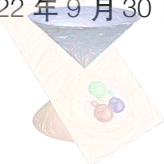


Confinement, chiral symmetry breaking and holography:
the 3D image of the pion

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2022 年 9 月 30 日



Contents

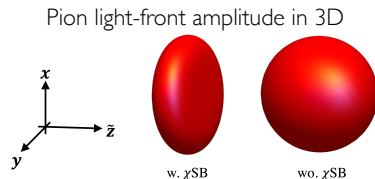
- Confinement & chiral symmetry breaking
- Light front and holography
- PCAC and chiral sum rule
- 3D image of the pion, form factor & parton distribution
- Summary

Based on:

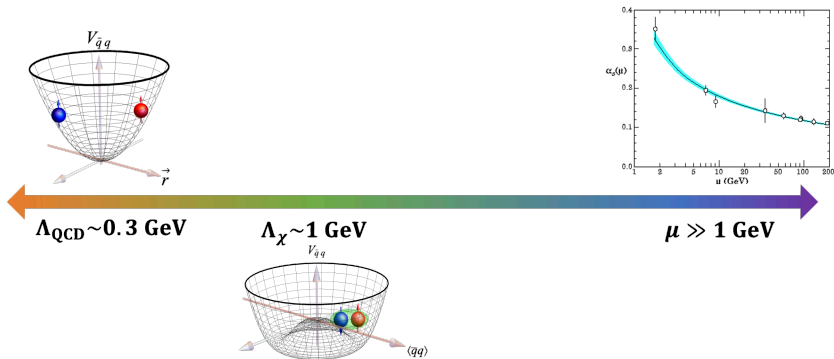
YL, Maris, Vary, arXiv:2203.14447 [hep-th];

YL, Vary, PLB (2022), arXiv:2103.09993 [hep-ph];

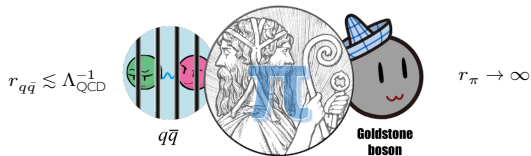
YL, Vary, PRD (2022), arXiv: 2202.05581 [hep-ph]



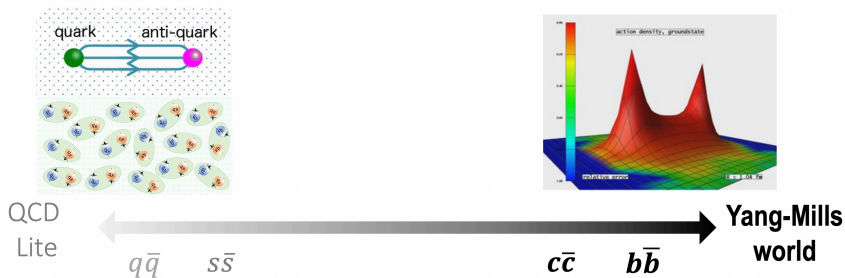
Non-perturbative QCD: last frontier within Standard Model



- Confinement
- Chiral symmetry breaking
- Pion: dichotomy 「矛盾的对立统一」



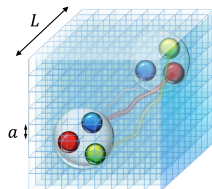
QCD vacuum



Tremendous progress in the past 20 years; yet many open questions remain:

- Confinement in QCD: various possibilities [Greensite:2016pfc]
 - *flux tube, dual superconductor, central vortices, Gribov horizon, ...*
- Chiral symmetry breaking: [Faber:2017alm]

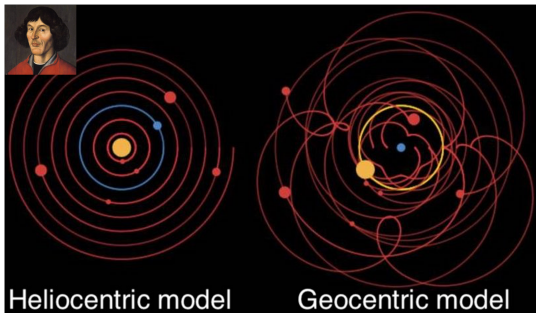
``Berlin wall": $\text{Cost} = K \left(\frac{M_\rho}{M_\pi} \right)^{2 \sim 6} a^{-(6 \sim 7)} L^5,$



[Ukawa:2002pc; cf. Schaefer:2012tq]

Is there a simplified description of the pion?

