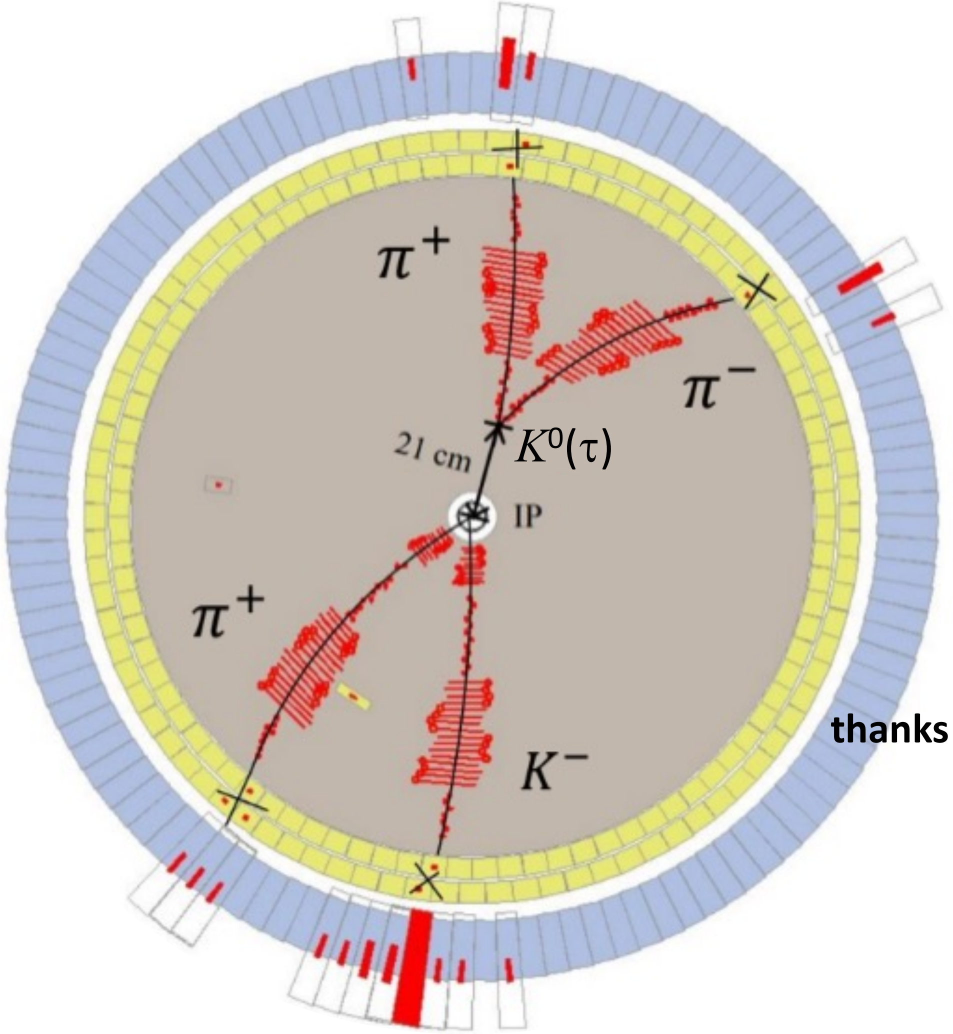
Testing CPT with neutral kaons from J/ψ decays



thanks to Jian-Yu Zhang

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Why CPT?

any Lorentz invariant local quantum field theory with a Hermitian Hamiltonian must have CPT symmetry.

what theory is a Lorentz invariant local quantum field theory with a Hermitian Hamiltonian?

the Standard Model

Why Kaons?

1) History:

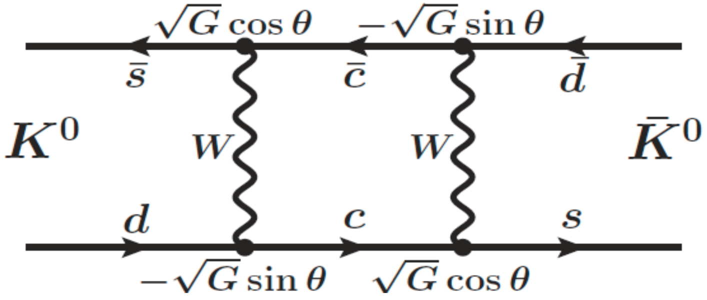
1956 kaons taught us that Parity is not conserved (Lee Yang Nobel prize)

1964 kaons taught us that CP is not conserved (Fitch-Cronin Nobel prize)

202? kaons will teach us that CPT is not conserved (???? Nobel prize)

2) Opportunity:

Kaons have this beautiful diagram that allow 2nd-order Weak Interaction effects influence 1st-order W.I. processes



3) Technology:

SCTF (or a specialized J/ψ factory?) will provide billion-event samples of high-purity strangeness-tagged neutral kaon decays (~100x previous kaon experiments).

CPT test with neutral kaon decays to two pions

-- in 3 easy steps --

4 CP parameters

(complex numbers)

h_{+-} η_{00} ε ε'

2 auxiliary parameters

(real numbers)

ΔM_K $\Delta\Gamma_K=\Gamma_{K_S}-\Gamma_{K_L}$

CPT test with neutral kaon decays to two pions

-- in 3 easy steps --

4 CP parameters

(complex numbers)

h_{+-} η_{00} ε ε'

1. measure these

2 auxiliary parameters

(real numbers)

ΔM_K $\Delta\Gamma_K = \Gamma_{K_S} - \Gamma_{K_L}$

and these

CPT test with neutral kaon decays to two pions

-- in 3 easy steps --

4 CP parameters
(complex numbers)

$\eta_{+-} \quad \eta_{00} \quad \varepsilon \quad \varepsilon'$

1. measure these

2 auxiliary parameters
(real numbers)

$\Delta M_K \quad \Delta\Gamma_K = \Gamma_{K_S} - \Gamma_{K_L}$

and these

2. compare

CPT: $\phi_{+-} = \phi_{00} = \phi_{\varepsilon} = \arctan \dfrac{2\Delta M_K}{\Delta\Gamma_K}$

CPT test with neutral kaon decays to two pions

-- in 3 easy steps --

4 CP parameters
(complex numbers)

h_{+-} η_{00} ε ε'

1. measure these

2 auxiliary parameters
(real numbers)

ΔM_K $\Delta\Gamma_K = \Gamma_{K_S} - \Gamma_{K_L}$

and these

2. compare **CPT:** $\phi_{+-} = \phi_{00} = \phi_\varepsilon = \text{arctan}\,\frac{2\Delta M_K}{\Delta\Gamma_K}$

3. if they are not equal **call Yifeng**

two issues

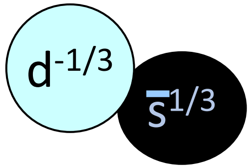
1. how valid is the relation **CPT:** $\phi_{+-} = \phi_{00} = \phi_{\varepsilon} = \arctan \frac{2\Delta M_K}{\Delta\Gamma_K}$

2. how to measure ϕ_{+-} (& ϕ_{00}?) with sub-0.1° precision

review of 60 year-old physics

neutral K mesons: K^0, $\bar{K}^0$

K^0:



d $\bar{s}$

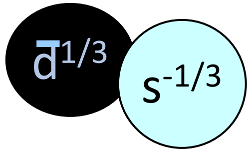
$Q = -1/3 + 1/3 = 0$

$\mathcal{S} = 0 + 1 = 1$

$\mathcal{S}(\bar{s}\text{-quark})=+1$

antiparticles

$\bar{K}^0$:



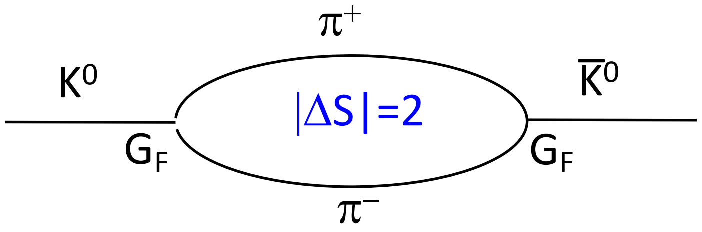
s $\bar{d}$

$Q = -1/3 + 1/3 = 0$

$\mathcal{S} = -1 + 0 = -1$

$\mathcal{S}(s\text{-quark})=-1$

K^0 & $\bar{K}^0$ only differ by strangeness; but strangeness is not conserved



$$\langle \bar{K}^0 | H | K^0 \rangle \neq 0$$

Some conventions

$$\mathcal{C}\left|\pi^0\right\rangle = +\left|\pi^0\right\rangle$$
$$\mathcal{C}P\left|\pi^0\right\rangle = -\left|\pi^0\right\rangle$$

fixed by Maxwell's equations

$$\mathcal{C}\left|\pi^+\right\rangle = +\left|\pi^-\right\rangle$$

arbitrary choices

$$\mathcal{C}\left|K^0\right\rangle = -\left|\bar{K}^0\right\rangle$$

$$\mathcal{C}P\left|\pi^+\right\rangle = -\left|\pi^-\right\rangle$$

$$\mathcal{C}P\left|K^0\right\rangle = +\left|\bar{K}^0\right\rangle$$

Two-state system with decays

-- Weisskopf-Wigner formulation of the Schrodinger equation --

not Hermitian

Hermitian

this is **not** a $\mathcal{CP}$-violating phase

Γ accounts for decays

$$H = M - \frac{i}{2}\Gamma$$

Mass matrix

Decay matrix

$$\langle K_j | H | K_i \rangle = \langle K_j | M | K_i \rangle - \tfrac{i}{2}\langle K_j | \Gamma | K_i \rangle = M_{ij} - \tfrac{i}{2}\Gamma_{ij} = X_{ij}$$

$$\boldsymbol{H} = \begin{pmatrix} \langle K^0 | H | K^0 \rangle & \langle K^0 | H | \overline{K}^0 \rangle \\ \langle \overline{K}^0 | H | K^0 \rangle & \langle \overline{K}^0 | H | \overline{K}^0 \rangle \end{pmatrix} = \begin{pmatrix} M_{11} - \tfrac{i}{2}\Gamma_{11} & M_{12} - \tfrac{i}{2}\Gamma_{12} \\ M_{21} - \tfrac{i}{2}\Gamma_{21} & M_{22} - \tfrac{i}{2}\Gamma_{22} \end{pmatrix}$$

$\mathcal{CPT}$ symmetry: $M_{11} = M_{22} \quad \Gamma_{11} = \Gamma_{22}$

Hermiticity: $X_{21} = M_{21} - \tfrac{i}{2}\Gamma_{21} = M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*$

note that $X_{21} \neq X_{12}^*$

with no assumptions about $\mathcal{CP}$:

$$\boldsymbol{H} = \begin{pmatrix} M_{11} - \tfrac{i}{2}\Gamma_{11} & M_{12} - \tfrac{i}{2}\Gamma_{12} \\ M_{12}^* - \tfrac{i}{2}\Gamma_{12}^* & M_{11} - \tfrac{i}{2}\Gamma_{11} \end{pmatrix} = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{11} \end{pmatrix}$$

find eigenstates (for $\mathcal{CP}$ conserved case)

$$\boldsymbol{H} = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{11} \end{pmatrix} \Longrightarrow \begin{pmatrix} X_{11} & \delta \\ \delta & X_{11} \end{pmatrix}$$

if $\mathcal{CP}$ is conserved

$(X_{21} = X_{12} \equiv \delta)$

eigenvalues:

$$\begin{aligned} \lambda_1 &\equiv M_S - \tfrac{i}{2}\Gamma_S = X_{11} - \delta = (M_{11} - M_{12}) - \tfrac{i}{2}(\Gamma_{11} - \Gamma_{12}) \\ \lambda_2 &\equiv M_L - \tfrac{i}{2}\Gamma_L = X_{11} + \delta = (M_{11} + M_{12}) - \tfrac{i}{2}(\Gamma_{11} + \Gamma_{12}) \end{aligned}$$

eigenstates:

$$|K_1(\tau)\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right)e^{iM_S\tau - \frac{1}{2}\Gamma_S\tau}$$

$$|K_2(\tau)\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right)e^{iM_L\tau - \frac{1}{2}\Gamma_L\tau}$$

Decay modes of the K_1 and K_2 states

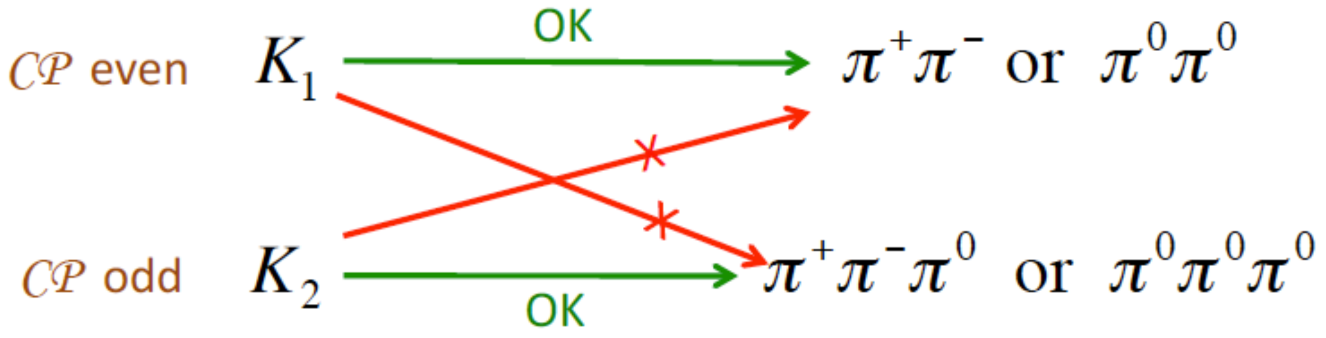
$$CP|\pi^{\pm}\rangle = -1|\pi^{\mp}\rangle$$

$$CP|\pi^{0}\rangle = -1|\pi^{0}\rangle$$

$$\boxed{\begin{array}{ll} CP\left(|\pi^{+}\rangle|\pi^{-}\rangle\right) = +1|\pi^{+}\rangle|\pi^{-}\rangle & \\ CP\left(|\pi^{+}\rangle|\pi^{-}\rangle|\pi^{0}\rangle\right) = -1|\pi^{+}\rangle|\pi^{-}\rangle|\pi^{0}\rangle & \end{array}}$$

$\mathcal{CP}$ even

$\mathcal{CP}$ odd



$\mathcal{CP}$ even K_1 OK $\pi^+\pi^-$ or $\pi^0\pi^0$

$\mathcal{CP}$ odd K_2 OK $\pi^+\pi^-\pi^0$ or $\pi^0\pi^0\pi^0$

Gell-Mann & Pais:
"…no more than half of all θ^0's…decay into two pions."

$K_1 \rightarrow \pi\pi \Leftarrow$ phase space large

lifetime is short : $\tau \approx 0.1\text{ns}$

500x different

$K_2 \rightarrow \pi\pi\pi \Leftarrow$ phase space small

lifetime is long : $\tau \approx 50\text{ns}$

$K_1 \Rightarrow K_S$ "*K-short*"

$K_2 \Rightarrow K_L$ "*K-long*"

Flavor states and mass eigenstates

-- if $\mathcal{CP}$ is conserved --

K_1-K_2
CP eigenstates

These have well defined
lifetime & mass

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

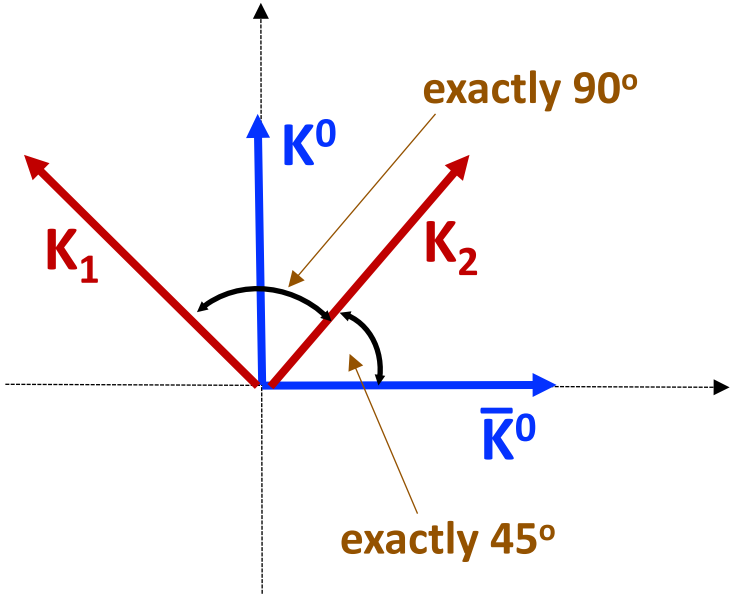
$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

$\bar{K}^0$-K^0
Flavor States

These have well defined
stangeness

$$|K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_1\rangle + |K_2\rangle\right)$$

$$|\bar{K}^0\rangle = \frac{1}{\sqrt{2}}\left(|K_1\rangle - |K_2\rangle\right)$$



$K^0 \leftrightarrow \overline{K^0}$ oscillations with lifetimes (but no $\mathcal{CP}$V)

include decay times

$$\begin{aligned} |K_S(t)\rangle &= e^{iM_S t - \frac{1}{2}\Gamma_S t}\,|K_S(0)\rangle \\ |K_L(t)\rangle &= e^{iM_L t - \frac{1}{2}\Gamma_L t}\,|K_L(0)\rangle \end{aligned}$$

mass eigenstates

start with K^0 at t=0:

$$\left|K^0(t=0)\right\rangle = \frac{1}{\sqrt{2}}\Big(\left|K_S(t=0)\right\rangle + \left|K_L(t=0)\right\rangle\Big)$$

Strangeness tagging

at a later time t:

$$\left|K^0(t)\right\rangle = \frac{1}{\sqrt{2}}\left(e^{iM_S t - \frac{1}{2}\Gamma_S t}\,|K_S(0)\rangle + e^{iM_L t - \frac{1}{2}\Gamma_L t}\,|K_L(0)\rangle\right)$$

use

$$\begin{aligned} |K_S(0)\rangle &= \tfrac{1}{\sqrt{2}}\left(|K^0(0)\rangle - |\bar{K}^0(0)\rangle\right) \\ |K_L(0)\rangle &= \tfrac{1}{\sqrt{2}}\left(|K^0(0)\rangle + |\bar{K}^0(0)\rangle\right) \end{aligned}$$

(assuming $\mathcal{CP}$ is conserved)

where:

$$|g_\pm(t)|^2 = e^{-\Gamma_L t} + e^{-\Gamma_S t} \pm 2e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t}\cos\Delta M_K t$$

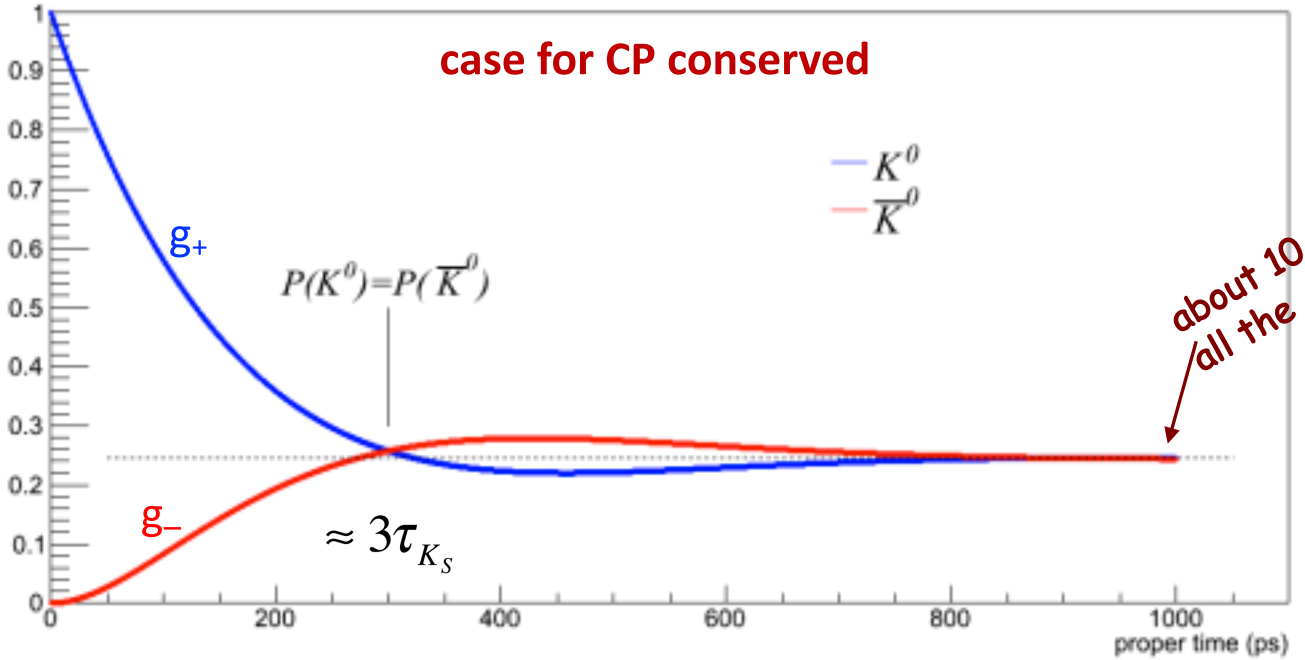
the measured K^0 and $\overline{K}^0$ rates:

$$\begin{aligned} I(K^0 \longrightarrow K^0; t) &= I_0|\left\langle K^0 \middle| K^0(t)\right\rangle|^2 = I_0|g_+(t)|^2 \\ I(K^0 \longrightarrow \bar{K}^0; t) &= I_0|\left\langle \bar{K}^0 \middle| K^0(t)\right\rangle|^2 = I_0|g_-(t)|^2 \end{aligned}$$

I_0 = beam intensity # of particles/s)

K^0 survival; $\bar{K}^0$ appearance

--strangeness oscillations--



about 10 K_S lifetimes,
all the K_S have decayed

$$I(K^0 \rightarrow K^0;t) = \tfrac{1}{4} I_0 \left[e^{-\Gamma_L t} + e^{-\Gamma_S t} + 2e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)t} \cos \Delta M_K t \right]$$

$$I(K^0 \rightarrow \overline{K}^0;t) = \tfrac{1}{4} I_0 \left[e^{-\Gamma_L t} + e^{-\Gamma_S t} - 2e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)t} \cos \Delta M_K t \right]$$

$\mathcal{CP}$ “tagging” vs Flavor “tagging”

$\mathcal{CP}$ “tagging”

usually, a neutral K decays either to $\pi\pi$ or $\pi\pi\pi$

if neutral *K* → $\pi^+\pi^-$ or → $\pi^0\pi^0$

$\pi\pi$ has $\mathcal{CP}$=+1
“tagged” as a K_1

if neutral *K* → $\pi^+\pi^-\pi^0$ or → $\pi^0\pi^0\pi^0$

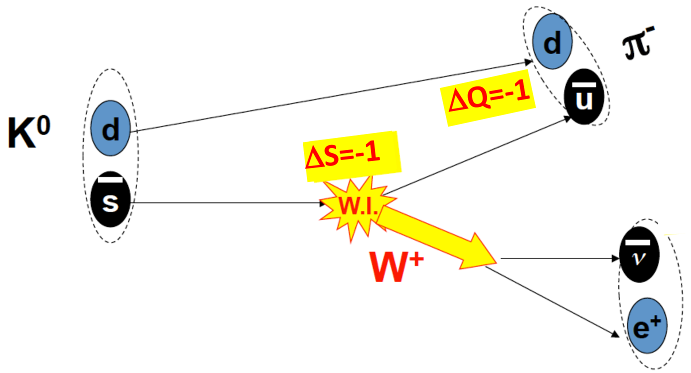
$\pi\pi\pi$ has $\mathcal{CP}$=-1
“tagged” as a K_2

Flavor “tagging”

sometimes, a neutral K decays semileptonicaly to either to πeν or πμν

ΔS=ΔQ rule

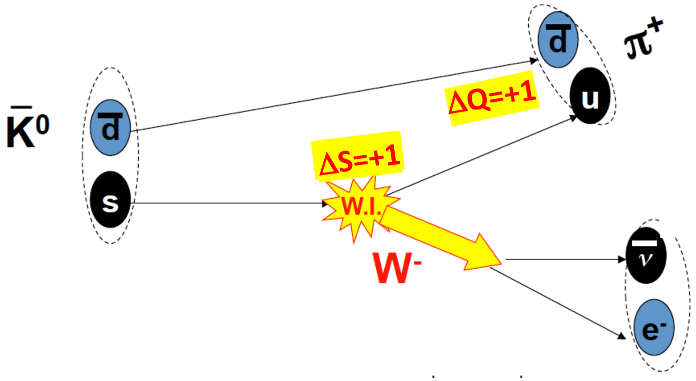
$K^0 \rightarrow \pi^- e^+ \nu$ and not $\pi^+ e^- \bar{\nu}$



neutral K → $\pi^- e^+ \nu$ or → $\pi^- \mu^+ \nu$

is “tagged” as a K^0 ($\mathcal{S}$=+1)

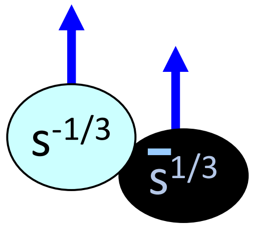
$\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}$ and not $\pi^- e^+ \nu$



neutral K → $\pi^+ e^- \nu$ or → $\pi^+ \mu^- \nu$

is “tagged” as a $\bar{K}^0$ ($\mathcal{S}$=-1)

The $\phi(1020)$ meson

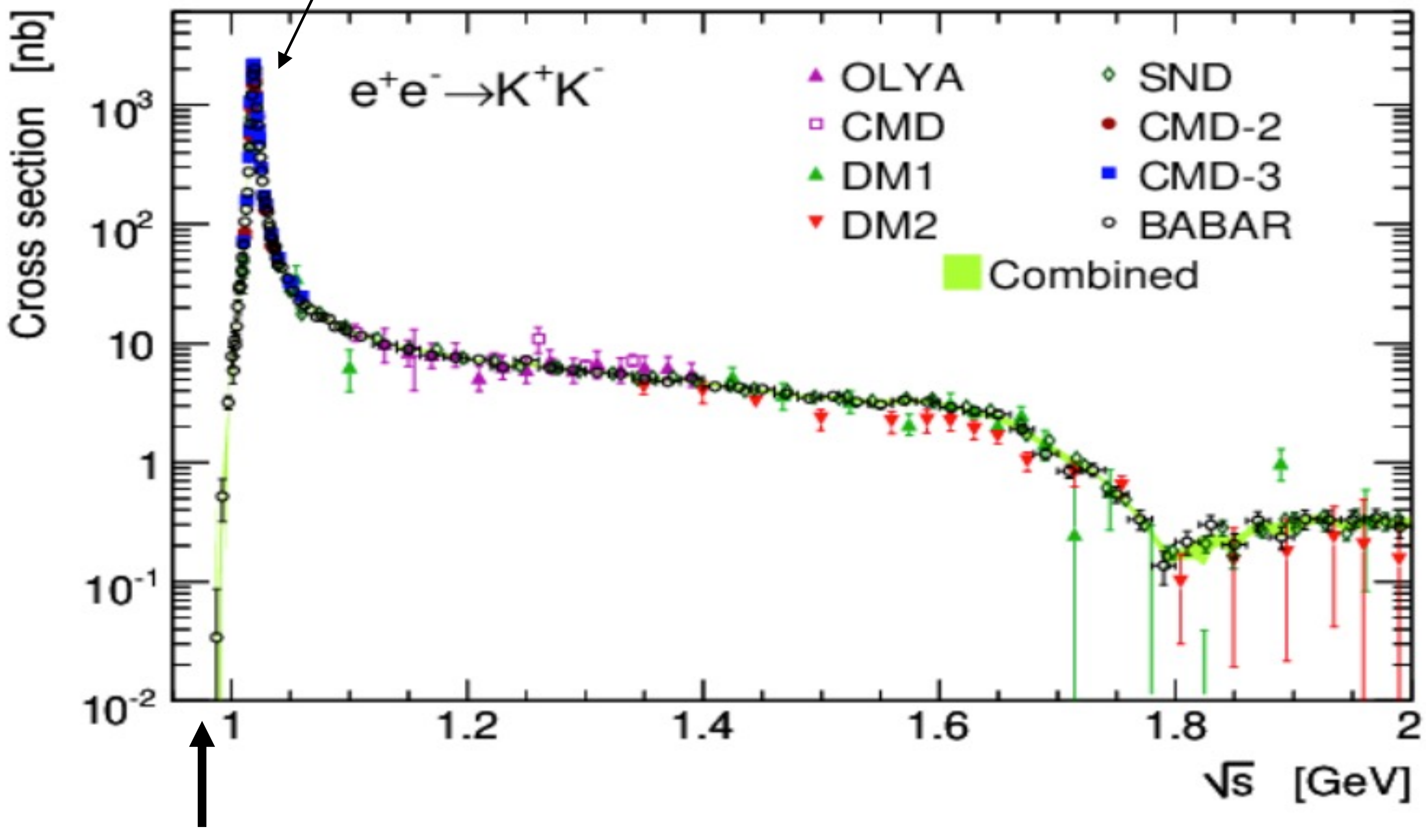


$Q=0$
$J=1$
$\mathcal{C}=-1$
$\mathcal{P}=-1$
$\mathcal{S}=0$

$J^{\mathcal{PC}}=1^{--}$

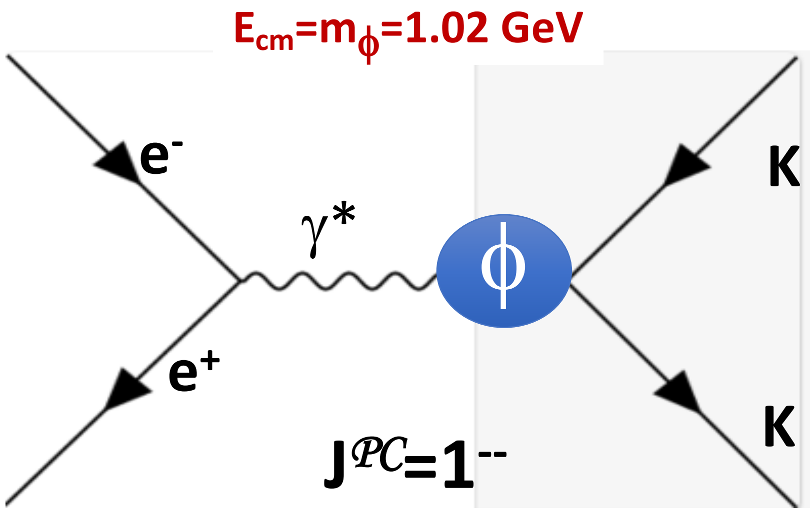
same as photon

well above the $\pi^+\pi^-\pi^0$ threshold and only barely above K^+K^- threshold but $Bf(\phi \rightarrow K^+K^-) \gg Bf(\phi \rightarrow \pi^+\pi^-\pi^0)$



$2m_K$

neutral K mesons produced via $e^+e^- \rightarrow \phi \rightarrow K\,K$



K (=K^0, K_S ,K_L,…?)

