Effective field theories for nuclear physics

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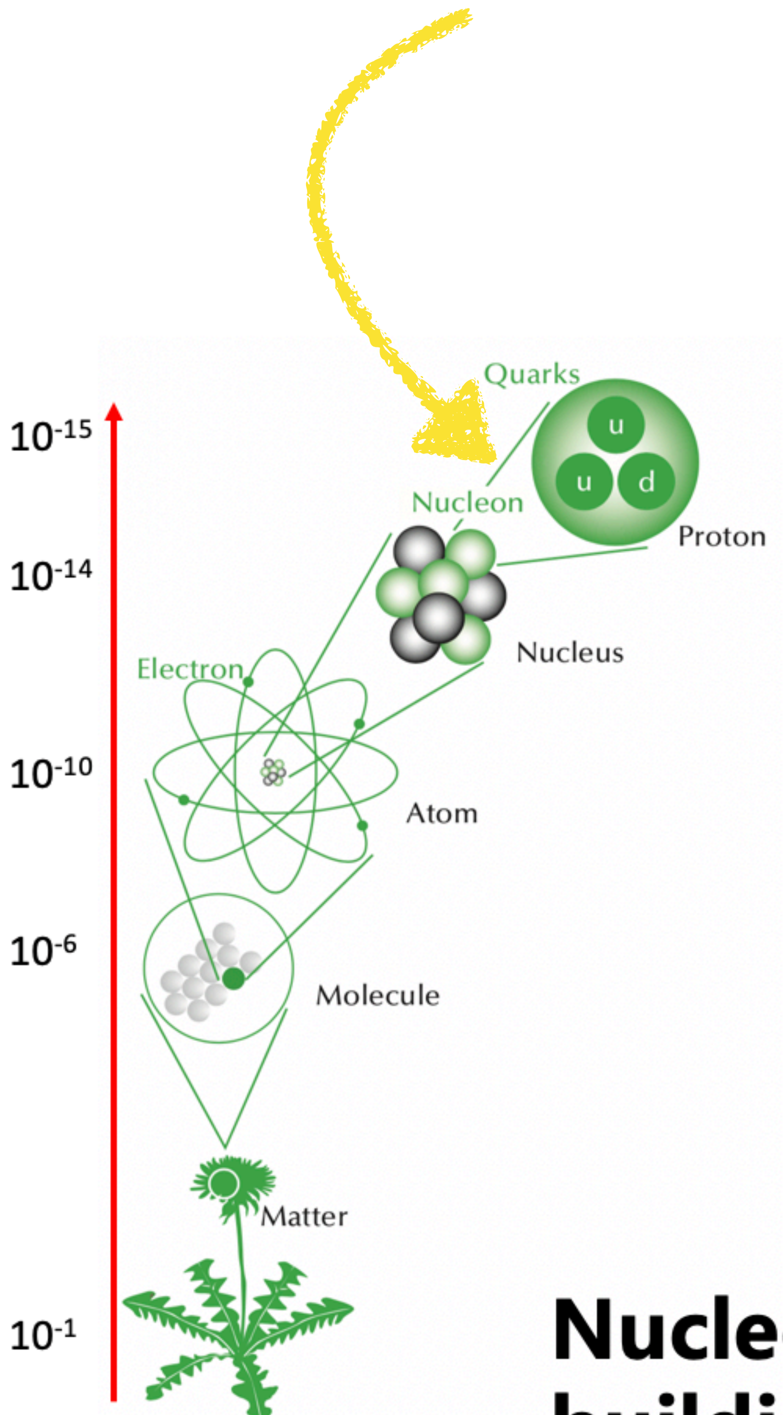
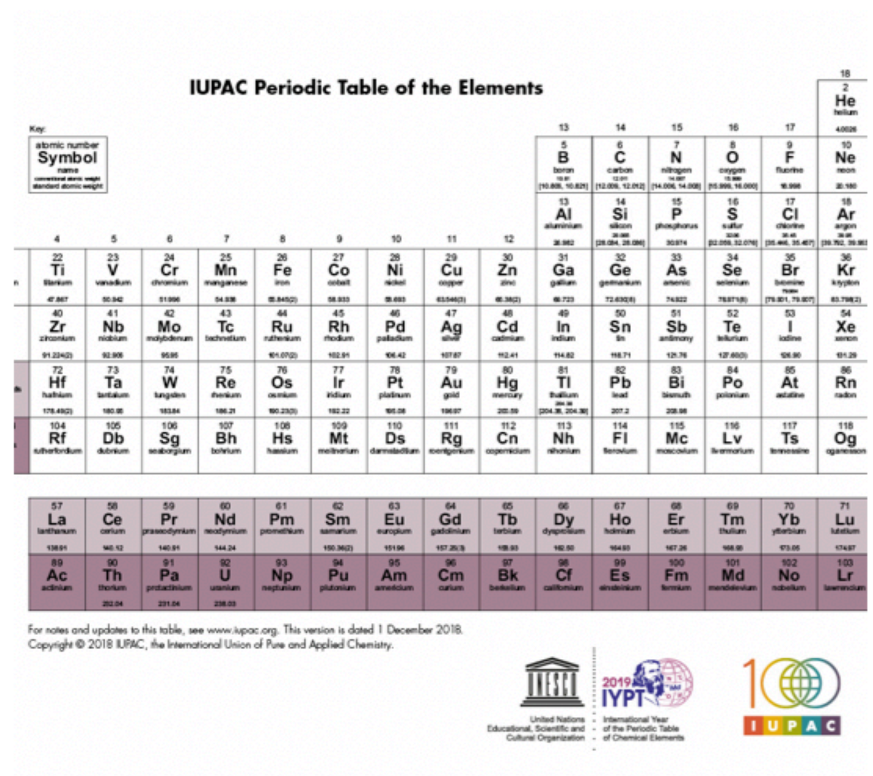
粒子物理与核物理结合，大有可为





Nucleons are the essential building blocks of Matter!

Outline

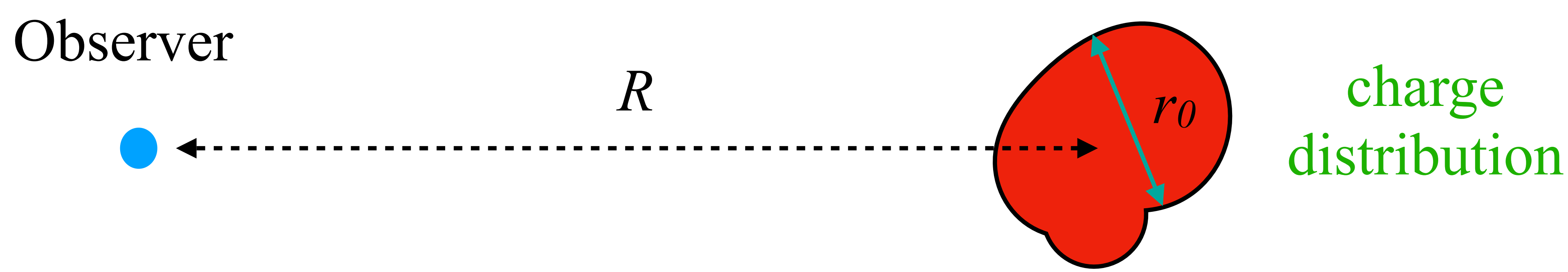
- Brief intro to nuclear effective field theories

- Expansion around unnatural leading-order interactions
  — Improving convergence of chiral nuclear forces

- Small-momentum fluctuations around fixed-energy configuration
  — bridging nuclear bound-state and reaction problems

Multipole expansion

A classical example of EFT



- Separation of scales: $R >> r_0$

- Controlled approximation, able to estimate uncertainty

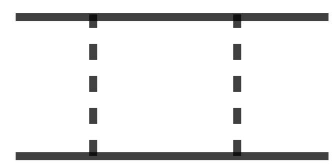
$$V = \frac{q}{R} + \frac{d_i R_i}{R^3} + \frac{Q_{ij} R_i R_j}{R^5} + \cdots$$

- Naturalness $|d_i| \sim q r_0$ $|Q_{ij}| \sim q r_0^2$ $\Rightarrow$ power counting based on naive dim. analysis (NDA)

- What if it is a rod?
  Slow convergence of a regular PC $\Rightarrow$ possible fine-tuning $\Rightarrow$ change PC

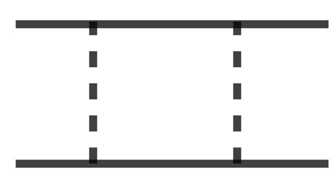
Renormalization in effective (field) theory

- UV regularization needed for intermediate states
  — rendering loops finite



- Observables independent of reg. scale Λ
  — guideline for power counting: RG invariance must be satisfied by power counting
  — can upset NDA (OPE tensor force $\propto$ $-1/r^3$)

