Possible molecular states in the $\pi\bar{K}K^*$ and $KK\bar{K}$ system

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Based on the following papers:

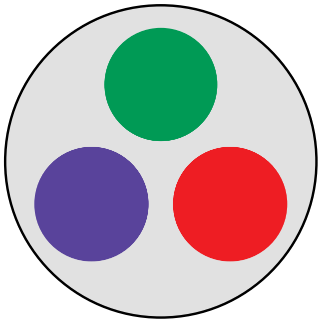
Xu Zhang, Ju-Jun Xie, Xurong Chen, Phys. Rev. D 95, 056014(2017).
Xu Zhang, Ju-Jun Xie, Chin. Phys. C 44, 054104(2020).
Xu Zhang, Christoph Hanhart, Ulf-G. Meiβner, Ju-Jun Xie, Eur. Phys. J. A 58, 20(2022).

Outline

1. Introduction
2. Theoretical Framework
3. Summary

Three-body problem using Faddeev equations

Triton is a bound state of one proton and two neutrons.
Whether there exist hadronic molecules from the three-hadron interactions.



$\Longrightarrow$ Deal with three-body problem

❑ Faddeev equations

Faddeev, Sov. Phys. JETP 12, 1014(1961)

$$\psi_{\alpha,1}(p,k) = \varphi_{\alpha,1}(p,k) + \int d^3k' d^3p' [A^{12}\,\psi_{\alpha,2}(p',k') + A^{13}\,\psi_{\alpha,3}(p',k')]$$

$$\psi_{\alpha,2}(p,k) = \int d^3k' d^3p' \,[A^{21}\,\psi_{\alpha,1}(p',k') + A^{23}\,\psi_{\alpha,3}(p',k')]$$

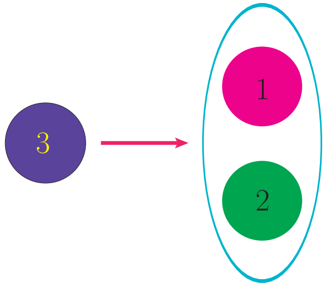
$$\psi_{\alpha,3}(p,k) = \int d^3k' d^3p' \,[A^{31}\,\psi_{\alpha,1}(p',k') + A^{32}\,\psi_{\alpha,2}(p',k')]$$

- Three coupled integral equations
- It is not easy to solve exactly

Approximations to Faddeev equations

❑ Fixed Center Approximation(FCA)

- Limitations: A heavy cluster formed by the first two particles and a light third particle



R. Chand et al., Ann. Phys. 20, 1 (1962).
R. C. Barrett et al., Phys. Rev. C 60, 025201 (1999).

❑ The isobar approach

- two-particle subsystems is assumed to be dominated by a finite number of bound states and resonances.
- It is a set of coupled Lippmann-Schwinger equations

C. Lovelace, Phys.Rev. 135, B1225 (1964)

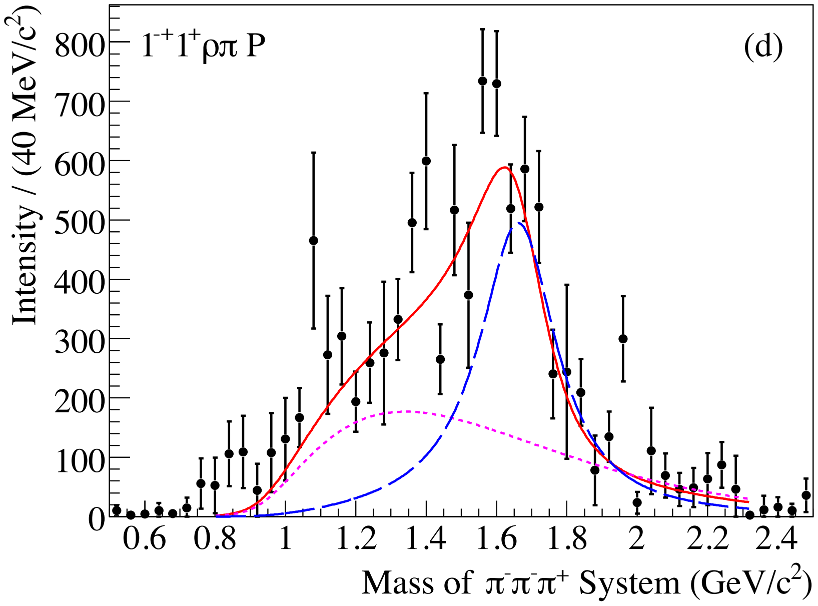
❑ Faddeev equations with chiral unitary approach

- The two-body scattering amplitudes are on shell.