K (=$\bar{K}^0$, K_S , K_L,…?)

2 neutral Kaons in a $J^{PC}=1^{--}$ state

- J=1 : must have L=1 (*P*-wave)
- $\mathcal{P}=-1$: $(-1)_K\,(-1)_K\,(-1)_{P\text{-wave}}$
- $\mathcal{C}=+1$: $(+1)_{KS}\,(-1)_{KL}$

only K_SK_L has $\mathcal{C}=-1$

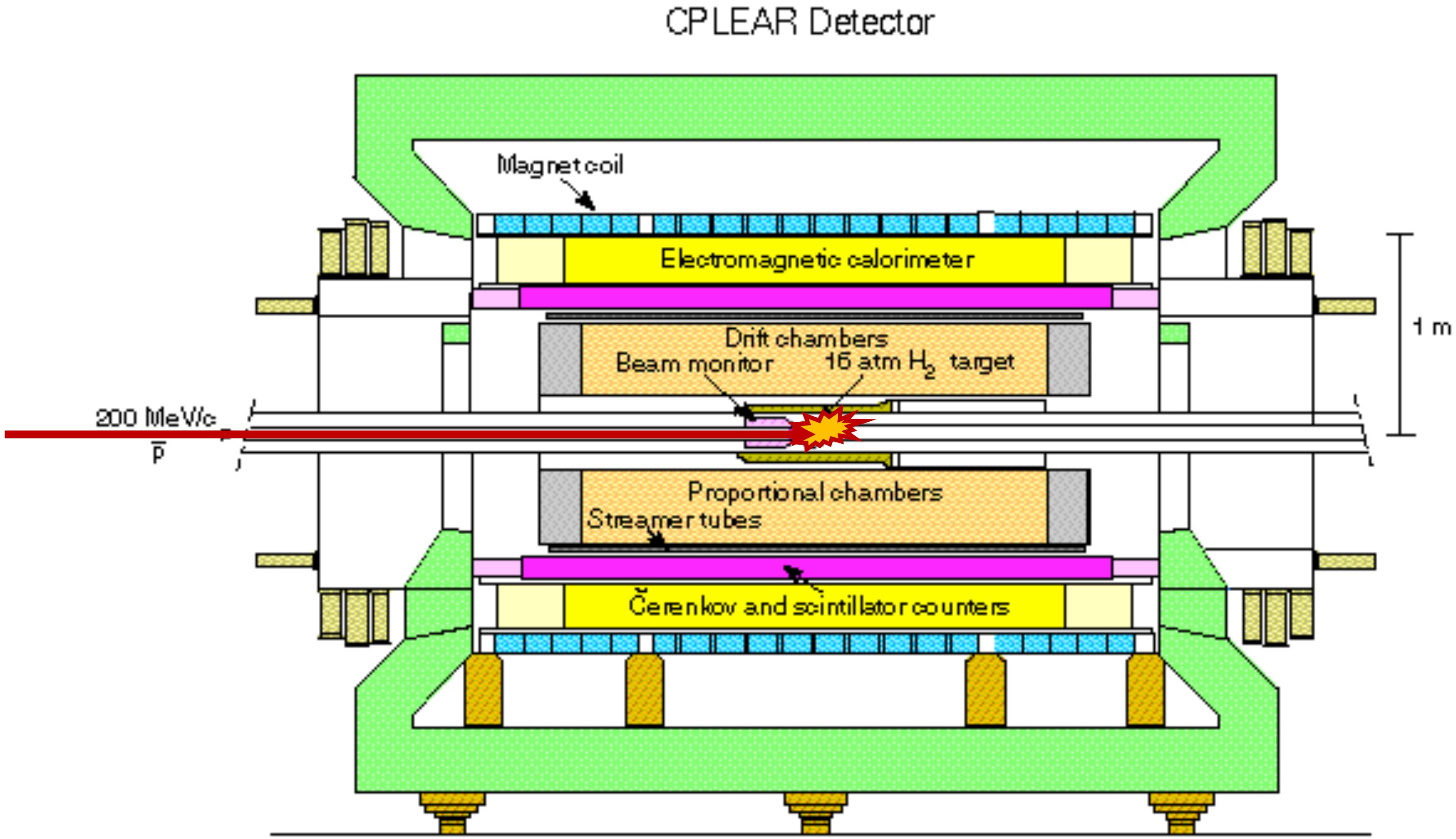
at an e^+e^- "ϕ-factory":

$$e^+e^- \rightarrow \phi \rightarrow K_SK_L$$

neutral *K* flavor-tagging with hadron beams

The CPLEAR anti-proton experiment at CERN

$\bar{p}$ beam stops in a H_2 target &
annihilates → $\bar{K}^0K^+\pi^-$ or $K^0K^-\pi^+$



Flavor-tagged production & Flavor-tagged decay

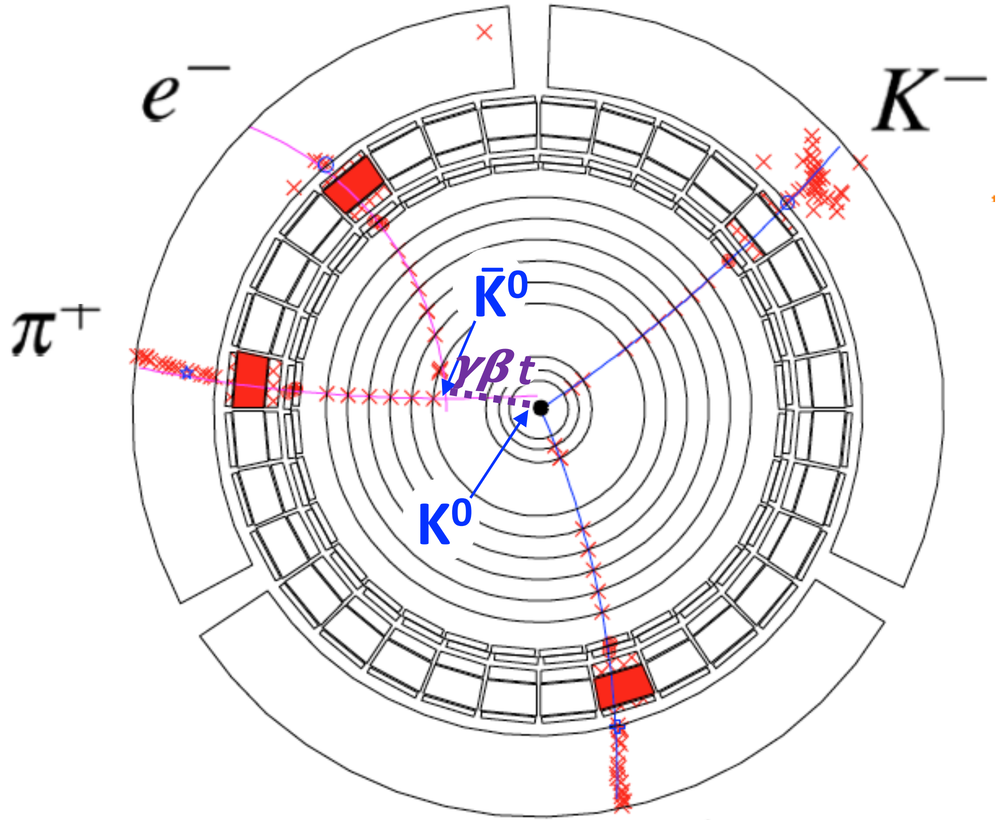
$\overline{K}^0 \rightarrow \boxed{\pi^+ e^-} \nu$

$K^0 \rightarrow \pi^- e^+ \nu$

Decay:

$$\overline{K}^0 \longrightarrow \boxed{\pi^+ e^-} \overline{\nu}_e$$

An example of a CPLEAR event



$\overline{p}p \rightarrow K^0 \boxed{K^- \pi^+}$

or

$\rightarrow \overline{K}^0 K^+ \pi^-$

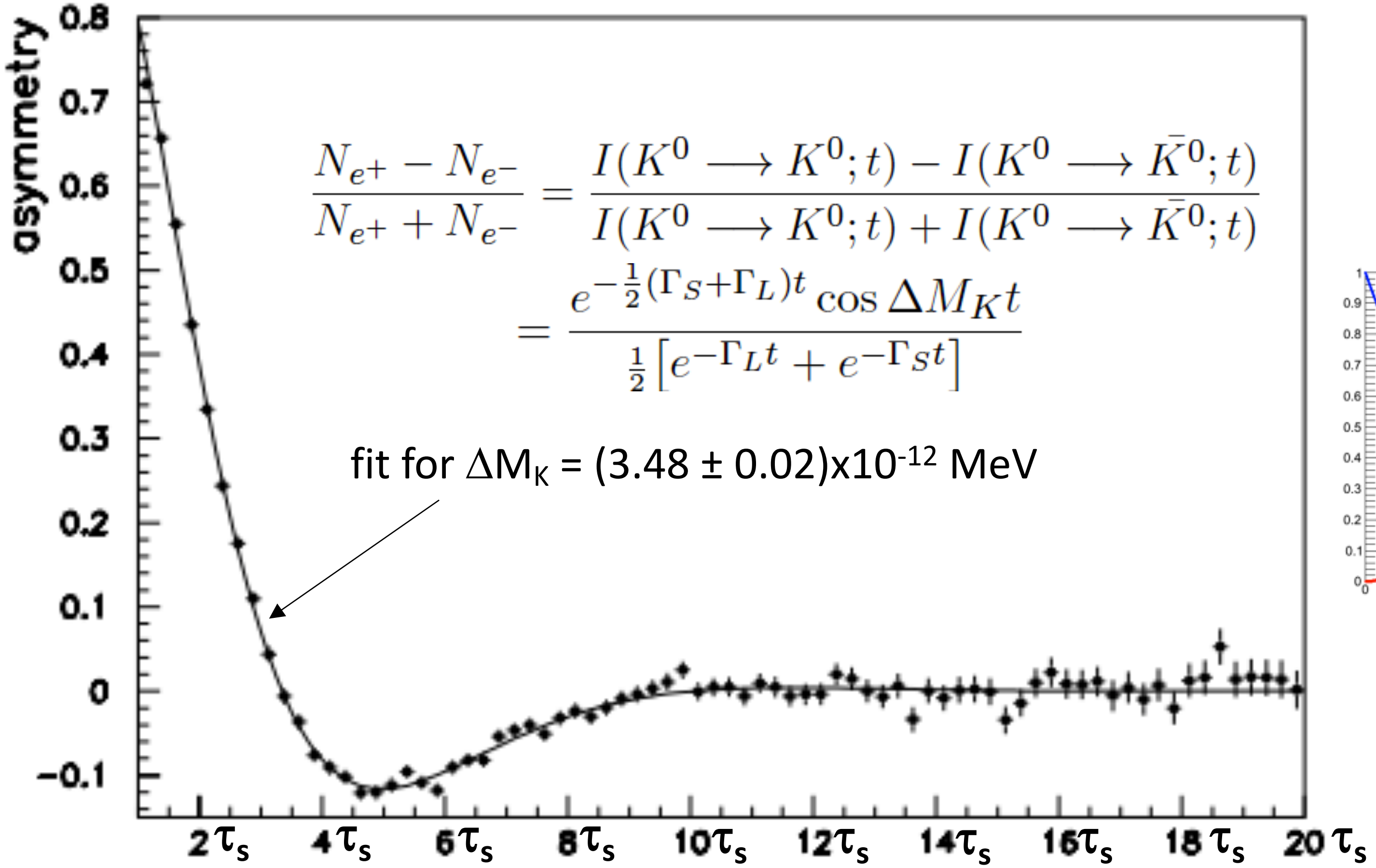
Production:

$$\overline{p}p \longrightarrow \boxed{K^- \pi^+} K^0$$

$\boxed{K^0(t=0) \rightarrow \overline{K}^0(t)}$

$K^0 \leftrightarrow \bar{K}^0$ mixing in CPLEAR

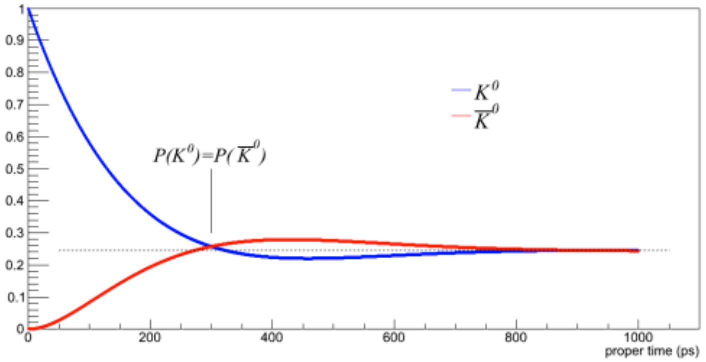
$$\frac{N_{e^+} - N_{e^-}}{N_{e^+} + N_{e^-}}$$



$$\frac{N_{e^+} - N_{e^-}}{N_{e^+} + N_{e^-}} = \frac{I(K^0 \longrightarrow K^0; t) - I(K^0 \longrightarrow \bar{K}^0; t)}{I(K^0 \longrightarrow K^0; t) + I(K^0 \longrightarrow \bar{K}^0; t)}$$

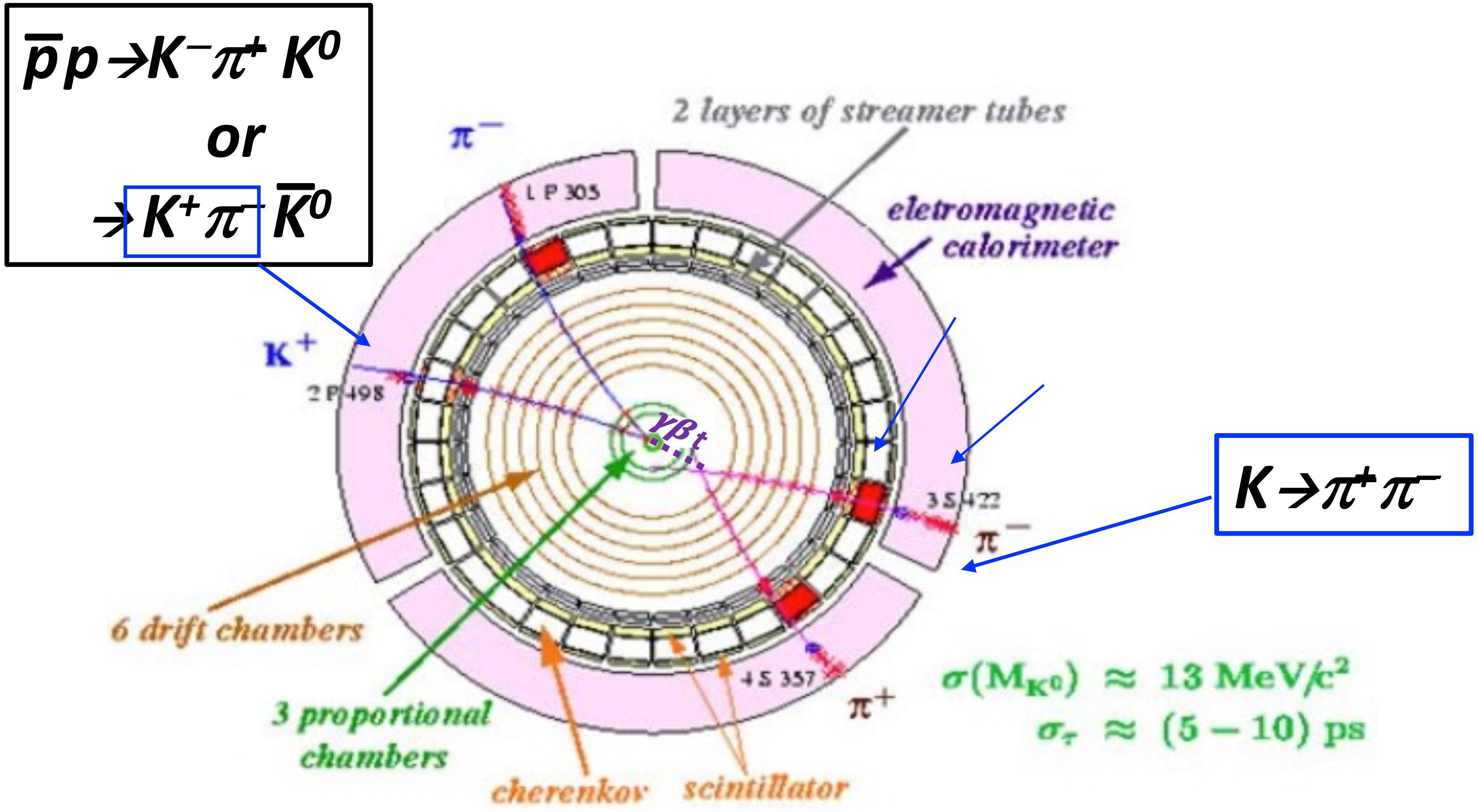
$$= \frac{e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)t} \cos \Delta M_K t}{\frac{1}{2}\left[e^{-\Gamma_L t} + e^{-\Gamma_S t}\right]}$$

fit for $\Delta M_K = (3.48 \pm 0.02)\mathrm{x}10^{-12}$ MeV



$\tau_S = 1/\Gamma_S = K_S$ **lifetime = 0.0895 ns**

Flavor-tagged production; $\mathcal{CP}$-tagged decay



$\bar{p}p \to K^-\pi^+ K^0$

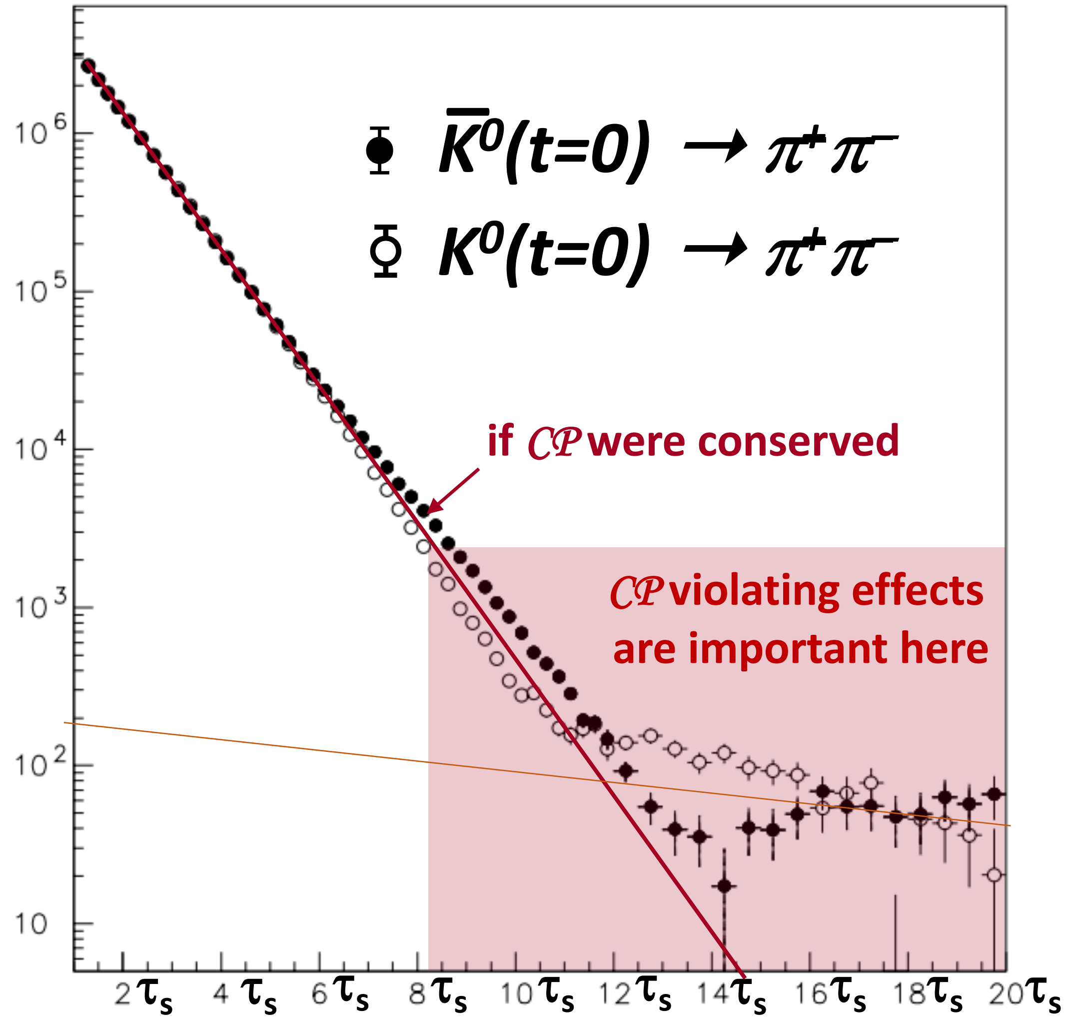
or

$\to \boxed{K^+\pi^-} \bar{K}^0$

$\boxed{K \to \pi^+\pi^-}$

$K^0(t=0) \to CP=+1$ at a later time=t

K^0 ($\bar{K}^0$) at production vertex; $\pi^+\pi^-$ at decay vertex



$\tau_S = 1/\Gamma_S = K_S$ lifetime = 0.895 ns

Let’s put in some numbers

$$\Delta M_K = M_L - M_S = (3.48 \pm 0.02) \times 10^{-12}\,\text{MeV} = \omega_K$$

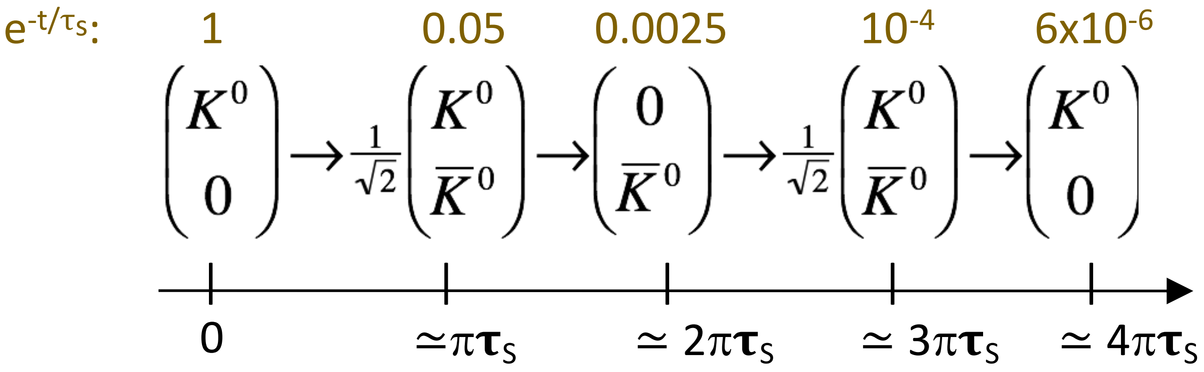
$$\Delta\Gamma_K = \Gamma_S - \Gamma_L = \frac{1}{\tau_S} - \frac{1}{\tau_L} = (7.28 \pm 0.01) \times 10^{-12}\,\text{MeV} \approx \frac{1}{\tau_\text{S}}$$

$$\frac{\Delta M_K}{\Delta\Gamma} = 0.48 \Rightarrow \omega_K \approx \frac{0.5}{\tau_S}$$

$$\omega_K T_{1-Oscillation} = \frac{0.5}{\tau_\text{S}} T_{1-Oscillation} = 2\pi \Rightarrow T_{1-Oscillation} \approx 4\pi \cdot \tau_S$$

miracle #1

$2\Delta M_K$≈DΓ



e^{-t/τ_S}: 1 0.05 0.0025 10^{-4} $6\text{x}10^{-6}$

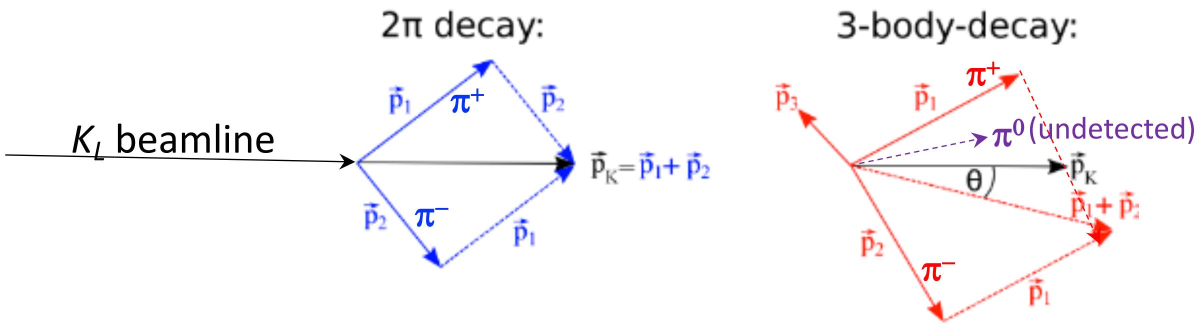
$$\begin{pmatrix} K^0 \\ 0 \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ \overline{K}^0 \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} \rightarrow \begin{pmatrix} K^0 \\ 0 \end{pmatrix}$$

