Open Quantum Systems for Quarkonia

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XY, T. Mehen, 1811.07027, 2009.02408
Review: XY, 2102.01736

Hadron Physics Online Forum
Aug. 19, 2022

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- Introduction: quark-gluon plasma (QGP) and heavy ion collision, hard probes of QGP: heavy quarks (bound states)
- Open quantum system framework for heavy quarks (quarkonia)
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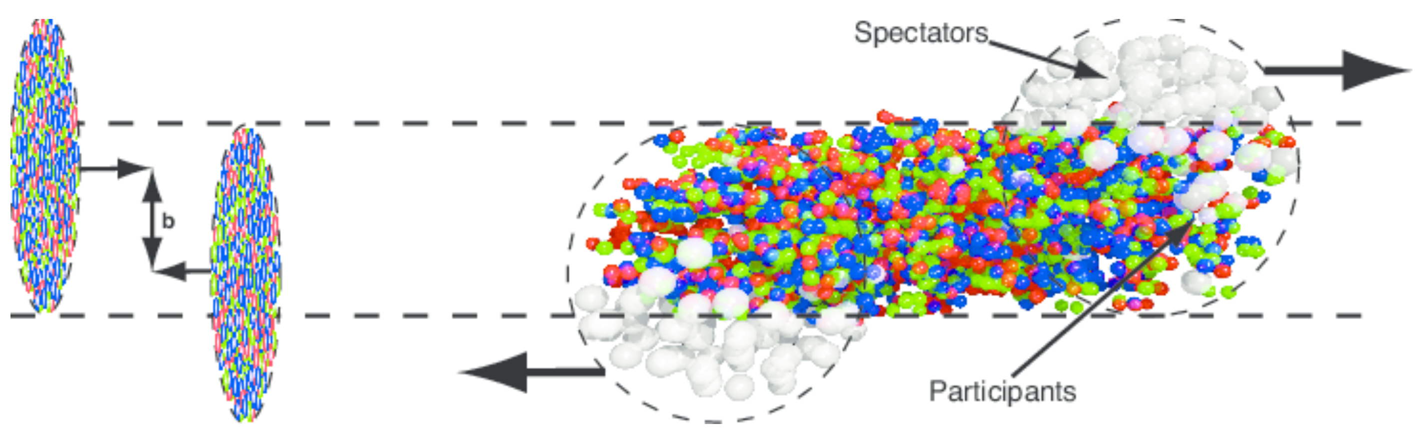
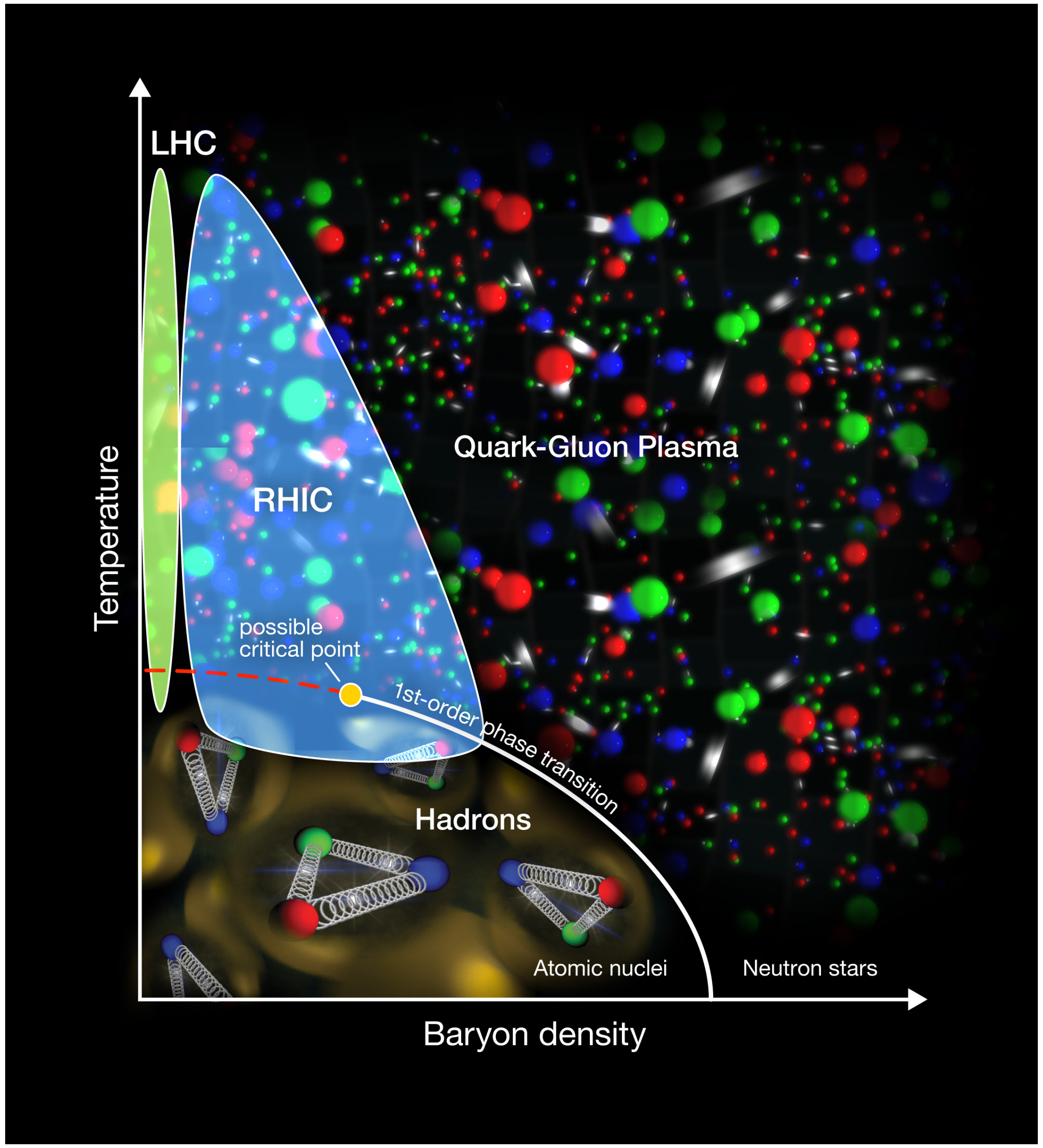
  - Connection with semiclassical transport equations
- Applications

I. Heavy Quarkonia in Heavy Ion Collisions

Quark-Gluon Plasma and Heavy Ion Collision

- Asymptotic freedom —> deconfined phase of QCD matter expected at high temperature / density —> QGP
- Study QGP: heavy ion collision experiments at RHIC and LHC



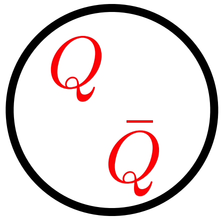


- QGP fireball: strongly coupled, lifetime ~10 fm/c, temperature 150—600 MeV

- Hard probes of QGP: large energy scale, heavy quarks, quarkonia and jets

Quarkonia inside QGP

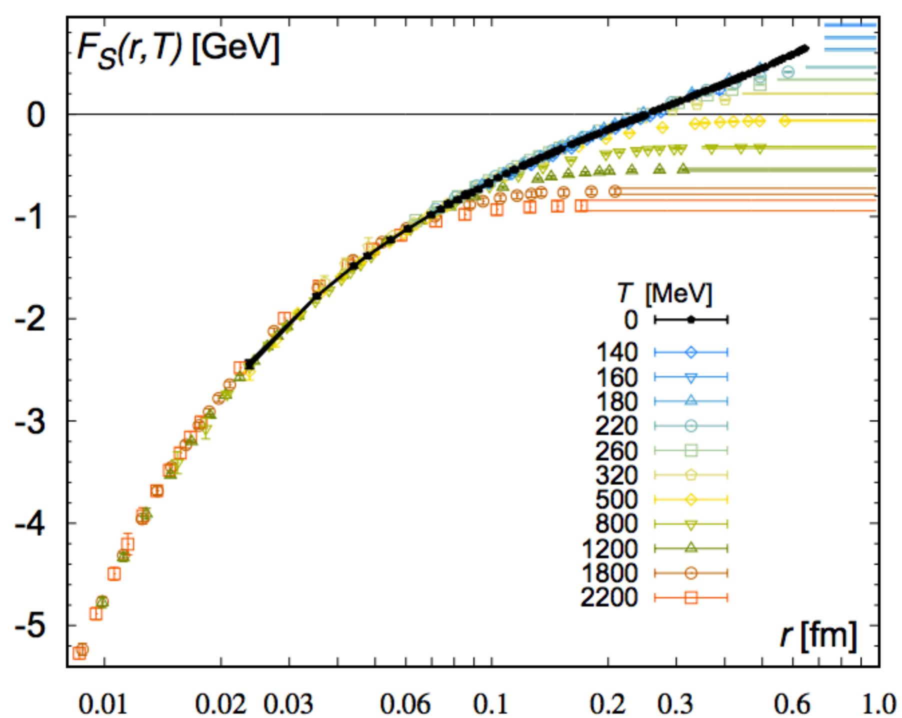
- **Quarkonium: screening**, dissociation and recombination



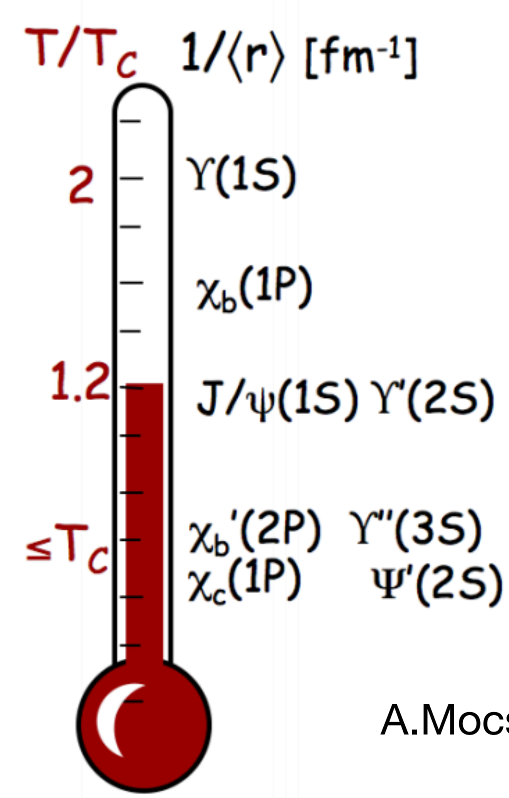
Ground and lower excited states described by Schrödinger equation