K. P. Khemchandani et al., Eur. Phys. J. A 37, 233 (2008).

The exotic state $\pi_1(1600)$

A state with quantum numbers $J^{PC} = 1^{-+}$ can not be described as simple quark antiquark pairs.



A lot of investigations interpret $\pi_1(1600)$ as a hybrid meson. And there are also other interpretations that $\pi_1(1600)$ as a four quark state.

Hua-Xing Chen et al., Phys. Rev. D 83, 014006 (2011).
C. A. Meyer et al., Phys. Rev. C 82, 025208(2010).
Bin Zhou et al., Chin. Phys. C 41, 043101(2017).

The COMPASS Collaboration showed evidence for $\pi_1(1600)$.
Phys. Rev. Lett. 104, 241803(2010)

The $\pi\bar{K}K^*$ interaction under FCA

$\bar{K}K^* \Rightarrow J^{PC} = 1^{++}$ $f_1(1285)$

$\pi f_1(1285) \Rightarrow J^{PC} = 1^{-+}$?

We study the $\pi\bar{K}K^*$ system by solving the Faddeev equation under fixed center approximation.

- The deep bound state $\bar{K}K^*$ forming a stable cluster $f_1(1285)$.
  Pei-Liang Lü and Jun He, Eur. Phys. J. A 52, 359 (2016).
  L. Roca, E. Oset, and J. Singh, Phys. Rev. D 72, 014002 (2005).

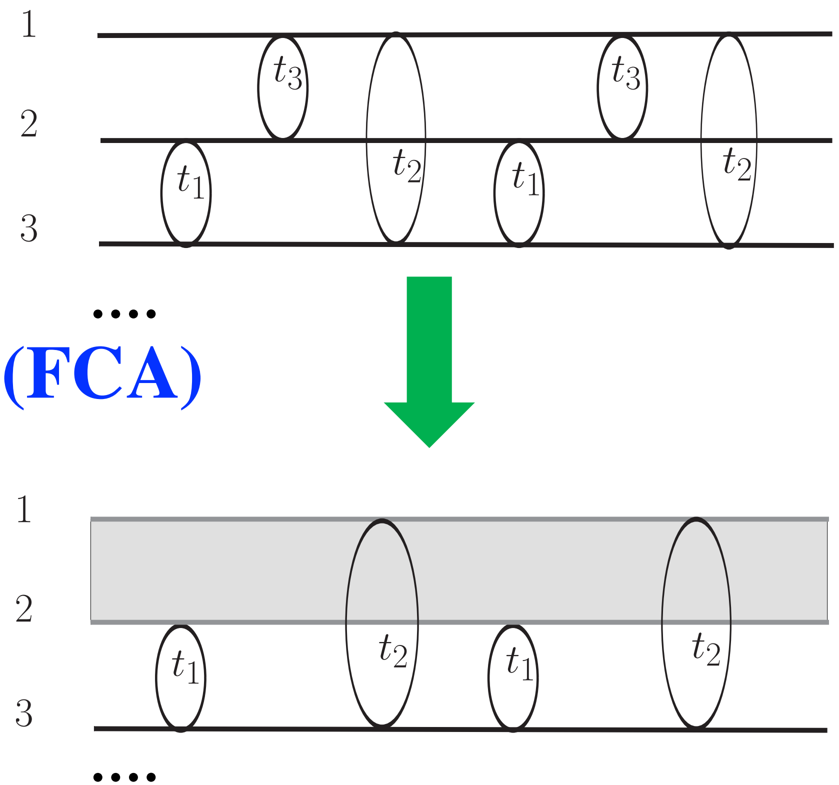
- The light particle π scatters off the cluster.

Fixed Center Approximation

❑ Faddeev equations

The T matrix of particles 1, 2, 3 scattering can be decomposed into three parts

$$T = \sum_{i=1,2,3} T_{i.} \qquad T_i = t_i + t_i G_0\, T_j + t_i G_0\, T_k.$$



❑ Fixed Center Approximation(FCA)

Three coupled equations are reduced into two coupled equations.

$$T = T_1 + T_2\,.$$

$$T_1 = t_1 + t_1 G_0 T_2\ ,$$

$$T_2 = t_2 + t_2 G_0 T_1\,,$$

Particle 1 and 2 form a stable cluster (1, 2).
And $m_3 \ll m_{12}$.

