Muon Collider at the Energy Frontier

缪子束加速和对撞技术及其应用
科学与技术前沿论坛
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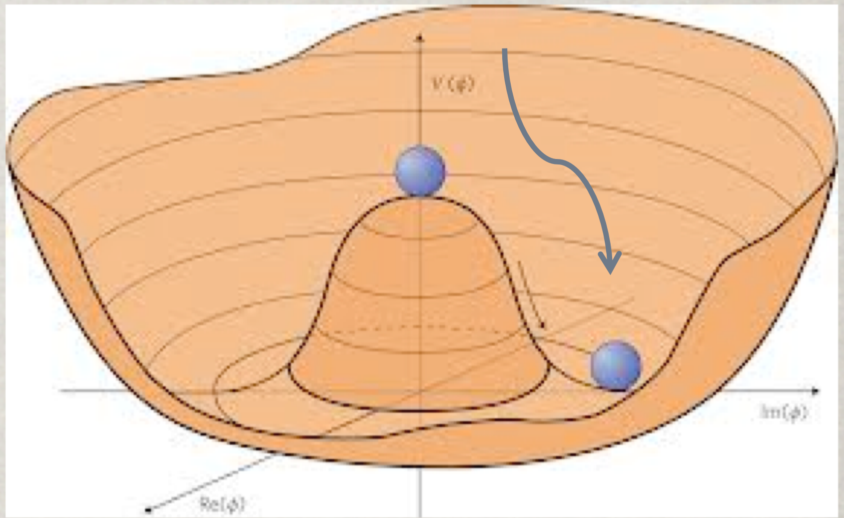
Motivation for Energy Frontier

1. Electroweak Symmetry Breaking, EW Superconductivity & Phase Transition

$$V(|\Phi|) \quad = \quad -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

You are here



It's like Landau-Ginzburg theory, but not!

$$F = \alpha(T)|\psi|^2 + \frac{\beta(T)}{2}|\psi|^4$$

$$|\psi|^2 = -\frac{\alpha(T)}{\beta(T)}$$

No EW analogue for BCS as the underlying theory to **understand** the dynamical mechanisms, to **calculate:**

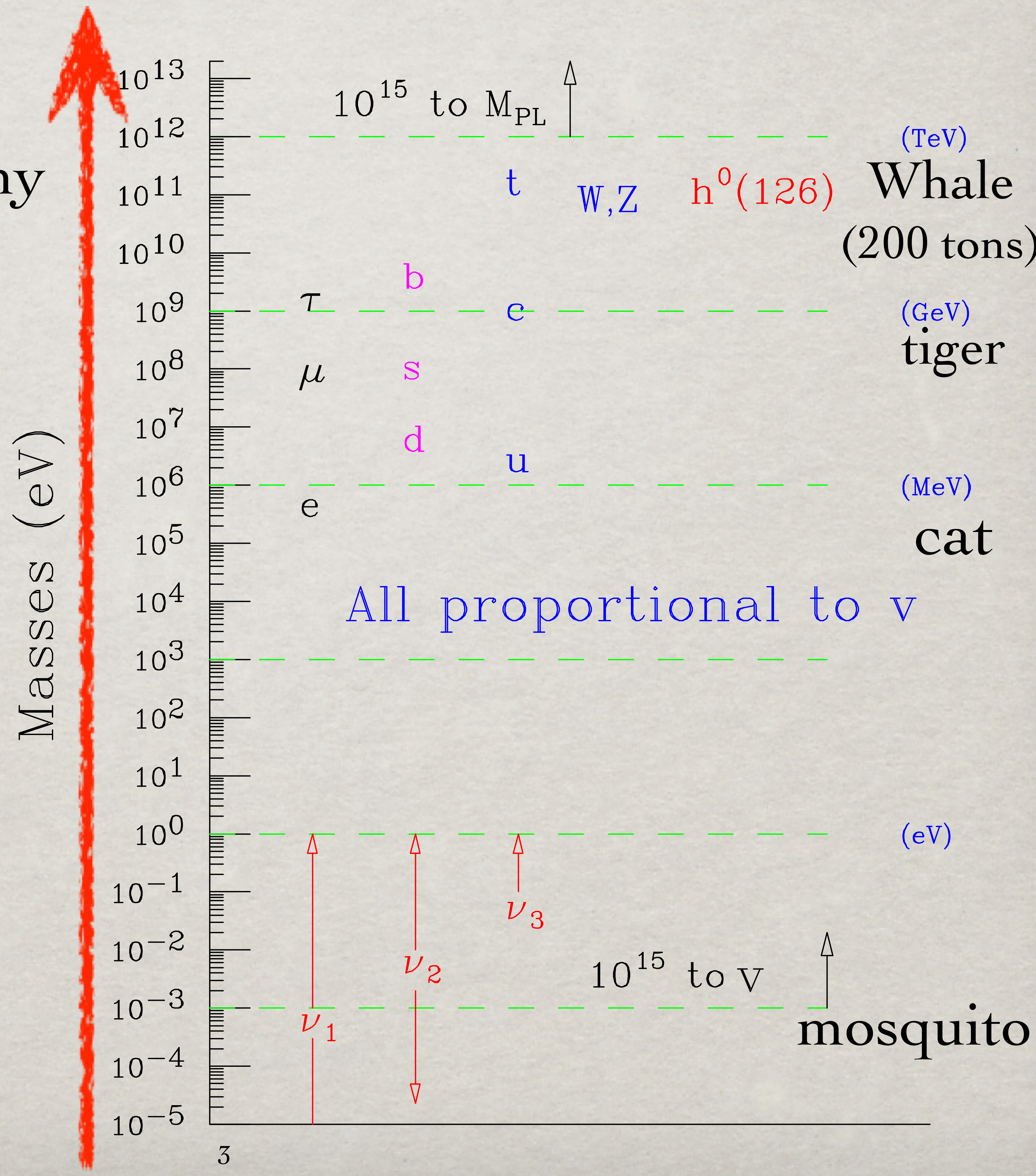
$\mu^2(\Lambda^2)$ & λ and potential shape → cosmology!

$$\to \frac{1}{2}\lambda(h^\dagger h)^2 \log\left[\frac{(h^\dagger h)}{m^2}\right]$$

2. The "Flavor Puzzle": fermion mass/mixing

- Particle mass hierarchy
- Patterns of quark, neutrino mixings
- Neutrino mass generation (seesaw)
- New CP-violation sources

Higgs is in a pivotal position.



3. The Dark Sector: Higgs portal?

The nature of DM is among the most pressing issue.

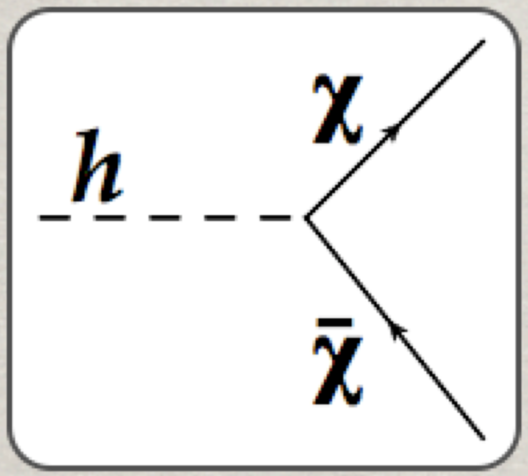
$H^\dagger H$ is a bi-linear SM gauge singlet to couple to anything.

Bad: May lead to hierarchy problem w.r.t. high-scale physics;

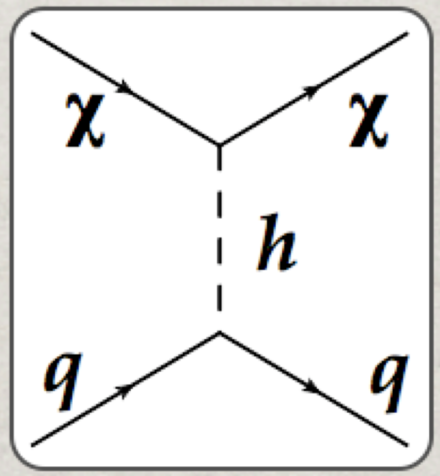
Good: May readily serve as a portal to the dark sector:

$$k_s H^\dagger H \; S^* S, \quad \frac{k_\chi}{\Lambda} H^\dagger H \; \bar{\chi}\chi.$$

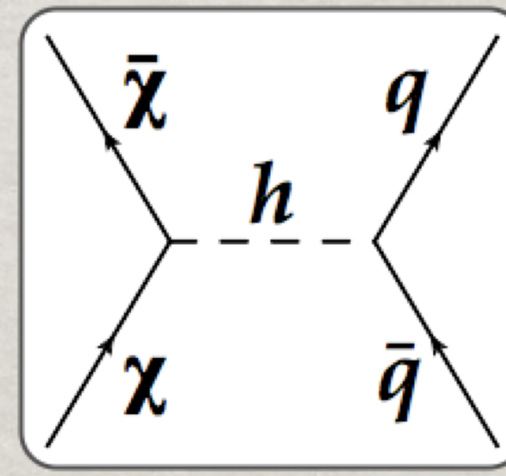
Dark matter at colliders

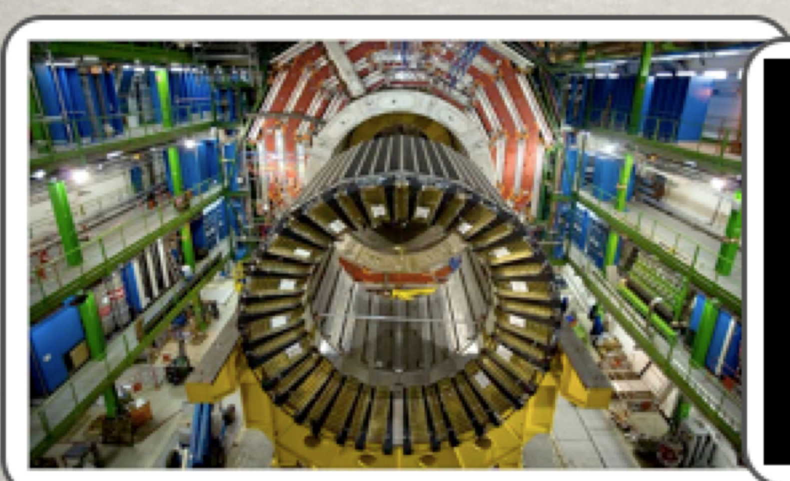


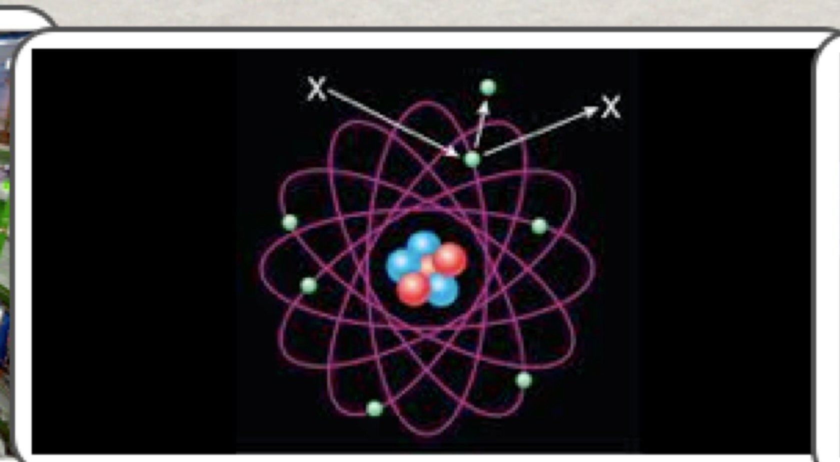
Direct detection

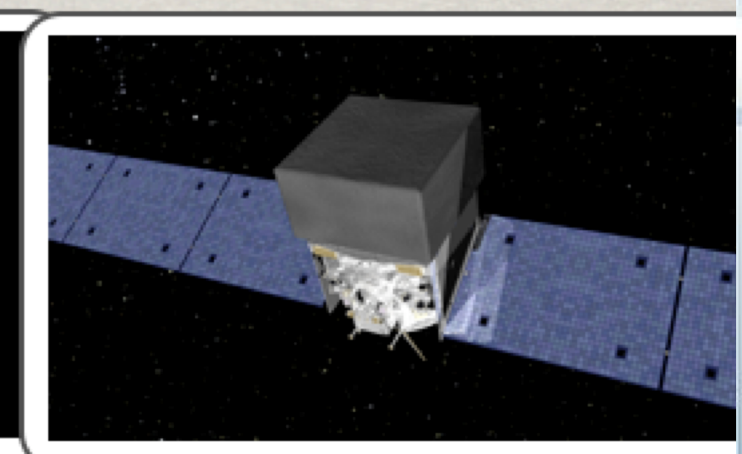


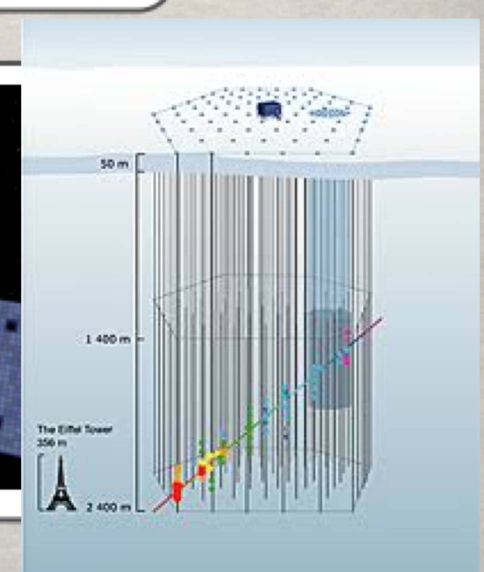
Indirect detection











Example 1:

Precision Higgs measurements, on g_i at the scale M:

$$\Delta_i \equiv \frac{g_i}{g_{SM}} - 1 \sim \mathcal{O}(v^2/M^2) \approx \text{a few \% for } M \approx 1 \text{ TeV}$$

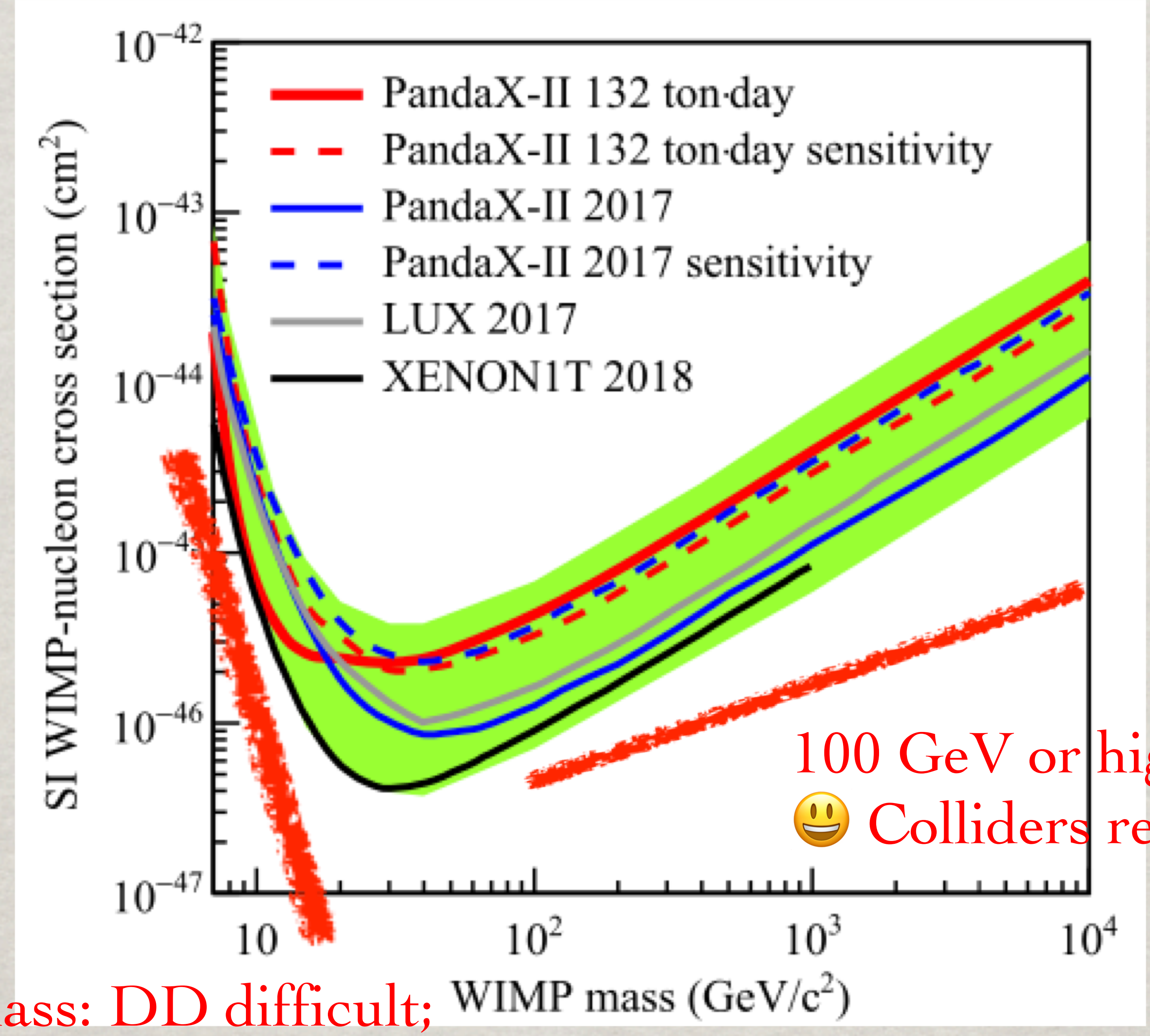
Higgs coupling deviations in theories:

Δ:	VVH	bbH,ττH	ggH,γγH	HHH	Inv.
Composite	(3-9)%	$(1\ \text{TeV}/f)^2$ (tree-level)		100%	
H^0, A^0 (SUSY)		6% $(500\ \text{GeV}/M_A)^2$			
T'			-10% $(1\ \text{TeV}/M_T)^2$ (loop)		

Observationally:

	VVH	bbH,ττH	ggH,γγH	HHH	Inv.
HL-LHC:	2%	4%	3%	50%	3%
27 TeV, 15 ab^{-1}:	<2%	<4%	1%	~ 20%	~3%
Higgs factory:	<0.2%	0.6%	2%	40% (indir)	1%
100 TeV, 30 ab^{-1}:	1%	a few%	<1%	7%	10^{-4}

Example 2: WIMP Dark Mater



100 GeV or higher mass:
😃 Colliders reach new threshold

GeV low mass: DD difficult;
😃 Colliders favor large p_T missing

A Muon Collider

Why muons?