$$\frac{d\mathcal{M}}{d\Lambda} = 0 \quad \Rightarrow$$



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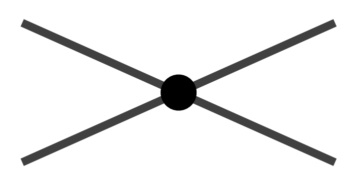
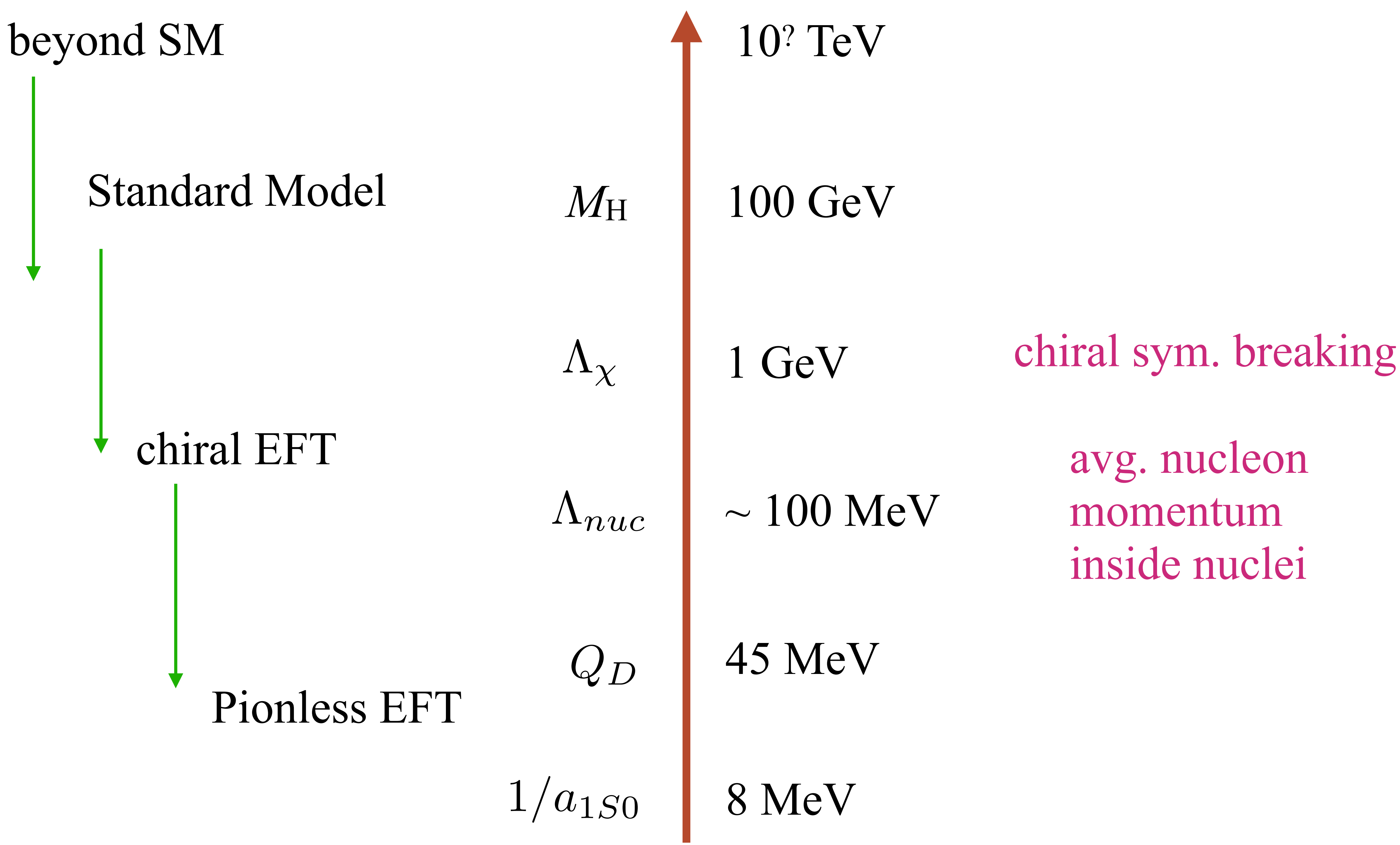
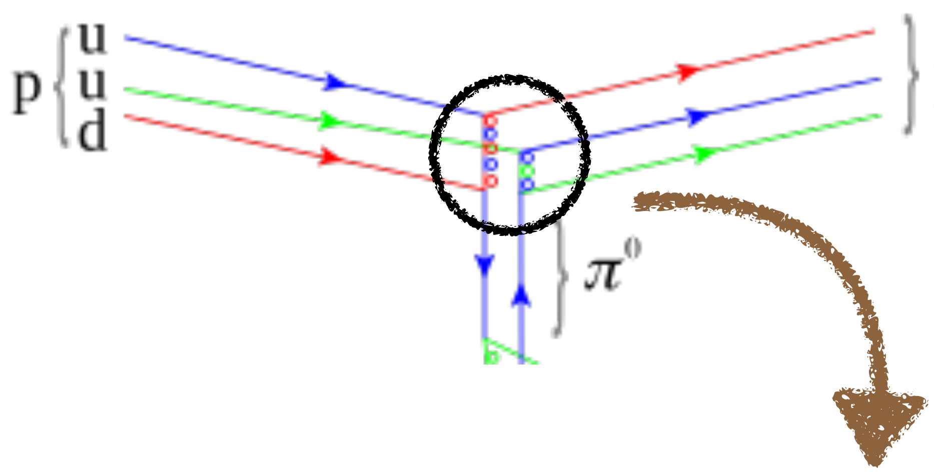


Hierarchy of EFTs



Chiral EFT

Dofs = nucleons + pions (Quark-gluon interactions encoded in low-energy constants (LECs))



$$-\frac{g_A}{2f_\pi} N^\dagger \tau_a \vec{\sigma} \cdot \vec{\nabla} \pi_a N$$

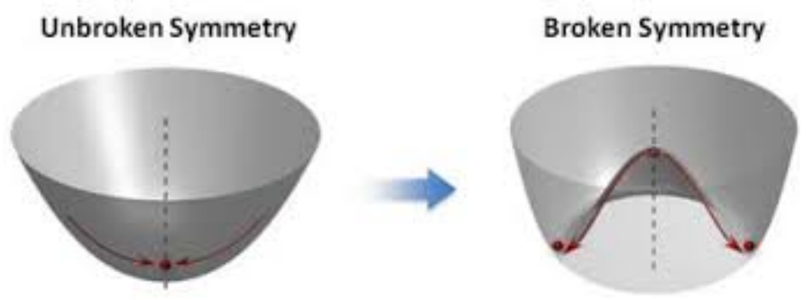
Chiral EFT

- Includes all symmetries of QCD, especially (approximate) chiral symmetry and its spontaneous breaking

$$\mathcal{L}_{\text{QCD}} = \sum_{f=u,d,s,\ldots} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4}\mathcal{G}_{a\mu\nu}\mathcal{G}_a^{\mu\nu}$$

$$q_L \equiv \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \mapsto \Big(\mathbf{SU(3)}_L\Big)\begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \qquad q_R \equiv \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix} \mapsto \Big(\mathbf{SU(3)}_R\Big)\begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix}$$

- Only two flavors used in our work



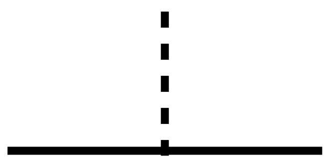
→ chiral symmetry nonlinearly realized by hadronic Dofs CCWZ; Weinberg; …

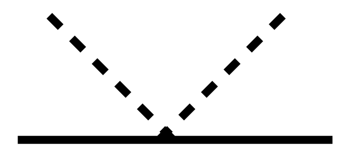
(see Weinberg's QFT Vol. 2)

Chiral EFT

- Nucleon-pion couplings are always $Q^n \Rightarrow$ long-range nuclear forces naturally ordered

O(Q)



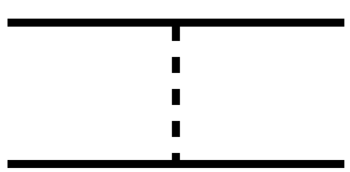


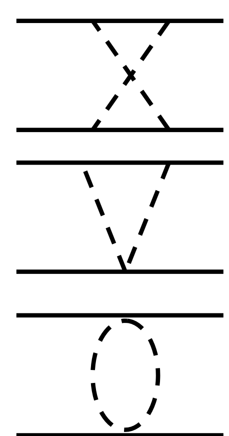
$N^\dagger(i\partial_t + \cdots)N$

$$-\frac{g_A}{2f_\pi}N^\dagger\tau_a\vec{\sigma}\cdot\vec{\nabla}\pi_a N \qquad -\frac{1}{4f_\pi^2}N^\dagger\epsilon_{abc}\tau_a\pi_b\dot{\pi}_c N$$

Derivative coupling accompanied by **chiral connection terms**

- NN long-range forces





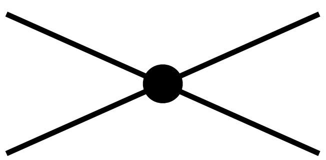
...

one-pion exchange = $O(1)$

two-pion exchanges = $O(Q^2)$

Weinberg '90

- Short-range interactions: large numbers of contact operators
- Organized by partial-wave quantum numbers
- In forms of momentum polynomials



$$-C^{(s)}(N^T P_i^{(s)} N)^\dagger (N^T P_i^{(s)} N) - C_2^{(s)} \left[(N^T P_i^{(s)} N)^\dagger (N^T P_i^{(s')} \overleftrightarrow{\nabla}^2 N) + h.c. \right] \quad \cdots$$