The $\pi\bar{K}K^*$ interaction under FCA

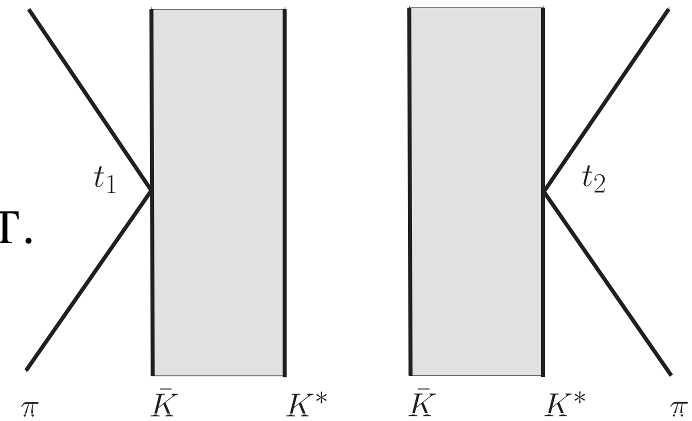
❑ Single scattering: t_i

$$\langle \pi f_1(1285)|\hat{t}_1|\pi f_1(1285)\rangle = t_1 = \frac{2}{3}\, t_{\pi K}^{I=3/2} + \frac{1}{3}\, t_{\pi K}^{I=1/2},$$

$$\langle \pi f_1(1285)|\hat{t}_2|\pi f_1(1285)\rangle = t_2 = \frac{2}{3}\, t_{\pi K^*}^{I=3/2} + \frac{1}{3}\, t_{\pi K^*}^{I=1/2}.$$

The two-body scattering amplitudes are obtained from UChPT.

F.-K. Guo et al., Nucl. Phys. A773, 78(2006)
L. S. Geng et al., Phys. Rev. D 75, 014017(2007).



❑ Double scattering: $t_i G_0 t_j$

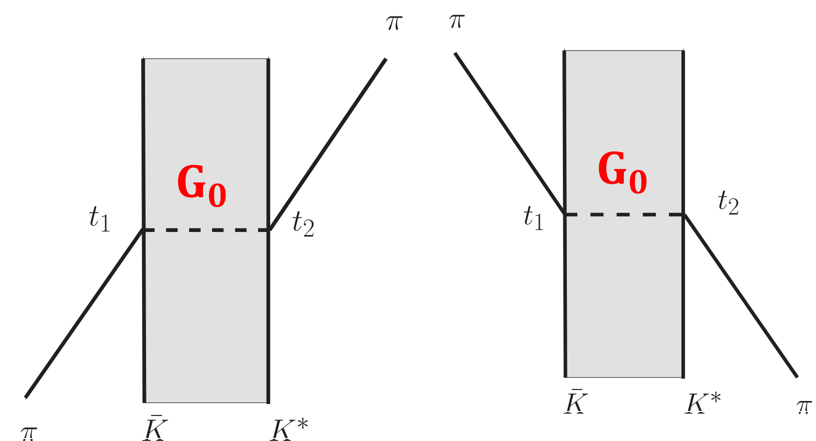
G_0: The π propagator inside the cluster $f_1(1285)$

$$G_0 = \frac{1}{m_{cls}} \int \frac{d^3q}{(2\pi)^3} F_{cls}(q) \frac{1}{{q^0}^2 - |\vec{q}|^2 - m_3^2 + i\varepsilon}$$

The form factor $F_{cls}(q)$ of $f_1(1285)$

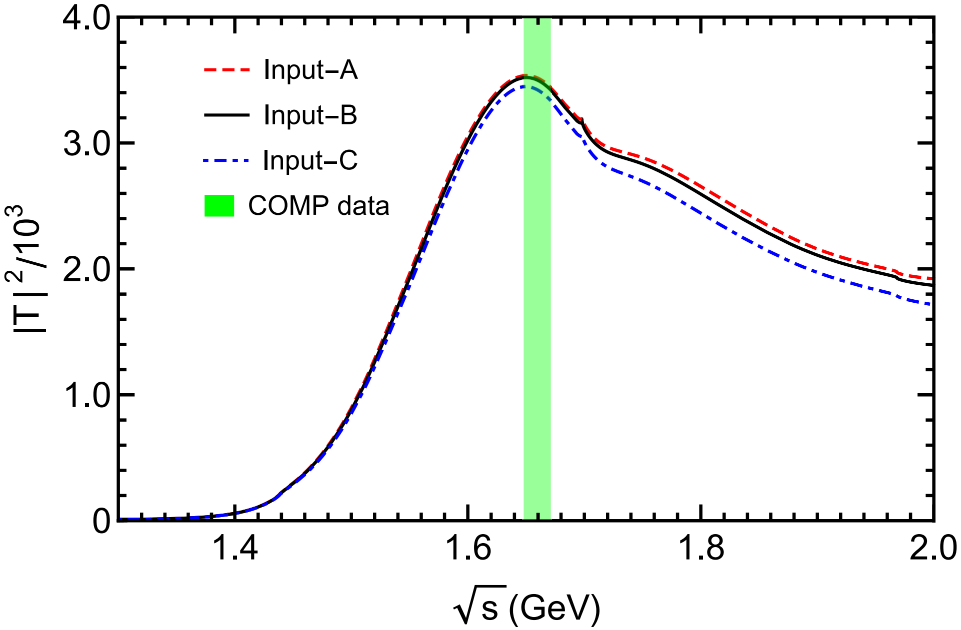
$$F_{cls}(q) = \frac{1}{N} \int_{|p|,|p-q|<\Lambda} d^3q\ f(p)f(p-q)$$

$$N = F_{cls}(0),\ f(p) = \frac{1}{\omega_K(p)\omega_{K^*}} \frac{1}{m_{cls} - \omega_K(p) - \omega_{K^*}(p)}$$