Although sharing the same EW interactions,
it isn't another electron:

$$m_\mu \approx 207\ m_e$$
$$\tau(\mu \to e\bar{\nu}_e\nu_\mu) \approx 2.2\ \mu s$$
$$c\tau \approx 660\ m.$$

It is these features: heavy mass, short lifetime
that dictate the physics.

Some early work:

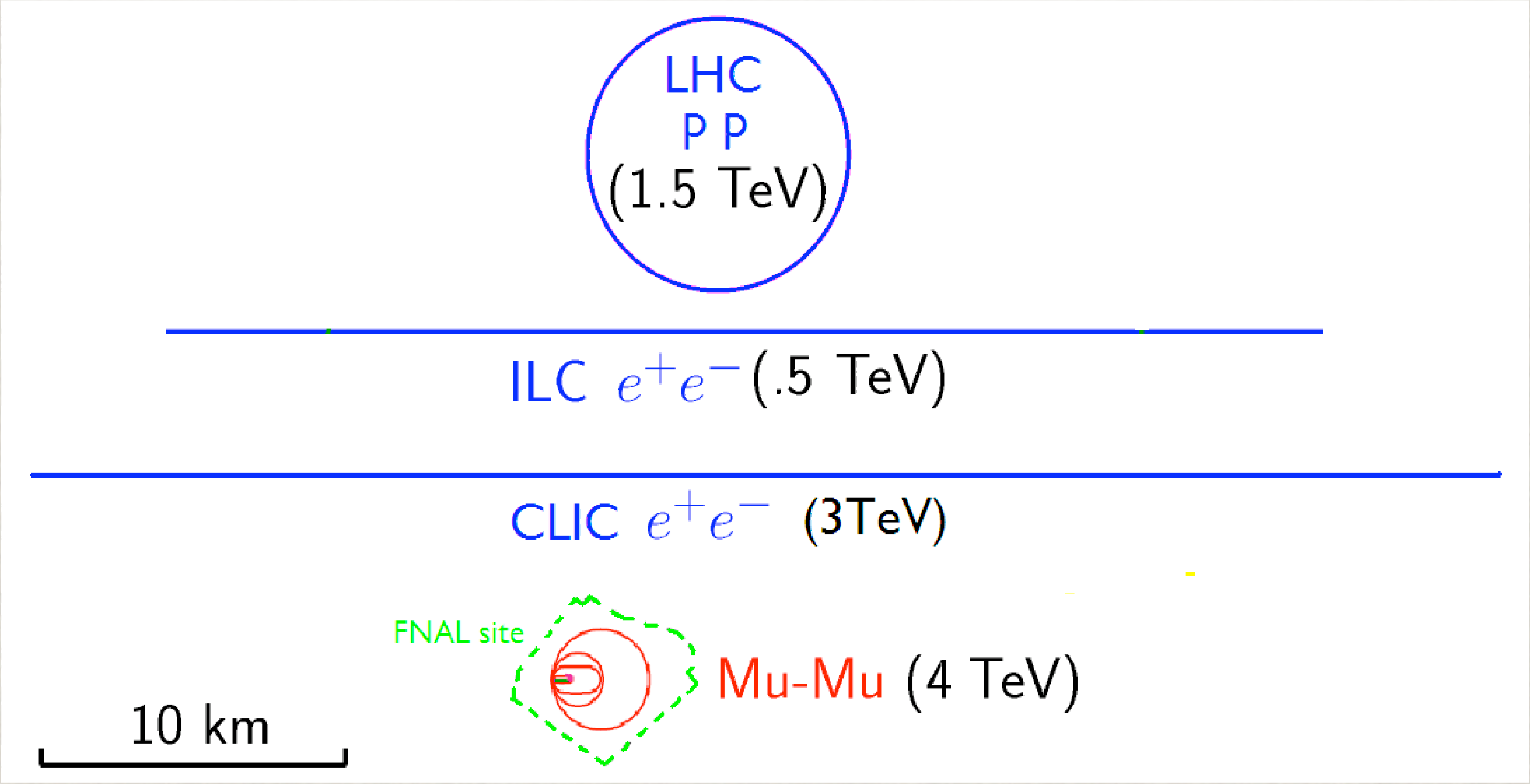
- *S-channel Higgs boson production at a muon collider,* Barger et al., PRL75 (1995).
- *$\mu^+\mu^-$ Collider: Feasibility study,* Muon collider collaboration (July, 1996).
- *Higgs boson physics in the s-channel muon collider,* Barger et al., Phys Rep. 186 (1997).
- *Status of muon collider research,* Muon collider collaboration (Aug., 1999).
- *Recent progress on neutrino factory and muon collider research,* Muon collider collaboration (July, 2003).

• Advantages of a muon collider

- Much less synchrotron radiation energy loss than e's:

$$\Delta E \sim \frac{1}{R}\,(\frac{E}{m_\mu})^4$$

which would allow a smaller and a circular machine:



- Unlike the proton as a composite particle, E_{CM} efficient in $\mu^+\mu^-$ annihilation
- Much smaller beam-energy spread:

$\Delta E/E \sim 0.01\% - 0.001\%$

• Disadvantages of a muon collider

- Production: Protons on target → pions → muons: Require sophisticated scheme for **μ** capture & transport

"Never play with an unstable thing!"

- Very short lifetime: in micro-second,

Muons cooling in (x,p) 6-dimensions

→ Difficult to make quality beams and a high luminosity

[Note: $E_{\mu} \sim 1$ TeV → $\gamma \sim 10^4$ → $\gamma\tau = 0.02$ s → d=6,000 km]

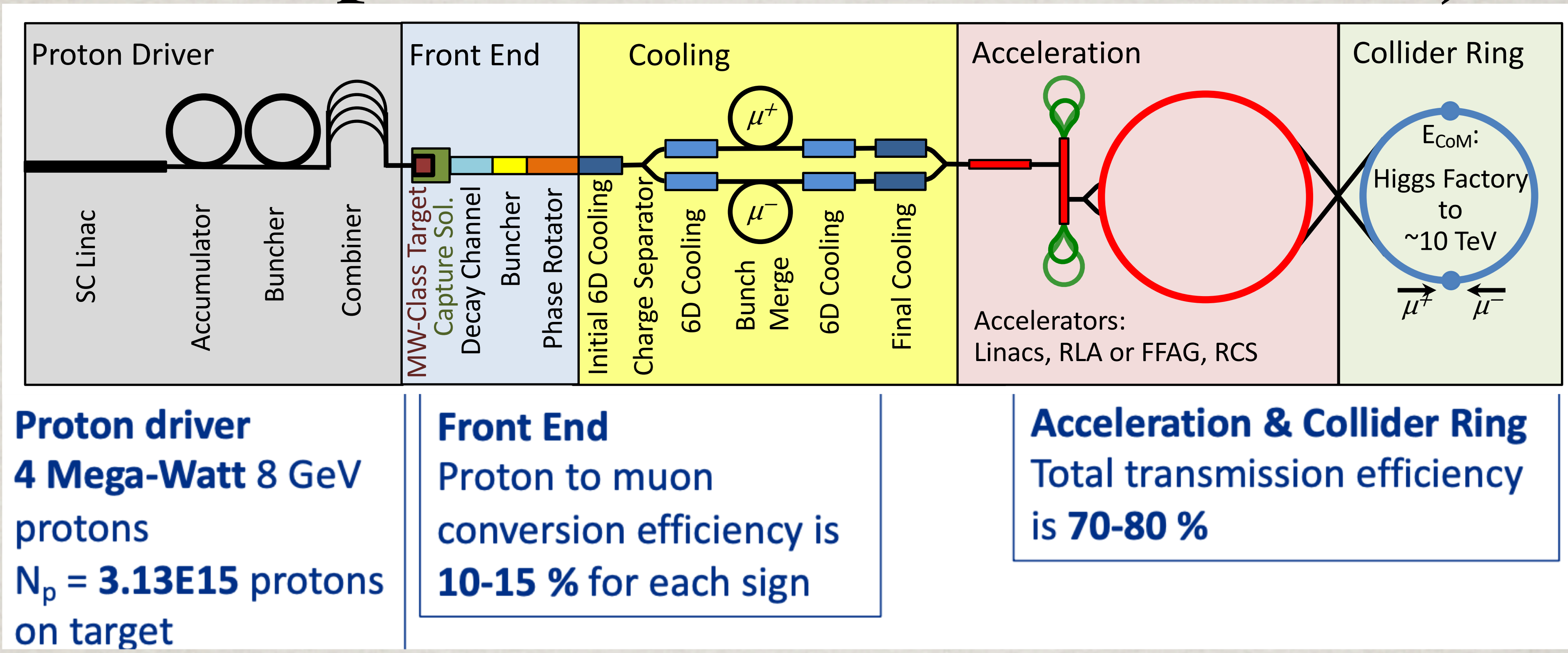
- Beam Induced Backgrounds (BIB) from the decays in the ring at the interacting point,

[Note: σ_{pp}(total)~100 mb; $\sigma_{\mu\mu}$(total)~100 nb]

- Neutrino beam dump (environmental hazard)

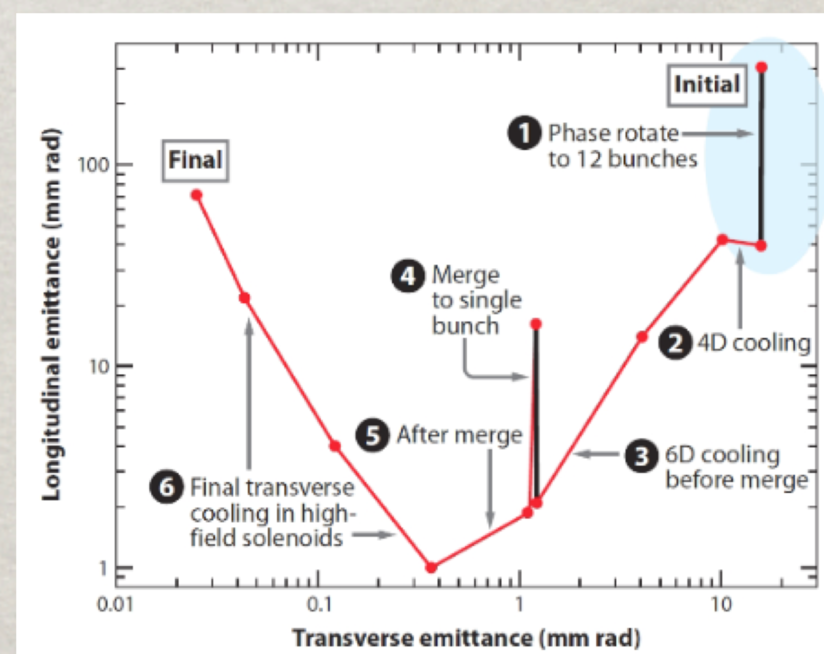
$\sigma_{\nu} \sim G_F^2 E^2$ → Shielding?

Proton Driver Option: Muon Accelerator Project (MAP)



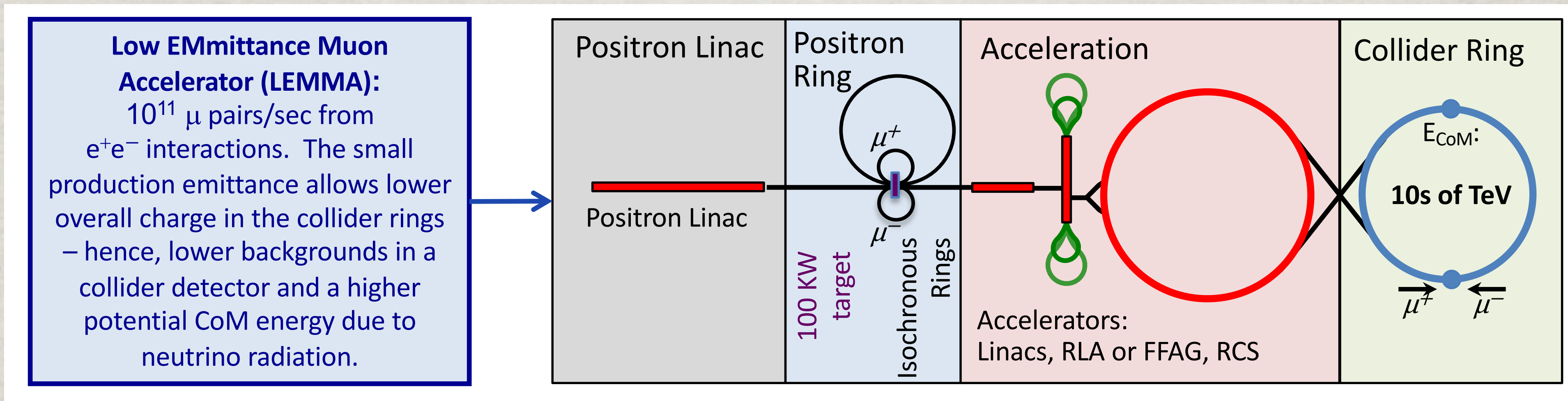
During 2011-2016, MAP collaboration formed:
to address key feasibility issues for **μ**C

- Protons → pions → muons
- Transverse ionization cooling achieved by MICE
- Muon emittance exchange demonstrated at FNAL/RAL
- 6D cooling of 5-6 orders needed

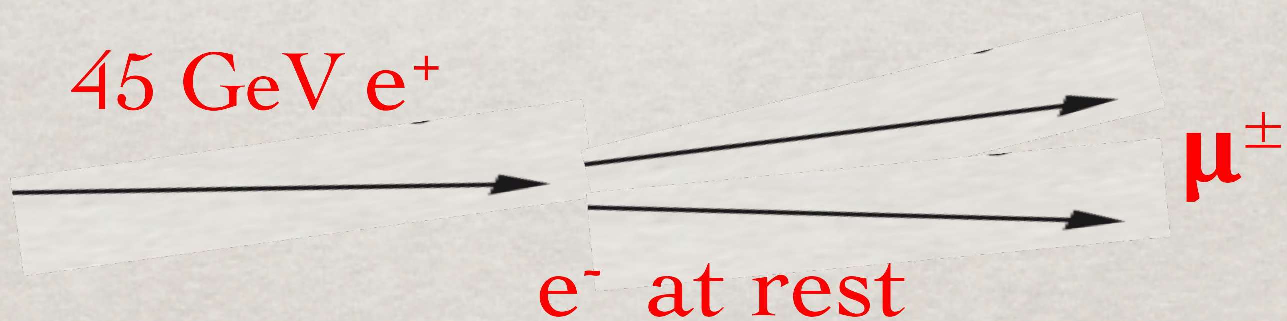


https://arxiv.org/abs/1907.08562, J.P. Delahauge et al., arXiv:1901.06150/

LEMMA: e^+e^- (at rest) → $\mu^+\mu^-$ (at threshold)



Low EMittance Muon Accelerator
web.infn.it/LEMMA



Cooling is not a problem;
but high luminosity is challenging!

J.P. Delahauge et al., arXiv:1901.06150

International Muon Collider Collaboration

https://muoncollider.web.cern.ch

Fermilab on site:

Site filler Accelerator

➢**Largest**
Radius is ~2.65 km

- **~16.5 km Circumference**
  - **~2/3 LHC**

~RCS accelerator

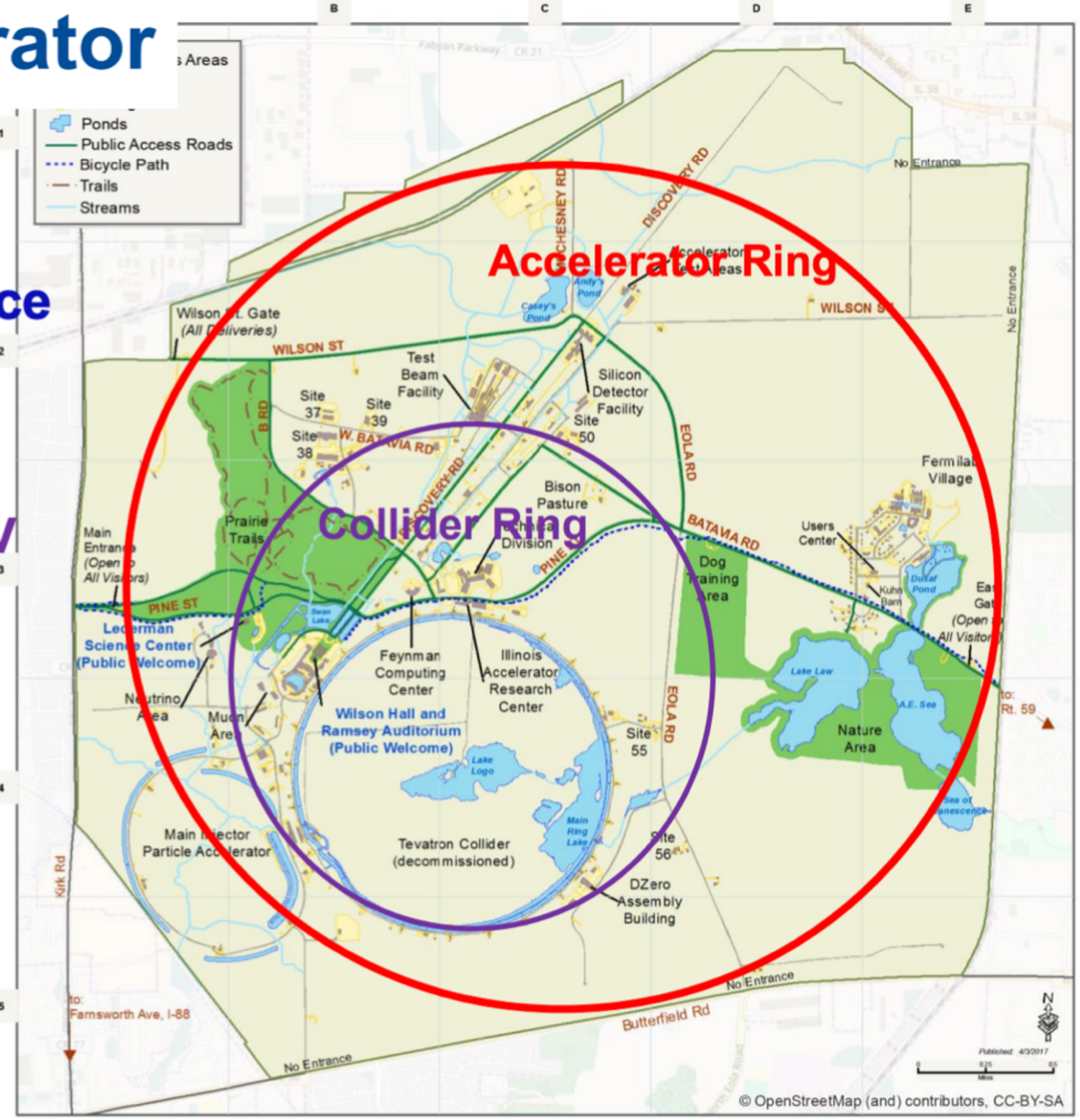
If B_{ave}= 3 T → E_{μ}= 2.4 TeV
(B_{max} = 8T, B_{pulse} =±2T)

Doubled ?