$$-C^{(ss')}(N^T P_i^{(s)} N)^\dagger (N^T P_i^{(s')} N) \quad \cdots$$

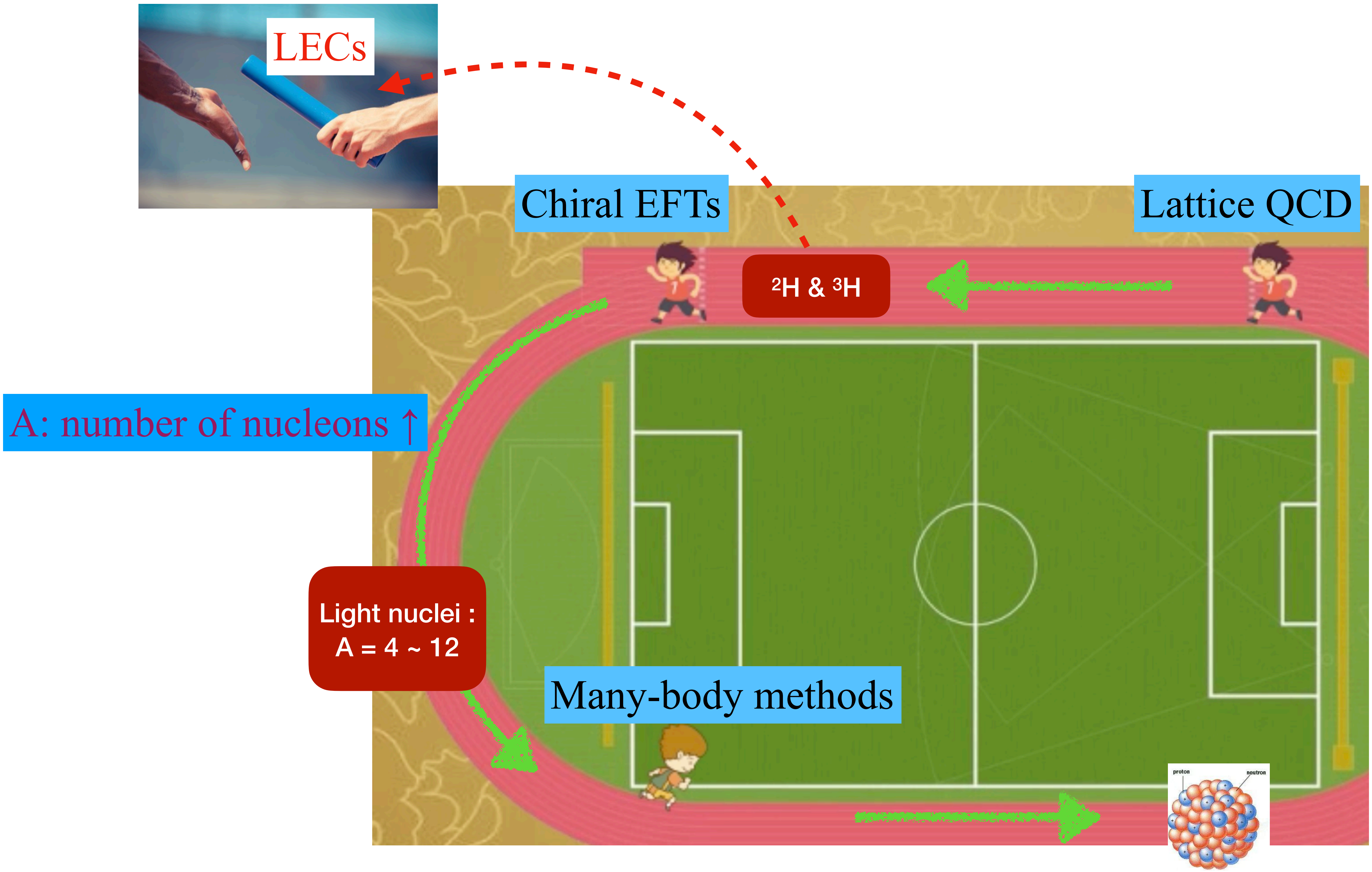
s, s' = $^1S_0$, $^3S_1$, $^3P_0$, …

$$P_i^{(^1S_0)} = \frac{(i\sigma_2)(i\tau_2\tau_i)}{2\sqrt{2}}$$

$$P_i^{(^3S_1)} = \frac{(i\sigma_2\sigma_i)(i\tau_2)}{2\sqrt{2}}$$

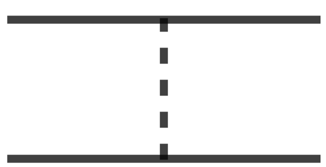
$$P_i^{(^3D_1)} = \left(\overleftrightarrow{\nabla}_i \overleftrightarrow{\nabla}_j - \frac{\delta_{ij}}{n} \overleftrightarrow{\nabla}^2 \right) P_j^{(^3S_1)}$$

Relay: from quarks & gluons to Uranium

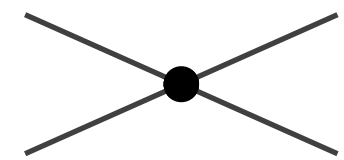


A Tale of Two Channels

- OPE (long-range) is much weaker in 1S0



$$V_{1\pi}(\vec{q}\,) = -\frac{1}{(2\pi)^3}\left(\frac{g_A}{2f_\pi}\right)^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{(\vec{\sigma}_1\cdot\vec{q}\,)(\vec{\sigma}_2\cdot\vec{q}\,)}{\vec{q}^{\,2}+m_\pi^2}$$

