_3^*(Q^2)+F_4^*(Q^2))i\gamma_\mu - \frac{F_4^*(Q^2)}{m_\Delta}K_\mu\right]\frac{Q_\alpha Q_\beta}{4m_\Delta^2}\,,\\
\Gamma^{\rm AX}_{5\mu,\alpha\beta}(Q) &= -\frac{1}{2}\gamma_5\left[\delta_{\alpha\beta}(g_1(Q^2)\gamma_\mu + ig_3(Q^2)\frac{Q_\mu}{m_\Delta}) - \frac{Q_\alpha Q_\beta}{4m_\Delta^2}(h_1(Q^2)\gamma_\mu + ih_3(Q^2)\frac{Q_\mu}{m_\Delta})\right]\,,\\
\Gamma^{\rm PS}_{5,\alpha\beta}(Q) &= -\frac{1}{2}\gamma_5\left[\delta_{\alpha\beta}\tilde{g}(Q^2) - \frac{Q_\alpha Q_\beta}{4m_\Delta^2}\tilde{h}(Q^2)\right].
\end{aligned}$$

Δ(1232) Form Factors

- **Δ(1232) -> Δ(1232)**

$$J^{\Delta\Delta}_{(5)(\mu),\alpha\beta}(K,Q) = \bar{u}^{\Delta}_{\alpha}(P_f)\Gamma_{(5)(\mu),\alpha\beta}(K,Q)u^{\Delta}_{\beta}(P_i)\,,$$

where

$$\Gamma^{\mathrm{EM}}_{\mu,\alpha\beta}(K,Q) = \left[(F_1^*(Q^2)+F_2^*(Q^2))i\gamma_\mu - \frac{F_2^*(Q^2)}{m_\Delta}K_\mu\right]\delta_{\alpha\beta} - \left[(F_3^*(Q^2)+F_4^*(Q^2))i\gamma_\mu - \frac{F_4^*(Q^2)}{m_\Delta}K_\mu\right]\frac{Q_\alpha Q_\beta}{4m_\Delta^2}\,,$$

$$\Gamma^{\mathrm{AX}}_{5\mu,\alpha\beta}(Q) = -\frac{1}{2}\gamma_5\left[\delta_{\alpha\beta}(g_1(Q^2)\gamma_\mu + ig_3(Q^2)\frac{Q_\mu}{m_\Delta}) - \frac{Q_\alpha Q_\beta}{4m_\Delta^2}(h_1(Q^2)\gamma_\mu + ih_3(Q^2)\frac{Q_\mu}{m_\Delta})\right],$$

$$\Gamma^{\mathrm{PS}}_{5,\alpha\beta}(Q) = -\frac{1}{2}\gamma_5\left[\delta_{\alpha\beta}\tilde{g}(Q^2) - \frac{Q_\alpha Q_\beta}{4m_\Delta^2}\tilde{h}(Q^2)\right].$$

$$\tilde{g}(Q^2) =: \frac{m_\pi^2}{Q^2+m_\pi^2}\frac{f_\pi}{m_q}G_{\pi\Delta\Delta}(Q^2)$$

$$\tilde{h}(Q^2) =: \frac{m_\pi^2}{Q^2+m_\pi^2}\frac{f_\pi}{m_q}H_{\pi\Delta\Delta}(Q^2)$$

Δ(1232) Form Factors

- **Δ(1232) -> Δ(1232)**

$$J^{\Delta\Delta}_{(5)(\mu),\alpha\beta}(K,Q) = \bar{u}^{\Delta}_{\alpha}(P_f)\Gamma_{(5)(\mu),\alpha\beta}(K,Q)u^{\Delta}_{\beta}(P_i)\,,$$

where

$$\begin{aligned}
\Gamma^{\mathrm{EM}}_{\mu,\alpha\beta}(K,Q) &= \left[(F_1^*(Q^2)+F_2^*(Q^2))i\gamma_\mu - \frac{F_2^*(Q^2)}{m_\Delta}K_\mu\right]\delta_{\alpha\beta} - \left[(F_3^*(Q^2)+F_4^*(Q^2))i\gamma_\mu - \frac{F_4^*(Q^2)}{m_\Delta}K_\mu\right]\frac{Q_\alpha Q_\beta}{4m_\Delta^2}\,,\\
\Gamma^{\mathrm{AX}}_{5\mu,\alpha\beta}(Q) &= -\frac{1}{2}\gamma_5\left[\delta_{\alpha\beta}(g_1(Q^2)\gamma_\mu + ig_3(Q^2)\frac{Q_\mu}{m_\Delta}) - \frac{Q_\alpha Q_\beta}{4m_\Delta^2}(h_1(Q^2)\gamma_\mu + ih_3(Q^2)\frac{Q_\mu}{m_\Delta})\right],\\
\Gamma^{\mathrm{PS}}_{5,\alpha\beta}(Q) &= -\frac{1}{2}\gamma_5\left[\delta_{\alpha\beta}\tilde{g}(Q^2) - \frac{Q_\alpha Q_\beta}{4m_\Delta^2}\tilde{h}(Q^2)\right].
\end{aligned}$$

$$\begin{aligned}
\tilde{g}(Q^2) &=: \frac{m_\pi^2}{Q^2+m_\pi^2}\frac{f_\pi}{m_q}G_{\pi\Delta\Delta}(Q^2)\\
\tilde{h}(Q^2) &=: \frac{m_\pi^2}{Q^2+m_\pi^2}\frac{f_\pi}{m_q}H_{\pi\Delta\Delta}(Q^2)
\end{aligned}$$

- **N(940) -> Δ(1232)**

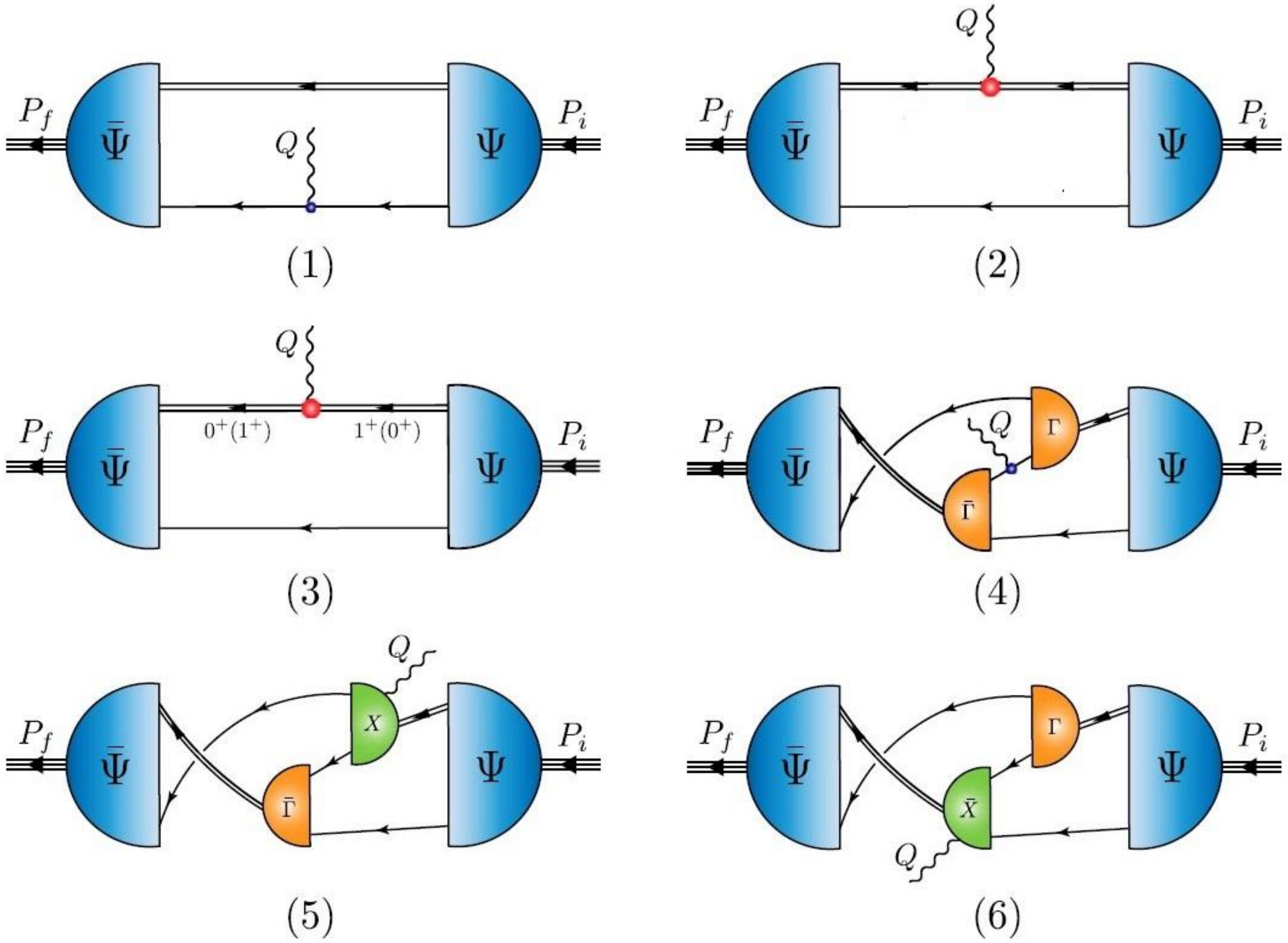
$$J^{\Delta N}_{(5)(\mu),\alpha\beta}(K,Q) = \bar{u}^{\Delta}_{\alpha}(P_f)\Gamma_{(5)(\mu),\alpha}(K,Q)u^{N}(P_i)\,,$$

where

$$\begin{aligned}
\Gamma^{\mathrm{EM}}_{\mu,\alpha}(K,Q) &= b\left[\frac{i\omega}{2\lambda_+}(G_M^*(Q^2)-G_E^*(Q^2))\gamma_5\varepsilon_{\alpha\mu\gamma\delta}K_\gamma\hat{Q}_\delta - G_E^*(Q^2)T^Q_{\alpha\gamma}T^K_{\gamma\mu} - \frac{i\tau}{\omega}G_C^*(Q^2)\hat{Q}_\alpha K_\mu\right],\\
\Gamma^{\mathrm{AX}}_{5\mu,\alpha}(Q) &= \sqrt{\frac{2}{3}}\left[i(\gamma_\mu Q_\lambda - \delta_{\mu\lambda}\not{Q})\frac{C_3^A(Q^2)}{m_N} - (\delta_{\mu\lambda}(P_f\cdot Q) - P_{f\,\mu}Q_\lambda)\frac{C_4^A(Q^2)}{m_N^2} + \delta_{\mu\lambda}C_5^A(Q^2) - Q_\mu Q_\lambda\frac{C_6^A(Q^2)}{m_N^2}\right],\\
\Gamma^{\mathrm{PS}}_{5,\alpha}(Q) &= \sqrt{\frac{2}{3}}\left[i\frac{Q_\lambda}{4m_N}\frac{m_\pi^2}{Q^2+m_\pi^2}\frac{f_\pi}{m_q}G_{\pi N\Delta}(Q^2)\right].
\end{aligned}$$

How to compute Form Factors?