B_{ave}= 6.3 T → E_{μ}= 5 TeV
(B_{max} = 16T, B_{pulse} =±4T)

10 TeV collider

Collider Ring ~10 km
B_{ave} = 10 T
τ_{μ} = 0.104 s



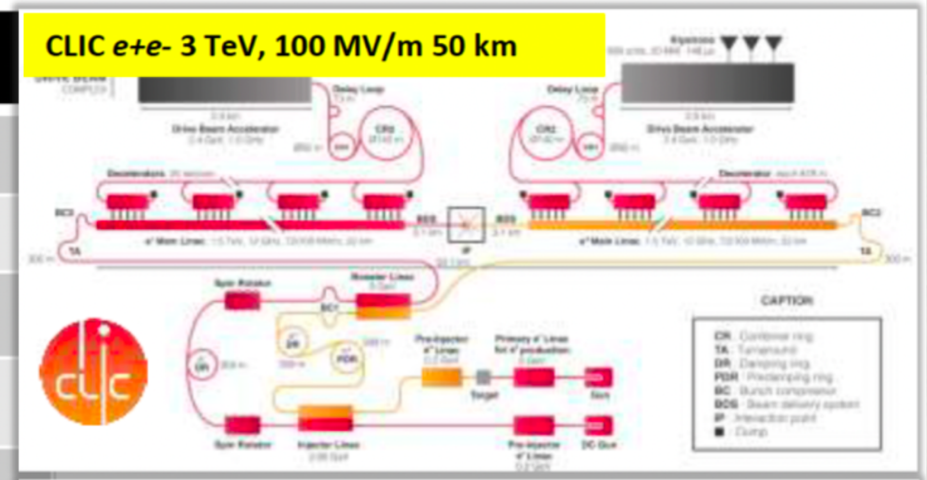
Daniel Schulte; Mark Palmer; Katsuya Yonehara talk, March 2022

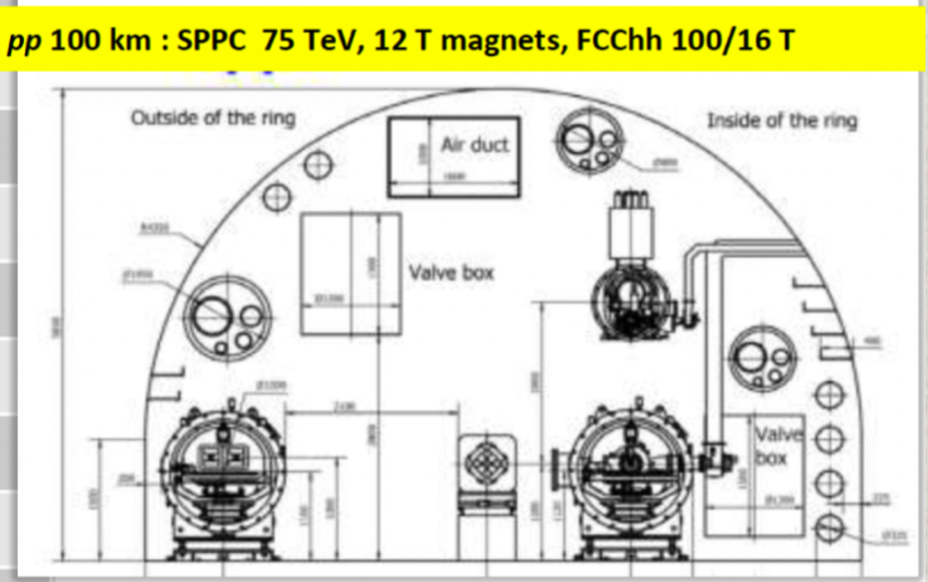
Much activities associated with Snowmass 2021

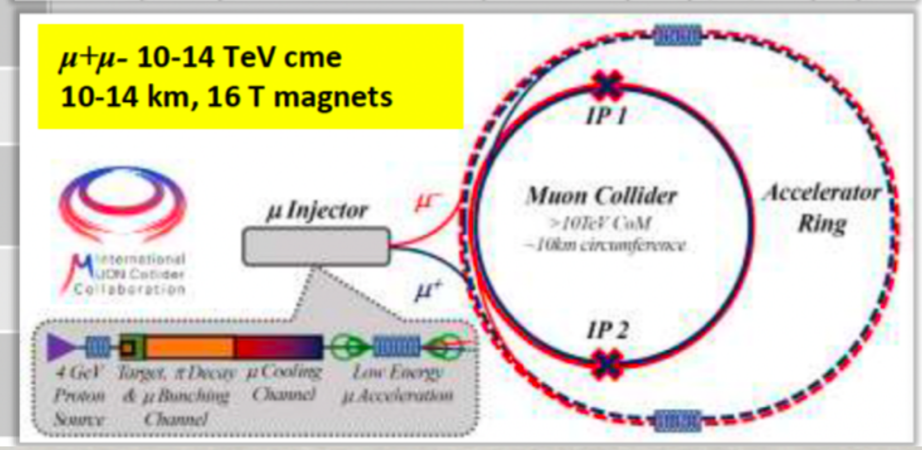
https://snowmass21.org

17 (!) High Energy Collider Concepts/Proposals

Name	Details
Cryo-Cooled Copper linac	e+e-, $\sqrt{s} = 2$ TeV, L= 4.5 $\times 10^{34}$
High Energy CLIC	e+e-, $\sqrt{s} = 1.5 - 3$ TeV, L= 5.9 $\times 10^{34}$
High Energy ILC	e+e-, $\sqrt{s} = 1 - 3$ TeV
FCC-hh	pp, $\sqrt{s} = 100$ TeV, L= 30 $\times 10^{34}$
SPPC	pp, $\sqrt{s} = 75/150$ TeV, L= 10 $\times 10^{34}$
Collider-in-Sea	pp, $\sqrt{s} = 500$ TeV, L= 50 $\times 10^{34}$
LHeC	ep, $\sqrt{s} = 1.3$ TeV, L= 1 $\times 10^{34}$
FCC-eh	ep, $\sqrt{s} = 3.5$ TeV, L= 1 $\times 10^{34}$
CEPC-SPPpC-eh	ep, $\sqrt{s} = 6$ TeV, L= 4.5 $\times 10^{33}$
VHE-ep	ep, $\sqrt{s} = 9$ TeV
MC – Proton Driver 1	$\mu\mu$, $\sqrt{s} = 1.5$ TeV, L= 1 $\times 10^{34}$
MC – Proton Driver 2	$\mu\mu$, $\sqrt{s} = 3$ TeV, L= 2 $\times 10^{34}$
MC – Proton Driver 3	$\mu\mu$, $\sqrt{s} = 10 - 14$ TeV, L= 20 $\times 10^{34}$
MC – Positron Driver	$\mu\mu$, $\sqrt{s} = 10 - 14$ TeV, L= 20 $\times 10^{34}$
LWFA-LC (e+e- and $\gamma\gamma$)	Laser driven; e+e-, $\sqrt{s} = 1 - 30$ TeV
PWFA-LC (e+e- and $\gamma\gamma$)	Beam driven; e+e-, $\sqrt{s} = 1 - 30$ TeV
SWFA-LC	Structure wakefields; e+e-, $\sqrt{s} = 1 - 30$ TeV







Collider benchmark points:

- The Higgs factory:

Parameter	Units	Higgs
CoM Energy	TeV	0.126
Avg. Luminosity	$10^{34}\text{cm}^{-2}\text{s}^{-1}$	0.008
Beam Energy Spread	%	0.004
Higgs Production/10^7 sec		13'500
Circumference	km	0.3

$E_{cm} = m_H$

$L \sim 1\ \text{fb}^{-1}/\text{yr}$

$\Delta E_{cm} \sim 5$ MeV

Current Snowmass 2021 point: 4 fb^{-1} / yr

- Multi-TeV colliders:

Lumi-scaling scheme: $\sigma L \sim$ const.

$$L \gtrsim \frac{5\ \text{years}}{\text{time}} \left(\frac{\sqrt{s_\mu}}{10\ \text{TeV}} \right)^2 2 \cdot 10^{35}\text{cm}^{-2}\text{s}^{-1}$$

1 ab^{-1} /yr

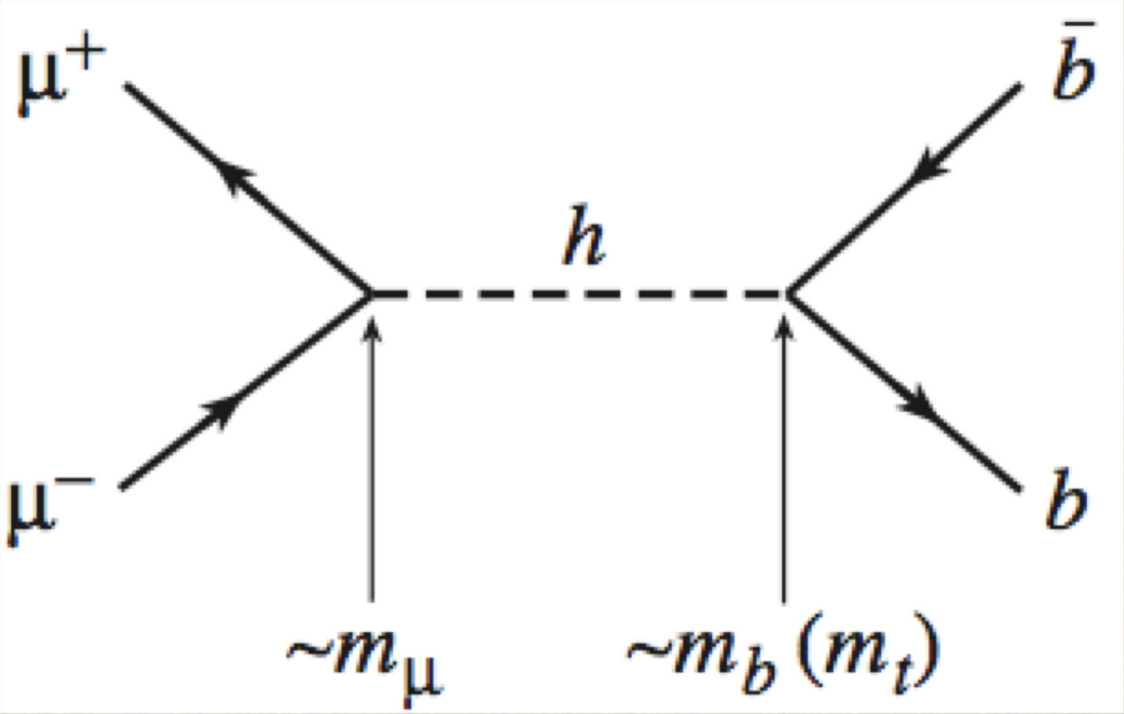
The aggressive choices:

$$\sqrt{s} = 3,\ 6,\ 10,\ 14,\ 30 \text{ and } 100 \text{ TeV}, \quad \mathcal{L} = 1,\ 4,\ 10,\ 20,\ 90, \text{ and } 1000\ \text{ab}^{-1}$$

European Strategy, arXiv:1910.11775; arXiv:1901.06150; arXiv:2007.15684.

A Higgs Factory

Resonant Production:



$$\sigma(\mu^+\mu^- \to h \to X) = \frac{4\pi\Gamma_h^2 \mathrm{Br}(h \to \mu^+\mu^-)\mathrm{Br}(h \to X)}{(\hat{s} - m_h^2)^2 + \Gamma_h^2 m_h^2}.$$

$$\sigma_{peak}(\mu^+\mu^- \to h) = \frac{4\pi}{m_h^2} BR(h \to \mu^+\mu^-)$$

$$\approx 71 \text{ pb at } m_h = 125 \text{ GeV}.$$

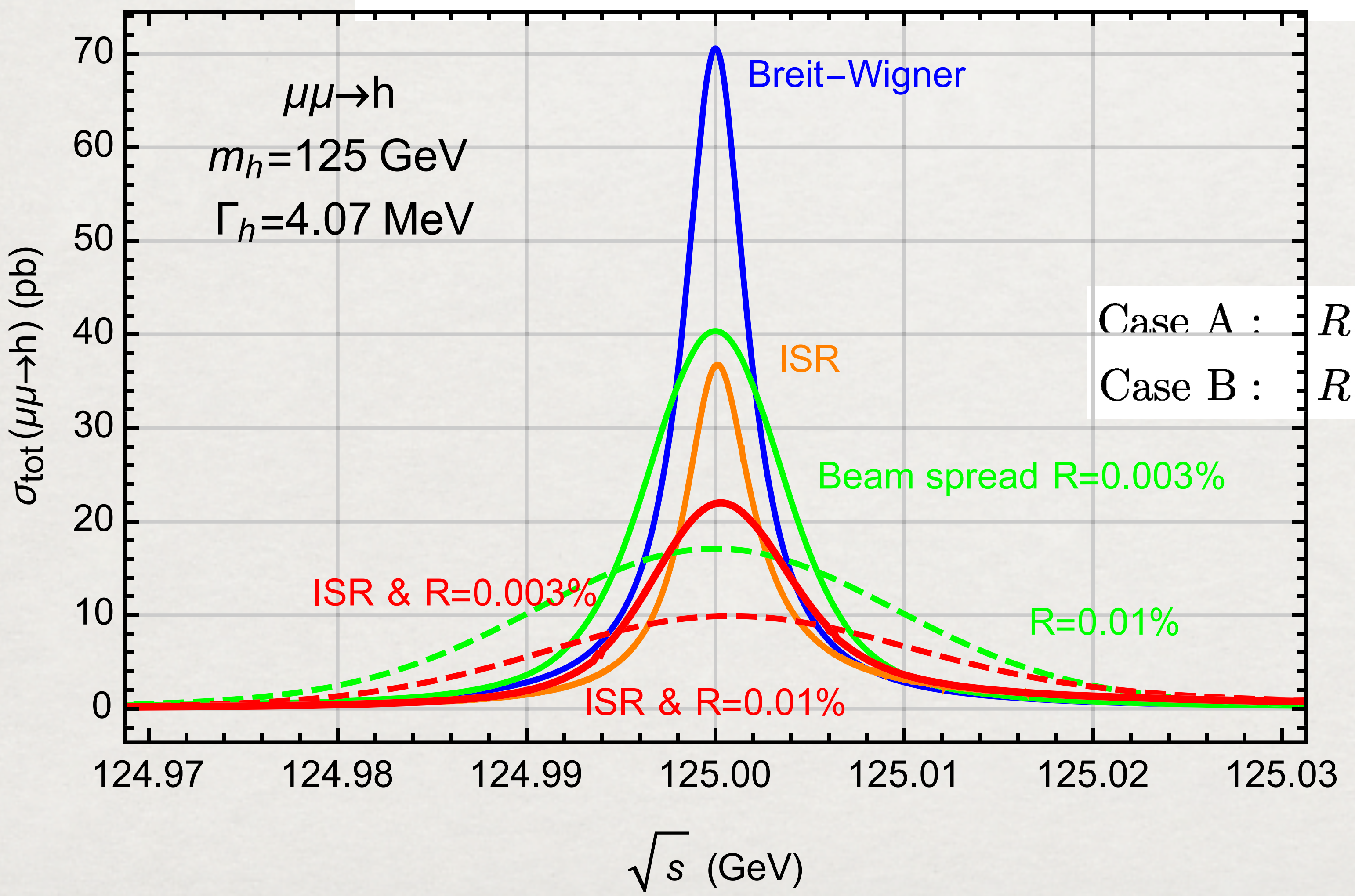
About O(70k) events produced per fb^{-1}

At m_h=125 GeV, Γ_h = 4.2 MeV

$$\frac{\exp[-(\sqrt{\hat{s}}-\sqrt{s})^2/(2\sigma^2_{\sqrt{s}})]}{\sqrt{2\pi}\sigma_{\sqrt{s}}} \qquad \frac{4\pi\Gamma(h\to\mu\mu)\,\Gamma(h\to X)}{(\hat{s}-m_h^2)^2+m_h^2[\Gamma_h^{\rm tot}]^2}$$

$$\sigma_{\rm eff}(s) = \int d\sqrt{\hat{s}}\,\frac{dL(\sqrt{s})}{d\sqrt{\hat{s}}}\sigma(\mu^+\mu^-\to h\to X)$$

$$\propto \begin{cases} \Gamma_h^2 B/[(s-m_h^2)^2+\Gamma_h^2 m_h^2] & (\Delta \ll \Gamma_h), \\ B\exp[\frac{-(m_h-\sqrt{s})^2}{2\Delta^2}](\frac{\Gamma_h}{\Delta})/m_h^2 & (\Delta \gg \Gamma_h). \end{cases}$$



"Muon Collider Quartet":

Barger-Berger-Gunion-Han

PRL & Phys. Report (1995)