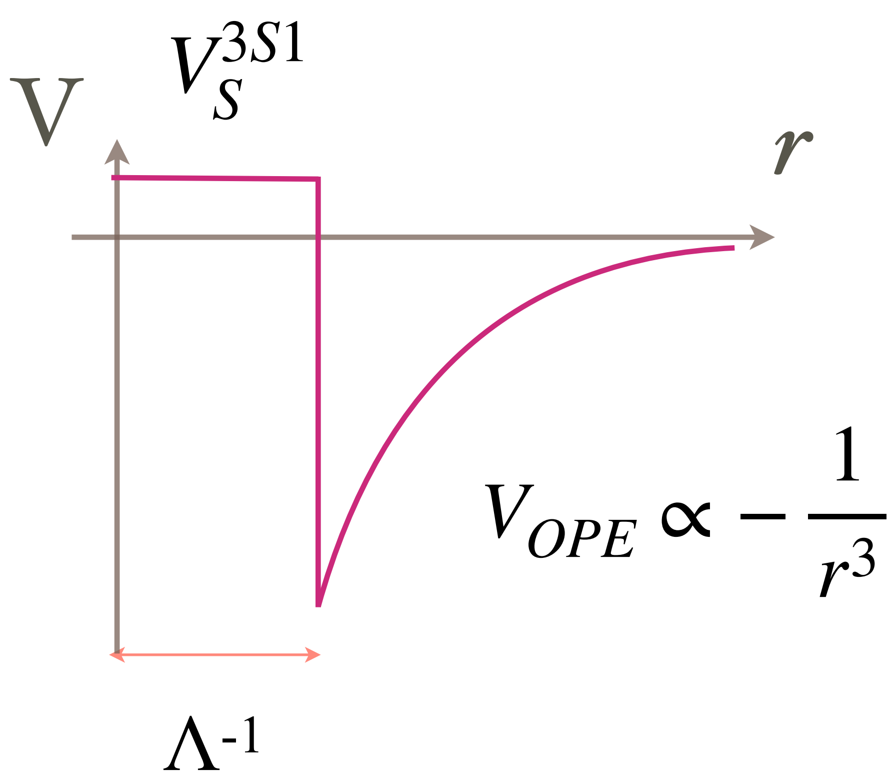


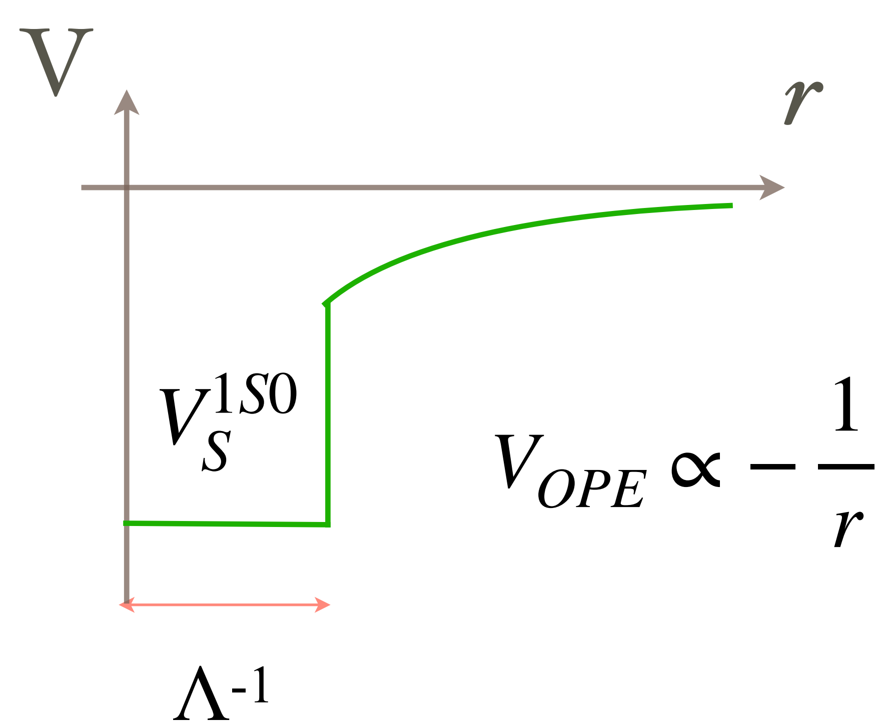
$$V_S^{3S1} = C_{3S1}$$

$$V_S^{1S0} = C_{1S0}$$

$^3S_1 - {}^3D_1$

$^1S_0$

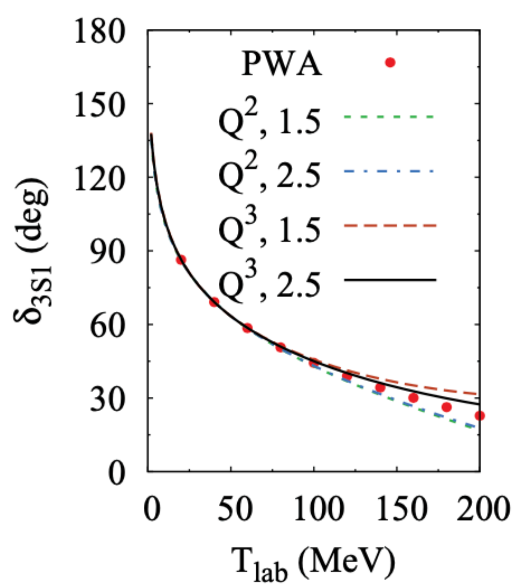


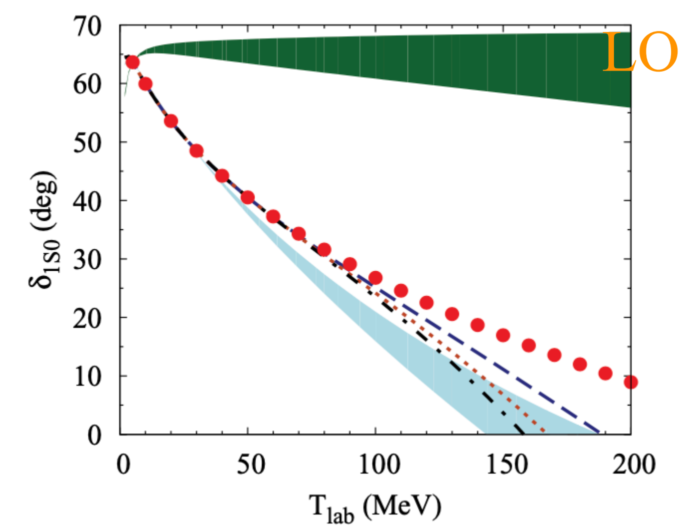


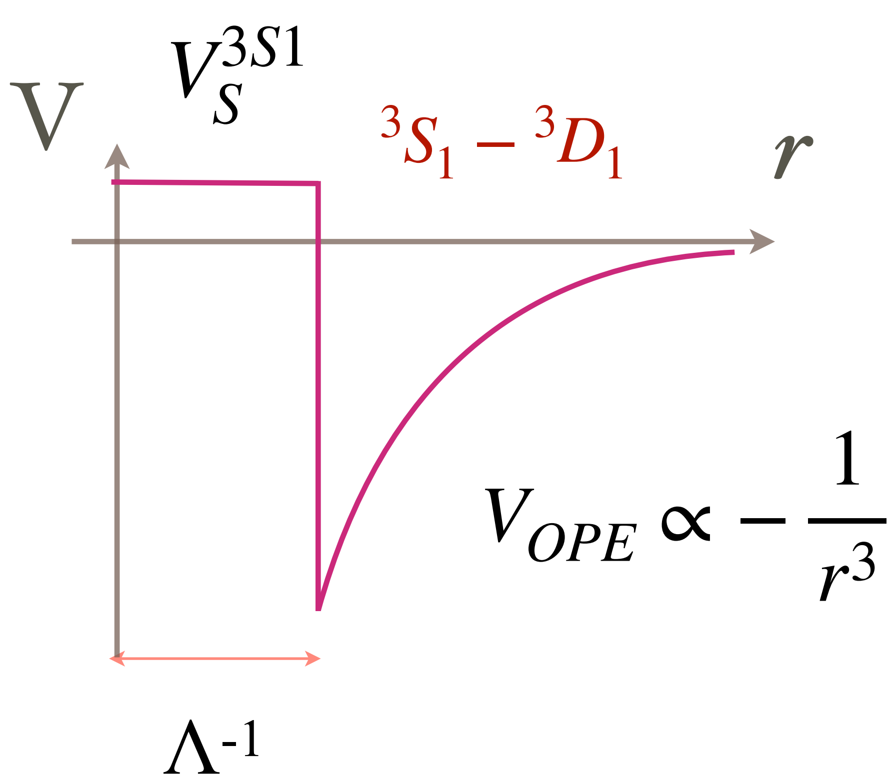
A Tale of Two Channels

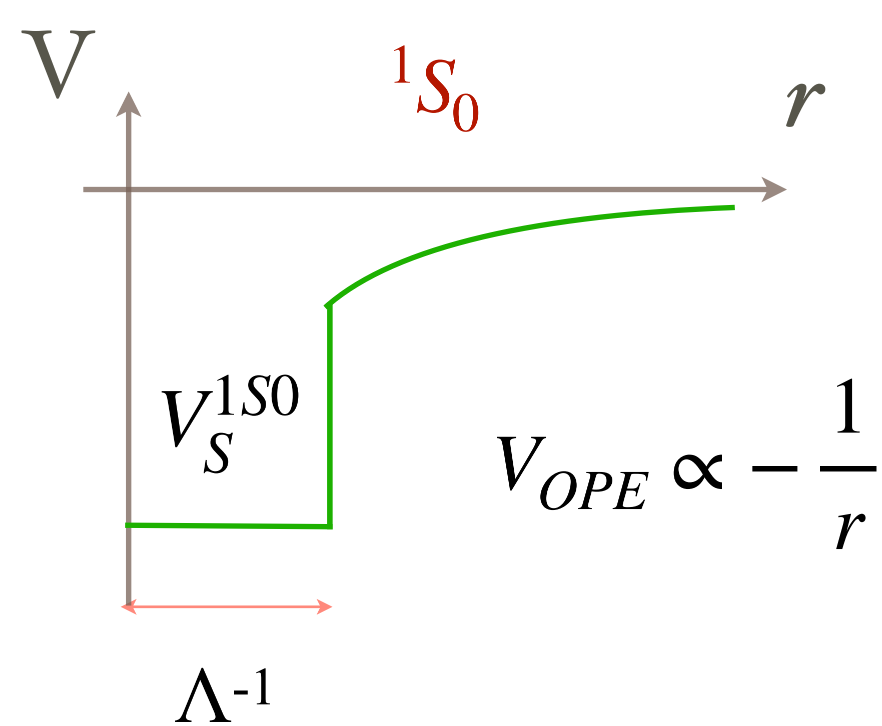
BwL & Yang PRC86, 024001
BwL & Yang PRC85, 034002

- One parameter in 1S0 cannot construct both short-range repulsion and long-range attraction





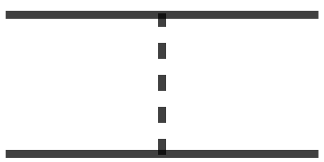




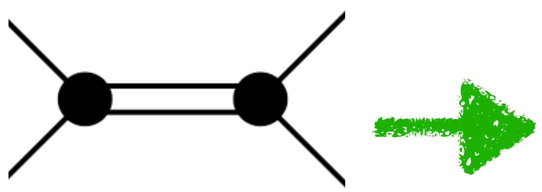
Unnatural LO forces

Peng, Lyu, Koenig & Long, arxiv 2112.00947

- Two parameters to describe non-pion forces:
  $\Rightarrow$ Two-dimensional surface in parameter space of LECs



OPE



$$V_{\rm spr}(p', p) = -\frac{4\pi}{m_N}\frac{\lambda}{\sqrt{p'^2 + m_N\Delta}\sqrt{p^2 + m_N\Delta}}$$

$$= \lambda\left(\# + \#\frac{p^2}{m_N\Delta} + \cdots\right)\left(\# + \#\frac{p'^2}{m_N\Delta} + \cdots\right)$$

$\sqrt{m_N\Delta} \simeq 150$ MeV

$\lambda \simeq 200$ MeV

Other efforts to improve 1S0:

"Energy-dependent," Long, PRC 88, 014002 ('13)

"Covariant ChPT," Xiu-Lei Ren et al. *CPL* 38 (2021) 6, 062101

"TPE as LO," Mishra et al. arxiv 2111.15515

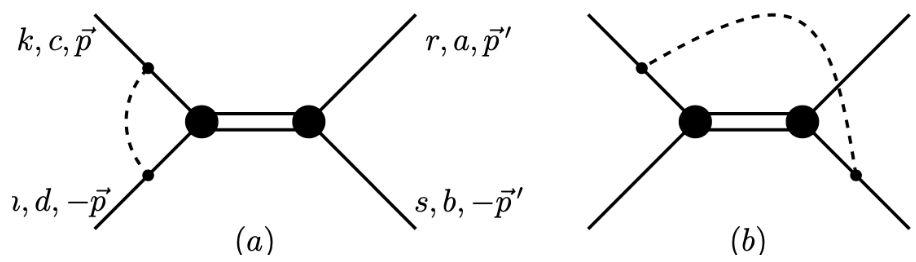
…

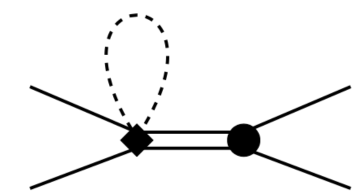
- How is this *not* just another model?

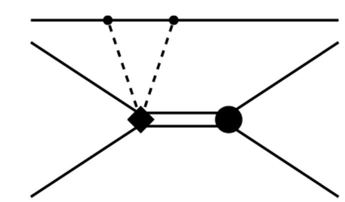
  * Only low-momenta scales at LO
  * Compatible with chiral Lagrangian (non-trivial)
  * Order-by-order convergence (NLO, NNLO…)
  * UV cutoff independence (satisfying RG invariance)

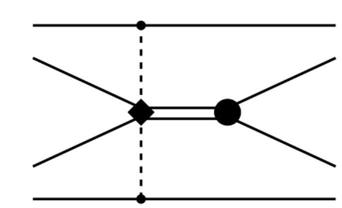
Compatible w/ chiral Lag.

- Non-trivial radiative corrections at NNLO
  - long-range forces
  - could have impact on triplet channels (3S1-3D1…)

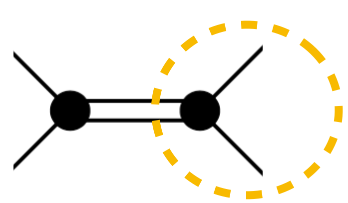




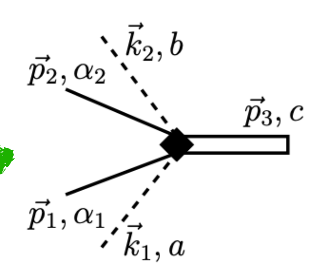




- Non-trivial chiral connections
  - able to derive additional forces
  - existing convergence preserved



$$\frac{\lambda}{\sqrt{p'^2 + m_N\Delta}\sqrt{p^2 + m_N\Delta}}$$



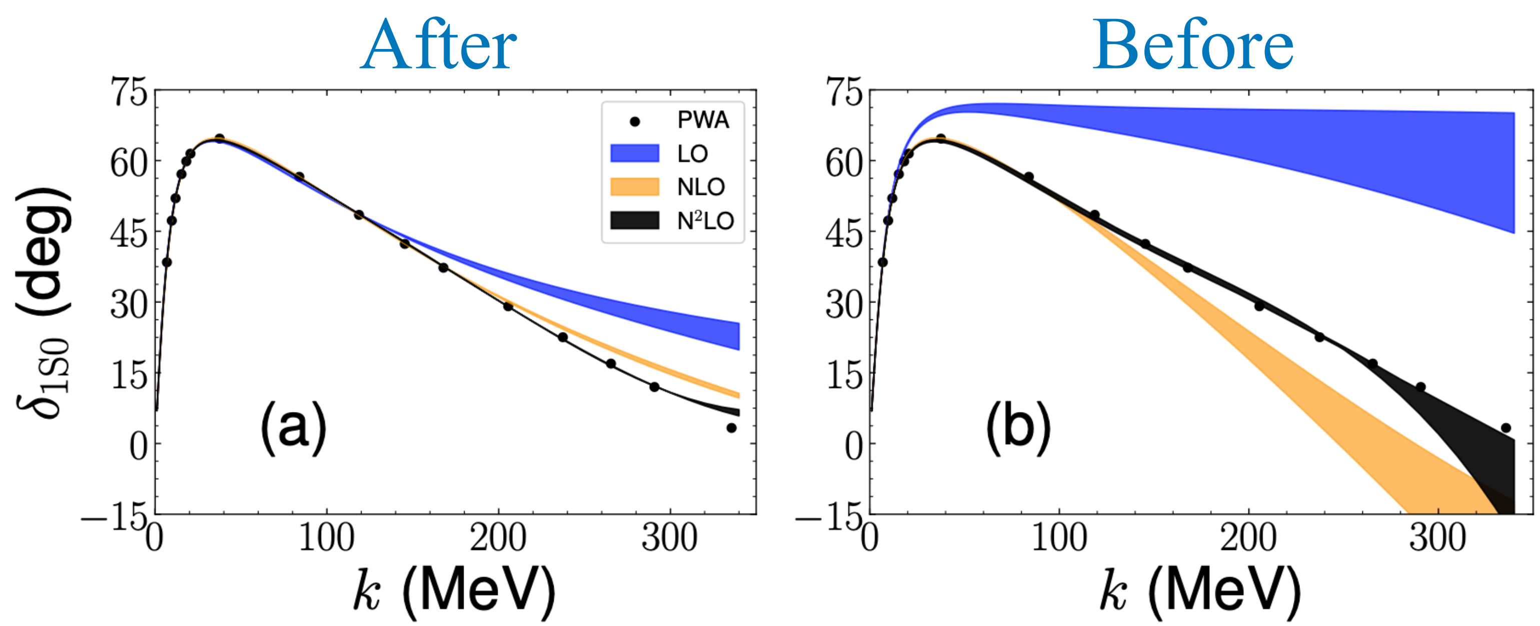
$$\mathcal{A}_{\phi NN\pi\pi} = \frac{ig}{4f_\pi^2}\sqrt{\frac{4\pi}{m_N}}\left\{i\left(\delta_{bc}\mathcal{P}_a - \delta_{ac}\mathcal{P}_b\right)_{\alpha_2\alpha_1}\mathcal{B}_+ + \frac{1}{\sqrt{8}}\epsilon_{abc}(\sigma_2\tau_2)_{\alpha_2\alpha_1}\mathcal{B}_-\right\}$$

$$\mathcal{B}_\pm \equiv u\left(\left|\vec{p}+\vec{k}_1/2\right|, \left|\vec{p}+\vec{k}_2/2\right|\right) \pm u\left(\left|\vec{p}-\vec{k}_1/2\right|, \left|\vec{p}-\vec{k}_2/2\right|\right),$$