Discovery $\mathcal{CP}$ violating $K_L \to \pi^+\pi^-$ decays

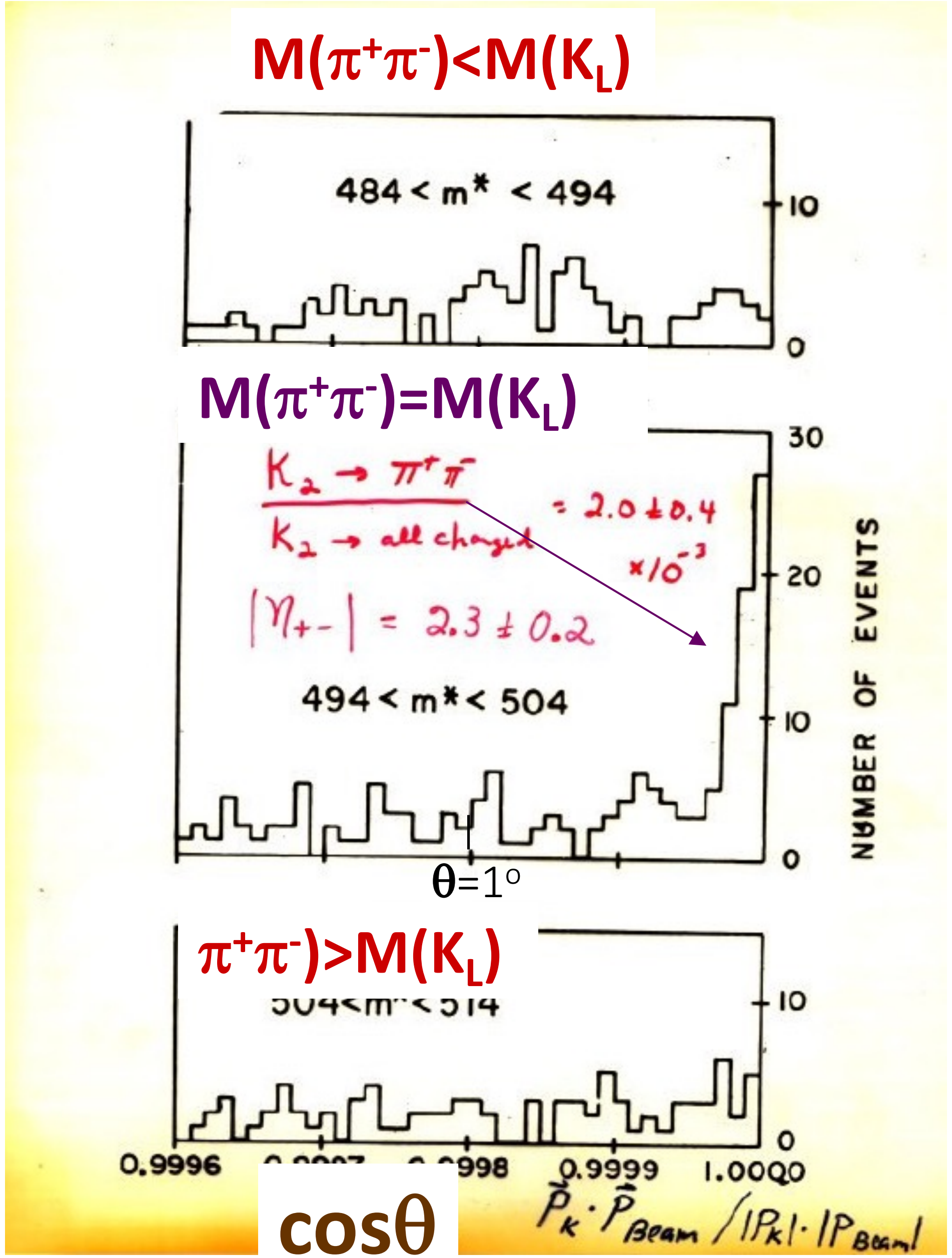
5211 $K_L \rightarrow \pi^+ + \pi^-$ candidates remeasured on a commercial bubble chamber measuring machine

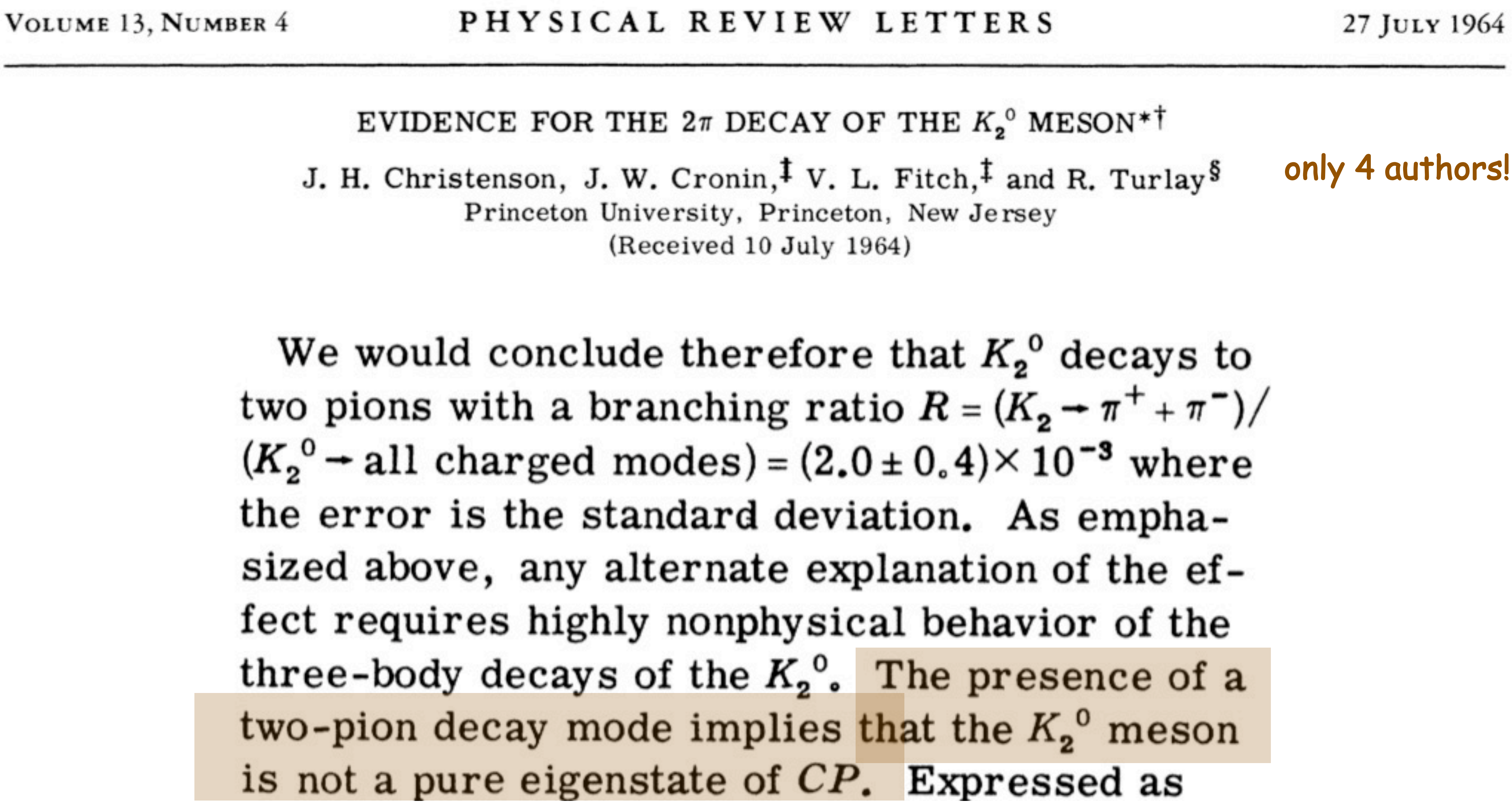
$\pi^+\pi^-$ "invariant mass"

$$M(\pi^+\pi^-) = \sqrt{(E_{\pi+}+E_{\pi-})^2 - (\vec{p}_{\pi+}+\vec{p}_{\pi-})^2}$$



see backup slides for more information about the experiment





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EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

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(Received 10 July 1964)

only 4 authors!

We would conclude therefore that K_2^0 decays to two pions with a branching ratio $R = (K_2 \rightarrow \pi^+ + \pi^-)/(K_2^0 \rightarrow \text{all charged modes}) = (2.0 \pm 0.4) \times 10^{-3}$ where the error is the standard deviation. As emphasized above, any alternate explanation of the effect requires highly nonphysical behavior of the three-body decays of the K_2^0. The presence of a two-pion decay mode implies that the K_2^0 meson is not a pure eigenstate of CP. Expressed as

Result

1964 experiment: $$\frac{Bf\,(K_L \rightarrow \pi^+\pi^-)}{Bf\,(K_L \rightarrow \text{charged particles})} = 2.0 \pm 0.4 \times 10^{-3}$$ **≈1/500**

these are not the same quantities

2020 World Average (PDG): $Bf(K_L \rightarrow \pi^+\pi^-) = 1.967 \pm 0.010 \times 10^{-3}$ **≈1/500**

Hamiltonian operator with $\mathcal{CP}$ violation

Hermitian

$$H = M - \frac{i}{2}\Gamma$$

-- now we will include decays --

$$\langle K_j|H|K_i\rangle = \langle K_j|M|K_i\rangle - \frac{i}{2}\langle K_j|\Gamma|K_i\rangle = M_{ij} - \frac{i}{2}\Gamma_{ij} = X_{ij}$$

$$\begin{pmatrix} \langle K^0|H|K^0\rangle & \langle K^0|H|\overline{K}^0\rangle \\ \langle \overline{K}^0|H|K^0\rangle & \langle \overline{K}^0|H|\overline{K}^0\rangle \end{pmatrix} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

$\mathcal{CPT}$ symmetry: $M_{11} = M_{22}$ $\Gamma_{11} = \Gamma_{22}$

Hermiticity: $M_{21} - \frac{i}{2}\Gamma_{21} = M_{12}^* - \frac{i}{2}\Gamma_{12}^*$

$X_{21} \neq X_{12}^*$

with no assumptions about $\mathcal{CP}$

$$\boldsymbol{H} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_{11} - \frac{i}{2}\Gamma_{11} \end{pmatrix} = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{11} \end{pmatrix}$$

Schrodinger's Equation

-- allowing for CP violation and including decays --

here I conform to standard notation:

$$X_{21} = \mathcal{A}_{K^0 \to \overline{K}^0} = -ip^2 \quad \left(= M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*\right)$$

$$X_{12} = \mathcal{A}_{\overline{K}^0 \to K^0} = -iq^2 \quad \left(= M_{12} - \tfrac{i}{2}\Gamma_{12}\right)$$

Schrodinger's equation

$$\boldsymbol{H}\psi(t) = -i\hbar\frac{\partial\psi(t)}{\partial t}$$

assume solutions of the form: $\psi(t) = \begin{pmatrix} a_i \\ b_i \end{pmatrix} e^{i\lambda_i t}$

solve for eigenvalues/states;

$(\hbar = 1)$

$$\begin{pmatrix} X_{11} & -iq^2 \\ -ip^2 & X_{11} \end{pmatrix} \begin{pmatrix} a_i \\ b_i \end{pmatrix} e^{i\lambda_i t} = -i\frac{\partial}{\partial t}\begin{pmatrix} a_i \\ b_i \end{pmatrix} e^{i\lambda_i t}$$

$$\Rightarrow \begin{pmatrix} X_{11} - \lambda_i & -iq^2 \\ -ip^2 & X_{11} - \lambda_i \end{pmatrix} \begin{pmatrix} a_i \\ b_i \end{pmatrix} = 0$$

Solutions for ψ(t)

eigenvalue equation:

$$\begin{vmatrix} X_{11}-\lambda_i & -iq^2 \\ -ip^2 & X_{11}-\lambda_i \end{vmatrix} = 0$$

$$X_{11} = M_K - \tfrac{i}{2}\Gamma_K$$

eigenvalues

$$i=1:\lambda_S = X_{11}+ipq = M_S - \tfrac{i}{2}\Gamma_S$$

$$i=2:\lambda_L = X_{11}-ipq = M_L - \tfrac{i}{2}\Gamma_L$$

$$\boxed{\lambda_S - \lambda_L = 2ipq = (M_S - M_L) - \tfrac{i}{2}(\Gamma_S - \Gamma_L)}$$

eigenstates

$$|K_S(t)\rangle = \tfrac{1}{\sqrt{p^2+q^2}}(p\,|K^0\rangle + q\,|\bar{K}^0\rangle\,)e^{iM_St-\frac{1}{2}\Gamma_St}$$

$$|K_L(t)\rangle = \tfrac{1}{\sqrt{p^2+q^2}}(p\,|K^0\rangle - q\,|\bar{K}^0\rangle\,)e^{iM_Lt-\frac{1}{2}\Gamma_Lt}$$

if *p* & *q* are not equal (i.e. $\mathcal{CP}$ is violated): $K_S \neq K_1$ & $K_L \neq K_2$

complex numbers

Solutions for ψ(t)

mass eigenstates: flavor basis

$$
\begin{aligned}
|K_S(t)\rangle &= \tfrac{1}{\sqrt{p^2+q^2}}\left(p\,|K^0\rangle + q\,|\bar{K}^0\rangle\,\right)e^{iM_S t-\frac{1}{2}\Gamma_S t} \\
|K_L(t)\rangle &= \tfrac{1}{\sqrt{p^2+q^2}}\left(p\,|K^0\rangle - q\,|\bar{K}^0\rangle\,\right)e^{iM_L t-\frac{1}{2}\Gamma_L t}
\end{aligned}
$$

Flavor → $\mathcal{CP}$ eigenstates:

$$
\begin{aligned}
|K^0\rangle &= \tfrac{1}{\sqrt{2}}\left(\,|K_1\rangle + |K_2\rangle\,\right) \\
|\bar{K}^0\rangle &= \tfrac{1}{\sqrt{2}}\left(\,|K_1\rangle - |K_2\rangle\,\right)
\end{aligned}
$$

mass eigenstates: $\mathcal{CP}$ basis

$$
\begin{aligned}
|K_S\rangle &= \tfrac{p+q}{\sqrt{p^2+q^2}}\left(|K_1\rangle + \tfrac{p-q}{p+q}|K_2\rangle\right) = \tfrac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K_1\rangle + \varepsilon|K_2\rangle\right) \\
|K_L\rangle &= \tfrac{p+q}{\sqrt{p^2+q^2}}\left(|K_2\rangle + \tfrac{p-q}{p+q}|K_1\rangle\right) = \tfrac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K_2\rangle + \varepsilon|K_1\rangle\right)
\end{aligned}
$$

$$\boxed{\varepsilon \equiv \frac{p-q}{p+q}}$$

the K_1 component decays to $\pi^+\pi^-$

K$_S$ & K$_L$ are not $\mathcal{CP}$ eigenstates

$$
\begin{aligned}
|K_S\rangle &= \tfrac{1}{\sqrt{1+|\epsilon|^2}}\left(\,|K_1\rangle + \epsilon\,|K_2\rangle\,\right)\\
|K_L\rangle &= \tfrac{1}{\sqrt{1+|\epsilon|^2}}\left(\,|K_2\rangle + \epsilon\,|K_1\rangle\,\right)
\end{aligned}
$$

≈1/500 again

$|K_L\rangle \neq |K_2\rangle$ it has a small (~0.2%) admixture of $|K_1\rangle$

and $|K_S\rangle$ has a similar admixture of $|K_2\rangle$

this means that $M_{21} - \frac{i}{2}\Gamma_{21} \neq M^*_{12} - \frac{i}{2}\Gamma^*_{12}$ *i.e.,* $\mathcal{CP}$ is violated

What is ε?

If $\langle \pi^+\pi^- | K_2 \rangle = 0$

good approximation,
but not exact

$$\begin{aligned} A(K_L \to \pi^+\pi^-) &= \Gamma(K_L \to \pi^+\pi^-) = \langle \pi^+\pi^- | K_L \rangle \\ &= \tfrac{1}{\sqrt{1+|\epsilon|^2}} \left(\cancel{\langle \pi^+\pi^- | K_2 \rangle} + \epsilon \langle \pi^+\pi^- | K_1 \rangle \right) \\ &= \tfrac{1}{\sqrt{1+|\epsilon|^2}} \epsilon \langle \pi^+\pi^- | K_1 \rangle \end{aligned}$$

$$\begin{aligned} A(K_S \to \pi^+\pi^-) &= \langle \pi^+\pi^- | K_S \rangle \\ &= \tfrac{1}{\sqrt{1+|\epsilon|^2}} \left(\langle \pi^+\pi^- | K_1 \rangle + \epsilon \cancel{\langle \pi^+\pi^- | K_2 \rangle} \right) \\ &= \tfrac{1}{\sqrt{1+|\epsilon|^2}} \langle \pi^+\pi^- | K_1 \rangle \end{aligned}$$

$$\epsilon = \frac{\mathcal{A}(K_L \to \pi^+\pi^-)}{\mathcal{A}(K_S \to \pi^+\pi^-)}$$ ←

amplitudes,
…not branching fractions

put in numbers

"partial widths"

"branching fractions"

$$|\epsilon| = \frac{|\mathcal{A}(K_L \to \pi^+\pi^-)|}{|\mathcal{A}(K_S \to \pi^+\pi^-)|} = \sqrt{\frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)}} = \sqrt{\frac{\mathcal{B}(K_L \to \pi^+\pi^-)\Gamma_L}{\mathcal{B}(K_S \to \pi^+\pi^-)\Gamma_S}} = \sqrt{\frac{\mathcal{B}(K_L \to \pi^+\pi^-)\tau_S}{\mathcal{B}(K_S \to \pi^+\pi^-)\tau_L}}$$

$= 2.28 \pm 0.01 \times 10^{-3}$

from PDG 2020:

$Bf(K_L \to \pi^+\pi^-) = 1.967 \pm 0.010 \times 10^{-3}$

$Bf(K_s \to \pi^+\pi^-) = 0.6920 \pm 0.0005$

$$\left.\begin{array}{l} \tau_S = 0.08954 \pm 0.0004 \text{ ns} \\ \tau_L = 51.16 \pm 0.21 \text{ ns} \end{array}\right\} \tau_S/\tau_L = 1.75 \pm 0.01 \times 10^{-3}$$

nearly the same as $Bf(K_L \to \pi^+\pi^-)$,
a coincidence due to similar values
of $Bf(K_L \to \pi^+\pi^-)$, τ_S/τ_L, & ε.

The K_S-K_L meson system is weird!

-- unlike any other particle systems--

→ **The K_S & K_L are not each others antiparticles**

-- **different mass and very different lifetimes**

-- **K^0 & $\bar{K}^0$ are particle-antiparticles; K_1 & K_2 are their own antiparticles**

→ **In fact, neither the K_S nor the K_L have antiparticles**

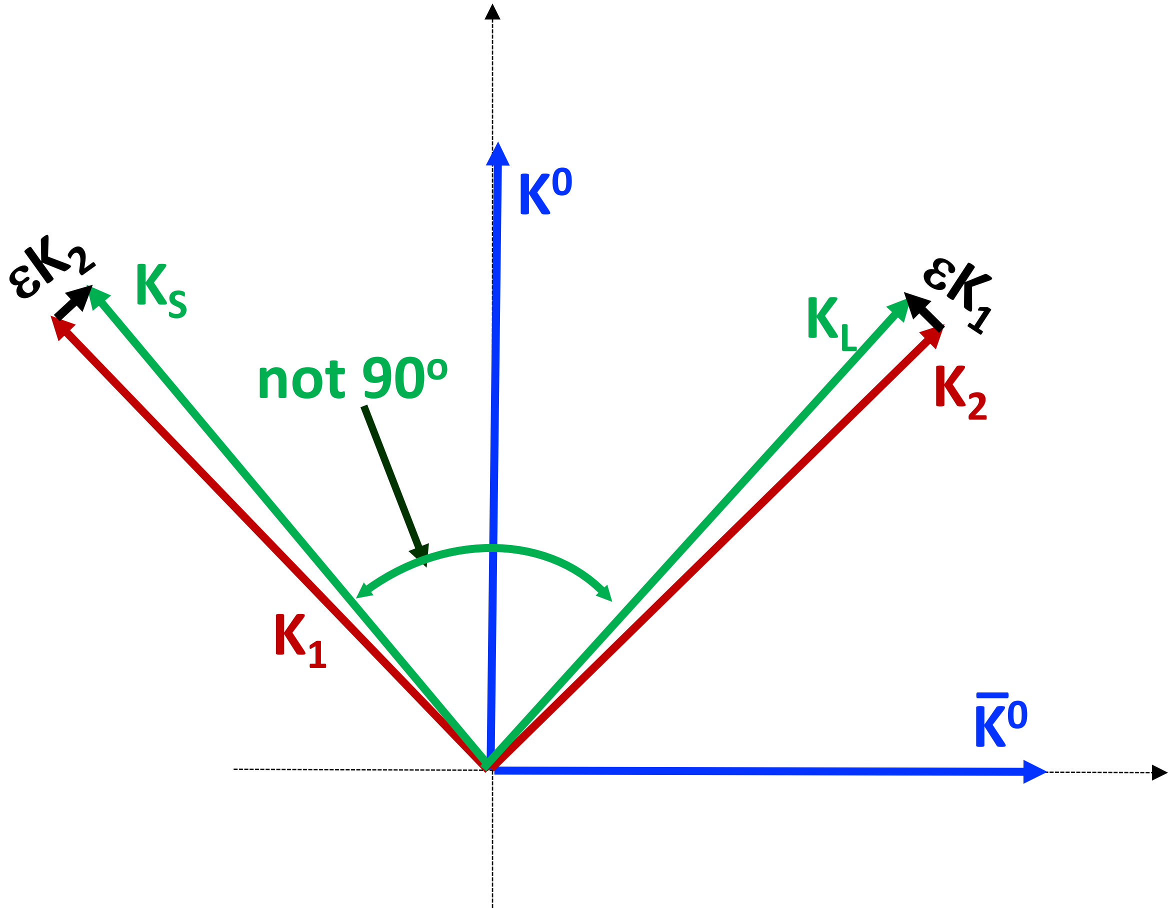
--
$$\begin{aligned} \mathcal{C}\,|K_S\rangle &= \mathcal{C}\big(\,|K_1\rangle + \epsilon\,|K_2\rangle\,\big) = +\big(\,|K_1\rangle - \epsilon\,|K_2\rangle\,\big) \\ \mathcal{C}\,|K_L\rangle &= \mathcal{C}\big(\,|K_2\rangle + \epsilon\,|K_1\rangle\,\big) = -\big(\,|K_2\rangle - \epsilon\,|K_1\rangle\,\big) \end{aligned}$$

these states do not occur in Nature

→ **The K_S & K_L are not even orthogonal**

-- $\langle K_S|K_L\rangle = \big(\,\langle K_1| + \epsilon\,|K_2\rangle\,\big)\big(\,|K_2\rangle + \epsilon\,|K_1\rangle\,\big) = 2\,\mathrm{Re}\,\epsilon \neq 0$

neutral Kaon system: 3 different basis systems



K^0-$\overline{K}^0$
Flavor States

These have well defined strangeness

K_1-K_2
$\mathcal{CP}$ eigenstates

These have well defined $\mathcal{CP}$ values

not the same!

K_S-K_L
Mass eigenstsate

These have well defined lifetime & mass

if $\varepsilon \neq 0$, the K_S & K_L mass eigenstates are not orthogonal

-- Lee, Omnes, Yang, Phys. Rev. 106, 340 (1957) --

Let’s look at ε:

definition of ε:

$$\varepsilon \equiv \frac{p-q}{p+q}$$

what we know about ***p*** & ***q***

$$-ip^2 = \left(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*\right) \Rightarrow \boxed{p^2 = iM_{12}^* - \tfrac{i}{2}\Gamma_{12}^*}$$

$$-iq^2 = \left(M_{12} - \tfrac{i}{2}\Gamma_{12}\right) \Rightarrow \boxed{q^2 = iM_{12} + \tfrac{i}{2}\Gamma_{12}}$$

$$\lambda_S - \lambda_L = 2ipq = (M_S - M_L) - \tfrac{i}{2}(\Gamma_S - \Gamma_L)$$

$$\Rightarrow \boxed{2pq = i(M_L - M_S) - \tfrac{1}{2}(\Gamma_S - \Gamma_L)}$$

rewrite ε in terms that we know:

$$\varepsilon \equiv \frac{p^2-q^2}{(p+q)^2} \Rightarrow \frac{p^2-q^2}{4pq + \underbrace{(p-q)^2}_{(p-q)^2 \sim \varepsilon^2 \Leftarrow small}} \approx \frac{p^2-q^2}{4pq}$$

$$\frac{p^2-q^2}{4pq} = \frac{-2\,\mathrm{Im}\,M_{12} + i\,\mathrm{Im}\Gamma_{12}}{2i(M_L - M_S) - (\Gamma_S - \Gamma_L)}$$

$$\boxed{\varepsilon = \frac{i\,\mathrm{Im}\,M_{12} + \tfrac{1}{2}\mathrm{Im}\Gamma_{12}}{(M_L - M_S) - \tfrac{i}{2}(\Gamma_S - \Gamma_L)}}$$

this is the key relation in this talk

Some comments

$$H \Rightarrow \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{12} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*} & M_{11} - \frac{i}{2}\Gamma_{11} \end{pmatrix}$$

$\mathcal{CP}$ is only violated if the off-diagonal terms are different

$$\varepsilon = \frac{i\,\mathrm{Im}\,M_{12} + \frac{1}{2}\mathrm{Im}\Gamma_{12}}{(M_L - M_S) - \frac{i}{2}(\Gamma_S - \Gamma_L)}$$

Hermiticity: $M_{21}=M_{12}^{*}$ & $\Gamma_{12}=\Gamma_{21}^{*}$
$\mathcal{CP}$V only driven by complex phases

Latest numbers:

$$\frac{Bf(K_L \to \pi^+\pi^-)}{Bf(K_L \to \mathrm{all})} = 1.97 \pm 0.01 \times 10^{-3}$$

$$|\eta_{+-}| = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \approx |\varepsilon| = 2.23 \pm 0.01 \times 10^{-3}$$

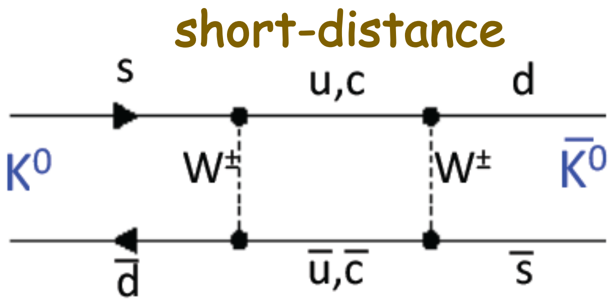
$\mathcal{CP}$V is very small, ~$10^{-3}G_F^2$, far from the maximum that is possible (unlike $\mathcal{P}$ and $\mathcal{C}$ violations)

What are M_{12} & Γ_{12}?

