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Impact of correlated noise on the parameter estimation of stochastic gravitational waves

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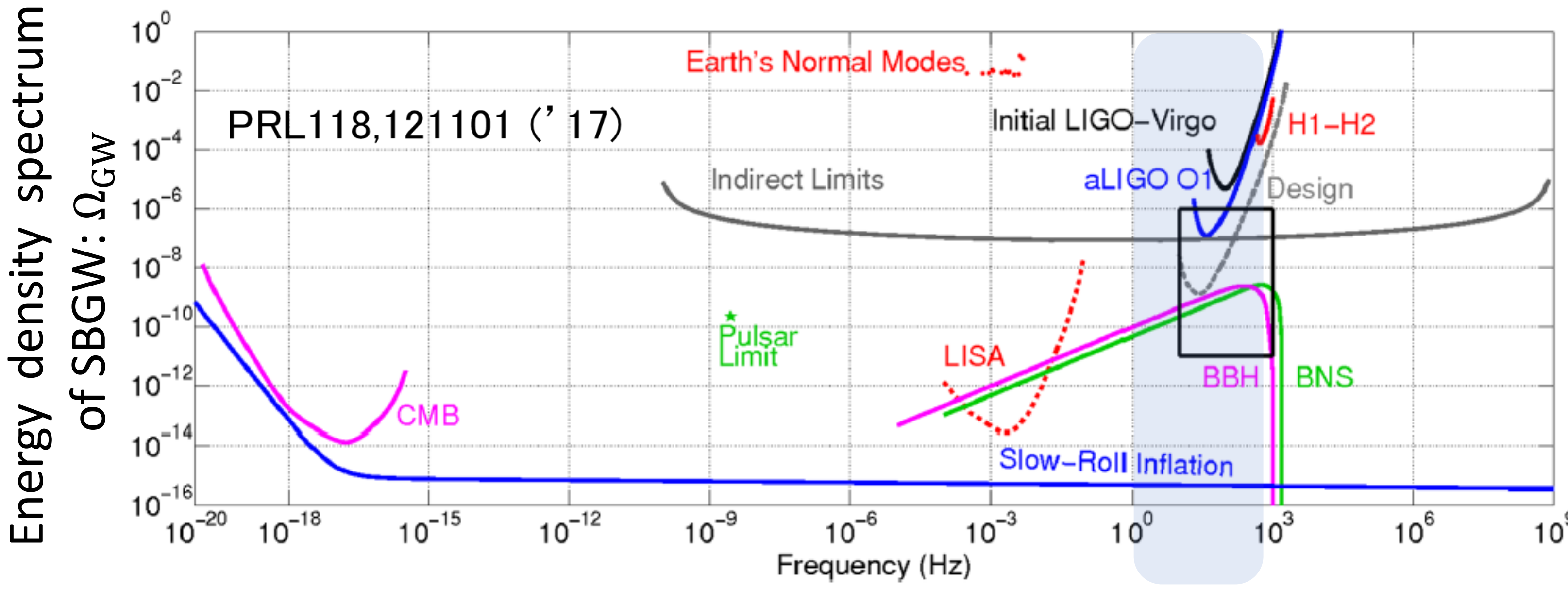
Stochastic Gravitational-Wave Background

Gravitational wave having a random phase, originated from

- Cosmological sources: Inflation, phase transition, ...
- Astrophysical sources: superposition of unresolved gravitational waves from compact binaries (BNS, BBH)



Brief review on detection method of SGWB

Cross-correlating between two data streams:

$$s_i = h_i + n_i \quad (i = 1, 2,\ h_i: \text{GW signal},\ n_i: \text{noise})$$

Cross correlation statistic

$$\langle s_1\, s_2 \rangle = \langle h_1\, h_2 \rangle + \langle h_1\, n_2 \rangle + \langle n_1\, h_2 \rangle + \langle n_1\, n_2 \rangle$$

SGWB can be detected if there is no cross talks between signal and noise (si & nj) and noise themselves (n1 & n2)