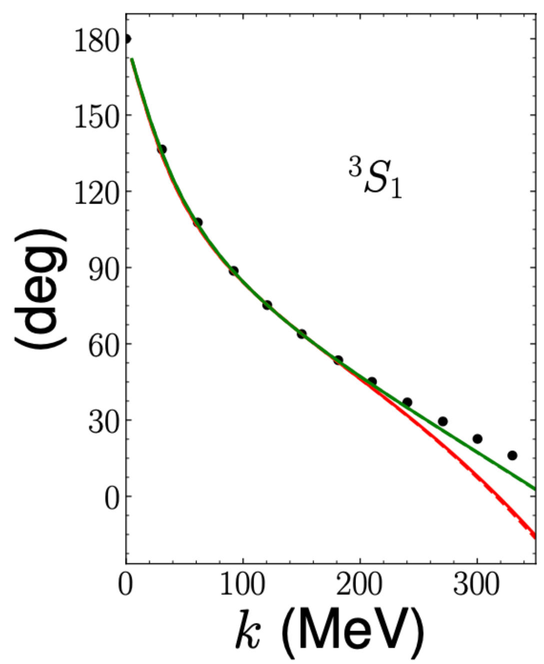
$$u(x,y) \equiv \left(1+\frac{x^2}{m_N\Delta}\right)^{-\frac{1}{2}} - \left(1+\frac{y^2}{m_N\Delta}\right)^{-\frac{1}{2}}$$

Up to NNLO

- Better convergence in 1S0



- 3S1 not spoiled by radiative corrections



Λ = 2400 MeV

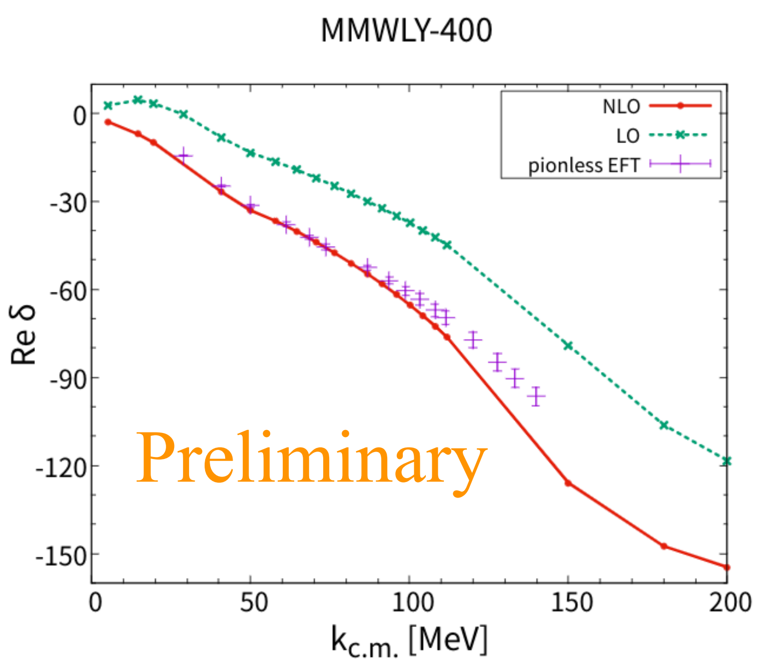
Λ = 600 MeV

3N systems

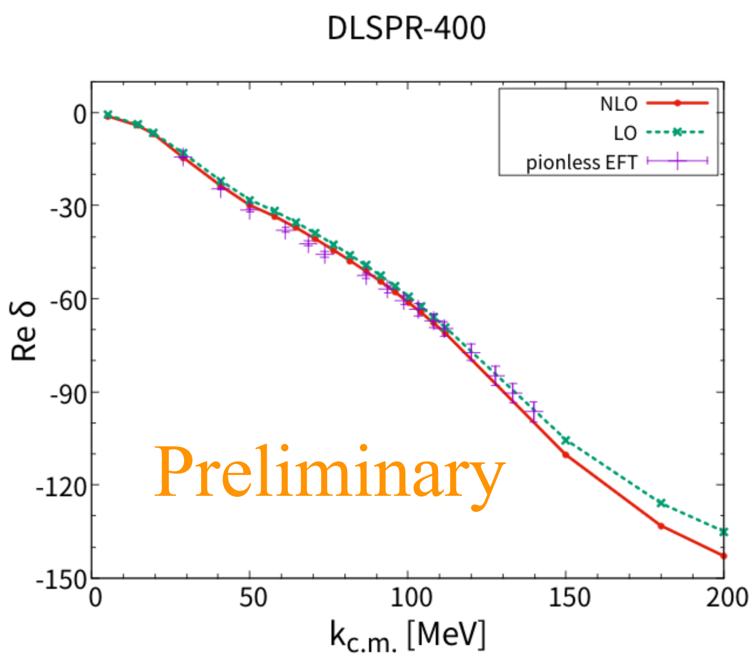
- Does it work in 3N systems?
- Solving the Faddeev equation (equivalent to the Schrodinger equation)
  - 3H binding energy, neutron-deuteron scattering phase shifts …

E.g., neuteron-deuteron scattering ($J^P = 1/2^+$, S wave)

Before

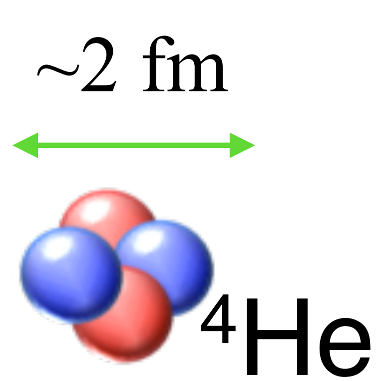


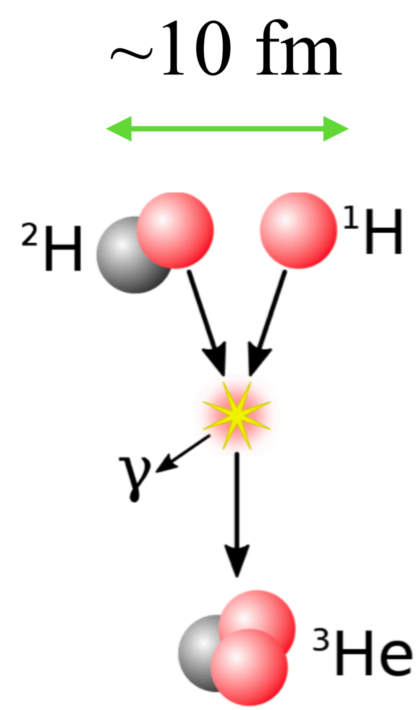
After

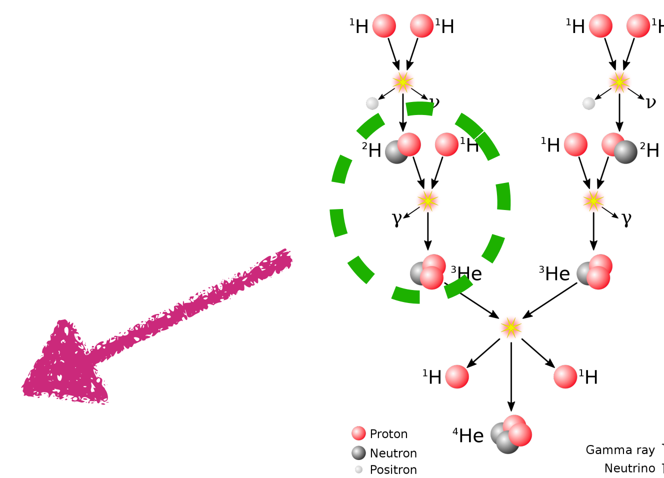


Ab initio calculations of nuclear reactions

- Ab initio ≈ diagonalizing nuclear Hamiltonian of A-nucleon systems
- Scattering and reactions normally involve larger configuration space







Chenbo Li (Sichuan U.), Jiexin Yu (Sichuan U.), Rui Peng (Sichuan U.), Songlin Lyu (Sichuan U.), Bingwei Long (Sichuan U.) (Jul 26, 2021)

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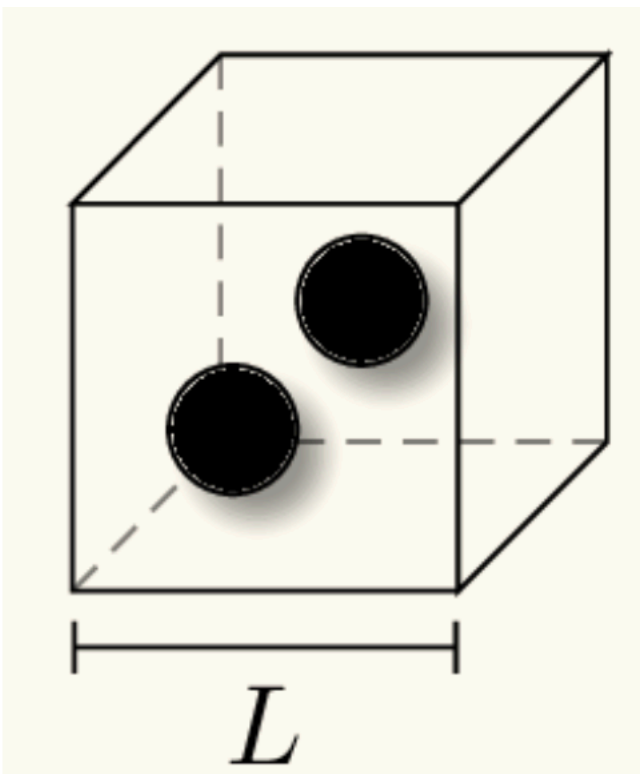
Lesson from LQCD



- Energy levels of particles confined in finite volume (provided by LQCD) tell us information about their interaction