Case A : $R = 0.01\%$ ($\Delta = 8.9$ MeV), $L = 0.5$ fb^{-1},

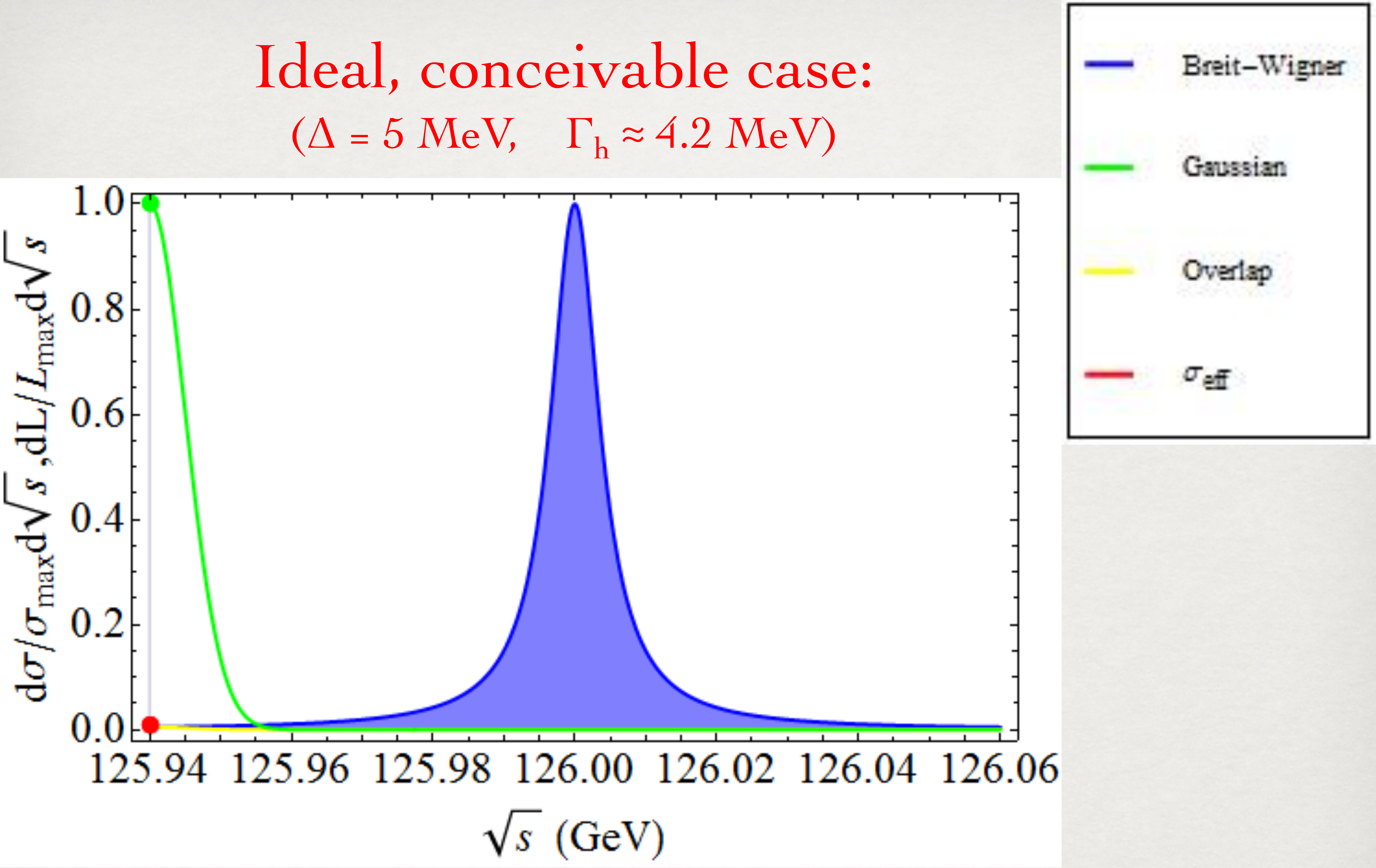
Case B : $R = 0.003\%$ ($\Delta = 2.7$ MeV), $L = 1$ fb^{-1}.

TH, Liu: 1210.7803;

Greco, TH, Liu: 1607.03210

Ideal, conceivable case:

(Δ = 5 MeV, $\Gamma_h \approx 4.2$ MeV)



An optimal fitting would reveal Γ_h

Achievable accuracy at the Higgs factory:

TABLE I. Effective cross sections (in pb) at the resonance $\sqrt{s} = m_h$ for two choices of beam energy resolutions R and two leading decay channels, with the SM branching fractions $\mathrm{Br}_{b\bar{b}} = 56\%$ and $\mathrm{Br}_{WW^*} = 23\%$ [9]. a cone angle cut: $10^o < \theta < 170^o$

	$\mu^+\mu^- \to h$	$h \to b\bar{b}$		$h \to WW^*$	
R (%)	σ_{eff} (pb)	σ_{Sig}	σ_{Bkg}	σ_{Sig}	σ_{Bkg}
0.01	16	7.6		3.7	
0.003	38	18	15	5.5	0.051

Good S/B>1, S/√B → % accuracies

Table 3

Fitting accuracies for one standard deviation of Γ_h, B and m_h of the SM Higgs with the scanning scheme for two representative luminosities per step and two bench-mark beam energy spread parameters.

$\Gamma_h = 4.07$ MeV	L_{step} (fb^{-1})	$\delta\Gamma_h$ (MeV)	δB	δm_h (MeV)
$R = 0.01\%$	0.05	0.79	3.0%	0.36
	0.2	0.39	1.1%	0.18
$R = 0.003\%$	0.05	0.30	2.5%	0.14
	0.2	0.14	0.8%	0.07

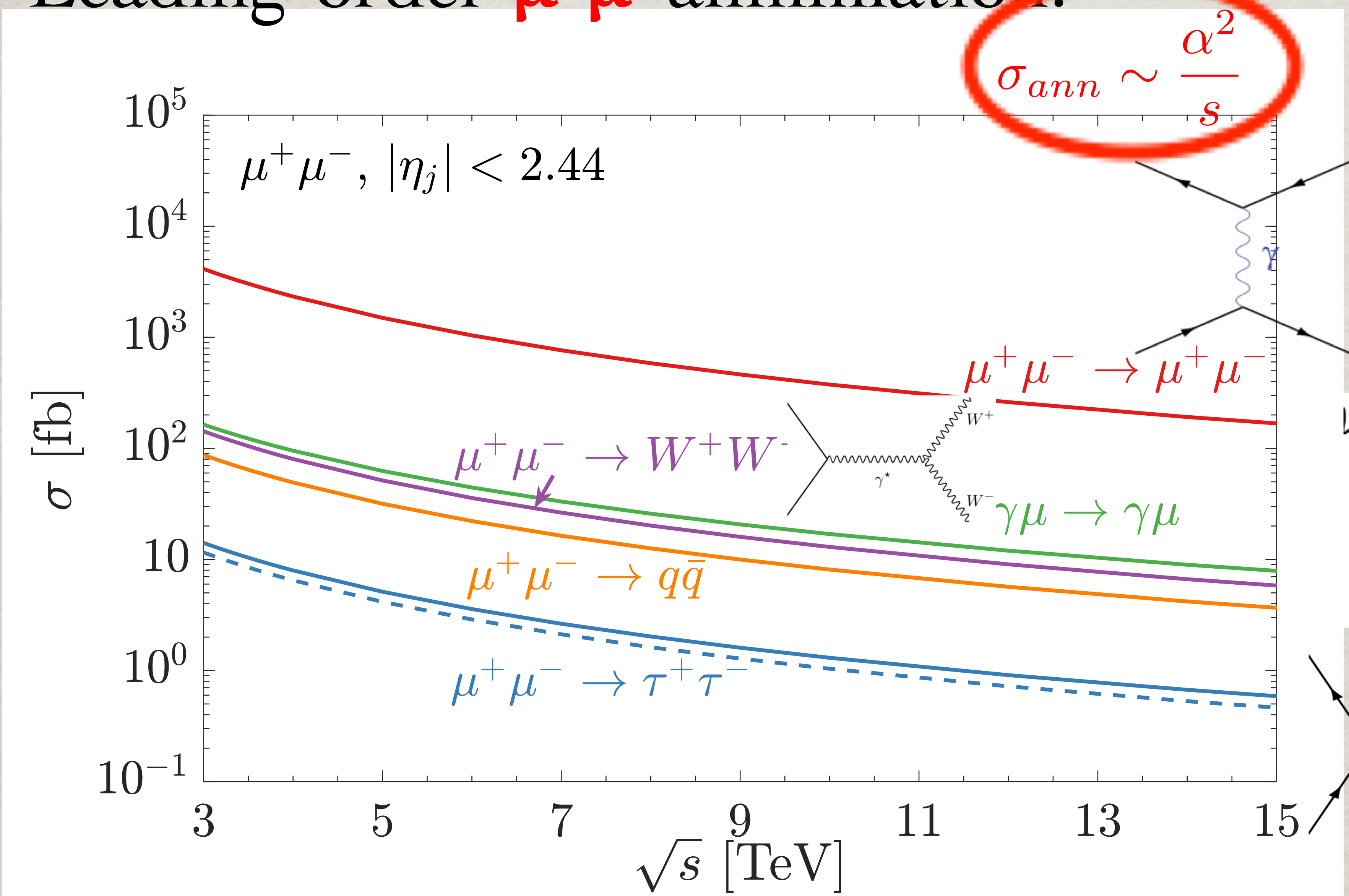
~ 3.5%

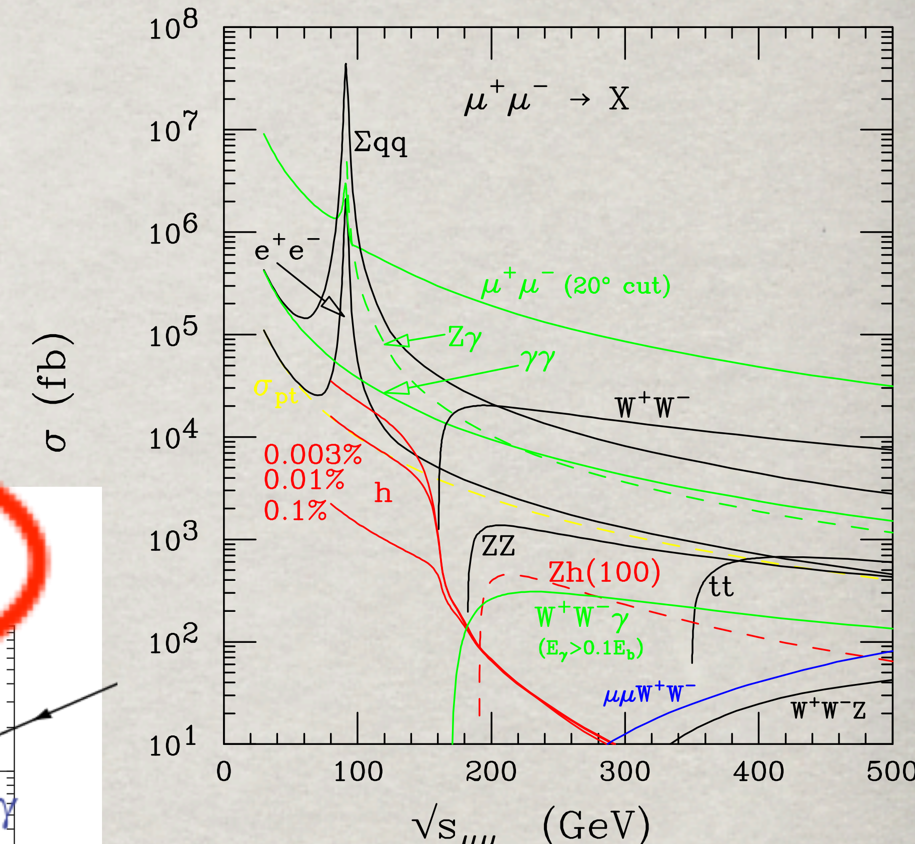
TH, Liu: 1210.7803;
Greco, TH, Liu: 1607.03210

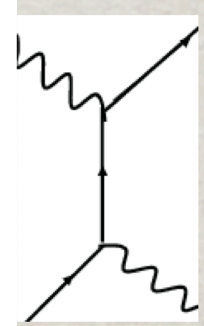
A Multi-TeV Muon Collider

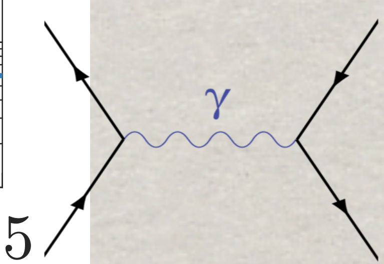
Exciting energy-frontier! What will happen when you turn on a $\mu^+\mu^-$ Smasher?

Leading-order $\mu^+\mu^-$ annihilation:



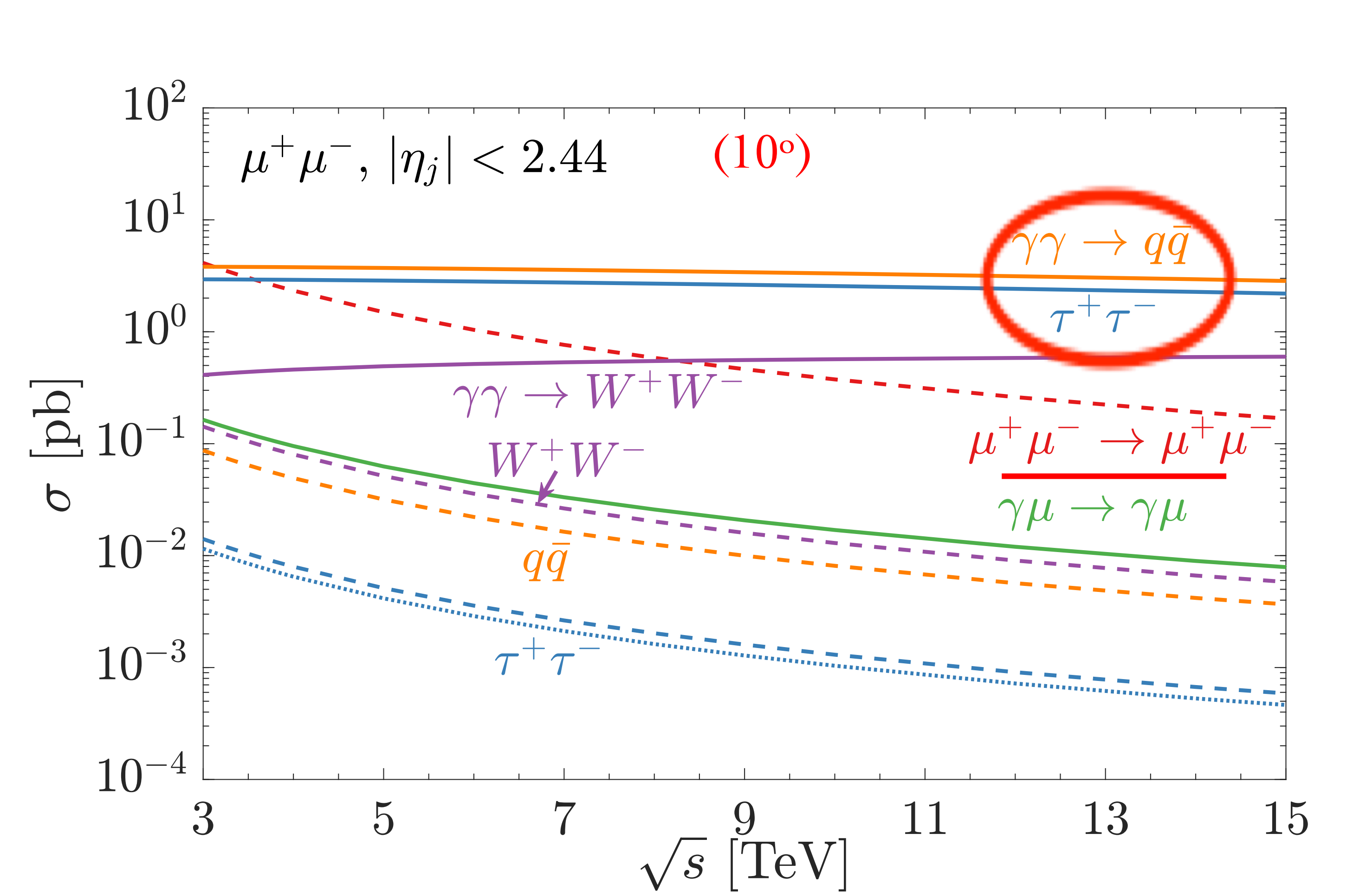


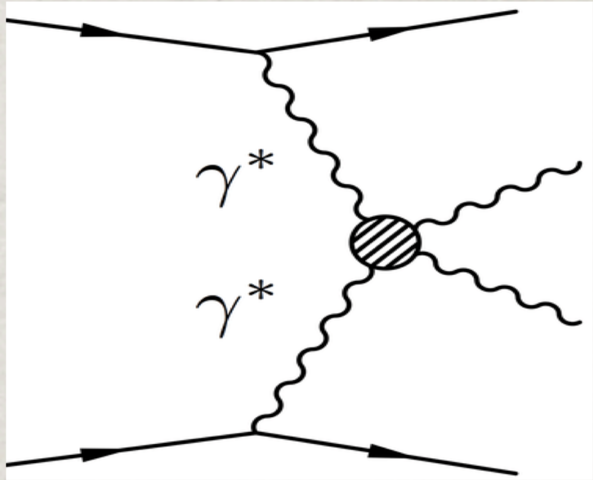




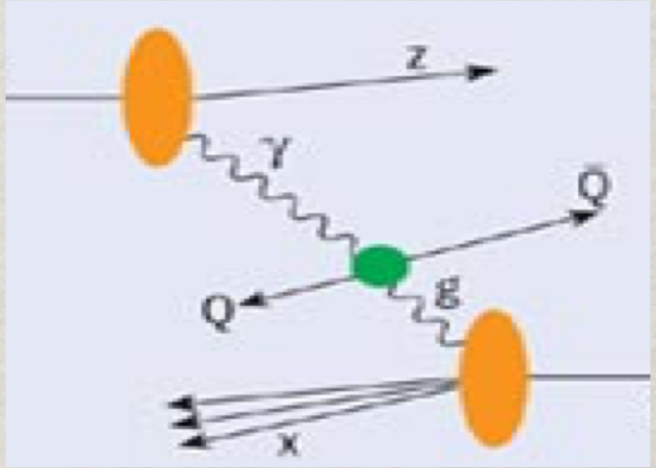
• Photon-induced QED cross sections

large rates $\sigma_{fusion} \sim \frac{\alpha^2}{m_{jj}^2} \log^2(\frac{Q^2}{m^2})$