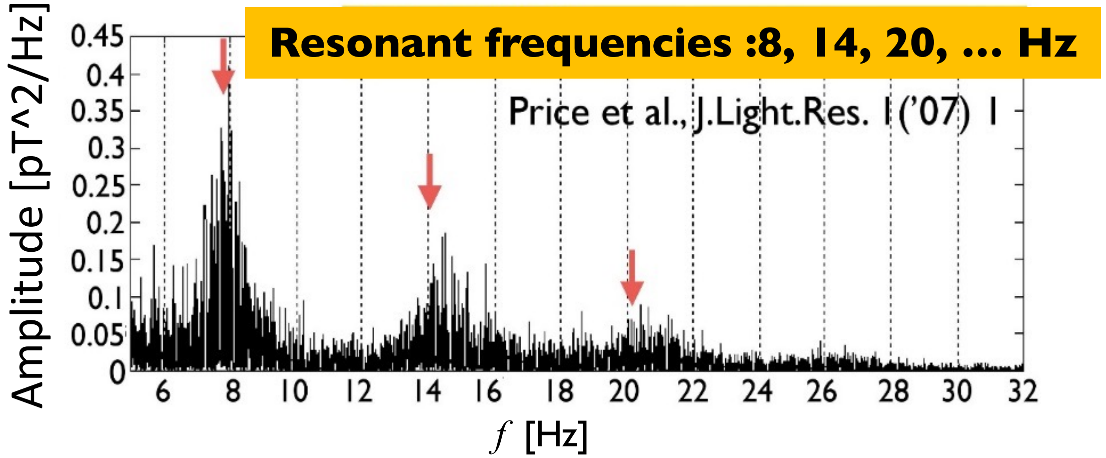
Q: is this assumption valid ?

NO! detector noise can have a non-negligible correlation even for a pair of two distant detectors due to a coupling with global disturbance (→next)

Correlated noise from Schumann resonance

Schumann resonance

Standing electromagnetic waves in the Earth-ionoshere cavity at ultra-low frequencies



Coupling it with detector (mirror system) induces a global noise correlation between two distant detectors

- ✓ A correlation was detected by magnetometers at LIGO and Virgo (Thrane et al.'13,14)
- ✓ Its impact on the detection of SGWB by LIGO/Virgo and ET was studied (Janssens et al.'21)

Present work How does the correlated noise affect the parameter estimation of SGWB ?

Setup of problem

We consider two possibilities:

- We have a good (parameterized) model for correlated noise:

  Marginalizing over the parameters of correlated noise, how are the constraints on the SGWB degraded ?

- We erroneously miss the presence of correlated noise:

  Ignoring correlated noise, how is the estimated parameters of SGWB biased ?

➡ Fisher matrix analysis for a network of LIGO, Virgo and KAGRA

See Meyers et al. ('20) for a similar work based on Bayesian MCMC analysis

Fisher matrix formalism

Fisher matrix (e.g., Seto '06, Kuroyanagi et al. '18)

$$F_{ab} = -\frac{\partial^2 \ln\mathcal{L}}{\partial\theta_a \partial\theta_b} = 2\,T_{\rm obs} \sum_{(I,J)} \int_0^\infty \frac{\partial_a U_{IJ}(f)\, \partial_b U_{IJ}(f)}{S_I(f) S_J(f)} df$$

Instrumental noise spectrum

Statistical error

$$\delta\theta_a = \sqrt{F^{-1}_{aa}}$$

$$U_{IJ}(f) = \langle s_I\, s_J \rangle = \langle h_I\, h_J \rangle + \langle n_I\, n_J \rangle$$

SGWB spectrum

$$\langle h_1\, h_2 \rangle = \frac{3H_0^2}{10\pi^2} \frac{\gamma_{ij}(f)\Omega_{\rm GW}(f)}{f^3}$$

γ_{ij} :overlap reduction function (e.g., Allen and Romano '99)

We consider a power-law spectrum:

$$\Omega_{\rm GW} = \Omega_{\Omega_{\rm GW,0}} \left(\frac{f}{25}\right)^{n_{\rm GW}}$$