- **In the quark-diquark framework, the associated symmetry-preserving current:**



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Nucleon axial and pseudoscalar form factors from the covariant Faddeev equation

Gernot Eichmann and Christian S. Fischer
Institut für Theoretische Physik, Justus-Liebig-Universität Giessen, D-35392 Giessen, Germany
(Dated: November 2, 2018)

We compute the axial and pseudoscalar form factors of the nucleon in the Dyson-Schwinger approach. To this end, we solve a covariant three-body Faddeev equation for the nucleon wave function and determine the matrix elements of the axialvector and pseudoscalar isotriplet currents. Our only input is a well-established and phenomenologically successful ansatz for the nonperturbative quark-gluon interaction. As a consequence of the axial Ward-Takahashi identity that is respected at the quark level, the Goldberger-Treiman relation is reproduced for all current-quark masses. We discuss the timelike pole structure of the quark-antiquark vertices that enters the nucleon matrix elements and determines the momentum dependence of the form factors. Our result for the axial charge underestimates the experimental value by 20 – 25% which might be a signal of missing pion-cloud contributions. The axial and pseudoscalar form factors agree with phenomenological and lattice data in the momentum range above $Q^2 \sim 1 \ldots 2$ GeV^2.

PACS numbers: 11.80.Jy 12.38.Lg, 11.40.Ha 14.20.Dh

I. INTRODUCTION

The nucleon's axial and pseudoscalar form factors are of fundamental significance for the properties of the nucleon that are probed in weak interaction processes. Their momentum dependence can be experimentally tested by (anti)neutino scattering off nucleons or nuclei, charged pion electroproduction and muon capture processes; see [1–3] for reviews. Both form factors are experimentally hard to extract and therefore considerably less well known than their electromagnetic counterparts. Precisely measured is only the low-momentum limit g_A of the axial form factor which is determined from neutron β-decay. Planned experiments at major facilities are expected to change this situation in the near future.

The theoretical calculation of the nucleon's axial and pseudoscalar form factors requires genuinely non-perturbative methods. Chiral perturbation theory has been successful in this respect [1, 4, 5] although it is generally limited to the region of low momentum transfer. Recent studies in lattice gauge theory are getting closer to the physical pion mass region [6–8] but finite-volume effects become increasingly important. Another non-perturbative approach is the one via functional meth-

The study of axial and pseudoscalar form factors in the functional approach has so far been limited to an approximation where the nucleon is treated as a bound object of a quark and a diquark that interact via quark exchange [12, 13]. The entire gluonic substructure appears here only implicitly within the dressing of quark and diquark propagators as well as diquark vertex functions. There are several conceptual issues that complicate the treatment of form factors in the quark-diquark model. First, the requirement of current conservation induces the appearance of intricate 'seagull' diagrams [14]. Such terms have been taken into account for electromagnetic form factors, but their implementation in the case of axial form factors has not yet been possible for technical reasons [13]. Second, to comply with chiral Ward identities, a current-conserving quark-diquark model requires vector diquarks in addition to the usual scalar and axialvector diquark degrees of freedom [15]. Such an elaborate treatment of the quark-diquark model has not yet been performed.

The situation is somewhat different when the nucleon is treated as a genuine three-body problem. The resulting Faddeev equation in rainbow-ladder truncation has been solved only recently for the nucleon and Δ

The study of axial and pseudoscalar form factors in the functional approach has so far been limited to an approximation where the nucleon is treated as a bound object of a quark and a diquark that interact via quark exchange [12, 13]. The entire gluonic substructure appears here only implicitly within the dressing of quark and diquark propagators as well as diquark vertex functions. There are several conceptual issues that complicate the treatment of form factors in the quark-diquark model. First, the requirement of current conservation induces the appearance of intricate 'seagull' diagrams [14]. Such terms have been taken into account for electromagnetic form factors, but their implementation in the case of axial form factors has not yet been possible for technical reasons [13]. Second, to comply with chiral Ward identities, a current-conserving quark-diquark model requires vector diquarks in addition to the usual scalar and axialvector diquark degrees of freedom [15]. Such an elaborate treatment of the quark-diquark model has not yet been performed.

Goldberger-Treiman relation and g pi N N from the three quark BS / Faddeev approach in the NJL model

Noriyoshi Ishii (Erlangen - Nuremberg U.) (Apr 28, 2000)

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PHYSICAL REVIEW D **105,** 094022 (2022)

Nucleon axial-vector and pseudoscalar form factors and PCAC relations

Chen Chen (陈晨),[1,2,3,4,*] Christian S. Fischer,[3,4,†] Craig D. Roberts,[5,6,‡] and Jorge Segovia[7,6,§]

[1]*Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei, Anhui 230026, China*

[2]*Peng Huanwu Center for Fundamental Theory, Hefei, Anhui 230026, China*

[3]*Institut für Theoretische Physik, Justus-Liebig-Universität Gießen, D-35392 Gießen, Germany*

[4]*Helmholtz Forschungsakademie Hessen für FAIR (HFHF), GSI Helmholtzzentrum für Schwerionenforschung, Campus Gießen, D-35392 Gießen, Germany*

[5]*School of Physics, Nanjing University, Nanjing, Jiangsu 210093, China*

[6]*Institute for Nonperturbative Physics, Nanjing University, Nanjing, Jiangsu 210093, China*

[7]*Dpto. Sistemas Físicos, Químicos y Naturales, Universidad Pablo de Olavide, E-41013 Sevilla, Spain*

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Large $Q2$ Nucleon Axial Form Factor

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Regular Article - Theoretical Physics

Nucleon axial form factor at large momentum transfers

Chen Chen[1,2,a], **Craig D. Roberts**[3,4,b]

[1] Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei 230026, Anhui, China
[2] Peng Huanwu Center for Fundamental Theory, Hefei 230026, Anhui, China
[3] School of Physics, Nanjing University, Nanjing 210093, Jiangsu, China
[4] Institute for Nonperturbative Physics, Nanjing University, Nanjing 210093, Jiangsu, China

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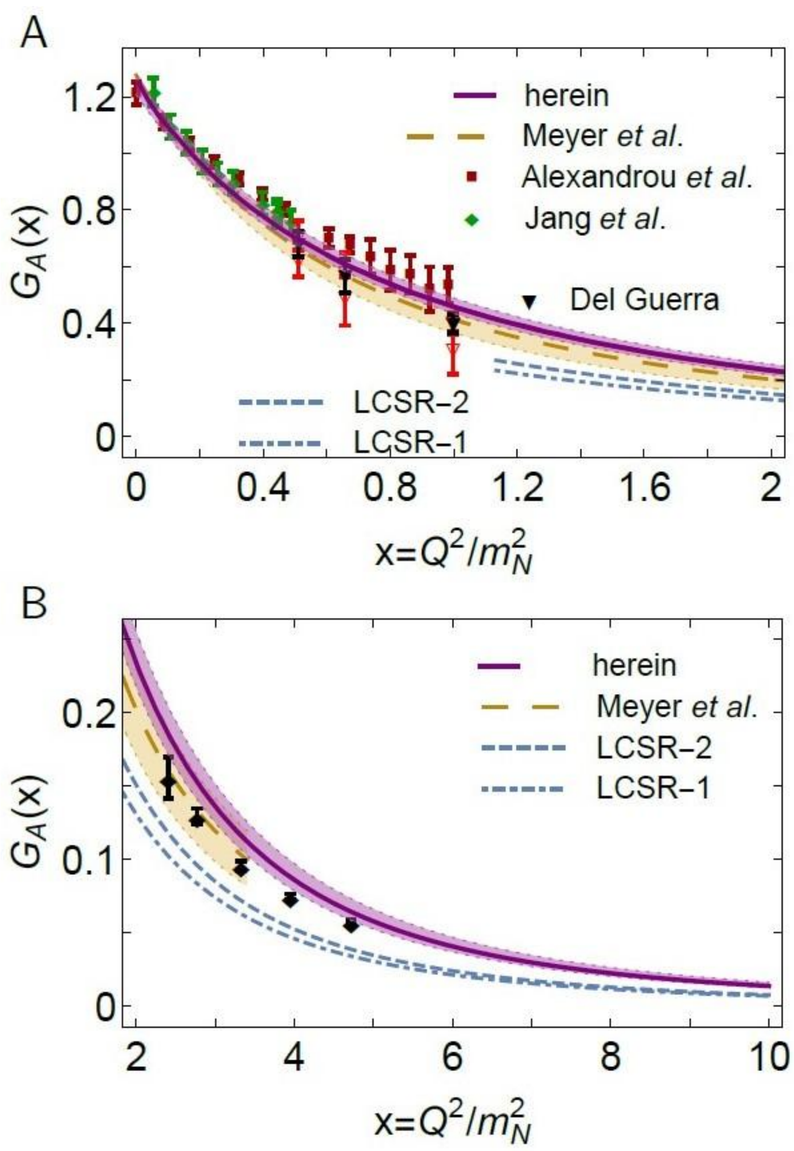
Large *Q2* Nucleon Axial Form Factor

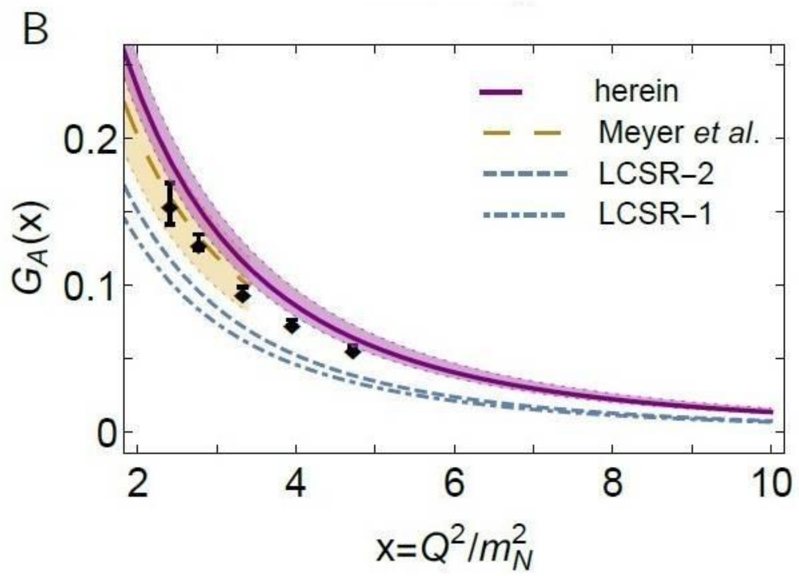
- **Parameter-free CSM predictions to *Q2* = 10*mN^2**
- **CSM prediction agrees with available data: small & large *Q2***



Large *Q2* Nucleon Axial Form Factor

- **Parameter-free CSM predictions to *Q2 = 10*mN^2***
- **CSM prediction agrees with available data: small & large *Q2***
- ***The dipole Ansatz:***

