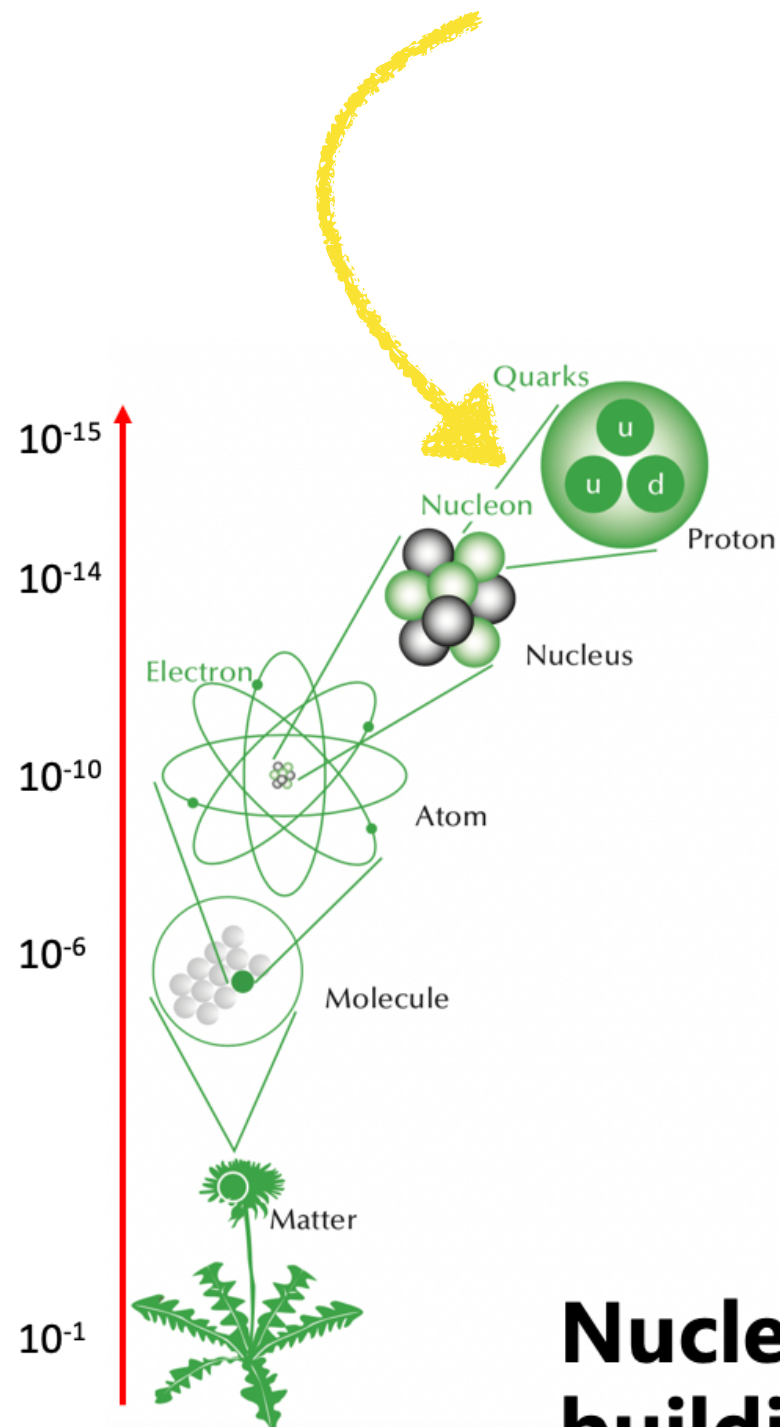


Effective field theories for nuclear physics

龙炳蔚



粒子物理与核物理结合，大有可为



IUPAC Periodic Table of the Elements

Key

atomic number

Symbol

name

standard atomic weight

													13	14	15	16	17	18
													5	6	7	8	9	10
													B	C	N	O	F	Ne
													boron	carbon	nitrogen	oxygen	fluorine	neon
													[10.806, 10.821]	[12.009, 12.012]	[14.006, 14.008]	[15.999, 16.000]	[18.998, 18.998]	[20.179, 20.180]
													13	14	15	16	17	18
													Al	Si	P	S	Cl	Ar
													aluminum	silicon	phosphorus	sulfur	chlorine	argon
													[26.981, 26.982]	[28.085, 28.086]	[30.973, 30.974]	[32.059, 32.061]	[35.45, 35.453]	[39.948, 39.949]
4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19			
22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37			
Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr	Rb			
titanium	vanadium	chromium	manganese	iron	cobalt	nickel	copper	zinc	gallium	germanium	arsenic	selenium	bromine	krypton	cesium			
[47.867, 47.867]	[50.942, 50.942]	[51.996, 51.996]	[54.938, 54.938]	[55.845, 55.845]	[58.933, 58.933]	[58.693, 58.693]	[63.546, 63.546]	[65.382, 65.382]	[69.723, 69.723]	[72.630, 72.630]	[74.922, 74.922]	[78.971, 78.971]	[79.904, 79.904]	[83.798, 83.798]	[85.468, 85.468]			
40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55			
Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe	Ba			
zirconium	niobium	molybdenum	technetium	ruthenium	rhodium	palladium	silver	cadmium	indium	tin	antimony	tellurium	iodine	xenon	barium			
[91.224, 91.224]	[92.906, 92.906]	[95.94, 95.94]	[98.906, 98.906]	[101.07, 101.07]	[102.91, 102.91]	[106.42, 106.42]	[107.868, 107.868]	[112.411, 112.411]	[114.818, 114.818]	[118.710, 118.710]	[121.757, 121.757]	[127.603, 127.603]	[126.905, 126.905]	[131.294, 131.294]	[137.327, 137.327]			
72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87			
Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn	Fr			
hafnium	tantalum	tungsten	rhenium	osmium	iridium	platinum	gold	mercury	thallium	lead	bismuth	polonium	astatine	radon	francium			
[178.49, 178.49]	[180.948, 180.948]	[183.84, 183.84]	[186.21, 186.21]	[190.23, 190.23]	[192.22, 192.22]	[195.084, 195.084]	[196.967, 196.967]	[200.598, 200.598]	[204.38, 204.38]	[207.2, 207.2]	[208.98, 208.98]	[209, 209]	[210, 210]	[222, 222]	[223, 223]			
104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119			
Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Nh	Fl	Mc	Lv	Ts	Og	Uue			
rutherfordium	dubnium	seaborgium	bohrium	hassium	meitnerium	darmstadtium	roentgenium	copernicium	nihonium	flerovium	moscovium	tennessine	oganesone	unbinilium	untridecium			
[261, 261]	[262, 262]	[263, 263]	[264, 264]	[265, 265]	[266, 266]	[267, 267]	[268, 268]	[269, 269]	[270, 270]	[271, 271]	[272, 272]	[273, 273]	[274, 274]	[275, 275]	[286, 286]			

57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu	Hf
lanthanum	cerium	praseodymium	neodymium	promethium	samarium	europium	gadolinium	terbium	dysprosium	holmium	erbium	thulium	ytterbium	lutetium	hafnium
[138.91, 138.91]	[140.12, 140.12]	[140.91, 140.91]	[144.24, 144.24]	[145, 145]	[150.36, 150.36]	[151.96, 151.96]	[157.25, 157.25]	[158.93, 158.93]	[162.50, 162.50]	[164.93, 164.93]	[167.26, 167.26]	[168.93, 168.93]	[173.05, 173.05]	[174.97, 174.97]	[178.49, 178.49]
89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104
Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr	Hf
actinium	thorium	protactinium	uranium	neptunium	plutonium	americium	curium	berkelium	californium	einsteinium	fermium	mendelevium	nobelium	lawrencium	hafnium
[227, 227]	[232.04, 232.04]	[231.04, 231.04]	[238.03, 238.03]	[237, 237]	[244, 244]	[243, 243]	[247, 247]	[247, 247]	[251, 251]	[252, 252]	[257, 257]	[258, 258]	[259, 259]	[260, 260]	[261, 261]

For notes and updates to this table, see www.iupac.org. This version is dated 1 December 2018.
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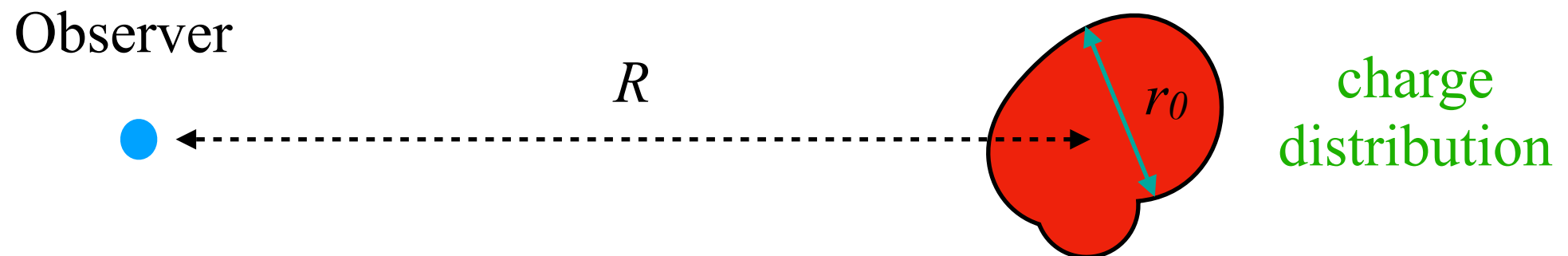
Nucleons are the essential building blocks of Matter!