2 parameters

$$\boldsymbol{\theta}_{\rm GW} = \{\Omega_{\rm GW,0}, n_{\rm GW}\}$$

Fisher matrix formalism

Fisher matrix (e.g., Seto '06, Kuroyanagi et al. '18)

$$F_{ab} = -\frac{\partial^2 \ln\mathcal{L}}{\partial\theta_a\partial\theta_b} = 2\,T_{\rm obs}\sum_{(I,J)}\int_0^\infty \frac{\partial_a U_{IJ}(f)\,\partial_b U_{IJ}(f)}{S_I(f)S_J(f)}df$$

Instrumental noise spectrum

Statistical error

$$\delta\theta_a = \sqrt{F_{aa}^{-1}}$$

$$U_{IJ}(f) = \langle s_I\, s_J\rangle = \langle h_I\, h_J\rangle + \langle n_I\, n_J\rangle$$

$$\tilde{n}_I\ (f) = r_I(f)m_I(f)$$

(projected) magnetic field

coupling function

Correlated noise spectrum

$$\langle n_I\, n_J\rangle = r_I(f)\, r_J(f) M_{IJ}(f)$$

$M_{IJ}(f)$: Magnetic noise spectrum (→ next slide)

$$r_I(f) = \kappa_I \times 10^{-23}\left(\frac{f}{10}\right)^{-\beta_{I_i}}\ \text{[strain/pT]}$$

ψ_I: projection angle of m_I

3 parameters for each detector

$$\boldsymbol{\theta}_{\rm Mag} = \{\kappa_I, \beta_I, \psi_I\}$$

Analytical model of correlated noise

Himemoto & Taruya ('17, '19)

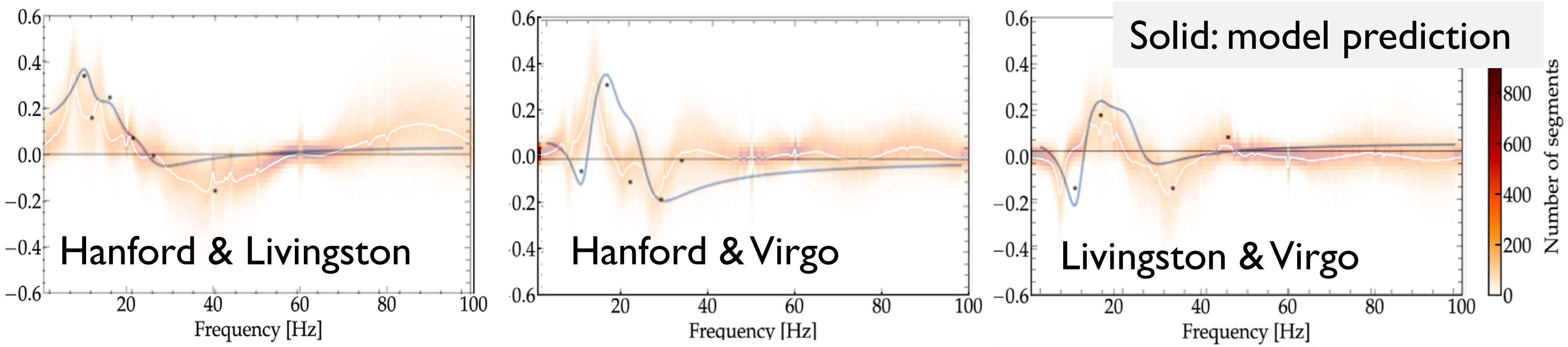
- Detector is linearly coupled with Earth magnetic field :

(projected) magnetic field

$$\tilde{n}_i^B(f) = r_i(f)\left[\widehat{\boldsymbol{X}}_i \cdot \widetilde{\boldsymbol{B}}(f,\hat{x}_i)\right] \equiv r_i(f)m_i(f)$$

- Schumann resonance is described by a random superposition of the axisymmetric transverse magnetic modes in the Earth-ionosphere cavity

The model reproduces major trends of measured results in Meyers et al. ('21)