[Brodsky et. al., Nature Rev. Phys. (2022)]



nature reviews physics

Artificial dynamical effects in quantum field theory

Stanley J. Brodsky , Alexandre Deur and Craig D. Roberts

Abstract | In Newtonian mechanics, studying a system in a non-Galilean reference frame can lead to inertial pseudoforces appearing, such as the centrifugal force that seems to arise in dynamics analysed in a rotating frame. Likewise, artificial

Today, no approach to QFT is perfect, but LF quantization possesses many merits. It often provides the simplest known description of nature. As always in physics, one can formulate a problem using any framework one desires. But if the wrong approach is chosen, the costs are high⁶⁵.

Light-front QCD

- Intrinsic structure of hadrons: [Miller:2018ybm; cf. Li:2022ldb]

- In-hadron condensate [Brodsky:2010xf; Casher:1974xd]

$$Q_5|0\rangle_{\text{LF}} = 0$$

- Axial-vector Ward-Takahashi identity in DSE/BSE [Maris:1997hd]

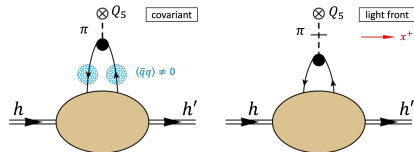
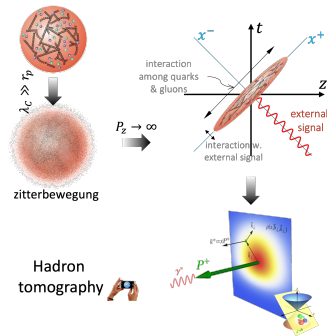
$$f_\pi E_\pi(k, 0) = B_q(k^2), \dots$$

- LF vacuum

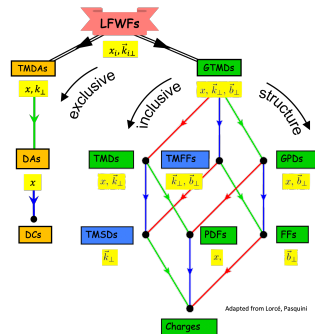
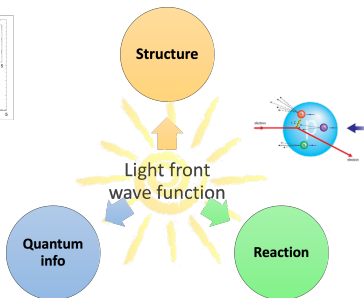
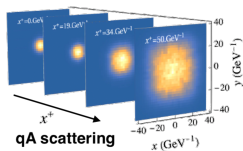
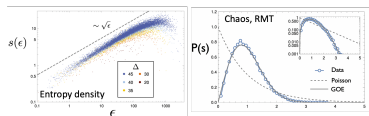
[Wu:2003vn; Beane:2013oia, Beane:2015ufo, Burkardt:1996pa]

$$\frac{m_q}{2} \langle 0 | \bar{q} \frac{\gamma^+}{i\partial^+} q | 0 \rangle_{\text{LF}} = \langle 0 | \bar{q} q | 0 \rangle_{\text{LF}}$$

Chiral symmetry breaking is encoded in the hadronic light-front wave functions



- Light-front physics underlines hadron structures measured in high-energy scattering experiments
- Light-front wave functions provide the full quantum information of hadrons



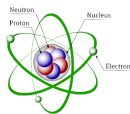
「道生一，一生二，二生三，三生万物」

Access to light front amplitudes

- Experimental measurements [E791:2000xcx]
- Global fits
- OPE (moments) [e.g. Chang:2013pq, Yang:2018nqn]
- Nakanishi representation & LF projection of BSA [dePaula:2022pcb]
- un-Wick rotation [Eichmann:2021vnj, Frederico:2019noo]
- Large momentum effective theory [ji:2013dva]
- Quasi-PDFs [Radyushkin:2017cyf]
- Interpolation between instant form and front form [Ma:2021yqx]
- Hamiltonian formalism: [review: Hiller:2016itl]

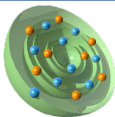
$$\underline{H}_{\text{LFQCD}}|\psi_h\rangle = M_h^2|\psi_h\rangle \quad \underline{H}_{\text{LFQCD}} = P^+ \underline{P}_{\text{LFQCD}}^- - \vec{P}_\perp^2$$