Outline

- Brief intro to nuclear effective field theories
- Expansion around unnatural leading-order interactions
 - Improving convergence of chiral nuclear forces
- Small-momentum fluctuations around fixed-energy configuration
 - bridging nuclear bound-state and reaction problems

Multipole expansion

A classical example of EFT



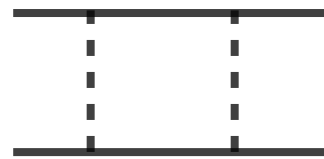
- Separation of scales: $R \gg r_0$
- Controlled approximation, able to estimate uncertainty

$$V = \frac{q}{R} + \frac{d_i R_i}{R^3} + \frac{Q_{ij} R_i R_j}{R^5} + \dots$$

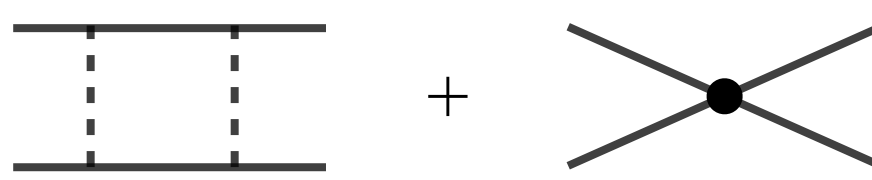
- Naturalness $|d_i| \sim q r_0$ $|Q_{ij}| \sim q r_0^2 \Rightarrow$ power counting based on naive dim. analysis (NDA)
- What if it is a rod?
Slow convergence of a regular PC $\Rightarrow$ possible fine-tuning $\Rightarrow$ change PC

Renormalization in effective (field) theory

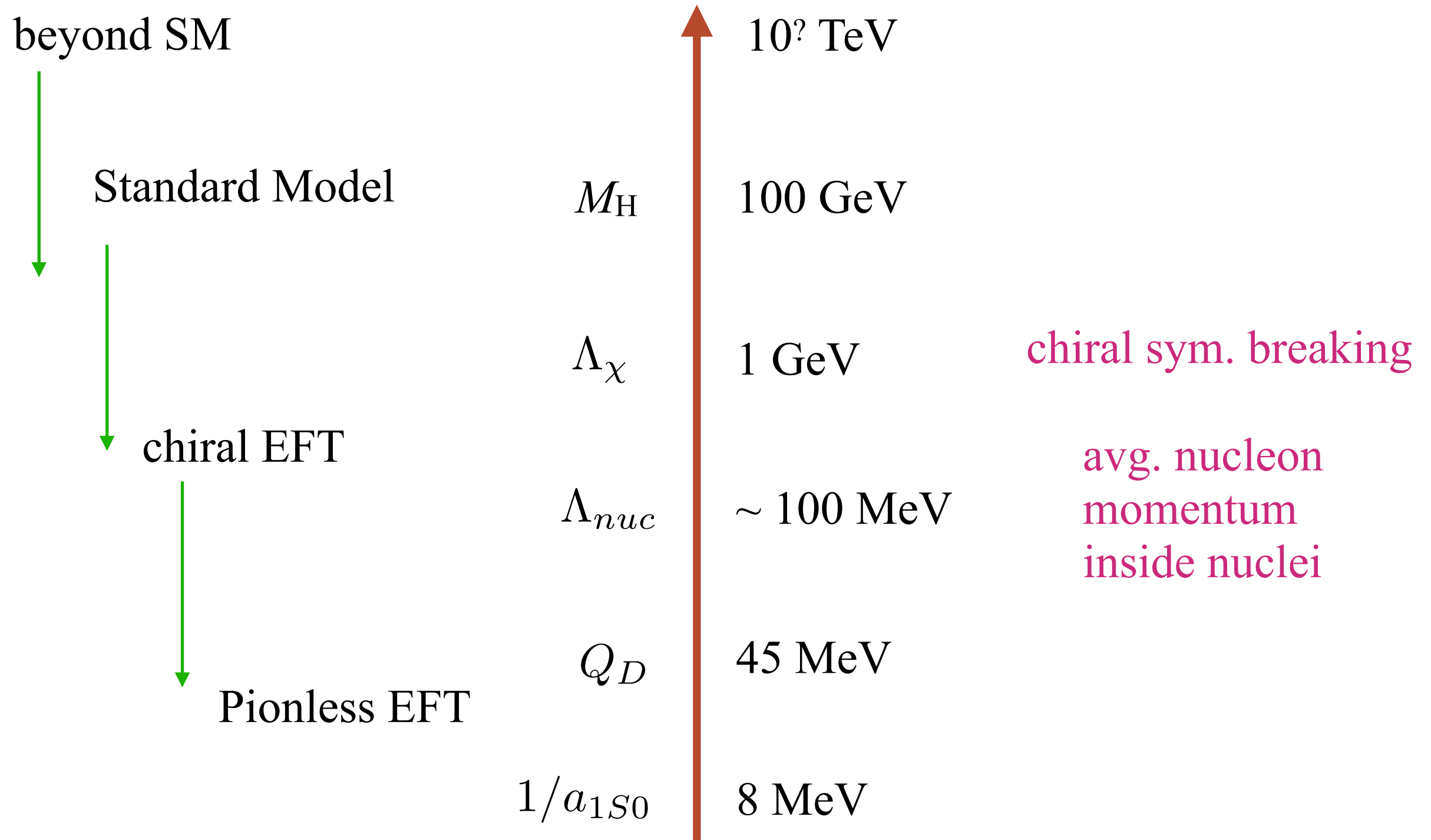
- UV regularization needed for intermediate states
 - rendering loops finite



- Observables independent of reg. scale Λ
 - guideline for power counting: RG invariance must be satisfied by power counting
 - can upset NDA (OPE tensor force $\propto -1/r^3$)

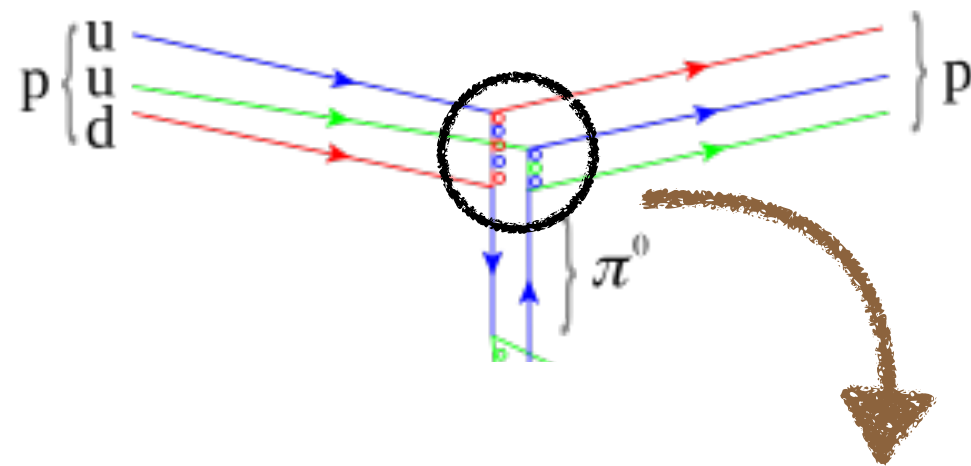
$$\frac{d\mathcal{M}}{d\Lambda} = 0 \quad \Rightarrow \quad \text{bubble loop} + \text{crossing}$$
The equation shows the derivative of the observable $\mathcal{M}$ with respect to the renormalization scale Λ is zero. This is equivalent to the sum of two Feynman diagrams: a bubble loop (two horizontal solid lines connected by two vertical dashed lines) and a crossing diagram (two solid lines crossing at a central black dot).

Hierarchy of EFTs



Chiral EFT

Dofs = nucleons + pions (Quark-gluon interactions encoded in low-energy constants (LECs))



$$-\frac{g_A}{2f_\pi} N^\dagger \tau_a \vec{\sigma} \cdot \vec{\nabla} \pi_a N$$

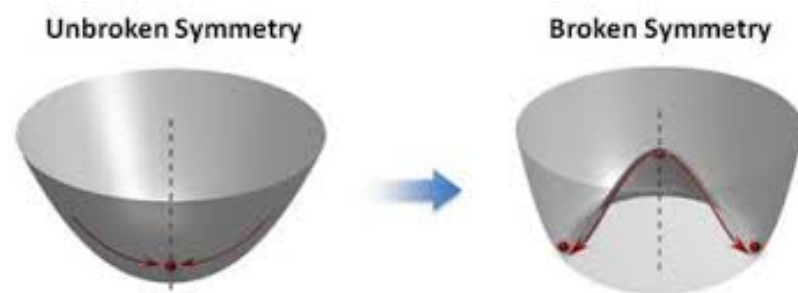
Chiral EFT

- Includes all symmetries of QCD, especially (approximate) chiral symmetry and its spontaneous breaking

$$\mathcal{L}_{\text{QCD}} = \sum_{f=u,d,s} \bar{q}_f (i \not{D} - m_f) q_f - \frac{1}{4} \mathcal{G}_{a\mu\nu} \mathcal{G}_a^{\mu\nu}$$

$$q_L \equiv \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \mapsto \left(\text{SU}(3)_L \right) \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \quad q_R \equiv \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix} \mapsto \left(\text{SU}(3)_R \right) \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix}$$

- Only two flavors used in our work



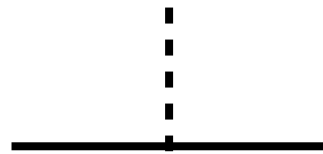
→ chiral symmetry nonlinearly realized by hadronic Dofs CCWZ; Weinberg; ...

(see Weinberg's QFT Vol. 2)

Chiral EFT

- Nucleon-pion couplings are always $Q^n \Rightarrow$ long-range nuclear forces naturally ordered

$O(Q)$



$$-\frac{g_A}{2f_\pi} N^\dagger \tau_a \vec{\sigma} \cdot \vec{\nabla} \pi_a N$$

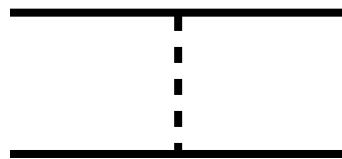


$$-\frac{1}{4f_\pi^2} N^\dagger \epsilon_{abc} \tau_a \pi_b \dot{\pi}_c N$$

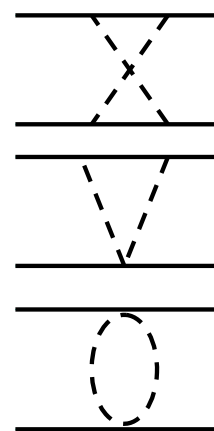
$$N^\dagger (i\partial_t + \dots) N$$

Derivative coupling accompanied by **chiral connection terms**

- NN long-range forces



one-pion exchange = $O(1)$

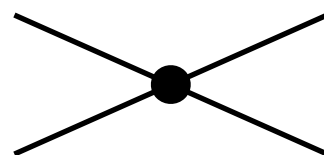


two-pion exchanges = $O(Q^2)$

...