The cut-off is fixed with the demand that $f_1(1285)$ is produced from the $\bar{K}K^*$ interaction.

The $\pi\bar{K}K^*$ scattering amplitude



Input-A: $m_{f_1(1285)} = 1231$ MeV
Input-B: $m_{f_1(1285)} = 1281$ MeV
Input-C: $m_{f_1(1285)} = 1331$ MeV

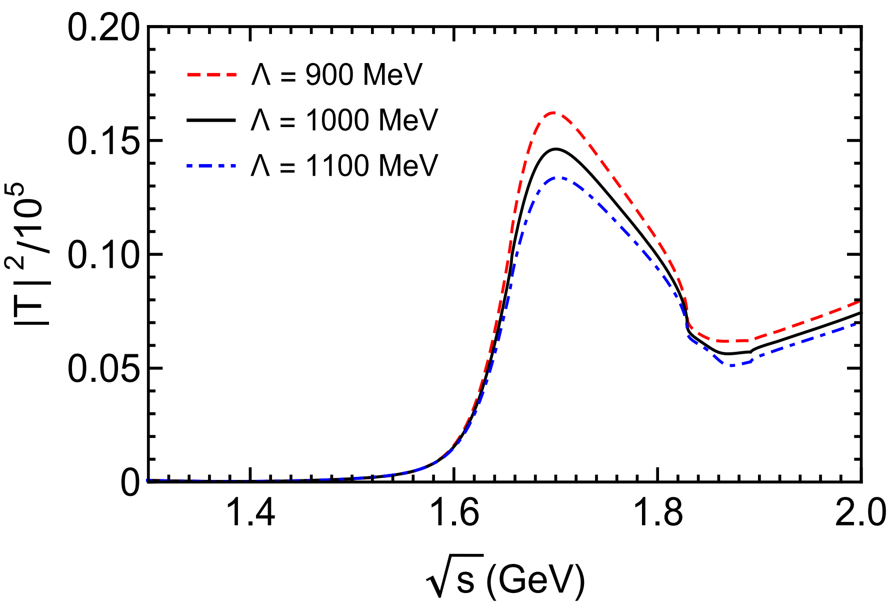
The resonant structure around 1650 MeV shows up in the modulus squared.
We suggest that this is the origin of the present $\pi_1(1600)$.

The $\eta \bar{K} K^*$ scattering amplitude

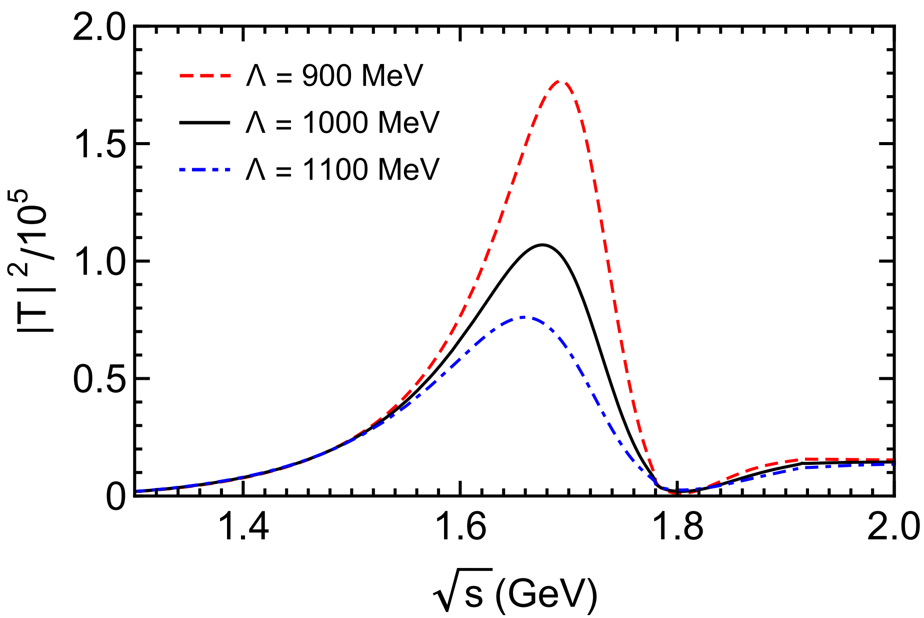
We also investigated the $\eta \bar{K} K^*$ system to look for possible isospin scalar $J^{PC} = 1^{-+}$ exotic state.