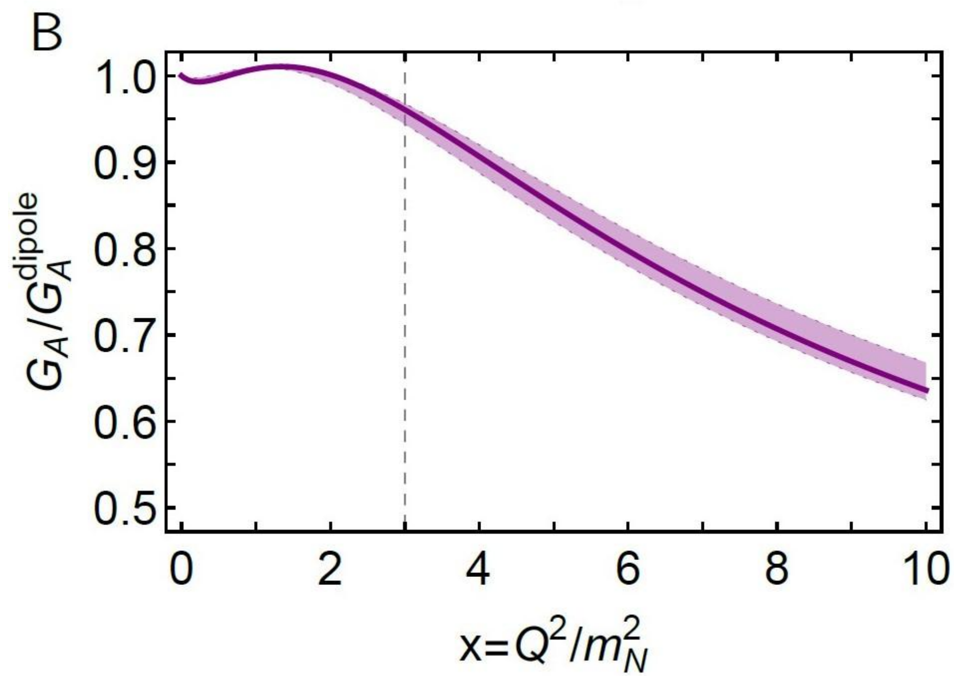
$$G_A(x) = \frac{G_A(0)}{\left(1 + x/(m_A^N/m_N)^2\right)^2}$$

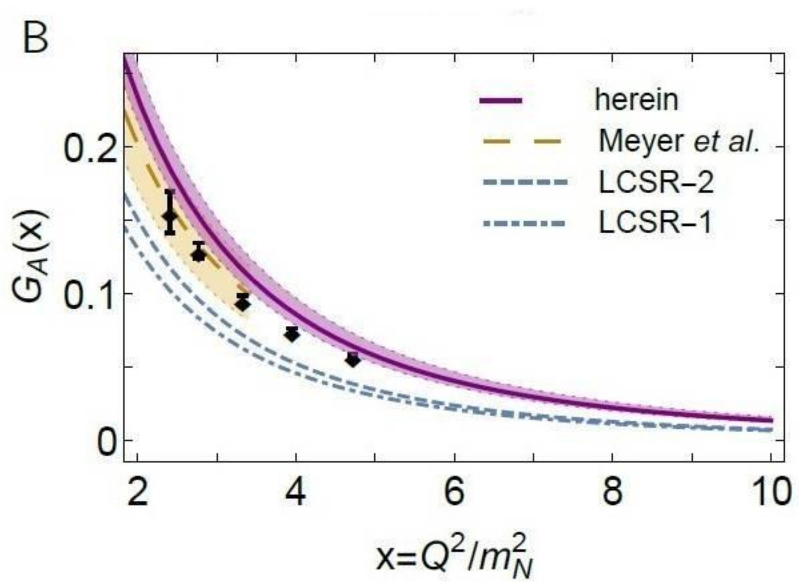


Large *Q2* Nucleon Axial Form Factor

- **Parameter-free CSM predictions to *Q2* = 10*mN^2**
- **CSM prediction agrees with available data: small & large *Q2***
- ***The dipole Ansatz:***
  - Fair representation of *GA*(*x*) on *x* ∈ [0, 3]
  - But outside fitted domain, quality of approximation deteriorates quickly
  - dipole overestimates true result by 56% at *x* = 10

$$G_A(x) = \frac{G_A(0)}{\left(1 + x/(m_A^N/m_N)^2\right)^2}$$



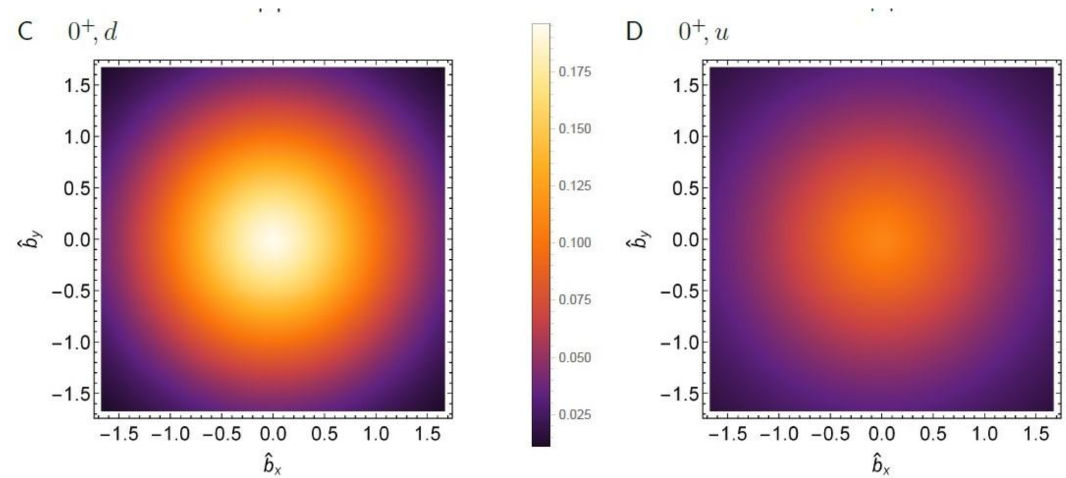


Large *Q2* Nucleon Axial Form Factor

- **Light-front transverse density profiles**

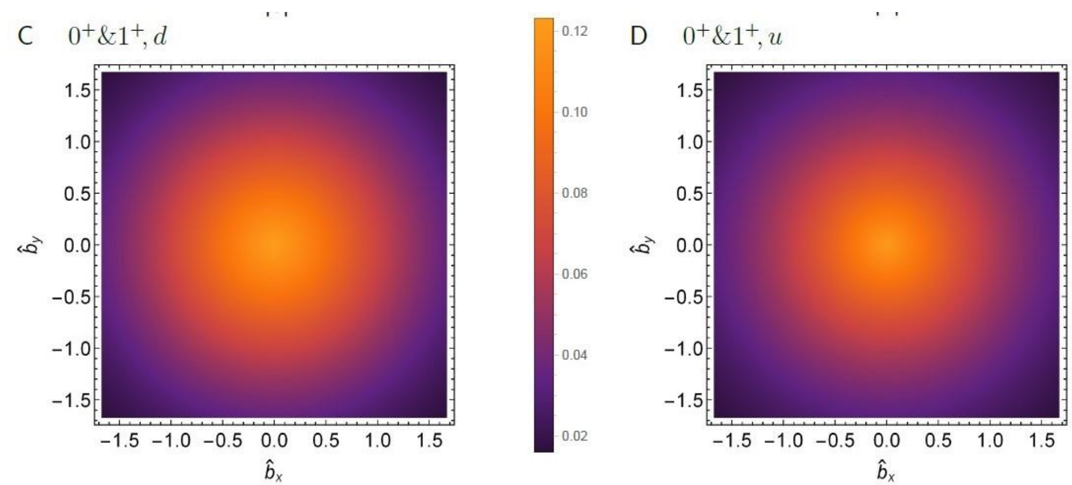
- **Scalar diquark only:**
  - magnitude of the *d* quark contribution to *GA* is just 10% of that from the *u* quark
  - *d* quark is also much more localized

$$r^{\perp}_{A_d} \approx 0.5\, r^{\perp}_{A_u}$$



- **Scalar + axial-vector diquarks:**
  - *d* and *u* quark transverse profiles are quite similar

$$r^{\perp}_{A_d} \approx 0.9\, r^{\perp}_{A_u}$$



Proton Spin Structure

- **Flavour separation of proton axial charge**
- ***d*-quark receives large contribution from probe+quark in presence of axialvector diquark**

$$\frac{g_A^d}{g_A^u} =^{0^+ \& 1^+} -0.32(2)$$

$$\frac{g_A^d}{g_A^u} =^{0^+ \text{ only}} -0.054(13)$$

- **Experiment: -0.27(4)**
- **Hadron scale:** $g_A^u + g_A^d \, (+g_A^s = 0) = 0.52(1)$
  - **quarks carry 52% of the proton spin**
  - **remaining 48% lodged with quark+diquark orbital angular momentum**
- **Contact interaction model: dressed-quarks carry 50(7)% of proton spin at hadron scale**

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Contact interaction analysis of octet baryon axial-vector and pseudoscalar form factors

Peng Cheng (程鹏),[1,2] Fernando E. Serna,[3,4] Zhao-Qian Yao (姚照千),[1,2] Chen Chen (陈晨),[5,6,*]
Zhu-Fang Cui (崔著钫),[1,2,†] and Craig D. Roberts[1,2,‡]

Δ(1232) Axial and Pseudoscalar Form Factors

- **Δ(1232) -> Δ(1232)**

$$J^{\Delta\Delta}_{(5)(\mu),\alpha\beta}(K,Q) = \bar{u}^{\Delta}_{\alpha}(P_f)\Gamma_{(5)(\mu),\alpha\beta}(K,Q)u^{\Delta}_{\beta}(P_i)\,,$$

where

$$\Gamma^{\rm EM}_{\mu,\alpha\beta}(K,Q) = \left[(F_1^*(Q^2)+F_2^*(Q^2))i\gamma_\mu - \frac{F_2^*(Q^2)}{m_\Delta}K_\mu\right]\delta_{\alpha\beta} - \left[(F_3^*(Q^2)+F_4^*(Q^2))i\gamma_\mu - \frac{F_4^*(Q^2)}{m_\Delta}K_\mu\right]\frac{Q_\alpha Q_\beta}{4m_\Delta^2}\,,$$

$$\Gamma^{\rm AX}_{5\mu,\alpha\beta}(Q) = -\frac{1}{2}\gamma_5\left[\delta_{\alpha\beta}\left(g_1(Q^2)\gamma_\mu + ig_3(Q^2)\frac{Q_\mu}{m_\Delta}\right) - \frac{Q_\alpha Q_\beta}{4m_\Delta^2}\left(h_1(Q^2)\gamma_\mu + ih_3(Q^2)\frac{Q_\mu}{m_\Delta}\right)\right],$$

$$\Gamma^{\rm PS}_{5,\alpha\beta}(Q) = -\frac{1}{2}\gamma_5\left[\delta_{\alpha\beta}\tilde{g}(Q^2) - \frac{Q_\alpha Q_\beta}{4m_\Delta^2}\tilde{h}(Q^2)\right].$$

$$\tilde{g}(Q^2) =: \frac{m_\pi^2}{Q^2+m_\pi^2}\frac{f_\pi}{m_q}G_{\pi\Delta\Delta}(Q^2)$$

$$\tilde{h}(Q^2) =: \frac{m_\pi^2}{Q^2+m_\pi^2}\frac{f_\pi}{m_q}H_{\pi\Delta\Delta}(Q^2)$$

