

A Lattice QCD study of the low-energy interactions of doubly charmed baryons

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Outline

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- Doubly charmed baryons
- Simulation in Lattice QCD

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- Ensembles
- Distillation quark smearing

Results

- Energy levels
- Scattering lengths

Summary and Outlook

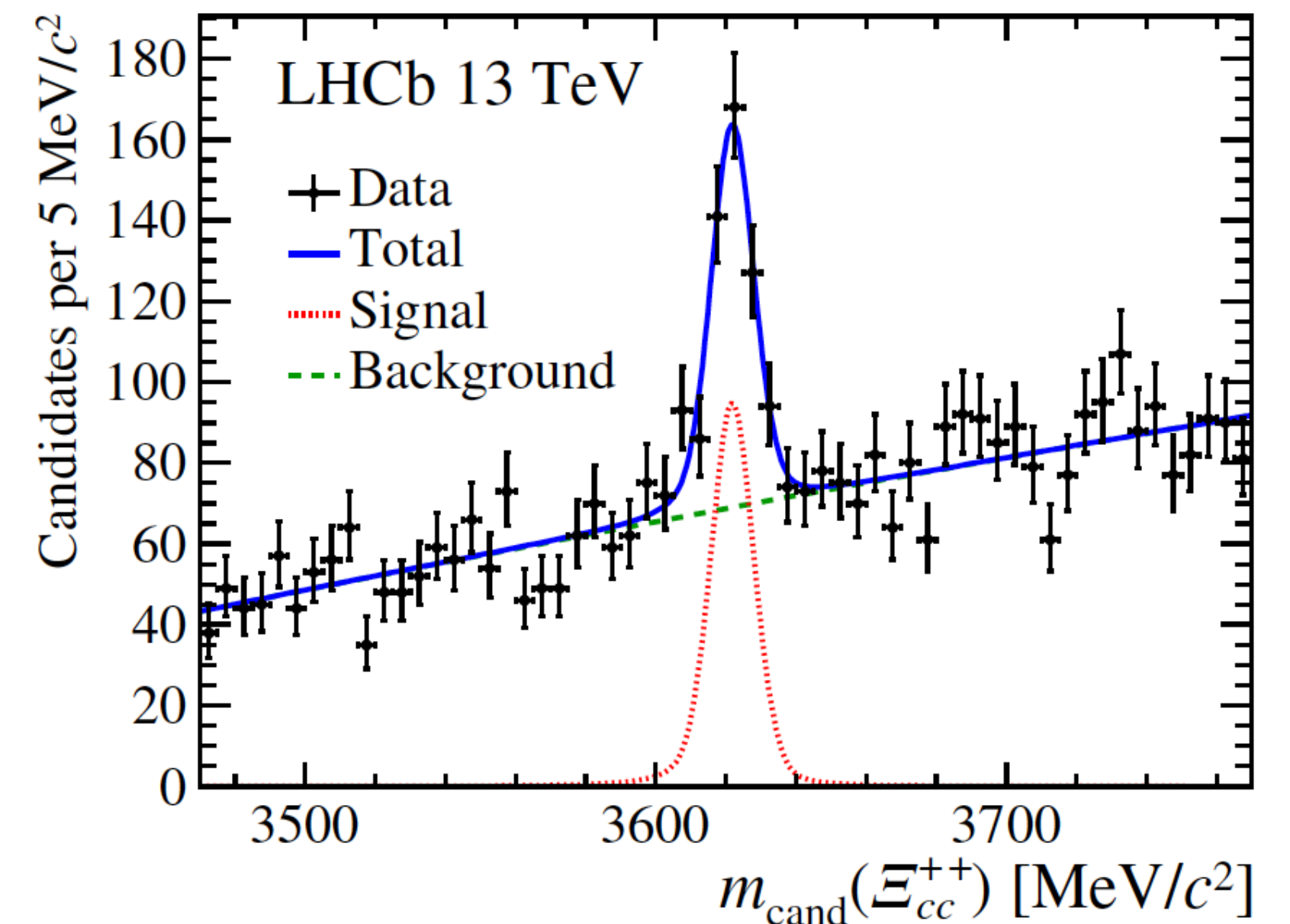
Doubly charmed baryons

Observation of doubly charmed baryons

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- **FOCUS** S. P. Ratti, Nucl. Phys. B Proc. Suppl. 115, 33 (2003).
- **Belle** R. Chistov and others, Phys. Rev. Lett. 97, 162001 (2006).
- **BABAR** B. Aubert and others, Phys. Rev. D 74, 011103 (2006).
- **LHCb** R. Aaij and others, Phys. Rev. Lett. 119, 112001 (2017).
- Searching for Ξ_{cc}^+ , Ω_{cc}^+ , ...

Theoretical studies

- Quark model
 - Q.-F. Lü, K.-L. Wang, L.-Y. Xiao, and X.-H. Zhong, Phys. Rev. D 96, 114006 (2017).
 - W. Wang, F.-S. Yu, and Z.-X. Zhao, Eur. Phys. J. C 77, 781 (2017).
 - H.-W. Ke, F. Lu, X.-H. Liu, and X.-Q. Li, Eur. Phys. J. C 80, 140 (2020).
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Doubly charmed baryons

Theoretical studies

- Lattice QCD
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 - Z. S. Brown, W. Detmold, S. Meinel, and K. Orginos, Phys. Rev. D 90, 094507 (2014).
 - N. Mathur and M. Padmanath, Phys. Rev. D 99, 031501 (2019).
 - H. Bahtiyar, K. U. Can, G. Erkol, P. Gubler, M. Oka, and T. T. Takahashi, Phys. Rev. D 102, 054513 (2020).
- QCD sum rules
 - H.-X. Chen, Q. Mao, W. Chen, X. Liu, and S.-L. Zhu, Phys. Rev. D 96, 031501 (2017).
 - Z.-G. Wang, Eur. Phys. J. C 78, 826 (2018).
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 - Y.-J. Shi, W. Wang, and Z.-X. Zhao, Eur. Phys. J. C 80, 568 (2020).
- Chiral perturbation theory
 - Z.-H. Guo, Phys. Rev. D 96, 074004 (2017).
 - Z.-F. Sun and M. J. Vicente Vacas, Phys. Rev. D 93, 094002 (2016).
 - A. N. Hiller Blin, Z.-F. Sun, and M. J. Vicente Vacas, Phys. Rev. D 98, 054025 (2018).
 - M.-J. Yan, X.-H. Liu, S. González-Solís, F.-K. Guo, C. Hanhart, U.-G. Meißner, and B.-S. Zou, Phys. Rev. D 98, 091502 (2018).
 - M.-Z. Liu, Y. Xiao, and L.-S. Geng, Phys. Rev. D 98, 014040 (2018).
 - D.-L. Yao, Phys. Rev. D 97, 034012 (2018).
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Doubly charmed baryons

Baryon chiral perturbation theory (BChPT)

$$\begin{aligned}\mathcal{L}_{\psi\phi}^{(1)} &= \bar{\psi} (i\not{D} - m) \psi + \frac{g}{2} \bar{\psi} \not{\psi} \gamma_5 \psi, \\ \mathcal{L}_{\psi\phi}^{(2)} &= b_1 \bar{\psi} \langle \chi_+ \rangle \psi + b_2 \bar{\psi} \tilde{\chi}_+ \psi + b_3 \bar{\psi} u^2 \psi + b_4 \bar{\psi} \langle u^2 \rangle \psi + \frac{b_5}{m^2} \bar{\psi} (\{u^\mu, u^\nu\} D_{\mu\nu} + H.c.) \psi \\ &\quad + \frac{b_6}{m^2} \bar{\psi} (\langle u^\mu u^\nu \rangle D_{\mu\nu} + H.c.) \psi + i b_7 \bar{\psi} [u^\mu, u^\nu] \sigma_{\mu\nu} \psi, \\ \mathcal{L}_{\psi\phi}^{(3)} &= i c_{11} \bar{\psi} [u_\mu, h^{\mu\nu}] \gamma_\nu \psi + \frac{c_{12}}{m^2} \bar{\psi} (i [u^\mu, h^{\nu\rho}] \gamma_\mu D_{\nu\rho} + H.c.) \psi + \frac{c_{13}}{m} \bar{\psi} (i \{u^\mu, h^{\nu\rho}\} \\ &\quad \times \sigma_{\mu\nu} D_\rho + H.c.) \psi + \frac{c_{14}}{m} \bar{\psi} (i \sigma_{\mu\nu} \langle u^\mu h^{\nu\rho} \rangle D_\rho + H.c.) \psi + c_{15} \bar{\psi} \{u^\mu, \tilde{\chi}_+\} \gamma_5 \gamma_\mu \psi \\ &\quad + c_{16} \bar{\psi} u^\mu \gamma_5 \gamma_\mu \langle \chi_+ \rangle \psi + c_{17} \bar{\psi} \gamma_5 \gamma_\mu \langle u^\mu \tilde{\chi}_+ \rangle \psi + i c_{18} \bar{\psi} \gamma_5 \gamma_\mu [D^\mu, \tilde{\chi}_-] \psi \\ &\quad + i c_{19} \bar{\psi} \gamma_5 \gamma_\mu \langle [D^\mu, \chi_-] \rangle \psi + c_{20} \bar{\psi} [\tilde{\chi}_-, u^\mu] \gamma_\mu \psi.\end{aligned}$$