Light-front Hamiltonian formalism



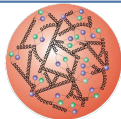
Non-relativistic,
weakly coupling

Bohr Model 



Non-relativistic,
strongly coupling

Shell Model 



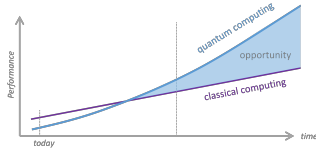
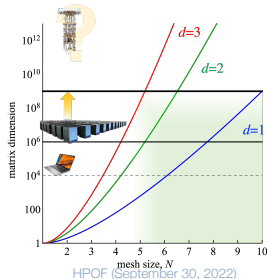
Relativistic,
strongly coupling

?

LFQCD as a strong-coupling relativistic quantum many-body problem

$$H_{\text{LFQCD}} = \sum_i \frac{\vec{p}_{i\perp}^2 + m_i^2}{x_i} + U_i + \sum_{ij} V_{ij}^{(\text{QCD})} - \delta_{ij} U_i$$

- Exponential wall: $\dim \mathcal{H} = N^{dN}$ [Hornbostel:1988fb, Anand:2020gnn]
- Quantum advantage? [Kreshchuk:2020dla]
- Need a semi-classical first approximation



Light-front Schrödinger wave equation with a factorization ansatz:

[Chabysheva:2012fe, Li:2021jqb, deTeramond:2021yyi]

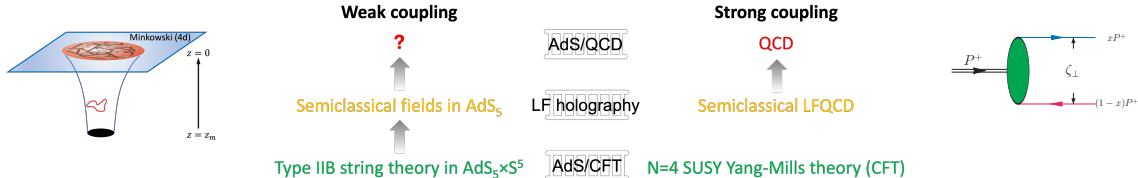
$$\left(\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + U \right) \psi(x, \vec{k}_\perp) = M^2 \psi(x, \vec{k}_\perp)$$

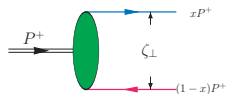
factorization ansatz $\Downarrow \psi = \varphi(\vec{\zeta}_\perp) \chi(x)$

$$\left[-\nabla_{\zeta_\perp}^2 + U_\perp(\vec{\zeta}_\perp) \right] \varphi(\vec{\zeta}_\perp) = M_\perp^2 \varphi(\vec{\zeta}_\perp), \quad \left[\cancel{\frac{m_q^2}{x}} + \frac{m_{\bar{q}}^2}{1-x} + U_\parallel(\tilde{z}) \right] \chi(x) = M_\parallel^2 \chi(x)$$

Light-front holography provides a unique correspondence between semiclassical LFQCD and semiclassical fields in 5D anti-de Sitter space

[Brodsky:2003px, Erlich:2005qh, Karch:2006pv]





semiclassical LFQCD

$$\zeta_{\perp} = \sqrt{x(1-x)r_{\perp}}$$

LF amplitude

confining potential

$$L^2 - (J-2)^2$$

$\leftrightarrow$

semiclassical field theory in AdS

$\leftrightarrow$

fifth coordinate z ,

$\leftrightarrow$

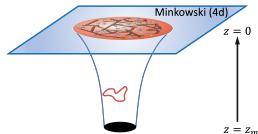
string amplitude

$\leftrightarrow$

dilation field Φ

$\leftrightarrow$

$$(\mu R)^2$$



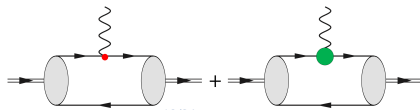
- Correspondence within equation of motions (hadron spectra)

$$U(z) = \frac{1}{4}\Phi'^2 + \frac{1}{2}\Phi'' + \frac{2J-3}{2z}\Phi'$$

- Correspondence within form factors [Drell:1969km, West:1970av, Hong:2004sa, Grigoryan:2007wn, Kwee:2007dd]

$$F_{\text{LF}}(Q^2) = \int \zeta_{\perp} d\zeta_{\perp} \varphi_{\pi}^2(\zeta_{\perp}) \zeta_{\perp} Q K_1(\zeta_{\perp} Q) + \text{high Fock sectors}$$

$$F_{\text{AdS}}(Q^2) = \int \frac{dz}{z^3} \left[z Q K_1(zQ) + z Q I_1(zQ) \frac{K_0(Qz_m)}{I_0(Qz_m)} \right] \phi_{\pi}^2(z)$$