Coherence function of magnetic field: $\dfrac{M_{IJ}(f)}{\sqrt{M_{II}M_{JJ}}}$

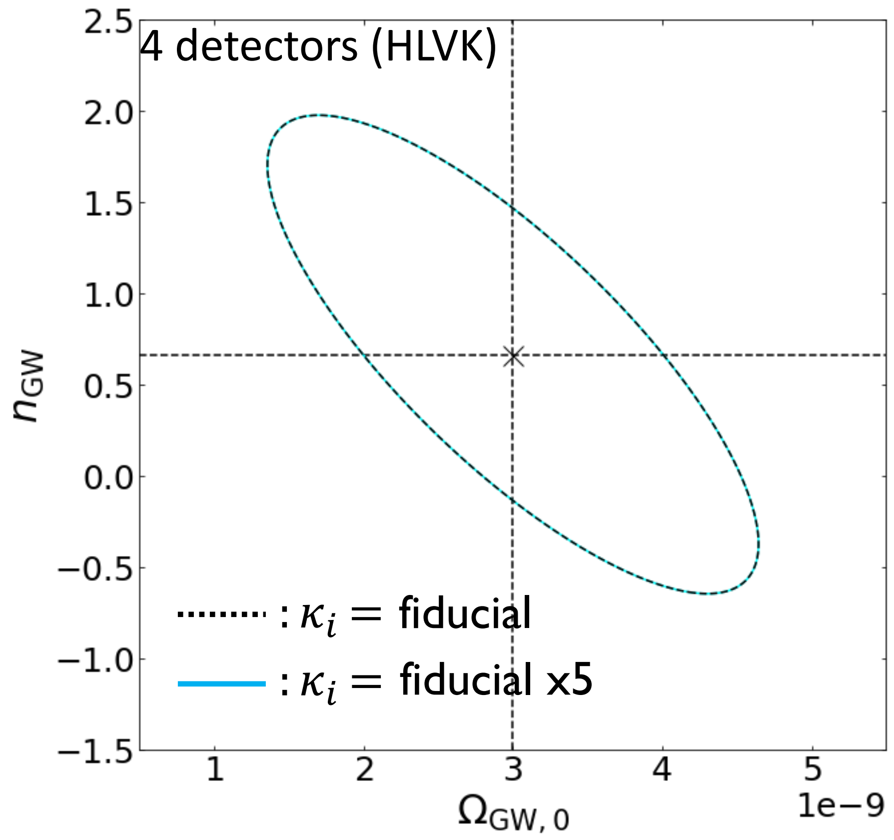
Meyers et al. ('21) modified

→ Use this model as a template of correlated magnetic noise in our subsequent analysis

Result 1: Impact on SGWB parameters

Adopting the '*realistic*' coupling parameters in Meyers et al. ('21),

Constraints on the SGWB parameters ($\Omega_{GW,0}$ & n_{GW}), with the correlated noise parameters marginalized over