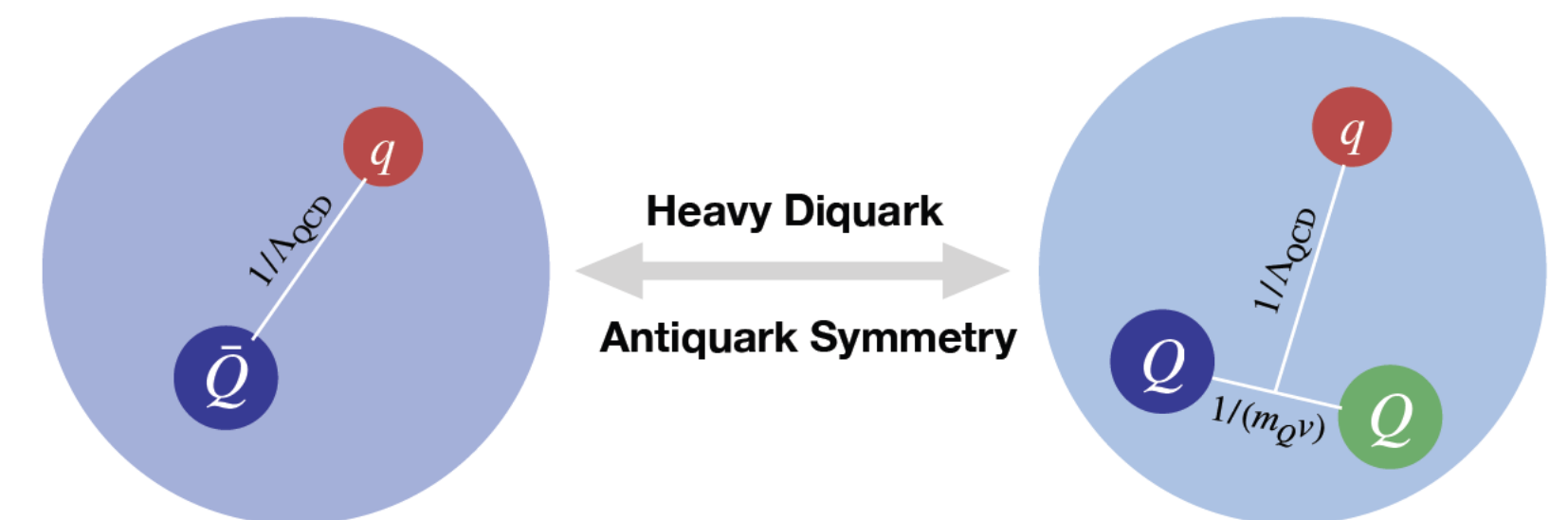


Unknown Low Energy Constants

- Experimental data
- Lattice QCD data
- Symmetries
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Heavy diquark-antiquark symmetry

- M. J. Savage and M. B. Wise, Phys. Lett. B 248, 177 (1990).
- M.-J. Yan, X.-H. Liu, S. González-Solís, F.-K. Guo, C. Hanhart, U.-G. Meißner, and B.-S. Zou, Phys. Rev. D 98, 091502 (2018).
- L. Meng and S.-L. Zhu, Phys. Rev. D 100, 014006 (2019).
- Z.-R. Liang, P.-C. Qiu, and D.-L. Yao, JHEP 07, 124 (2023).
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Doubly charmed baryons

Scattering lengths in BChPT Z.-R. Liang, P.-C. Qiu, and D.-L. Yao, JHEP 07, 124 (2023).

$$a_{0+}^{(S,I)} = \frac{m_\psi}{4\pi(m_\psi + m_\phi)} \left\{ [A^{(S,I)}(s,0)]_{\mathbf{q}^2=0} + m_\phi [B^{(S,I)}(s,0)]_{\mathbf{q}^2=0} \right\}$$

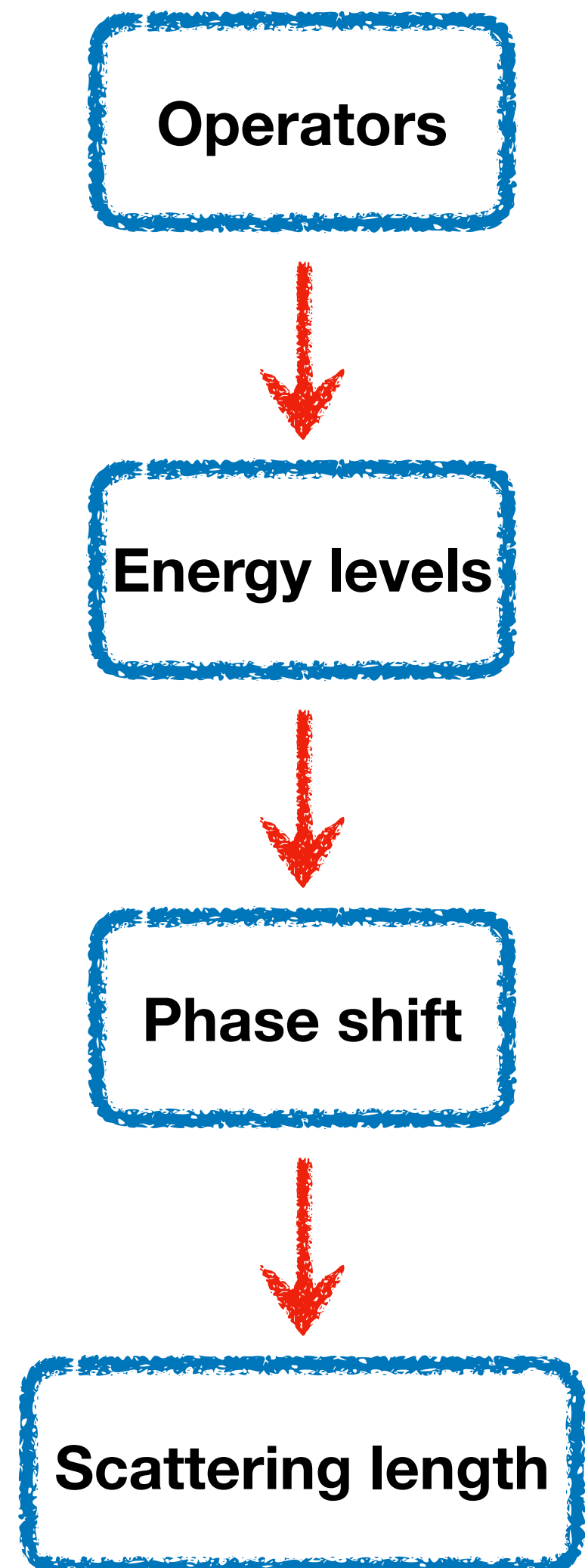
$$a_{1+}^{(S,I)} = \frac{m_\psi}{6\pi(m_\psi + m_\phi)} \left\{ [\partial_t A^{(S,I)}(s,t)]_{t=0, \mathbf{q}^2=0} + m_\phi [\partial_t B^{(S,I)}(s,t)]_{t=0, \mathbf{q}^2=0} \right\}$$

$$a_{1-}^{(S,I)} = a_{1+}^{(S,I)} - \frac{1}{16\pi m_\psi(m_\psi + m_\phi)} \left\{ [A^{(S,I)}(s,0)]_{\mathbf{q}^2=0} - (2m_\psi + m_\phi) [B^{(S,I)}(s,0)]_{\mathbf{q}^2=0} \right\}$$

Scattering lengths in Lattice QCD

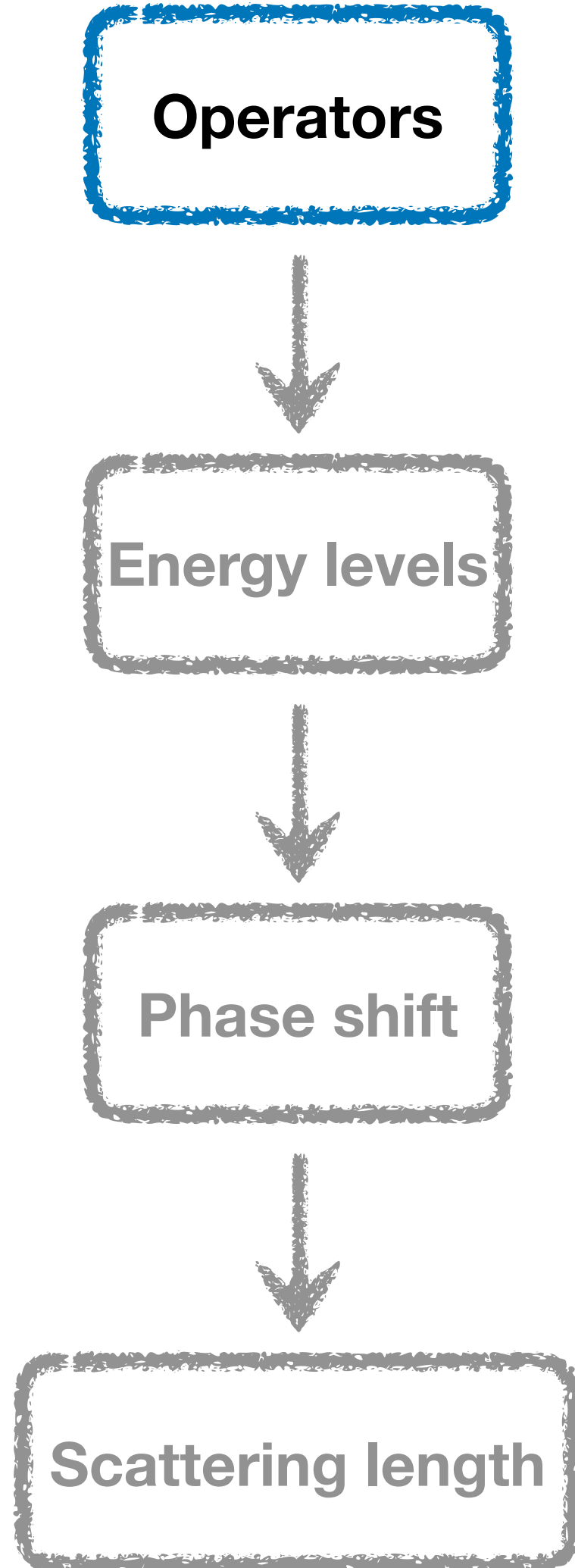
- Liu et al., “Interactions of Charmed Mesons with Light Pseudoscalar Mesons from Lattice QCD and Implications on the Nature of the $D_{s0}^*(2317)$.”
- Lyu et al., “Doubly Charmed Tetraquark T_{cc}^+ from Lattice QCD near Physical Point.”
- Xing et al., “First Observation of the Hidden-Charm Pentaquarks on Lattice.”
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Determination of LECs on Lattice