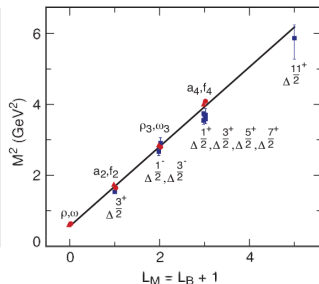
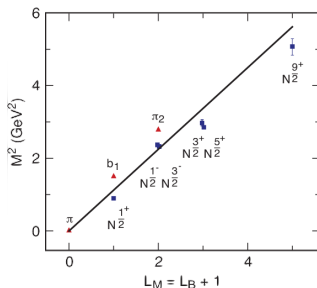


[Review: Brodsky:2014yha]

[Erlich:2005qh]

-
- The diagram illustrates a 4D Minkowski space. A blue plane at the top is labeled "Minkowski (4d)". Below it, a red, irregularly shaped cross-section is shown on a horizontal plane labeled $z = 0$. A vertical axis with an upward arrow is labeled $z = z_m$. The cross-section is depicted as a 2D slice of a 3D volume, which is shown as a funnel-like shape narrowing downwards.

[Brodsky:2013ar, deAlfaro:1976vlx]



What about chiral symmetry breaking? (cf. dynamical mass generation)

PRL **95**, 261602 (2005)

QCD and a Holographic Model of Hadrons

Joshua Erlich,¹ Emanuel Katz,² Dam T. Son,³ and Mikhail A. Stephanov⁴

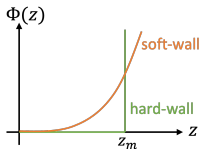
Hard-wall model: with chiral symmetry breaking, but no Regge trajectory

PHYSICAL REVIEW D **74**, 015005 (2006)

Linear confinement and AdS/QCD

Andreas Karch,^{1,*} Emanuel Katz,^{2,†} Dam T. Son,^{3,‡} and Mikhail A. Stephanov^{4,§}

Soft-wall model: with Regge trajectory, but no chiral symmetry breaking



tion between bulk and boundary theories [12,19]. The action at quadratic order in the fields and derivatives reads

$$I = \int d^5x e^{-\Phi(z)} \sqrt{g} \left\{ -|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2} (F_L^2 + F_R^2) \right\}$$

asymptotics $e^{z^2} \rightarrow \infty$ and $\exp\{-(3/4)z^{-2}\} \rightarrow 1$. Since the equation is linear, selecting one of the solutions in the IR (the $X < \infty$ one, of course) gives Σ simply proportional to M . This is not what one wants in a theory with spontaneous symmetry breaking such as QCD. It is clear that one has to consider higher order terms in the potential $U(X, \dots)$ for X and all other scalar condensates. Such a potential would

Various implementations of χ SB in soft-wall AdS/QCD:

[Gherghetta:2009ac, Zuo:2009dz, Sui:2009xe, Sui:2010ay, Iatrakis:2010zf, Iatrakis:2010jb, Vega:2010ne, Jarvinen '12, Afonin:2012jn, Li:2012ay,

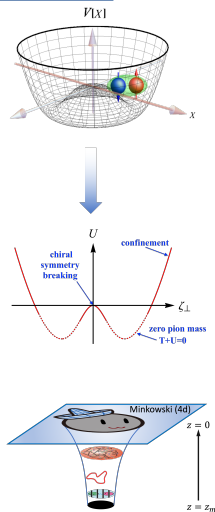
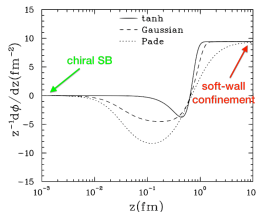
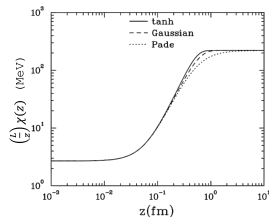
Li:2013oda, Cui:2013xva, Chelabi:2015cwn, Fang:2016nfj, Ballon-Bayona:2021ibm, Rinaldi:2022dyh, ...]