- **PCAC relations**

$$g_1 - \frac{Q^2}{4m_\Delta^2}g_3 = \frac{m_q}{m_\Delta}\tilde{g}\,,$$

$$h_1 - \frac{Q^2}{4m_\Delta^2}h_3 = \frac{m_q}{m_\Delta}\tilde{h}\,.$$

- **Goldberger-Treiman relations**

$$g_1(0) = \frac{f_\pi}{m_\Delta}G_{\pi\Delta\Delta}(0)\,,$$

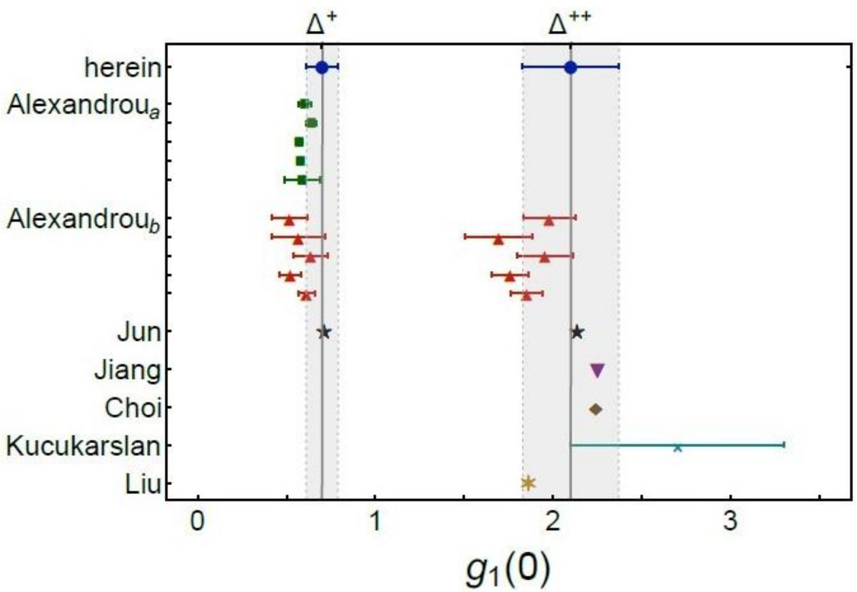
$$h_1(0) = \frac{f_\pi}{m_\Delta}H_{\pi\Delta\Delta}(0)\,.$$

✓ C. Alexandrou et al., Phys.Rev.D 87 (2013) 11

✓ Yu-Son Jun, Jung-Min Suh, Hyun-Chul Kim, Phys.Rev.D 102 (2020) 5, 054011

Δ(1232) Axial and Pseudoscalar Form Factors

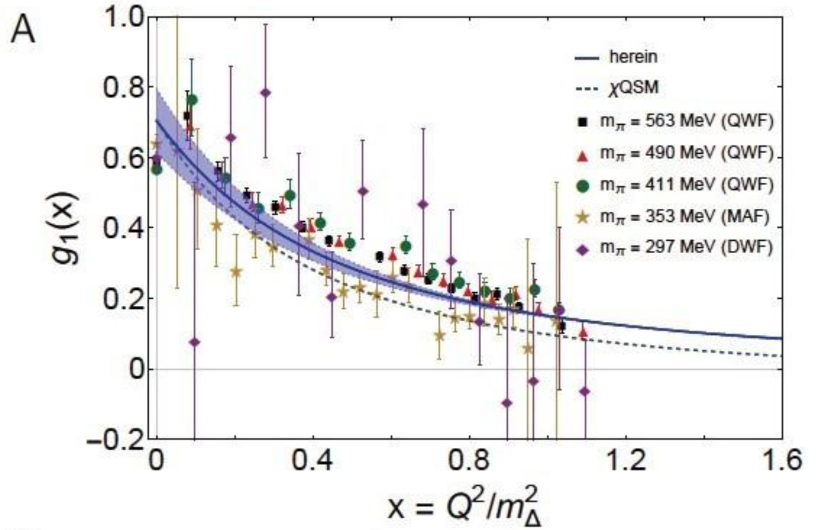
- **Axial charge: *g1*(0) = 0.71(9)**

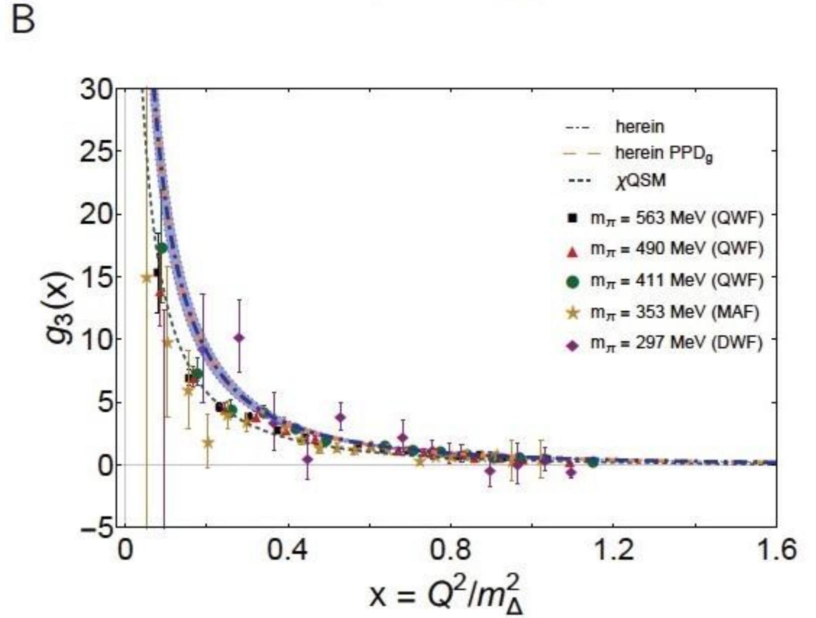


- **Dipole Ansatz:**

$$g_1(x) = \frac{g_1(0)}{\left(1 + x/(m_A^{\Delta}/m_{\Delta})^2\right)^2}$$

- ***mA* = 0.95(2)**mΔ***





Δ(1232) Axial and Pseudoscalar Form Factors

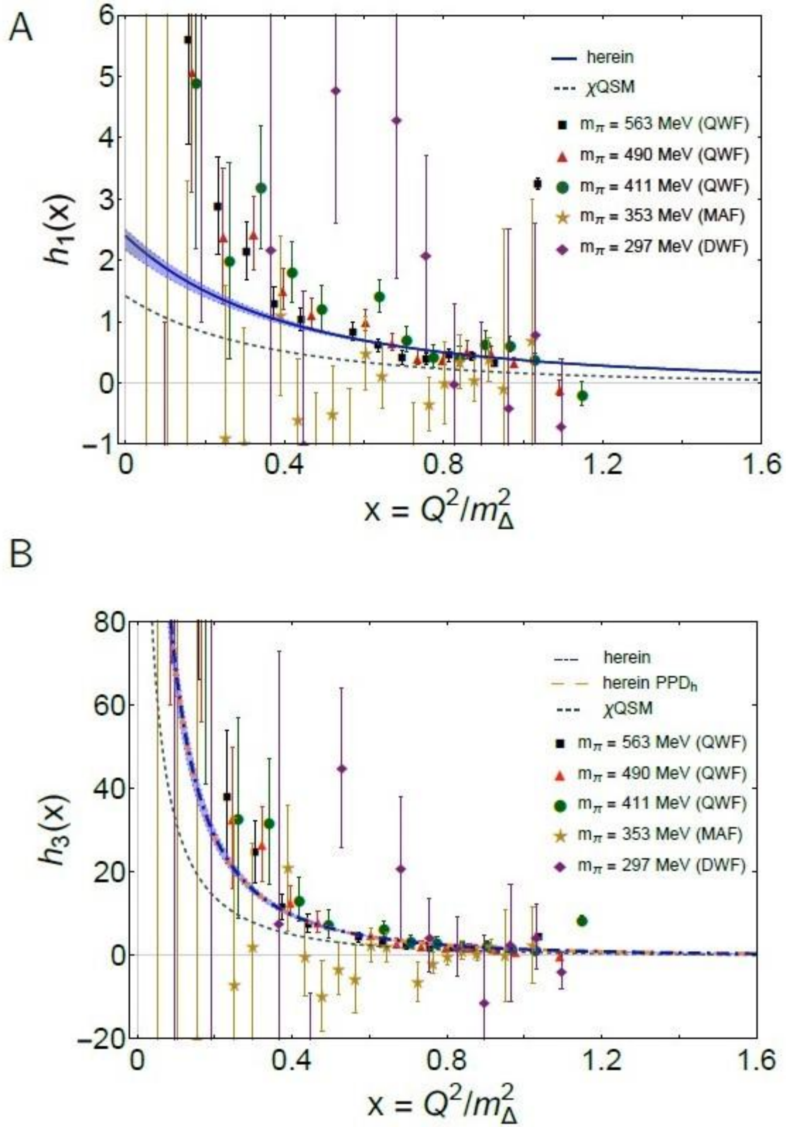
- ***h1***(x): regular
- ***h1***(0) = 2.25(17)

- lQCD:

$$h_1^{\mathrm{lQCD}}(Q^2) \propto \frac{1}{Q^2+m_\pi^2}\,,$$
$$h_3^{\mathrm{lQCD}}(Q^2) \propto \frac{1}{(Q^2+m_\pi^2)^2}\,.$$

- Projections:

$$s_1 := i\mathrm{tr}_{\mathrm{D}}[J_{5\mu,\lambda\omega}\gamma_5]\hat{Q}_\mu\hat{Q}_\lambda\hat{Q}_\omega\,,$$
$$s_2 := i\mathrm{tr}_{\mathrm{D}}[J_{5\mu,\lambda\lambda}\gamma_5]\hat{Q}_\mu\,,$$
$$s_3 := \mathrm{tr}_{\mathrm{D}}[J_{5\mu,\lambda\omega}\gamma_\mu^T\gamma_5]\hat{Q}_\lambda\hat{Q}_\omega\,,$$
$$s_4 := \mathrm{tr}_{\mathrm{D}}[J_{5\mu,\lambda\lambda}\gamma_\mu^T\gamma_5]\,,$$