m_{jj}

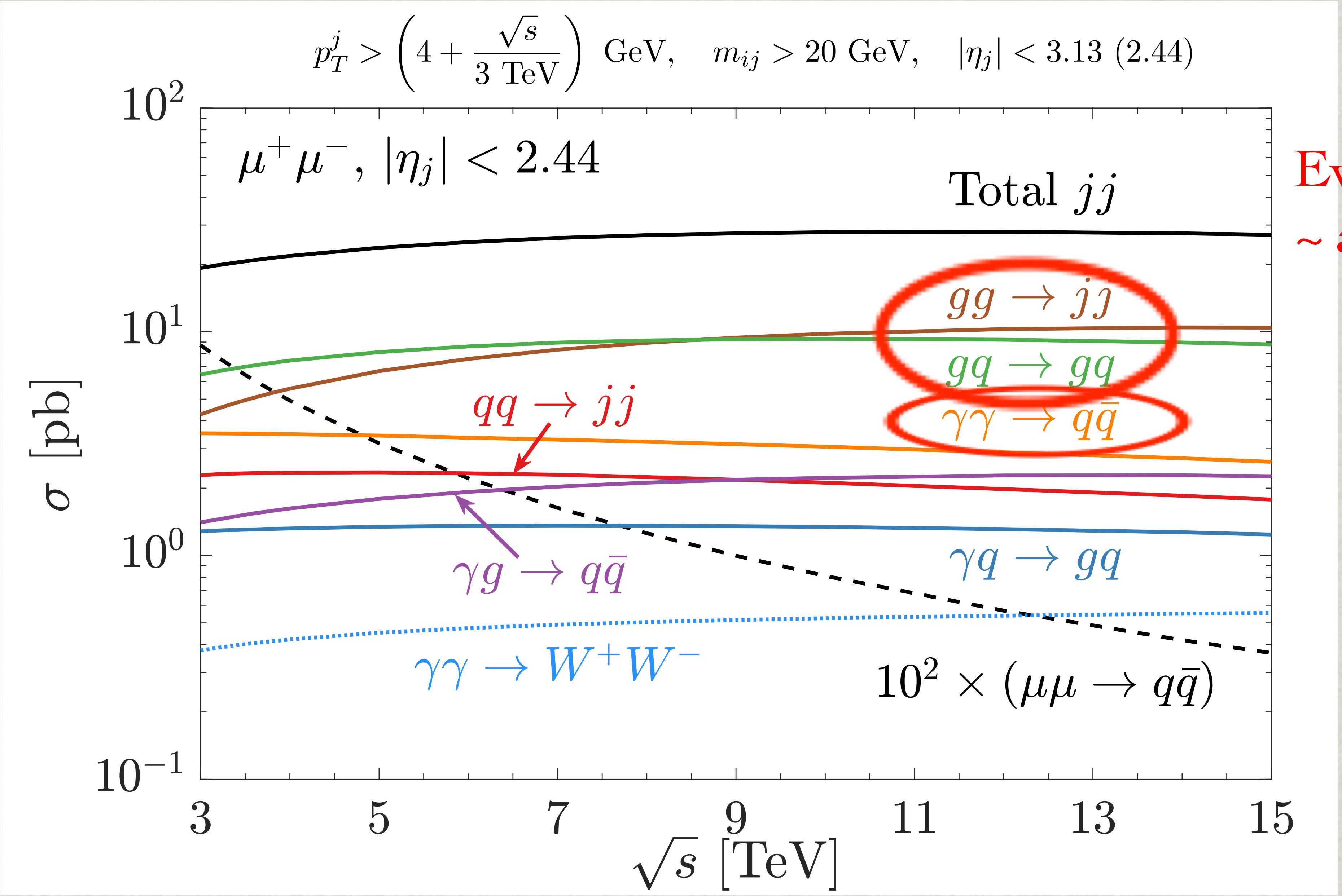


$$p_T^j > \left(4 + \frac{\sqrt{s}}{3\text{ TeV}}\right)\text{ GeV}, \quad m_{ij} > 20\text{ GeV}, \quad |\eta_j| < 3.13\ (2.44)$$

Quarks/gluons come into the picture via SM DGLAP:

$$\frac{\mathrm{d}}{\mathrm{d}\log Q^2}\begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix} = \begin{pmatrix} P_{\ell\ell} & 0 & 0 & 2N_\ell P_{\ell\gamma} & 0 \\ 0 & P_{uu} & 0 & 2N_u P_{u\gamma} & 2N_u P_{ug} \\ 0 & 0 & P_{dd} & 2N_d P_{d\gamma} & 2N_d P_{dg} \\ P_{\gamma\ell} & P_{\gamma u} & P_{\gamma d} & P_{\gamma\gamma} & 0 \\ 0 & P_{gu} & P_{gd} & 0 & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix}$$

Di-jet production: $\gamma\gamma \to q\bar{q},\ \gamma g \to q\bar{q},\ \gamma q \to gq,$ $qq \to qq(gg),\ gq \to gq,$ and $gg \to gg(q\bar{q})$

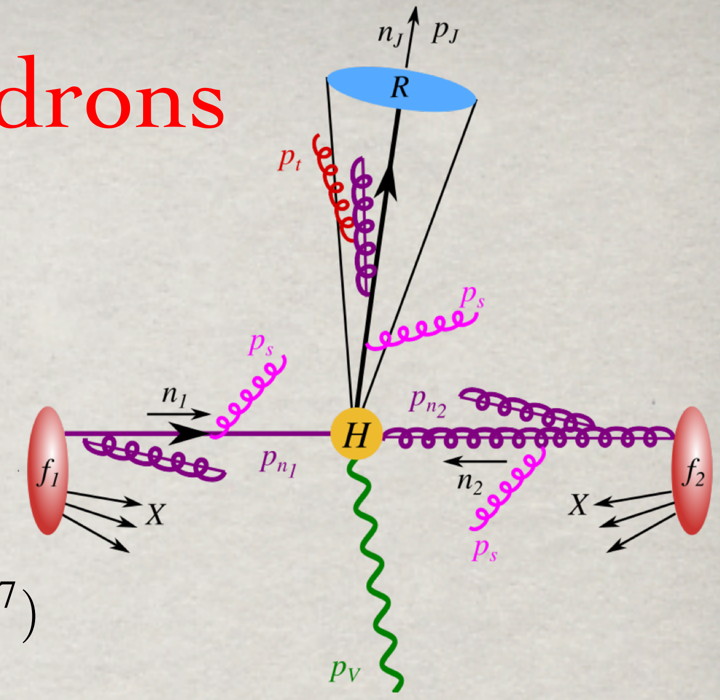


Event rate ~ a few Hz

- Jet production dominates at low energies
- EW processes take over for $p_T > 60$ GeV

TH, Yang Ma, Keping Xie, arXiv:2103.09844.

Total cross sections: γγ and $\mu^+\mu^-$ → hadrons

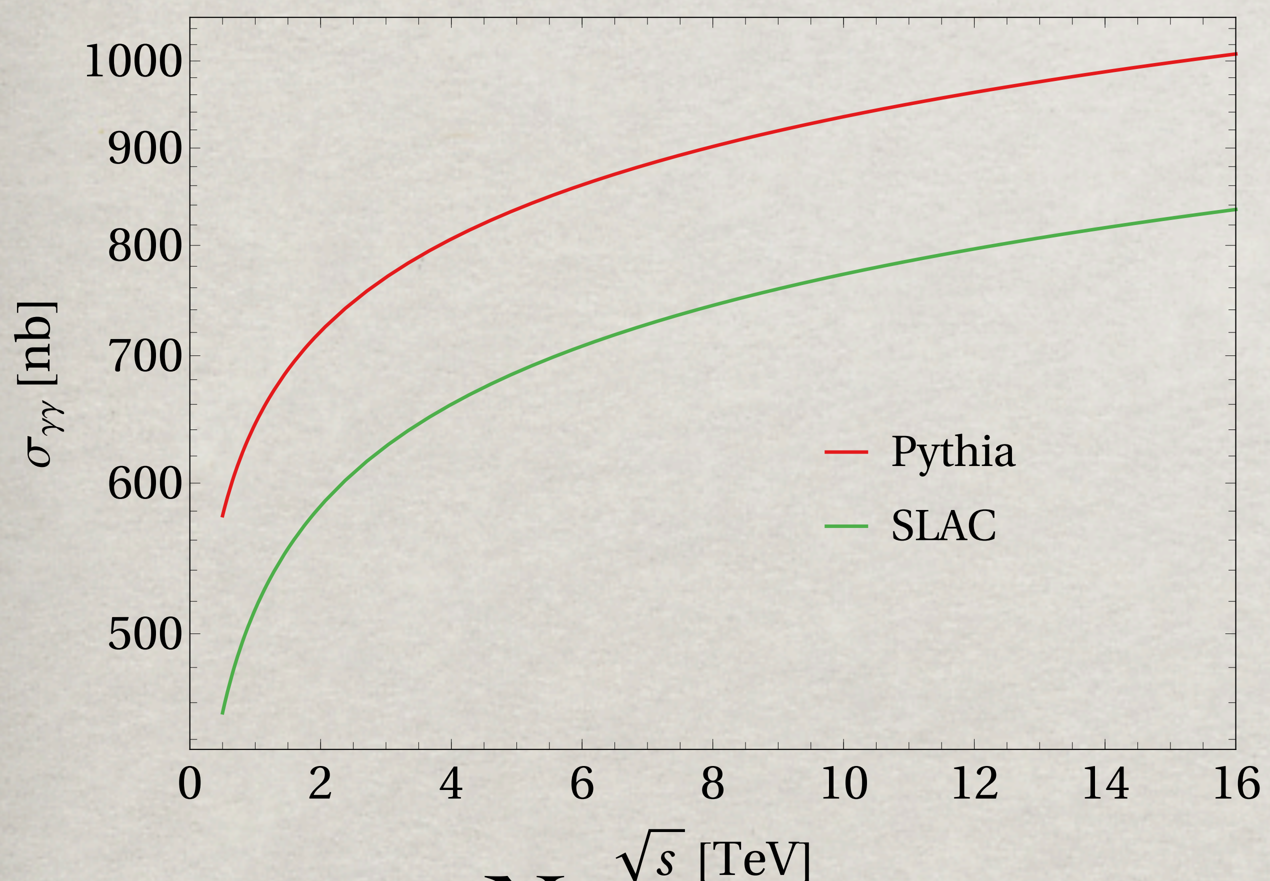


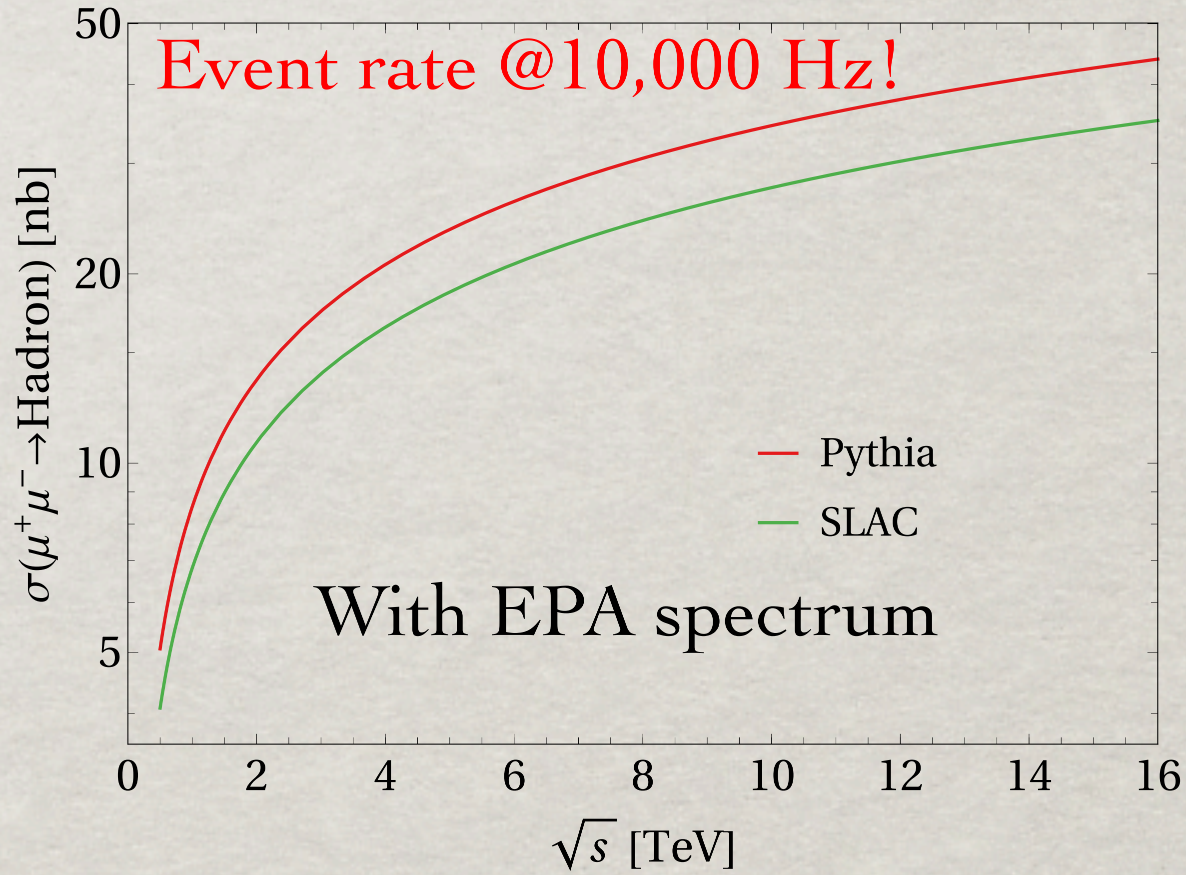
PYTHIA parameterization:

$$\sigma_{\gamma\gamma}(E_{cm}^2) = 211\ \text{nb}(E_{cm}^2\text{GeV}^{-2})^{0.0808} + 215\ \text{nb}(E_{cm}^2\text{GeV}^{-2})^{-0.4525}$$

SLAC parameterization:

$$\sigma_{\gamma\gamma}(E_{cm}^2) = 200\ \text{nb}(1 + 0.0063[\ln(E_{cm}^2\text{GeV}^{-2})]^{2.1} + 1.96(E_{cm}^2\text{GeV}^{-2})^{-0.37})$$





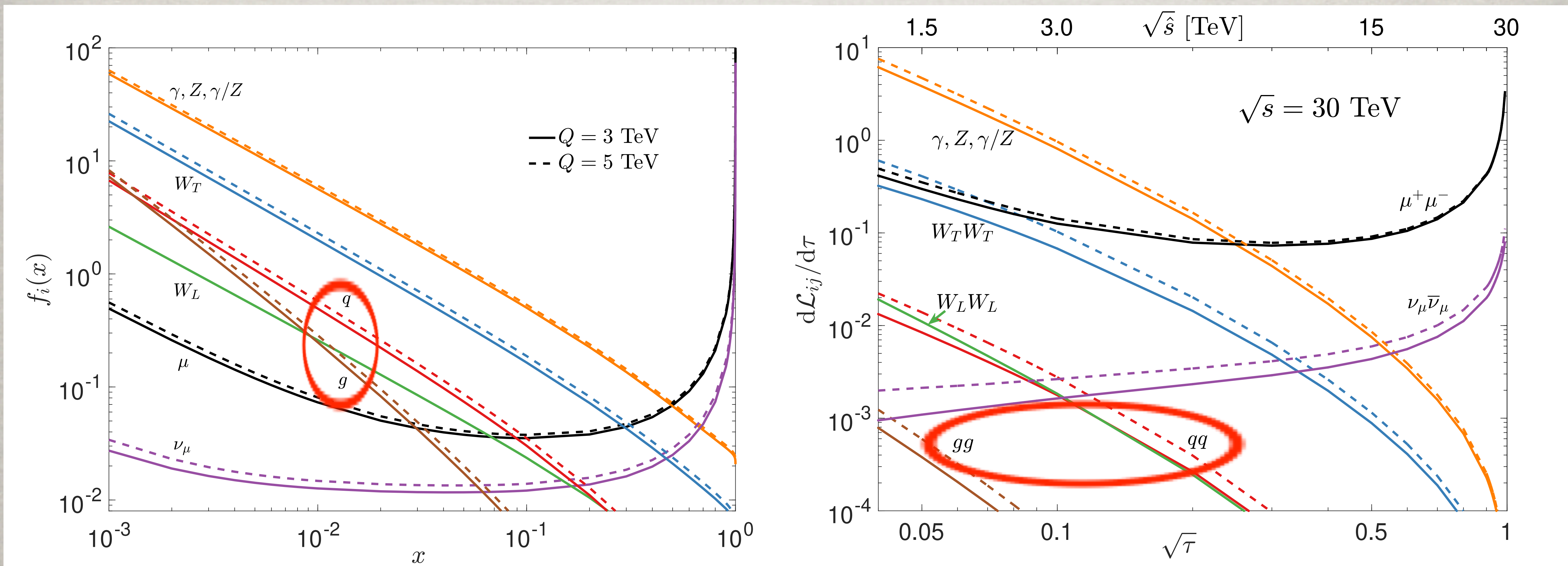
Note: σ_{pp}(total) ~ 100 mb; $\sigma_{\mu\mu}$ (total)~ 50 nb

Events populated at $p_T^{hadrons}$< a few GeV

T. Barklow, D. Dannheim, M.O. Sahin & D. Schulte, LCD-Note-2011-020.

• EW PDFs at a muon collider:

"partons" dynamically generated $\frac{\mathrm{d} f_i}{\mathrm{d}\ln Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P^I_{i,j} \otimes f_j$



$\mu^{\pm}$: the valance. ℓ_R, ℓ_L, ν_L and $B, W^{\pm,3}$: LO sea.