In vacuum: $V(r) = -\frac{A}{r} + Br \longrightarrow$ In QGP: $V(r) = -\frac{A}{r} e^{-m_D r}$

Screening: potential too weak to support bound state, melting of state, suppression



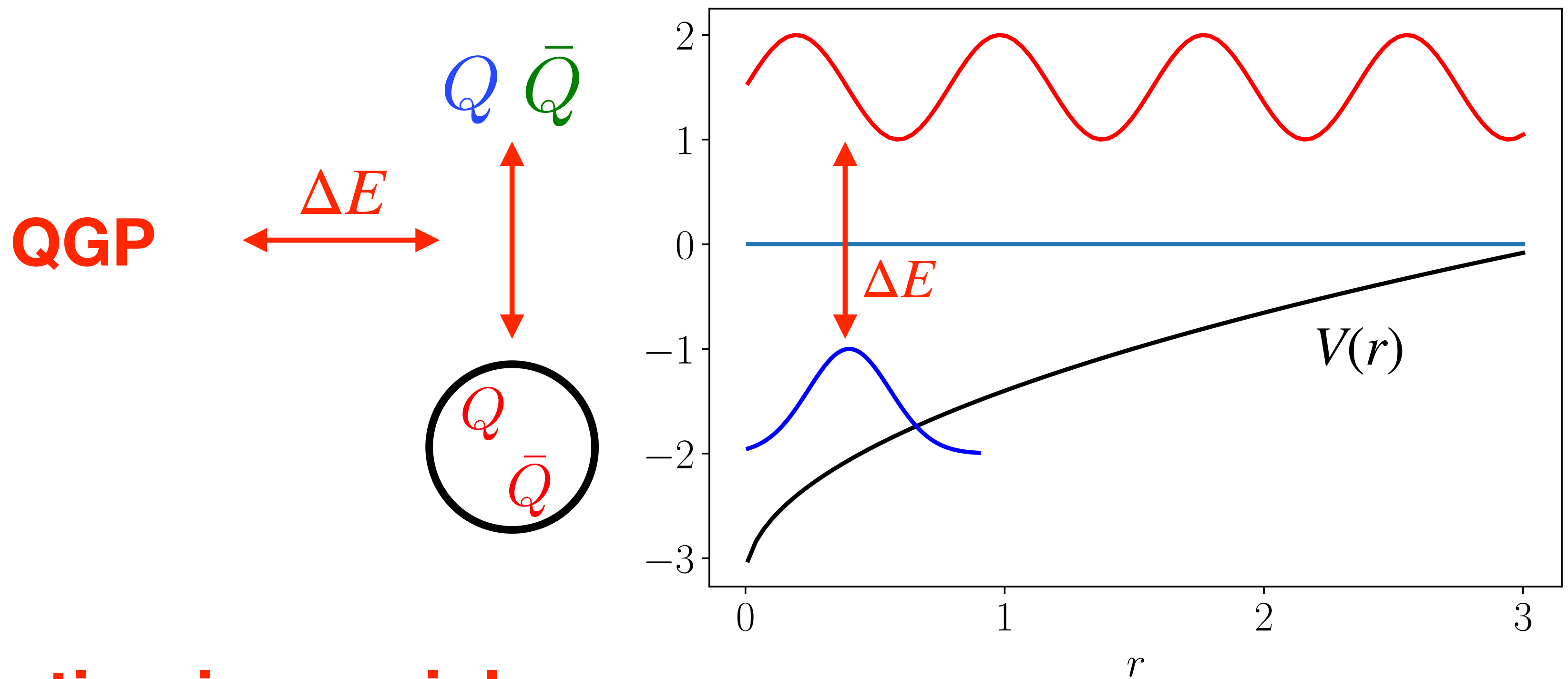
A.Bazavov, N.Brambilla, P.Petreczky, A.Vairo, J.H.Weber, 1804.10600



A.Mocsy, 0811.0337

Quarkonia inside QGP

- **Quarkonium:** screening, **dissociation and recombination**

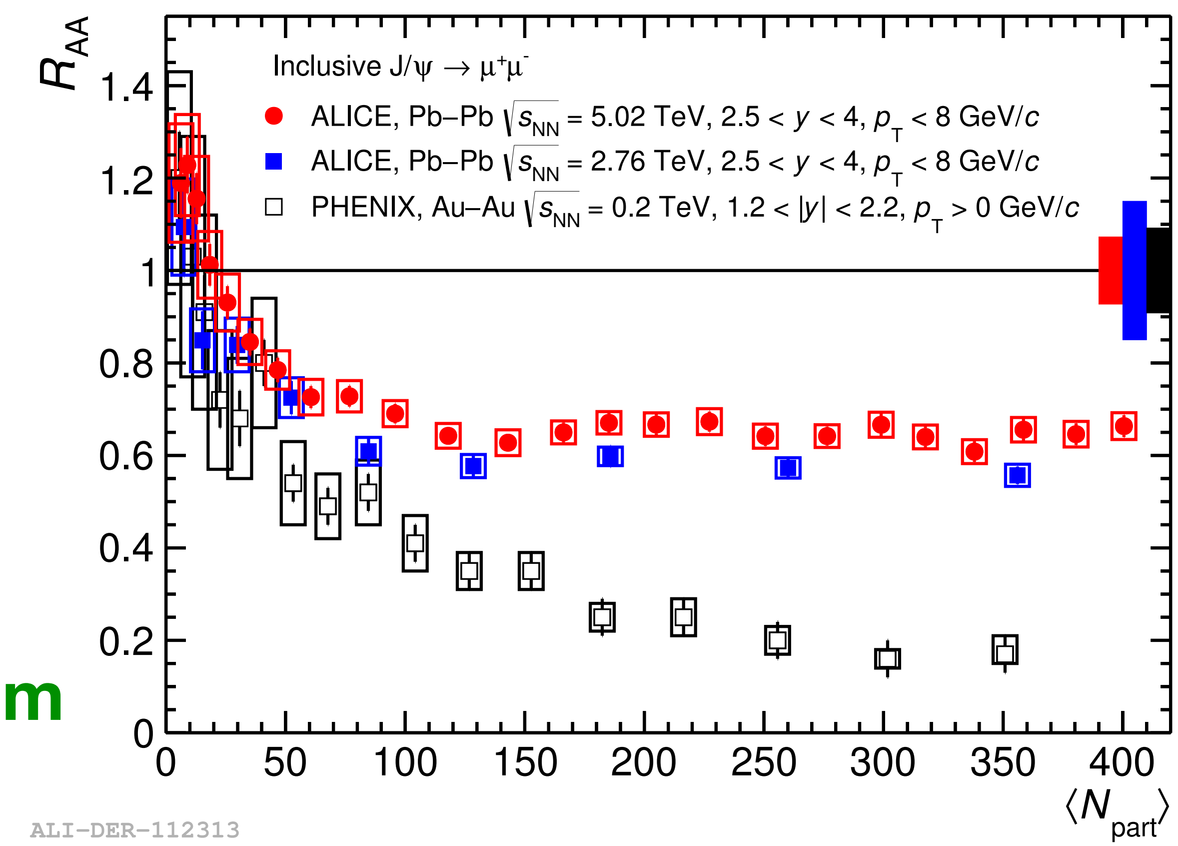


- **Recombination is crucial**

J/ψ less suppressed at higher energy: enhanced recombination from HQs produced from different hard collisions

Semiclassical transport equations model recombination, **go beyond semiclassical approach to understand recombination**

Treat heavy quarks as open quantum system



II. Open Quantum Systems

Open Quantum System

Total system = subsystem + environment: $H = H_S + H_E + H_I$

$\rho(t=0) = \rho_S \otimes \rho_E$

Subsystem & environment

(Heavy quarks & QGP)

→ **Unitary evolution** / **Time reversible** →

$U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)$

Subsystem & environment

↓ **Trace out (integrate out) environment** ↓

$\rho_S(t=0)$

Subsystem

→ **Non-unitary** / **Time irreversible** →

$\mathrm{Tr}_E\left[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\right]$

Review: XY, 2102.01736

Towards Lindblad Equation: Two Limits

Subsystem: non-unitary, time-irreversible evolution

$$\mathrm{Tr}_E\Big[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\Big]$$

Markovian (weak coupling)

Quantum optical limit (low T) → **Lindblad equation**

Quantum Brownian motion (high T) → **Lindblad equation**

$$\frac{d\rho_S(t)}{dt} = -i\big[H_{S,\mathrm{eff}}, \rho_S(t)\big] + \sum_n \Big(L_n \rho_S(t) L_n^\dagger - \frac{1}{2}\{L_n^\dagger L_n, \rho_S(t)\}\Big)$$

Wigner transform (smearing for positivity)

$$f(\boldsymbol{x},\boldsymbol{k},t) \equiv \int \frac{d^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \Big\langle \boldsymbol{k}+\frac{\boldsymbol{k}'}{2}\Big|\rho_S(t)\Big|\boldsymbol{k}-\frac{\boldsymbol{k}'}{2}\Big\rangle$$

Semiclassical (gradient expansion)

Lindblad equation (low T) → **Boltzmann equation**

Lindblad equation (high T) → **Langevin/Fokker-Planck equation**

Towards Lindblad Equation

- Assume **weak coupling** between subsystem/environment

$$H = H_S + H_E + \boxed{H_I} \qquad H_I = \sum_\alpha O_\alpha^{(S)} \otimes O_\alpha^{(E)}$$

- Expand unitary evolution operator in interaction picture

- Trace out environment —> **finite-difference equation, not well-defined differential equation**

$$\rho_S^{(\mathrm{int})}(t) = \rho_S^{(\mathrm{int})}(0) - i \int_0^t \mathrm{d}t' \sum_\alpha D_\alpha(t') \big[O_\alpha^{(S)}(t'), \rho_S^{(\mathrm{int})}(0)\big]$$

$$- \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 \frac{\mathrm{sign}(t_1 - t_2)}{2} \sum_{\alpha,\beta} D_{\alpha\beta}(t_1, t_2) \big[O_\alpha^{(S)}(t_1) O_\beta^{(S)}(t_2), \rho_S^{(\mathrm{int})}(0)\big]$$

$$+ \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 \sum_{\alpha,\beta} D_{\alpha\beta}(t_1, t_2) \Big(O_\beta^{(S)}(t_2) \rho_S^{(\mathrm{int})}(0) O_\alpha^{(S)}(t_1)$$

$$- \frac{1}{2}\big\{O_\alpha^{(S)}(t_1) O_\beta^{(S)}(t_2), \rho_S^{(\mathrm{int})}(0)\big\}\Big) + \mathcal{O}\big((tH_I^{(\mathrm{int})})^3\big)$$

$$D_\alpha(t) = \mathrm{Tr}_E\big(\rho_E\, O_\alpha^{(E)}(t)\big) \qquad D_{\alpha\beta}(t_1, t_2) = \mathrm{Tr}_E\big(\rho_E\, O_\alpha^{(E)}(t_1) O_\beta^{(E)}(t_2)\big)$$