Scalar field X dual to $\bar{q}q$, with a non-vanishing VEV: $\langle X \rangle = \frac{1}{2}\chi(z)$

$$S = - \int d^5x \sqrt{-g} e^{-\Phi(z)} \text{Tr} \left\{ |DX|^2 - V[X] + \frac{1}{4g_5^2} F^2 \right\},$$

Higgs potential: $V[X] = -m_X^2 |X|^2 + \kappa |X|^4$, $m_X^2 = -3$

- At the CFT boundary ($z \rightarrow 0$): $\chi(z) \sim m_q z + \Sigma z^3$ [Klebanov:1999tb]
- (Non-linear) Eq. of Motion $\Rightarrow \Phi(z) \sim z^6 \Rightarrow U(z) \sim -z^4$ at $z \rightarrow 0$
- χ SB & confinement dictate the UV & IR behaviors of $U(\zeta_\perp)$, respectively



Model independent constraints from light-front dynamics?

Partially conserved axial-vector current (PCAC)

$$\partial_\mu J_5^\mu = 2im_q \bar{q}\gamma_5 q \Rightarrow \langle 0 | \partial_\mu J_5^\mu - 2im_q \bar{q}\gamma_5 q | P(p) \rangle = 0.$$

- $J_5^\mu = \bar{q}\gamma^\mu\gamma_5 q$ is the full axial-vector current, not the LF axial-vector current $\hat{J}_5^\mu$

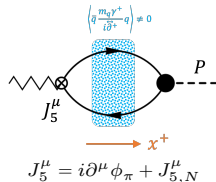
$J_5^+ = \hat{J}_5^+$, but $\hat{J}_5^\mu$ does not create the pion from the vacuum

- Fock space expansion: only valence LFWFs contribute
- The most general covariant structure of the pion valence LFWFs: covariant LFD

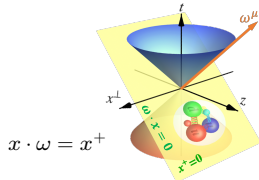
$$\psi_{s\bar{s}/P}(x, \vec{k}_\perp) = \bar{u}_s(p_1) \left[\gamma_5 \phi_1(x, k_\perp) + \hat{f}_\chi \frac{\gamma_5 \not{\omega}}{\omega \cdot p} \phi_2(x, k_\perp) \right] v_{\bar{s}}(p_2),$$

- ω -term: direction of the light front - conformal structure $\omega^\mu \rightarrow \lambda \omega^\mu$
- ω -term: allowed by kinematical symmetries, required by rotational invariance
- ω -term does not satisfy the "covariant LF condition" and is often neglected
- Chiral broken phase: $f_\pi \neq 0 \Rightarrow \hat{f}_\chi \neq 0$

$$\frac{f_P}{2\sqrt{2}N_C} = \int \frac{dx}{2\sqrt{x(1-x)}} \int \frac{d^2 k_\perp}{(2\pi)^3} \psi_{\uparrow\downarrow-\downarrow\uparrow/P}(x, \vec{k}_\perp).$$



[Carbonell:1998rj]



In the chiral limit $m_q \rightarrow 0$,

$$\int \frac{dx}{2\sqrt{x(1-x)}} \int \frac{d^2k_\perp}{(2\pi)^3} \frac{k_\perp^2}{x(1-x)} \psi_{\uparrow\downarrow-\downarrow\uparrow/P}^{(0)}(x, \vec{k}_\perp) = 0$$

- **Exact relation:** no Fock sector truncations, no approximations
- If $\hat{f}_\chi = 0$, this sum rule is automatically satisfied. However, $f_\pi = 0 \rightarrow$ chiral symmetric phase (not QCD)
- Gell-Mann-Oakes-Renner relation: $f_P^{(0)2} M_P^2 = 2m_q g_P^{(0)} + O(m_q^2)$, where $g_P = \langle 0 | j_5 | P(p) \rangle$
- Additional sum rules from further light-front current algebra [Beane:2013oia, Beane:2015ufa]