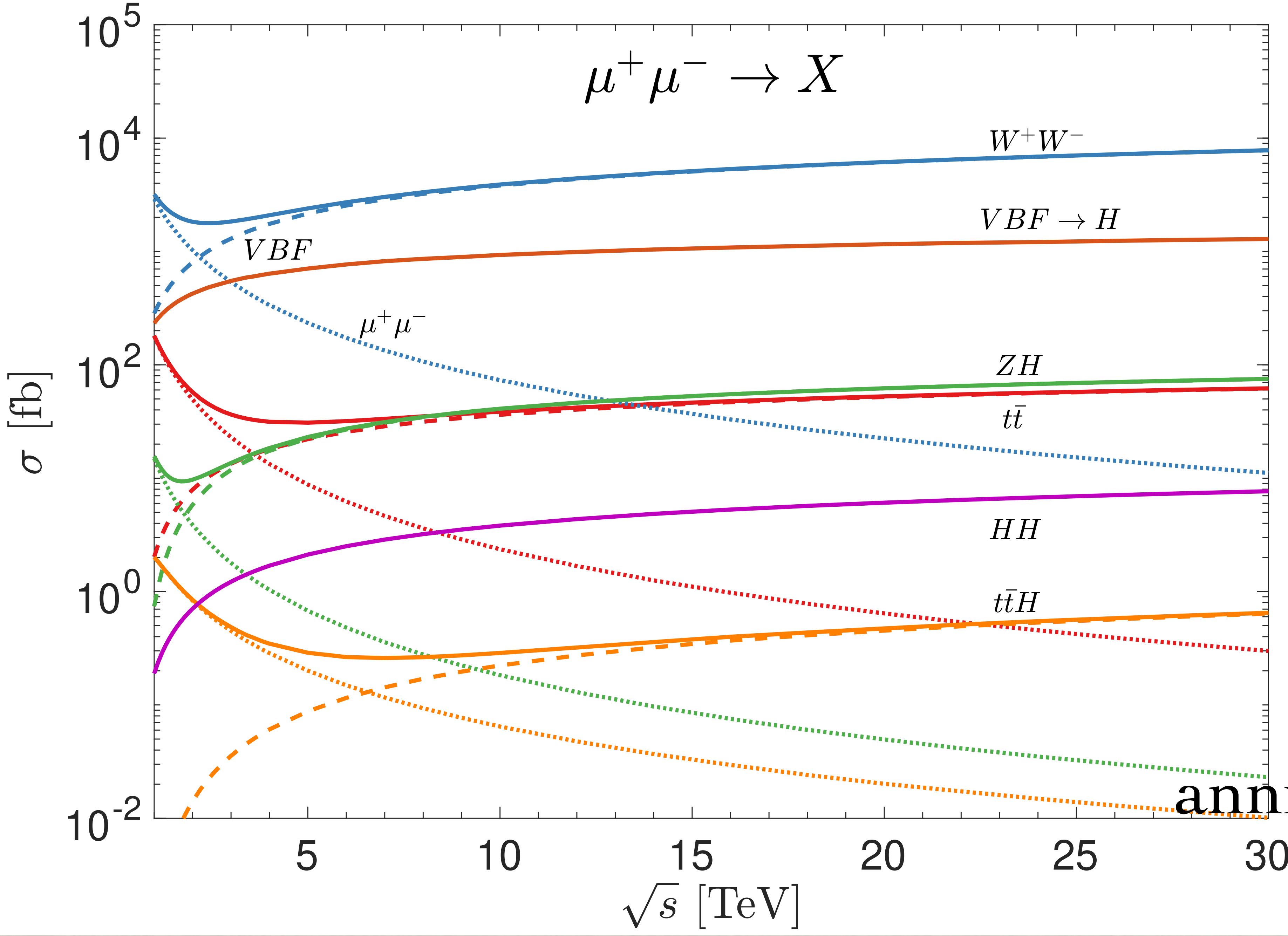
Quarks: NLO; gluons: NNLO.

TH, Yang Ma, Keping Xie, arXiv:2007.14300

• "Semi-inclusive" processes

Just like in hadronic collisions:

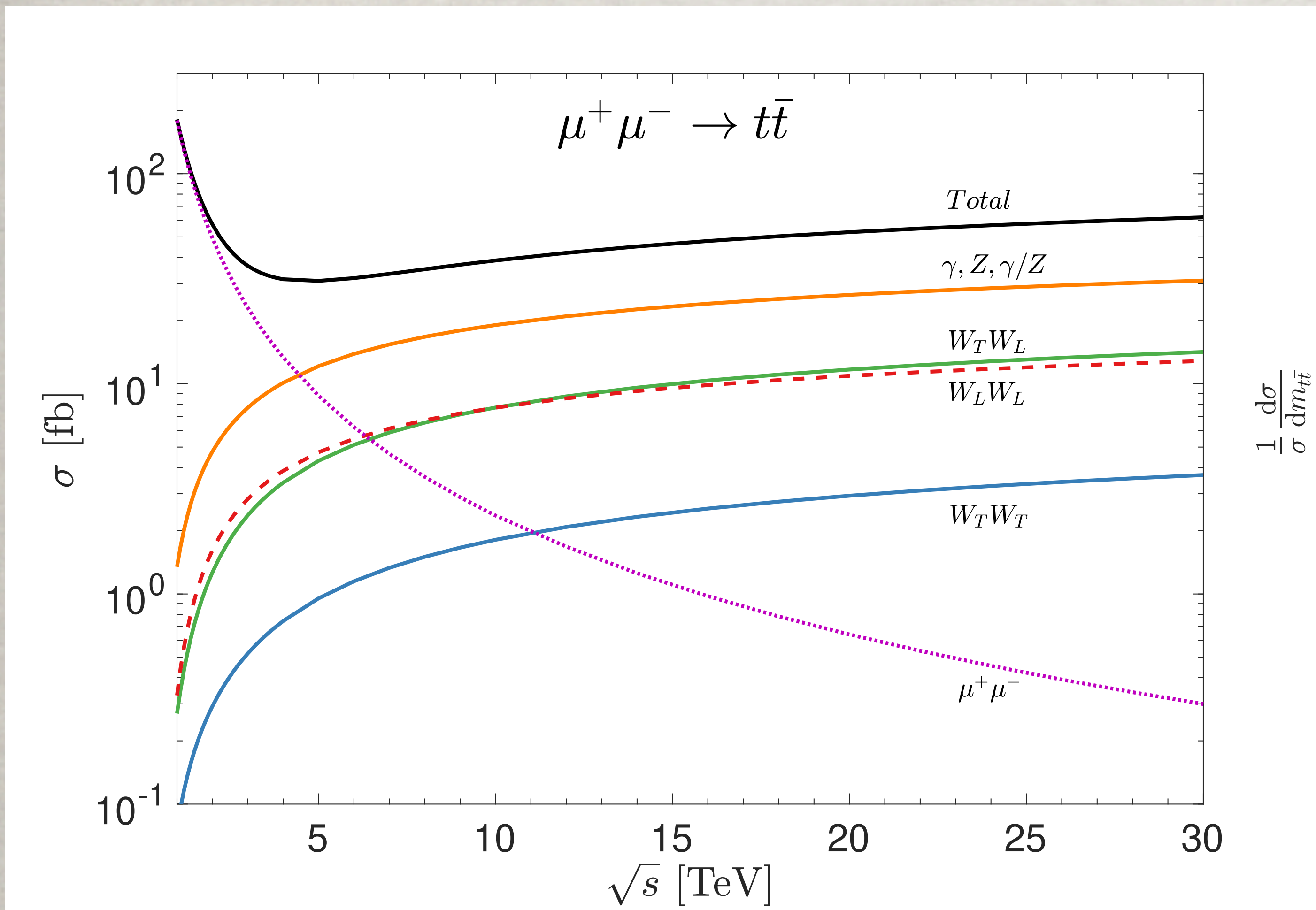
$\mu^+\mu^-$ → exclusive particles + remnants

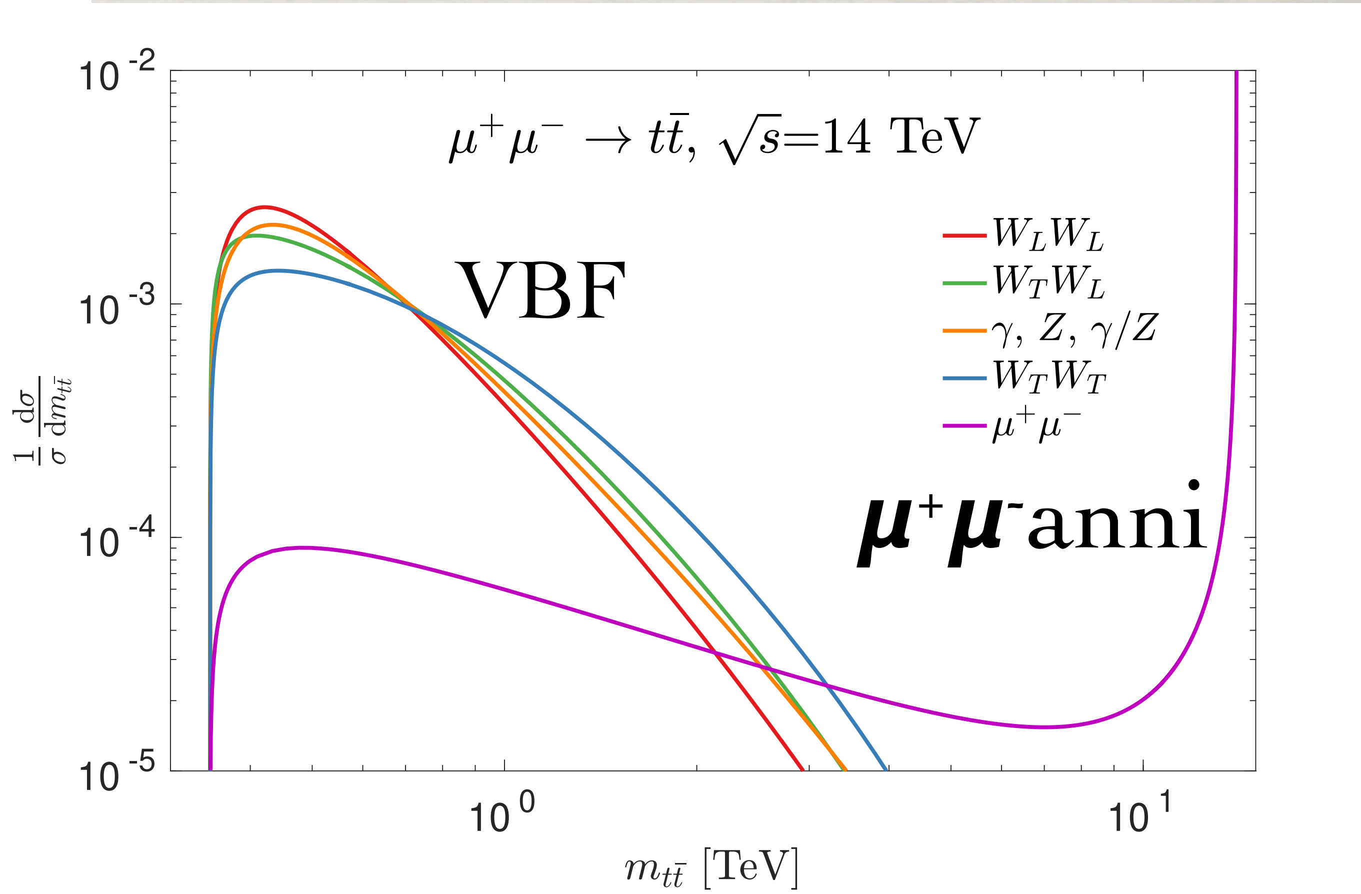


VBF

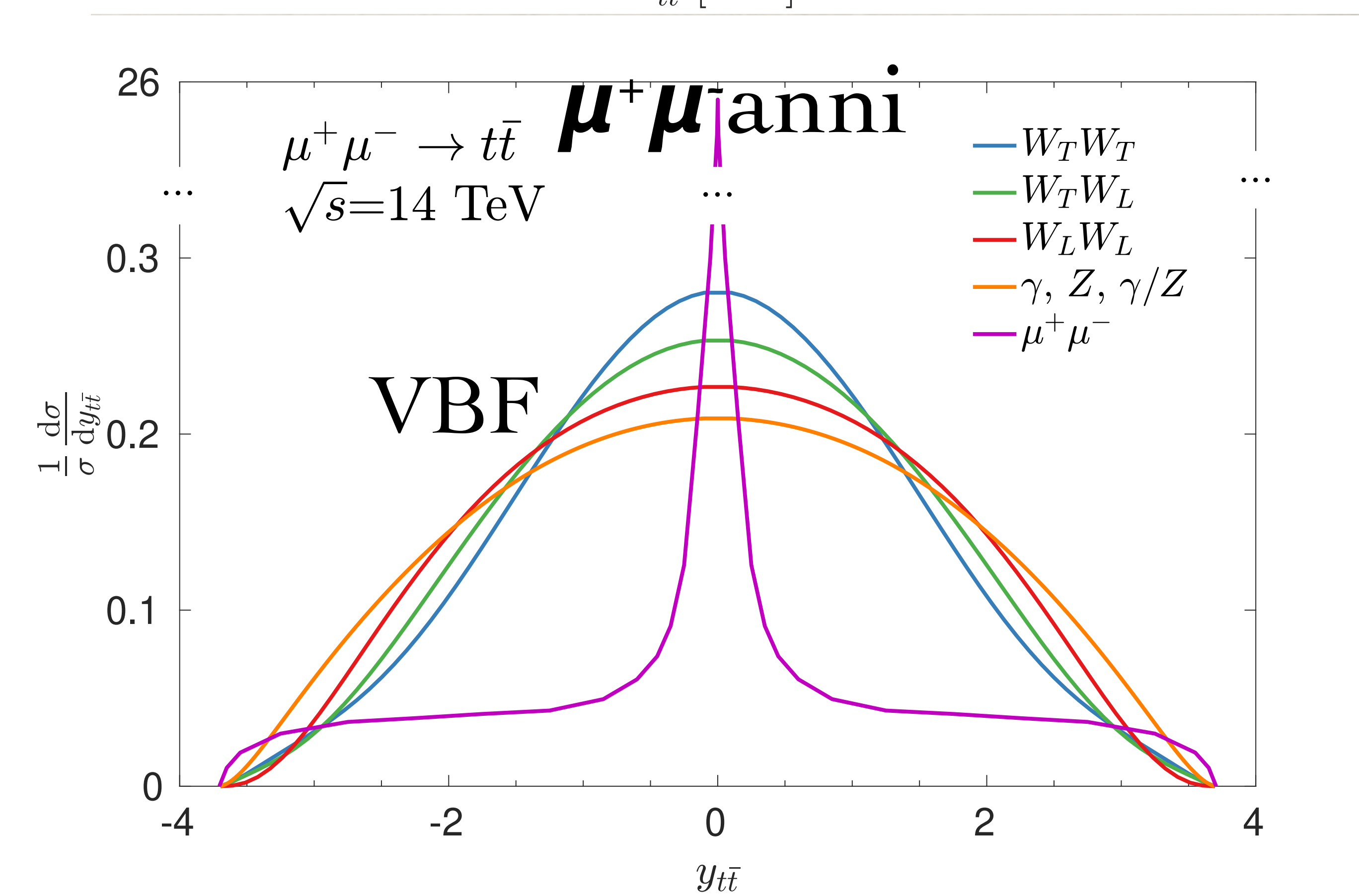
$\mu^+\mu^-$ annihilations

- # Underlying sub-processes:





Partonic contributions

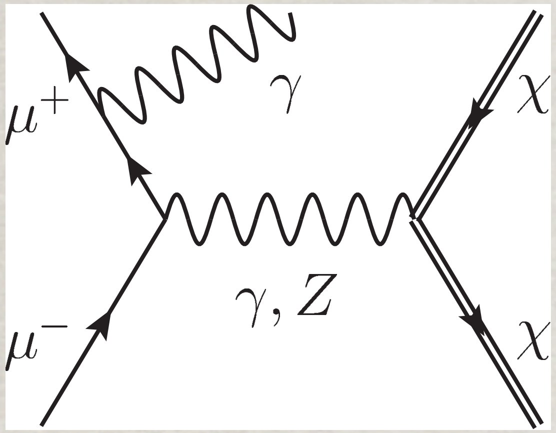


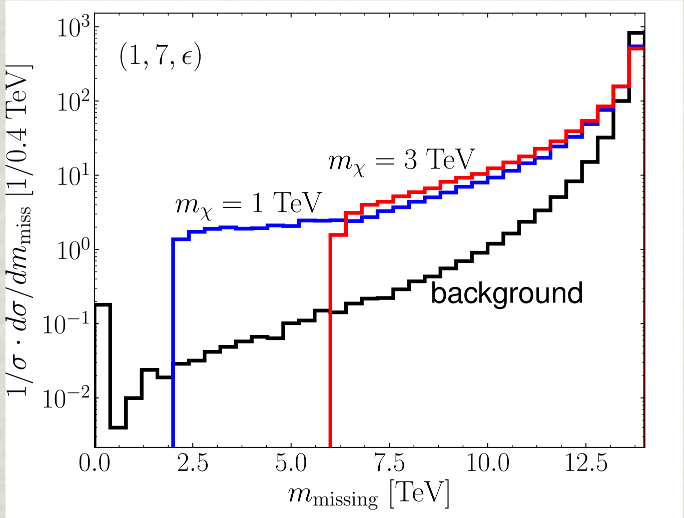
$\mu^+\mu^-$ Collider:
"Buy one, get one free"
Annihilation +VBF

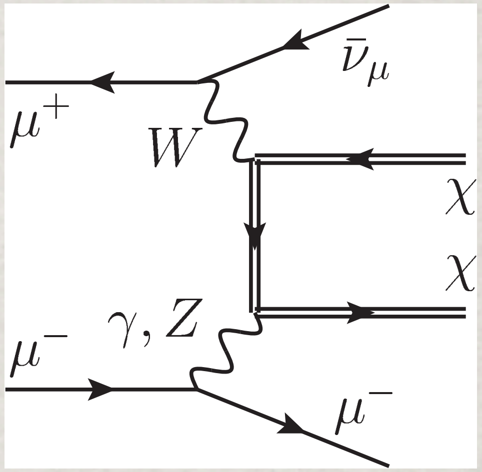
• Unique kinematic features:

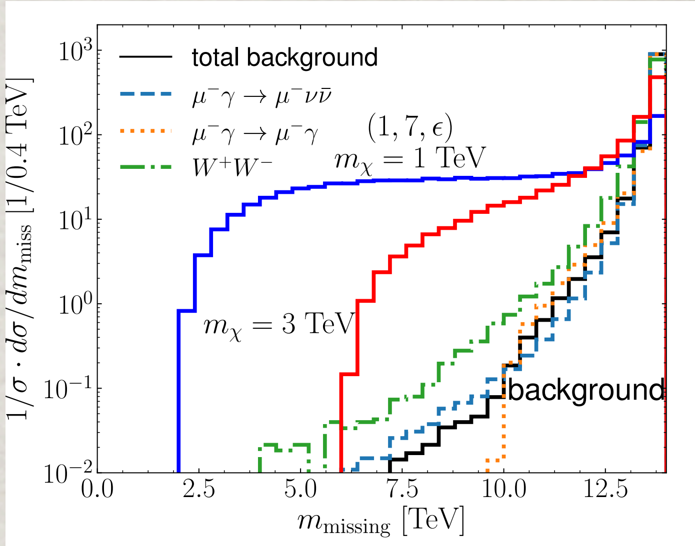
- "Recoil mass" → "missing mass": $m^2_{\text{missing}} \equiv (p_{\mu^+} + p_{\mu^-} - \sum_i p_i^{\text{obs}})^2$

$m^2_{\text{missing}} \equiv (p_{\mu^+} + p_{\mu^-} - p_\gamma)^2 > 4m_\chi^2$ $\qquad$ $m^2_{\text{missing}} = (p^{\text{in}}_{\mu^+} + p^{\text{in}}_{\mu^-} - p^{\text{out}}_{\mu^\pm})^2 > 4m_\chi^2.$



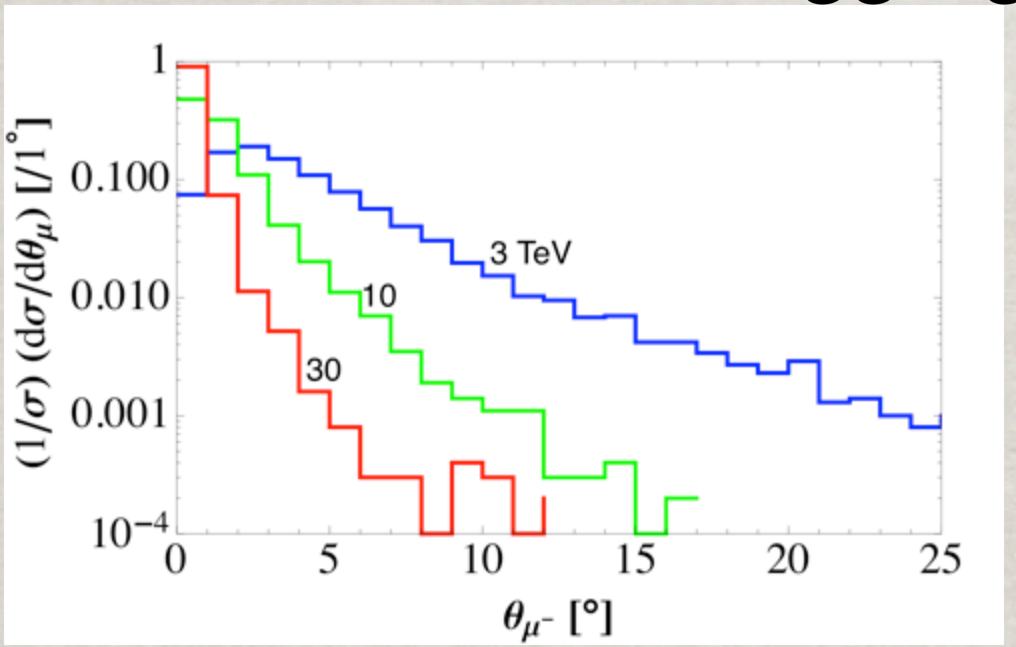


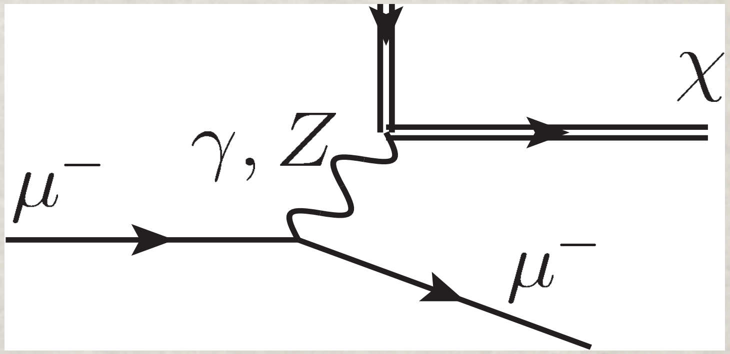




Unavailable in hadronic collisions!

- Forward tagging:





$\theta_\mu \approx M_Z/E_\mu \quad \theta_\mu \sim 0.02 \approx 1.2^\circ$ at 10 TeV.

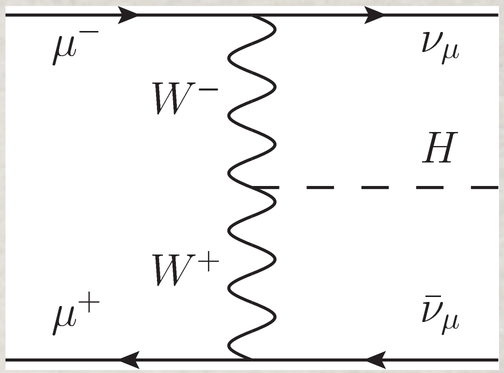
Tagging is costly: forward detector ?

TH, Z. Liu, L.T. Wang, X. Wang: arXiv:2009.11287

TH, D. Liu, I. Low, X. Wang, arXiv:2008.12204

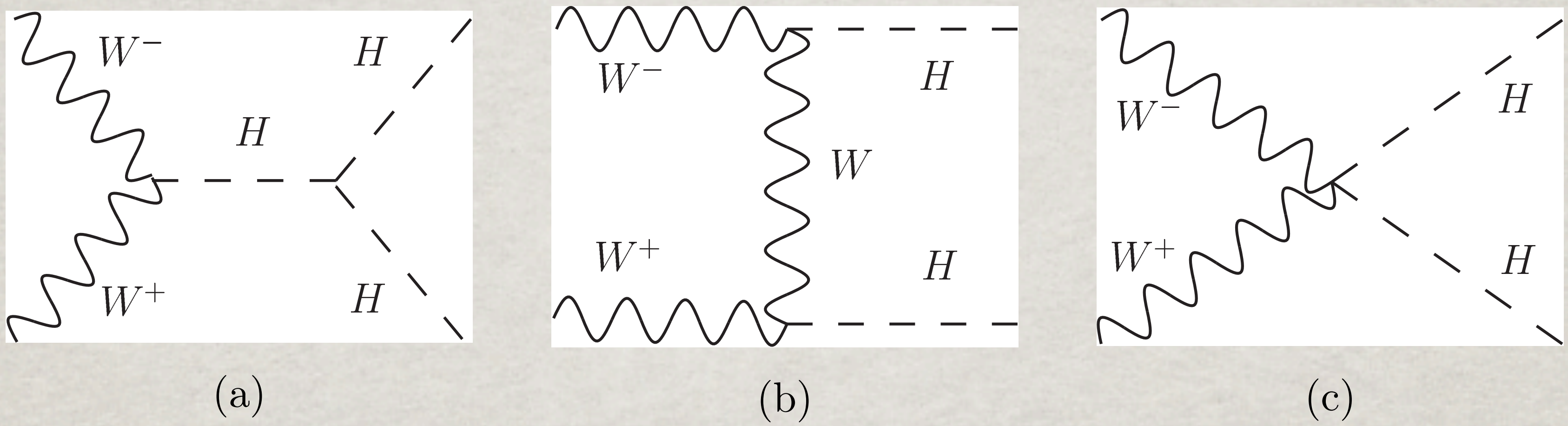
• Precision Higgs Physics

$$\mu^+\mu^- \to \nu_\mu \bar{\nu}_\mu \, H \qquad (WW \text{ fusion}),$$
$$\mu^+\mu^- \to \mu^+\mu^- \, H \qquad (ZZ \text{ fusion}).$$



WWH / ZZH couplings

HHH / WWHH couplings:



$\sqrt{s}$ (TeV)	3	6	10	14	30
benchmark lumi (ab^{-1})	1	4	10	20	90
σ (fb): $WW \to H$	490	700	830	950	1200
$ZZ \to H$	51	72	89	96	120
$WW \to HH$	0.80	1.8	3.2	4.3	6.7
$ZZ \to HH$	0.11	0.24	0.43	0.57	0.91
$WW \to ZH$	9.5	22	33	42	67
$WW \to t\bar{t}H$	0.012	0.046	0.090	0.14	0.28
$WW \to Z$	2200	3100	3600	4200	5200
$WW \to ZZ$	57	130	200	260	420

10M H

500k HH

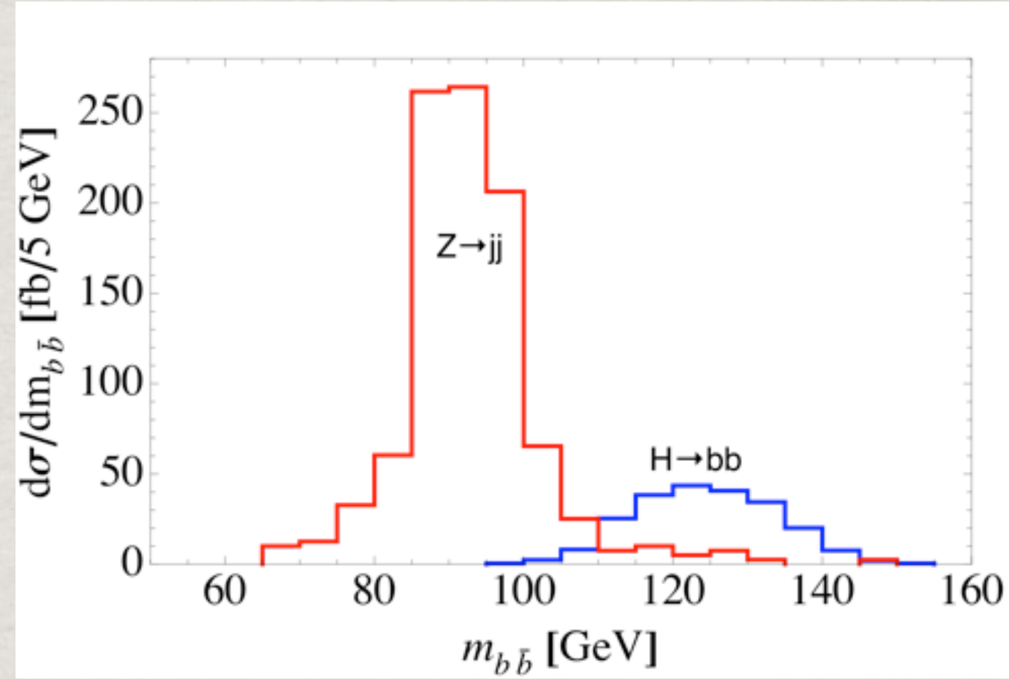
TH, D. Liu, I. Low,
X. Wang, arXiv:2008.12204

Achievable accuracies

Leading channel H → bb:

$$\Delta E/E = 10\%.$$

$$10^\circ < \theta_{\mu^\pm} < 170^\circ.$$



$$\mathcal{L} \supset \left(M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu\right)\left(\kappa_V \frac{2H}{v} + \kappa_{V_2}\frac{H^2}{v^2}\right) - \frac{m_H^2}{2v}\left(\kappa_3 H^3 + \frac{1}{4v}\kappa_4 H^4\right)$$

$\sqrt{s}$ (lumi.)	3 TeV (1 ab^{-1})	6 (4)	10 (10)	14 (20)	30 (90)	Comparison
WWH ($\Delta\kappa_W$)	0.26%	0.12%	0.073%	0.050%	0.023%	0.1% [41]
$\Lambda/\sqrt{c_i}$ (TeV)	4.7	7.0	9.0	11	16	(68% C.L.)
ZZH ($\Delta\kappa_Z$)	1.4%	0.89%	0.61%	0.46%	0.21%	0.13% [17]
$\Lambda/\sqrt{c_i}$ (TeV)	2.1	2.6	3.2	3.6	5.3	(95% C.L.)
$WWHH$ ($\Delta\kappa_{W_2}$)	5.3%	1.3%	0.62%	0.41%	0.20%	5% [36]
$\Lambda/\sqrt{c_i}$ (TeV)	1.1	2.1	3.1	3.8	5.5	(68% C.L.)
HHH ($\Delta\kappa_3$)	25%	10%	5.6%	3.9%	2.0%	5% [22, 23]
$\Lambda/\sqrt{c_i}$ (TeV)	0.49	0.77	1.0	1.2	1.7	(68% C.L.)

Table 7: Summary table of the expected accuracies at 95% C.L. for the Higgs couplings at a variety of muon collider collider energies and luminosities.

 TH, D. Liu, I. Low, X. Wang, arXiv:2008.12204

• WIMP Dark Matter (a conservative SUSY scenario)

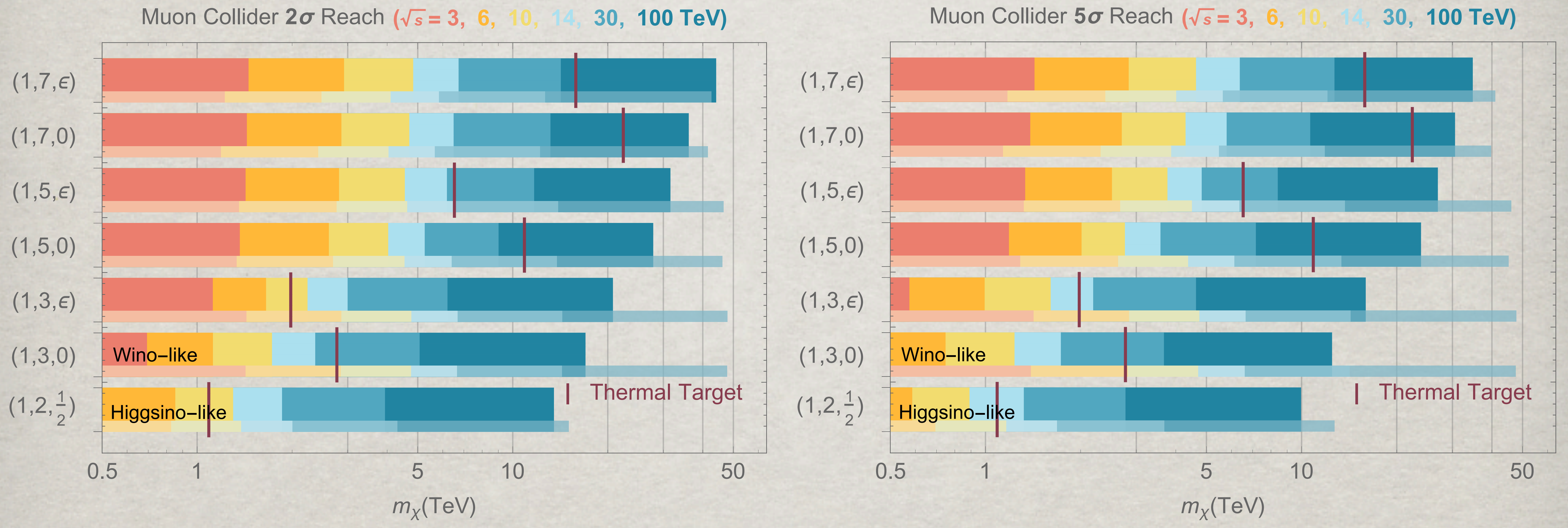
Consider the "minimal EW dark matter": an EW multi-plet

- The lightest neutral component as DM
- Interactions well defined → pure gauge
- Mass upper limit predicted → thermal relic abundance

Model (color, n, Y)		Therm. target
(1,2,1/2)	Dirac	1.1 TeV
(1,3,0)	Majorana	2.8 TeV
(1,3,ϵ)	Dirac	2.0 TeV
(1,5,0)	Majorana	14 TeV
(1,5,ϵ)	Dirac	6.6 TeV
(1,7,0)	Majorana	23 TeV
(1,7,ϵ)	Dirac	16 TeV

Cirelli, Fornengo and Strumia: hep-ph/0512090, 0903.3381;
TH, Z. Liu, L.T. Wang, X. Wang: arXiv:2009.11287

The mass reach for minimal WIMP DM:



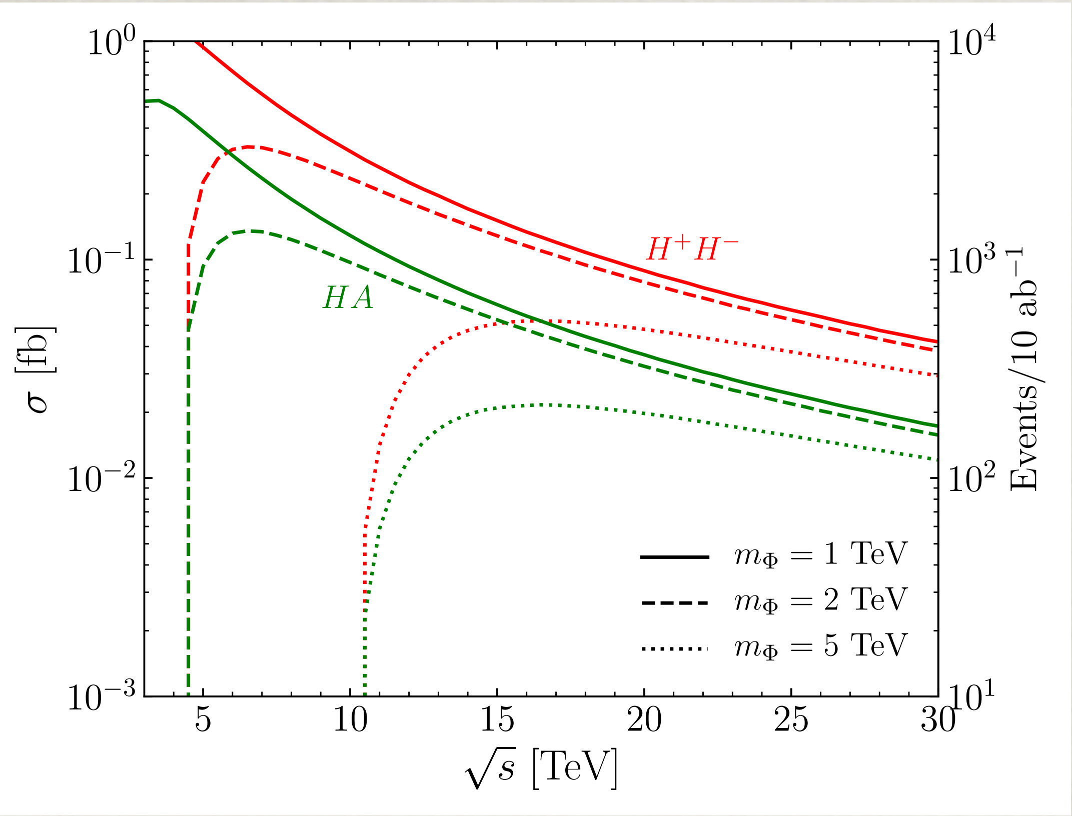
E_{CM} ≈ 14 TeV enough to cover n≤3 multiplets.