Relevant Time Scales

$$D_{\alpha\beta}(t_1, t_2) = D_{\alpha\beta}(t_1 - t_2) = \int \frac{\mathrm{d}\omega}{2\pi}\, e^{-i\omega(t_1 - t_2)} D_{\alpha\beta}(\omega)$$

$$\mathrm{sign}(t_1 - t_2) D_{\alpha\beta}(t_1 - t_2) = i \int \frac{\mathrm{d}\omega}{2\pi}\, e^{-i\omega(t_1 - t_2)} \Sigma_{\alpha\beta}(\omega)$$

τ_E: **environment correlation time**, time domain of environment correlator

$$D_{\alpha\beta}(t_1, t_2) = \mathrm{Tr}_E\big(\rho_E\, O_\alpha^{(E)}(t_1) O_\beta^{(E)}(t_2)\big) \qquad \rho_E = \frac{e^{-\beta H_E}}{Z} \qquad \tau_E \sim \frac{1}{T}$$

If $t_1 - t_2 \gg \tau_E \sim \frac{1}{T} \sim \frac{1}{\omega}$, then the integral vanishes

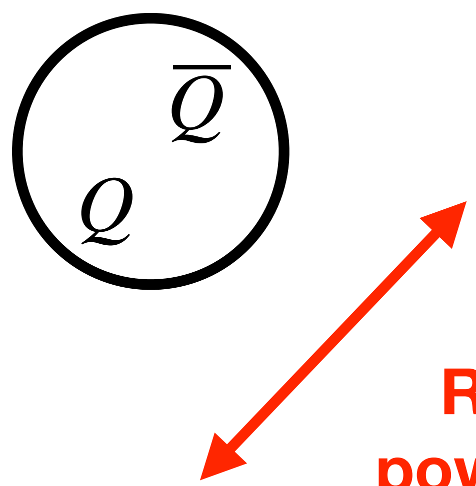
τ_S: **subsystem intrinsic time**, inverse of typical energy gap $\qquad \tau_S \sim \frac{1}{\Delta H_S}$

τ_R: **relaxation time**, depends on interaction strength between subsystem and environment

Physical Pictures of Two Hierarchies of Times

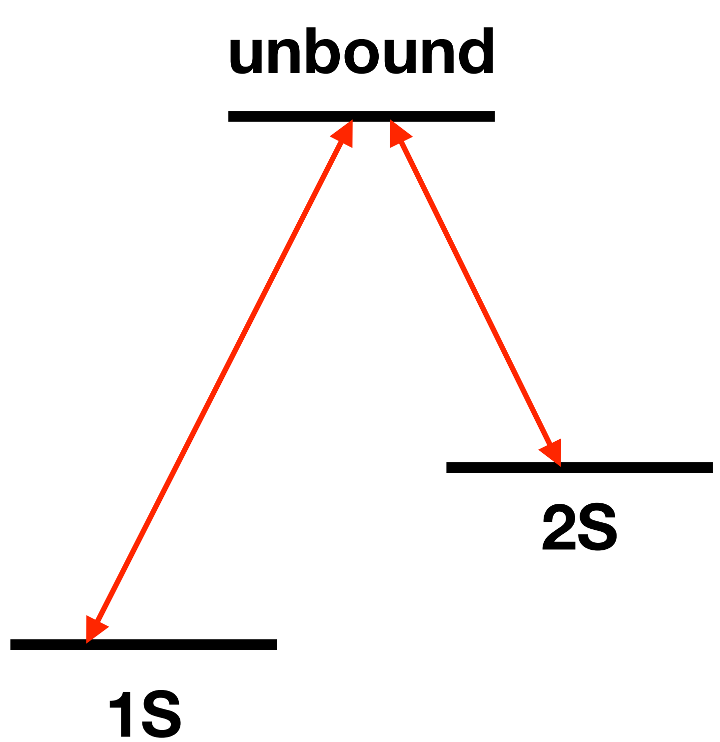
- **Quantum optical limit (low T)**

$$\tau_R \gg \tau_E,\ \tau_R \gg \tau_S$$



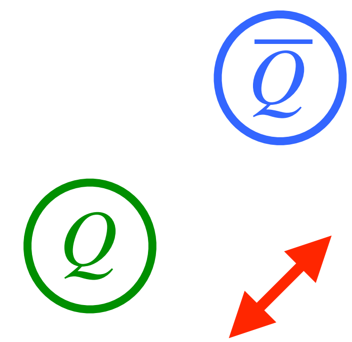
Resolving
power of QGP

Transitions between levels



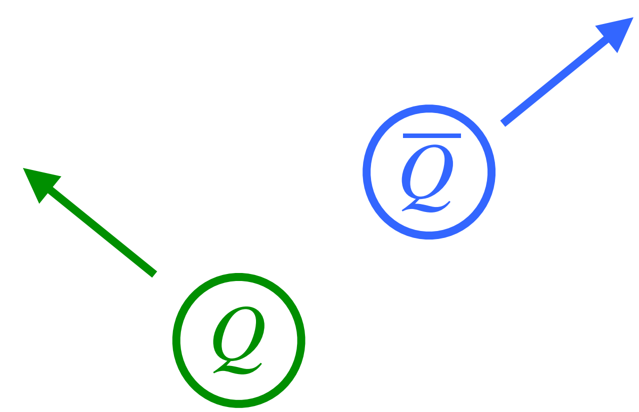
- **Quantum Brownian motion (high T)**

$$\tau_R \gg \tau_E,\ \tau_S \gg \tau_E$$



Resolving
power of QGP

Diffusion of heavy Q pair



Wavefunction decoherence
—> dissociation

Quantum Brownian Motion Limit

$$\tau_R \gg \tau_E,\ \tau_S \gg \tau_E$$

$$\rho_S^{(\mathrm{int})}(t) = \rho_S^{(\mathrm{int})}(0) - \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 \int \frac{\mathrm{d}\omega}{2\pi}\, e^{-i\omega(t_1-t_2)}\, \frac{i}{2} \sum_{\alpha,\beta} \Sigma_{\alpha\beta}(\omega) \left[O_\alpha^{(S)}(t_1) O_\beta^{(S)}(t_2), \rho_S^{(\mathrm{int})}(0)\right] + \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 \int \frac{\mathrm{d}\omega}{2\pi}\, e^{-i\omega(t_1-t_2)} \sum_{\alpha,\beta} D_{\alpha\beta}(\omega) \Big(O_\beta^{(S)}(t_2) \rho_S^{(\mathrm{int})}(0) O_\alpha^{(S)}(t_1) - \frac{1}{2}\left\{O_\alpha^{(S)}(t_1) O_\beta^{(S)}(t_2), \rho_S^{(\mathrm{int})}(0)\right\}\Big)$$

$$\int_0^t \mathrm{d}t_1\, e^{-i\omega t_1} O_\alpha^{(S)}(t_1) = \int_0^t \mathrm{d}t_1\, e^{-i\omega t_1} e^{iH_S t_1} O_\alpha^{(S)}(0)\, e^{-iH_S t_1}$$

$$\tau_E \sim \frac{1}{T} \sim \frac{1}{\omega} \qquad \tau_S \sim \frac{1}{H_S}$$

$\xrightarrow{\tau_S \gg \tau_E}$ Expand in H_S

At LO $\quad O_\alpha^{(S)}(0) \int_0^t \mathrm{d}t_1\, e^{-i\omega t_1} = O_\alpha^{(S)}(0)\, 2e^{-i\omega t/2} \dfrac{\sin(\frac{\omega t}{2})}{\omega}$

$\xrightarrow[\text{Set } t \to +\infty]{\tau_R \gg \tau_E}$ $O_\alpha^{(S)}(0) 2\pi\delta(\omega)$

Lindblad Equation in Quantum Brownian Motion Limit

$$\rho_S^{(\mathrm{int})}(t) = \rho_S^{(\mathrm{int})}(0) - t\frac{i}{2}\sum_{\alpha,\beta}\Sigma_{\alpha\beta}(\omega=0)\big[O_\alpha^{(S)}(0)O_\beta^{(S)}(0), \rho_S^{(\mathrm{int})}(0)\big]$$
$$+ t\sum_{\alpha,\beta}D_{\alpha\beta}(\omega=0)\Big(O_\beta^{(S)}(0)\rho_S^{(\mathrm{int})}(0)O_\alpha^{(S)}(0) - \frac{1}{2}\big\{O_\alpha^{(S)}(0)O_\beta^{(S)}(0), \rho_S^{(\mathrm{int})}(0)\big\}\Big)$$

Divide by t, then take $t \to 0$

$$\frac{\mathrm{d}\rho_S^{(\mathrm{int})}(t)}{\mathrm{d}t}\bigg|_{t=0} = -\frac{i}{2}\big[\sum_{\alpha,\beta}\Sigma_{\alpha\beta}(\omega=0)O_\alpha^{(S)}(0)O_\beta^{(S)}(0), \rho_S^{(\mathrm{int})}(0)\big]$$
$$+ \sum_{\alpha,\beta}D_{\alpha\beta}(\omega=0)\Big(O_\beta^{(S)}(0)\rho_S^{(\mathrm{int})}(0)O_\alpha^{(S)}(0) - \frac{1}{2}\big\{O_\alpha^{(S)}(0)O_\beta^{(S)}(0), \rho_S^{(\mathrm{int})}(0)\big\}\Big)$$

Lindblad equation in quantum Brownian motion limit at LO, NLO similarly worked out

The two limits $t \to 0$ and $t \to +\infty$ are not contradictory!