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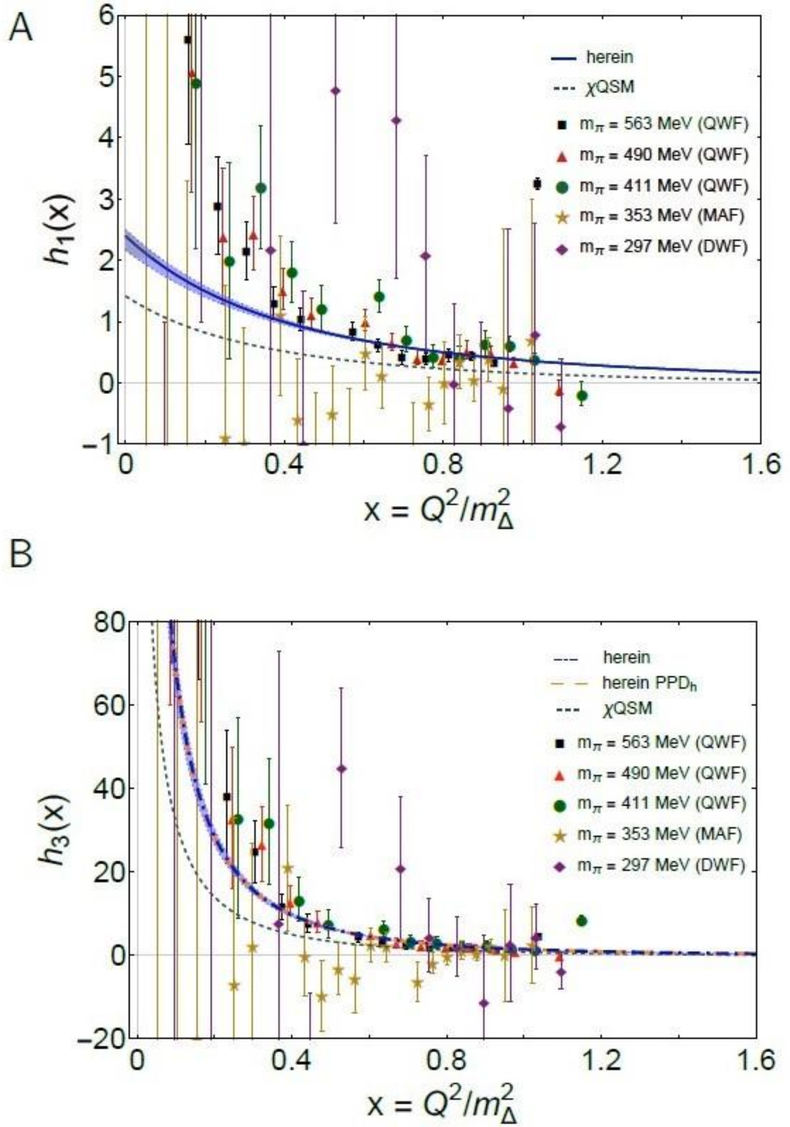
$$s_4 := {\rm tr_D}[J_{5\mu,\lambda\lambda}\gamma_\mu^T\gamma_5]\,,$$



Δ(1232) Axial and Pseudoscalar Form Factors

- **π-Δ couplings:**

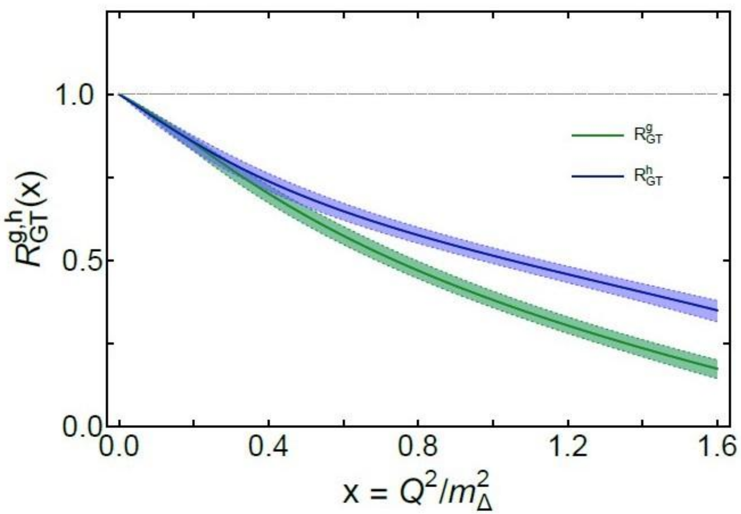
$$g_{\pi\Delta\Delta} := G_{\pi\Delta\Delta}(Q^2 = -m_\pi^2) = 10.52(1.98)\,,$$
$$h_{\pi\Delta\Delta} := H_{\pi\Delta\Delta}(Q^2 = -m_\pi^2) = 35.17(4.62)\,.$$

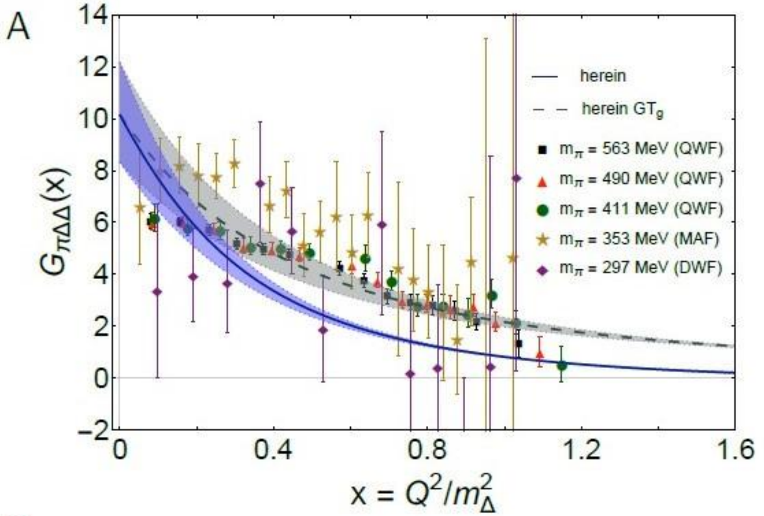
- **Goldberger-Treiman relations:**

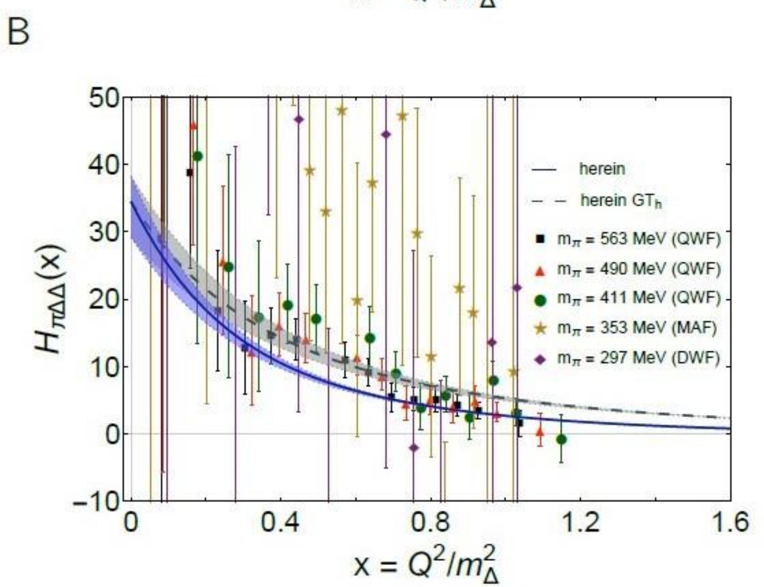
$$g_1(0) = \frac{f_\pi}{m_\Delta} G_{\pi\Delta\Delta}(0)\,,$$
$$h_1(0) = \frac{f_\pi}{m_\Delta} H_{\pi\Delta\Delta}(0)\,.$$

- **Goldberger-Treiman ratios:**

$$R^g_{\mathrm{GT}}(x) := \frac{f_\pi G_{\pi\Delta\Delta}(x)}{m_\Delta g_1(x)}$$
$$R^h_{\mathrm{GT}}(x) := \frac{f_\pi H_{\pi\Delta\Delta}(x)}{m_\Delta h_1(x)}$$







Summary & Perspective

- **Nucleon Axial Form Factor at large momentum transfer**
- **Δ(1232) Axial and Pseudoscalar Form Factors**
- **Next:**
  - **Compute the axial N-> Delta, N->N(1535)…**
  - **Studying the axial form factors in the RL framework (quark-diquark and three-body).**