Weinberg '90

- Short-range interactions: large numbers of contact operators
- Organized by partial-wave quantum numbers
- In forms of momentum polynomials



$$\begin{aligned}
 & -C^{(s)} (N^T P_i^{(s)} N)^\dagger (N^T P_i^{(s)} N) - C_2^{(s)} \left[(N^T P_i^{(s)} N)^\dagger (N^T P_i^{(s')} \overleftrightarrow{\nabla}^2 N) + h.c. \right] \dots \\
 & -C^{(ss')} (N^T P_i^{(s)} N)^\dagger (N^T P_i^{(s')} N) \dots
 \end{aligned}$$

$$\mathbf{s}, \mathbf{s}' = {}^1\mathbf{S}_0, {}^3\mathbf{S}_1, {}^3\mathbf{P}_0, \dots$$

$$P_i^{({}^1S_0)} = \frac{(i\sigma_2)(i\tau_2\tau_i)}{2\sqrt{2}}$$

$$P_i^{({}^3S_1)} = \frac{(i\sigma_2\sigma_i)(i\tau_2)}{2\sqrt{2}}$$

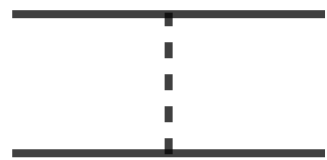
$$P_i^{({}^3D_1)} = \left(\overleftrightarrow{\nabla}_i \overleftrightarrow{\nabla}_j - \frac{\delta_{ij}}{n} \overleftrightarrow{\nabla}^2 \right) P_j^{({}^3S_1)}$$

Relay: from quarks & gluons to Uranium

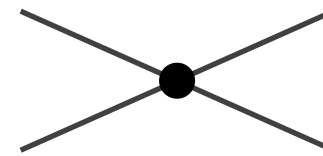


A Tale of Two Channels

- OPE (long-range) is much weaker in $1S0$



$$V_{1\pi}(\vec{q}) = -\frac{1}{(2\pi)^3} \left(\frac{g_A}{2f_\pi} \right)^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{\vec{q}^2 + m_\pi^2}$$

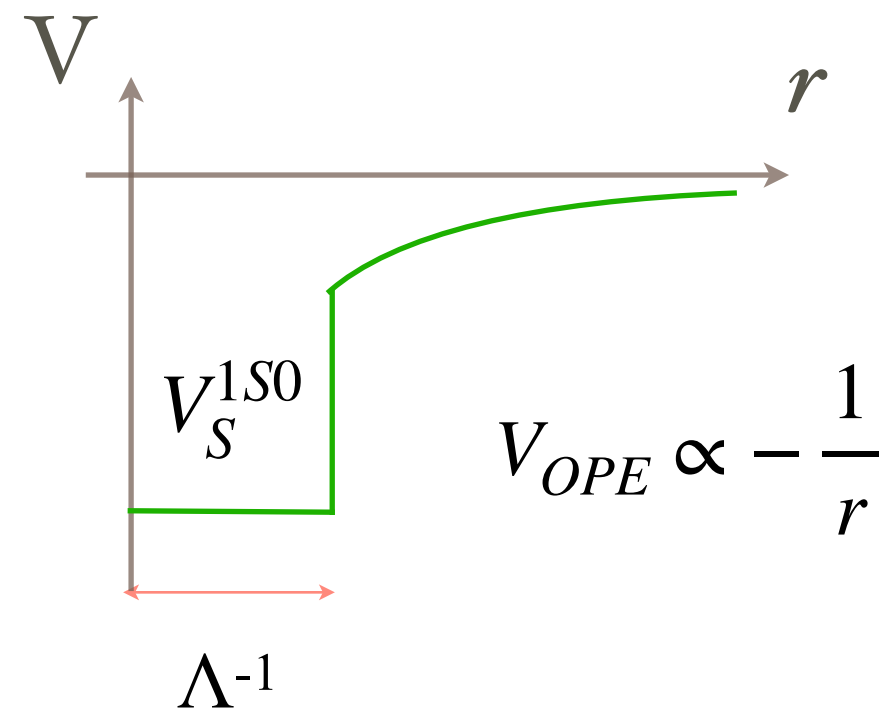
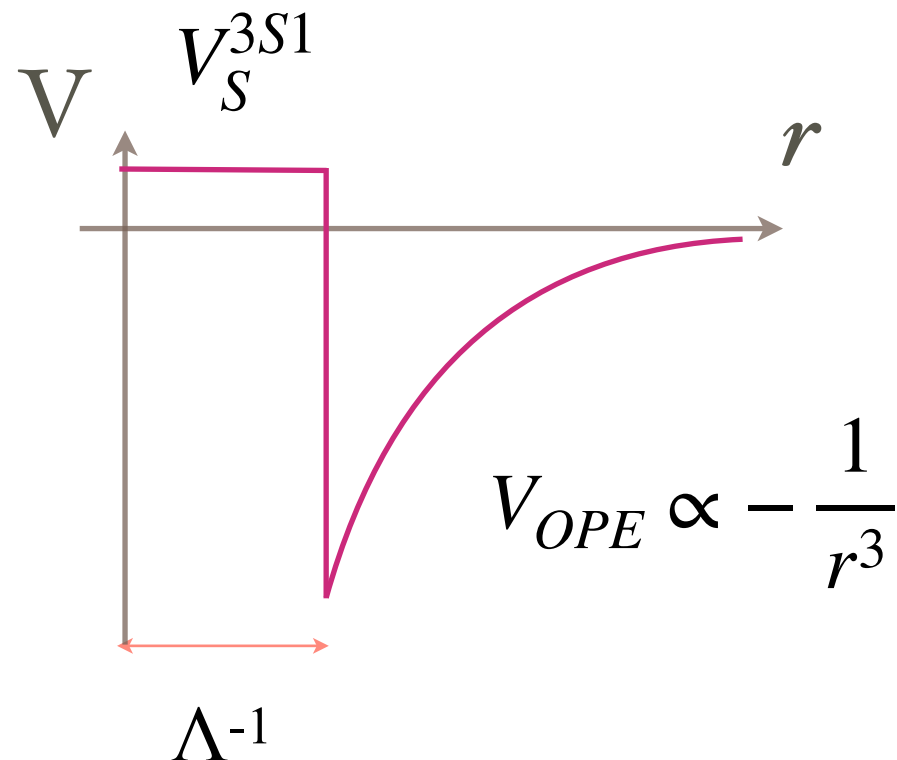


$$V_S^{3S1} = C_{3S1}$$

$$V_S^{1S0} = C_{1S0}$$

${}^3S_1 - {}^3D_1$

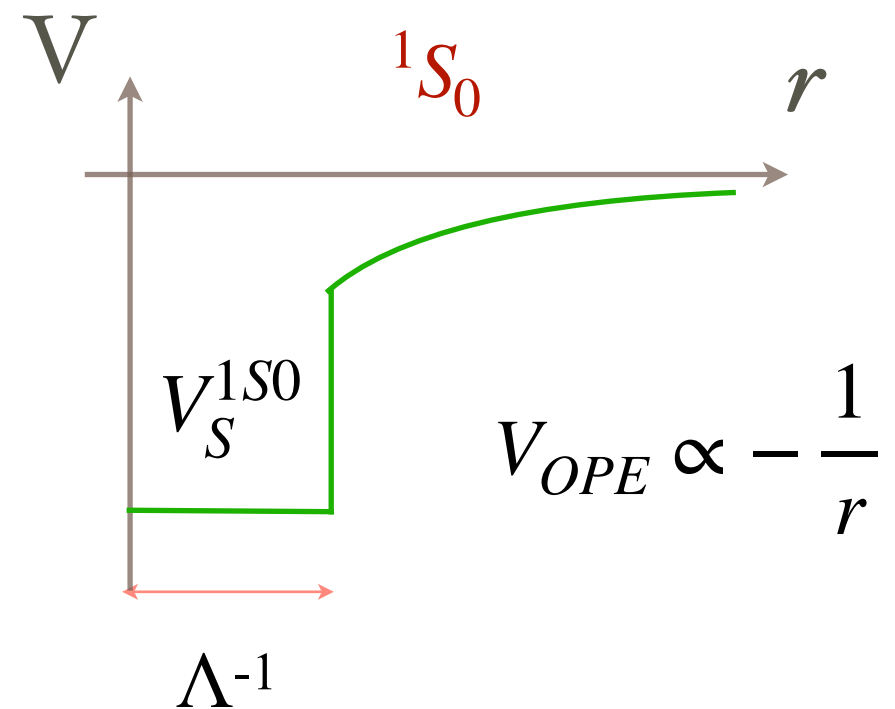
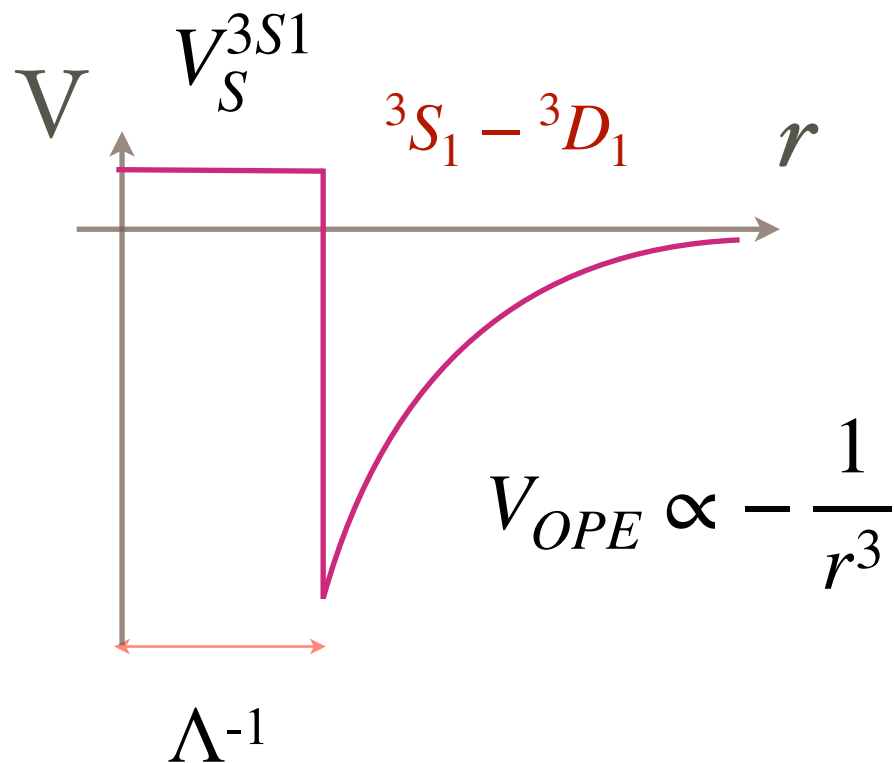
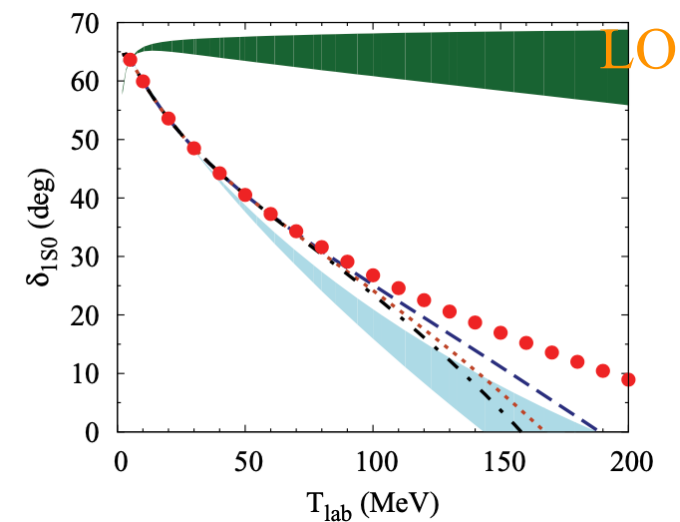
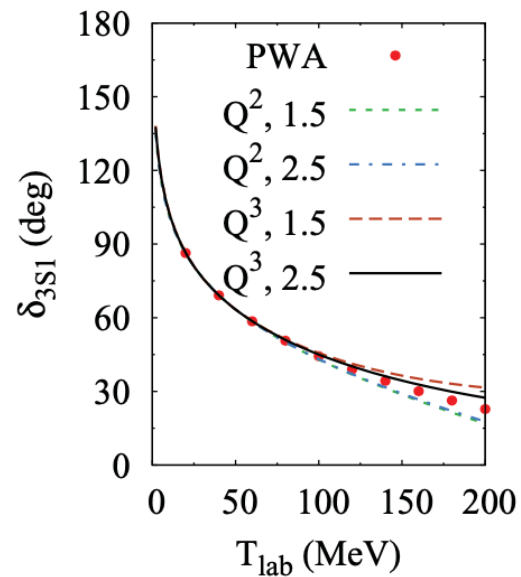
1S_0