For the $\eta \bar{K} K^*$ system, $\bar{K} K^*$ can form a cluster $f_1(1285)$ and ηK^* can form a cluster $K_1(1270)$.

$\eta f_1(1285)$ scattering amplitude



$\bar{K} K_1(1270)$ scattering amplitude



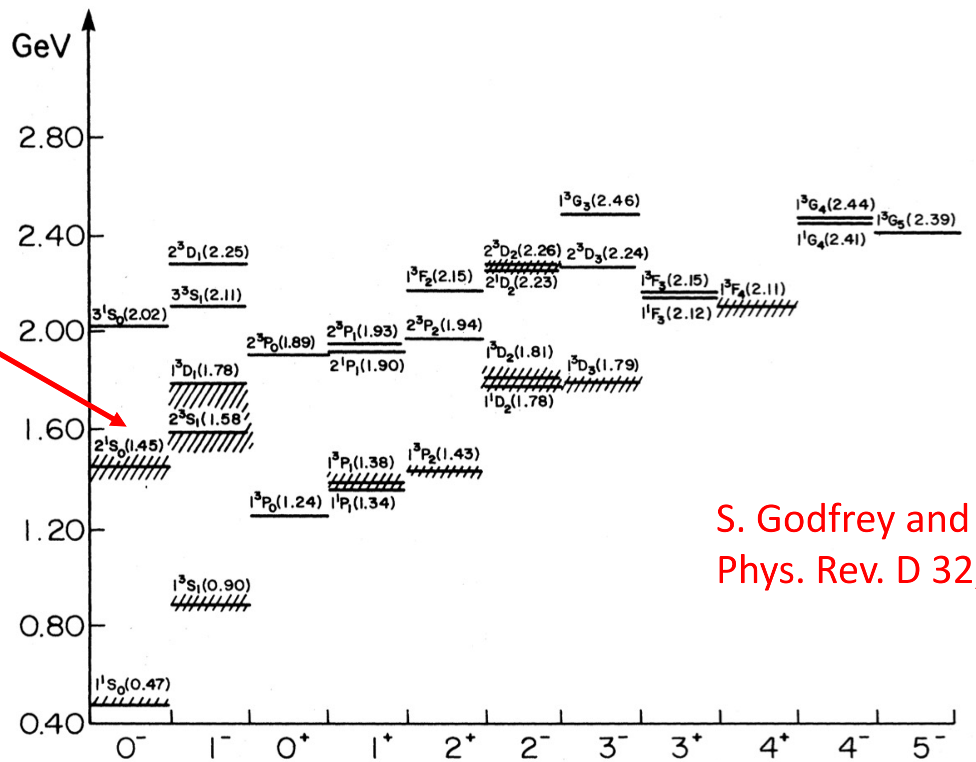
We find that there is a resonance with $I(J^{PC}) = 0(1^{-+})$ from $\eta \bar{K} K^*$ interaction, and $\bar{K} K_1(1270)$ component is dominant.

Experimental and theoretical status of K(1460)

In experiment, its mass and width has not been determined

1482.4±3.48 MeV	335.6±6.2 MeV	LHCb	EPJ	C78	443
~ 1460 MeV	~ 260 MeV	CNTR	NP	B187	1
~ 1400 MeV	~ 250 MeV	SLAC	PRL	36	1239

In quark model, it can be interpreted as 2^1S_0 excitation of the kaon.



S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).

FIG. 4. The strange mesons ($-u\bar{s}, -d\bar{s}$). The legend is as for Fig. 3. Significant spectroscopic mixing in this sector:

(a) With $\begin{bmatrix} Q_{\text{low}} \\ Q_{\text{high}} \end{bmatrix} \simeq \begin{bmatrix} \cos\theta_{nL} & \sin\theta_{nL} \\ -\sin\theta_{nL} & \cos\theta_{nL} \end{bmatrix} \begin{bmatrix} n\,^1L_L \\ n\,^3L_L \end{bmatrix}$ we find $\theta_{1P}\simeq 34^0$, $\theta_{1D}\simeq 33^0$, $\theta_{2P}\simeq 15^0$, $\theta_{1F}\simeq 32^0$, $\theta_{2D}\simeq 25^0$, $\theta_{1G}\simeq 33^0$;

(b) $1^{--}(1.58)\simeq 1.00(2\,^3S_1)+0.04(1\,^3D_1)$.