Simulation in Lattice QCD

Determination of LECs on Lattice



Goldstone boson operators

$$\mathcal{O}_M(x) \equiv \bar{\psi}^{f_1}(x) \Gamma \psi^{f_2}(x)$$

$$\mathcal{O}_{\pi^+}(x) = \bar{d}(x)_\alpha^a (\gamma_5)_{\alpha\beta} u(x)_\beta^a$$

$$\mathcal{O}_{K^+}(x) = \bar{s}(x)_\alpha^a (\gamma_5)_{\alpha\beta} u(x)_\beta^a$$

$$\mathcal{O}_{\pi^0}(x) = \frac{1}{\sqrt{2}} \left(\bar{u}(x)_\alpha^a (\gamma_5)_{\alpha\beta} u(x)_\beta^a - \bar{d}(x)_\alpha^a (\gamma_5)_{\alpha\beta} d(x)_\beta^a \right)$$

$$\mathcal{O}_{K^0}(x) = \bar{s}(x)_\alpha^a (\gamma_5)_{\alpha\beta} d(x)_\beta^a$$

$$\mathcal{O}_\eta(x) = \frac{1}{\sqrt{2}} \left(\bar{u}(x)_\alpha^a (\gamma_5)_{\alpha\beta} u(x)_\beta^a + \bar{d}(x)_\alpha^a (\gamma_5)_{\alpha\beta} d(x)_\beta^a \right)$$

Doubly Charmed Baryon operators

$$\mathcal{O}_\gamma^B(x) = \epsilon^{ijk} P_\pm \left[\psi_\alpha^{f_1,i}(x) (C\gamma_5)_{\alpha\beta} \psi_\beta^{f_2,j}(x) \right] \psi_\gamma^{f_3,k}(x)$$

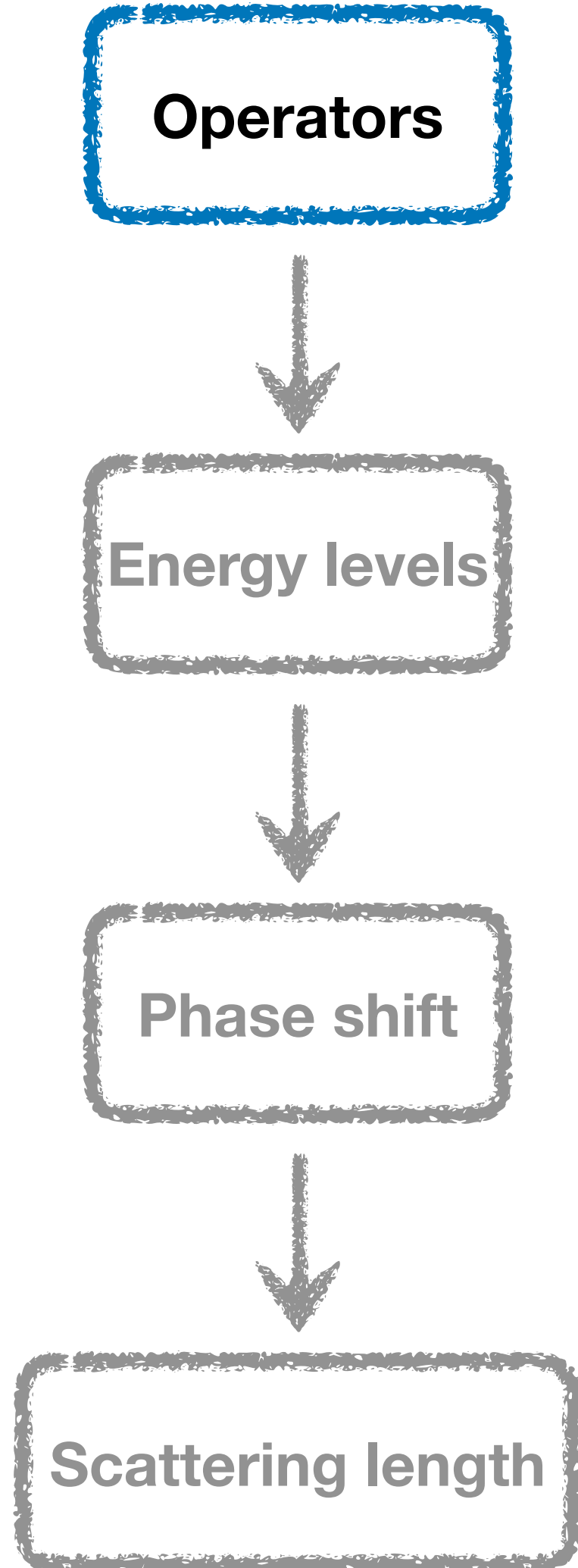
$$\mathcal{O}_{\Xi_{cc}^{++}(ccu)}(x) = \epsilon^{ijk} P_+ \left[Q_c^{iT}(x) C\gamma_5 q_u^j(x) \right] Q_c^k(x)$$

$$\mathcal{O}_{\Xi_{cc}^+(ccd)}(x) = \epsilon^{ijk} P_+ \left[Q_c^{iT}(x) C\gamma_5 q_d^j(x) \right] Q_c^k(x)$$

$$\mathcal{O}_{\Omega_{cc}^+(ccs)}(x) = \epsilon^{ijk} P_+ \left[Q_c^{iT}(x) C\gamma_5 q_s^j(x) \right] Q_c^k(x)$$

Simulation in Lattice QCD

Determination of LECs on Lattice



Two-particle operators

$$\mathcal{O}_{\Xi_{cc}K}^{I=0}(t) = \frac{1}{\sqrt{2}}\mathcal{O}_{\Xi_{cc}^{++}}(t)\mathcal{O}_{K^0}(t) - \frac{1}{\sqrt{2}}\mathcal{O}_{\Xi_{cc}^+}(t)\mathcal{O}_{K^+}(t)$$

$$\mathcal{O}_{\Xi_{cc}K}^{I=1}(t) = \mathcal{O}_{\Xi_{cc}^{++}}(t)\mathcal{O}_{K^+}(t) \quad \mathcal{O}_{\Xi_{cc}\pi}^{I=3/2}(t) = \mathcal{O}_{\Xi_{cc}^{++}}(t)\mathcal{O}_{\pi^+}(t)$$

$$\mathcal{O}_{\Omega_{cc}\bar{K}}^{I=1/2}(t) = \mathcal{O}_{\Omega_{cc}^+}(t)\mathcal{O}_{\bar{K}^0}(t)$$

For s-wave scattering, $J^P = \frac{1}{2}^- \longrightarrow G_1^-$

$$\mathcal{O}_{\mathbf{p}_1, \mathbf{p}_2}^{\Xi_{cc}K, I=0} = \sum_{\alpha, \mathbf{p}_1, \mathbf{p}_2} C_{\alpha, \mathbf{p}_1, \mathbf{p}_2} \left(\frac{1}{\sqrt{2}}\mathcal{O}_{\Xi_{cc}^{++}, \alpha}(\mathbf{p}_1)\mathcal{O}_{K^0}(\mathbf{p}_2) - \frac{1}{\sqrt{2}}\mathcal{O}_{\Xi_{cc}^+, \alpha}(\mathbf{p}_1)\mathcal{O}_{K^+}(\mathbf{p}_2) \right)$$

$$\mathcal{O}_{\mathbf{p}_1, \mathbf{p}_2}^{\Xi_{cc}K, I=1} = \sum_{\alpha, \mathbf{p}_1, \mathbf{p}_2} C_{\alpha, \mathbf{p}_1, \mathbf{p}_2} \left(\mathcal{O}_{\Xi_{cc}^{++}, \alpha}(\mathbf{p}_1)\mathcal{O}_{K^+}(\mathbf{p}_2) \right)$$

$$\mathcal{O}_{\mathbf{p}_1, \mathbf{p}_2}^{\Omega_{cc}\bar{K}, I=1/2} = \sum_{\alpha, \mathbf{p}_1, \mathbf{p}_2} C_{\alpha, \mathbf{p}_1, \mathbf{p}_2} \left(\mathcal{O}_{\Omega_{cc}^+, \alpha}(\mathbf{p}_1)\mathcal{O}_{\bar{K}^0}(\mathbf{p}_2) \right)$$

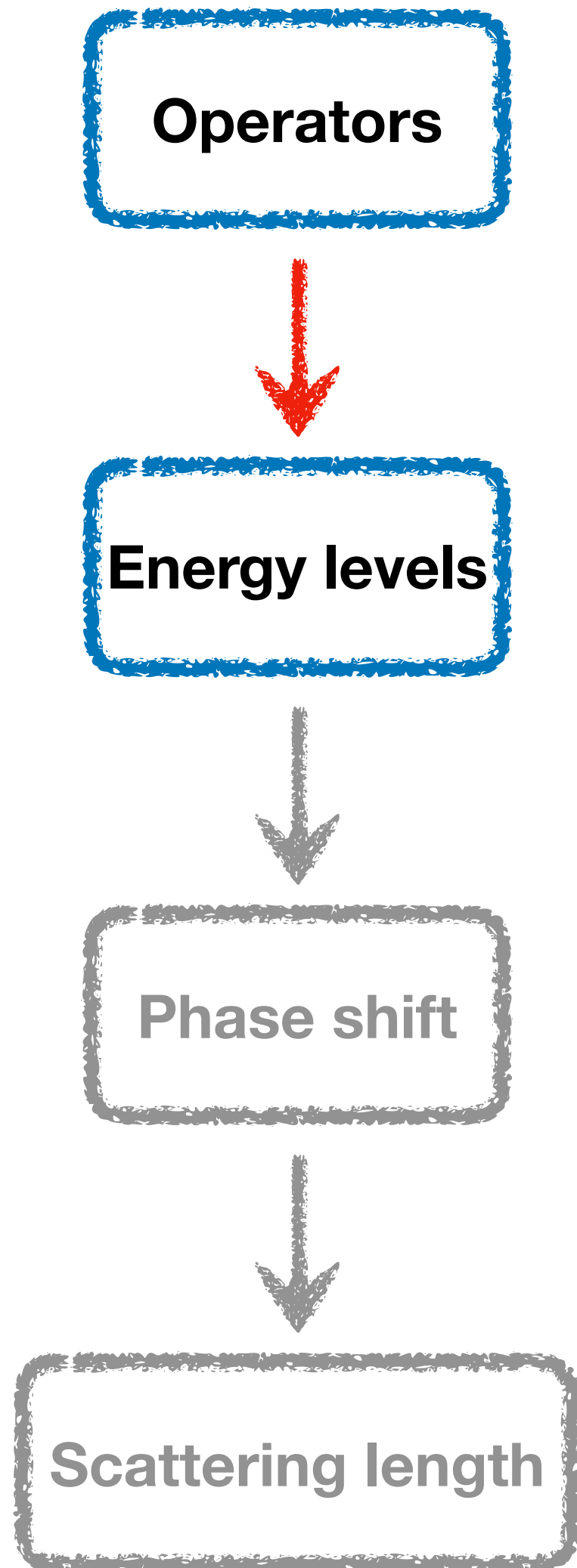
$$\mathcal{O}_{\mathbf{p}_1, \mathbf{p}_2}^{\Xi_{cc}\pi, I=3/2} = \sum_{\alpha, \mathbf{p}_1, \mathbf{p}_2} C_{\alpha, \mathbf{p}_1, \mathbf{p}_2} \left(\mathcal{O}_{\Xi_{cc}^{++}, \alpha}(\mathbf{p}_1)\mathcal{O}_{\pi^+}(\mathbf{p}_2) \right)$$