$\tau_R \gg t \gg \tau_E$ **Dynamics coarse grained**

Lindblad Equation in Quantum Optical Limit

$$\rho_S^{(\mathrm{int})}(t) = \rho_S^{(\mathrm{int})}(0) - \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 \frac{\mathrm{sign}(t_1 - t_2)}{2} \sum_{\alpha,\beta} D_{\alpha\beta}(t_1, t_2)$$

$$H_S|n\rangle = E_n|n\rangle$$

$$\times \sum_{n,m,k} \left[|n\rangle\langle n|O_\alpha^{(S)}(t_1)|m\rangle\langle m|O_\beta^{(S)}(t_2)|k\rangle\langle k|, \rho_S^{(\mathrm{int})}(0)\right]$$

$$+ \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 \sum_{\alpha,\beta} D_{\alpha\beta}(t_1, t_2) \sum_{n,m,k,l} \Big(|n\rangle\langle n|O_\beta^{(S)}(t_2)|m\rangle\langle m|\rho_S^{(\mathrm{int})}(0)|k\rangle\langle k|O_\alpha^{(S)}(t_1)|l\rangle\langle l|$$

$$- \frac{1}{2}\left\{|k\rangle\langle k|O_\alpha^{(S)}(t_1)|l\rangle\langle l|n\rangle\langle n|O_\beta^{(S)}(t_2)|m\rangle\langle m|, \rho_S^{(\mathrm{int})}(0)\right\}\Big)$$

$$\langle n|O_\alpha^{(S)}(t)|m\rangle = \langle n|e^{iH_S t} O_\alpha^{(S)} e^{-iH_S t}|m\rangle = e^{i(E_n - E_m)t}\langle n|O_\alpha^{(S)}|m\rangle$$

$$\tau_R \gg t \gg \tau_E, \tau_S$$

$$\left(\lim_{t\to+\infty} 2e^{-i(\omega - E_k + E_l)t/2} \frac{\sin(\frac{(\omega - E_k + E_l)t}{2})}{\omega - E_k + E_l}\right) 2e^{i(\omega + E_n - E_m)t/2} \frac{\sin(\frac{(\omega + E_n - E_m)t}{2})}{\omega + E_n - E_m}$$

$$= 2\pi\delta(\omega - E_k + E_l)\, e^{i(E_k - E_l + E_n - E_m)t/2} \frac{2\sin(\frac{(E_k - E_l + E_n - E_m)t}{2})}{E_k - E_l + E_n - E_m}$$

Lindblad Equation in Quantum Optical Limit

If eigenenergies are discrete: $2\pi\delta(\omega - E_k + E_l)\, t\, \delta_{E_k - E_l, E_m - E_n}$

$$\frac{\mathrm{d}\rho_S^{(\mathrm{int})}(t)}{\mathrm{d}t}\bigg|_{t=0} = -i \sum_{n,k} \sigma_{nk} \Big[|n\rangle\langle k|, \rho_S^{(\mathrm{int})}(0) \Big]$$

$$+ \sum_{n,m,k,l} \gamma_{nm,kl} \Big(|n\rangle\langle m|\rho_S^{(\mathrm{int})}(0)|k\rangle\langle l| - \frac{1}{2}\big\{ |k\rangle\langle l|n\rangle\langle m|, \rho_S^{(\mathrm{int})}(0) \big\} \Big)$$

$$\sigma_{nk} = \frac{1}{2}\sum_{\alpha,\beta}\sum_m \Sigma_{\alpha\beta}(E_n - E_m)\delta_{E_n,E_k}\langle n|O_\alpha^{(S)}|m\rangle\langle m|O_\beta^{(S)}|k\rangle$$

$$\gamma_{nm,kl} = \sum_{\alpha,\beta} D_{\alpha\beta}(E_m - E_n)\delta_{E_k - E_l, E_m - E_n}\langle k|O_\alpha^{(S)}|l\rangle\langle n|O_\beta^{(S)}|m\rangle$$

Subtlety when applying to quarkonium: octet heavy quark pair has continuous energy spectrum, time scale hierarchy may be violated, Lindblad equation in quantum optical limit may not exist!

However, in semiclassical limit, the issue can be solved by gradient expansion

Review: XY, 2102.01736

Boltzmann equation of quarkonium dissociation and regeneration exists!

Separation of Scales and NREFT

- **Separation of scales**

$v^2 \sim 0.3$ charmonium

$v^2 \sim 0.1$ bottomonium

$$M \gg Mv \gg Mv^2,\ \Lambda_{QCD}$$

QCD

Hard M — perturbative matching

HQET/NRQCD

Heavy quark physics, A.Manohar, M.Wise

hep-ph/9407339, G.Bodwin, E.Braaten, G.Lepage

Soft Mv — perturbative / non-perturbative matching

potential NRQCD

hep-ph/9907240, hep-ph/0410047,
N.Brambilla A.Pineda, J.Soto, A.Vairo

Ultrasoft

$Mv^2 \sim 500\ \mathrm{MeV}$

Different descriptions depending on where T fits into the hierarchy

High Temperature: NRQCD $M \gg T \gg Mv^2$

- **Lindblad equation in limit of quantum Brownian motion**

NRQCD motivated Hamiltonian

$$H = \frac{\hat{\boldsymbol{p}}_Q^2}{2M} + \frac{\hat{\boldsymbol{p}}_{\bar{Q}}^2}{2M} + H_{q+A} + \int d^3x \big(\delta^3(\boldsymbol{x} - \hat{\boldsymbol{x}}_Q)T_F^a - \delta^3(\boldsymbol{x} - \hat{\boldsymbol{x}}_{\bar{Q}})T_F^{*a}\big) g A_0^a(\boldsymbol{x})$$

Lindblad equation

$$\frac{d\rho_S(t)}{dt} = -i\big[H_S + \Delta H_S, \rho_S(t)\big] + \frac{1}{N_c^2 - 1}\int \frac{d^3q}{(2\pi)^3} D^{>}(q_0 = 0, \boldsymbol{q})$$
$$\times \Big(\widetilde{O}^a(\boldsymbol{q})\rho_S(t)\widetilde{O}^{a\dagger}(\boldsymbol{q}) - \frac{1}{2}\big\{\widetilde{O}^{a\dagger}(\boldsymbol{q})\widetilde{O}^a(\boldsymbol{q}), \rho_S(t)\big\}\Big)$$

Zero frequency

Environment correlator $D^{>ab}(x_1, x_2) = g^2 \mathrm{Tr}_E\big(\rho_E A_0^a(t_1, \boldsymbol{x}_1) A_0^b(t_2, \boldsymbol{x}_2)\big)$

$$\widetilde{O}^a(\boldsymbol{q}) = e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_Q}\Big(1 - \frac{\boldsymbol{q}\cdot\hat{\boldsymbol{p}}_Q}{4MT}\Big)e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_Q}T_F^a - e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_{\bar{Q}}}\Big(1 - \frac{\boldsymbol{q}\cdot\hat{\boldsymbol{p}}_{\bar{Q}}}{4MT}\Big)e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_{\bar{Q}}}T_F^{*a}$$

Dissipation effect, important for thermalization, from expanding βH_S

Solving this Lindblad equation expensive at 3D

J.-P. Blaizot, M.A.Escobedo, 1711.10812

T.Miura, Y.Akamatsu, M.Asakawa, A.Rothkopf, 1908.06293

Intermediate Temperature: pNRQCD $Mv \gg T \gg Mv^2$

- **Lindblad equation in limit of quantum Brownian motion**

pNRQCD motivated Hamiltonian

N.Brambilla, M.A.Escobedo, M.Strickland, A.Vairo, P.V.Griend, J.H.Weber, 2012.01240, 2205.10289

$$H_S = \frac{\boldsymbol{p}_{\rm rel}^2}{M} - \frac{C_F \alpha_s}{r}|s\rangle\langle s| + \frac{\alpha_s}{2N_c r}|a\rangle\langle a|$$

$$H_I = r_i \left(\sqrt{\frac{T_F}{N_c}} \left(|s\rangle\langle a| + |a\rangle\langle s|\right) + \frac{1}{2} d^{abc} |b\rangle\langle c| \right) g\widetilde{E}_i^a(\boldsymbol{R}=0)$$

$$\frac{\mathrm{d}\rho_S(t)}{\mathrm{d}t} = -i\left[H_S + \Delta H_S,\, \rho_S(t)\right] + \frac{D(\omega=0, \boldsymbol{R}=0)}{N_c^2-1}\left(\widetilde{O}_i^a \rho_S(t) \widetilde{O}_i^{a\dagger} - \frac{1}{2}\left\{\widetilde{O}_i^{a\dagger}\widetilde{O}_i^a,\, \rho_S(t)\right\}\right)$$

$$\Delta H_S = \frac{1}{2}\Sigma_{ij}^{ab}(\omega=0, \boldsymbol{R}=0)\, O_i^a O_j^b$$

$$\widetilde{O}_i^a = O_i^a - \frac{1}{4T}\left[H_S,\, O_i^a\right] = \sqrt{\frac{T_F}{N_c}}\left(r_i + \frac{1}{2MT}\nabla_i + \frac{N_c}{8T}\frac{\alpha_s r_i}{r}\right)|s\rangle\langle a| + \sqrt{\frac{T_F}{N_c}}\left(r_i + \frac{1}{2MT}\nabla_i - \frac{N_c}{8T}\frac{\alpha_s r_i}{r}\right)|a\rangle\langle s| + \frac{1}{2}\left(r_i + \frac{1}{2MT}\nabla_i\right) d^{abc}|b\rangle\langle c|$$

Low Temperature: pNRQCD $Mv \gg Mv^2 \gtrsim T$

- **The potential NRQCD and quantum optical limit**

$$\mathcal{L}_{\mathrm{pNRQCD}} = \int d^3r\, \mathrm{Tr}\Big(\mathrm{S}^\dagger(i\partial_0 - H_s)\mathrm{S} + \mathrm{O}^\dagger(iD_0 - H_o)\mathrm{O} + \boxed{V_A(\mathrm{O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.})} + \frac{V_B}{2}\mathrm{O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, \mathrm{O}\} + \cdots\Big)$$

D.o.f. heavy quark pairs in color singlet (S) or octet (O)

Bound/unbound transition —> singlet/octet transition

Dipole interaction $r \sim \frac{1}{Mv}$

When at rest in medium, $rT \sim v$ suppressed

Weak coupling between quarkonium and QGP: quarkonium small in size

At leading (nontrivial) order in v, sum all interactions not suppressed

Low Temperature: pNRQCD $Mv \gg Mv^2 \gtrsim T$

$$\rho_S(t) = \rho_S(0) - i\Big[tH_S + \sum_{a,b}\sigma_{ab}(t)L_{ab}, \rho_S(0)\Big] + \sum_{a,b,c,d}\gamma_{ab,cd}\Big(L_{ab}\rho_S(0)L^\dagger_{cd} - \frac{1}{2}\{L^\dagger_{cd}L_{ab}, \rho_S\}\Big)$$