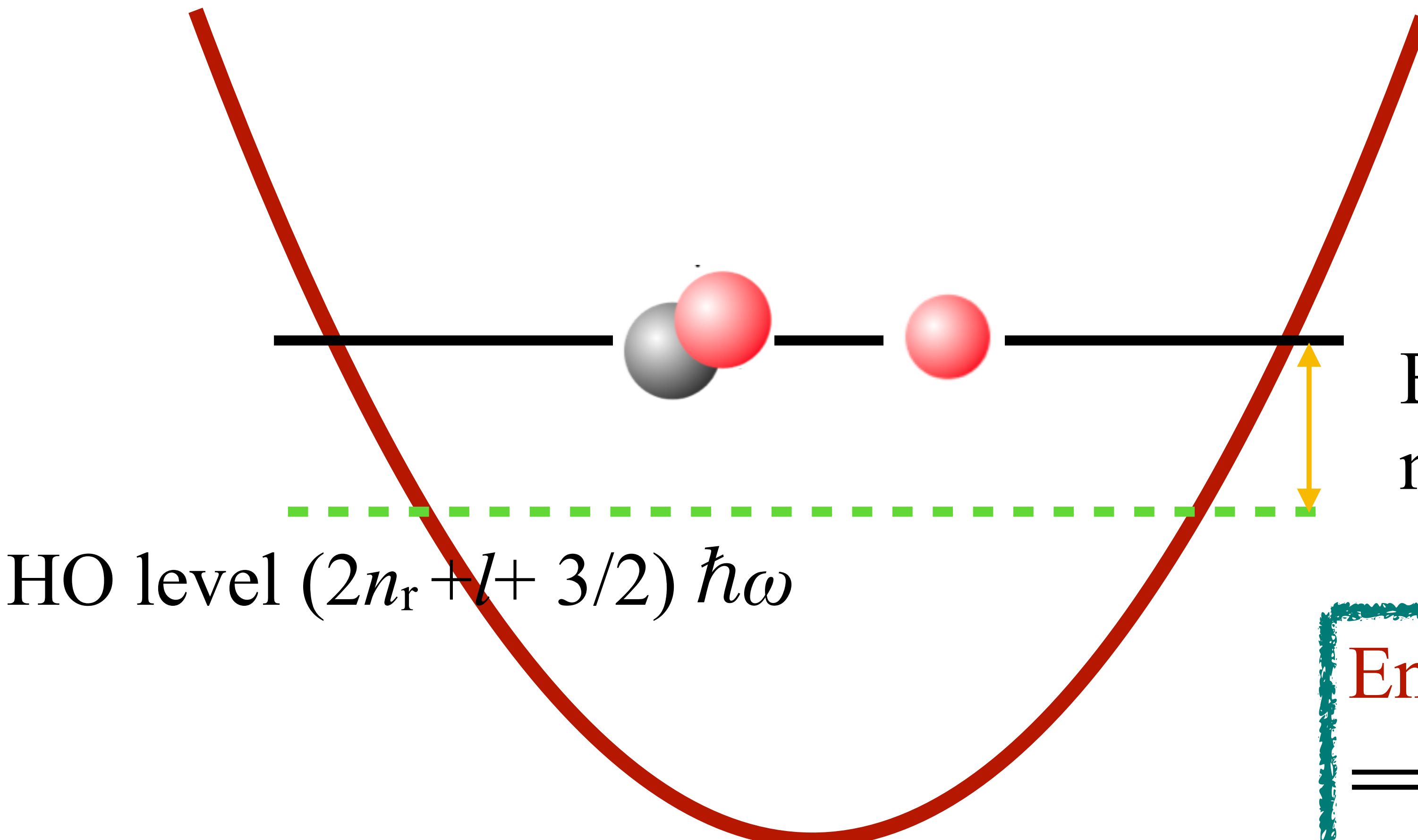
- Use energy eigenvalues as inputs to model-independently determine phase shifts at the same energies

Luscher's formula: $$\det[\mathcal{M}^{-1}(E_L) + F^{(P)}(E_L, L)] = 0$$

M. Luscher, Nucl. Phys. B **354**, 531 (1991)

Harmonic-oscillator trap

Busch et al. ’98
Luu, Savage & Vary ’10
Rotureau, et al. ’10 ’12
Xilin Zhang, et al. ’19 ’20



Energies of confined particles
$\Longrightarrow$ Amplitude

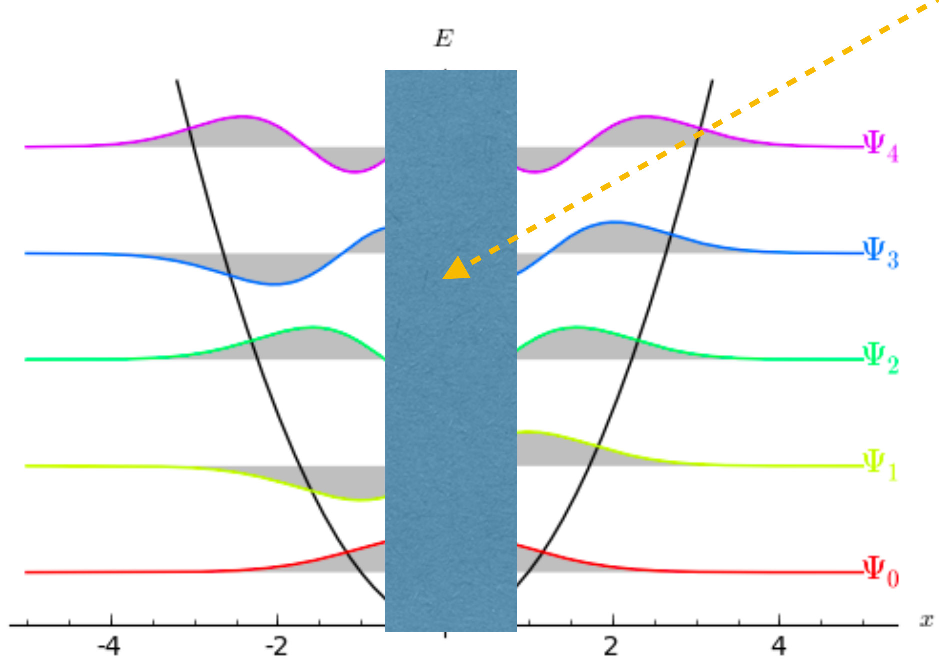
- HO potential is isotropic => angular momentum remains good quantum number
- HO w.f. analytically known
- Available software packages

Trick: matching w.f.

Outside wf : HO trapped

$$\propto aR(r;E) + bY(r;E)$$

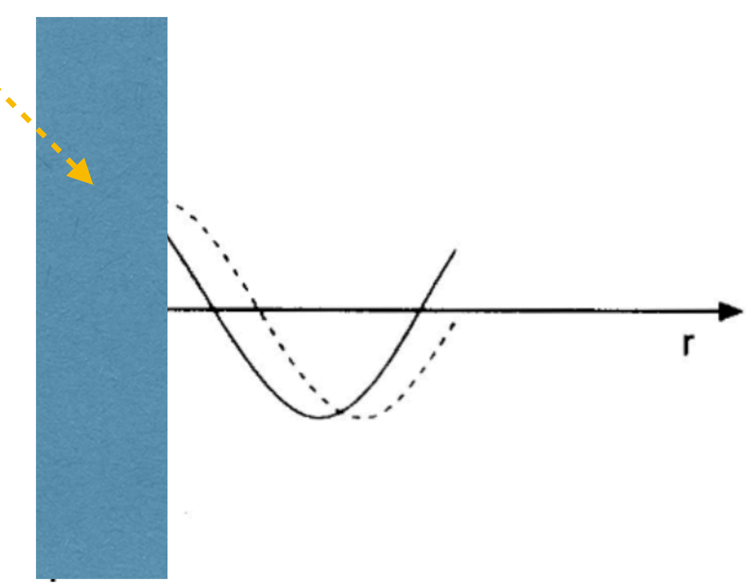
R & Y: solutions to
HO Schrodinger eq.



Intrinsic potential

Outside wf : scattering

$$\propto \sin(kr + \delta)$$

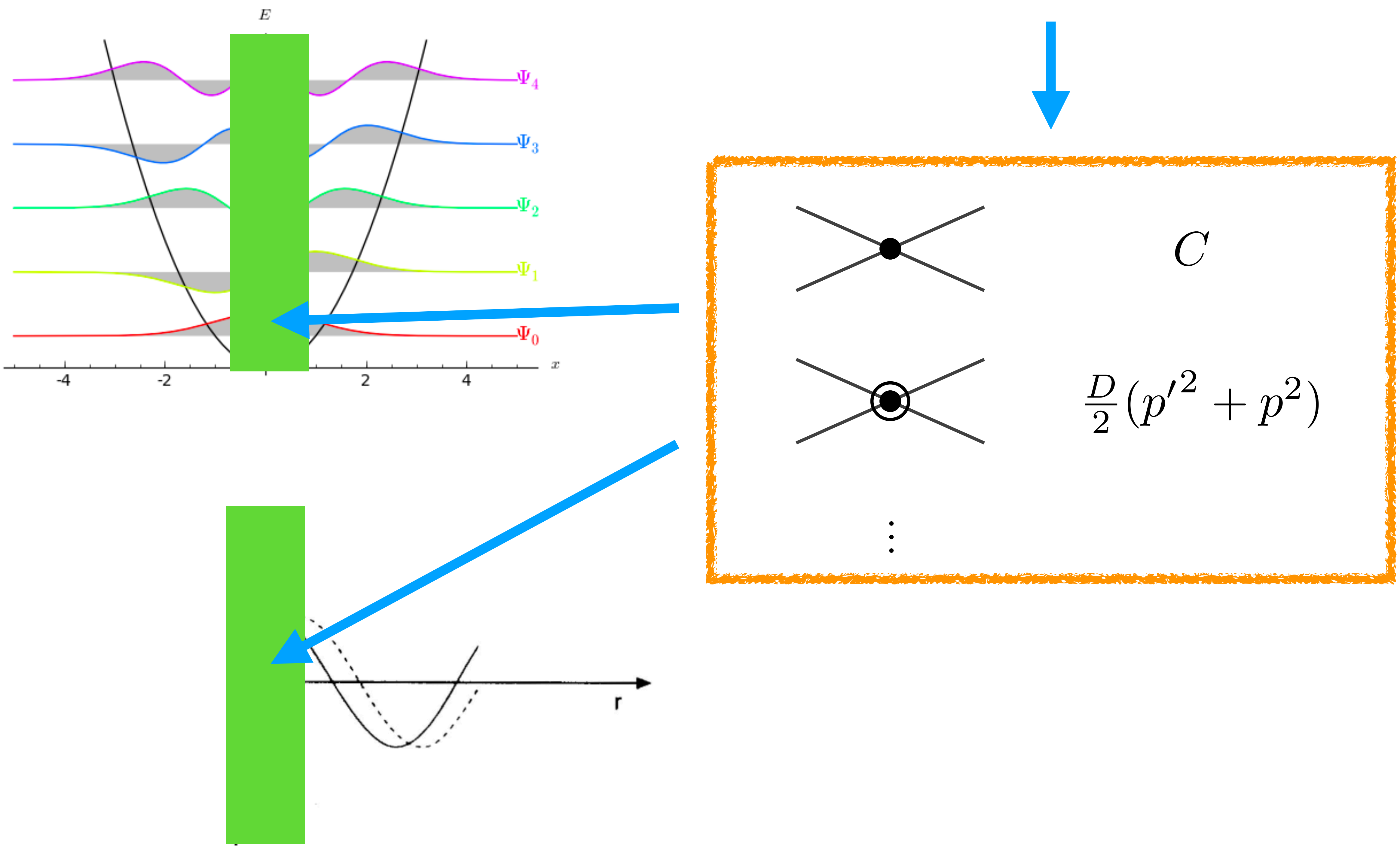


- Both wfs must match at the edge of the intrinsic potential V_i
- To construct outside wfs, detail of V_i does not matter $\Rightarrow$ use EFT !