	α	$\mathbf{p}_1$	$\mathbf{p}_2$	$C_{\alpha, \mathbf{p}_1, \mathbf{p}_2}$
$ \mathbf{p}_{1,2} = 0$	1	(0,0,0)	(0,0,0)	1
$ \mathbf{p}_{1,2} = 1$	1	(-1,0,0)	(1,0,0)	1
	1	(1,0,0)	(-1,0,0)	1
	1	(0,-1,0)	(0,1,0)	1
	1	(0,1,0)	(0,-1,0)	1
	1	(0,0,-1)	(0,0,1)	1
	1	(0,0,1)	(0,0,-1)	1
$ \mathbf{p}_{1,2} = \sqrt{2}$	1	(-1,-1,0)	(1,1,0)	1
	1	(1,1,0)	(-1,-1,0)	1
	1	(-1,0,-1)	(1,0,1)	1
	1	(1,0,1)	(-1,0,-1)	1
	1	(0,-1,-1)	(0,1,1)	1
	1	(0,1,1)	(0,-1,-1)	1
	1	(-1,1,0)	(1,-1,0)	1
	1	(1,-1,0)	(-1,1,0)	1
	1	(-1,0,1)	(1,0,-1)	1
	1	(1,0,-1)	(-1,0,1)	1
	1	(0,1,-1)	(0,-1,1)	1
	1	(0,-1,1)	(0,1,-1)	1

H. Xing, J. Liang, L. Liu, P. Sun, and Y.-B. Yang,
arXiv:2210.08555 [hep-lat] (2022).

Simulation in Lattice QCD

Determination of LECs on Lattice



Two point correlation function

$$\begin{aligned}
 & \left\langle \mathcal{O}_M(x) \mathcal{O}_M^\dagger(0) \right\rangle \\
 &= \left\langle \bar{\psi}_\alpha^{f_1,a}(x) \Gamma_{\alpha\beta} \psi_\beta^{f_2,a}(x) \bar{\psi}_{\alpha'}^{f_2,b}(0) \Gamma_{\alpha'\beta'} \psi_{\beta'}^{f_1,b}(0) \right\rangle \\
 &= -\Gamma_{\alpha\beta} \Gamma_{\alpha'\beta'} G_{\beta'\alpha}^{f_1,ba}(0,x) G_{\beta'\alpha'}^{f_2,ab}(x,0) \\
 &= -\text{tr} [\Gamma G^{f_2}(x,0) \Gamma G^{f_1}(0,x)]
 \end{aligned}$$

$$\left\langle \mathcal{O}(\vec{x}, t) \mathcal{O}^\dagger(0) \right\rangle = \sum_{n=0}^{\infty} \frac{\langle 0 | \mathcal{O} | n \rangle \langle n | \mathcal{O}^\dagger | 0 \rangle}{2E_n} e^{-E_n t} = c_0 e^{-E_0 t} + c_1 e^{-E_1 t} + \dots$$

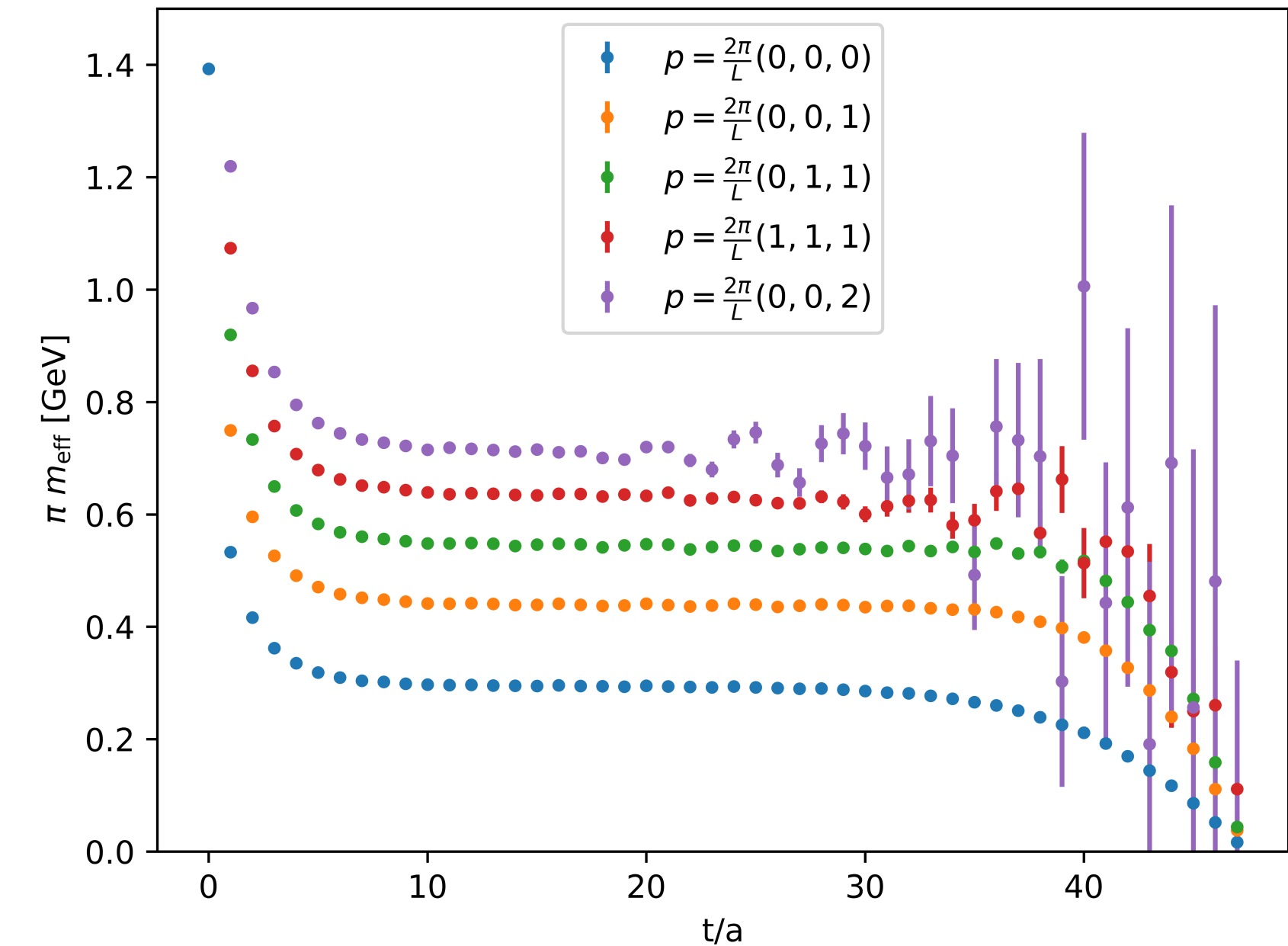
Excited states are suppressed at large t



$$\left\langle \mathcal{O}(\vec{x}, t) \mathcal{O}^\dagger(0) \right\rangle \approx c_0 e^{-E_0 t}$$

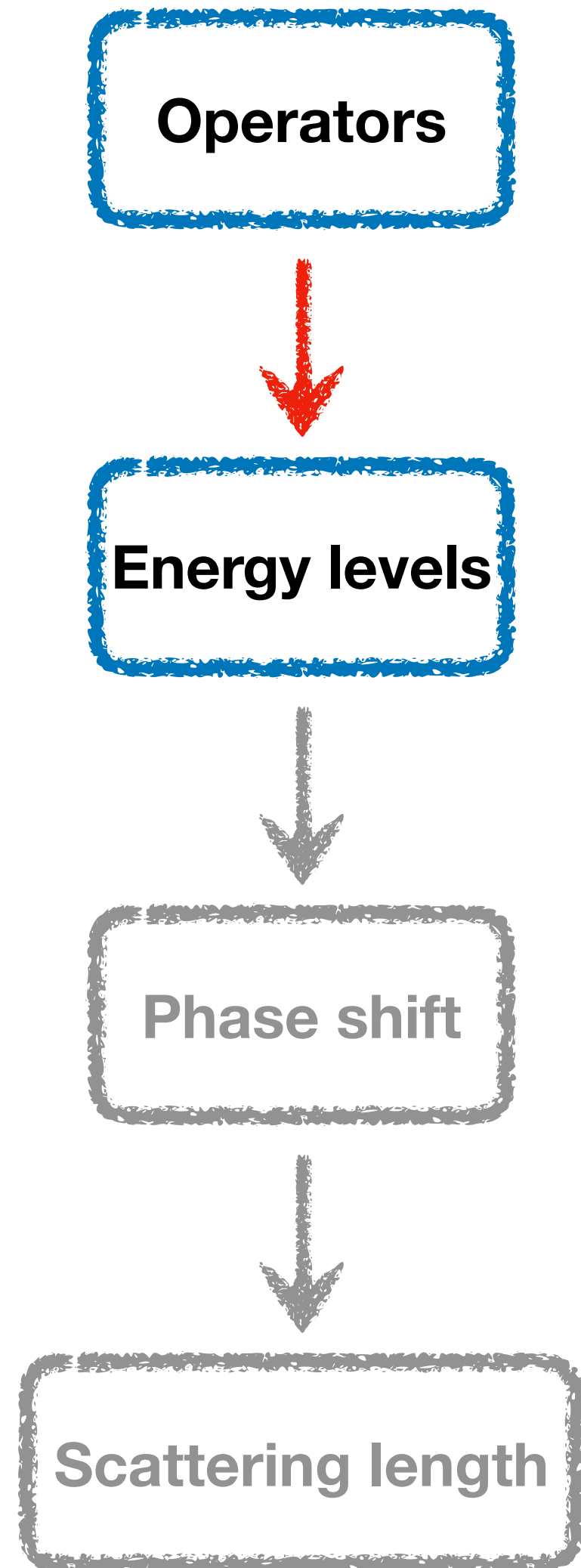
for one state fit

$$c_0 e^{-E_0 t} + c_1 e^{-E_1 t} \quad \text{for two state fit}$$



Simulation in Lattice QCD

Determination of LECs on Lattice



Four point correlation function

- Solve generalized eigenvalue problem (GEVP) M. Luscher and U. Wolff, Nucl. Phys. B 339, 222 (1990).