Static screening

- **Boltzmann equation**

$$\frac{\partial}{\partial t}f_{nl}(\boldsymbol{x},\boldsymbol{k},t) + \frac{\boldsymbol{k}}{2M}\cdot\nabla_{\boldsymbol{x}}f_{nl}(\boldsymbol{x},\boldsymbol{k},t) = \mathcal{C}^+_{nl}(\boldsymbol{x},\boldsymbol{k},t) - \mathcal{C}^-_{nl}(\boldsymbol{x},\boldsymbol{k},t)$$

Recombination Dissociation

T.Mehen, XY, 1811.07027, 2009.02408

$$\mathcal{C}^-_{nl}(\boldsymbol{x},\boldsymbol{k},t) = g^2\frac{T_F}{N_c}\sum_{i_1,i_2}\int\frac{d^3p_{\rm cm}}{(2\pi)^3}\frac{d^3p_{\rm rel}}{(2\pi)^3}\frac{d^4q}{(2\pi)^4}(2\pi)^4\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\rm cm}+\boldsymbol{q})\delta\Big(E_{nl}-\frac{p^2_{\rm rel}}{M}-q_0\Big)$$
$$\times\langle\psi_{nl}|r_{i_1}|\Psi_{\boldsymbol{p}_{\rm rel}}\rangle\langle\Psi_{\boldsymbol{p}_{\rm rel}}|r_{i_2}|\psi_{nl}\rangle\,[g_E^{++}]^>_{i_1i_2}(q_0,\boldsymbol{q})f_{nl}(\boldsymbol{x},\boldsymbol{k},t)$$

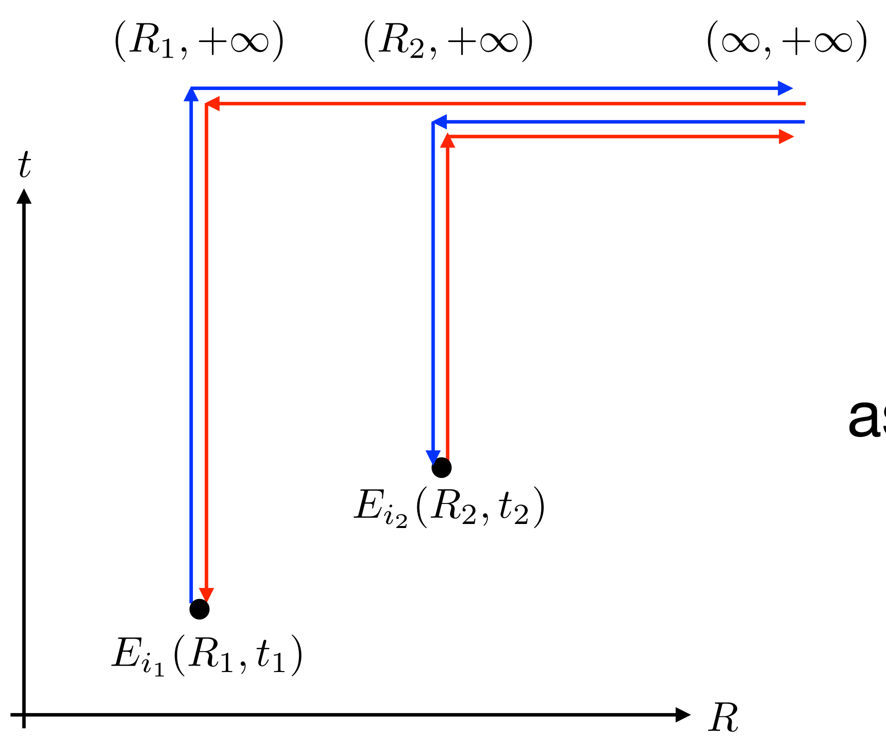
$$\mathcal{C}^+_{nl}(\boldsymbol{x},\boldsymbol{k},t) = g^2\frac{T_F}{N_c}\sum_{i_1,i_2}\int\frac{d^3p_{\rm cm}}{(2\pi)^3}\frac{d^3p_{\rm rel}}{(2\pi)^3}\frac{d^4q}{(2\pi)^4}(2\pi)^4\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\rm cm}-\boldsymbol{q})\delta\Big(E_{nl}-\frac{p^2_{\rm rel}}{M}+q_0\Big)$$
$$\times\langle\psi_{nl}|r_{i_1}|\Psi_{\boldsymbol{p}_{\rm rel}}\rangle\langle\Psi_{\boldsymbol{p}_{\rm rel}}|r_{i_2}|\psi_{nl}\rangle\,[g_E^{--}]^>_{i_2i_1}(q_0,\boldsymbol{q})f_O(\boldsymbol{x},\boldsymbol{p}_{\rm cm},\boldsymbol{r}=0,\boldsymbol{p}_{\rm rel},t)$$

See also XY, B.Mueller 1811.09644 for diagrammatic approach

Chromoelectric Field Correlator

$$[g_E^{++}]^{>}_{ji}(y,x) \equiv \Big\langle \big[E_j(y)\mathcal{W}_{[(y^0,\boldsymbol{y}),(+\infty,\boldsymbol{y})]}\mathcal{W}_{[(+\infty,\boldsymbol{y}),(+\infty,\boldsymbol{\infty})]}\big]^a \times \big[\mathcal{W}_{[(+\infty,\boldsymbol{\infty}),(+\infty,\boldsymbol{x})]}\mathcal{W}_{[(+\infty,\boldsymbol{x}),(x^0,\boldsymbol{x})]}E_i(x)\big]^a \Big\rangle_T$$

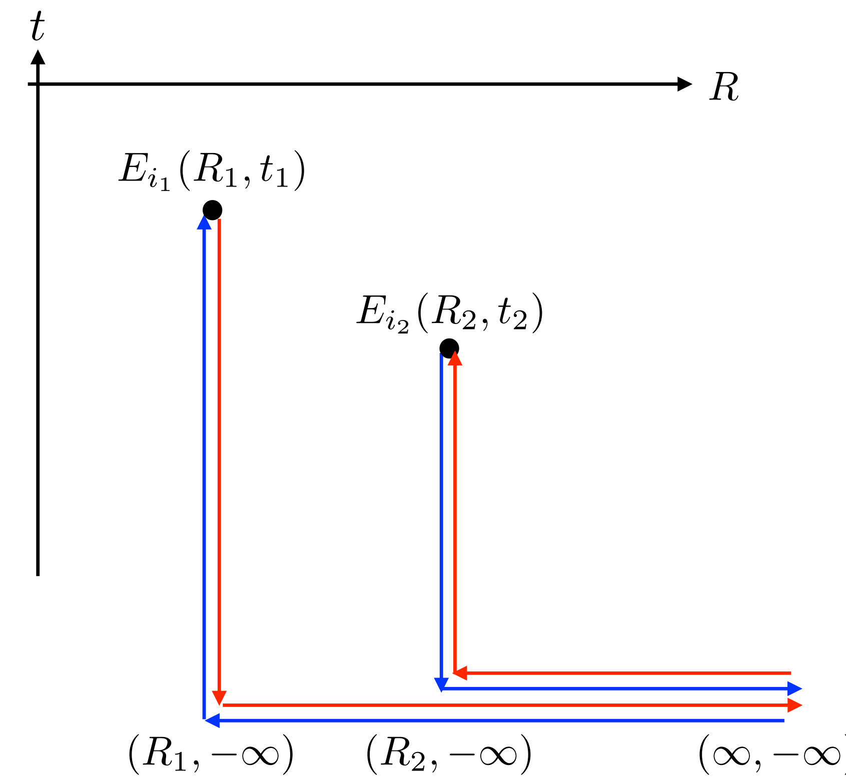
$$[g_E^{--}]^{>}_{ji}(y,x) \equiv \Big\langle \big[\mathcal{W}_{[(-\infty,\boldsymbol{\infty}),(-\infty,\boldsymbol{y})]}\mathcal{W}_{[(-\infty,\boldsymbol{y}),(y^0,\boldsymbol{y})]}E_j(y)\big]^a \times \big[E_i(x)\mathcal{W}_{[(x^0,\boldsymbol{x}),(-\infty,\boldsymbol{x})]}\mathcal{W}_{[(-\infty,\boldsymbol{x}),(-\infty,\boldsymbol{\infty})]}\big]^a \Big\rangle_T$$



PT transformation, assume state invariant

⟷

KMS relation



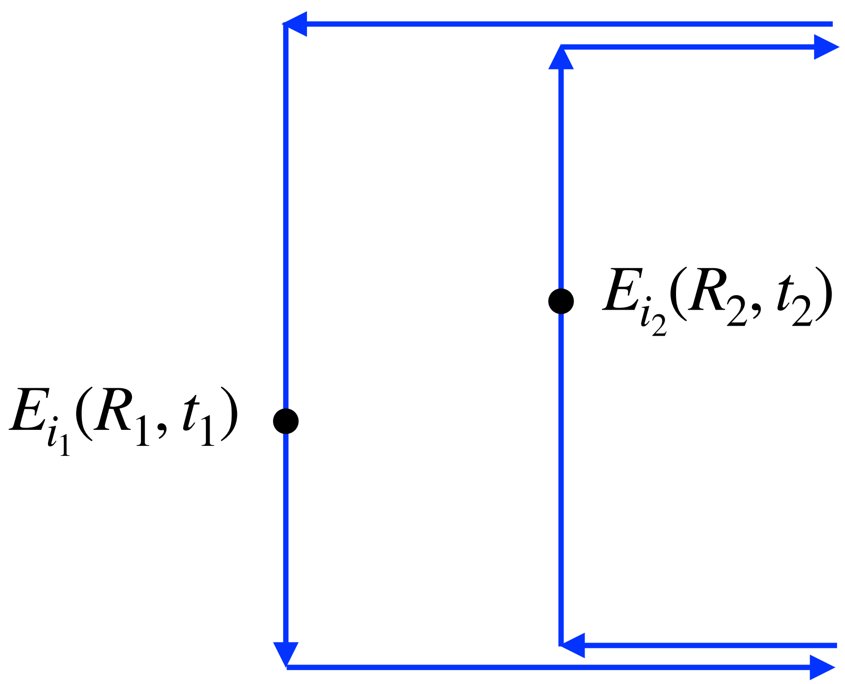
Dissociation: final-state interaction

Recombination: initial-state interaction

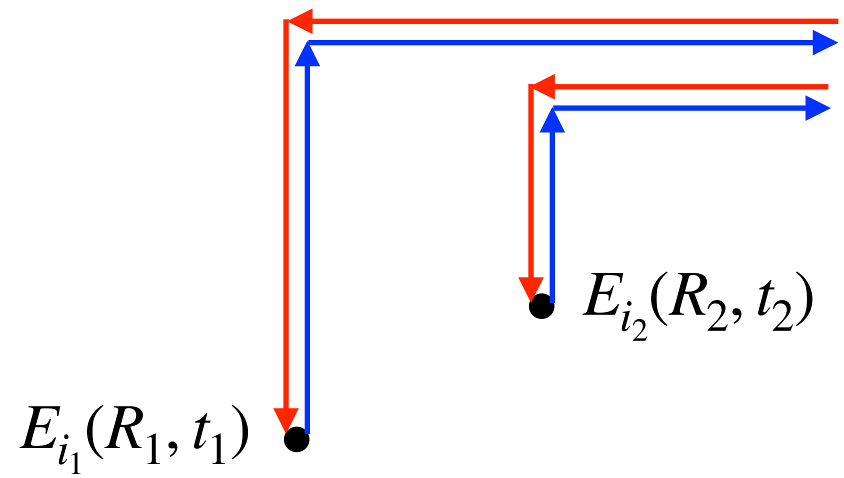
For total reaction rates, integrating over final momentum gives setting $R_1 \to R_2$, the correlator becomes momentum independent