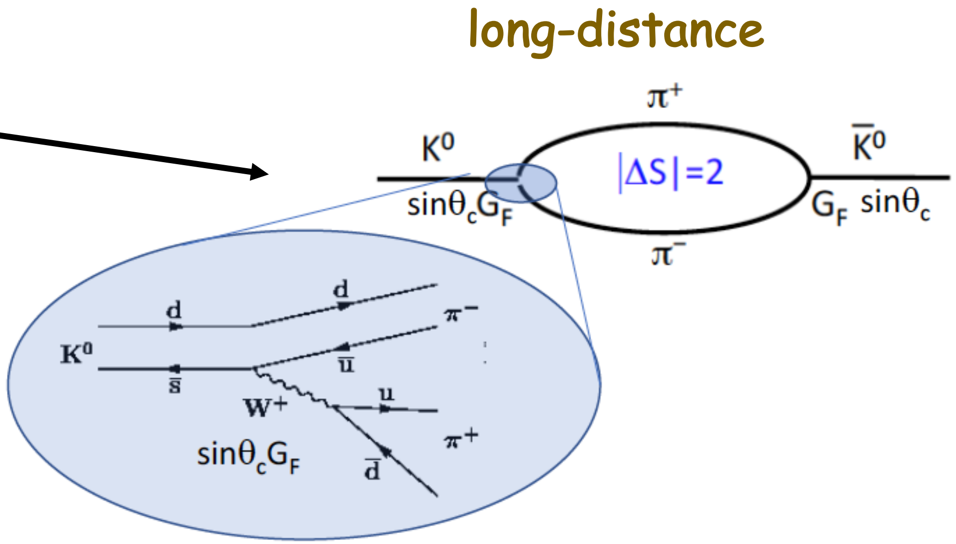
virtual & on-shell K^0-$\overline{K}^0$ couplings

non-zero ImM_{12} would imply new heavy virtual particles

--"Mass-Matrix $\mathcal{CP}$V"--



$$\varepsilon = \frac{i\,\mathrm{Im}M_{12} + \frac{1}{2}\mathrm{Im}\Gamma_{12}}{(M_L - M_S) - \frac{i}{2}(\Gamma_S - \Gamma_L)}$$



non-zero ImΓ_{12} would imply $A(\overline{K}^0 \to \pi\pi) \neq A^*(K^0 \to \pi\pi)$

--"Direct $\mathcal{CP}$V"--

Some notation/terminology:

$$\epsilon = \frac{i\mathrm{Im}M_{12} + \frac{1}{2}\mathrm{Im}\Gamma_{12}}{(M_L - M_S) - \frac{i}{2}(\Gamma_S - \Gamma_L)} = \frac{i\mathrm{Im}M_{12} + \frac{1}{2}\mathrm{Im}\Gamma_{12}}{\Delta M - \frac{i}{2}\Delta\Gamma}$$

$$\eta_{+-} \equiv \frac{\mathcal{A}(K_L \to \pi^+\pi^-)}{\mathcal{A}(K_S \to \pi^+\pi^-)}$$

$$\eta_{00} \equiv \frac{\mathcal{A}(K_L \to \pi^0 + \pi^0)}{\mathcal{A}(K_S \to \pi^0\pi^0)}$$

$$\eta_{00} \approx \eta_{+-} \approx \epsilon$$

almost --but not exactly-- equal

phase of ε

-- the "Superweak" phase --

$$\epsilon = \frac{i\mathrm{Im}M_{12} + \frac{1}{2}\mathrm{Im}\Gamma_{12}}{(M_L - M_S) - \frac{i}{2}(\Gamma_S - \Gamma_L)} = \frac{i\mathrm{Im}M_{12} + \frac{1}{2}\mathrm{Im}\Gamma_{12}}{\Delta M - \frac{i}{2}\Delta\Gamma}$$

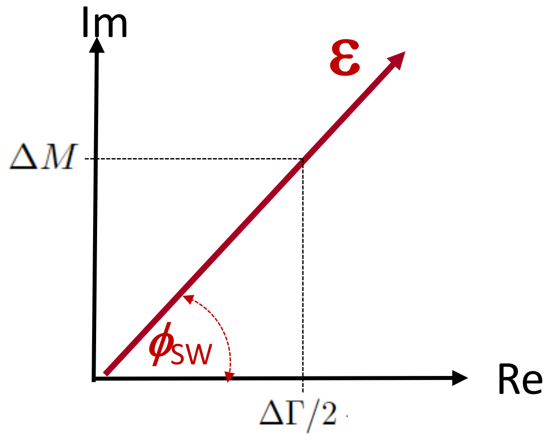
if ImΓ_{12}=0:

$$\Longrightarrow \frac{i\mathrm{Im}M_{12}}{\Delta M - \frac{i}{2}\Delta\Gamma} = \underbrace{\frac{\mathrm{Im}M_{12}}{(\Delta\Gamma/2)^2 + \Delta M^2}}_{\text{real number}}(\Delta\Gamma/2 + i\Delta M)$$

$$\phi_{\mathrm{SW}} = \tan^{-1}\frac{2\Delta m}{\Delta\Gamma} = 43.30^\circ \pm 0.16^\circ,$$

↑

using PDG values



measuring the phase of η_{+-}

K^0 -- $\bar{K}^0$ basis states with $\mathcal{CP}$V

$$
\begin{aligned}
|K_S\rangle &= \tfrac{1}{\sqrt{1+|\epsilon|^2}}\big(\,|K_1\rangle + \epsilon\,|K_2\rangle\,\big) = \tfrac{1}{\sqrt{2(1+|\epsilon|^2)}}\Big((1+\epsilon)\left|K^0\right\rangle + (1-\epsilon)\left|\bar{K^0}\right\rangle\Big) \\
|K_L\rangle &= \tfrac{1}{\sqrt{1+|\epsilon|^2}}\big(\,|K_2\rangle + \epsilon\,|K_1\rangle\,\big) = \tfrac{1}{\sqrt{2(1+|\epsilon|^2)}}\Big((1+\epsilon)\left|K^0\right\rangle - (1-\epsilon)\left|\bar{K^0}\right\rangle\Big)
\end{aligned}
$$

solve for K^0 and $\bar{K}^0$

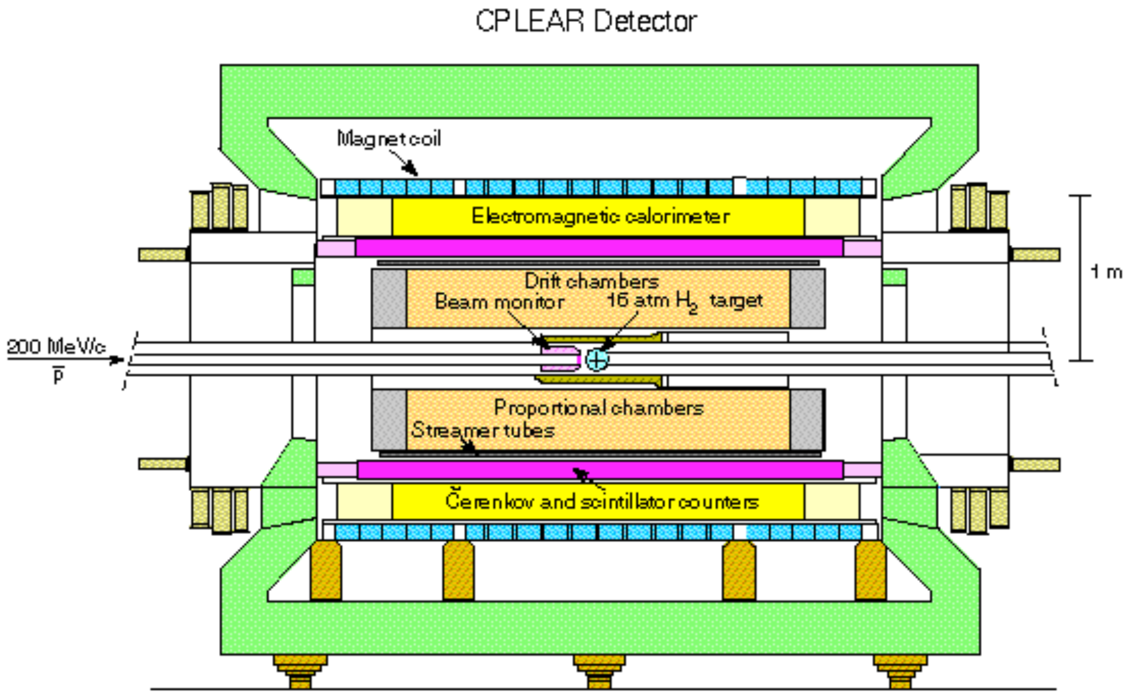
$$
\begin{aligned}
\left|K^0(\tau)\right\rangle &\propto (1-\epsilon)\big(\,|K_S(\tau)\rangle + |K_L(\tau)\rangle\,\big) \\
\left|\bar{K^0}(\tau)\right\rangle &\propto (1+\epsilon)\big(\,|K_S(\tau)\rangle - |K_L(\tau)\rangle\,\big)
\end{aligned}
$$

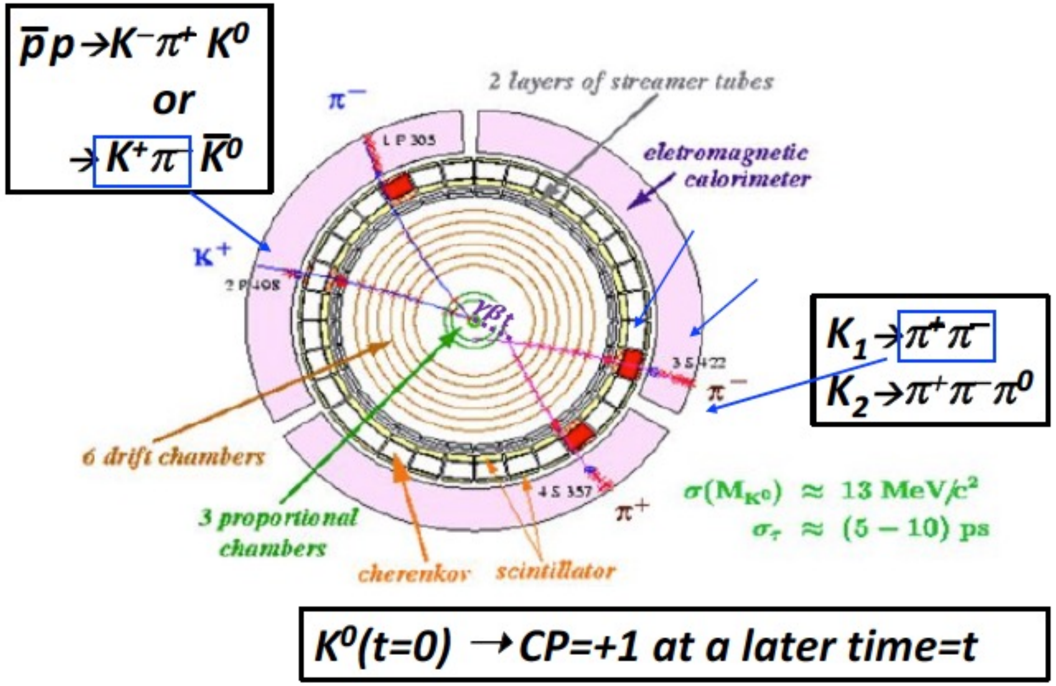
strangeness-tagged K^0 and $\bar{K}^0$

$$
\begin{aligned}
\left|K^0 \to \pi^+\pi^-(\tau)\right\rangle &\propto (1-\epsilon)\Big(\,\overbrace{|K_S(\tau)\rangle}^{|K_S(0)\rangle\,(e^{-i(M_S-\frac{1}{2}\Gamma_S)\tau})} + \overbrace{\eta_{+-}\,|K_L(\tau)\rangle}^{|\eta_{+-}|e^{i\phi_{+-}}\,|K_L(0)\rangle\,(e^{-i(M_L-\frac{1}{2}\Gamma_L)\tau})}\,\Big) \\
\left|\bar{K^0} \to \pi^+\pi^-(\tau)\right\rangle &\propto (1+\epsilon)\Big(\,|K_S(\tau)\rangle - \eta_{+-}\,|K_L(\tau)\rangle\,\Big)
\end{aligned}
$$

K^0 ($\bar{K}^0$) $\rightarrow \pi^+\pi^-$ vs τ at CPLEAR

Use favor-tagged events that decay to $\pi^+\pi^-$





$\bar{p}p \rightarrow K^-\pi^+ K^0$

or

$\rightarrow \boxed{K^+\pi^-} \bar{K}^0$

$K_1 \rightarrow \boxed{\pi^+\pi^-}$

$K_2 \rightarrow \pi^+\pi^-\pi^0$

$\sigma(M_{K^0}) \approx 13$ MeV/c²

$\sigma_\tau \approx (5-10)$ ps

$K^0(t=0) \rightarrow CP=+1$ at a later time=t

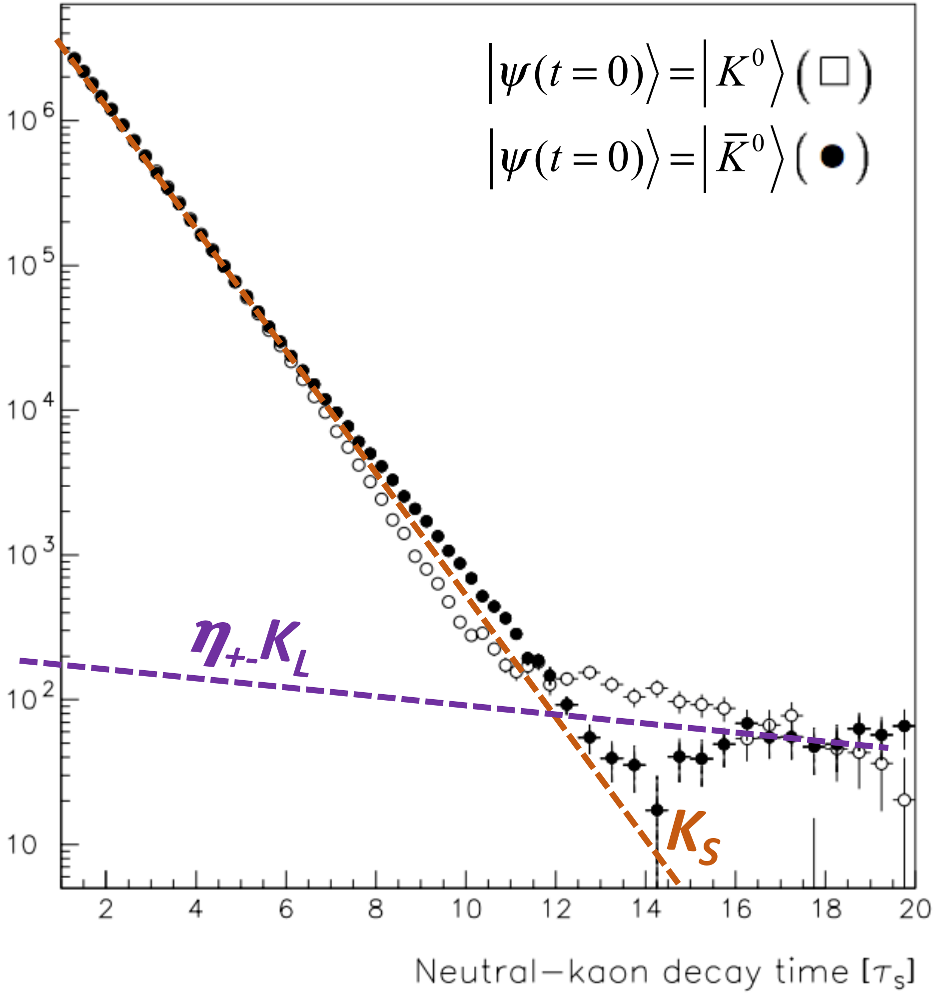
Time-dependence of $K^0(\bar{K}^0) \rightarrow \pi^+\pi^-$

$$\left|K^0 \to \pi^+\pi^-(\tau)\right\rangle \quad \propto \quad (1-\epsilon)\Big(\left|K_S(\tau)\right\rangle + \eta_{+-}\left|K_L(\tau)\right\rangle\Big)$$

$$\left|\bar{K}^0 \to \pi^+\pi^-(\tau)\right\rangle \quad \propto \quad (1+\epsilon)\Big(\left|K_S(\tau)\right\rangle - \eta_{+-}\left|K_L(\tau)\right\rangle\Big)$$

$$\begin{bmatrix} R(\tau) \\ \bar{R}(\tau) \end{bmatrix} \quad \propto \quad (1 \mp 2\mathcal{R}e(\epsilon))\big[e^{-\Gamma_s\tau} + |\eta_{+-}|^2 e^{-\Gamma_L\tau}$$

$$\pm\, 2|\eta_{+-}|e^{\frac{1}{2}(\Gamma_S+\Gamma_L)\tau}\cos(\Delta M_K\tau - \phi_{+-})\big]$$



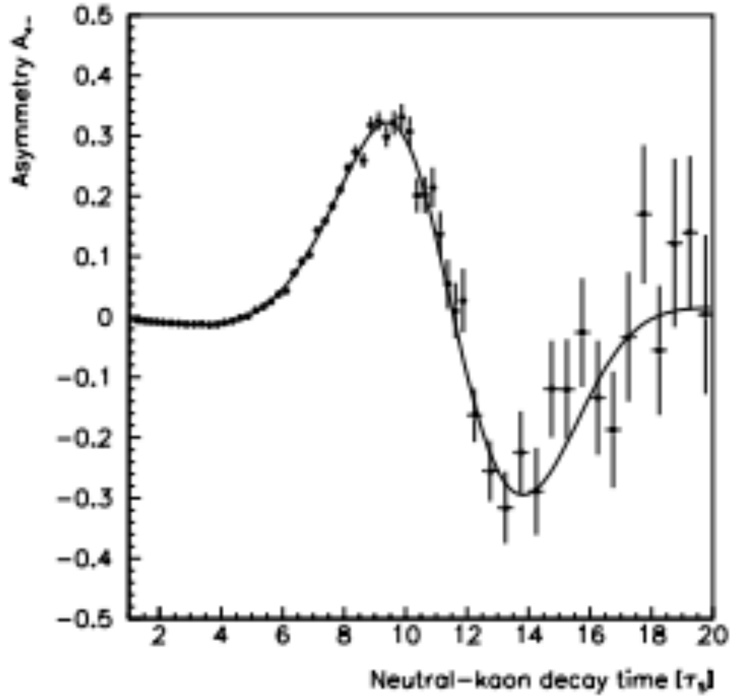
Phase of (η_{+-}) from CPLEAR

$$A_{+-}(\tau) = \frac{\bar{R}(\tau) - R(\tau)}{\bar{R}(\tau) + R(\tau)}$$

$$= -2\frac{|\eta_{+-}|e^{\frac{1}{2}(\tau/\tau_{\mathrm{S}}-\tau/\tau_{\mathrm{L}})}\cos(\Delta m\tau - \phi_{+-})}{1 + |\eta_{+-}|^2 e^{(\tau/\tau_{\mathrm{S}}-\tau/\tau_{\mathrm{L}})}},$$

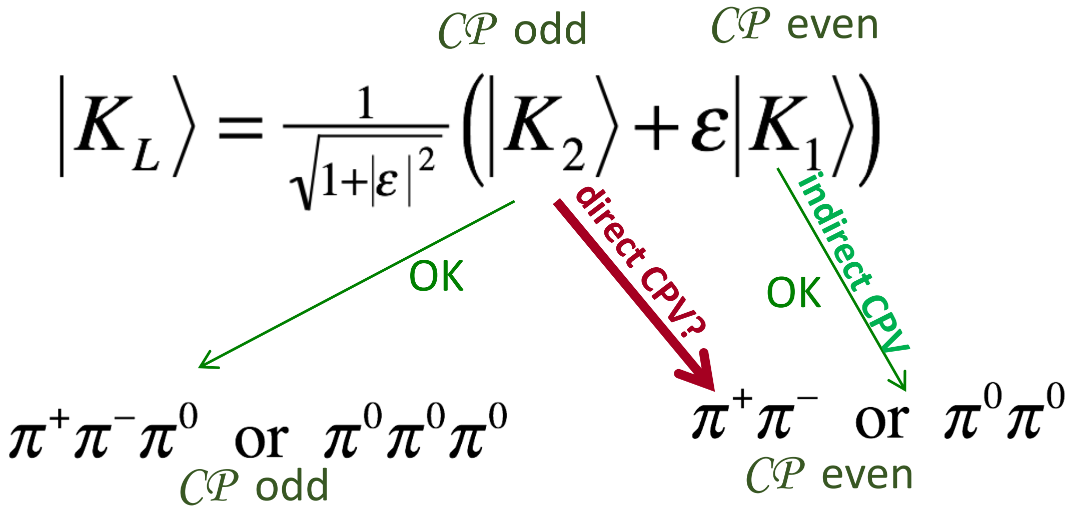
$$|\eta_{+-}| = (2.264 \pm 0.023_{\mathrm{stat}} \pm 0.026_{\mathrm{syst}} \pm 0.007_{\tau_{\mathrm{S}}}) \times 10^{-3}$$

$$\varphi_{+-} = 43.19^\circ \pm 0.53^\circ{}_{\mathrm{stat}} \pm 0.28^\circ{}_{\mathrm{syst}} \pm 0.42^\circ{}_{\Delta\mathrm{m}}$$



CPLEAR Phys. Lett. B458, 545 (1999)

big question 1



$\mathcal{CP}$ odd　　$\mathcal{CP}$ even

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K_2\rangle + \varepsilon|K_1\rangle\right)$$

$\pi^+\pi^-\pi^0$ or $\pi^0\pi^0\pi^0$

$\mathcal{CP}$ odd

$\pi^+\pi^-$ or $\pi^0\pi^0$

$\mathcal{CP}$ even

Is there a direct decay $K_{\mathcal{CP}\text{-odd}}$ to $\pi^+\pi^-$ or $\pi^0\pi^0$?

i.e., differences between $A(K^0 \to \pi\pi)$ and $A(\bar{K}^0 \to \pi\pi)$

η_{+-} and η_{00} with direct decays

direct CP?

$$\eta_{+-} \equiv \frac{\langle \pi^+\pi^- | H | K_L \rangle}{\langle \pi^+\pi^- | H | K_S \rangle} = \frac{\langle \pi^+\pi^- | H | K_2 \rangle + \varepsilon \langle \pi^+\pi^- | H | K_1 \rangle}{\langle \pi^+\pi^- | H | K_1 \rangle} = \varepsilon + \frac{\langle \pi^+\pi^- | H | K_2 \rangle}{\langle \pi^+\pi^- | H | K_1 \rangle}$$

direct CP?