A Tale of Two Channels

BwL & Yang PRC86, 024001
BwL & Yang PRC85, 034002

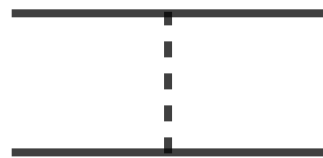
- One parameter in $1S0$ cannot construct both short-range repulsion and long-range attraction



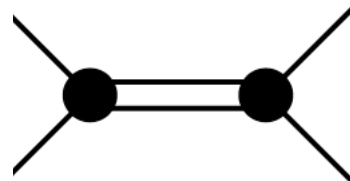
Unnatural LO forces

Peng, Lyu, Koenig & Long,
arxiv 2112.00947

- Two parameters to describe non-pion forces:
⇒ Two-dimensional surface in parameter space of LECs



OPE



$$V_{\text{spr}}(p', p) = -\frac{4\pi}{m_N} \frac{\lambda}{\sqrt{p'^2 + m_N \Delta} \sqrt{p^2 + m_N \Delta}}$$

$$= \lambda \left(\# + \# \frac{p^2}{m_N \Delta} + \dots \right) \left(\# + \# \frac{p'^2}{m_N \Delta} + \dots \right)$$

$$\sqrt{m_N \Delta} \simeq 150 \text{ MeV}$$

$$\lambda \simeq 200 \text{ MeV}$$

- How is this *not* just another model?
 - * Only low-momenta scales at LO
 - * Compatible with chiral Lagrangian (non-trivial)
 - * Order-by-order convergence (NLO, NNLO...)
 - * UV cutoff independence (satisfying RG invariance)

Other efforts to improve 1S0:

“Energy-dependent,” Long, PRC 88, 014002 ('13)

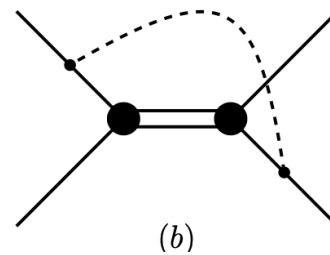
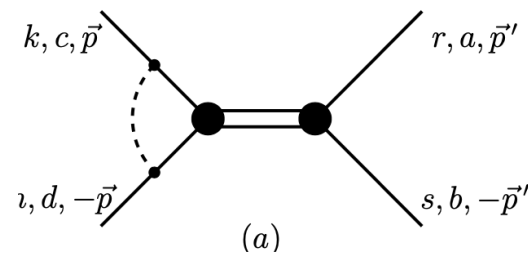
“Covariant ChPT,” Xiu-Lei Ren et al. CPL 38 (2021) 6, 062101

“TPE as LO,” Mishra et al. arxiv 2111.15515

...

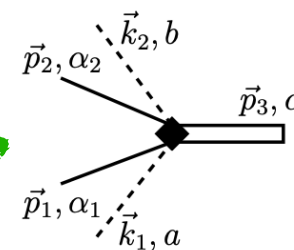
Compatible w/ chiral Lag.

- Non-trivial radiative corrections at NNLO
 - long-range forces
 - could have impact on triplet channels (3S1-3D1...)



- Non-trivial chiral connections
 - able to derive additional forces
 - existing convergence preserved

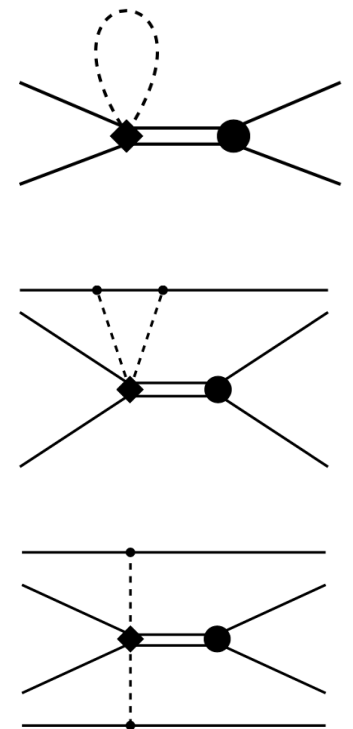
$$\frac{\lambda}{\sqrt{p'^2 + m_N \Delta} \sqrt{p^2 + m_N \Delta}}$$



$$\mathcal{A}_{\phi NN\pi\pi} = \frac{ig}{4f_\pi^2} \sqrt{\frac{4\pi}{m_N}} \left\{ i(\delta_{bc}\mathcal{P}_a - \delta_{ac}\mathcal{P}_b)_{\alpha_2\alpha_1} \mathcal{B}_+ + \frac{1}{\sqrt{8}} \epsilon_{abc} (\sigma_2 \tau_2)_{\alpha_2\alpha_1} \mathcal{B}_- \right\}$$

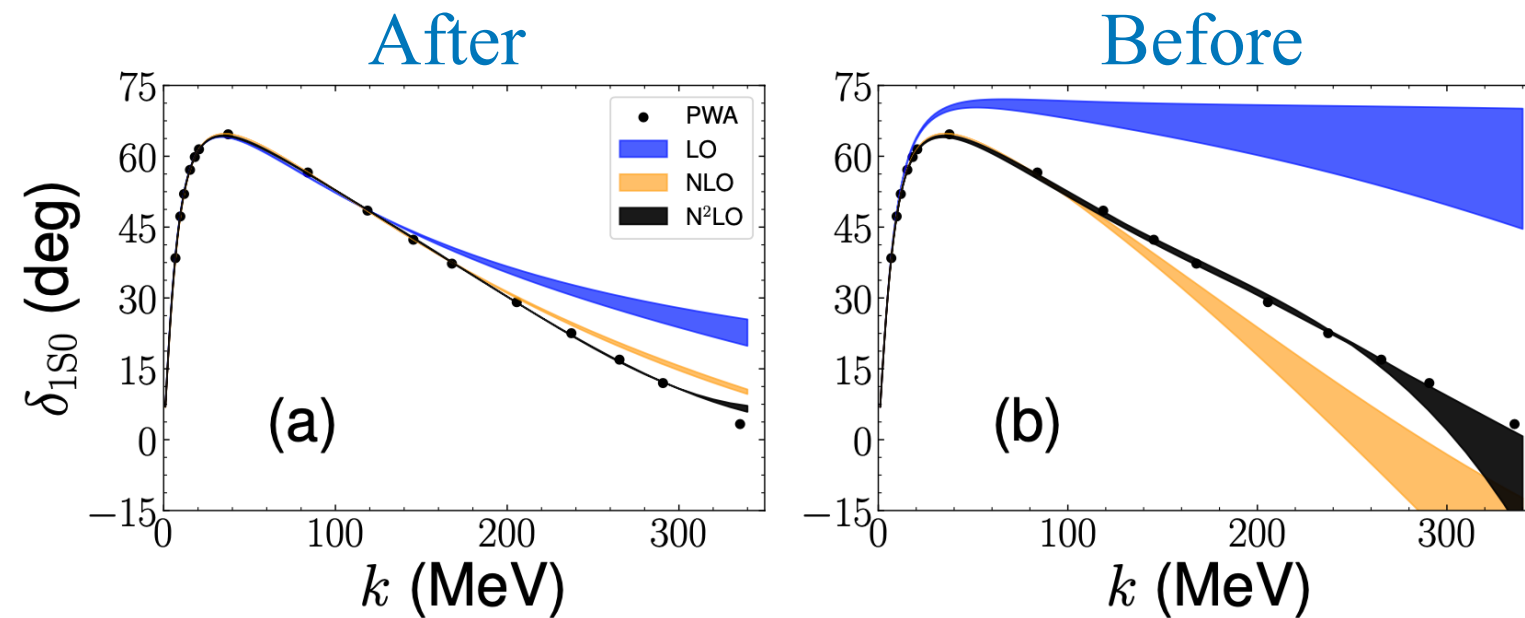
$$\mathcal{B}_\pm \equiv u \left(\left| \vec{p} + \vec{k}_1/2 \right|, \left| \vec{p} + \vec{k}_2/2 \right| \right) \pm u \left(\left| \vec{p} - \vec{k}_1/2 \right|, \left| \vec{p} - \vec{k}_2/2 \right| \right),$$

$$u(x, y) \equiv \left(1 + \frac{x^2}{m_N \Delta} \right)^{-\frac{1}{2}} - \left(1 + \frac{y^2}{m_N \Delta} \right)^{-\frac{1}{2}}$$