$$C_{ij}(t) = \sum_{t_{src}} \left\langle \mathcal{O}_i(t + t_{src}) \mathcal{O}_j^\dagger(t_{src}) \right\rangle \quad C(t)v^n(t) = \lambda^n(t)C(t_0)v^n(t)$$

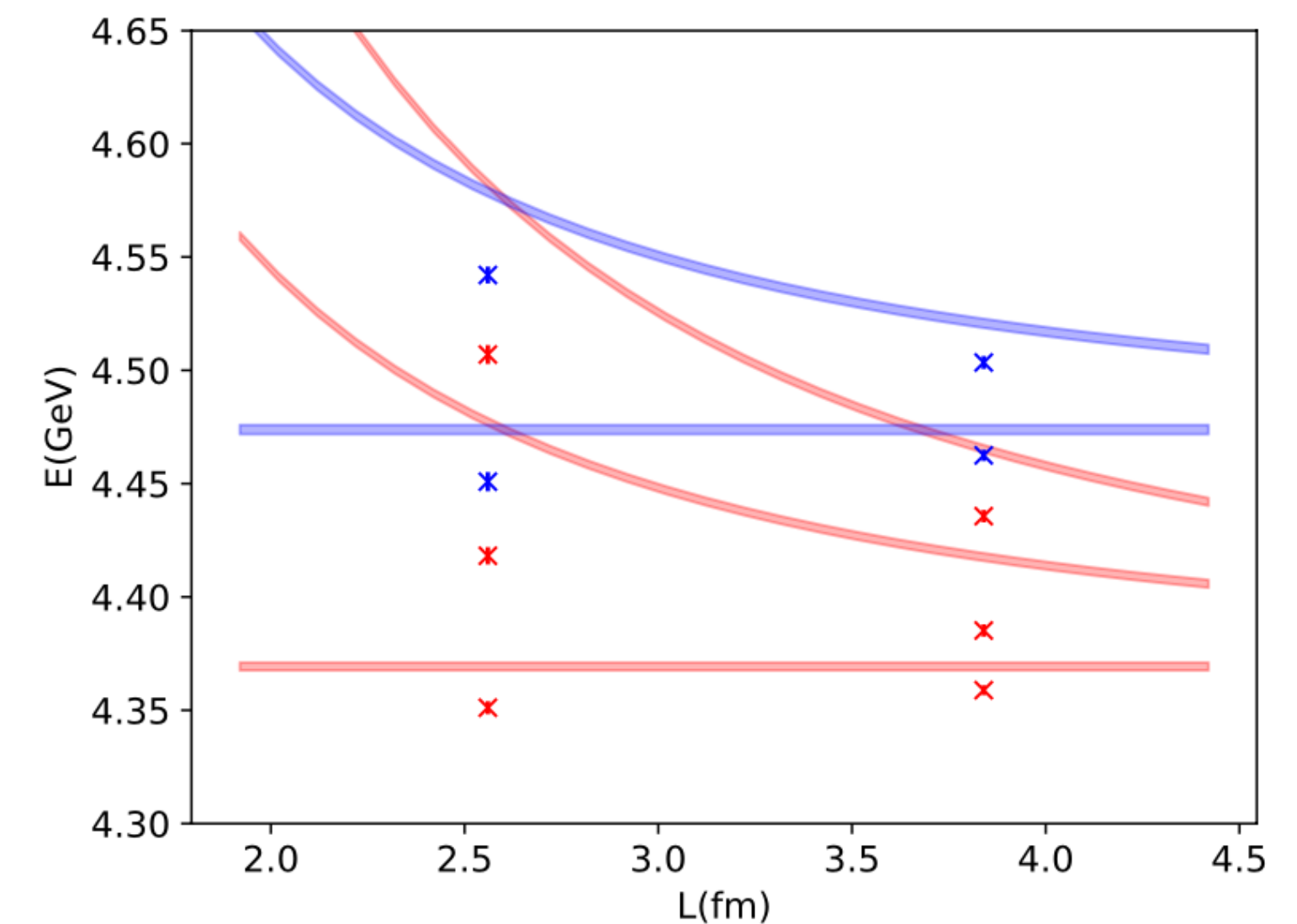
- Extract effective mass by two-exponential fit

$$\lambda^n(t) = (1 - A_n) e^{-E_n(t - t_0)} + A_n e^{-E'_n(t - t_0)}$$

- Compare with non-interacting energy

$$E^{\text{free}} = \sqrt{m_1^2 + \mathbf{p}_1^2} + \sqrt{m_2^2 + \mathbf{p}_2^2}$$