Summary & Perspective

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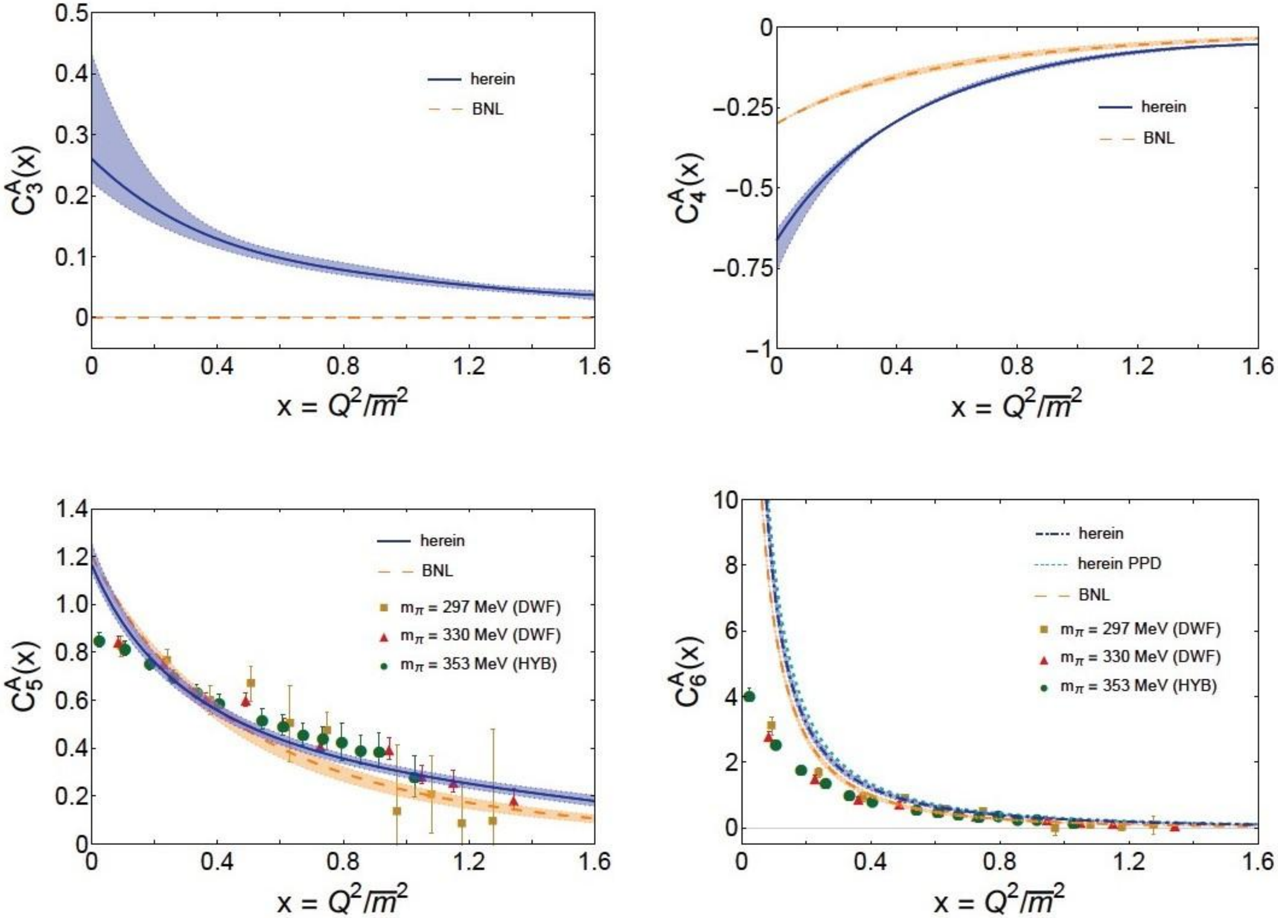
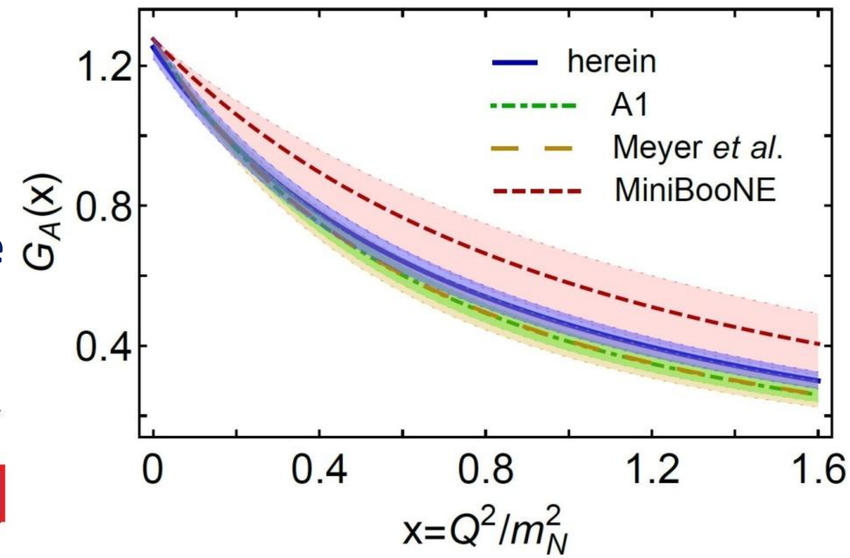


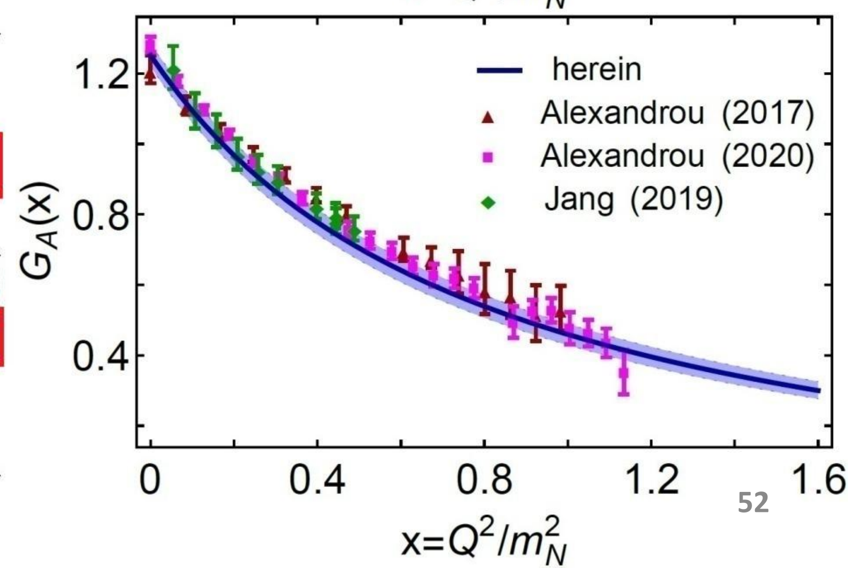
The axial current – G_A & G_P

$$J^j_{5\mu}(K,Q) = \bar{u}(P_f)\frac{\tau^j}{2}\gamma_5\left[\gamma_\mu G_A(Q^2) + i\frac{Q_\mu}{2m_N}G_P(Q^2)\right]u(P_i)$$

- **Two form factors:**
  - ***G_A* – axial form factor**
  - ***G_P* – induced pseudoscalar form factor**
- ***G_A* can reliably be represented by dipole characterised by mass-scale *m_A***



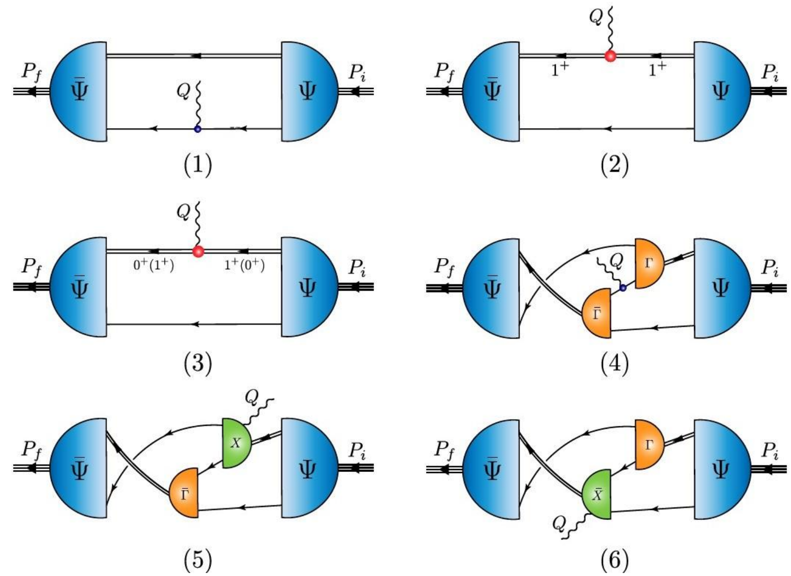
	g_A	$m_N\langle r_A^2\rangle^{1/2}$	m_A/m_N
Herein	1.25(03)	3.25(04)	1.23(03)
Faddeev$_3$ [31]	0.99(02)	2.63(06)	1.32(03)
Exp [4]	1.2756(13)	–	–
Exp [13]	–	3.02(11)	1.15(04)
Exp [14]	–	3.23(72)	1.15(08)
Exp [17]	–	2.41(31)	1.44(18)
lQCD [57]	1.21(3)(2)	2.45(08)(03)	1.41(04)(02)
lQCD [58]	1.30(6)	3.57(30)	0.97(16)
lQCD$_d$ [59]	1.23(3)	2.48(15)	1.39(09)
lQCD$_z$ [59]	1.30(9)	3.19(30)	1.09(11)



Fractions of $G_A(0)$, $G_P(0)$ and $G_5(0)$

TABLE I. Referring to Fig. 3, separation of $G_A(0)$, $G_P(0)$ and $G_5(0)$ into contributions from various diagrams, listed as a fraction of the total $Q^2 = 0$ value. Diagram (1): $\langle J\rangle_{\rm q}^{S}$ – weak-boson strikes dressed-quark with scalar diquark spectator; and $\langle J\rangle_{\rm q}^{A}$ – weak-boson strikes dressed-quark with axial-vector diquark spectator. Diagram (2): $\langle J\rangle_{\rm qq}^{AA}$ – weak-boson interacts strikes axial-vector diquark with dressed-quark spectator. Diagram (3): $\langle J\rangle_{\rm dq}^{SA+AS}$ – weak-boson mediates transition between scalar and axial-vector diquarks, with dressed-quark spectator. Diagram (4): $\langle J\rangle_{\rm ex}$ – weak-boson strikes dressed-quark "in-flight" between one diquark correlation and another. Diagrams (5) and (6): $\langle J\rangle_{\rm sg}$ – weak-boson couples inside the diquark correlation amplitude. The listed uncertainty in these results reflects the impact of ±5% variations in the diquark masses in Eq. (16), *e.g.* $0.71 1_{\mp} \Rightarrow 0.71 \mp 0.01$.

	$\langle J\rangle_{\rm q}^{S}$	$\langle J\rangle_{\rm q}^{A}$	$\langle J\rangle_{\rm qq}^{AA}$	$\langle J\rangle_{\rm qq}^{SA+AS}$	$\langle J\rangle_{\rm ex}$	$\langle J\rangle_{\rm sg}$
$G_A(0)$	$0.71 4_{\mp}$	$0.064 2_{\pm}$	$0.025 5_{\pm}$	$0.13 0_{\mp}$	$0.072 32_{\pm}$	0
$G_P(0)$	$0.74 4_{\mp}$	$0.070 5_{\pm}$	$0.025 5_{\pm}$	$0.13 0_{\mp}$	$0.22 4_{\pm}$	$-0.19 1_{\mp}$
$G_5(0)$	$0.74 4_{\mp}$	$0.069 5_{\pm}$	$0.025 5_{\pm}$	$0.13 0_{\mp}$	$0.22 4_{\pm}$	$-0.19 1_{\mp}$



- **Projections:**

$$G_A = -\frac{1}{4(1+\tau)} {\rm tr_D}[J_{5\mu}\gamma_5\gamma_\mu^T],$$

$$G_P = \frac{1}{\tau}\left(G_A - \frac{Q_\mu}{4im_N\tau}{\rm tr_D}[J_{5\mu}\gamma_5]\right),$$

$$G_5 = \frac{1}{2\tau}{\rm tr_D}[J_5\gamma_5],$$

- **$G_P(0)$ ~ $G_5(0)$**

$$G_P \sim \frac{Q_\mu}{\tau^2}{\rm tr_D}\left[J_{5\mu}\gamma_5\right] \sim \frac{1}{\tau}{\rm tr_D}\left[J_5\gamma_5\right] \sim G_5\,,$$

when $Q^2 \sim 0\,{\rm GeV}^2$.