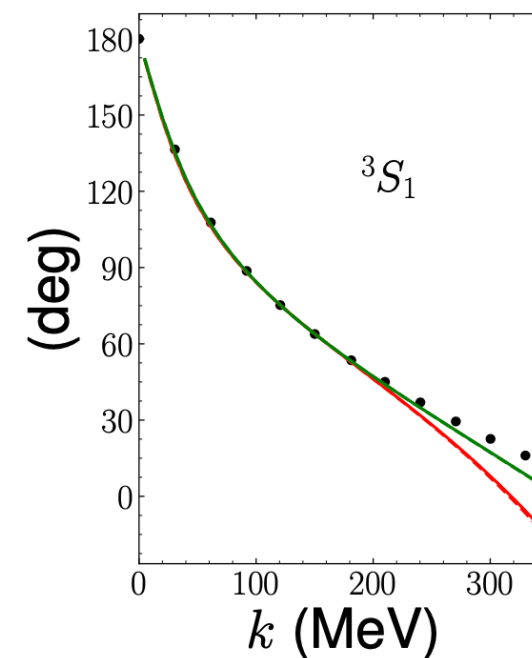


Up to NNLO

- Better convergence in 1S0



- 3S1 not spoiled by radiative corrections



$\Lambda = 2400$ MeV

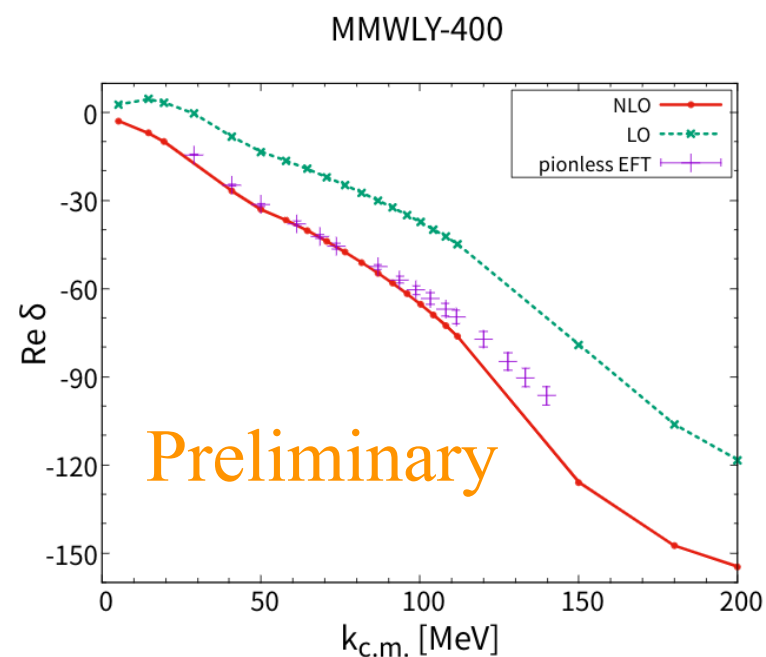
$\Lambda = 600$ MeV

3N systems

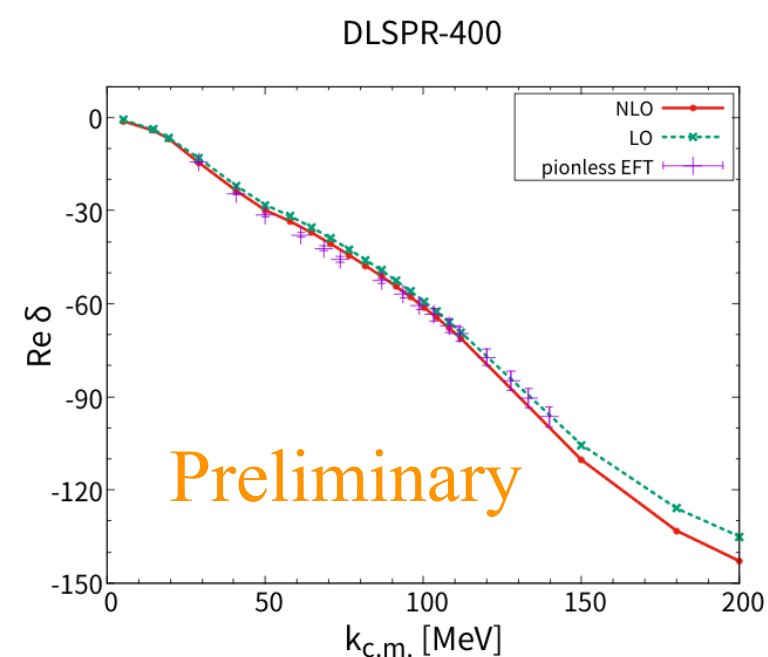
- Does it work in 3N systems?
- Solving the Faddeev equation (equivalent to the Schrodinger equation)
 - 3H binding energy, neutron-deuteron scattering phase shifts ...

E.g., neutron-deuteron scattering ($J^P = 1/2^+$, S wave)

Before

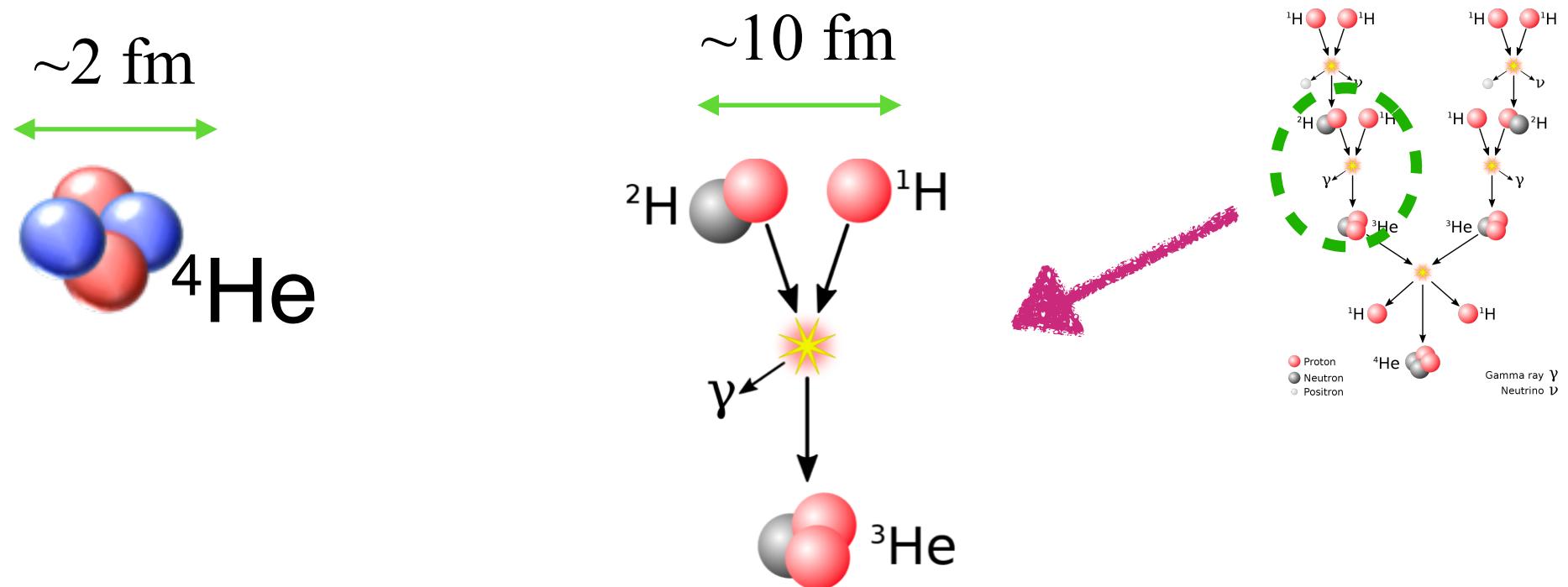


After



Ab initio calculations of nuclear reactions

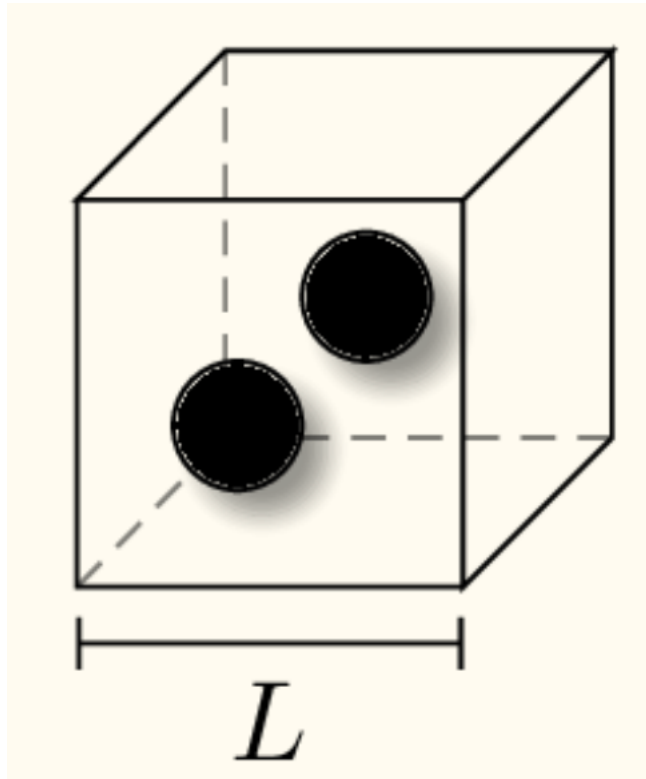
- Ab initio $\approx$ diagonalizing nuclear Hamiltonian of A -nucleon systems
- Scattering and reactions normally involve larger configuration space



Chenbo Li (Sichuan U.), Jiexin Yu (Sichuan U.), Rui Peng (Sichuan U.), Songlin Lyu (Sichuan U.),
Bingwei Long (Sichuan U.) (Jul 26, 2021)

Published in: *Phys.Rev.C* 104 (2021) 4, 044001 • e-Print: [2107.12273](https://arxiv.org/abs/2107.12273) [nucl-th]

Lesson from LQCD

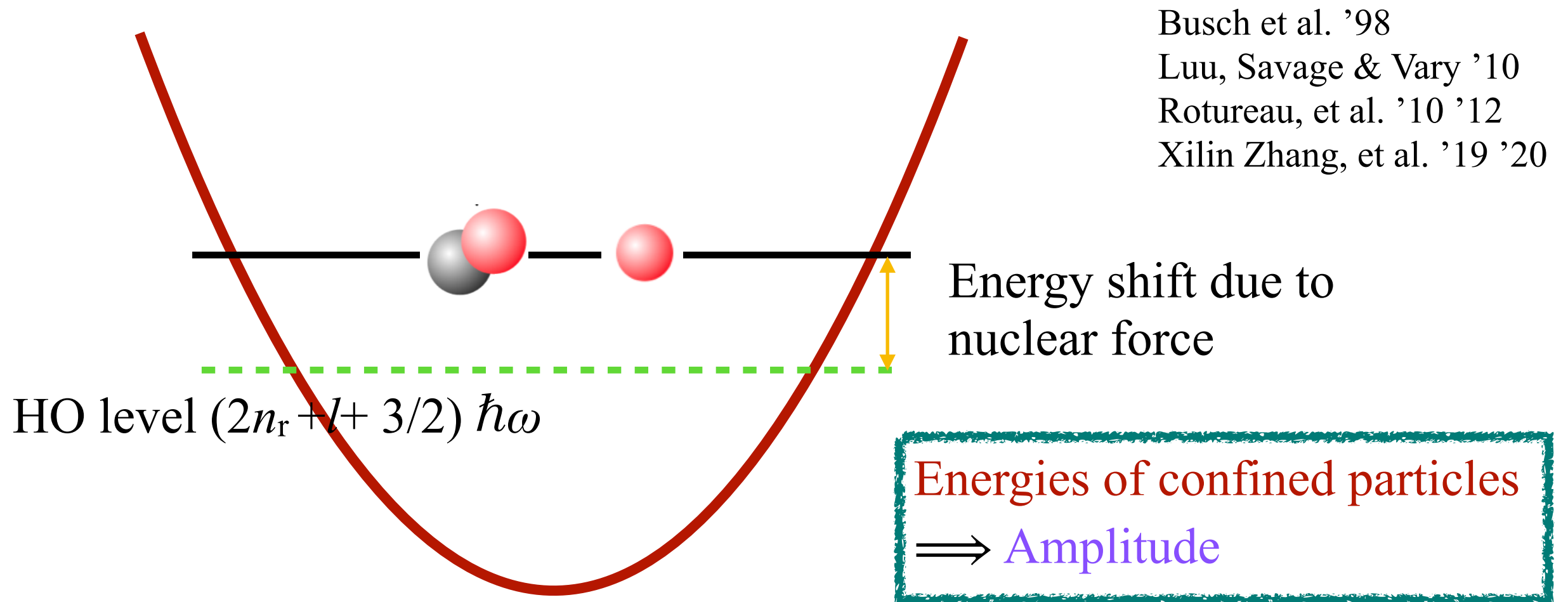


- Energy levels of particles confined in finite volume (provided by LQCD) tell us information about their interaction
- Use energy eigenvalues as inputs to model-independently determine phase shifts at the same energies