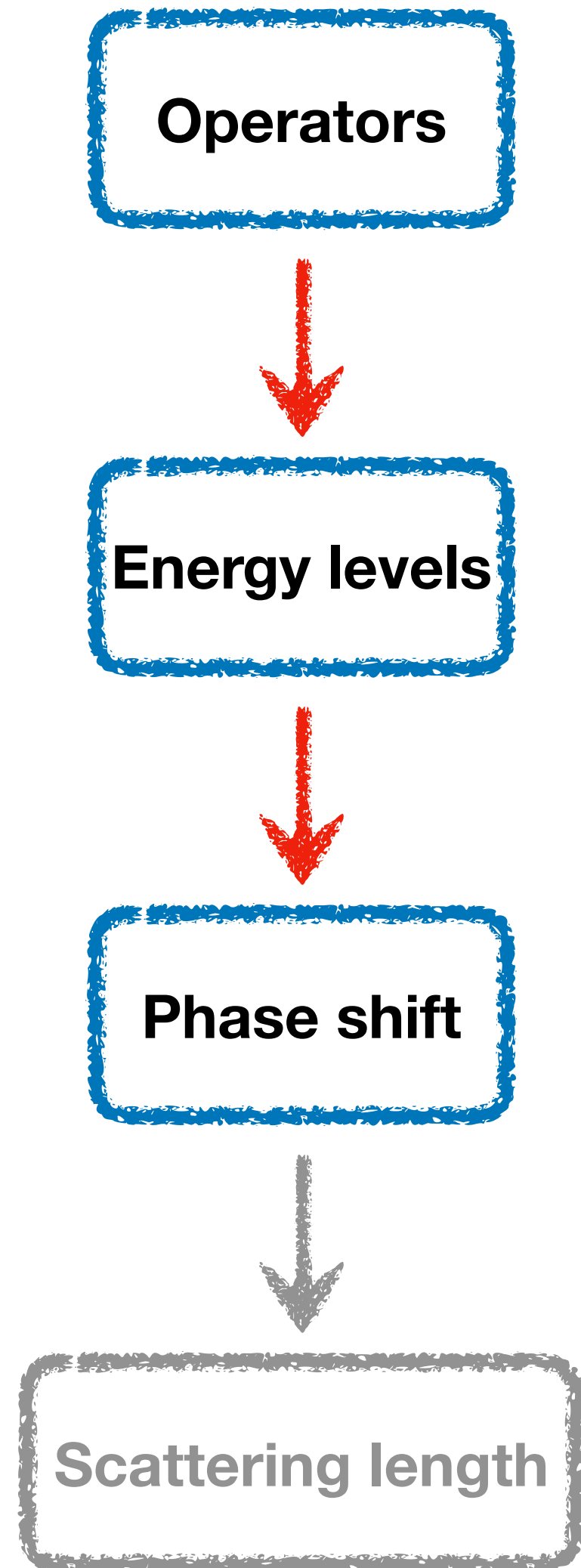
→ Energy shift ΔE



H. Xing, J. Liang, L. Liu, P. Sun, and Y.-B. Yang,
arXiv:2210.08555 [hep-lat] (2022).

Simulation in Lattice QCD

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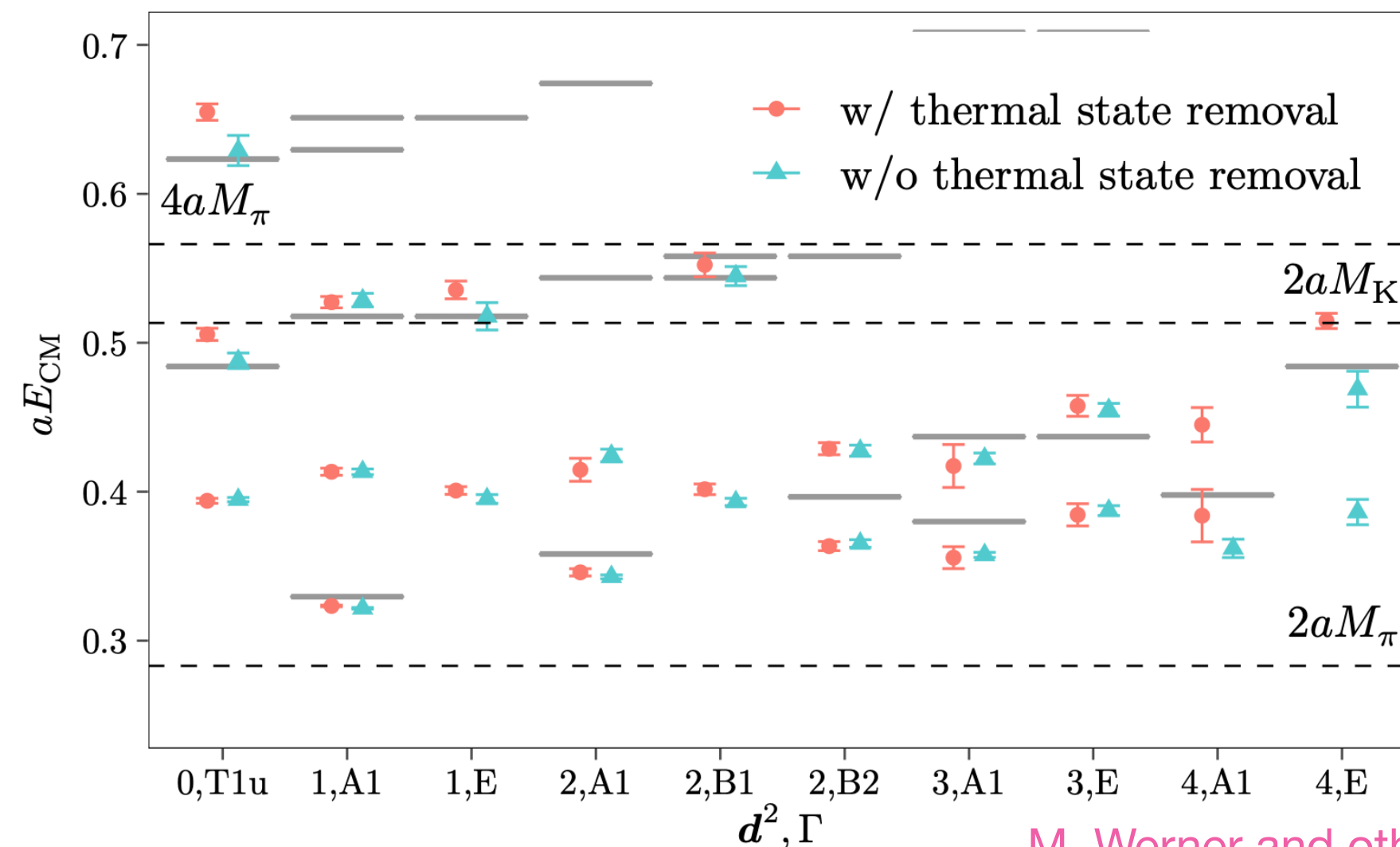
Luscher's formula for two-body scattering M. Luscher, Nucl. Phys. B 354, 531 (1991).

$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (1 + i\mathbf{M})] = 0$$

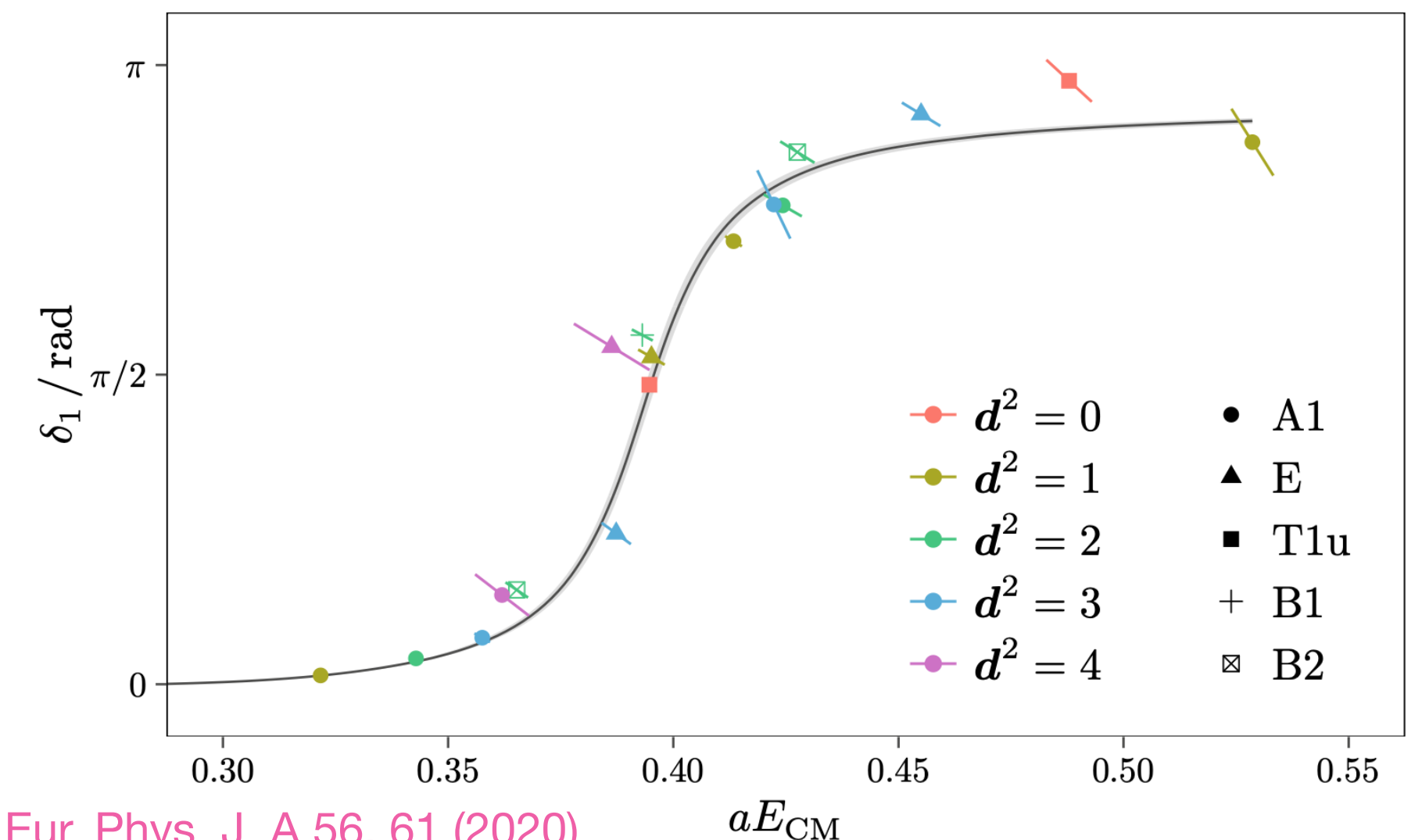
For single channel s-wave scattering

$$p \cot \delta_0(p) = \frac{2}{L\sqrt{\pi}} \mathcal{Z}_{00}(1; q^2)$$

$$E = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2} \quad q = \frac{pL}{2\pi}$$