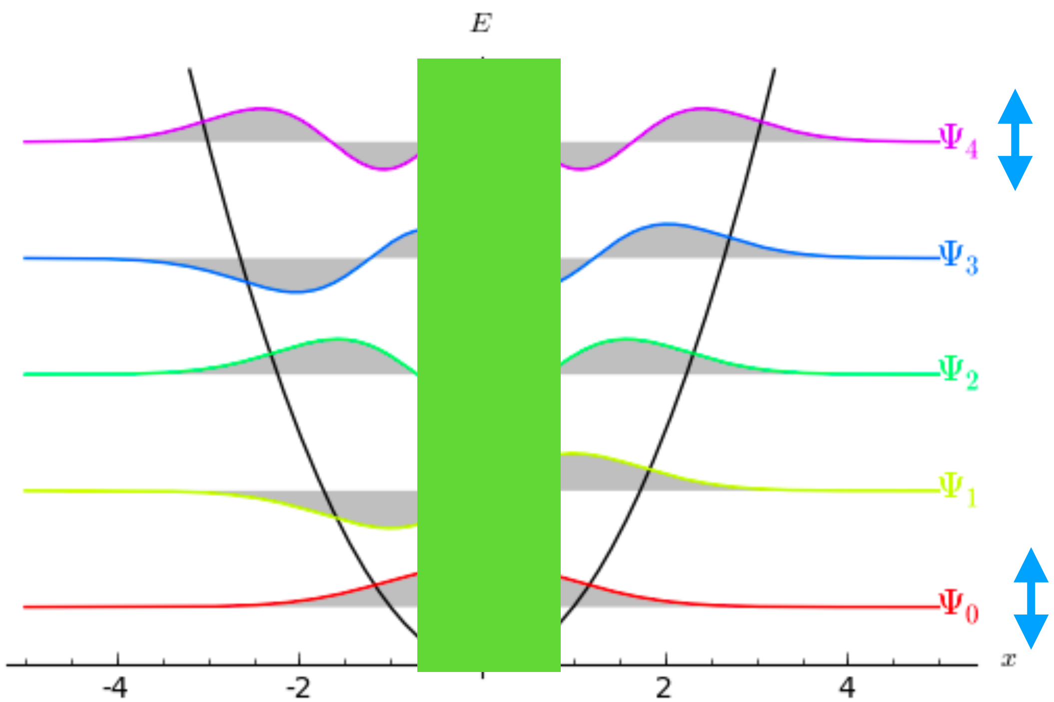
Contact EFT

- Use *NN* as example

$$\mathcal{L}_{NN} = -\frac{1}{2}C_s(N^\dagger N)^2 - \frac{1}{2}C_t(N^\dagger \vec{\sigma} N)^2 + \cdots$$



"Manifold" of EFTs



$$\mathcal{L}_{NN} = -\frac{1}{2}C_s(E_4)\,(N^\dagger N)^2 - \frac{1}{2}C_t(E_4)\,(N^\dagger \vec{\sigma} N)^2 + \cdots$$

$$\vdots$$

$$\mathcal{L}_{NN} = -\frac{1}{2}C_s(E_0)\,(N^\dagger N)^2 - \frac{1}{2}C_t(E_0)\,(N^\dagger \vec{\sigma} N)^2 + \cdots$$

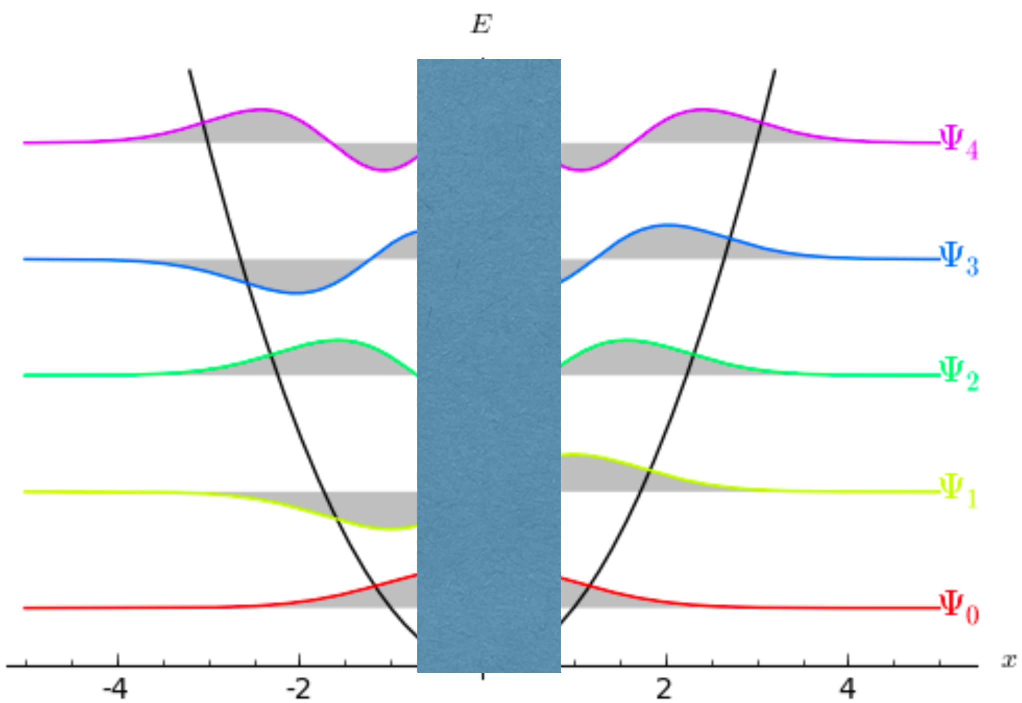
- One EFT in charge of a small domain around each eigen energy
  effective-range expansion around each energy

$$k\cot\delta = \alpha_0(\mathcal{E}_r) + \alpha_1(\mathcal{E}_r)(E-\mathcal{E}_r) + \alpha_2(\mathcal{E}_r)(E-\mathcal{E}_r)^2 + \cdots$$

- Weak predictive power, but it's OK

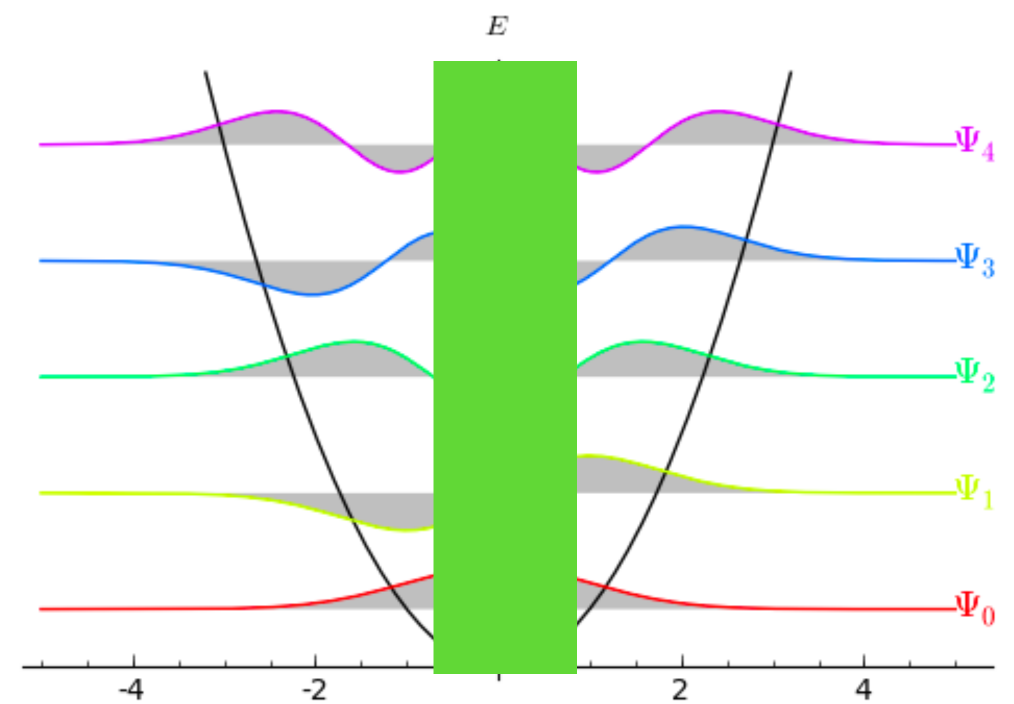
Recipe

$V_{realistic}$ (ab initio many-body)

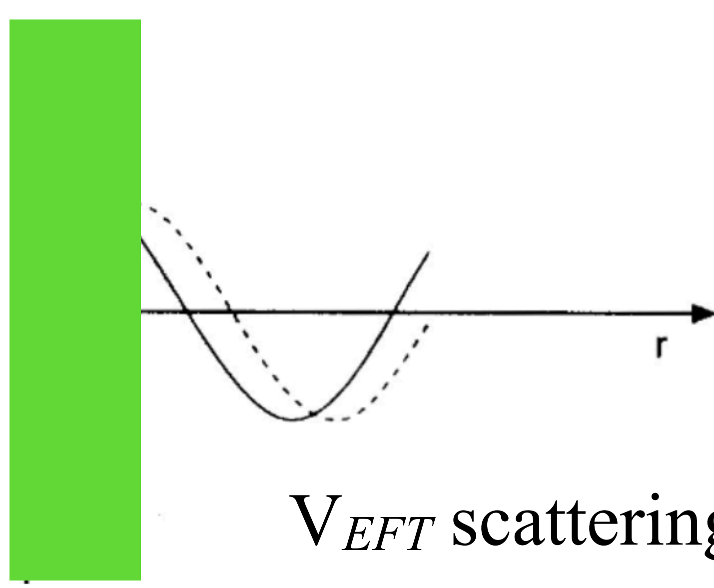


inputs

V_{EFT} (2B cal.)