QCD-kindred model

- **The dressed-quark propagator**

$$S(p) = -i\gamma \cdot p\sigma_V(p^2) + \sigma_S(p^2)$$

- **algebraic form:**

$$\bar{\sigma}_S(x) = 2\bar{m}\mathcal{F}(2(x+\bar{m}^2)) + \mathcal{F}(b_1 x)\mathcal{F}(b_3 x)[b_0 + b_2\mathcal{F}(\epsilon x)], \quad \text{(A3a)}$$

$$\bar{\sigma}_V(x) = \frac{1}{x+\bar{m}^2}[1 - \mathcal{F}(2(x+\bar{m}^2))], \quad \text{(A3b)}$$

with $x = p^2/\lambda^2$, $\bar{m} = m/\lambda$,

$$\mathcal{F}(x) = \frac{1-e^{-x}}{x}, \quad \text{(A4)}$$

$\bar{\sigma}_S(x) = \lambda\sigma_S(p^2)$ and $\bar{\sigma}_V(x) = \lambda^2\sigma_V(p^2)$. The mass scale, $\lambda = 0.566$ GeV, and parameter values,

$\bar{m}$	b_0	b_1	b_2	b_3
0.00897	0.131	2.90	0.603	0.185

(A5)

associated with Eq. (A3) were fixed in a least-squares fit to light-meson observables [79,80]. [$\epsilon = 10^{-4}$ in Eq. (A3a) acts only to decouple the large- and intermediate-p^2 domains.]

QCD-kindred model

- **The dressed-quark propagator**

$$S(p) = -i\gamma \cdot p\sigma_V(p^2) + \sigma_S(p^2)$$

  - Based on solutions to the gap equation that were obtained with a dressed gluon-quark vertex.
  - Mass function has a real-world value at $p^2 = 0$, NOT the highly inflated value typical of ~~RL~~ truncation.
  - Propagators are entire functions, consistent with sufficient condition for confinement and completely unlike known results from ~~RL~~ truncation.
  - Parameters in quark propagators were fitted to a diverse array of meson observables. **ZERO** parameters changed in study of baryons.
  - Compare with that computed using the DCSB-improved gap equation kernel (DB). The parametrization is a sound representation numerical results, although simple and introdu long beforehand.

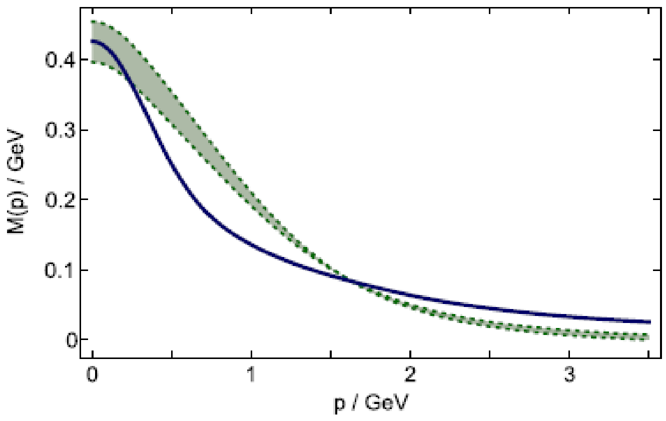


FIG. 6. Solid curve (blue)—quark mass function generated by the parametrization of the dressed-quark propagator specified by Eqs. (A3) and (A4) (A5); and band (green)—exemplary range of numerical results obtained by solving the gap equation with the modern DCSB-improved kernels described and used in Refs. [16,81–83].

QCD-kindred model

- **Diquark amplitudes: five types of correlation are possible in a *J=1/2* bound state: isoscalar scalar(I=0,J^P=0^+), isovector pseudovector, isoscalar pseudoscalar, isoscalar vector, and isovector vector.**
- **The LEADING structures in the correlation amplitudes for each case are, respectively (Dirac-flavor-color),**

$$\Gamma^{0^+}(k;K) = g_{0^+}\gamma_5 C\tau^2 \vec{H}\mathcal{F}(k^2/\omega_{0^+}^2),$$

$$\vec{\Gamma}_\mu^{1^+}(k;K) = ig_{1^+}\gamma_\mu C\vec{t}\vec{H}\,\mathcal{F}(k^2/\omega_{1^+}^2),$$

$$\Gamma^{0^-}(k;K) = ig_{0^-}C\tau^2\vec{H}\mathcal{F}(k^2/\omega_{0^-}^2),$$

$$\Gamma_\mu^{1^-}(k;K) = g_{1^-}\gamma_\mu\gamma_5 C\tau^2\vec{H}\mathcal{F}(k^2/\omega_{1^-}^2),$$

$$\vec{\Gamma}_\mu^{\bar{1}^-}(k;K) = ig_{\bar{1}^-}[\gamma_\mu, \gamma\cdot K]\gamma_5 C\vec{t}\vec{H}\,\mathcal{F}(k^2/\omega_{\bar{1}^-}^2),$$

- **Simple form. Just one parameter: diquark masses.**
- **Match expectations based on solutions of meson and diquark Bethe-Salpeter amplitudes.**

QCD-kindred model

- **The diquark propagators**

$$\Delta^{0^\pm}(K) = \frac{1}{m_{0^\pm}^2}\mathcal{F}(k^2/\omega_{0^\pm}^2),$$

$$\Delta_{\mu\nu}^{1^\pm}(K) = \left[\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1^\pm}^2}\right]\frac{1}{m_{1^\pm}^2}\mathcal{F}(k^2/\omega_{1^\pm}^2).$$

- **The *$\mathcal{F}$-functions*: Simplest possible form that is consistent with infrared and ultraviolet constraints of confinement (IR) and 1/q^2 evolution (UV) of meson propagators.**

- **Diquarks are confined.**
  - **free-particle-like at spacelike momenta**
  - **pole-free on the timelike axis**
  - **This is NOT true of ~~RL~~ studies. It enables us to reach arbitrarily high values of momentum transfer.**

QCD-kindred model

- **The Faddeev amplitudes:**

$$\begin{aligned}\psi^{\pm}(p_i,\alpha_i,\sigma_i) = & [\Gamma^{0^+}(k;K)]^{\alpha_1\alpha_2}_{\sigma_1\sigma_2}\Delta^{0^+}(K)[\varphi^{\pm}_{0^+}(\ell;P)u(P)]^{\alpha_3}_{\sigma_3} \\ & + [\Gamma^{1^+j}_{\mu}]\Delta^{1^+}_{\mu\nu}[\varphi^{j\pm}_{1^+\nu}(\ell;P)u(P)] \\ & + [\Gamma^{0^-}]\Delta^{0^-}[\varphi^{\pm}_{0^-}(\ell;P)u(P)] \\ & + [\Gamma^{1^-}_{\mu}]\Delta^{1^-}_{\mu\nu}[\varphi^{\pm}_{1^-\nu}(\ell;P)u(P)], \qquad (9)\end{aligned}$$

- **Quark-diquark vertices:**

$$\varphi^{\pm}_{0^+}(\ell;P) = \sum_{i=1}^{2} \mathfrak{s}^{\pm}_i(\ell^2,\ell\cdot P)\mathcal{S}^i(\ell;P)\mathcal{G}^{\pm},$$

$$\varphi^{j\pm}_{1^+\nu}(\ell;P) = \sum_{i=1}^{6} \mathfrak{a}^{j\pm}_i(\ell^2,\ell\cdot P)\gamma_5\mathcal{A}^i_\nu(\ell;P)\mathcal{G}^{\pm},$$

$$\varphi^{\pm}_{0^-}(\ell;P) = \sum_{i=1}^{2} \mathfrak{p}^{\pm}_i(\ell^2,\ell\cdot P)\mathcal{S}^i(\ell;P)\mathcal{G}^{\mp},$$

$$\varphi^{\pm}_{1^-\nu}(\ell;P) = \sum_{i=1}^{6} \mathfrak{v}^{\pm}_i(\ell^2,\ell\cdot P)\gamma_5\mathcal{A}^i_\nu(\ell;P)\mathcal{G}^{\mp},$$

where $\mathcal{G}^{+(-)} = \mathbf{I}_{\rm D}(\gamma_5)$ and

$$\mathcal{S}^1 = \mathbf{I}_{\rm D}, \qquad \mathcal{S}^2 = i\gamma\cdot\hat{\ell} - \hat{\ell}\cdot\hat{P}\mathbf{I}_{\rm D}$$

$$\mathcal{A}^1_\nu = \gamma\cdot\ell^\perp\hat{P}_\nu, \qquad \mathcal{A}^2_\nu = -i\hat{P}_\nu\mathbf{I}_{\rm D}, \qquad \mathcal{A}^3_\nu = \gamma\cdot\hat{\ell}^\perp\hat{\ell}^\perp_\nu$$

$$\mathcal{A}^4_\nu = i\hat{\ell}^\perp_\nu\mathbf{I}_{\rm D}, \quad \mathcal{A}^5_\nu = \gamma^\perp_\nu - \mathcal{A}^3_\nu, \quad \mathcal{A}^6_\nu = i\gamma^\perp_\nu\gamma\cdot\hat{\ell}^\perp - \mathcal{A}^4_\nu,$$

QCD-kindred model

- **Both the Faddeev amplitude and wave function are Poincare covariant, i.e. they are qualitatively identical in all reference frames.**
- **Each of the scalar functions that appears is frame independent, but the frame chosen determines just how the elements should be combined.**
- **In consequence, the manner by which the dressed quarks' spin, *S*, and orbital angular momentum, *L*, add to form the total momentum *J,* is frame dependent: *L*, *S* are not independently Poincare invariant.**
- **The set of baryon rest-frame quark-diquark angular momentum identifications:**

$$^2S:\ \mathcal{S}^1, \mathcal{A}_\nu^2, (\mathcal{A}_\nu^3 + \mathcal{A}_\nu^5),$$

$$^2P:\ \mathcal{S}^2, \mathcal{A}_\nu^1, (\mathcal{A}_\nu^4 + \mathcal{A}_\nu^6),$$

$$^4P:\ (2\mathcal{A}_\nu^4 - \mathcal{A}_\nu^6)/3,$$

$$^4D:\ (2\mathcal{A}_\nu^3 - \mathcal{A}_\nu^5)/3,$$

- **The scalar functions associated with these combinations of Dirac matrices in a Faddeev wave function possess the identified angular momentum correlation between the quark and diquark.**

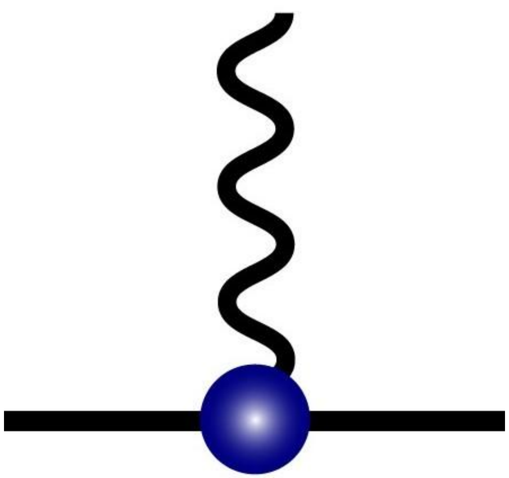
Building blocks (I)

- **The current-quark vertices**
  - **The axial-vector Ward-Takahashi identity:**

$$Q_\mu \Gamma^j_{5\mu}(k_+,k_-) + 2im_q\Gamma^j_5(k_+,k_-) = S^{-1}(k_+)i\gamma_5\frac{\tau^j}{2} + \frac{\tau^j}{2}i\gamma_5 S^{-1}(k_-)$$