Higher energy needed to cover higher multiplets.

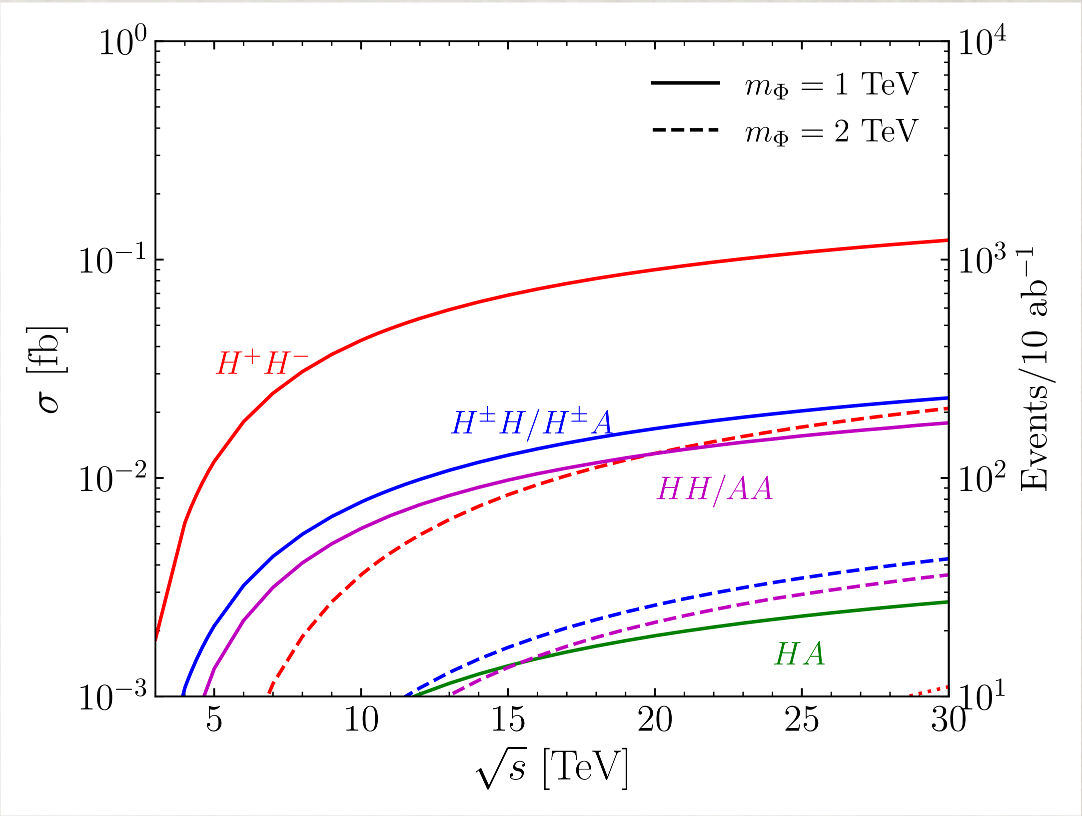
TH, Z. Liu, L.T. Wang, X. Wang: arXiv:2009.11287

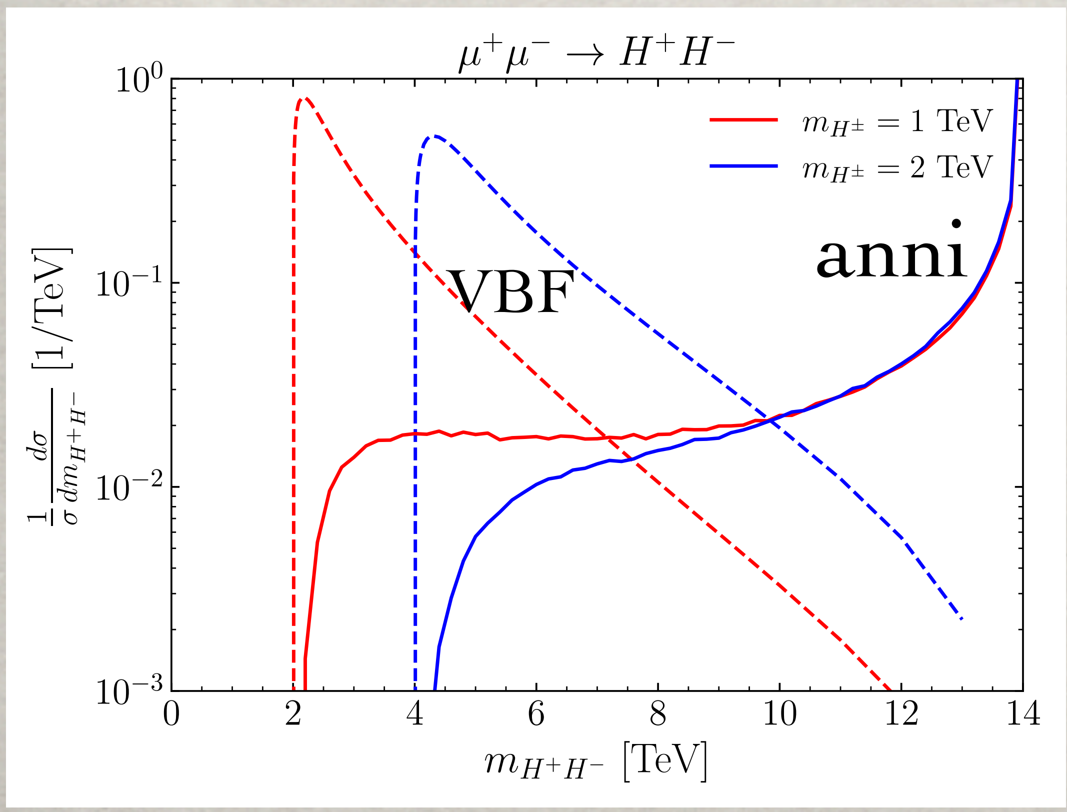
• Heavy Higgs Bosons Production

annihilation



VBF

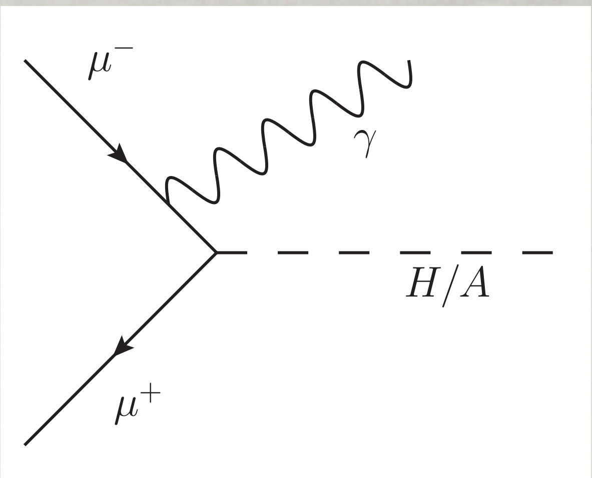


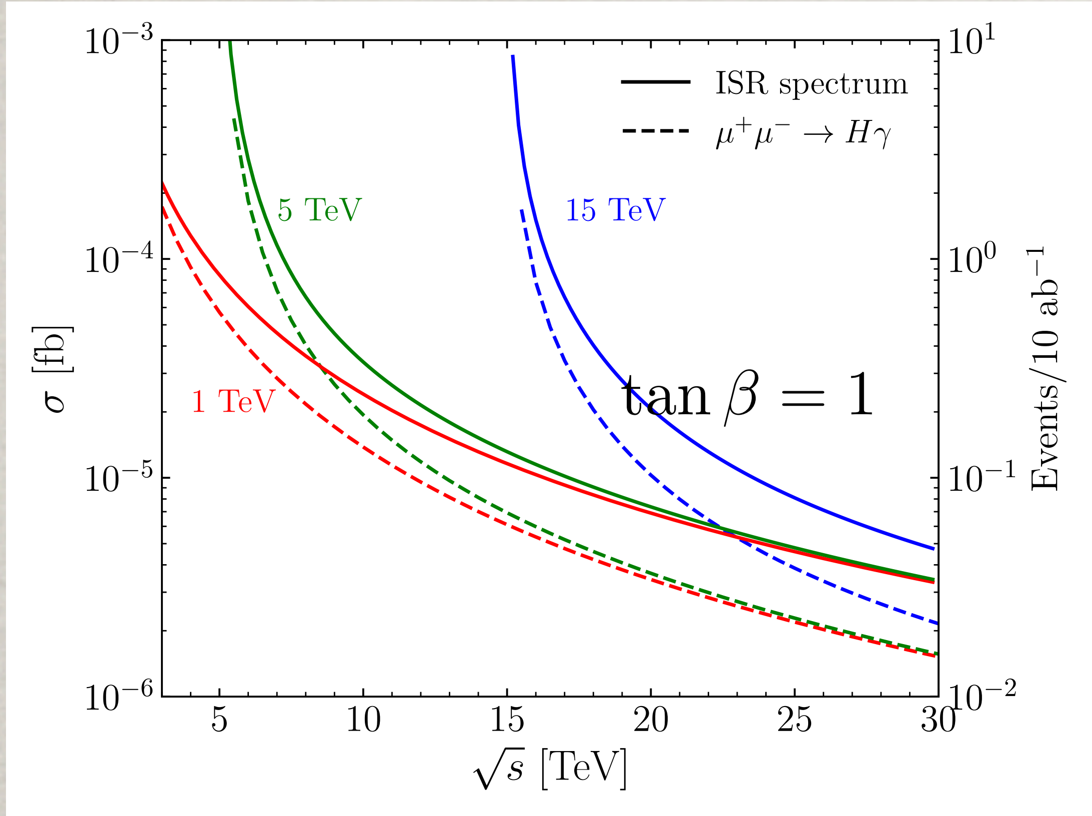


	production	Type-I	Type-II	Type-F	Type-L
small $\tan\beta < 5$	H^+H^-	$t\bar{b}, \bar{t}b$			
	$HA/HH/AA$	$t\bar{t}, t\bar{t}$			
	$H^\pm H/A$	$tb, t\bar{t}$			
intermediate $\tan\beta$	H^+H^-	$t\bar{b}, \bar{t}b$			$tb, \tau\nu_\tau$
	$HA/HH/AA$	$t\bar{t}, t\bar{t}$	$t\bar{t}, b\bar{b}$		$t\bar{t}, \tau^+\tau^-$
	$H^\pm H/A$	$tb, t\bar{t}$	$tb, t\bar{t}$; $tb, b\bar{b}$		$tb, t\bar{t}$; $tb, \tau^+\tau^-$; $\tau\nu_\tau, t\bar{t}$; $\tau\nu_\tau, \tau^+\tau^-$
large $\tan\beta > 10$	H^+H^-	$t\bar{b}, \bar{t}b$	$tb, tb(\tau\nu_\tau)$	$t\bar{b}, \bar{t}b$	$\tau^+\nu_\tau, \tau^-\nu_\tau$
	$HA/HH/AA$	$t\bar{t}, t\bar{t}$	$b\bar{b}, b\bar{b}(\tau^+\tau^-)$	$b\bar{b}, b\bar{b}$	$\tau^+\tau^-, \tau^+\tau^-$
	$H^\pm H/A$	$tb, t\bar{t}$	$tb(\tau\nu_\tau), b\bar{b}(\tau^+\tau^-)$	$tb, b\bar{b}$	$\tau^\pm\nu_\tau, \tau^+\tau^-$

TH, S. Li, S. Su, W. Su, Y. Wu, arXiv:2102.08386.

Radiative returns:

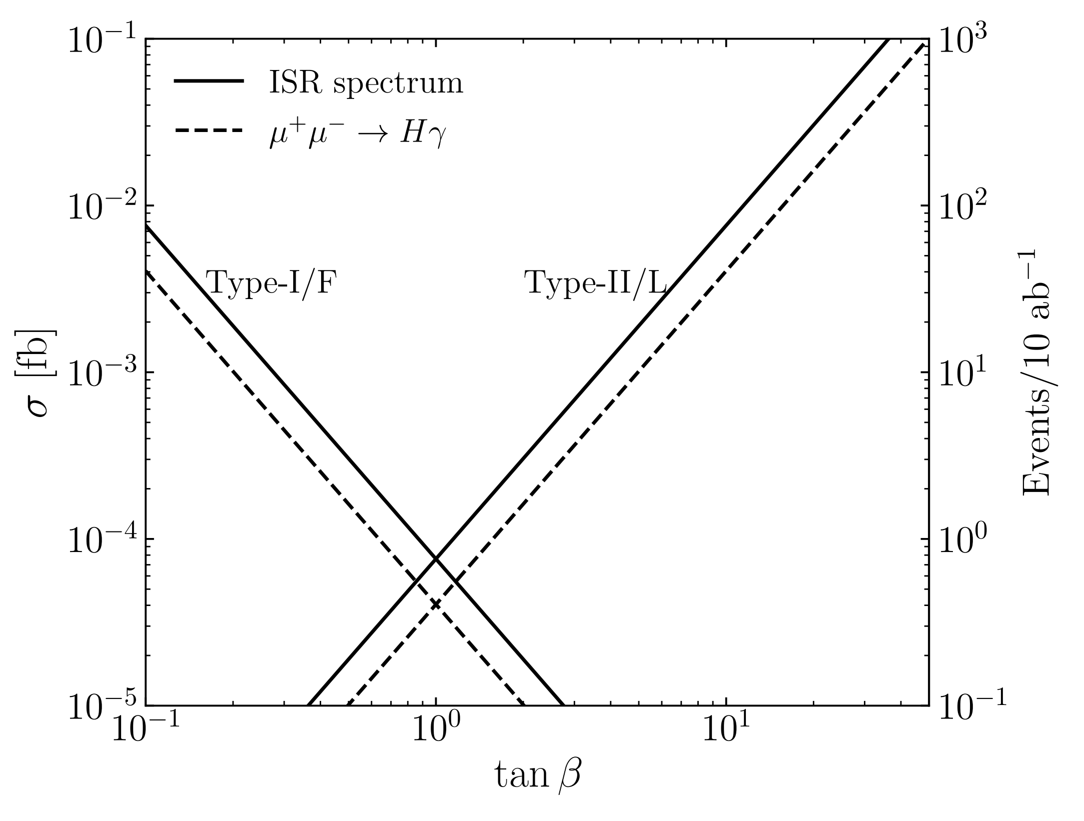




$$\hat{\sigma}(\mu^+\mu^- \to H) = \frac{\pi Y_\mu^2}{4}\delta(\hat{s} - m_H^2) = \frac{\pi Y_\mu^2}{4s}\delta(\tau - \frac{m_H^2}{s})$$

$$f_{\ell/\ell}(x) = \frac{\alpha}{2\pi}\frac{1+x^2}{1-x}\log\frac{s}{m_\mu^2}$$

$$\sigma = 2\int dx_1 f_{\ell/\ell}(x_1)\hat{\sigma}(\tau = x_1) = \frac{\alpha Y_\mu^2}{4s}\frac{s + m_H^4/s}{s - m_H^2}\log\frac{s}{m_\mu^2}$$



Depending on the coupling,

$M_H \sim E_{cm}$

TH, S. Li, S. Su, W. Su, Y. Wu, arXiv:2102.08386;
TH, Z. Liu et al., arXiv:1408.5912.

Lots of recent works!
-- my apologies not to cover properly

D. Buttazzo, D. Redogolo, F. Sala, arXiv:1807.04743 (VBF to Higgs)
A. Costantini, F. Maltoni, et al., arXiv:2005.10289 (VBF to NP)
M. Chiesa, F. Maltoni, L. Mantani, B. Mele, F. Piccinini, and X. Zhao, arXiv:2005.10289 (SM Higgs)
R. Capdevilla, D. Curtin, Y. Kahn, G. Krnjaic, arXiv:2006.16277; arXiv:2101.10334 (g-2, flavor)
P. Bandyopadhyay, A. Costantini et al., arXiv:2010.02597 (Higgs)
D. Buttazzo, P. Paradisi, arXiv:2012.02769 (g-2)
W. Yin, M. Yamaguchi, arXiv:2012.03928 (g-2)
R. Capdevilla, F. Meloni, R. Simoniello, and J. Zurita, arXiv:2012.11292 (MD)
D. Buttazzo, F. Franceschini, A. Wulzer, arXiv:2012.11555 (general)
G.-Y. Huang, F. Queiroz, W. Rodejohann, arXiv:2101.04956; arXiv:2103.01617 (flavor)
W. Liu, K.-P. Xie, arXiv:2101.10469 (EWPT)
H. Ali, N. Arkani-Hamed, et al, arXiv:2103.14043 (Muon Smasher's Guide)
Richard Ruiz et al., arXiv:2111.02442 (MadGraph5)
… …

Anything for us to do?

- **Accelerator:**
  Need high-field magnets
  High-energy proton source? **μ**-storage? **ν**-flux?
  New ideas for muon-cooling?
- **Detector:**
  Beam-induced background suppression (BIB)?
- **Physics:**
  EW PDFs; EW fragmentation functions …
  MC simulation / event generator
  New physics coverage; muon-flavor specific?

Summary

- **High energy muon-collider is a new endeavor:**

Challenging technology; interdisciplinary to other fields; great physics potential!

- **s-channel Higgs factory:**
  - Direct measurements on Y_μ & Γ_H
  - Other BRs comparable to e^+e^- Higgs factories

- **Multi-TeV colliders:**
  - Unprecedented accuracies for WWH, WWHH, H^3, H^4
  - Bread & butter SM EW physics in the new territory
  - New particle (Q,H…) mass coverage $M_H \sim (0.5 – 1)E_{cm}$
  - Decisive coverage for minimal WIMP DM $M \sim 0.5\ E_{cm}$
  - Complementary to Astro/Cosmo/GW & to FCC-hh:

Exciting journey ahead!