Application to semiclassical light-front QCD:

$$f_P \nabla_{\perp}^2 \varphi_P(\zeta_{\perp} = 0) = 0$$

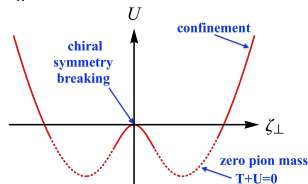
for any pseudo-scalar P . Recall,

$$\left[-\nabla_{\perp}^2 + U(\zeta_{\perp}) \right] \varphi_P(\zeta_{\perp}) = M_P^2 \varphi_P(\zeta_{\perp})$$

The pion: $M_{\pi} = 0$,

$$\Rightarrow U(\zeta_{\perp} = 0) = 0 \quad (\text{cf. } U_{\text{Higgs}} \sim -\zeta_{\perp}^4 \rightarrow 0)$$

For excited pions, the decay constants $f_{\pi^n} = 0$. [Maris:1997hd, Ballon-Bayona:2014oma]



Let's build a chiral pion:

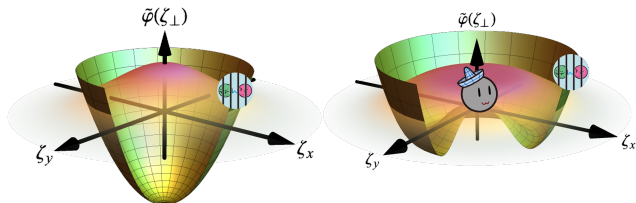
$$\left[-\nabla_{\perp}^2 + U(\zeta_{\perp}) \right] \varphi_P(\zeta_{\perp}) = M_P^2 \varphi_P(\zeta_{\perp}), \quad \nabla_{\perp}^2 \varphi_{\pi}(\zeta_{\perp} = 0) = 0, \quad U(\zeta_{\perp}) \rightarrow \begin{cases} \zeta_{\perp}^2 & \zeta_{\perp} \rightarrow \infty \\ -\zeta_{\perp}^4 & \zeta_{\perp} \rightarrow 0 \end{cases}$$

We propose the following pion wave function based on light-front holography:

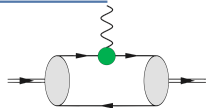
$$\varphi_{\pi}(\zeta_{\perp}) = \underbrace{\left(1 + \frac{1}{2}\zeta_{\perp}^2 + \frac{c}{8}\zeta_{\perp}^4 \right)}_{\text{chiral symmetry breaking}} \overbrace{e^{-\frac{\zeta_{\perp}^2}{2}}}^{\text{confinement}}$$

Given the pion wave function, the potential is ($c = 1$),

$$U(\zeta_{\perp}) = \frac{\nabla_{\perp}^2 \varphi_{\pi}(\zeta_{\perp})}{\varphi_{\pi}(\zeta_{\perp})} = \frac{\zeta_{\perp}^4 (\zeta_{\perp}^2 - 6)}{\zeta_{\perp}^4 + 4\zeta_{\perp}^2 + 8}$$



$$F_\pi(q^2) = \int \zeta_\perp d\zeta_\perp \rho_\pi(\zeta_\perp) V_{\text{SW}}(q^2, \zeta_\perp) = \int dx H_\pi(x, t = q^2)$$



Veneziano amplitudes

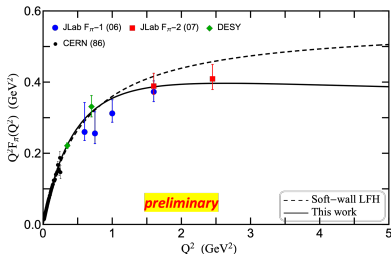
- Soft-wall AdS/QCD: only leading-twist contribution $\sim Q^{-2}$

$$\rho_\pi(\zeta_\perp) = N \exp(-\zeta_\perp^2) \Rightarrow F_\pi(q^2) = F_{\tau=2}(q^2)$$

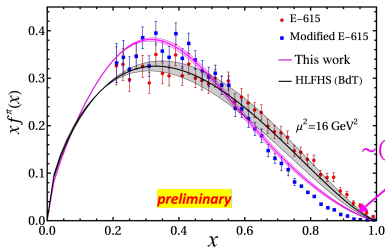
- χ SB modifies the large- Q^2 behavior: incorporating high-twist ($\tau = N$) contributions