Chromoelectric Field Correlator

- **Relation to the correlator defining heavy quark diffusion coefficient**



Single heavy quark



Heavy quark antiquark pair

- **At NLO: temperature-dependent parts of spectral functions agree**
  vacuum parts differ by a constant

$$\frac{149}{36} - \frac{2}{3}\pi^2$$

Y.Burnier, M.Laine, J.Langelage, L.Mether, 1006.0867

$$\frac{149}{36} + \frac{1}{3}\pi^2$$

T.Binder, K.Mukaida, B.Scheihing-Hitschfeld, XY, 2107.03945

M.Eidemuller, M.Jamin, hep-ph/9709419 (only vacuum)

III. Applications

Coupled Transport Equations of Heavy Flavors

Open heavy quark antiquark

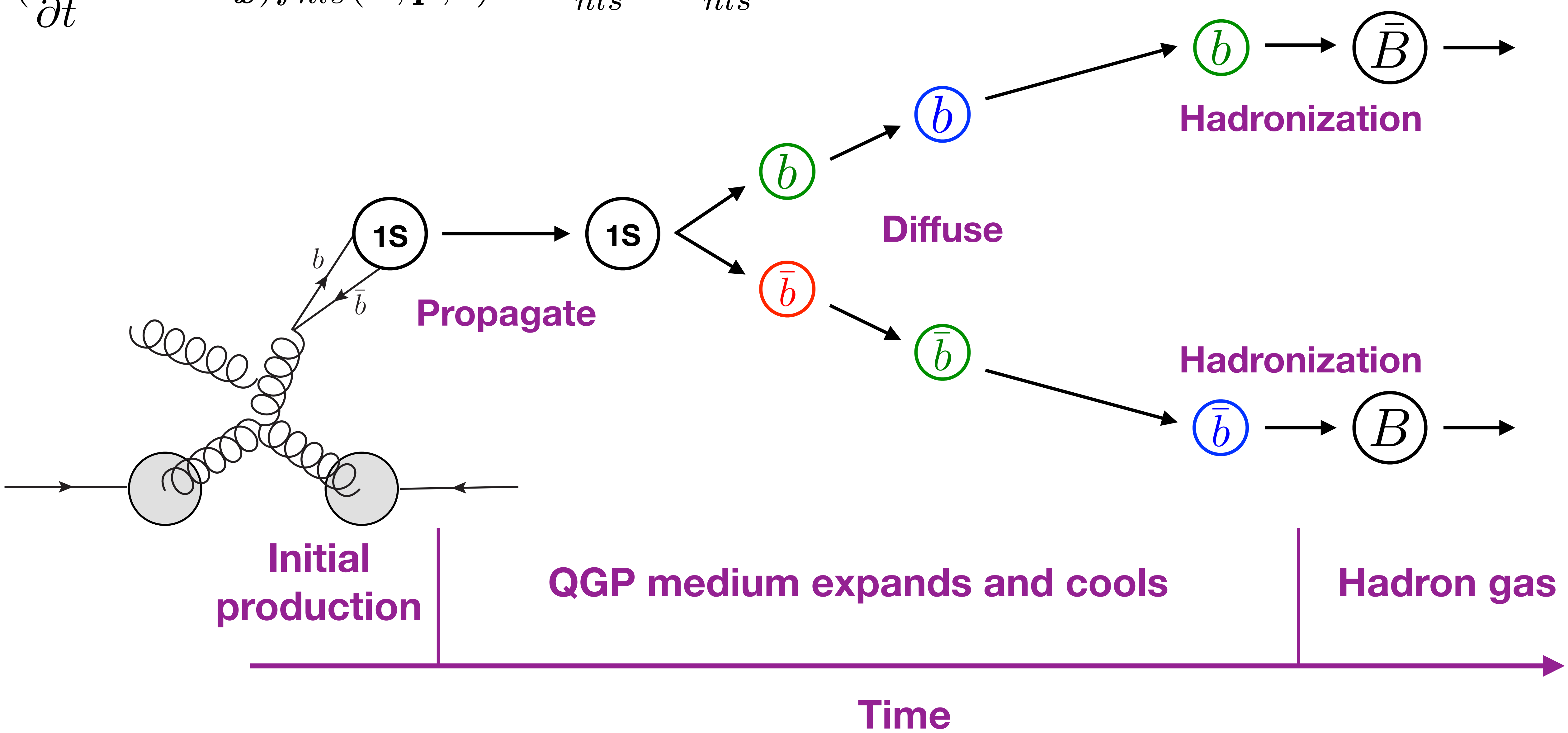
$C_{Q\bar{Q}}$: HQ scattering; +: recombination; −: dissociation

$$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}}_Q \cdot \nabla_{\boldsymbol{x}_Q} + \dot{\boldsymbol{x}}_{\bar{Q}} \cdot \nabla_{\boldsymbol{x}_{\bar{Q}}}) f_{Q\bar{Q}}(\boldsymbol{x}_Q, \boldsymbol{p}_Q, \boldsymbol{x}_{\bar{Q}}, \boldsymbol{p}_{\bar{Q}}, t) = \mathcal{C}_{Q\bar{Q}} - \mathcal{C}^{+}_{Q\bar{Q}} + \mathcal{C}^{-}_{Q\bar{Q}}$$

Each quarkonium state, nl = 1S, 2S,1P etc.

XY, W.Ke, Y.Xu, S.A.Bass, B.Mueller, 2004.06746

$$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}^{+}_{nls} - \mathcal{C}^{-}_{nls}$$



Coupled Transport Equations of Heavy Flavors

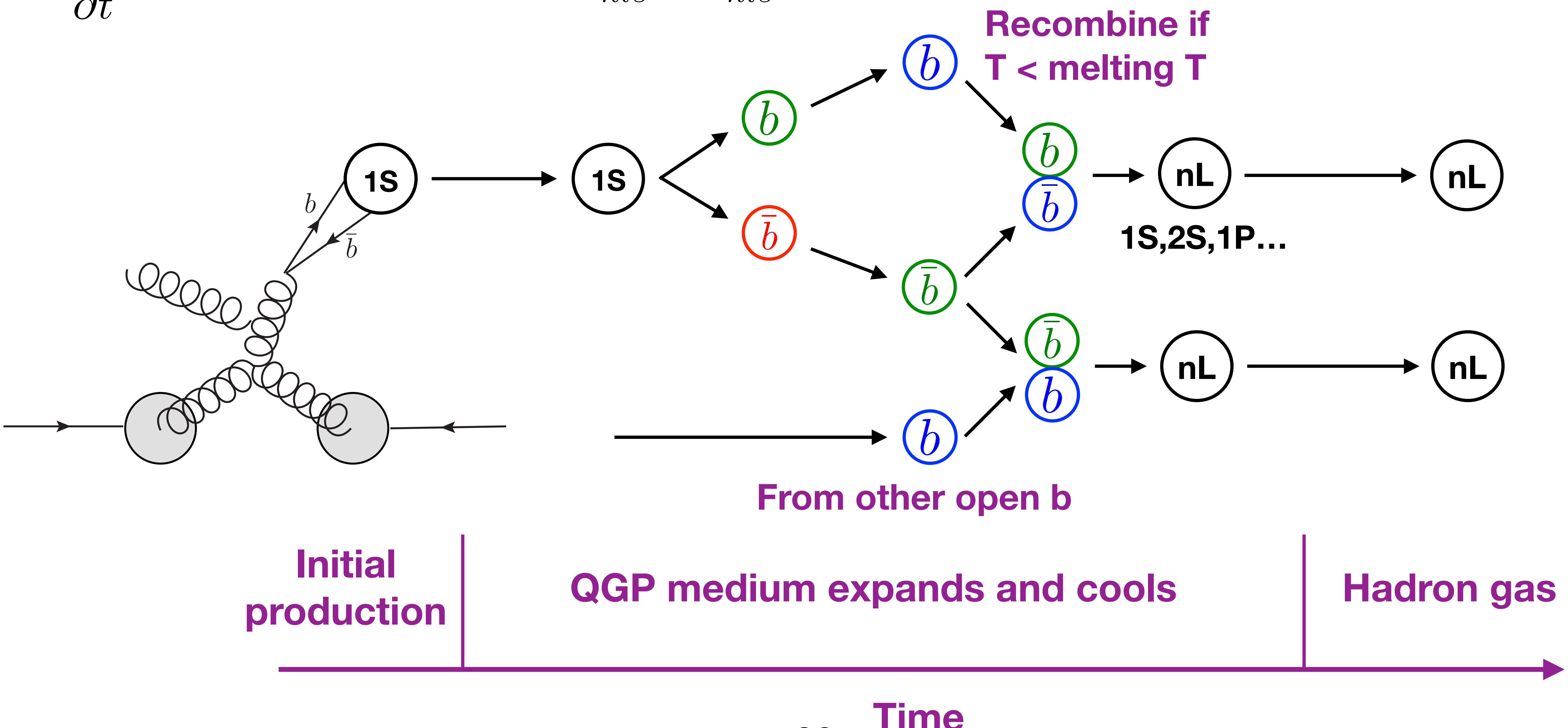
Open heavy quark antiquark

$C_{Q\bar{Q}}$: HQ scattering; +: recombination; −: dissociation

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}}_Q \cdot \nabla_{\boldsymbol{x}_Q} + \dot{\boldsymbol{x}}_{\bar{Q}} \cdot \nabla_{\boldsymbol{x}_{\bar{Q}}}\right) f_{Q\bar{Q}}(\boldsymbol{x}_Q, \boldsymbol{p}_Q, \boldsymbol{x}_{\bar{Q}}, \boldsymbol{p}_{\bar{Q}}, t) = \mathcal{C}_{Q\bar{Q}} - \mathcal{C}^{+}_{Q\bar{Q}} + \mathcal{C}^{-}_{Q\bar{Q}}$$

Each quarkonium state, nl = 1S, 2S,1P etc.