K(1460) can be interpreted as a molecule state from the $KK\bar{K}$ interaction

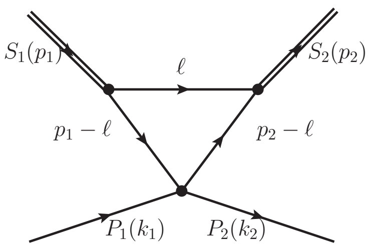
1. On shell Faddeev equation (all particles are on shell)

$$T^{ij} = t^i g^{ij} t^j + t^j [G^{iji} T^{ji} + G^{ijk} T^{jk}]$$

Phys. Rev. D 82, 094019 (2010)
Phys. Rev. C 83, 065205 (2011)
Phys. Rev. D 102, 094027 (2020)

2. The triangle loop method

f_0/a_0 →



K →

3. Solving the Faddeev equation in coordinate space

Isobar model

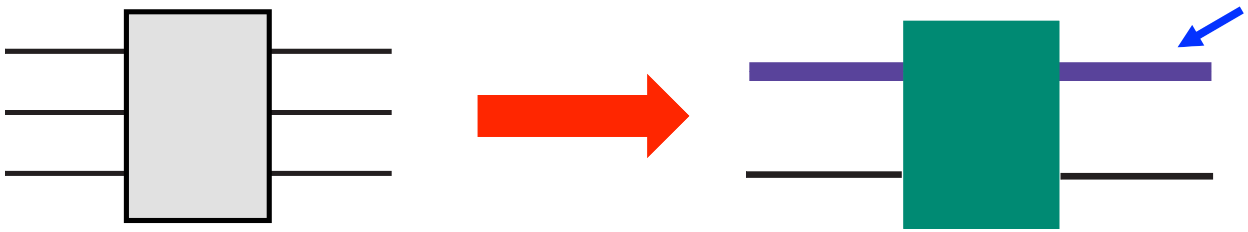
Under isobar model, the three-body interaction is reduced to the interaction of one particle and the isobar

Isobar

$\pi\pi \rightarrow \rho$

$K\bar{K} \rightarrow f_0/a_0$

…



$3 \rightarrow 3$ interaction

$2 \rightarrow 2$ interaction

This method has been used to study

The $a_1(1260)$ in $\pi^+\pi^+\pi^-$ scattering

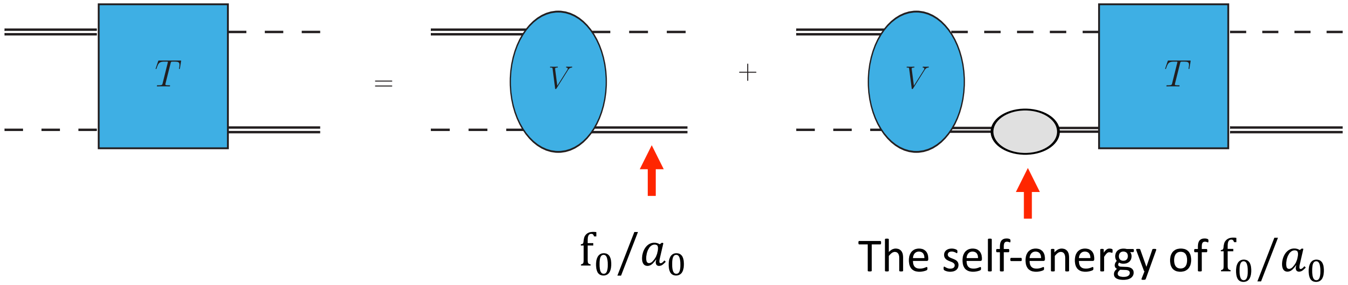
Phys. Rev. D 101, 094018 (2020)
Phys. Rev. D 49, 2763 (1994)

Three-body $D\bar{D}\pi$ dynamics for the X(3872)

Phys. Rev. D 84, 074029 (2011)

The $KK\bar{K}$ interaction under isobar model

Under isobar model, the $KK\bar{K}$ interaction is reduced to $Kf_0(980) - Ka_0(980)$ interaction