[Brodsky:1973kr]

$$\rho_\pi(\zeta_\perp) = \left[1 + \frac{1}{2}\zeta_\perp^2 + \frac{c}{8}\zeta_\perp^4\right]^2 \exp(-\zeta_\perp^2) \Rightarrow F_\pi(q^2) = N \left\{ \underbrace{F_{\tau=2}(q^2)}_{[q\bar{q}]} + \underbrace{F_{\tau=3}(q^2)}_{[q\bar{q}g]} + c_1 \underbrace{F_{\tau=4}(q^2)}_{[q\bar{q}q\bar{q}]} + \dots \right\}$$



[See also, Chang:2020kjj, Dosch:2022mop, Gurjar:2022jxx]



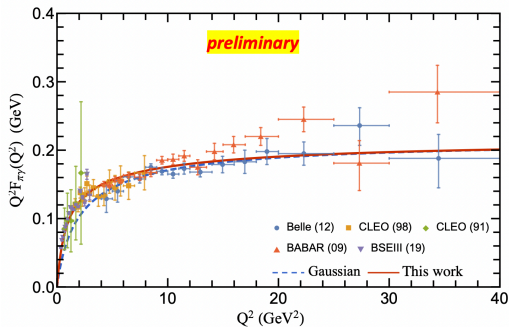
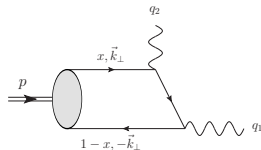
Global analysis: $\sim (1-x)^{1.0 \sim 1.3}$

[Barry:2021osv]

Singly-virtual two-photon transition form factor:

$$F_{\pi\gamma}(Q^2) = e_f^2 2\sqrt{2N_C} \int \frac{dx}{2\sqrt{x(1-x)}} \int \frac{d^2k_\perp}{(2\pi)^3} \frac{\psi_\pi(x, \vec{k}_\perp)}{k_\perp^2 + x(1-x)Q^2}$$

- VMD: $Q^2 \rightarrow Q^2 + M_\rho^2$
- pQCD normalization: $Q^2 F_{\pi\gamma}(Q^2) \rightarrow 2f_\pi, Q^2 \rightarrow \infty$



$$\tilde{\psi}_\pi(\vec{z}_\perp, \tilde{z}) = \int_0^1 \frac{dx}{2\sqrt{x(1-x)}} \int \frac{d^2 k_\perp}{(2\pi)^3} e^{ix\tilde{z} - i\vec{k}_\perp \cdot \vec{z}_\perp} \psi_\pi(x, \vec{k}_\perp) = \langle 0 | \bar{q}(-\frac{1}{2}z) \frac{\gamma^+ \gamma_5}{p^+} q(+\frac{1}{2}z) | P(p) \rangle_{z^+=0}$$

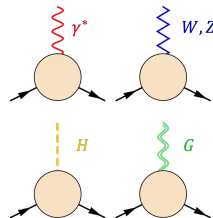
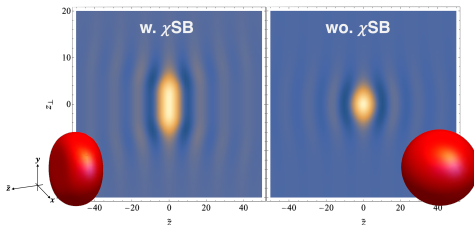
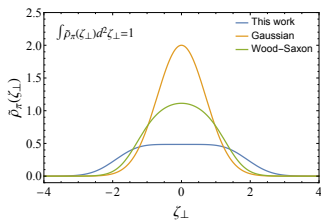
where $\tilde{z} = p \cdot z$ is the *loffe time* of Miller and Brodsky. [Miller:2019ysh]

- Infinitely long in the longitudinal direction, $r_\pi \propto -\ln M_\pi^2 \rightarrow \infty$ [Li:2022izo, Weller:2021wog]

$$r_{\text{em}}^2 = -6F'_{\text{ch}}(0) = \frac{3}{2} \langle \vec{b}_\perp^2 \rangle, \quad r_{\text{grav}}^2 = -6A'(0) = \frac{3}{2} \langle \vec{\zeta}_\perp^2 \rangle \quad \vec{b}_\perp = (1-x)\vec{r}_\perp$$