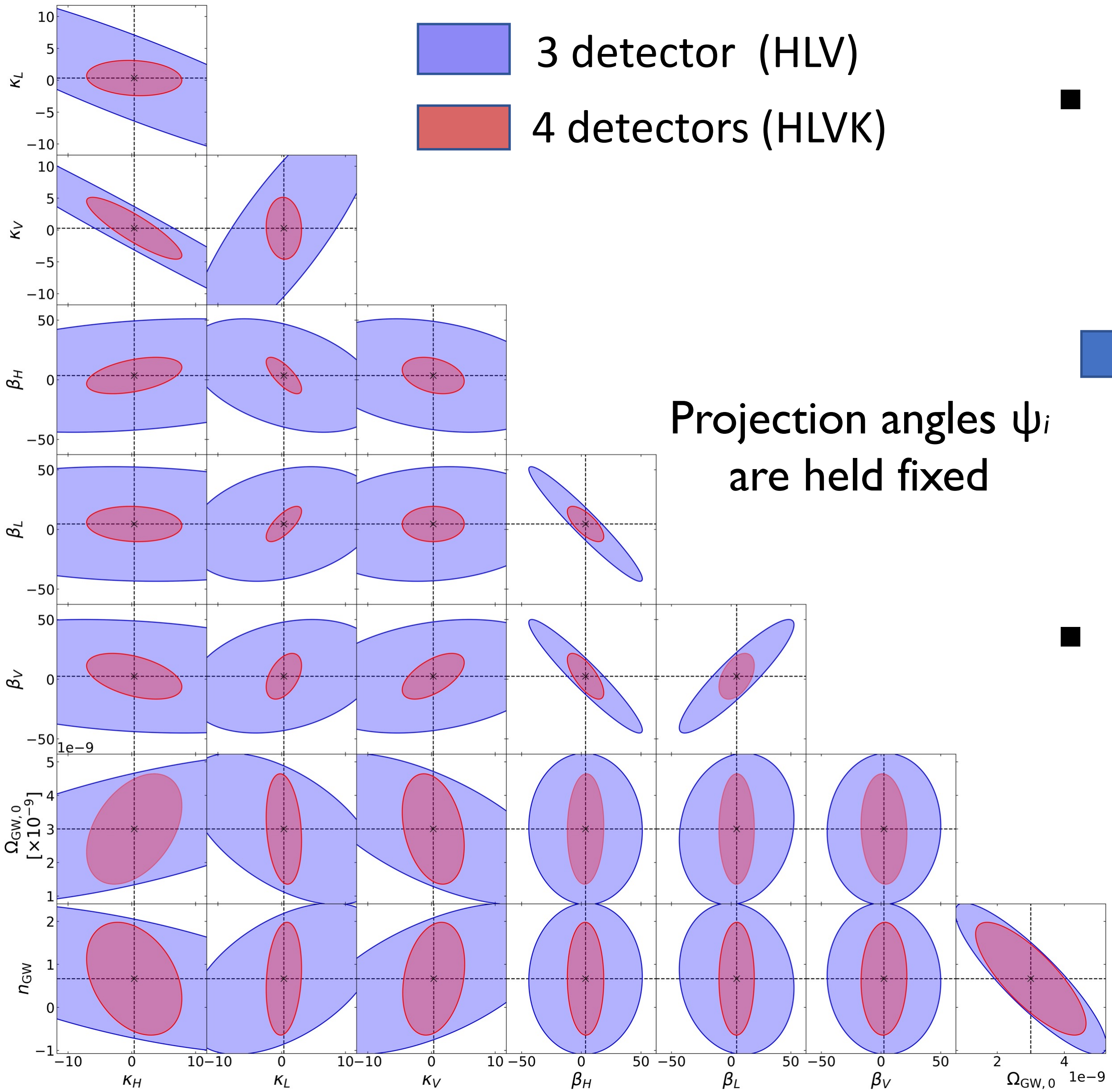
Despite many nuisance parameters introduced (12 parameters (!) for 4 detector case, HLVK),

The impact of the size of coupling on the parameter estimation of SGWB is small.

Fiducial values of correlated noise parameters

	κ_i	β_i	ψ_i(rad)
Hanford	0.38	3.55	6.09
Livingston	0.35	4.61	0.74
Virgo	0.275	2.50	1.37
KAGRA	0.34	3.55	2.74

Result 2: constraining correlated noise parameters



Projection angles ψ_i are held fixed

- There is little degeneracy between correlated noise and SGWB parameters

Good news !

➡ Even with a large statistical error on the coupling parameters, the SGWB is well-constrained.

- Constraining noise parameters needs *more than 3 detectors*, and constraining power gets increased when adding more detectors

➡ A role of KAGRA is important ! (red)