The Lippmann-Schwinger Equation

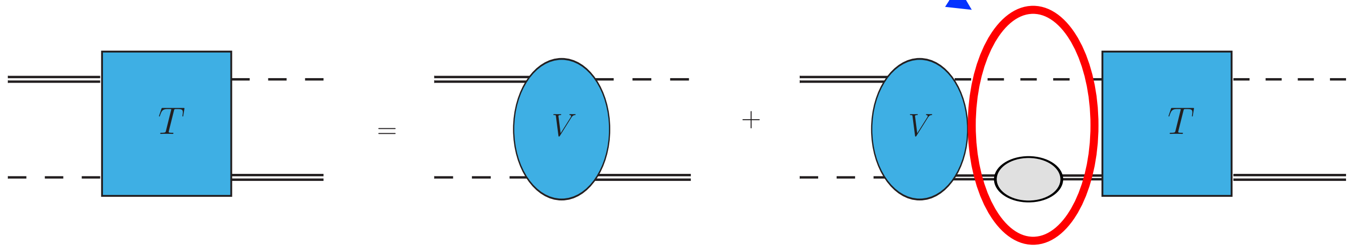
$$T(E,p',p) = V(E,p',p) + \int_0^{\Lambda} \frac{4\pi k^2 dk}{(2\pi)^3} V(E,p',k)G(E,k)T(E,k,p)$$

The scattering potential $V = \begin{bmatrix} V^{11} & V^{12} \\ V^{21} & V^{22} \end{bmatrix}$, $V^{\lambda'\lambda}$ can be obtained from time ordered perturbation theory (TOPT).

The bound state : $\det[1 - VG] = 0$.

The $KK\bar{K}$ interaction under isobar model

The Kf_0/Ka_0 propagator



The Kf_0/Ka_0 propagator is

$$G(E,k) = [Z\left(E - \omega^{(\lambda)}(k) - \omega_K(k)\right) - \Sigma_R(E,k)]^{-1}$$

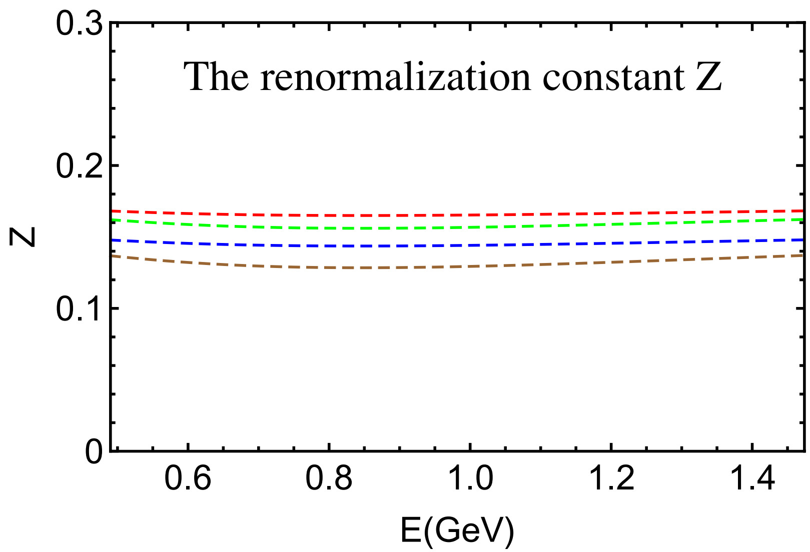
The self-energy has a strong effect on the propagator

To have Kf_0/Ka_0 cut, we use the subtraction

$$\Sigma_R(E,k) = \Sigma\,(E,k) \text{ - } \Sigma\,(E,k_{on})$$

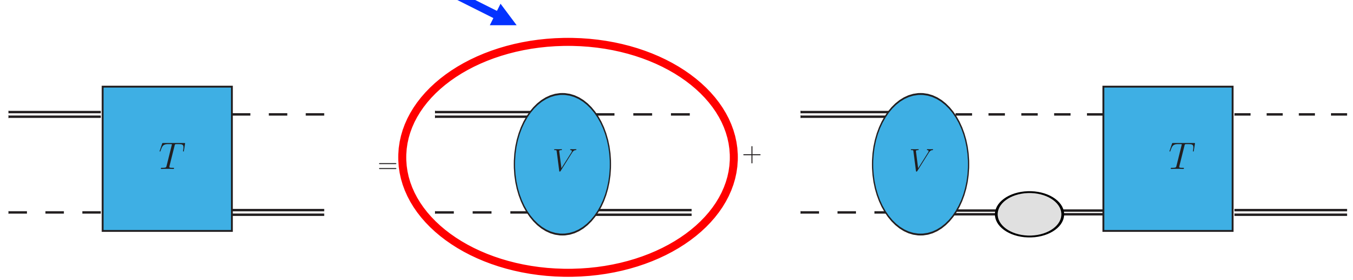
By definition the residue to be one at the pole position