  - **The Bethe-Salpeter Amplitude of the pion:**

$$\Gamma^j_\pi(k,Q) = \tau^j\gamma_5\left[iE_\pi(k,Q) + \gamma\cdot Q F_\pi(k,Q) + \gamma\cdot k\, k\cdot Q G_\pi(k,Q) + \sigma_{\mu\nu}k_\mu H_\pi(k,Q)\right]$$



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  - **The Bethe-Salpeter Amplitude of the pion:**

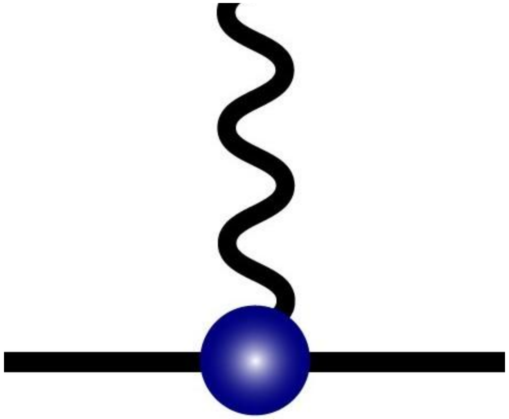
$$\Gamma^j_\pi(k, Q) = \tau^j \gamma_5 \left[iE_\pi(k, Q) \right]$$

  - **One *Ansatz*:** $E_\pi(k, Q) = \frac{1}{2f_\pi}\left(B(k_+^2) + B(k_-^2)\right)$

$$S^{-1}(k) = i\gamma \cdot k A(k^2) + B(k^2)$$

in the chiral limit:

$$E_\pi(k, 0) = \frac{B(k^2)}{f_\pi}$$



Therefore, we finally arrive at

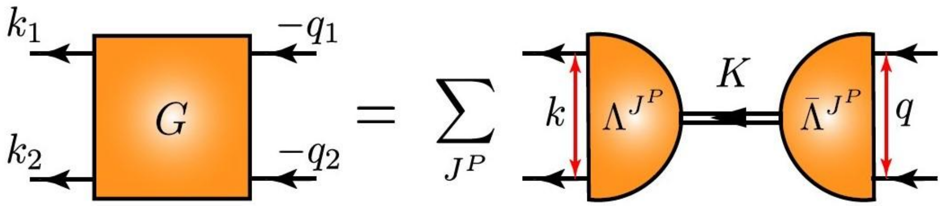
$$\Gamma^j_{5\mu}(k_+, k_-) = \frac{\tau^j}{2}\gamma_5 \Big[\gamma_\mu \Sigma_A(k_+^2, k_-^2) + 2\gamma \cdot k k_\mu \Delta_A(k_+^2, k_-^2) + 2i \frac{Q_\mu}{Q^2 + m_\pi^2} \Sigma_B(k_+^2, k_-^2) \Big], \quad (28)$$

and

$$i\Gamma^j_5(k_+, k_-) = \frac{m_\pi^2}{Q^2 + m_\pi^2} \frac{f_\pi}{2m_q} \Gamma^j_\pi(k, Q) \equiv \frac{\tau^j}{2} \frac{m_\pi^2}{Q^2 + m_\pi^2} \frac{1}{m_q} i\gamma_5 \Sigma_B(k_+^2, k_-^2), \quad (29)$$

Building blocks (II)

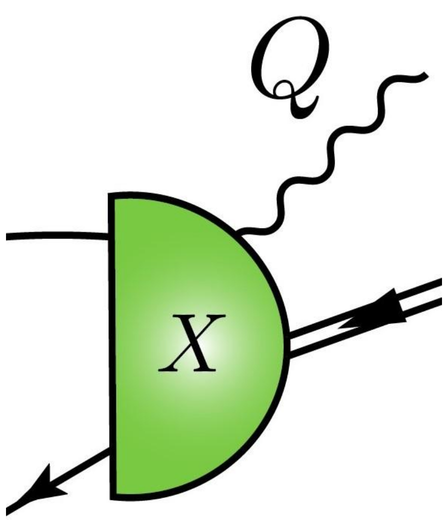
- **The seagull terms**
  - **The diquark *Ansatz* for the 4-point Green's function of the quark-quark correlations:**



  - **The equaltime commutators of the axial current operator:**

$$[\mathcal{A}^j_{5\mu=4}(x), \psi(y)]_{x_4=y_4} = \frac{\tau^j}{2}\gamma_5\psi(x)\delta^{(4)}(x-y)$$

$$[\mathcal{A}^j_{5\mu=4}(x), \bar{\psi}(y)]_{x_4=y_4} = \bar{\psi}(x)\gamma_5\frac{\tau^j}{2}\delta^{(4)}(x-y)$$



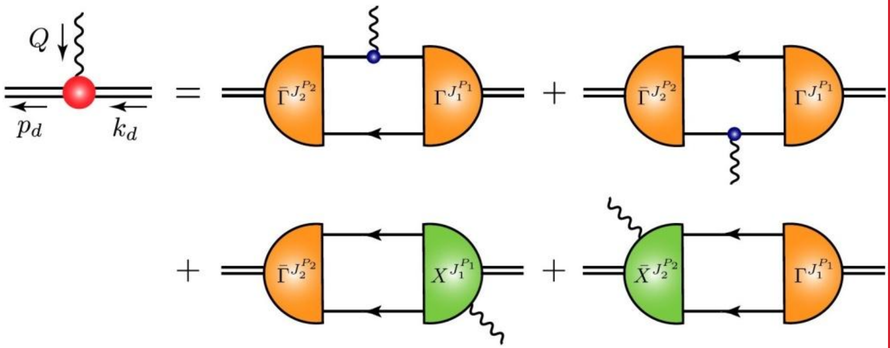
$$\chi^{j,J^P}_{5\mu,[\mathrm{sg}]}(k,Q) = -\frac{Q_\mu}{Q^2+m_\pi^2}\left[\frac{\tau^j}{2}i\gamma_5\Gamma^{J^P}(k-Q/2)+ \Gamma^{J^P}(k+Q/2)(i\gamma_5\frac{\tau^j}{2})^{\mathrm{T}}\right], \quad (57)$$

and

$$i\chi^{j,J^P}_{5,[\mathrm{sg}]}(k,Q) = -\frac{1}{2m_q}\frac{m_\pi^2}{Q^2+m_\pi^2}\left[\frac{\tau^j}{2}i\gamma_5\Gamma^{J^P}(k-Q/2)+ \Gamma^{J^P}(k+Q/2)(i\gamma_5\frac{\tau^j}{2})^{\mathrm{T}}\right]. \quad (58)$$

Building blocks (III)

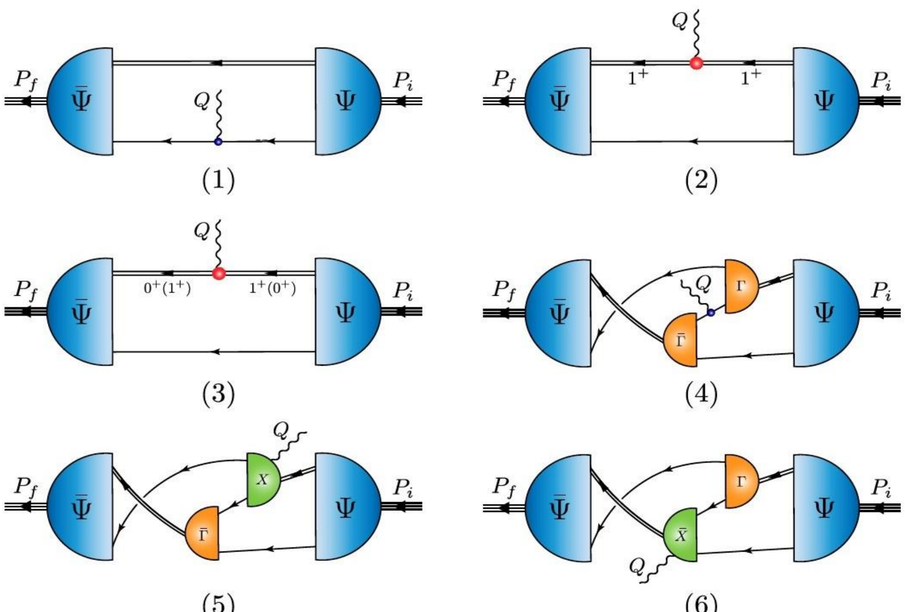
➢ **The current-diquark vertices**



➢ **AXWTIs:**

$$Q_\mu \, \Gamma^{aa}_{5\mu,\alpha\beta}(p_d, k_d) + 2im_q \, \Gamma^{aa}_{5,\alpha\beta}(p_d, k_d) = 0$$

$$Q_\mu \, \Gamma^{sa}_{5\mu,\beta}(p_d, k_d) + 2im_q \, \Gamma^{sa}_{5,\beta}(p_d, k_d) = 0$$



i) The $\{qq\}_{1^+}$–pseudoscalar-current vertex

$$\Gamma^{aa}_{5,\alpha\beta}(p_d, k_d) = = \frac{1}{2m_q} \frac{m_\pi^2}{Q^2 + m_\pi^2} \left(\kappa^{aa}_{\rm ps} \frac{M_q^E}{m_N} \epsilon_{\alpha\beta\gamma\delta}(p_d + k_d)_\gamma Q_\delta \right) d(\tau^{aa}) \,, \quad (61)$$

ii) The $\{qq\}_{1^+}$–axial-current vertex

$$\Gamma^{aa}_{5\mu,\alpha\beta}(p_d, k_d) = \left(\frac{\kappa^{aa}_{\rm ax}}{2} \epsilon_{\mu\alpha\beta\nu}(p_d + k_d)_\nu + + \frac{Q_\mu}{Q^2 + m_\pi^2} \left(\kappa^{aa}_{\rm ps} \frac{M_q^E}{m_N} \epsilon_{\alpha\beta\gamma\delta}(p_d + k_d)_\gamma Q_\delta \right) \right) d(\tau^{aa}) \,, \quad (62)$$

iii) The pseudoscalar-current induced $0^+ \leftarrow 1^+$ transition vertex

$$\Gamma^{sa}_{5,\beta}(p_d, k_d) = = \frac{1}{2m_q} \frac{m_\pi^2}{Q^2 + m_\pi^2} \left(-2i\kappa^{sa}_{\rm ps} M_q^E Q_\beta \right) d(\tau^{sa}) \,, \quad (63)$$

iv) The axial-current induced $0^+ \leftarrow 1^+$ transition vertex

$$\Gamma^{sa}_{5\mu,\beta}(p_d, k_d) = \left(im_N \kappa^{sa}_{\rm ax} \delta_{\mu\beta} + + \frac{Q_\mu}{Q^2 + m_\pi^2} \left(-2i\kappa^{sa}_{\rm ps} M_q^E Q_\beta \right) \right) d(\tau^{sa}) \,. \quad (64)$$