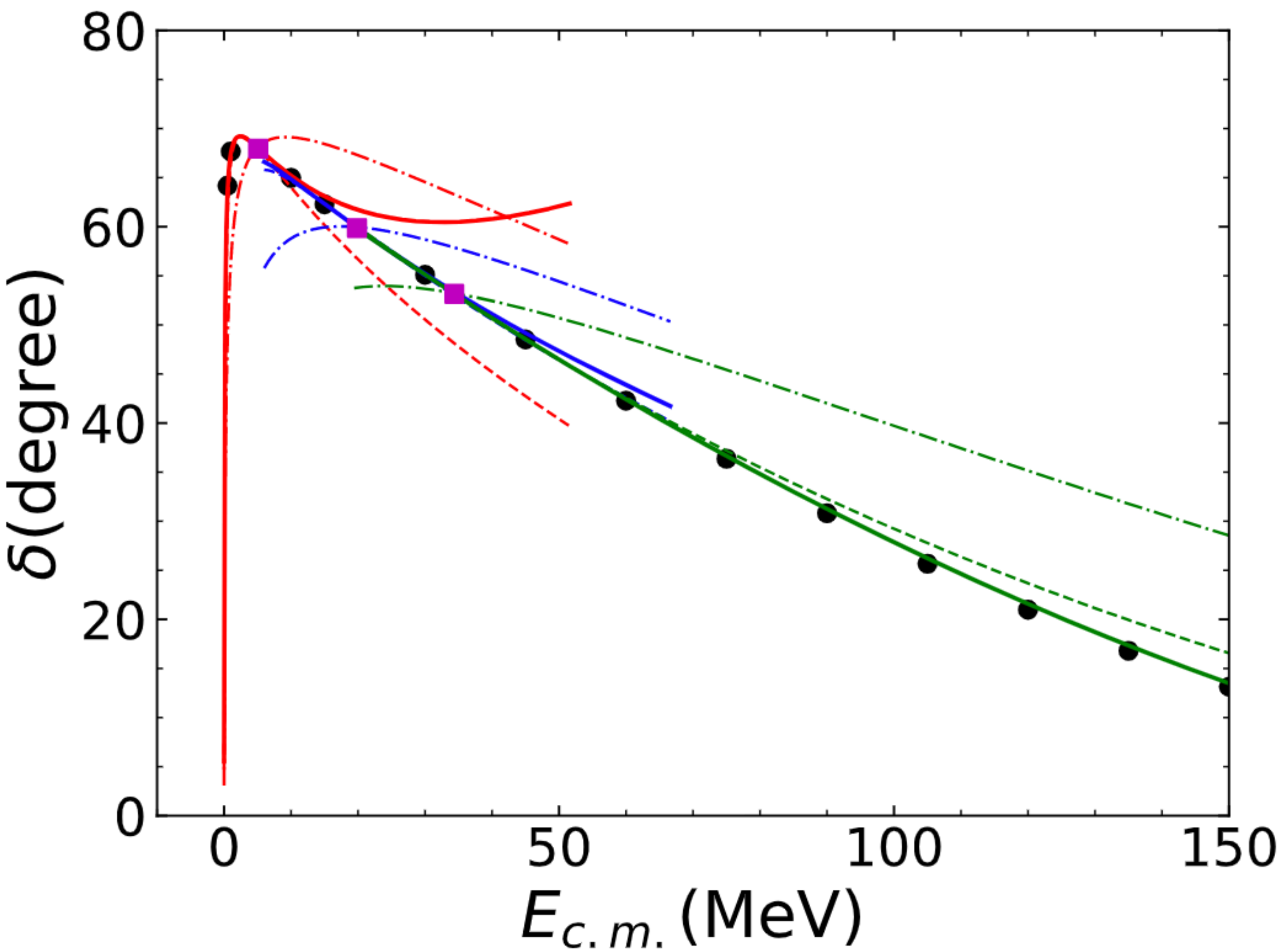
- Fix C_0, C_2, … of EFT



V_{EFT} scattering (2B cal.)

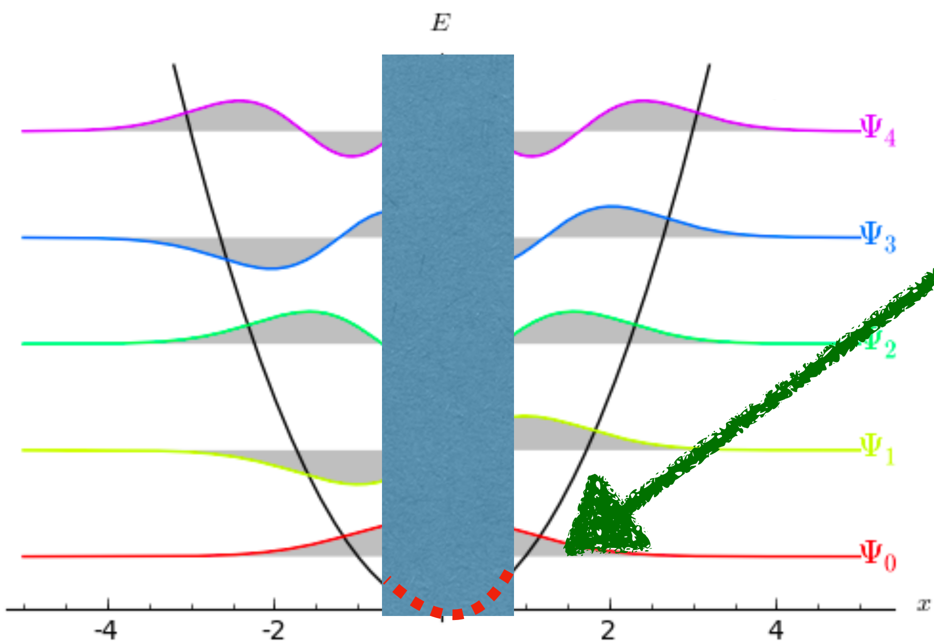
Exercise

- With three levels ($\hbar\omega$ = 7 MeV), NN 1S0 phase shifts are covered up to Ecm = 150 MeV



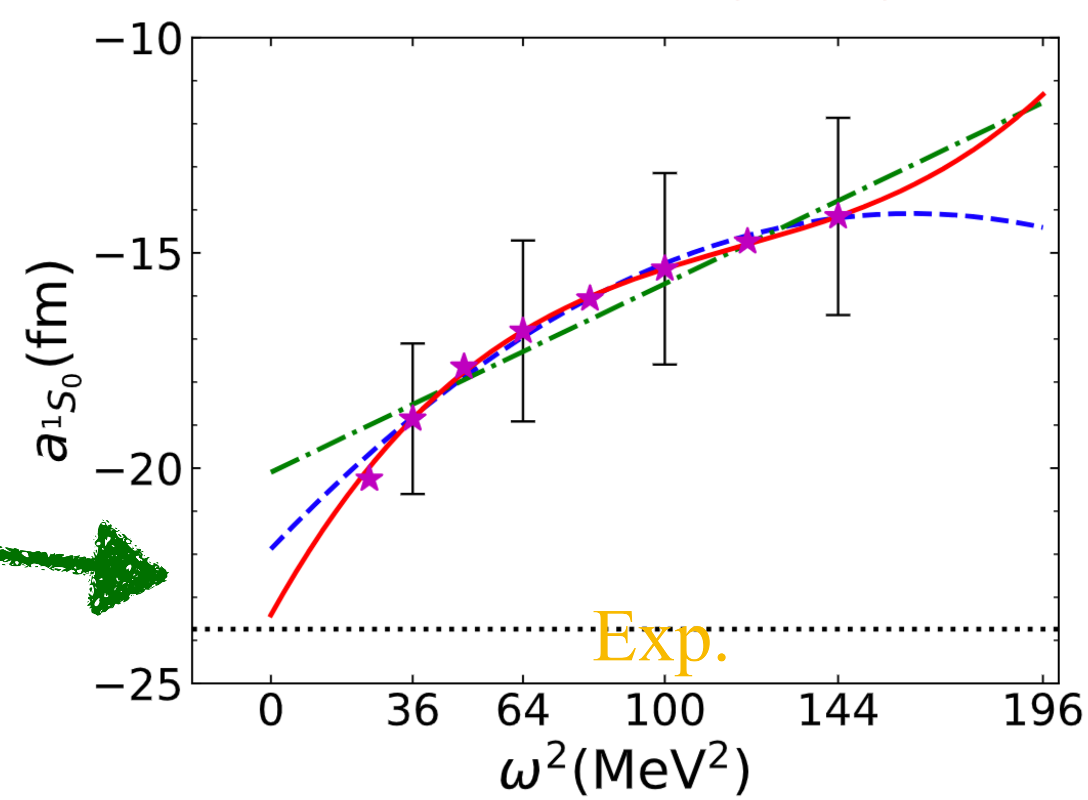
Li, Yu, Peng, Lyu, BwL, PRC 104, 044001

Continuum limit



- Unlike a box, HO trap does not vanish at short distance.
- EFT LECs contaminated by omega dependence
- Need to extrapolate scattering observables to $\hbar\omega = 0$

1S0 scattering length



- For small omega

$$T(E;\omega^2) = T_\infty(E) + c_1\omega^2 + c_2\omega^4 + \cdots ,$$

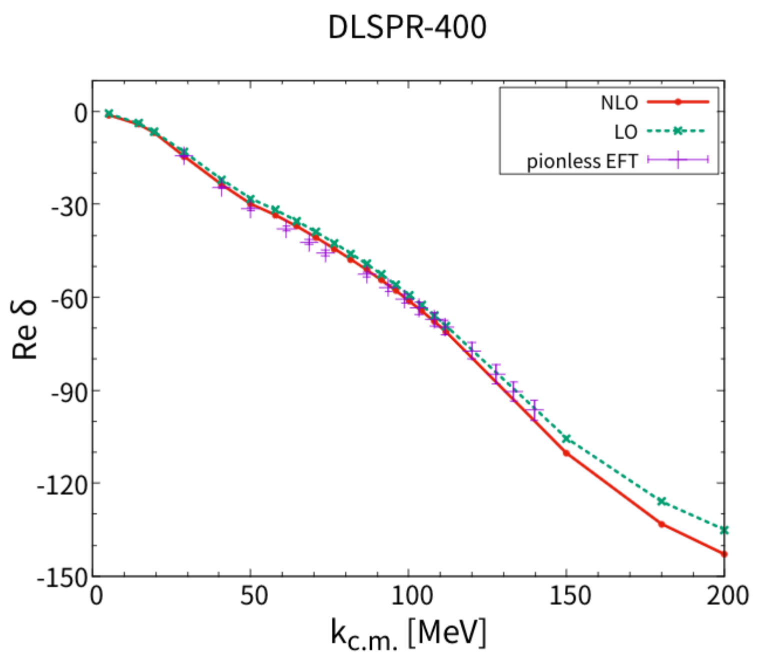
More Q's than A's

- Inelastic threshold (optical potentials)?
- Size effects of clusters?
- Electroweak currents induced reactions?
  Importance of unknown contact operators

Outlook

- Use neutron-deuteron scattering, breakup reaction as advanced test

- neutron-deuteron elastic scattering by Faddeev equation



VS.

- conversion formula + 3N levels in HO trap (to be worked out!)