Lüscher's formula:

$$\det[\mathcal{M}^{-1}(E_L) + F^{(P)}(E_L, L)] = 0$$

Harmonic-oscillator trap



- HO potential is isotropic $\Rightarrow$ angular momentum remains good quantum number
- HO w.f. analytically known
- Available software packages

Trick: matching w.f.

Outside wf : HO trapped

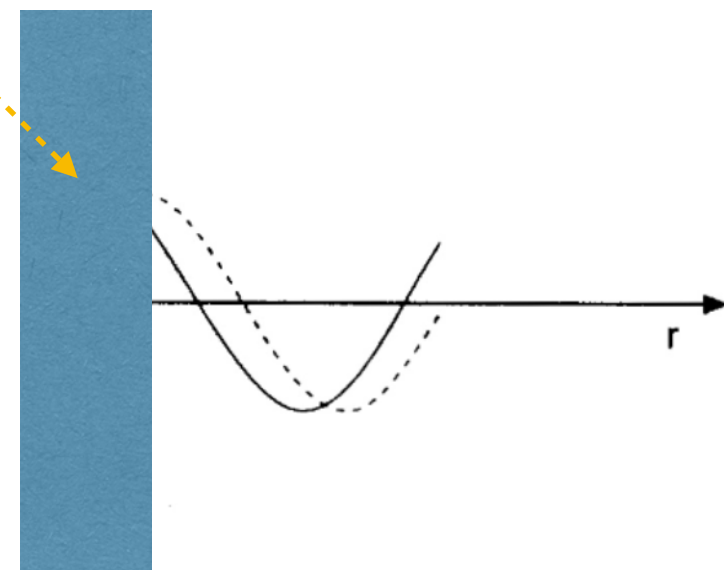
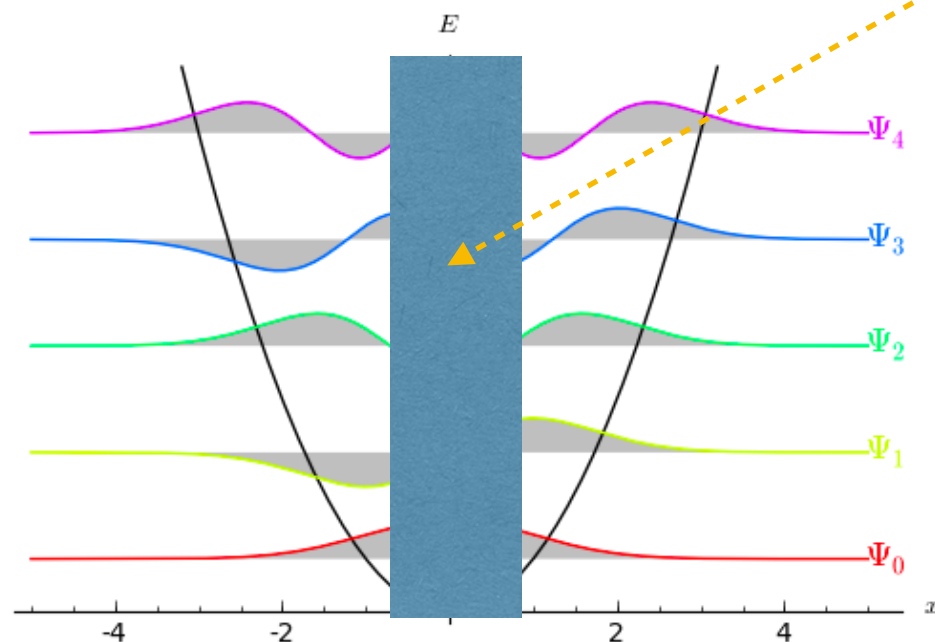
$$\propto aR(r; E) + bY(r; E)$$

R & Y: solutions to
HO Schrodinger eq.

Outside wf : scattering

$$\propto \sin(kr + \delta)$$

Intrinsic
potential

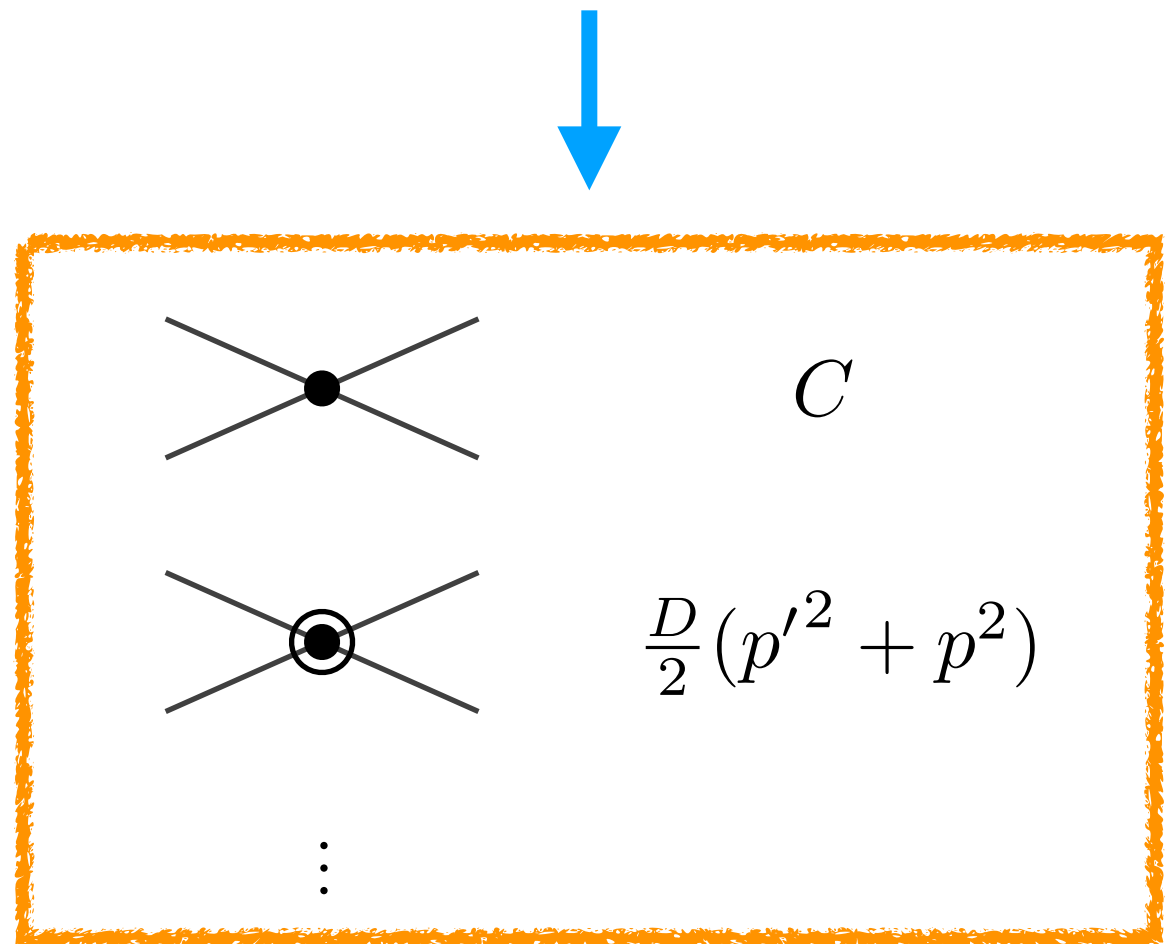
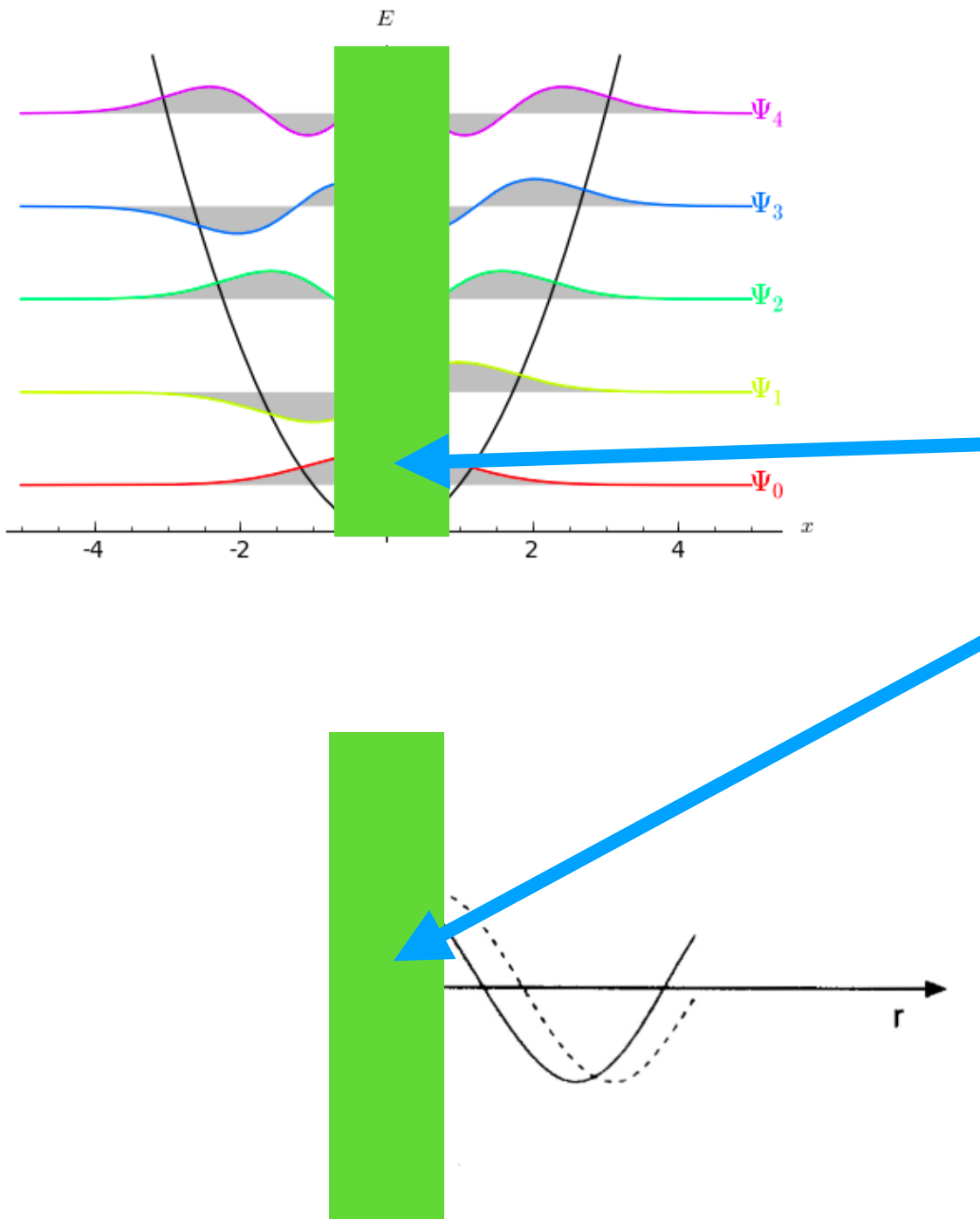