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}\right) f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}^{+}_{nls} - \mathcal{C}^{-}_{nls}$$



Coupled Transport Equations of Heavy Flavors

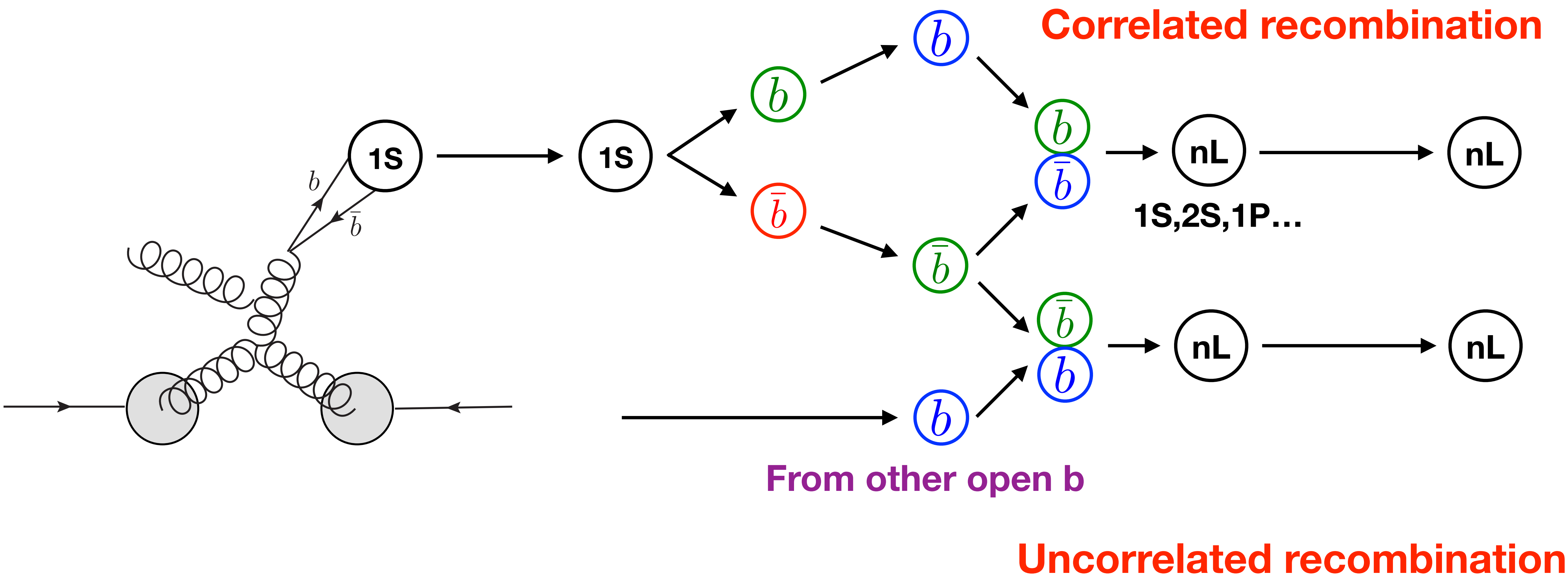
Open heavy quark antiquark

$C_{Q\bar{Q}}$: HQ scattering; +: recombination; −: dissociation

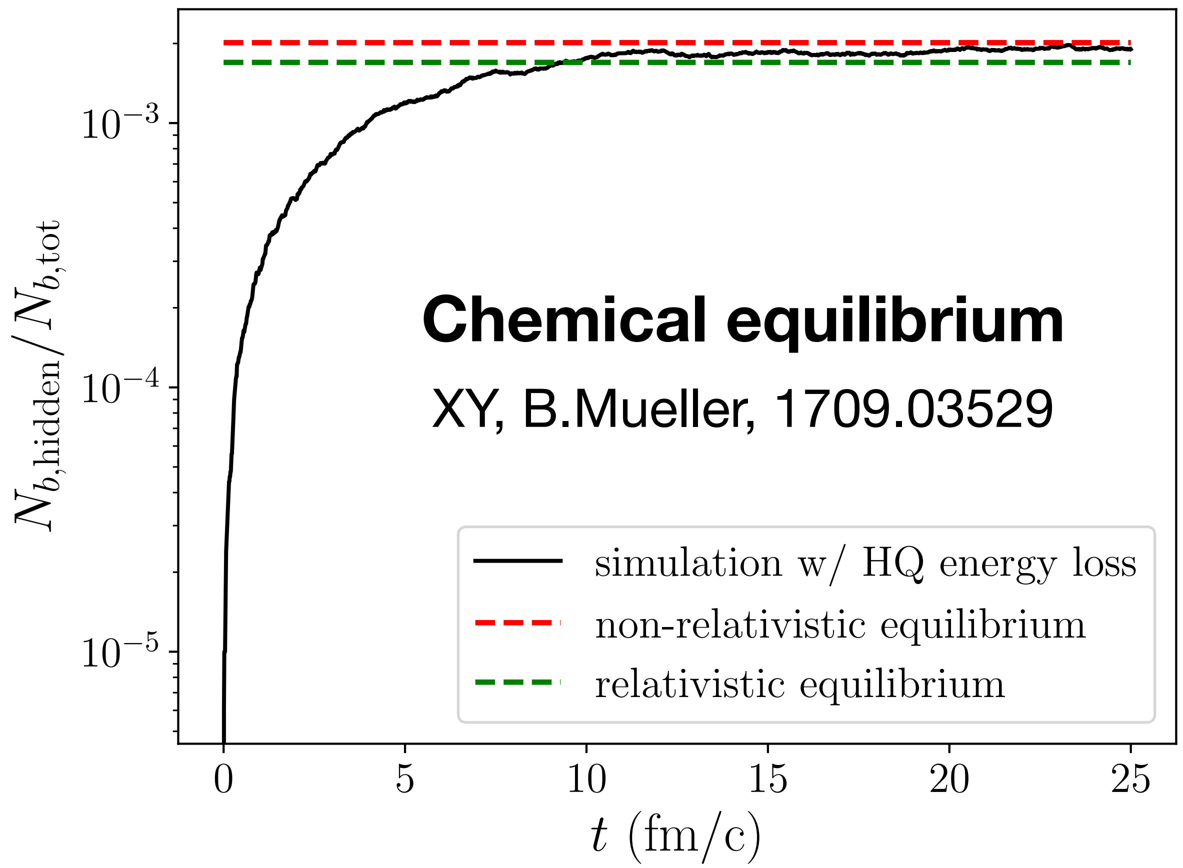
$$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}}_Q \cdot \nabla_{\boldsymbol{x}_Q} + \dot{\boldsymbol{x}}_{\bar{Q}} \cdot \nabla_{\boldsymbol{x}_{\bar{Q}}}) f_{Q\bar{Q}}(\boldsymbol{x}_Q, \boldsymbol{p}_Q, \boldsymbol{x}_{\bar{Q}}, \boldsymbol{p}_{\bar{Q}}, t) = \mathcal{C}_{Q\bar{Q}} - \mathcal{C}^+_{Q\bar{Q}} + \mathcal{C}^-_{Q\bar{Q}}$$

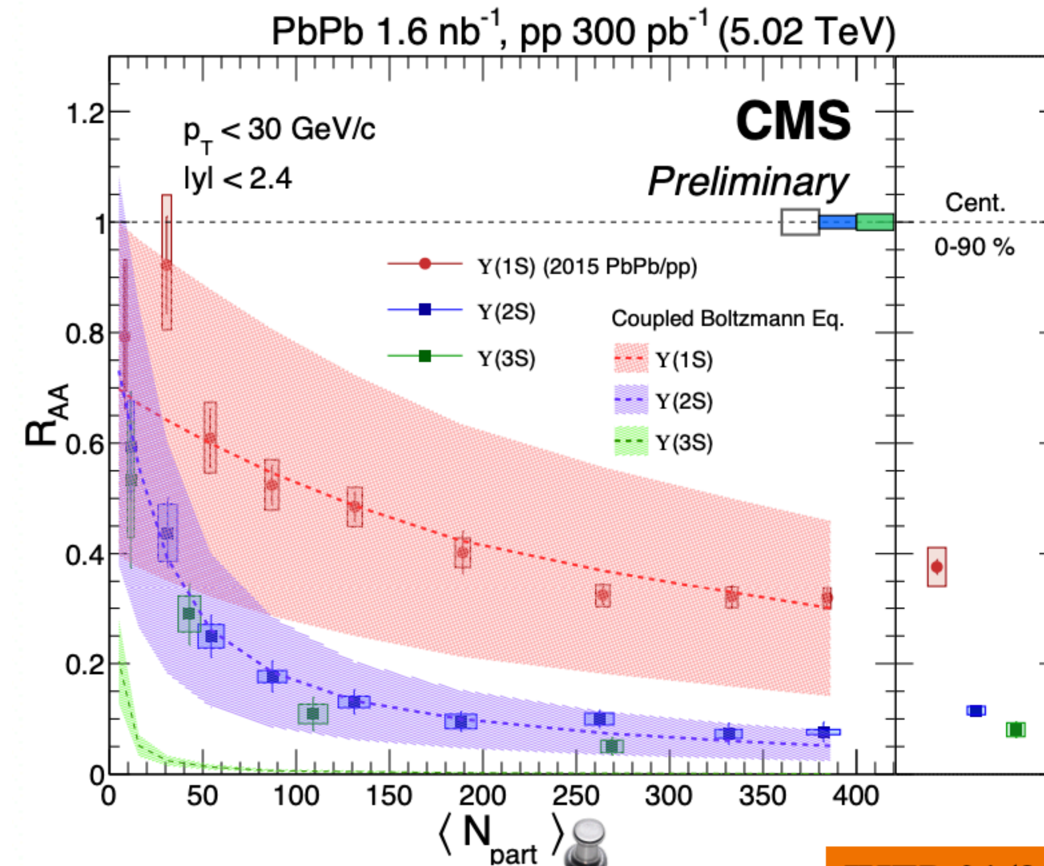
Each quarkonium state, nl = 1S, 2S,1P etc.