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0 | H | K_L \rangle}{\langle \pi^0\pi^0 | H | K_S \rangle} = \frac{\langle \pi^0\pi^0 | H | K_2 \rangle + \varepsilon \langle \pi^0\pi^0 | H | K_1 \rangle}{\langle \pi^0\pi^0 | H | K_1 \rangle} = \varepsilon + \frac{\langle \pi^0\pi^0 | H | K_2 \rangle}{\langle \pi^0\pi^0 | H | K_1 \rangle}$$

direct $K_2 \rightarrow \pi\pi$ effects $\pi^+\pi^-$ & $\pi^0\pi^0$ differently

express $K^0(\overline{K}^0)\rightarrow\pi\pi$ in Isospin states

since we use CP(K^0)=+$\overline{K}^0$

$$\langle \pi\pi;I=0|H|K^0\rangle = A_0e^{i\delta_0} \qquad \langle \pi\pi;I=0|H|\overline{K}^0\rangle = -A_0^*e^{i\delta_0}$$

$$\langle \pi\pi;I=2|H|K^0\rangle = A_2e^{i\delta_2} \qquad \langle \pi\pi;I=2|H|\overline{K}^0\rangle = -A_2^*e^{i\delta_2}$$

δ_0 & $\delta_2 = \pi\pi$ strong int. phase shifts; same for K^0 & $\overline{K}^0$

Clebsch-Gordon coefficients

$$|\pi^+\pi^-\rangle = \sqrt{\tfrac{2}{3}}|\pi\pi;I=0\rangle + \sqrt{\tfrac{1}{3}}|\pi\pi;I=2\rangle \qquad |\pi^0\pi^0\rangle = -\sqrt{\tfrac{1}{3}}|\pi\pi;I=0\rangle + \sqrt{\tfrac{2}{3}}|\pi\pi;I=2\rangle$$

$$\begin{aligned}\langle\pi^+\pi^-|H|K_2\rangle &= \tfrac{1}{\sqrt{2}}\left[\langle\pi^+\pi^-|H|K^0\rangle + \langle\pi^+\pi^-|H|\overline{K}^0\rangle\right]\\ &= \tfrac{1}{\sqrt{2}}\left(\sqrt{\tfrac{2}{3}}(A_0-A_0^*)e^{i\delta_0} + \sqrt{\tfrac{1}{3}}(A_2-A_2^*)e^{i\delta_2}\right)\\ &= \mathrm{i}\sqrt{\tfrac{2}{3}}\left(\sqrt{2}\,\mathrm{Im}\,A_0e^{i\delta_0} + \mathrm{Im}\,A_2e^{i\delta_2}\right)\end{aligned}$$

$$\langle\pi^+\pi^-|H|K_1\rangle = \sqrt{\tfrac{2}{3}}\left(\sqrt{2}\,\mathrm{Re}\,A_0e^{i\delta_0} + \mathrm{Re}\,A_2e^{i\delta_2}\right)$$

$$\begin{aligned}\langle\pi^0\pi^0|H|K_2\rangle &= \tfrac{1}{\sqrt{2}}\left[\langle\pi^0\pi^0|H|K^0\rangle - \langle\pi^0\pi^0|H|\overline{K}^0\rangle\right]\\ &= \tfrac{1}{\sqrt{2}}\left(+\sqrt{\tfrac{1}{3}}(A_0-A_0^*)e^{i\delta_0} - \sqrt{\tfrac{2}{3}}(A_2-A_2^*)e^{i\delta_2}\right)\\ &= \mathrm{i}\sqrt{\tfrac{2}{3}}\left(+\mathrm{Im}\,A_0e^{i\delta_0} - \sqrt{2}\,\mathrm{Im}\,A_2e^{i\delta_2}\right)\end{aligned}$$

$$\langle\pi^0\pi^0|H|K_1\rangle = \sqrt{\tfrac{2}{3}}\left(+\mathrm{Re}\,A_0e^{i\delta_0} - \sqrt{2}\,\mathrm{Re}\,A_2e^{i\delta_2}\right)$$

Important point:

The "Wu-Yang" phase-convention

Physics is in the difference between A_0 & A_2 phases. It is customary* to chose A_0 to be real

$$\langle\pi^+\pi^-|H|K_2\rangle = i\sqrt{\tfrac{2}{3}}\,\mathrm{Im}\,A_2e^{i\delta_2} \qquad \langle\pi^0\pi^0|H|K_2\rangle = -i\sqrt{\tfrac{4}{3}}\,\mathrm{Im}\,A_2e^{i\delta_2}$$

$$\langle\pi^+\pi^-|H|K_1\rangle = \sqrt{\tfrac{4}{3}}A_0e^{i\delta_0}\left(1+\frac{\mathrm{Re}\,A_2}{\sqrt{2}A_0}e^{i(\delta_2-\delta_0)}\right) \qquad \langle\pi^0\pi^0|H|K_1\rangle = \sqrt{\tfrac{2}{3}}A_0e^{i\delta_0}\left(1-\frac{\sqrt{2}\,\mathrm{Re}\,A_2}{A_0}e^{i(\delta_2-\delta_0)}\right)$$

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*T. T. Wu and C. N. Yang, *Phys. Rev. Lett.* **13**, 380 (1964)

$$\langle \pi^+\pi^- | H | K_2 \rangle = i\sqrt{\tfrac{2}{3}}\,\mathrm{Im}\,A_2 e^{i\delta_2}$$

$$\langle \pi^+\pi^- | H | K_1 \rangle = \sqrt{\tfrac{4}{3}}\,A_0 e^{i\delta_0}\left(1+\frac{\mathrm{Re}\,A_2}{\sqrt{2}A_0}e^{i(\delta_2-\delta_0)}\right)$$

from previous slide

$$\langle \pi^0\pi^0 | H | K_2 \rangle = -\,i\sqrt{\tfrac{4}{3}}\,\mathrm{Im}\,A_2 e^{i\delta_2}$$

$$\langle \pi^0\pi^0 | H | K_1 \rangle = \sqrt{\tfrac{2}{3}}\,A_0 e^{i\delta_0}\left(1-\frac{\sqrt{2}\,\mathrm{Re}\,A_2}{A_0}e^{i(\delta_2-\delta_0)}\right)$$

define:

$$\boxed{\varepsilon' = \frac{i}{\sqrt{2}}\frac{\mathrm{Im}\,A_2}{A_0}e^{i(\delta_2-\delta_0)} \qquad \omega = \frac{\mathrm{Re}\,A_2}{A_0}e^{i(\delta_2-\delta_0)}}$$

$$\frac{\langle \pi^+\pi^- | H | K_2 \rangle}{\langle \pi^+\pi^- | H | K_1 \rangle} = \frac{\varepsilon'}{1+\frac{1}{\sqrt{2}}\omega}$$

$$\frac{\langle \pi^0\pi^0 | H | K_2 \rangle}{\langle \pi^0\pi^0 | H | K_1 \rangle} = -\frac{2\varepsilon'}{1-\sqrt{2}\omega}$$

ΔI=1/2 rule →

$$\boxed{|\omega| = \frac{\mathrm{Re}\,A_2}{A_0} \approx \frac{\sqrt{Bf(K^+\to\pi^+\pi^0)\tau_{KS}}}{\sqrt{Bf(K_S\to\pi^+\pi^-)\tau_{K+}}} = \sqrt{\frac{0.21\times 0.1\mathrm{ns}}{0.69\times 12\mathrm{ns}}} \simeq \frac{1}{22}}$$

$$\frac{\langle \pi^+\pi^- | H | K_2 \rangle}{\langle \pi^+\pi^- | H | K_1 \rangle} \simeq \varepsilon'$$

$$\frac{\langle \pi^0\pi^0 | H | K_2 \rangle}{\langle \pi^0\pi^0 | H | K_1 \rangle} \simeq -2\varepsilon'$$

$$\eta_{+-} = \varepsilon + \varepsilon'$$ ← **different** → $$\eta_{00} = \varepsilon - 2\varepsilon'$$

Phase of ε′

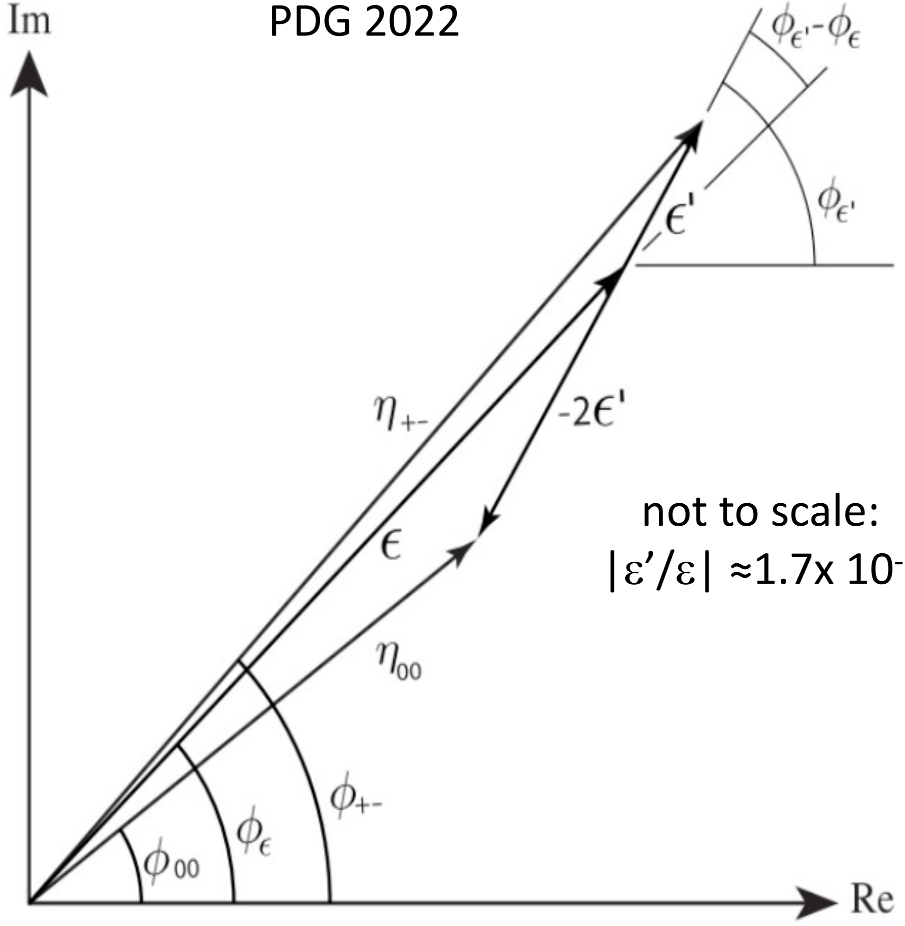
$$\varepsilon' = \frac{i}{\sqrt{2}} \frac{\operatorname{Im} A_2}{A_0} e^{i(\delta_2 - \delta_0)}$$

$$\delta_2 - \delta_0 = -47.7^\circ \pm 1.5^\circ$$

G. Colangelo, J. Gasser and H. Leutwyler, $\pi\pi$ scattering, *Nucl. Phys. B* **603**, pp. 12
(2001), doi:10.1016/S0550-3213(01)00147-X, `arXiv:hep-ph/0103088`.

$$\phi_{\varepsilon'} = \delta_2 - \delta_0 + \frac{\pi}{2} \approx 42.3^\circ \pm 1.5^\circ$$

$$\phi_{\rm SW} = \tan^{-1}\frac{2\Delta m}{\Delta\Gamma} = 43.30^\circ \pm 0.16^\circ,$$



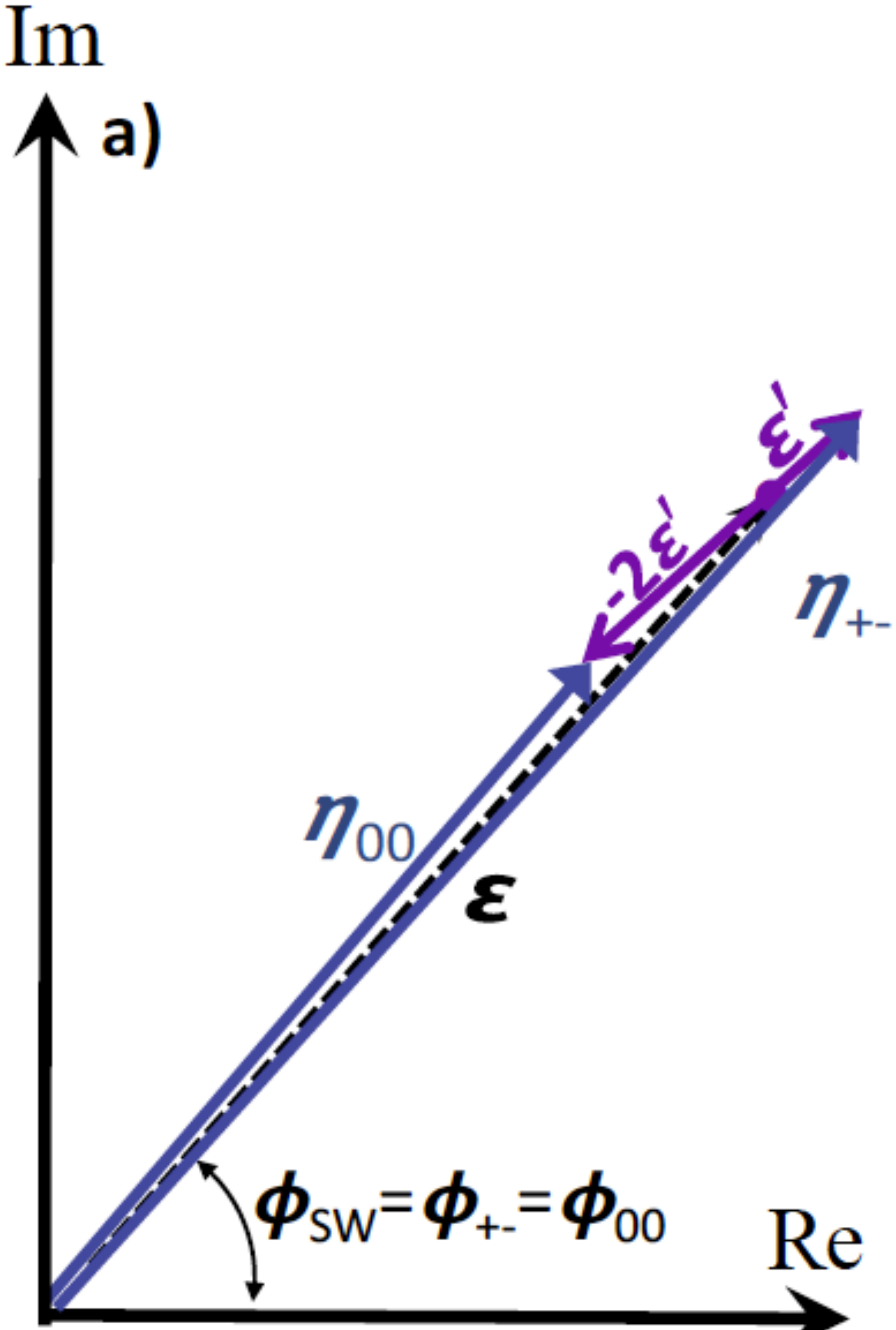
PDG 2022

not to scale:
$|\varepsilon'/\varepsilon| \approx 1.7\text{x } 10^{-3}$

Miracle #2

ε' and ε are parallel (to within ~< 1.5°)

phases of η_{+-} and η_{00} are insensitive to to uncertainty in the length of ε'



Big question 2: is the phase of ε really ϕ_{sw}?

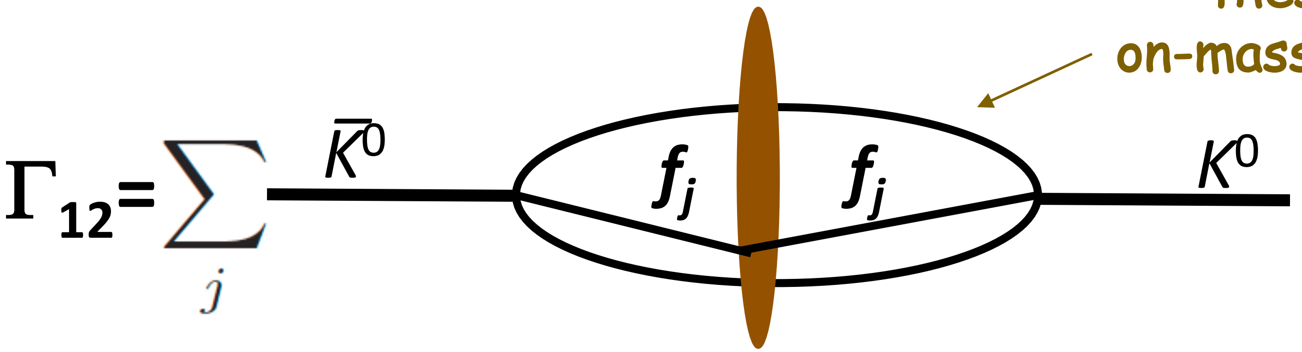
$$\epsilon = \frac{i\mathrm{Im}M_{12} + \frac{1}{2}\mathrm{Im}\Gamma_{12}}{(M_L - M_S) - \frac{i}{2}(\Gamma_S - \Gamma_L)} = \frac{i\mathrm{Im}M_{12} + \frac{1}{2}\mathrm{Im}\Gamma_{12}}{\Delta M - \frac{i}{2}\Delta\Gamma}$$

if $\mathrm{Im}\Gamma_{12}$=0: $$\Longrightarrow \frac{i\mathrm{Im}M_{12}}{\Delta M - \frac{i}{2}\Delta\Gamma} = \underbrace{\frac{\mathrm{Im}M_{12}}{(\Delta\Gamma/2)^2 + \Delta M^2}}_{\text{real number}}(\Delta\Gamma/2 + i\Delta M)$$

$$\phi_{\mathrm{SW}} = \tan^{-1}\frac{2\Delta m}{\Delta\Gamma} = 43.30^\circ \pm 0.16^\circ,$$

can we really ignore $\mathrm{Im}\Gamma_{12}$?

Big question 2: What is the Im Γ_{12}?



these are alll on-mass-shell states

$$\Gamma_{12} = \sum_j \langle K^0 | H_w | f_j \rangle \langle f_j | H_w | \bar{K}^0 \rangle$$

in terms of the K_S & K_L basis

$$\Gamma_{12} = \sum_j \left(|\langle f_j | H_w | K_S \rangle|^2 - |\langle f_j | H_w | K_L \rangle|^2 - 2i\,\mathrm{Im}\left(\langle f_j | H_w | K_S \rangle^* \langle f_j | H_w | K_L \rangle \right) \right)$$

$$\mathrm{Im}\,\Gamma_{12} = 2 \sum_j \mathrm{Im}(\langle K_S | H_w | f_j \rangle^* \langle f_j | H_w | K_L \rangle)$$

have to consider decay modes common to K_S and K_L

Relevant K_S decay modes

mode	Branching fraction	comment
$\pi^+\pi^-$	69.2%	CP conserving (real) CP violating (complex) $0.95\,\langle K\vert\, H_w\,\vert \pi\pi_{I=0}\rangle + 0.05\,\langle K\vert\, H_w\,\vert \pi\pi_{I=2}\rangle$
$\pi^0\pi^0$	30.7%	
$\pi^+\pi^-\gamma_{\text{non-brems}}$	1.8 x 10^{-3}	
$\pi^+\pi^-\pi^0$	3.5 x 10^{-7}	
$\pi^0\pi^0\pi^0$	< 2.6 x 10^{-8}	
$\pi^\pm e^\mp \nu$	7.0 x 10^{-7}	ΔQ=-1 $\langle K\vert\, H_w\,\vert \pi^-\ell^+\nu\rangle$ $\xcancel{\langle \bar{K}\vert\, H_w\,\vert \pi^-\ell^+\nu\rangle}$ ΔQ=+1 $\xcancel{\langle K\vert\, H_w\,\vert \pi^+\ell^-\bar{\nu}\rangle}$ $\langle \bar{K}\vert\, H_w\,\vert \pi^+\ell^-\bar{\nu}\rangle$ ⟸ ΔS=ΔQ rule ΔS=-1 ΔS=+!
$\pi^\pm \mu^\mp \nu$	less than $\pi^\pm e^\mp \nu$	

$\mathcal{CP}$ violation in $K_S \rightarrow \pi^+\pi^-\pi^0$ decay?

analog to the $K_L \rightarrow \pi^+\pi^-$ discussion the K_2 component of K_S can decay to $\pi^+\pi^-\pi^0$
What is the expected rate?

$$|\epsilon| \overset{?}{=} \frac{|\mathcal{A}(K_S \to \pi^+\pi^-\pi^0)|}{|\mathcal{A}(K_L \to \pi^+\pi^-\pi^0)|} = \sqrt{\frac{\mathcal{B}(K_S \to \pi^+\pi^-\pi^0)\tau_L}{\mathcal{B}(K_L \to \pi^+\pi^-\pi^0)\tau_S}} = 2.28 \pm 0.01 \times 10^{-3}?$$

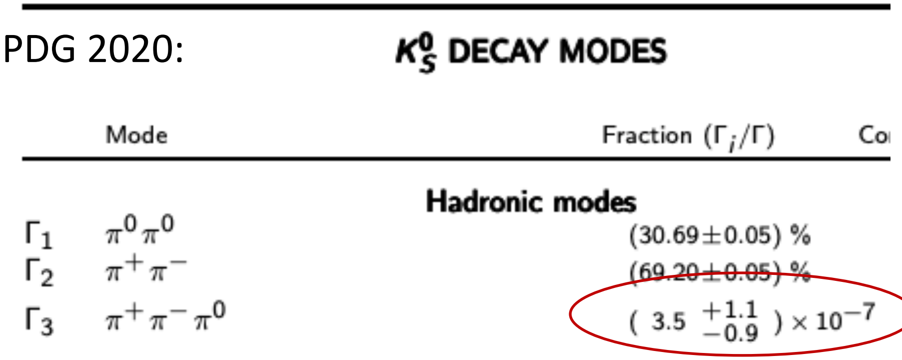
$$\mathcal{B}(K_S \to \pi^+\pi^-\pi^0) \overset{?}{=} |\epsilon|^2 \frac{\tau_S}{\tau_L} \mathcal{B}(K_L \to \pi^+\pi^-\pi^0) \approx 10^{-9}$$

$\sim(5 \times 10^{-6}) \times (2 \times 10^{-3}) \times 10^{-1}$

from PDG 2020:
$Bf(K_L \rightarrow \pi^+\pi^-\pi^0) = 0.1254 \pm 0.0005 \times 10^{-3}$
$\tau_S/\tau_L = 1.75 \pm 0.01 \times 10^{-3}$

100x larger?

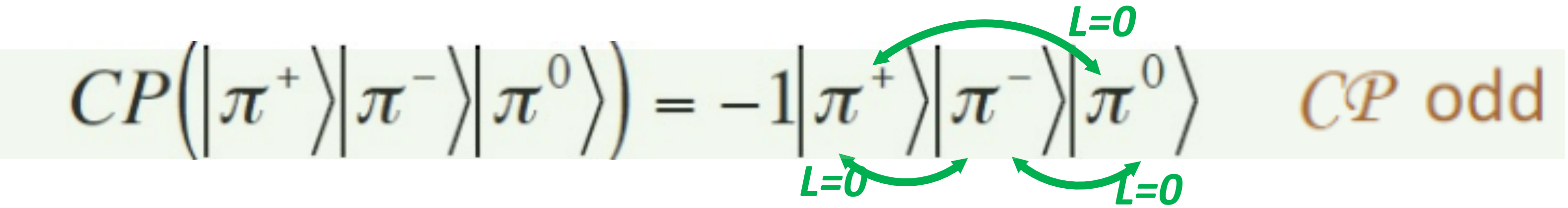
PDG 2020:



K_S^0 DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Co
	Hadronic modes		
Γ_1	$\pi^0\pi^0$	(30.69 ± 0.05) %	
Γ_2	$\pi^+\pi^-$	(69.20 ± 0.05) %	
Γ_3	$\pi^+\pi^-\pi^0$	$(3.5^{+1.1}_{-0.9}) \times 10^{-7}$	

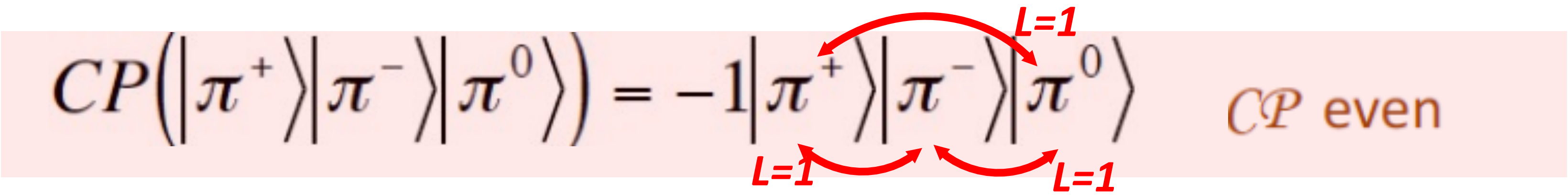
$\mathcal{CP}$ of the $\pi^+\pi^-\pi^0$ system revisited



$$CP\left(|\pi^+\rangle|\pi^-\rangle|\pi^0\rangle\right) = -1|\pi^+\rangle|\pi^-\rangle|\pi^0\rangle$$

$\mathcal{CP}$ odd

only true if all 3 pion pairs are in an *S*-wave, *i.e.* *L*=0



$$CP\left(|\pi^+\rangle|\pi^-\rangle|\pi^0\rangle\right) = -1|\pi^+\rangle|\pi^-\rangle|\pi^0\rangle$$

$\mathcal{CP}$ even

if all 3 pion pairs are in *P*-waves: $\mathcal{CP} = (-1)^{3+1} = +1$ ⟸ $\mathcal{CP}$ even

strongly suppressed by "centrifugal barriers," but not zero (~100x expected $\mathcal{CPV}$ level)

need to do Dalitz plot analysis of $K_S \rightarrow 3\pi$ & $K_L \rightarrow 3\pi$ interference. See:

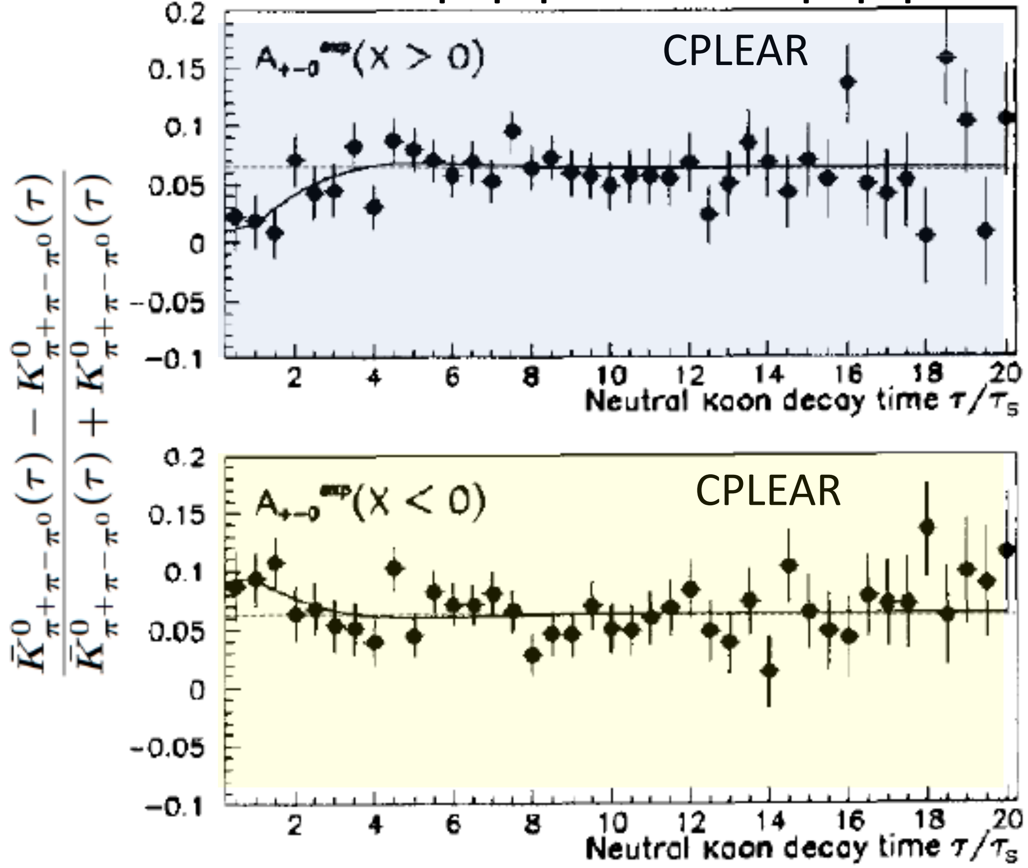
Experiment: *Phys.Lett.B* 630 (2005) 31
Theory: Phys. Rev. D 46 (1992) 252

Measurements of SM $\mathcal{CP}$V effects in $K_S \rightarrow \pi^+\pi^-\pi^0$ are probably hopeless

Measuring $K_S \rightarrow p^+p^-p^0$

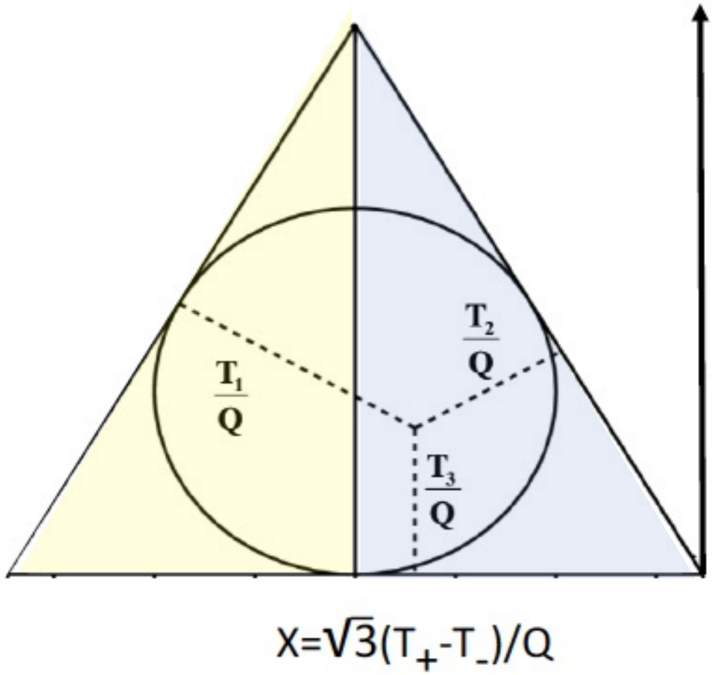
current best results from CPLEAR: time-dependent $\bar{K}^0 \rightarrow p^+p^-p^0 - K^0 \rightarrow p^+p^-p^0$ differences