- Both wfs must match at the edge of the intrinsic potential V_i
- To construct outside wfs, detail of V_i does not matter $\Rightarrow$ use **EFT** !

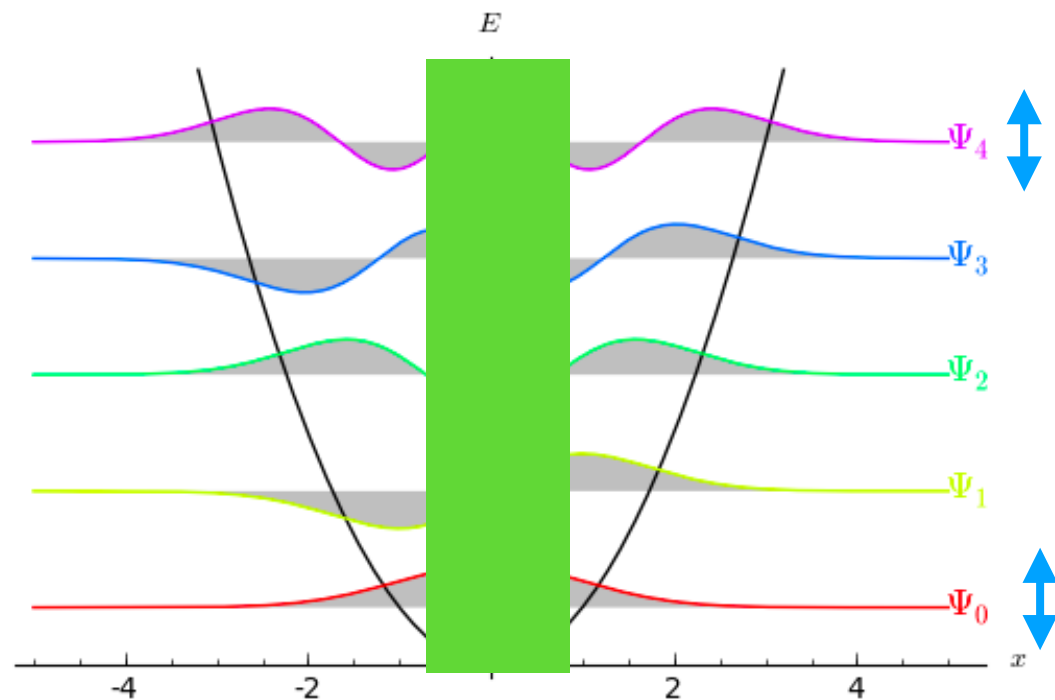
Contact EFT

- Use NN as example

$$\mathcal{L}_{NN} = -\frac{1}{2}C_s(N^\dagger N)^2 - \frac{1}{2}C_t(N^\dagger \vec{\sigma} N)^2 + \dots$$



“Manifold” of EFTs



$$\mathcal{L}_{NN} = -\frac{1}{2}C_s(\mathbf{E}_4) (N^\dagger N)^2 - \frac{1}{2}C_t(\mathbf{E}_4) (N^\dagger \vec{\sigma} N)^2 + \dots$$

⋮

$$\mathcal{L}_{NN} = -\frac{1}{2}C_s(\mathbf{E}_0) (N^\dagger N)^2 - \frac{1}{2}C_t(\mathbf{E}_0) (N^\dagger \vec{\sigma} N)^2 + \dots$$

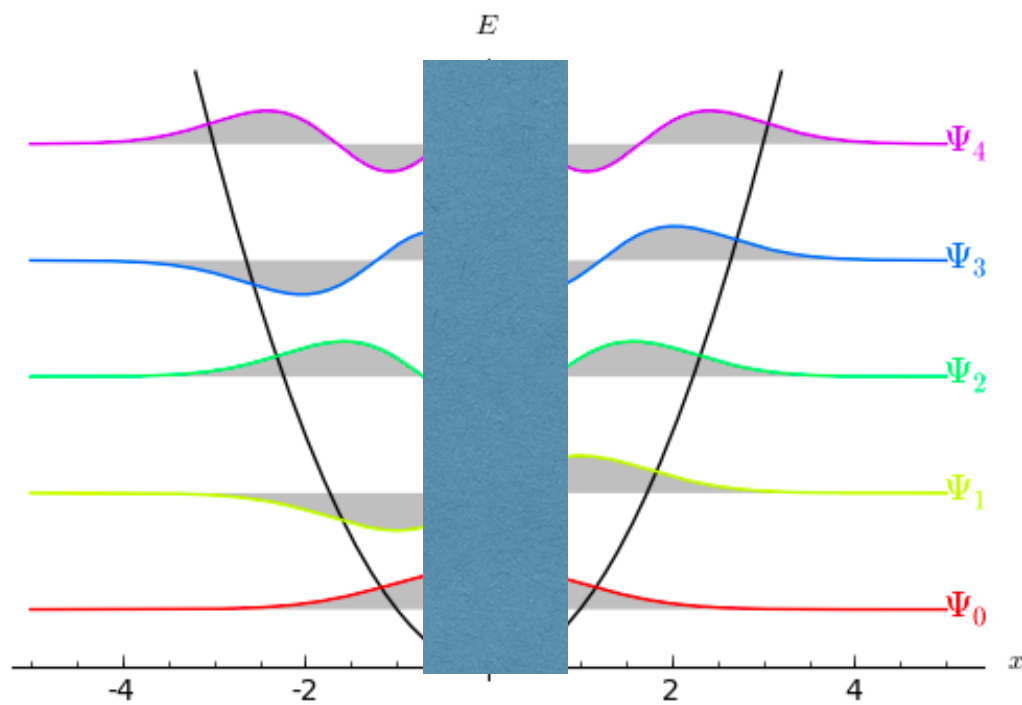
- One EFT in charge of a small domain around each eigen energy
effective-range expansion around each energy

$$k \cot \delta = \alpha_0(\mathcal{E}_r) + \alpha_1(\mathcal{E}_r)(E - \mathcal{E}_r) + \alpha_2(\mathcal{E}_r)(E - \mathcal{E}_r)^2 + \dots$$

- Weak predictive power, but it's OK

Recipe

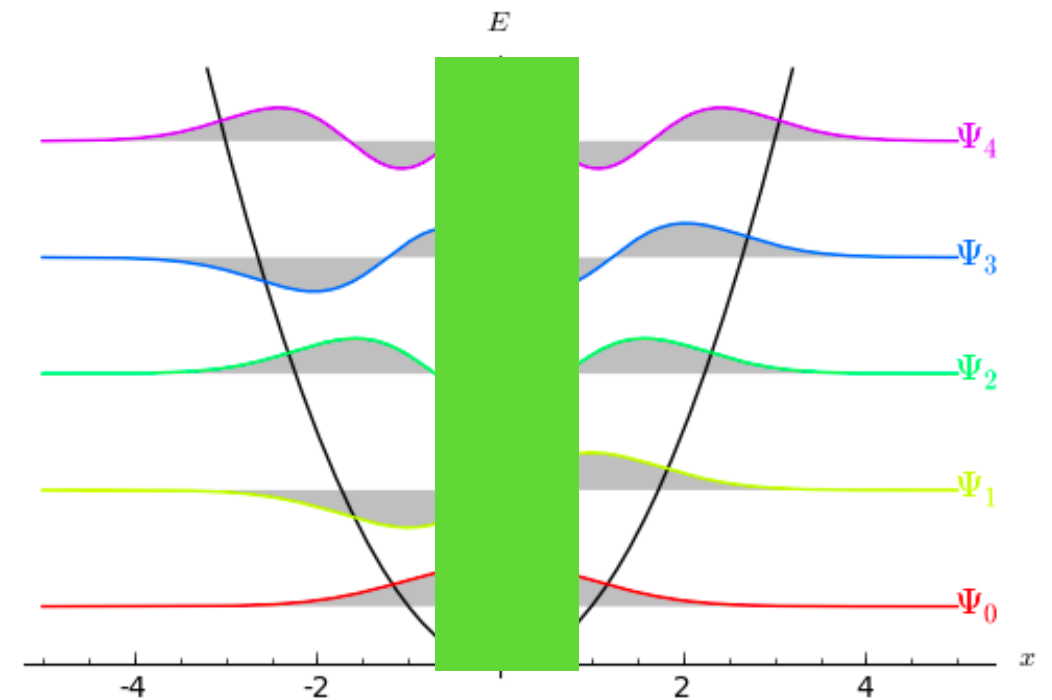
$V_{realistic}$ (ab initio many-body)



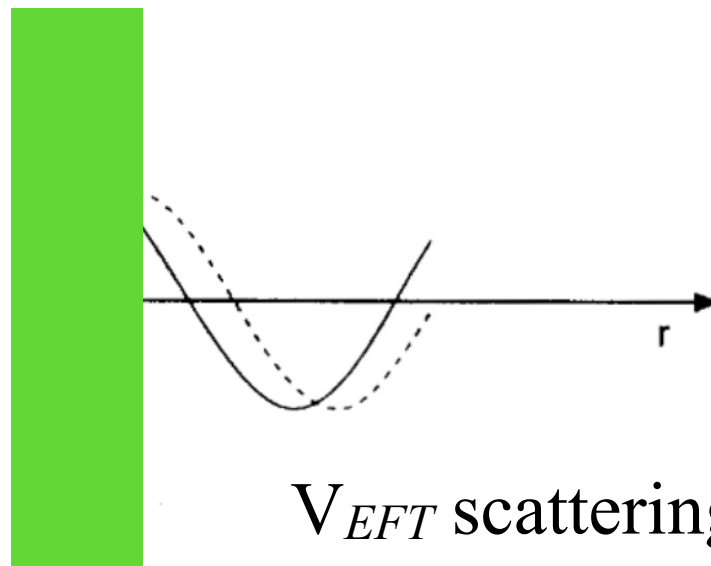
inputs



V_{EFT} (2B cal.)