$$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}^+_{nls} - \mathcal{C}^-_{nls}$$



Thermalization and Phenomenology





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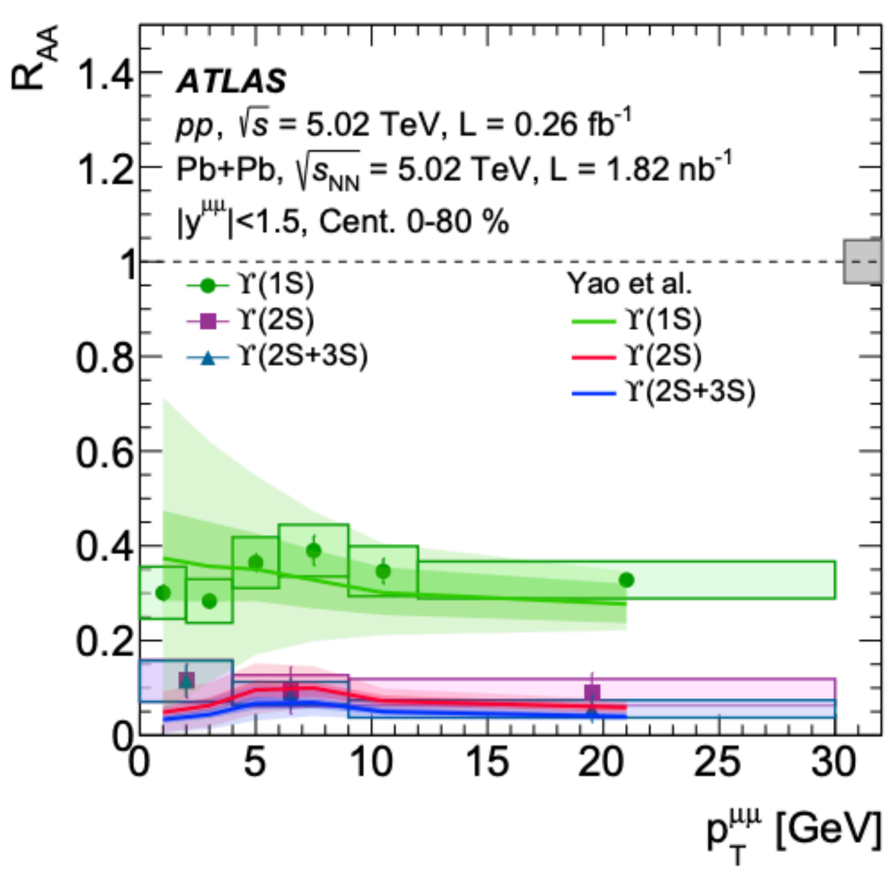
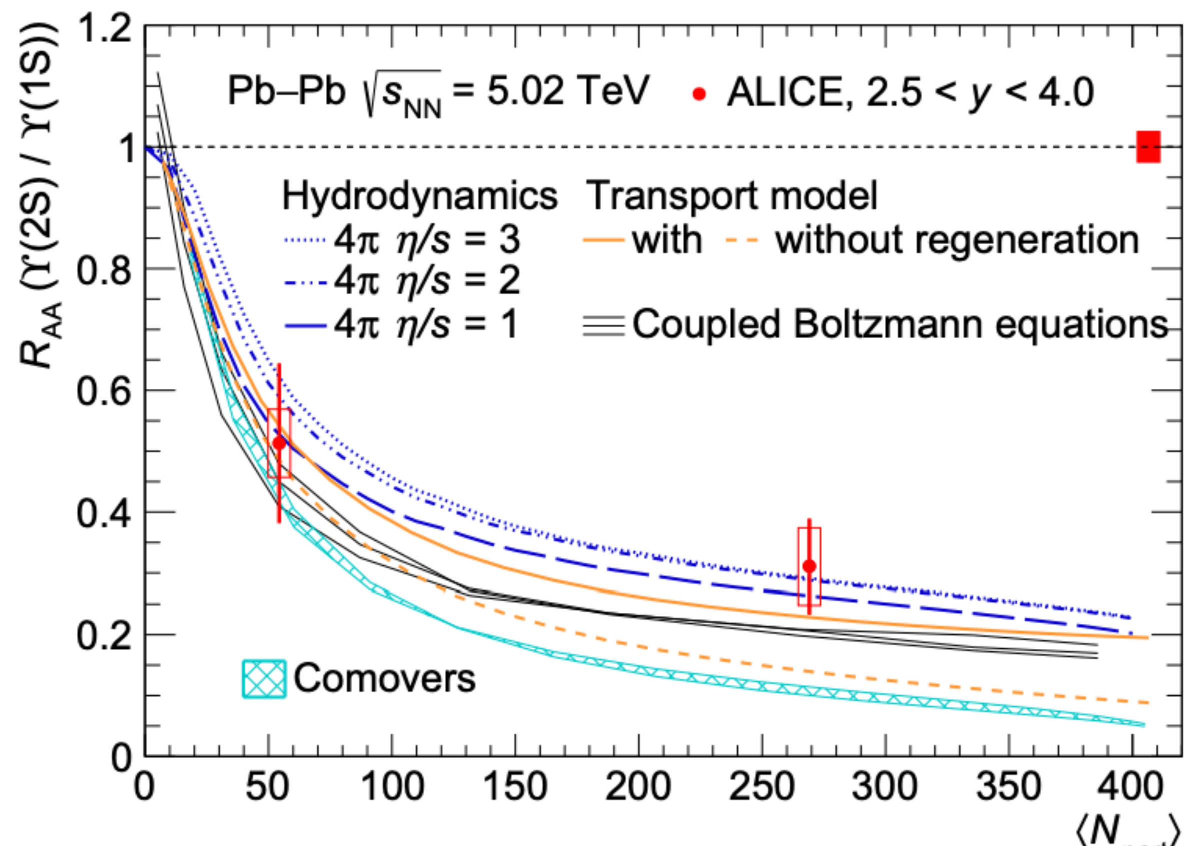
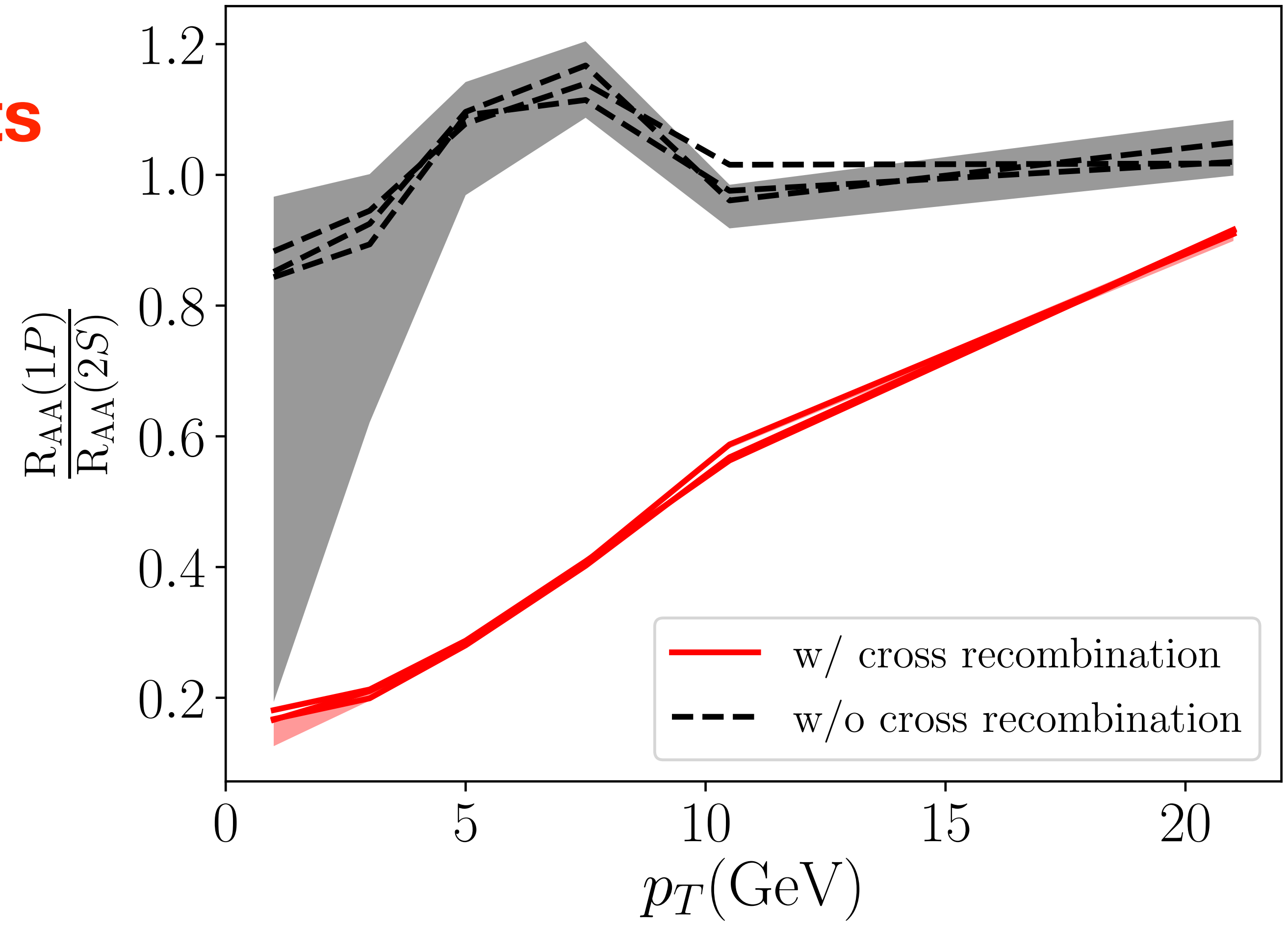




Experimental Test of Correlated Recombination

Correlated recombination predicts 1P more suppressed than 2S



Traditional sequential suppression argument based on hierarchy of binding energy or size —> R_{AA}(2S) ~ R_{AA}(1P), since their binding energies are close

Correlated recombination rates (2S—>1P) ~ (1P—>2S) because of similar binding energy, but primordial production cross section

$$\frac{\sigma_{1P}}{\sigma_{2S}} \sim 4.5$$

Conclusion

- Open quantum system framework for quarkonium transport in quark-gluon plasma
  - Lindblad equations in two limits
  - Hierarchy of time scales, EFT, semiclassical counterparts
- Chromoelectric correlators, nonperturbative calculation
- Coupled Boltzmann equations

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