$\bar{K}^0 \rightarrow p^+p^-p^0 - K^0 \rightarrow p^+p^-p^0$



Re $a_{p+p-p0} \propto A_{+-0}(X>0) - A_{+-0}(X<0)$

Im $a_{p+p-p0} \propto A_{+-0}(X>0) - A_{+-0}(X<0)$

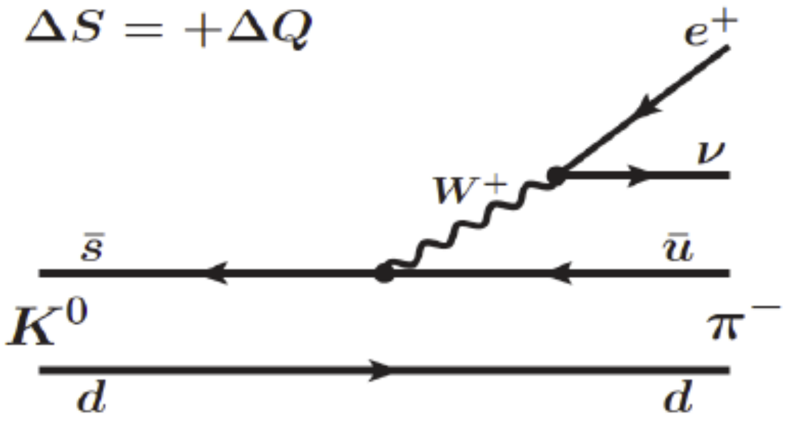


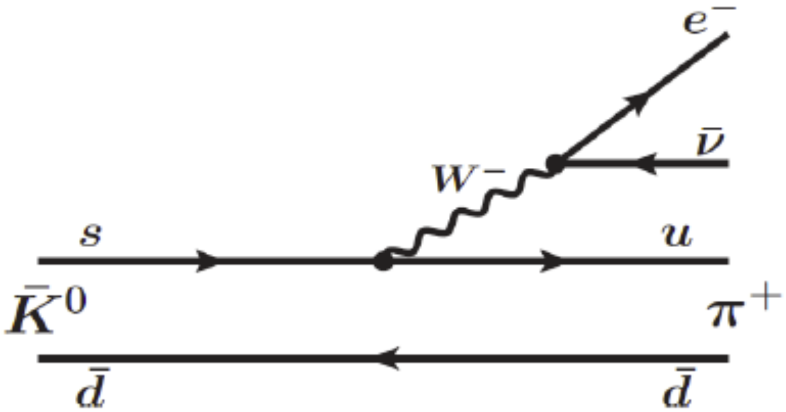
What about the semileptonic decays?

mode	Branching fraction	comment
$\pi^+\pi^-$	69.2%	CP conserving (real) CP violating (complex) $0.95 \langle K\vert H_w \vert\pi\pi_{I=0}\rangle + 0.05 \langle K\vert H_w \vert\pi\pi_{I=2}\rangle$
$\pi^0\pi^0$	30.7%	
$\pi^+\pi^-\gamma_{\text{non-brems}}$	1.8×10^{-3}	~CP conserving (real) to >1 part in 10^4
$\pi^+\pi^-\pi^0$	3.5×10^{-7}	mostly CP conserving
$\pi^0\pi^0\pi^0$	$< 2.6 \times 10^{-8}$	entirely CP violating
$\pi^\pm e^\mp\nu$	7.0×10^{-7}	$\begin{array}{lcc} \Delta Q=-1 & \langle K\vert H_w \vert\pi^-\ell^+\nu\rangle & \cancel{\langle \bar{K}\vert H_w \vert\pi^-\ell^+\nu\rangle} \\ \Delta Q=+1 & \cancel{\langle K\vert H_w \vert\pi^+\ell^-\bar{\nu}\rangle} & \langle \bar{K}\vert H_w \vert\pi^+\ell^-\bar{\nu}\rangle \\ & \Delta S=-1 & \Delta S=+ \end{array}$ ⇐ ΔS=ΔQ rule
$\pi^\pm\mu^\mp\nu$	less than $\pi^\pm e^\mp\nu$	

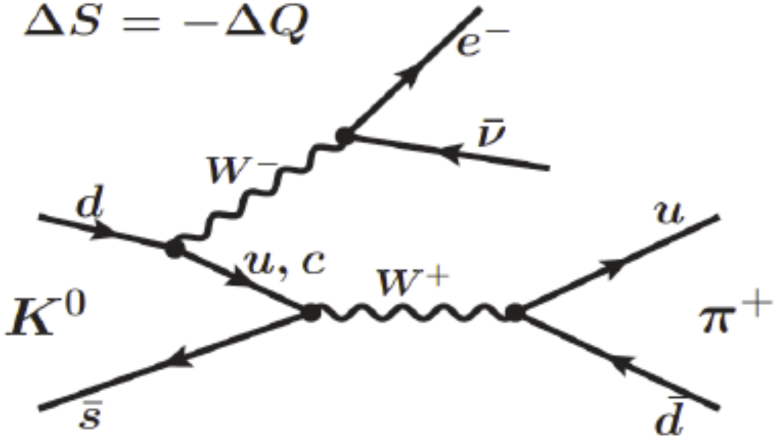
SM violations of the ΔS=ΔQ rule

-- immeasurably small --

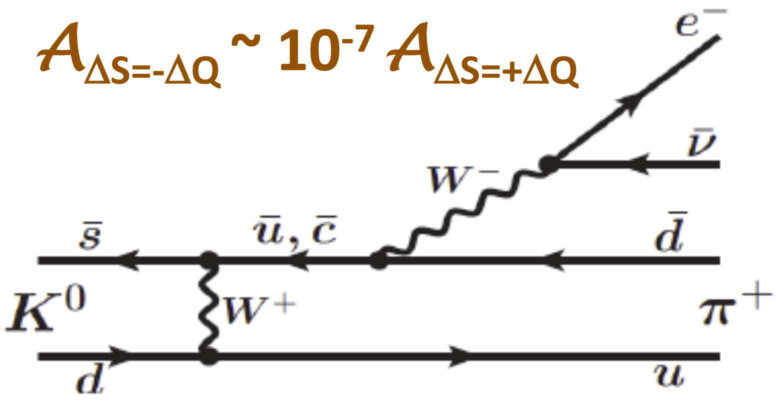




2nd-order Weak-Interaction



$A_{\Delta S=-\Delta Q} \sim 10^{-7} A_{\Delta S=+\Delta Q}$



some arithmetic

from an earlier slide: $\mathrm{Im}\,\Gamma_{12} = 2\sum_j \mathrm{Im}(\langle K_S|\, H_w\, |f_j\rangle^* \langle f_j|\, H_w\, |K_L\rangle)$

multiply and divide by $\langle j|\, H_w\, |K_S\rangle$ use $\mathcal{B}(K_S \to f_j) = \dfrac{|\,\langle f_j|\, H_w\, |K_S\rangle\,|^2}{\Gamma_S}$

$$\tan\phi_{\Gamma_{12}} = \frac{\mathrm{Im}\,\Gamma_{12}}{\Gamma_S} = \sum_j \frac{\langle j|\, H_w\, |K_L\rangle}{\langle j|\, H_w\, |K_S\rangle}\mathcal{B}(K_S \to j) = \sum_j \mathrm{Im}\,\alpha_j$$

$$\alpha_j \equiv \overset{=\eta_j}{\boxed{\frac{\langle f_j|\, H_w\, |K_L\rangle}{\langle f_j|\, H_w\, |K_S\rangle}}}\mathcal{B}(K_S \to f_j) = \eta_j \mathcal{B}(K_S \to f_j)$$

calculate phase of Γ_{12} term-by-term

dominant mode:

$K \rightarrow \pi\pi$ (I=0): Wu-Yang phase convention $\langle K^0 | H_w | \pi\pi_{I=0} \rangle \langle \pi\pi_{I=0} | H_w | \bar{K}^0 \rangle$ is real

sub-dominant mode'

$$K \rightarrow \pi\pi\ (\text{I=2}):\ \alpha_{\pi+\pi-} - 2\alpha_{\pi0\pi0} = (\eta_{+-} - \eta_{00}) Bf(K_S \rightarrow \pi\pi_{I=2}) = 3\varepsilon' Bf(K_S \rightarrow \pi\pi_{I=2}) \approx 3\overset{\text{phase}\approx\phi_{SW}}{e'} \times \overset{\Delta\text{I=1/2 rule}}{\frac{1}{500}} \approx 3 \times 10^{-8}$$

other modes	experimental values	
$\alpha_{\pi^+\pi^-\pi^0}$	$((0 \pm 2) + i(0 \pm 2)) \times 10^{-6}$	CPLEAR
$\alpha_{\pi^0\pi^0\pi^0}$	$< 1.5 \times 10^{-6}$	KLOE
$\alpha_{\pi\ell\nu}$	$((-0.1 \pm 0.2) + i(-0.1 \pm 0.5)) \times 10^{-6}$	CPLEAR

$$\sum \text{Im}\, \alpha_j \lesssim 5\text{x } 10^{-6}$$

$$\delta\phi_\varepsilon \approx \frac{\phi_{\Gamma_{12}}}{2\text{Re}\varepsilon}: \quad \boxed{\phi_\varepsilon^{\text{Data}} - \phi_{\text{SW}} < 0.1^\circ}$$

These limits can all be improved with 10^{12} J/ys

difference between ϕ_{ε} & ϕ_{sw} in the SM

here I assumesd

$\eta'_{3\pi} = \varepsilon'_{3\pi}/\varepsilon$ <1

Bf($K \rightarrow \pi^+\pi^-\pi^0$ (CPV) ≈ 10^{-9} $\alpha_{\pi0\pi0\pi0} \approx 10^{-7}$

Bf($K \rightarrow \pi^0\pi^0\pi^0$ (CPV) ≈ $2\mathrm{x}10^{-9}$ $\alpha_{\pi+\pi-\pi0} \approx 2\mathrm{x}10^{-7}$

$\langle K|\, H_w \,|\pi\ell\nu\rangle^* \,\langle\pi\ell\nu|\, H_w \,|\bar{K}\rangle$ = 0 ← ΔS=ΔQ rule

$\phi_{\Gamma_{12}} \approx 3 \times 10^{-7}$

$$\Delta\phi_{\varepsilon} \approx \frac{\phi_{\Gamma_{12}}}{2\mathrm{Re}\,\varepsilon}:$$

$$\boxed{\phi_{\varepsilon}^{\mathrm{SM}} - \phi_{\mathrm{SW}} \approx 0.01^{\circ}}$$

Neutral Kaon system without a CPT constraint

$M_{11} \neq M_{22}$ & $\Gamma_{11} \neq \Gamma_{22}$:

$$\boldsymbol{H} = \begin{pmatrix} \langle K^0 | H | K^0 \rangle & \langle K^0 | H | \overline{K}^0 \rangle \\ \langle \overline{K}^0 | H | K^0 \rangle & \langle \overline{K}^0 | H | \overline{K}^0 \rangle \end{pmatrix} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

Schrodinger's eqn:

$$\begin{pmatrix} X_{11} & -ip^2 \\ -iq^2 & X_{22} \end{pmatrix} \begin{pmatrix} a_i \\ b_i \end{pmatrix} e^{i\lambda_i \tau} = i\frac{d}{d\tau} \begin{pmatrix} a_i \\ b_i \end{pmatrix} e^{-i\lambda_i \tau} \quad \Rightarrow \begin{pmatrix} X_{11} - \lambda_i & -ip^2 \\ -iq^2 & X_{22} - \lambda_i \end{pmatrix} \begin{pmatrix} a_i \\ b_i \end{pmatrix} = 0$$

$e_S = e + d$

$$|K_S\rangle = \left(p\overset{\approx 1+\delta}{\sqrt{1+2\delta}}\,|K^0\rangle + q\overset{\approx 1-\delta}{\sqrt{1-2\delta}}\,|\bar{K}^0\rangle\right) = (1+\varepsilon_S)\,|K^0\rangle + (1-\varepsilon_S)\,|\bar{K}^0\rangle$$

$$|K_L\rangle = \left(p\sqrt{1-2\delta}\,|K^0\rangle - q\sqrt{1+2\delta}\,|\bar{K}^0\rangle\right) = (1+\varepsilon_L)\,|K^0\rangle - (1-\varepsilon_L)\,|\bar{K}^0\rangle$$

$e_L = e - d$

$$\delta = \frac{(M_{\bar{K}^0} - M_{K^0}) - i(\Gamma_{\bar{K}^0} - \Gamma_{K^0})/2}{2\Delta M - i\Delta\Gamma}$$

$$\langle K_S | K_L \rangle = 2\,\mathrm{Re}\,\varepsilon - 2i\,\mathrm{Im}\,\delta$$

Miracle #3

$$\varepsilon = \frac{i\mathrm{Im}M_{12} + \frac{1}{2}\mathrm{Im}\Gamma_{12}}{\Delta M - \frac{i}{2}\Delta\Gamma} \qquad \delta = \frac{(M_{\bar{K}^0} - M_{K^0}) - i(\Gamma_{\bar{K}^0} - \Gamma_{K^0})/2}{2\Delta M - i\Delta\Gamma}$$

this term and this term

differ in phase by 90°

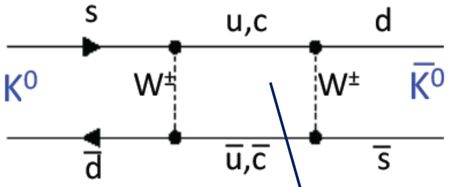
$$\delta \approx \frac{i(M_{\bar{K}^0} - M_{K^0}) + (\Gamma_{\bar{K}^0} - \Gamma_{K^0})/2}{2\sqrt{2}\Delta M} e^{i\phi_{\mathrm{SW}}}$$

$$= (i\delta_\perp + \delta_{||})e^{i\phi_{\mathrm{SW}}}$$

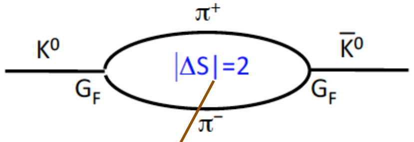
(ϕ_{00} still equals ϕ_{+-})

effect of δ on ϕ_{+-} (ϕ_{00})

short-distance physics



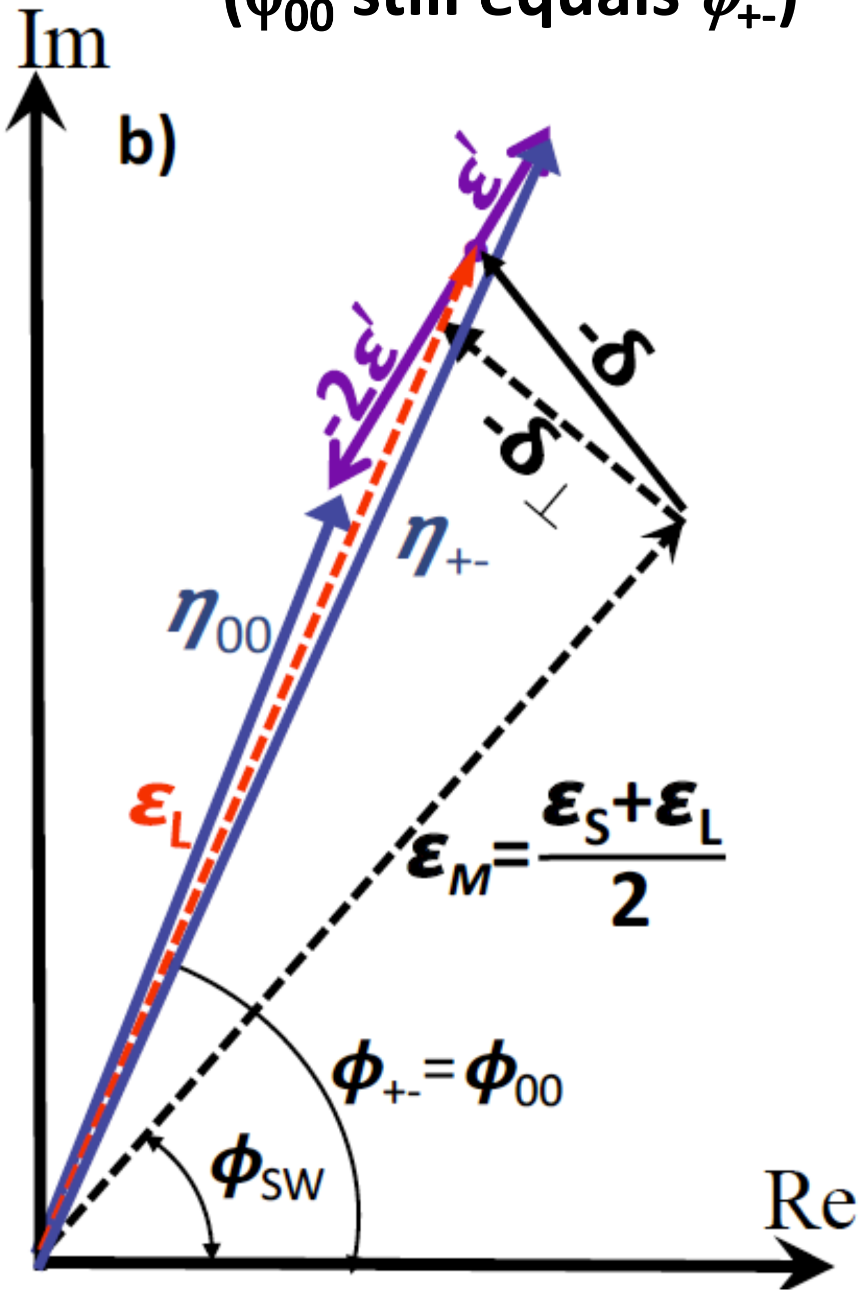
long-distance physics



$$\delta \approx \frac{i(M_{\bar{K}^0}-M_{K^0})+(\Gamma_{\bar{K}^0}-\Gamma_{K^0})/2}{2\sqrt{2}\Delta M}e^{i\phi_{\rm SW}}$$

$$= (i\delta_{\perp}+\delta_{||})e^{i\phi_{\rm SW}}$$

$d_{\perp}\approx(f_{+-}-f_{SW})e$



ϕ_{+-}: maximum sensitivity to short-distance physics

Wu-Yang Triangle with indirect + direct CPTV

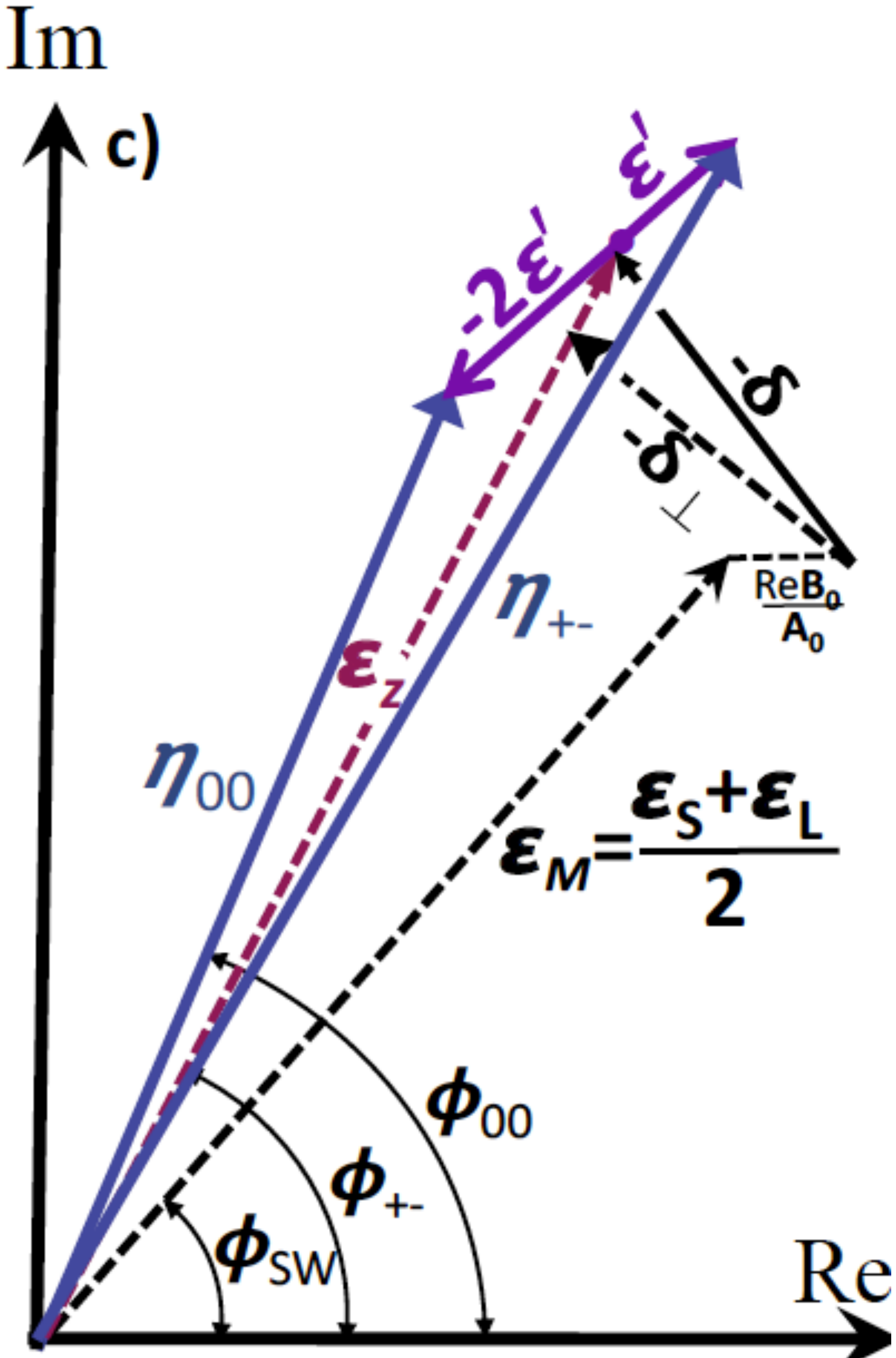
$$\langle \pi\pi; I=0 |\, H_W \,| K^0 \rangle = (A_0 + B_0) e^{i\delta_0}$$
$$\langle \pi\pi; I=2 |\, H_W \,| K^0 \rangle = (A_2 + B_2) e^{i\delta_2}$$

$$\langle \pi\pi; I=0 |\, H_W \,| \bar{K}^0 \rangle = (A_0^* - B_0^*) e^{i\delta_0}$$
$$\langle \pi\pi; I=0 |\, H_W \,| \bar{K}^0 \rangle = (A_2^* - B_2^*) e^{i\delta_2}$$

- Wu-Yang phase convention: A_0 is real but not B_0 or B_2

- Re B_0/A_0 common to $\pi^+\pi^-$ & $\pi^0\pi^0$

– ε' gets rotated: $\phi_{00} \neq \phi_{+-}$



Phase of (η_{+-}) from CPLEAR

CPLEAR Phys. Lett. B458, 545 (1999)

$$A_{+-}(\tau) = \frac{\bar{R}(\tau) - R(\tau)}{\bar{R}(\tau) + R(\tau)}$$

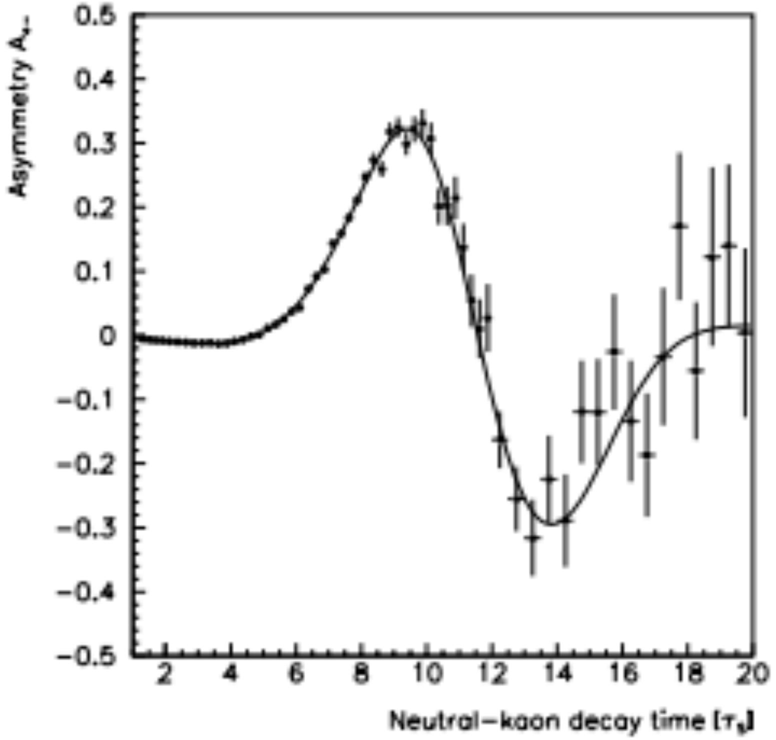
$$= -2\frac{|\eta_{+-}|e^{\frac{1}{2}(\tau/\tau_{\mathrm{S}}-\tau/\tau_{\mathrm{L}})}\cos(\Delta m\tau - \phi_{+-})}{1 + |\eta_{+-}|^2 e^{(\tau/\tau_{\mathrm{S}}-\tau/\tau_{\mathrm{L}})}}.$$

$$|\eta_{+-}| = (2.264 \pm 0.023_{\mathrm{stat}} \pm 0.026_{\mathrm{syst}} \pm 0.007_{\tau_S})\times 10^{-3}$$

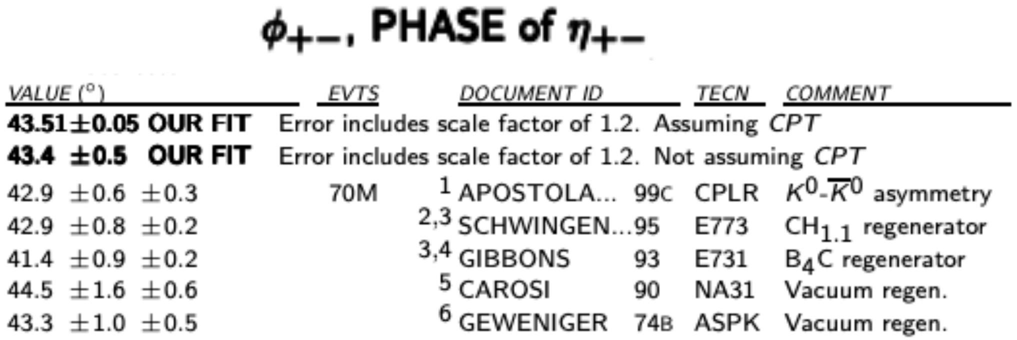
$$\varphi_{+-} = 43.19^\circ \pm 0.53^\circ{}_{\mathrm{stat}} \pm 0.28^\circ{}_{\mathrm{syst}} \pm 0.42^\circ{}_{\Delta\mathrm{m}}$$

in good agreement with the "Superweak phase"

$$\varphi_{\mathrm{sw}} = \arctan[2\Delta\mathrm{m}\Delta\Gamma] = 43.50^\circ \pm 0.08^\circ$$