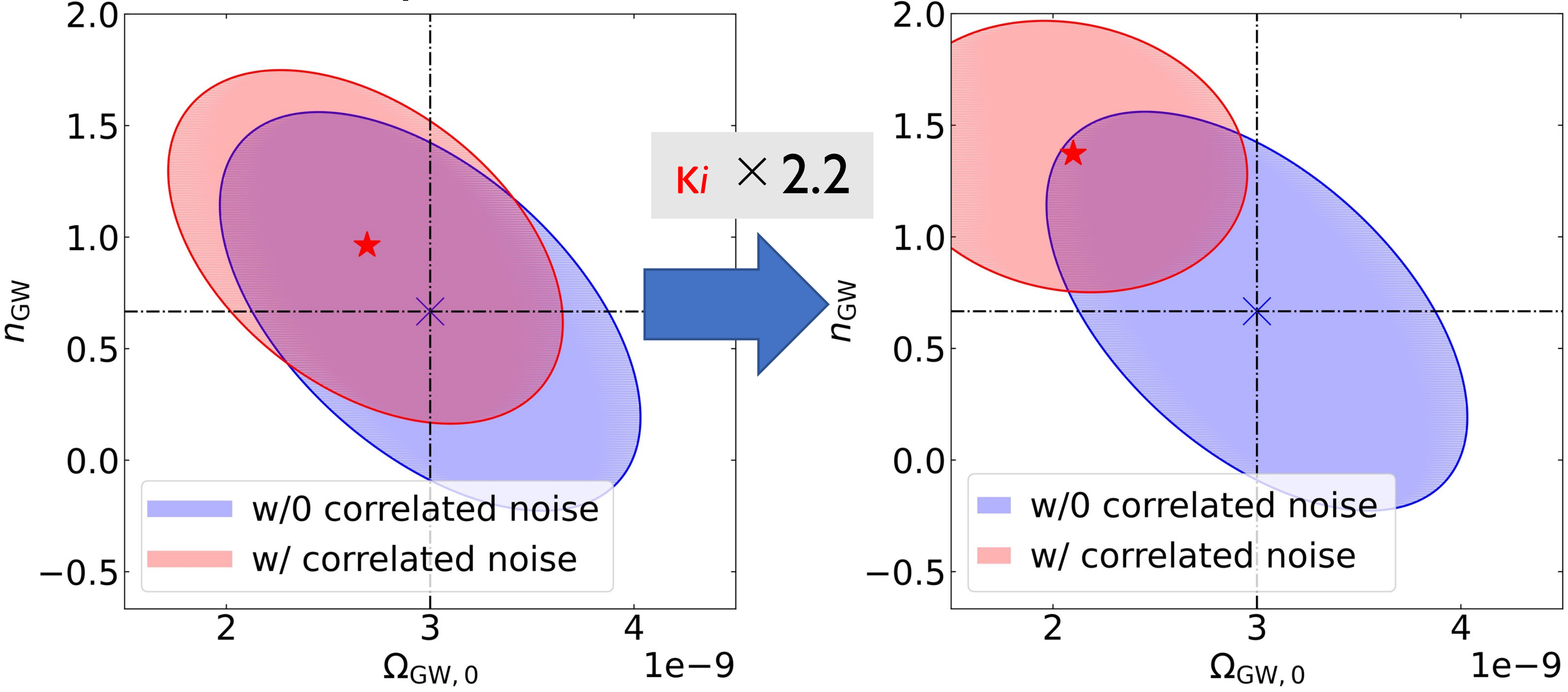
Result 3: systematic bias in SGWB parameters

If we ignore the correlated noise, the estimated SGWB parameters are biased

For our fiducial setup, the bias seems insignificant ...

However,

Fiducial setup



The result is sensitive to the coupling parameter κ_i, and it can become serious when κ_i is twice larger than the fiducial setup

Result ignoring the correlated noise

Result taking the correlated noise into account

Summary

Fisher matrix analysis to clarify the impact of correlated noise on the parameter estimation of stochastic gravitational-wave background (SGWB)

Adopting the analytical model by Himemoto & Taruya ('17, '19) that successfully describes measured Schumann resonances as a template of correlated noise,

- Parameter degeneracy between the correlated noise and SGWB is generally (very) weak. If the magnetic correlation noise is well modeled, it does not affect the parameter estimation of the SGWB.
- Ignoring the correlated noise yields a biased parameter estimation of the SGWB, and even the realistic noise coupling would give a substantial bias
- A network observation combining more than 3 detectors is quite essential, and KAGRA will play an important role to better constrain the correlated noise.