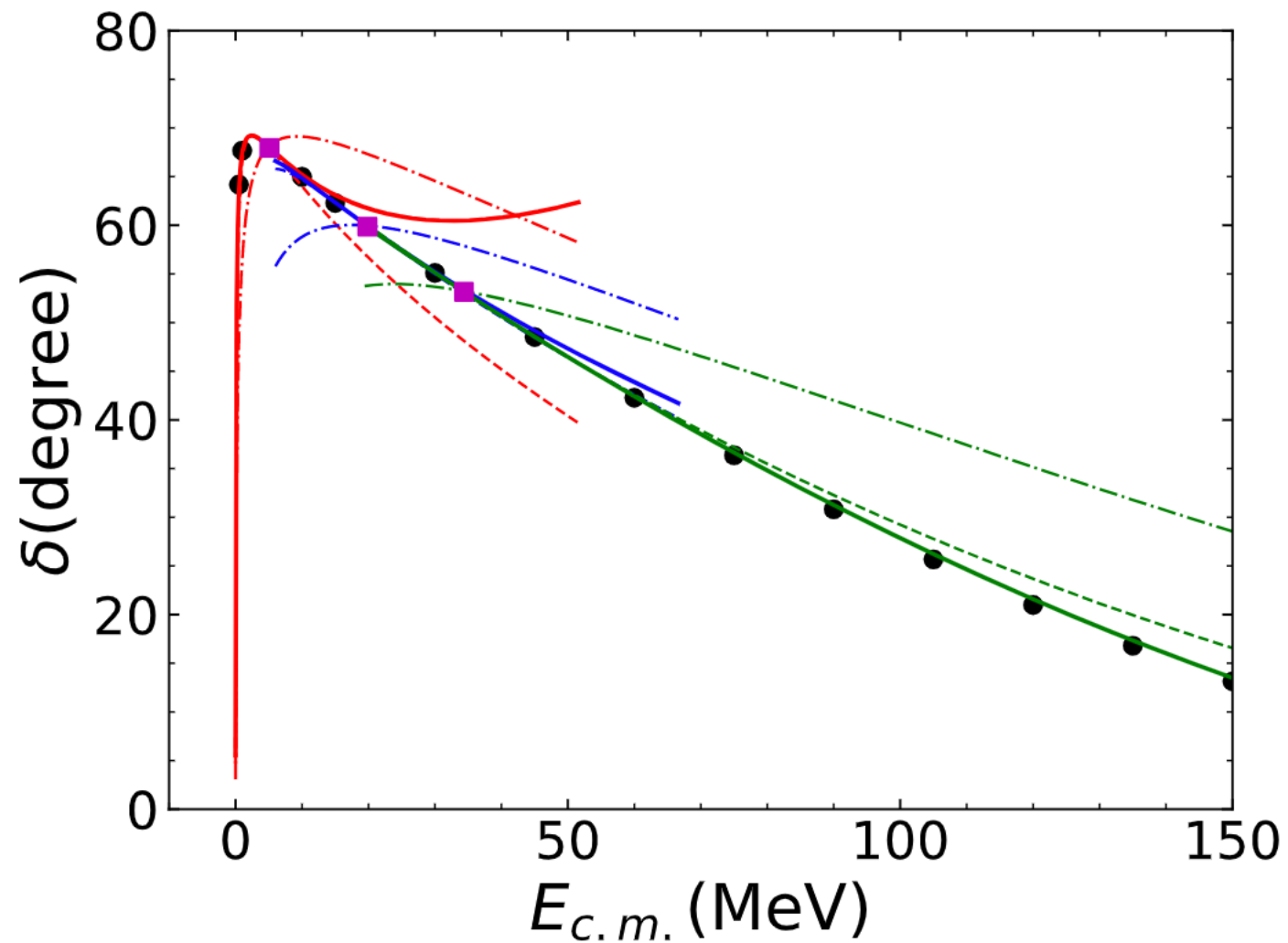
- Fix $C_0, C_2, \dots$ of EFT



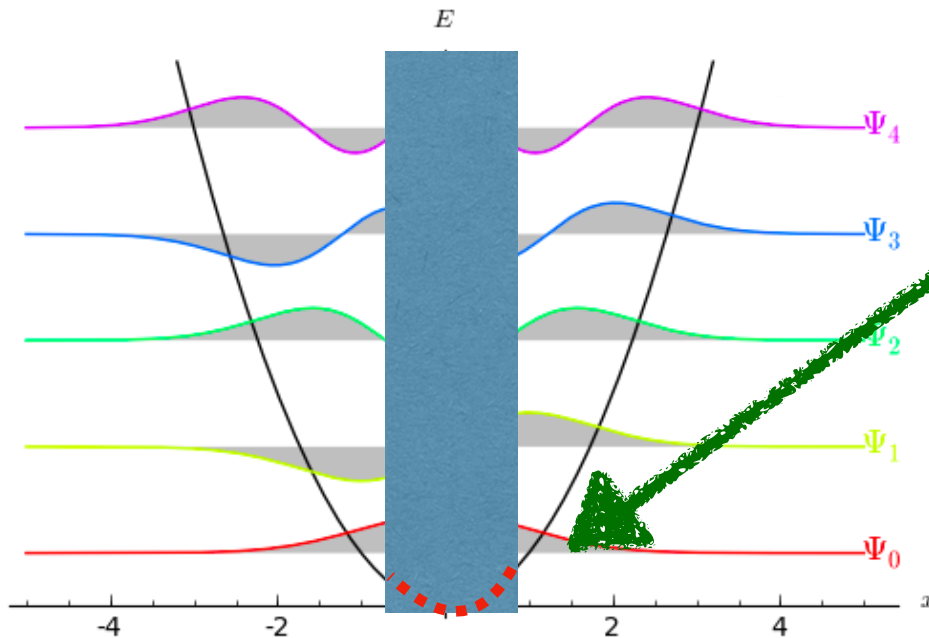
V_{EFT} scattering (2B cal.)

Exercise

- With three levels ($\hbar\omega = 7$ MeV), NN 1S0 phase shifts are covered up to $E_{cm} = 150$ MeV



Continuum limit

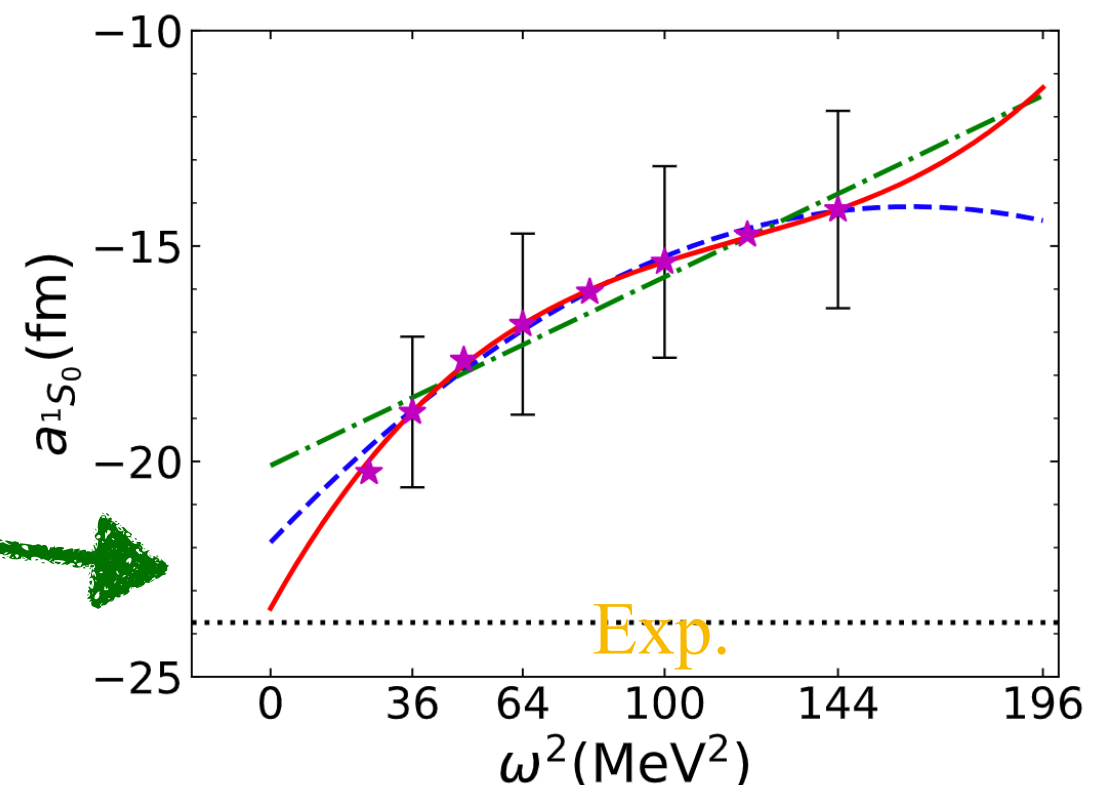


- Unlike a box, HO trap does not vanish at short distance.
- EFT LECs contaminated by omega dependence
- Need to extrapolate scattering observables to $\hbar\omega = 0$

- For small omega

$$T(E; \omega^2) = \boxed{T_\infty(E)} + c_1 \omega^2 + c_2 \omega^4 + \dots,$$

1S0 scattering length



More Q's than A's

- Inelastic threshold (optical potentials)?
- Size effects of clusters?
- Electroweak currents induced reactions?
Importance of unknown contact operators

Outlook

- Use neutron-deuteron scattering, breakup reaction as advanced test

- neutron-deuteron elastic scattering by Faddeev equation

VS.

- conversion formula + 3N levels in HO trap (to be worked out!)