$d_\perp$ with PDG 2022 averages



ϕ_{+-}, PHASE of η_{+-}

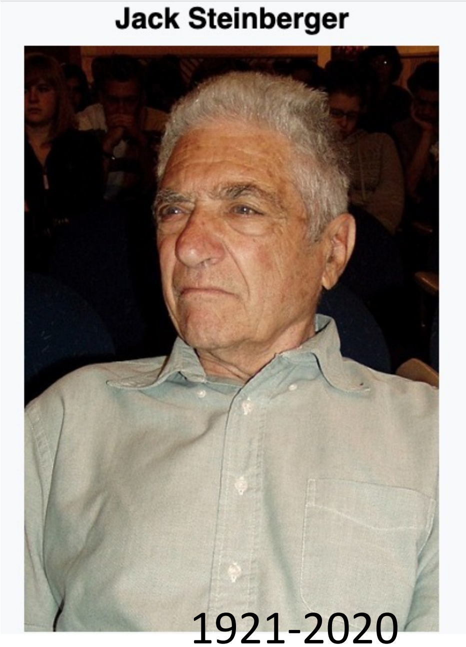
VALUE (°)	EVTS	DOCUMENT ID	TECN	COMMENT
43.51±0.05 OUR FIT	Error includes scale factor of 1.2. Assuming *CPT*			
43.4 ±0.5 OUR FIT	Error includes scale factor of 1.2. Not assuming *CPT*			
42.9 ±0.6 ±0.3	70M	[1] APOSTOLA... 99C	CPLR	K^0-$\overline{K}^0$ asymmetry
42.9 ±0.8 ±0.2		[2,3] SCHWINGEN...95	E773	$CH_{1.1}$ regenerator
41.4 ±0.9 ±0.2		[3,4] GIBBONS 93	E731	B_4C regenerator
44.5 ±1.6 ±0.6		[5] CAROSI 90	NA31	Vacuum regen.
43.3 ±1.0 ±0.5		[6] GEWENIGER 74B	ASPK	Vacuum regen.

$$\phi_{\rm SW} = \tan^{-1}\frac{2\Delta m}{\Delta\Gamma} = 43.30^\circ \pm 0.16^\circ,$$

$$\boxed{d_\perp \approx (f_{+-} - f_{SW})e = (0.4 \pm 2.0) \times 10^{-5}}$$

This was a “bottom→up” approach to CPT

Most theorists use a top→down approach called the Bell Steinberger relation


Jack Steinberger

1921-2020


John Stewart Bell

1928-1990

Bell-Steinberger relation

Schrodinger eqn:

$$\psi(\tau) = \alpha e^{-i\lambda_S \tau} \left|K_S\right\rangle + \beta e^{-i\lambda_L \tau} \left|K_L\right\rangle \quad (|\alpha|^2 + |\beta^2 = 1)$$

$$|\psi(\tau)|^2 = |\alpha|^2 e^{-\Gamma_S \tau} + |\beta|^2 e^{-\Gamma_L \tau} + 2\,\mathrm{Re}\left(\alpha^* \beta e^{-\frac{1}{2}(\Gamma_S + \Gamma_L + 2i\Delta M)\tau} \left\langle K_S | K_L \right\rangle\right) = |\psi(0)|^2 e^{-\Gamma_{\mathrm{tot}} \tau}$$

$$-\left.\frac{d|\psi(\tau)|^2}{d\tau}\right|_{\tau=0} = |\alpha|^2 \Gamma_S + |\beta|^2 \Gamma_L + \mathrm{Re}\left(\alpha^* \beta (\Gamma_S + \Gamma_L + 2i\Delta M) \left\langle K_S | K_L \right\rangle\right),$$

must be true for all values of *a* and *b*

$$(|\alpha|^2 + |\beta^2 = 1)$$

unitarity:

$$-\left.\frac{d|\psi(\tau)|^2}{d\tau}\right|_{\tau=0} = \Gamma_{\mathrm{tot}} = \sum_j \Gamma_j = \sum_j \left| \left\langle f_j | \psi(0) \right\rangle \right|^2$$

$$-\left.\frac{d|\psi(\tau)|^2}{d\tau}\right|_{\tau=0} = \sum_j \left| \alpha \left\langle f_j | K_S \right\rangle + \beta \left\langle f_j | K_L \right\rangle \right|^2$$

$$= \sum_j |\alpha|^2 \left| \left\langle f_j | K_S \right\rangle \right|^2 + |\beta|^2 \left| \left\langle f_j | K_L \right\rangle \right|^2 + 2\,\mathrm{Re}\left(\alpha^* \beta \left\langle f_j | K_S \right\rangle^* \left\langle f_j | K_L \right\rangle\right).$$

Bell-Steinberger relation

Schrodinger eqn:

$$\psi(\tau) = \alpha e^{-i\lambda_S \tau} |K_S\rangle + \beta e^{-i\lambda_L \tau} |K_L\rangle \quad (|\alpha|^2 + |\beta^2 = 1)$$

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$$-\left.\frac{d|\psi(\tau)|^2}{d\tau}\right|_{\tau=0} = |\alpha|^2 \Gamma_S + |\beta|^2 \Gamma_L + \mathrm{Re}\left(\alpha^* \beta (\Gamma_S + \Gamma_L + 2i\Delta M) \langle K_S|K_L\rangle\right)$$

$$\boxed{(\Gamma_S + \Gamma_L + 2i\Delta M) \langle K_S|K_L\rangle\,) = 2\sum_j \langle f_j|K_S\rangle^* \langle f_j|K_L\rangle}$$

unitarity:

$$-\left.\frac{d|\psi(\tau)|^2}{d\tau}\right|_{\tau=0} = \Gamma_{\mathrm{tot}} = \sum_j \Gamma_j = \sum_j |\langle f_j|\psi(0)\rangle|^2$$

$$-\left.\frac{d|\psi(\tau)|^2}{d\tau}\right|_{\tau=0} = \sum_j |\alpha \langle f_j|K_S\rangle + \beta \langle f_j|K_L\rangle|^2$$

$$= \sum_j |\alpha|^2 |\langle f_j|K_S\rangle|^2 + |\beta|^2 \langle f_j|K_L\rangle|^2 + 2\,\mathrm{Re}\left(\alpha^* \beta \langle f_j|K_S\rangle^* \langle f_j|K_L\rangle\right).$$

Bell-Steinberger relation

$$(\Gamma_S + \Gamma_L + 2i\Delta M)\,\langle K_S|K_L\rangle\,) = 2\sum \langle f_j|K_S\rangle^* \langle f_j|K_L\rangle$$

$$\langle K_S|K_L\rangle = 2\,\mathrm{Re}\,\varepsilon - 2i\,\mathrm{Im}\,\delta$$

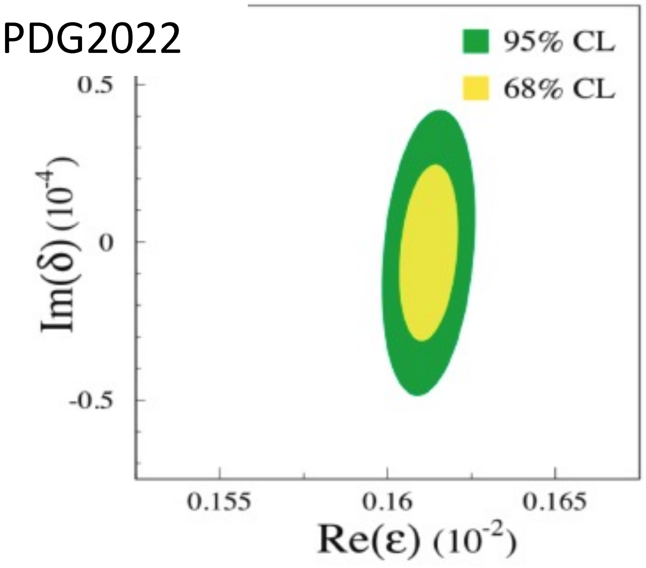
this is it: $$\boxed{\mathrm{Re}\,\varepsilon - i\,\mathrm{Im}\,\delta = \frac{\sum_j \langle f_j|K_S\rangle^* \langle f_j|K_L\rangle}{\Gamma_S + \Gamma_L + 2i\Delta M},}$$

not G$_S$-Γ_L!

$$\approx \frac{\sum_j \alpha_j}{1 + i\tan\phi_{\mathrm{SW}}} = \frac{\cos\phi_{\mathrm{SW}} \sum_j \alpha_j}{e^{i\phi_{\mathrm{SW}}}}$$

Bell-Steinberger deconstructed

PDG2022



top

down

$\text{Im}\, d = (0.3 \pm 1.4) \times 10^{-5}$

full BS analysis
(reviewed in PDG)

sin f_{SW}=1/√2

up

bottom

$$\boldsymbol{d_\perp \approx (f_{+-} - f_{SW}) e = (0.4 \pm 2.0) \times 10^{-5}}$$

f_{+-} and f_{SW}
comparison

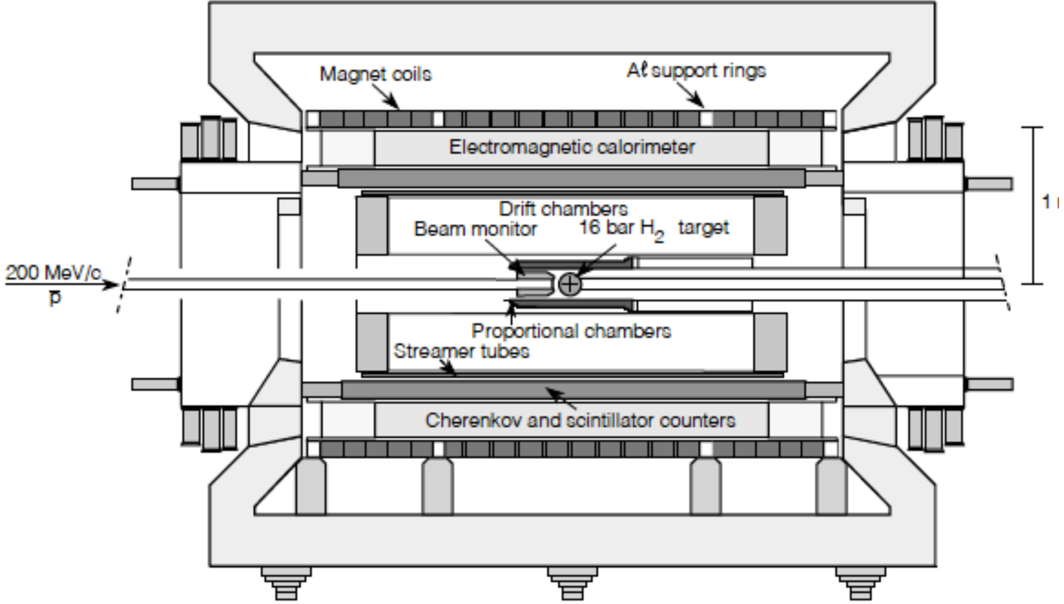
my interpretation of BS :

-- $\boldsymbol{d_\perp = e(f_{+-}}\text{(or } f_{00}) - \text{atan}\ \frac{2DM}{DG})$

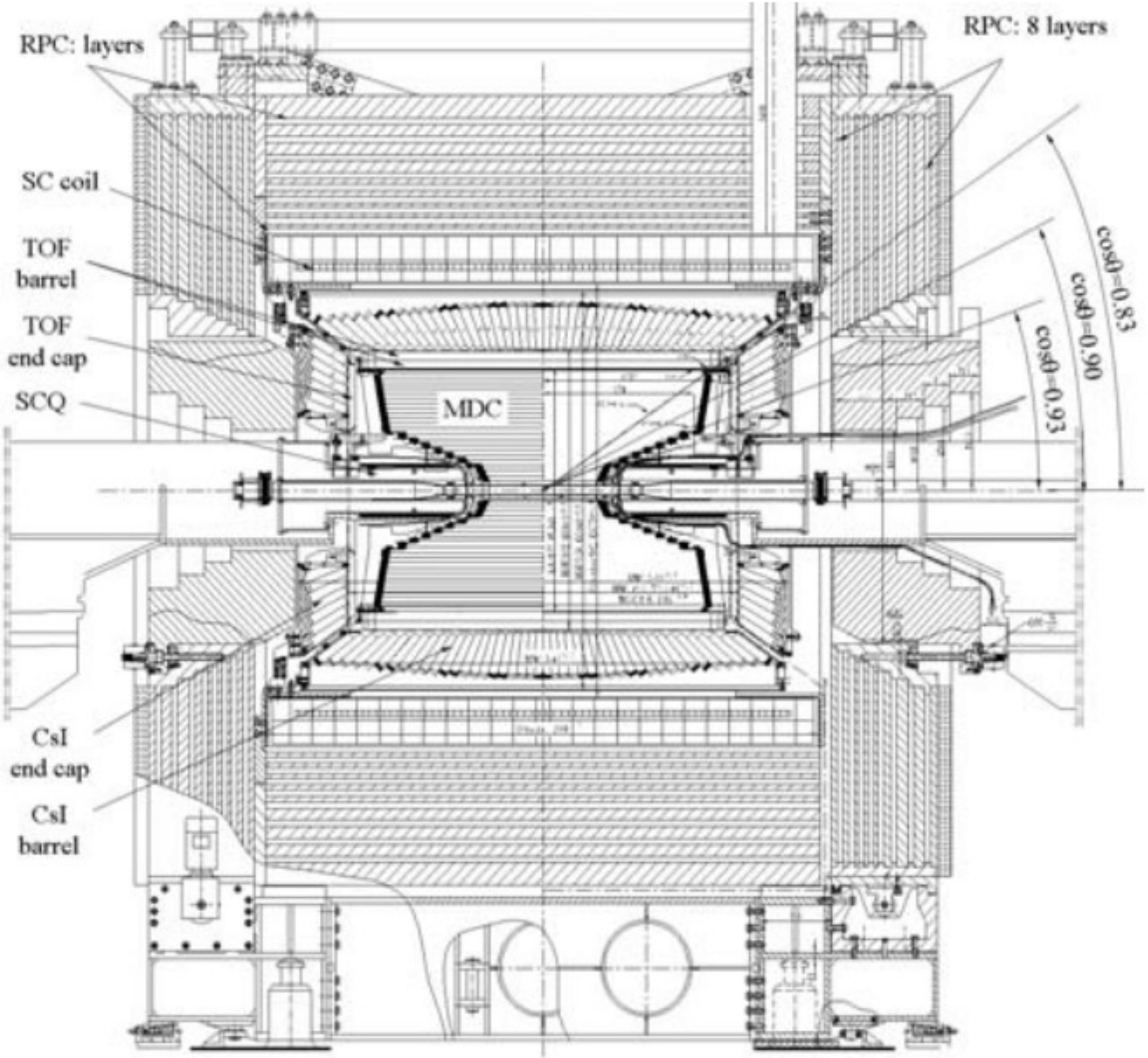
-- corrections to this are small

Experimental issues:

CPLEAR: best experiment to date
with strangeness-tagged K & $\overline{K}$
-- designed in the 1980s --

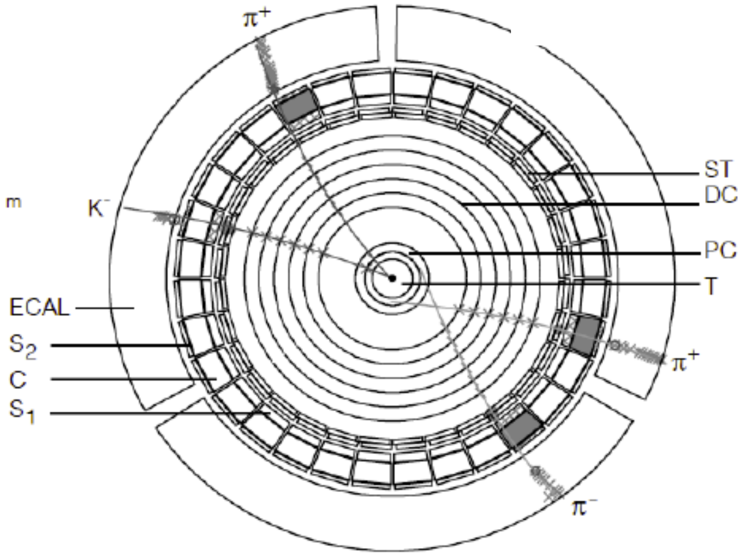


BESIII
-- designed in the 2000s --



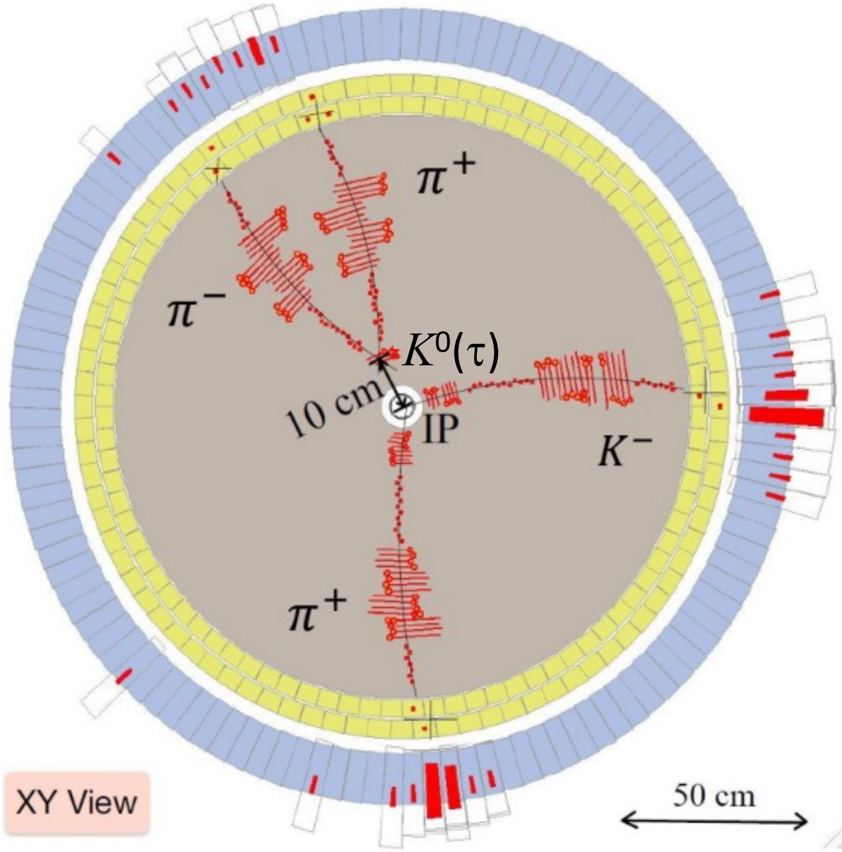
CPLEAR: $p\bar{p} \rightarrow K^+\pi^-\bar{K}^0/ K^-\pi^=K^0$

STCF: $J/\psi \rightarrow K^+\pi^-\bar{K}^0/ K^-\pi^=K^0$



E_{cm} 1.877 GeV

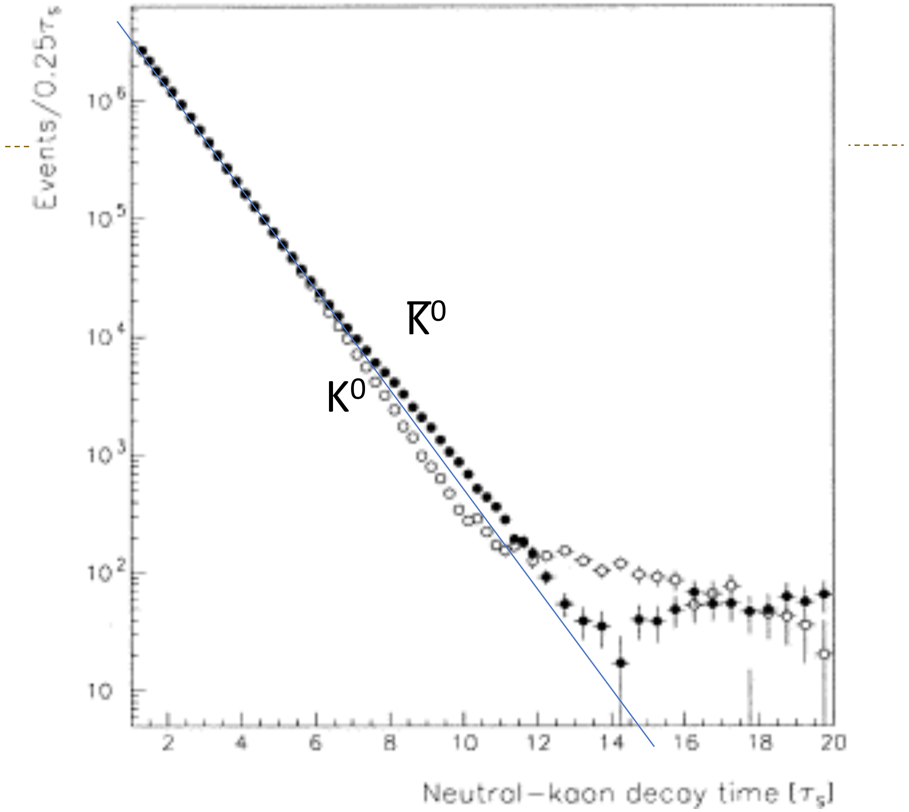
x 1.7

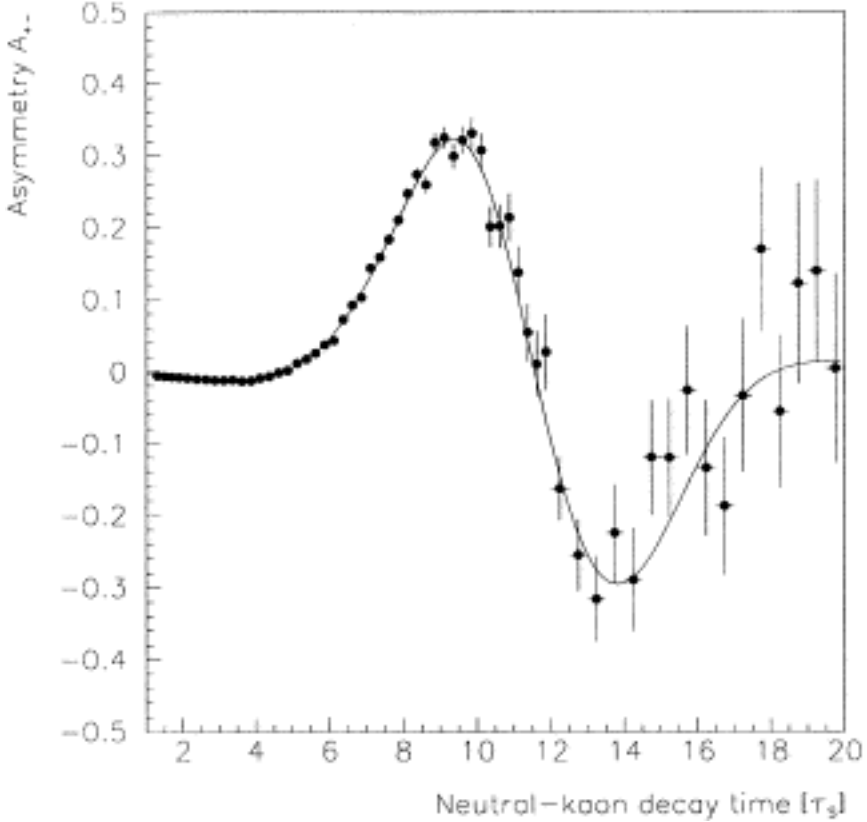


CPLEAR is 60%-sized prototype

E_{cm} 3.097 GeV

CPLEAR: ~70M tagged $K^0 \to \pi^+\pi^-$ events



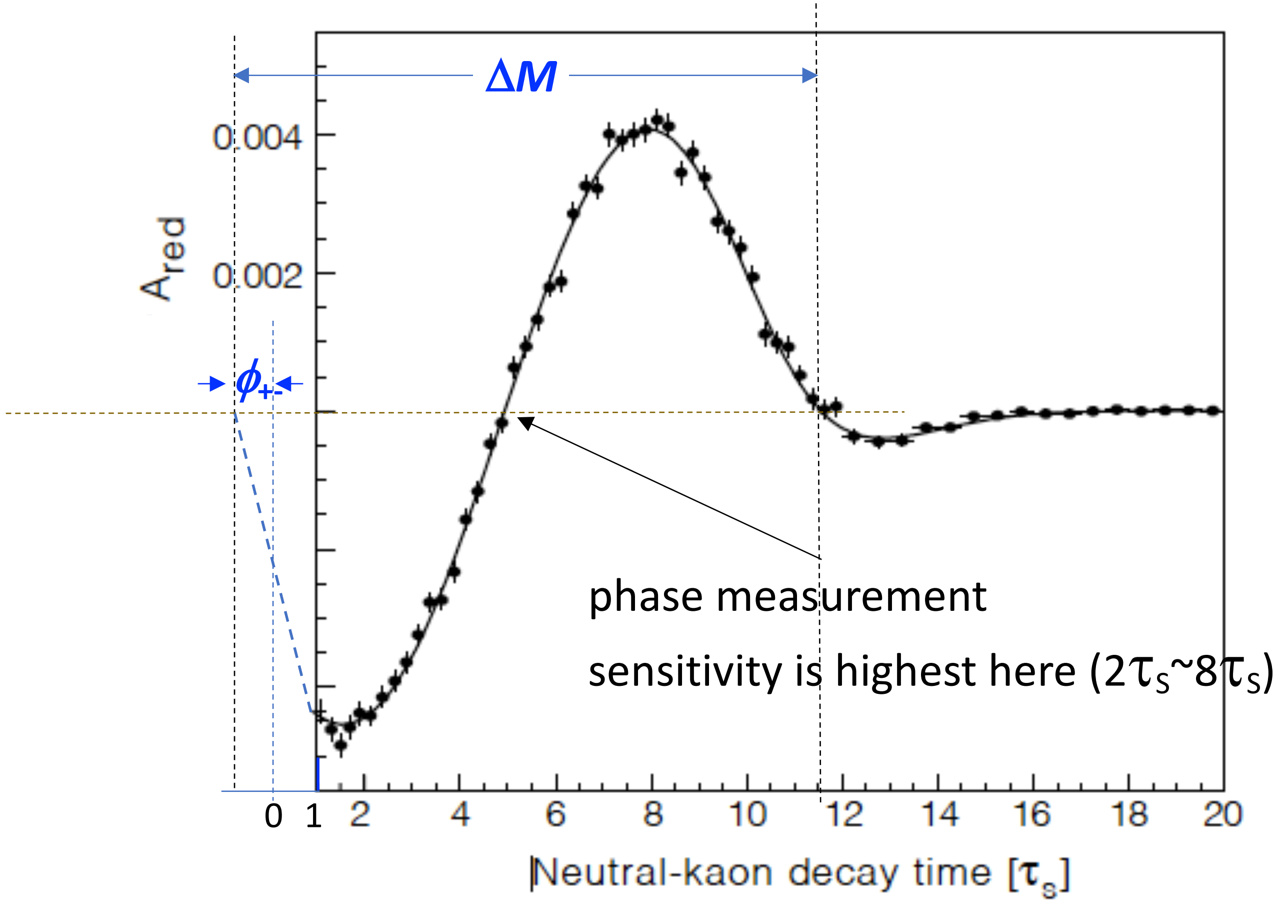


$$|\eta_{+-}| = (2.264 \pm 0.023_{\mathrm{stat}} \pm 0.026_{\mathrm{syst}} \pm 0.007_{\tau_S}) \times 10^{-3}$$

$$\varphi_{+-} = 43.19^\circ \pm 0.53^\circ{}_{\mathrm{stat}} \pm 0.28^\circ{}_{\mathrm{syst}} \pm 0.42^\circ{}_{\Delta m}$$

weight events according to “usefulness”