$$Z = 1 + \frac{d}{dE}\Sigma_R(E,k)$$

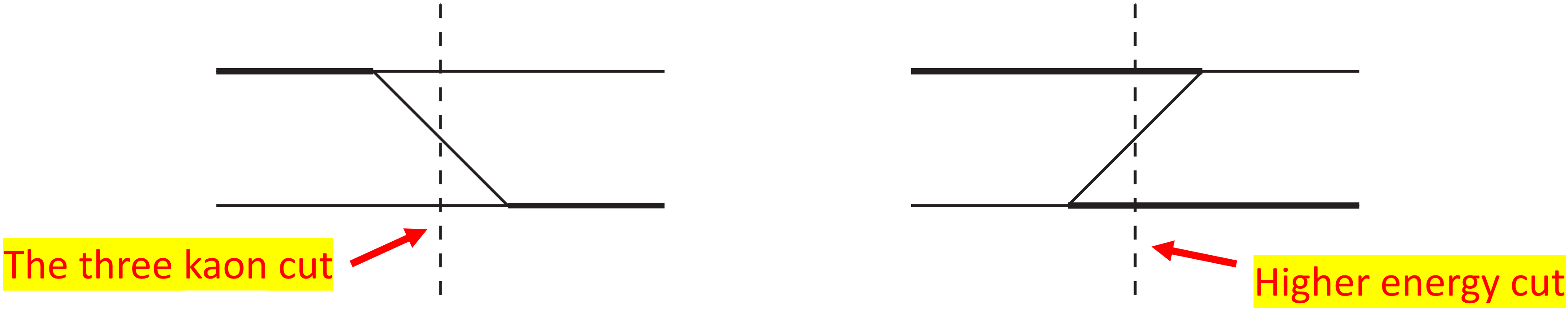


The $KK\bar{K}$ interaction under isobar model

The interaction potential



In TOPT, the t-channel acquires two contributions



The t-channel one kaon exchange is

$$V^{\lambda'\lambda}(E, p', p) = f_S^2 \cdot IF \cdot N \frac{1}{2\omega_K}\left[\frac{1}{E - \omega_{1'} - \omega_K - \omega_1} + \frac{1}{E - \omega_{2'} - \omega_K - \omega_2}\right]$$

We have also considered s-channel contribution

The $KK\bar{K}$ interaction under isobar model

In TOPT, the two terms corresponding to t-channel can be combined to

$$V^{\lambda'\lambda}(E, p', p) = f_S^2 \cdot IF \cdot N \frac{1}{\omega_K} \left[\frac{\omega_K - E^{off}}{(\Delta E)^2 - (\omega_K - E^{off})^2} \right]$$

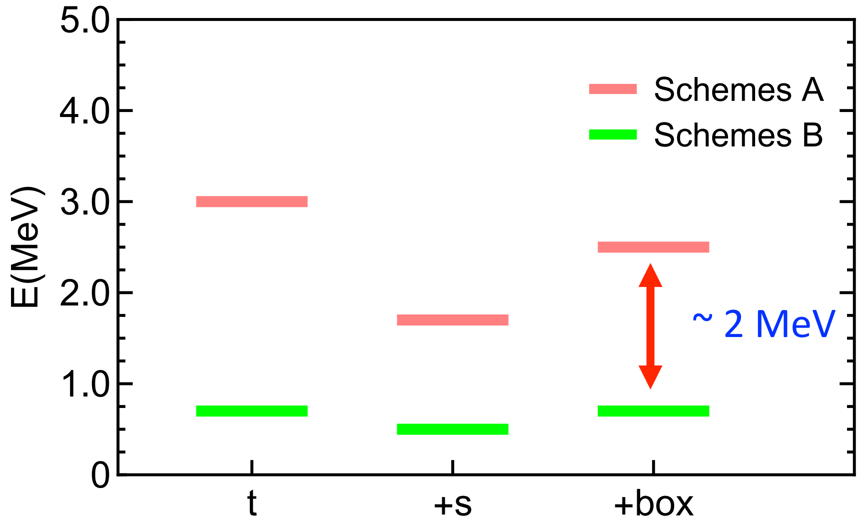
It is different from a covariant form $\frac{1}{t^2 - m_K{}^2}$

To recover the covariance, we have also considered the stretched boxes

And at one loop order, there are also crossed boxes contribution. Here I have not shown all the diagrams explicitly.

Numerical results

Schemes A represents the self-energy of f_0/a_0 has been considered
Schemes B represents the self-energy of f_0/a_0 has not been considered



$K(1460)$ can be interpreted as a molecule state from the $KK\bar{K}$ interaction

$K(1460)$ can be interpreted as a molecule state from the $KK\bar{K}$ interaction.
The self-energy of f_0/a_0 plays an important role.
The boxes contribution is not large.

Summary

1、The $\pi_1(1600)$ can be interpreted as a molecular state from $\pi\bar{K}K^*$ interaction within FCA approach. And also we predict an isospin scalar $J^{PC} = 1^{-+}$ exotic state in the $\eta\bar{K}K^*$ system.

2、Using isobar approach, we find that K(1460) can be interpreted as a molecular state from $KK\bar{K}$ interaction.

Thank you very much!