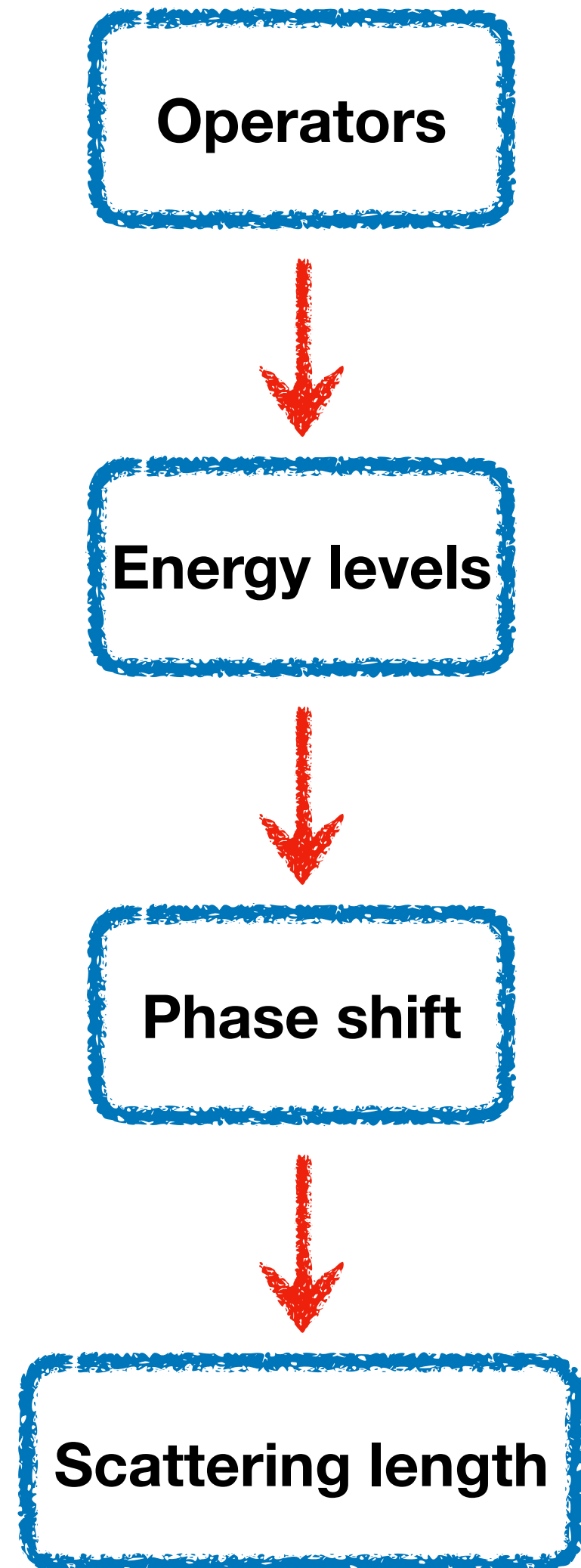


M. Werner and others, Eur. Phys. J. A 56, 61 (2020).



Simulation in Lattice QCD

Determination of LECs on Lattice



Effective range expansion

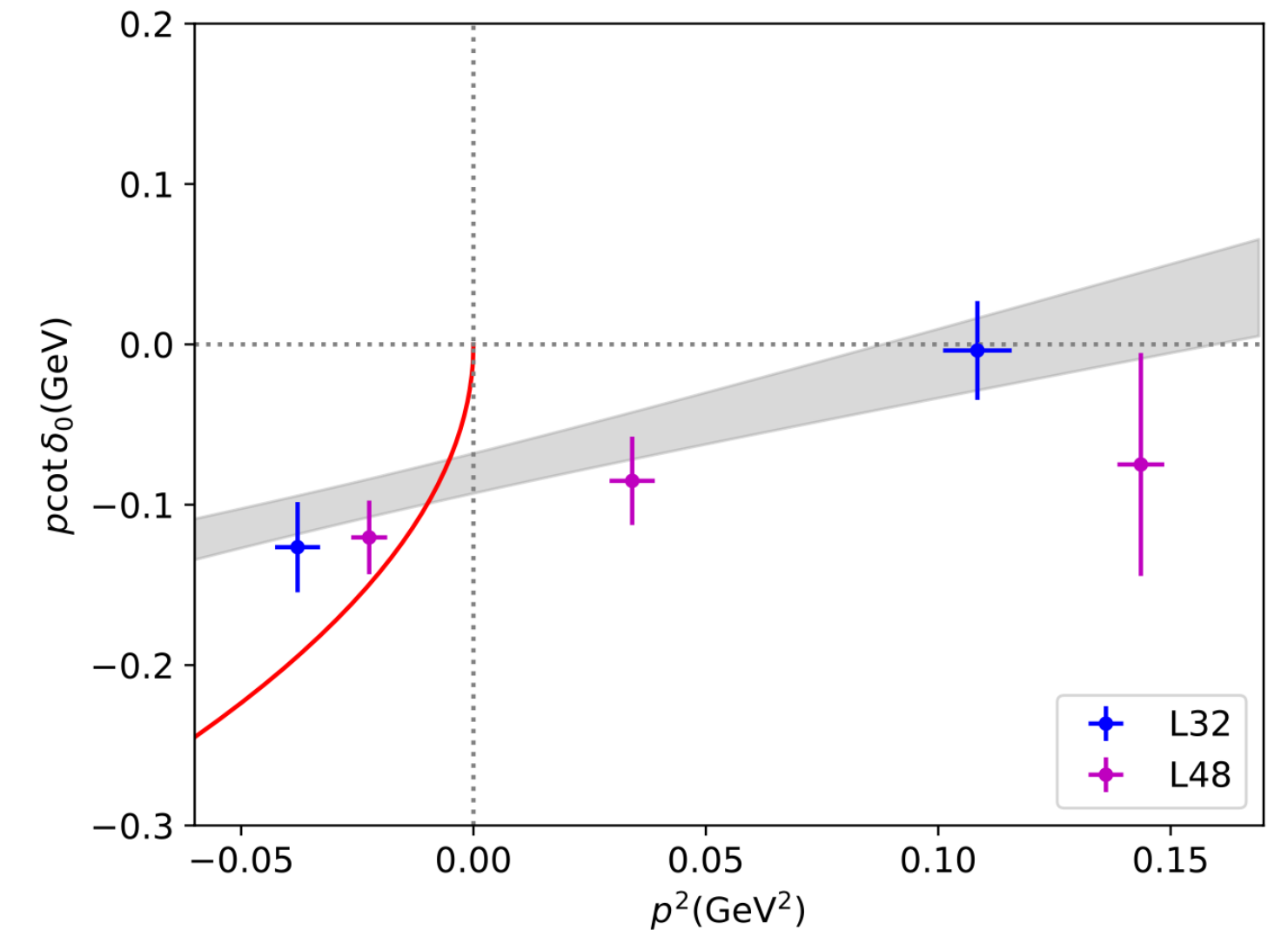
$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

Scattering amplitude

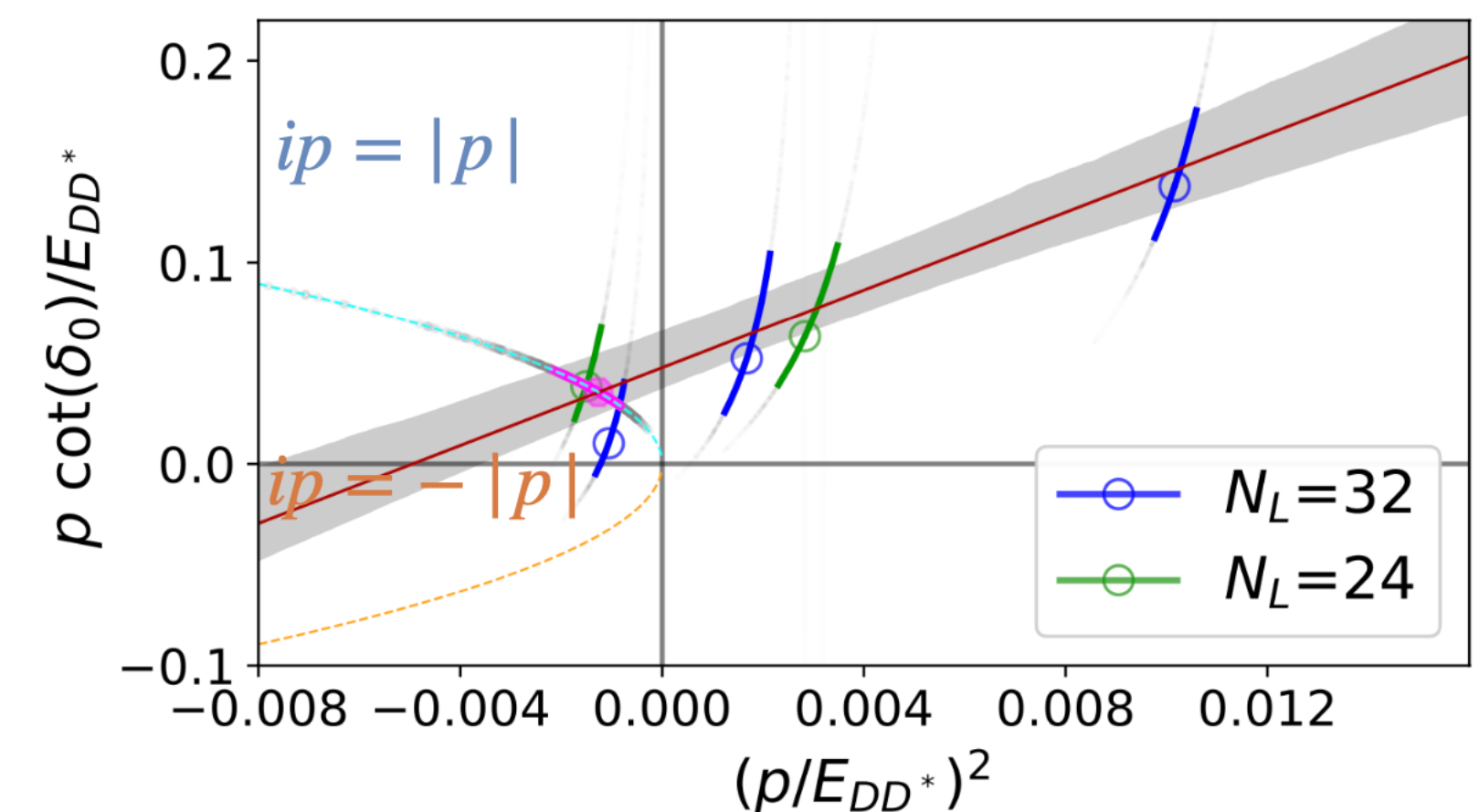
$$t_l^{(J)} \sim \frac{1}{p \cot \delta_l^{(J)} - ip}$$

For partial wave l and total angular momentum J

- L. Liu, K. Orginos, F.-K. Guo, C. Hanhart, and U.-G. Meissner, Phys. Rev. D 87, 014508 (2013).
- Y. Lyu, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, and J. Meng, arXiv:2302.04505 [hep-lat] (2023).
- H. Xing, J. Liang, L. Liu, P. Sun, and Y.-B. Yang, arXiv:2210.08555 [hep-lat] (2022).
- M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 129, 032002 (2022).
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H. Xing, J. Liang, L. Liu, P. Sun, and Y.-B. Yang, arXiv:2210.08555 [hep-lat] (2022).



M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 129, 032002 (2022).