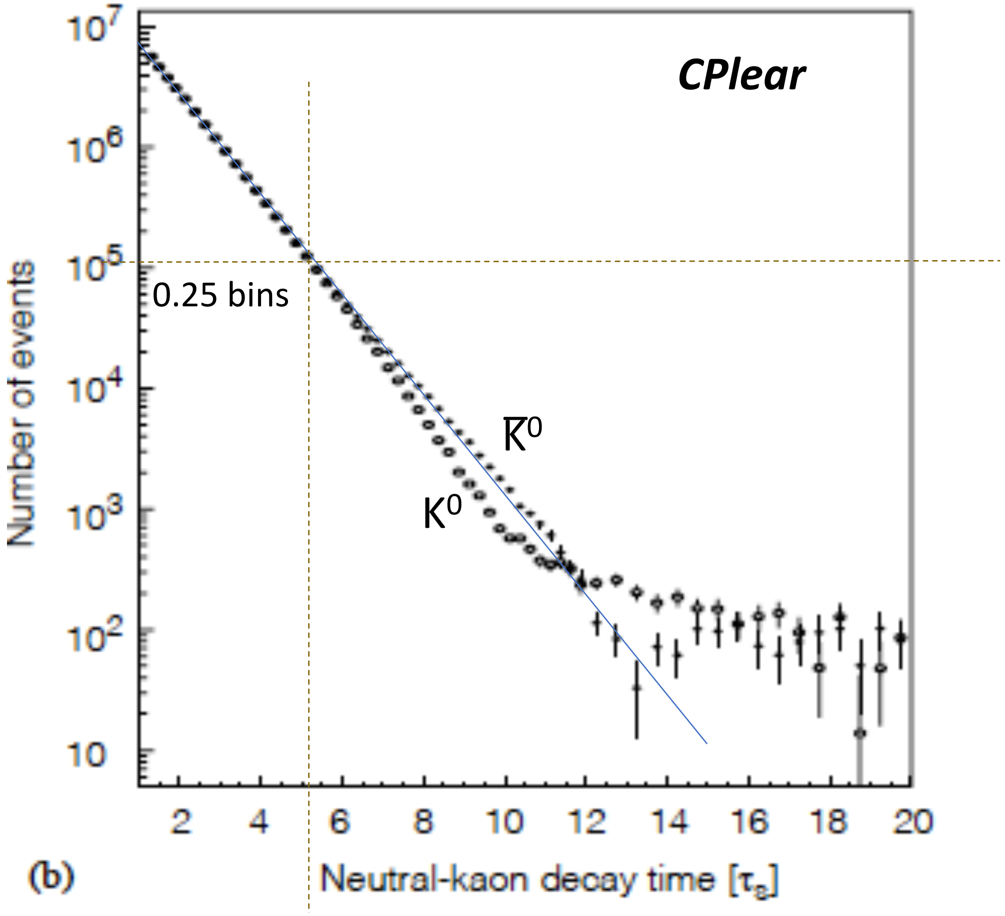
$$A_{red}(\tau)=A_{+-}(\tau)\times e^{-12(\Gamma_S-\Gamma_L)\tau}$$



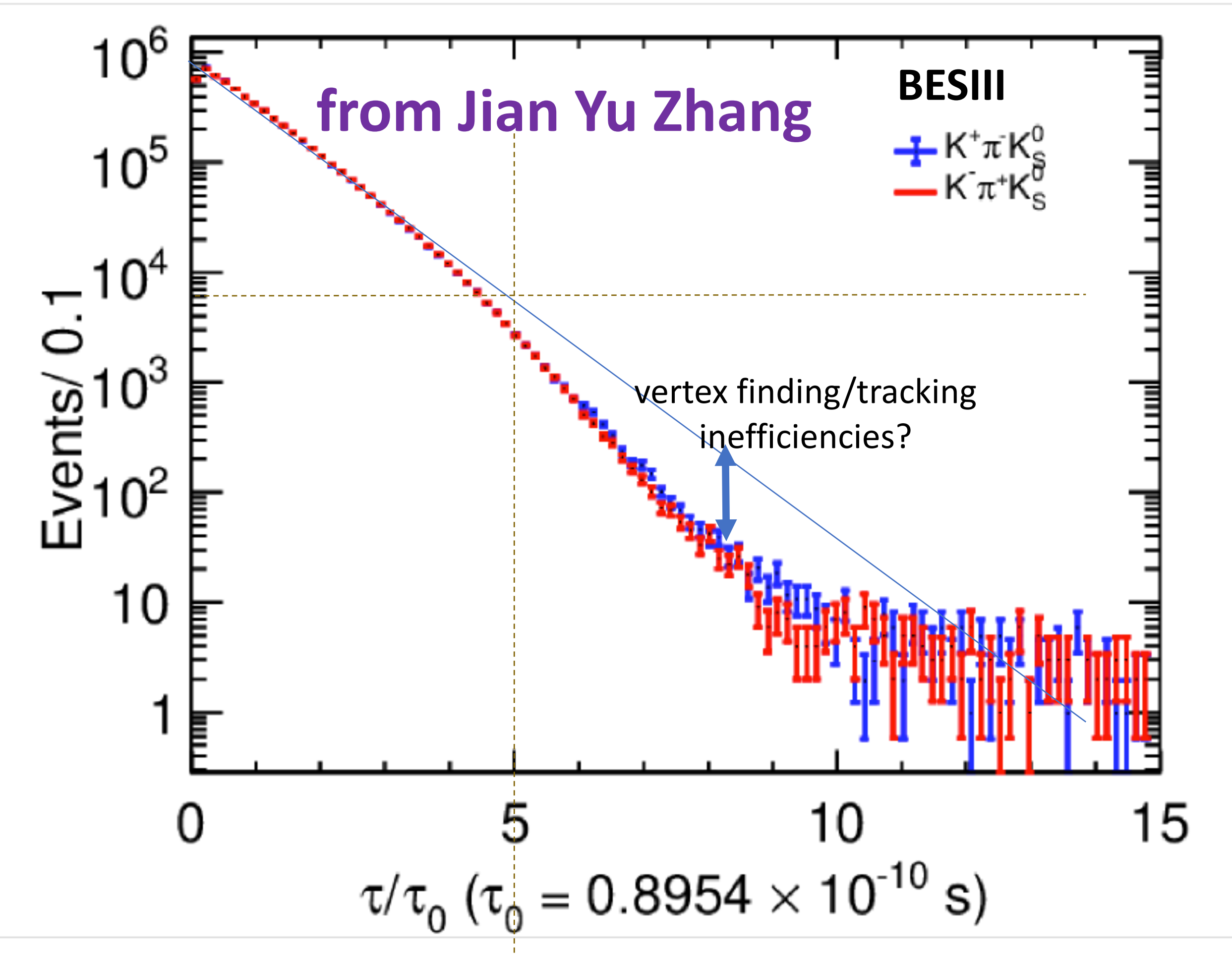
phase measurement sensitivity is highest here ($2\tau_S$~$8\tau_S$)

BESIII (first peek) vs CPlear (10 years of data)

Flavor-tagged K^0 and $\overline{K}^0$ decays to $\pi^+\pi^-$



0.25 bins



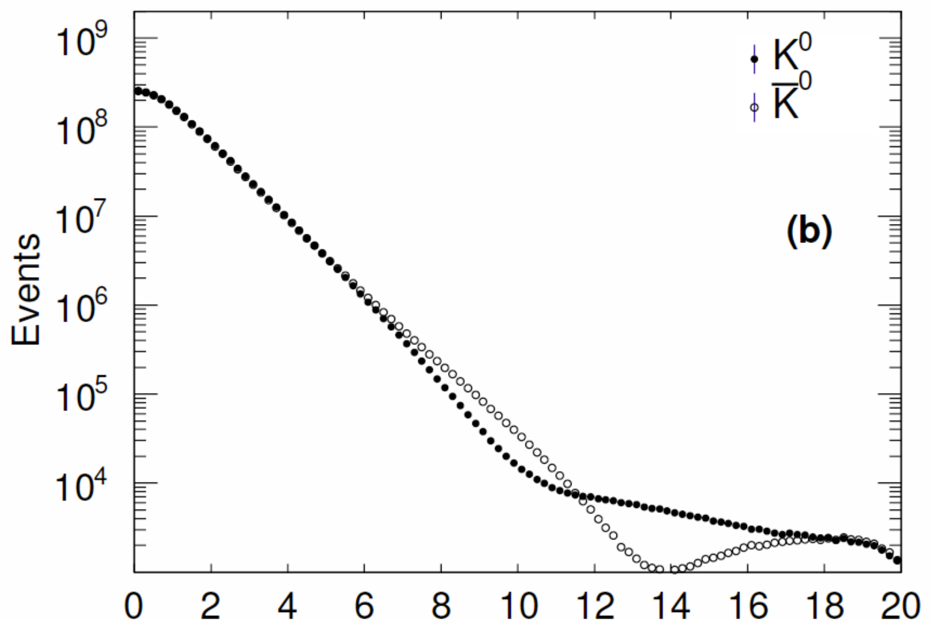
from Jian Yu Zhang

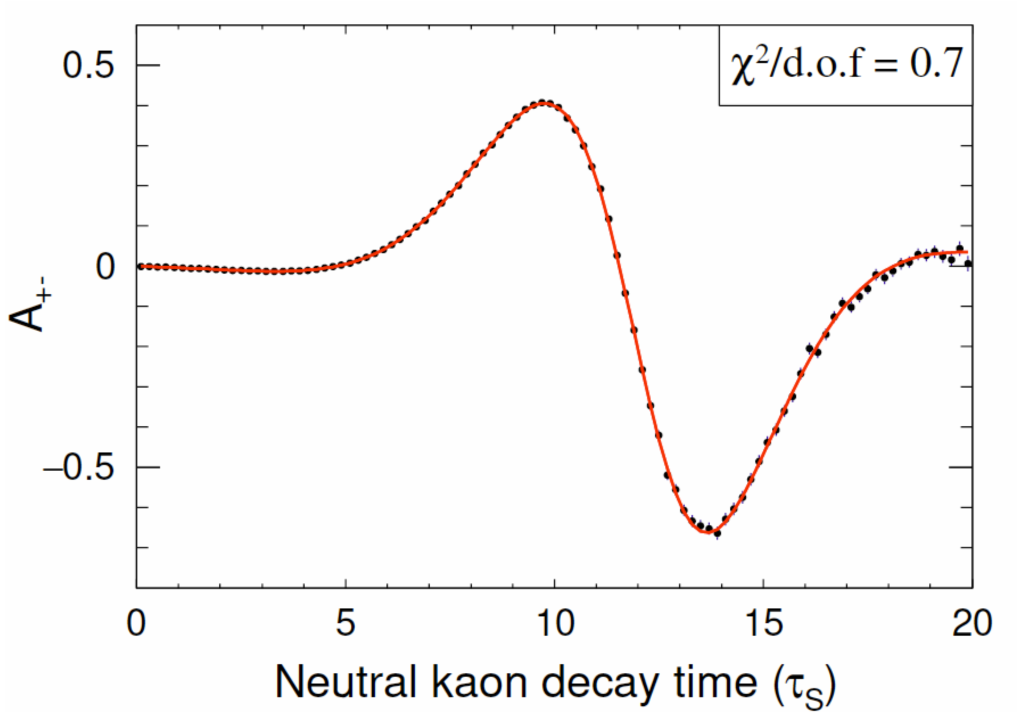
vertex finding/tracking inefficiencies?

CPLEAR measurements had about 7x as much data as BESIII has

SCTF with 10^{12} J/ψ events

-- from Jian-Yu Zhang --





~30x as much data as CPLEAR had ⇒ 10x reduction in errors

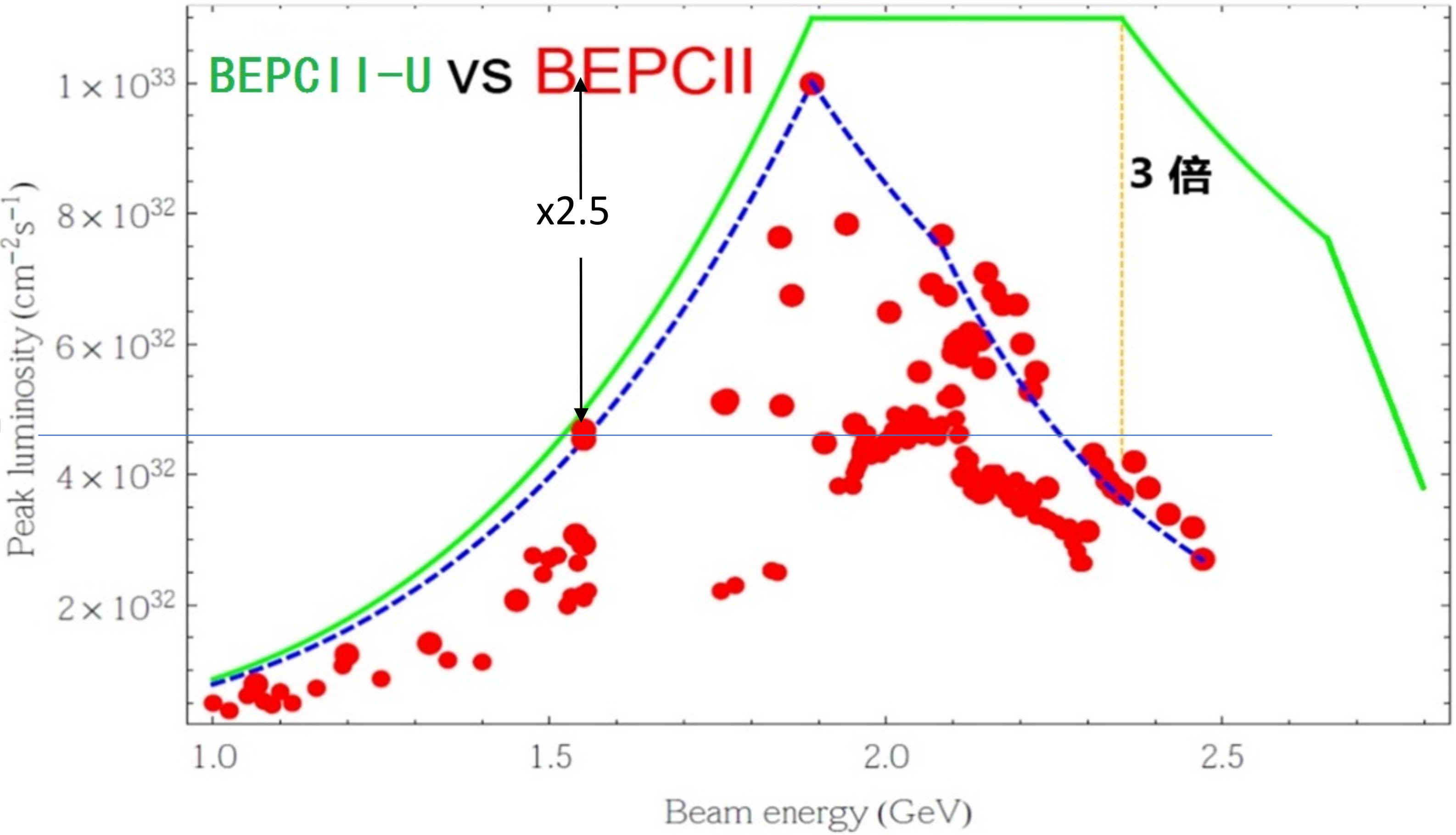
Par.	$\lvert\eta_{+-}\rvert(10^{-3})$	ϕ_{+-}(degree)
PDG	2.232 ± 0.011	43.4 ± 0.5
STCF	$2.2320 \pm 0.0025 \pm 0.0027$	$43.510 \pm 0.051 \pm 0.059$

how to get more *J*/ψ events:

1. factor of ~2 by re-optimizing the lattice to E_{beam}=1.55 GeV

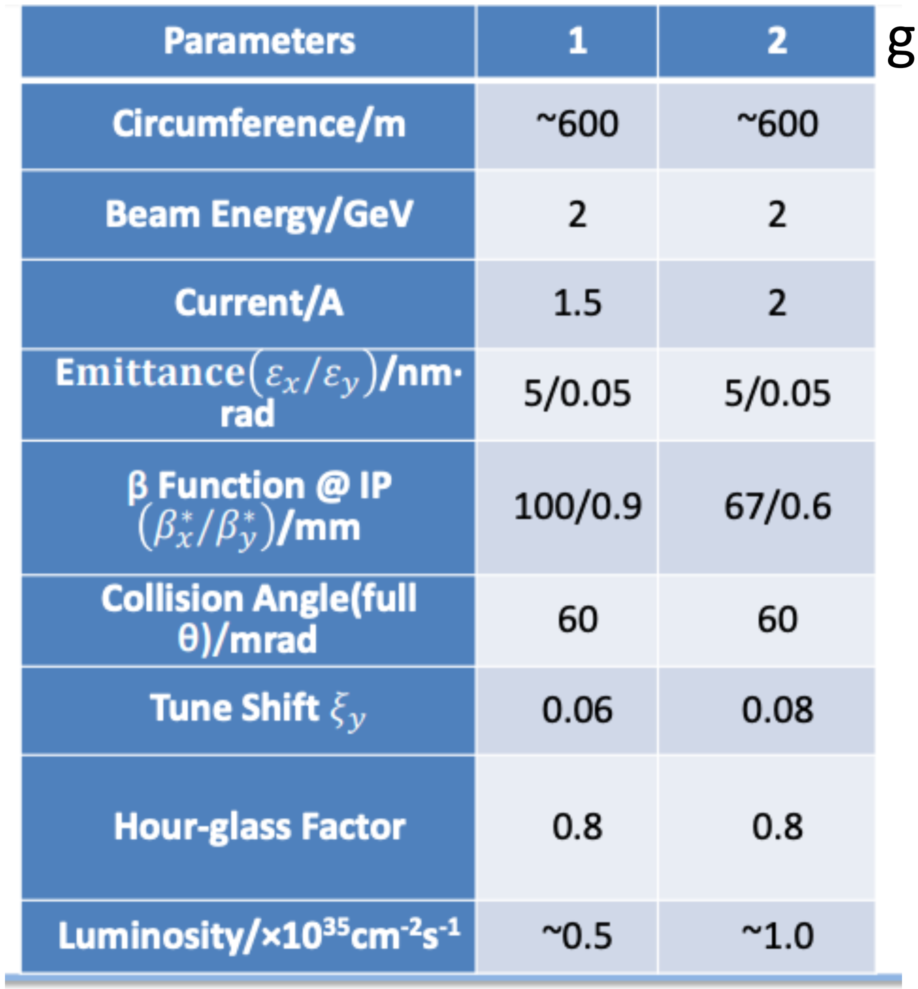
BEPCII lattice design is optimized for the ψ''

BESII: 10^9 evts/month
@ ~$4x10^{32}$ $cm^{-2}s^{-1}$



SCTF parameter list

$$L = \frac{\gamma n_b I_b}{2 e r_e \beta_y^*} \xi_y H$$



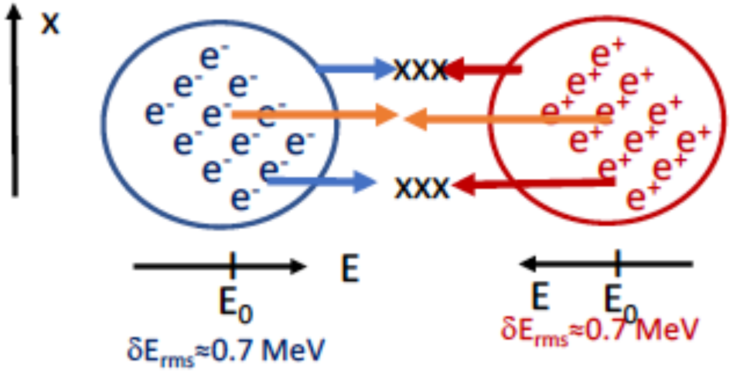
Parameters	1	2
Circumference/m	~600	~600
Beam Energy/GeV	2	2
Current/A	1.5	2
Emittance$(\varepsilon_x/\varepsilon_y)$/nm·rad	5/0.05	5/0.05
β Function @ IP (β_x^*/β_y^*)/mm	100/0.9	67/0.6
Collision Angle(full θ)/mrad	60	60
Tune Shift ξ_y	0.06	0.08
Hour-glass Factor	0.8	0.8
Luminosity/×10^{35}cm^{-2}s^{-1}	~0.5	~1.0

luminosity
gain vs. BEPCII

2x
4x
2.5x
2x

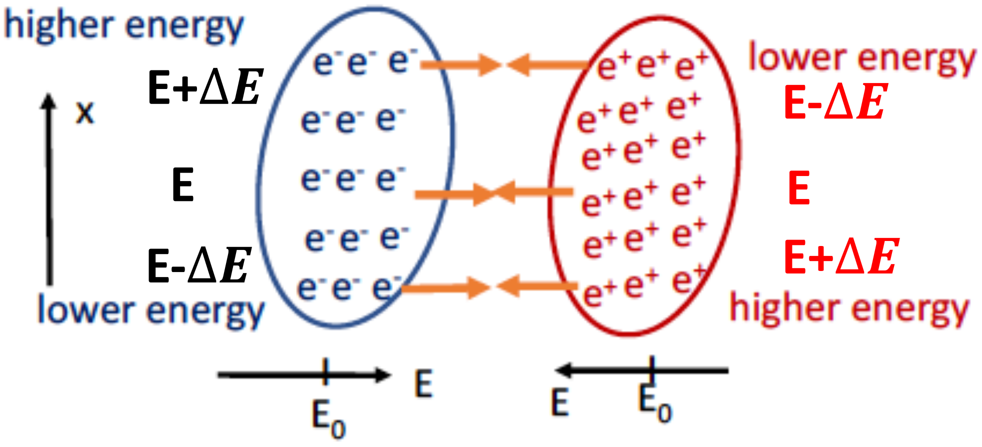
~40x

Monochromater: factor of 10 from reduction of e^+e^- CM spread



only e^+e^- pairs with E_{cm}=3096 ± 0.14 MeV can produce a J/ψ, ~1/30th of the total

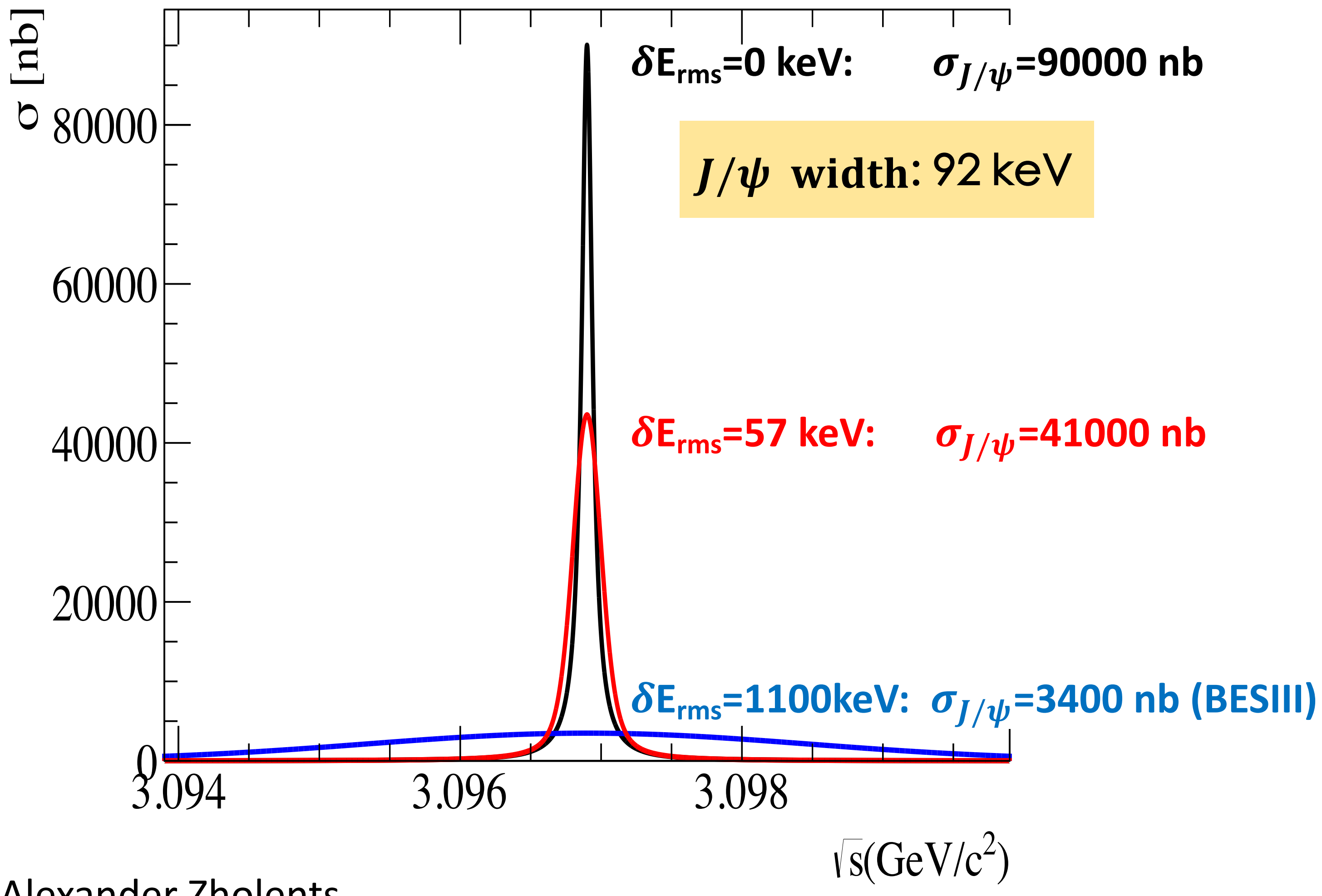
introduce dispersion



more e^+e^- pairs with E_{cm}=3096 ± 0.14 MeV

J/ψ production cross-section

Xiaoshuai Qin



Alexander Zholents
CERN SL/92-27/AP

Comments

-- In CP studies with $K\to\pi\pi$ decays the number ≈1/500 keeps popping up

Γ_L/Γ_S =1/575; ε=1/448 ; ε'/ε=1/425; $Bf(K\to\pi\pi_{(I=2)})/Bf(K\to\pi\pi_{(I=0)})$=1/484; …

-- The current limit on $\delta_\perp/\varepsilon \lesssim$ 1/50, an order-of-magnitude away from *magicland*

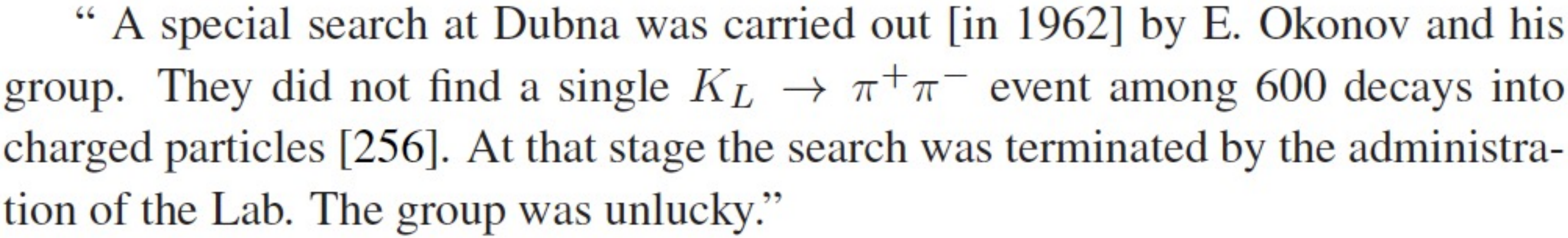
this was 1990s state-of-the-art

-- Current accelerator & detector technology suggests a 10x improvement may be possible

shouldn't we try for it?

Lev Okun

1929-2015



" A special search at Dubna was carried out [in 1962] by E. Okonov and his group. They did not find a single $K_L \to \pi^+\pi^-$ event among 600 decays into charged particles [256]. At that stage the search was terminated by the administration of the Lab. The group was unlucky."