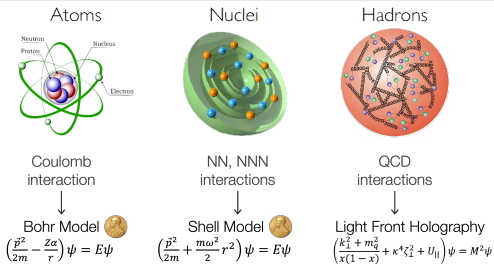
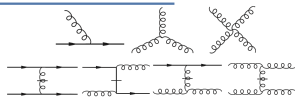
- Plateau in the transverse direction: the pion is uniform within up to $r_\perp \sim \kappa_{\text{confin}}^{-1}$

$$\nabla_\perp^2 \varphi_\pi(\zeta_\perp = 0) = 0$$

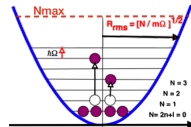


Light-front QCD in
light cone gauge $A^+ = 0$

$$H_{\text{LFQCD}} = \sum_i \frac{\vec{p}_{i\perp}^2 + m_i^2}{x_i} + U_i + \sum_{ij} V_{ij}^{(\text{QCD})} - \delta_{ij} U_i$$



- It is important to **preserve all kinematical symmetries under the truncation**
- Covariant LFD gets too complicated for many-body ($A \leq 3$) system [Carbonell:1998rj]
- Discretized momentum basis (DLCQ): choose between rotational symmetry and boost symmetry
- Basis light-front quantization (BLFQ) [Vary:2009gt]



[Review: Barrett:2013nh (NCSM)]

■ Semi-classical LF wave equations: [Reviews: Brodsky:2014yha]

[Li:2021jqb, Li:2022izo, deTeramond:2021yyi, Ahmady:2021lsh,
Ahmady:2021yzh, Li:2022izo, Lyubovitskij:2022rod, Ahmady:2022dfv]

■ Realistic QQ interaction:

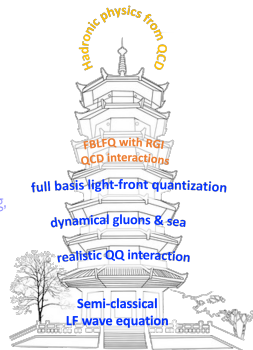
- QED: [Wiecki:2014ola, Hu:2020arv, Nair:2022evk]
- heavy flavor [Li:2015zda, Li:2017mlw, Leita:2017esb, Li:2018uif,
Adhikari:2018umb, Tang:2018myz, Lan:2019img, Tang:2019gvn,
Tang:2020org, Li:2021ejv, Li:2021cwv, Shuryak:2021fsu,
Shuryak:2021hng, Shuryak:2021mlh, Shuryak:2022thi, Shuryak:2022wtk]
- light mesons [Jia:2018ary, Lan:2019vui, Lan:2019rba, Qian:2020utg,
Mondal:2021czk, Adhikari:2021jrh, Peng:2022lte]
- nucleons [Mondal:2019jdg, Xu:2021wwj, Liu:2022fm, Hu:2022ctr]
- tetraquarks [Kuang:2022vdy]

■ Dynamical gluons & sea:

[Lan:2021wok, Xu:2022abw]

■ Full basis light-front quantization

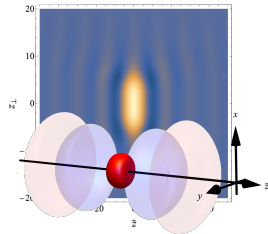
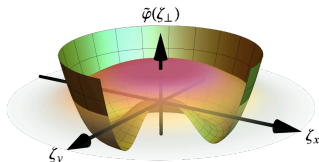
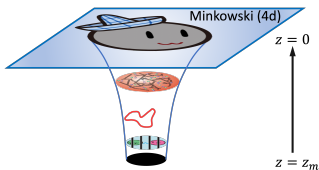
[Vary:2009gt]

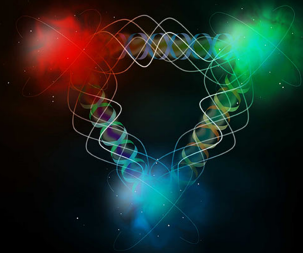


Jacob's ladder in quantum chemistry



- The pion is the key to understand confinement and chiral symmetry breaking in QCD
- Light-front wave functions provide the direct access to the parton structure of the pion
- Obtained an exact sum rule for the valence sector wave function based on the most general covariant structure and PCAC
- This chiral sum rule is consistent with chiral symmetry breaking in soft-wall AdS/QCD and leads to a remarkable feature of the pion structure





fin