Lattice setup

Gauge configurations from CLQCD

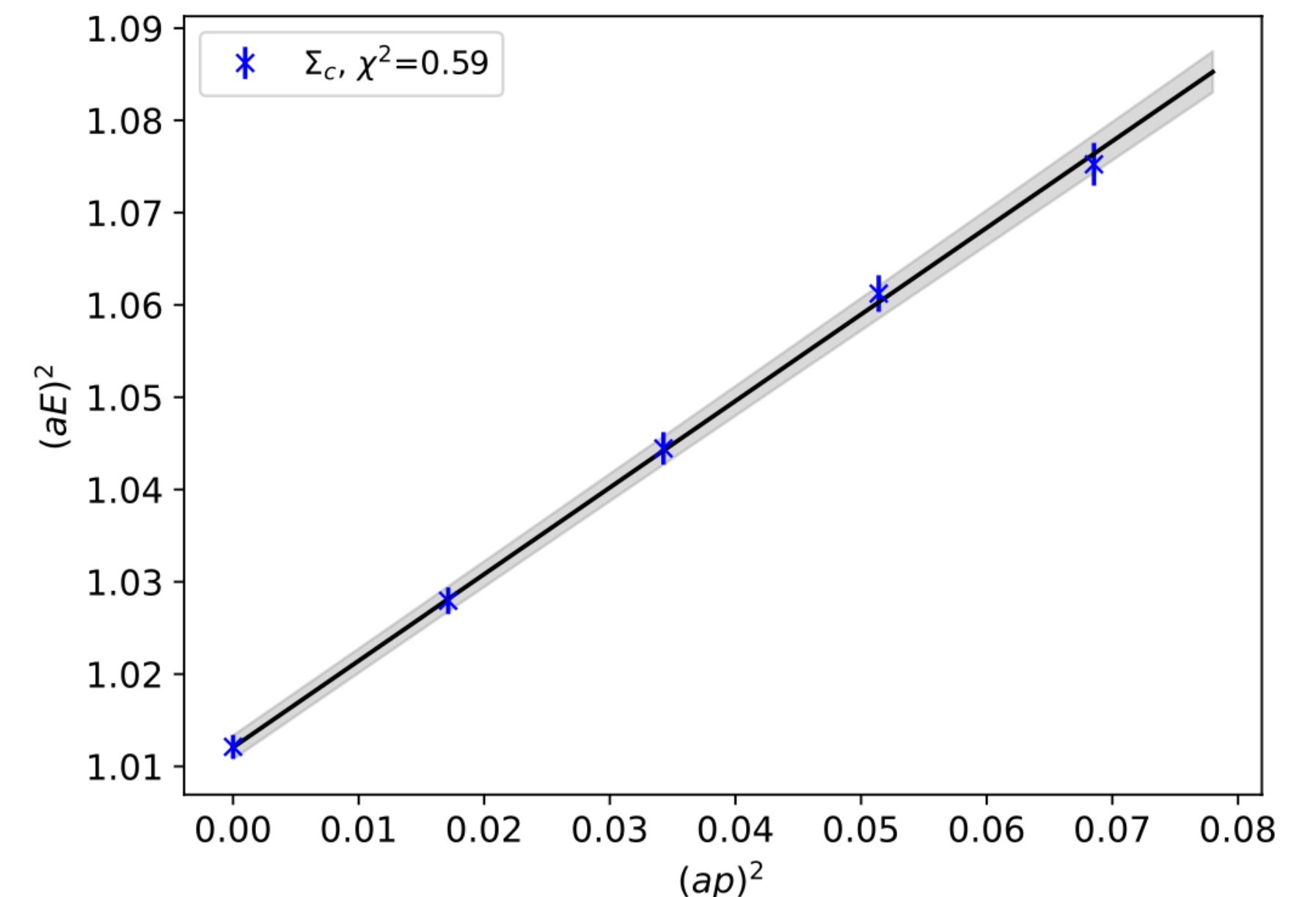
Ensembles	β	$L^3 \times T$	lattice space(fm)	am_l	am_s	$N_{\text{conf.}}$
1	6.2	$24^3 \times 72$	0.108	-0.2770	-0.2400	
2	6.2	$32^3 \times 64$	0.108	-0.2770	-0.2400	
3	6.2	$32^3 \times 64$	0.108	-0.2790	-0.2400	
4	6.2	$48^3 \times 96$	0.108	-0.2790	-0.2400	
5	6.41	$32^3 \times 96$	0.080	-0.2295	-0.2050	371
6	6.41	$48^3 \times 96$	0.080	-0.2295	-0.2050	201
7	6.41	$32^3 \times 64$	0.080	-0.2320	-0.2050	
8	6.41	$48^3 \times 96$	0.080	-0.2320	-0.2050	
9	6.72	$48^3 \times 144$	0.050	-0.1850	-0.1700	

The results presented in this talk are based on

- Two ensembles of gauge configurations with 2+1 dynamical quark flavors.
- Pion mass ~ 295 MeV.
- tree-level Symanzik-improved gauge action.
- Shekholeslami-Wohlert fermion action with tree-level tadpole improvement
- Distillation quark smearing

Charm quark mass

- Determined by physical spin-averaged mass of η_c and J/Ψ
- DCB mass deviate from continuum dispersion relation due to lattice artifacts



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Lattice setup

Smearing

- Signal-to-noise ratio
- Quark smearing method: Jacobi smearing, distillation smearing,

Distillation quark smearing method

$$-\nabla_{xy}^2(t) = 6\delta_{xy} - \sum_{j=1}^3 \left(\tilde{U}_j(x, t)\delta_{x+\hat{j}, y} + \tilde{U}_j^\dagger(x - \hat{j}, t)\delta_{x-\hat{j}, y} \right) \quad \square(t) = V(t)V^\dagger(t) \longrightarrow \square_{xy}(t) = \sum_{k=1}^{N_{ev}} \mathbf{v}_x^{(k)}(t)\mathbf{v}_y^{(k)\dagger}(t)$$

$$N_t \times N_x^3 \times N_{color} \times N_{spin} \longrightarrow N_t \times N_{spin} \times N_{ev}$$

- M. Peardon, J. Bulava, J. Foley, C. Morningstar, J. Dudek, R. G. Edwards, B. Joo, H.-W. Lin, D. G. Richards, and K. J. Juge, Phys. Rev. D 80, 054506 (2009).
- R. G. Edwards, J. J. Dudek, D. G. Richards, and S. J. Wallace, Phys. Rev. D 84, 074508 (2011).
- C. Egerer, D. Richards, and F. Winter, Phys. Rev. D 99, 034506 (2019).
- H. Xing, J. Liang, L. Liu, P. Sun, and Y.-B. Yang, arXiv:2210.08555 [hep-lat] (2022).
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- Perambulators and eigenvectors are generated from CLQCD ensembles.
- Number of eigenvectors are set to $N_{ev} = 100$.
- Contraction done by opt_einsum package in Python.

Results

BChPT scattering length results

Z.-R. Liang, P.-C. Qiu, and D.-L. Yao, JHEP 07, 124 (2023), L. Meng and S.-L. Zhu, Phys. Rev. D 100, 014006 (2019).

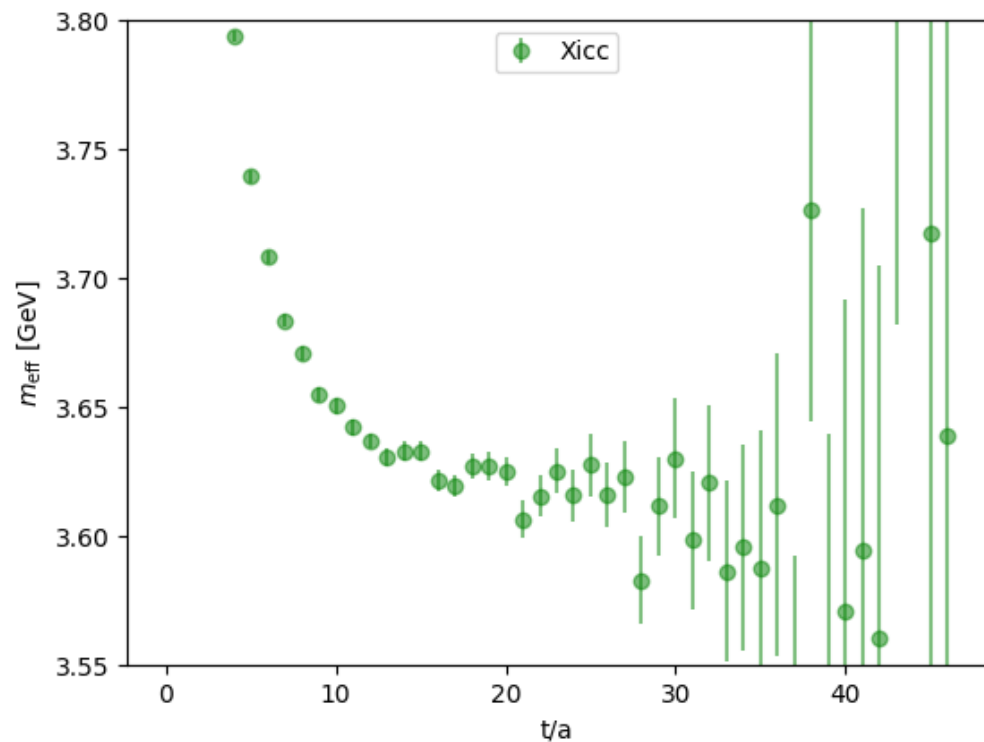
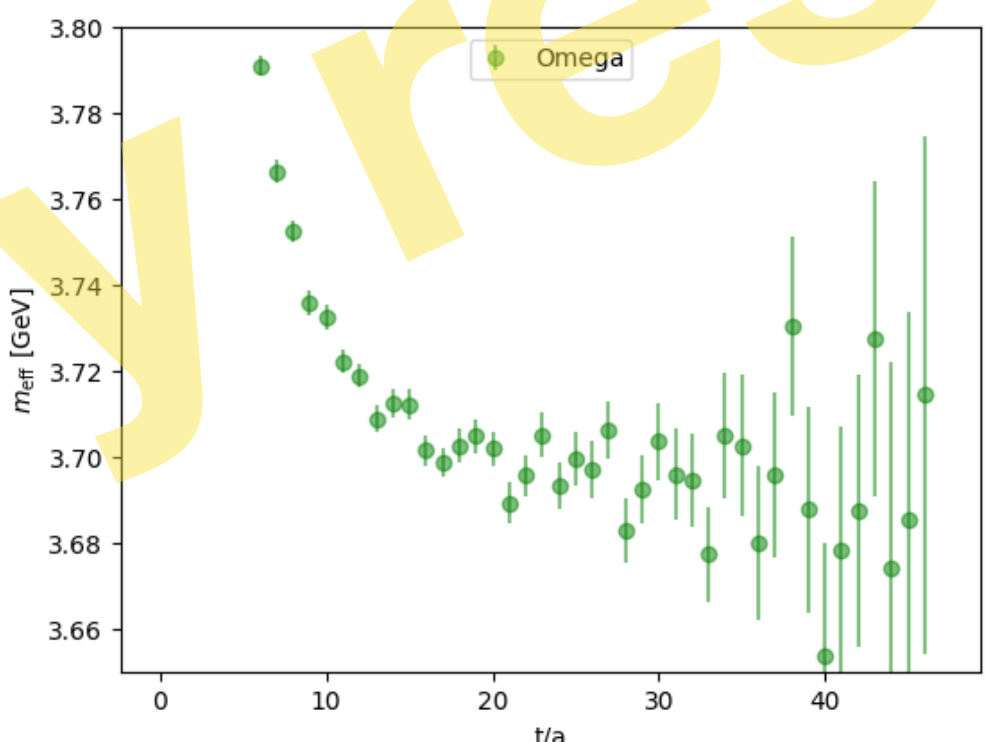
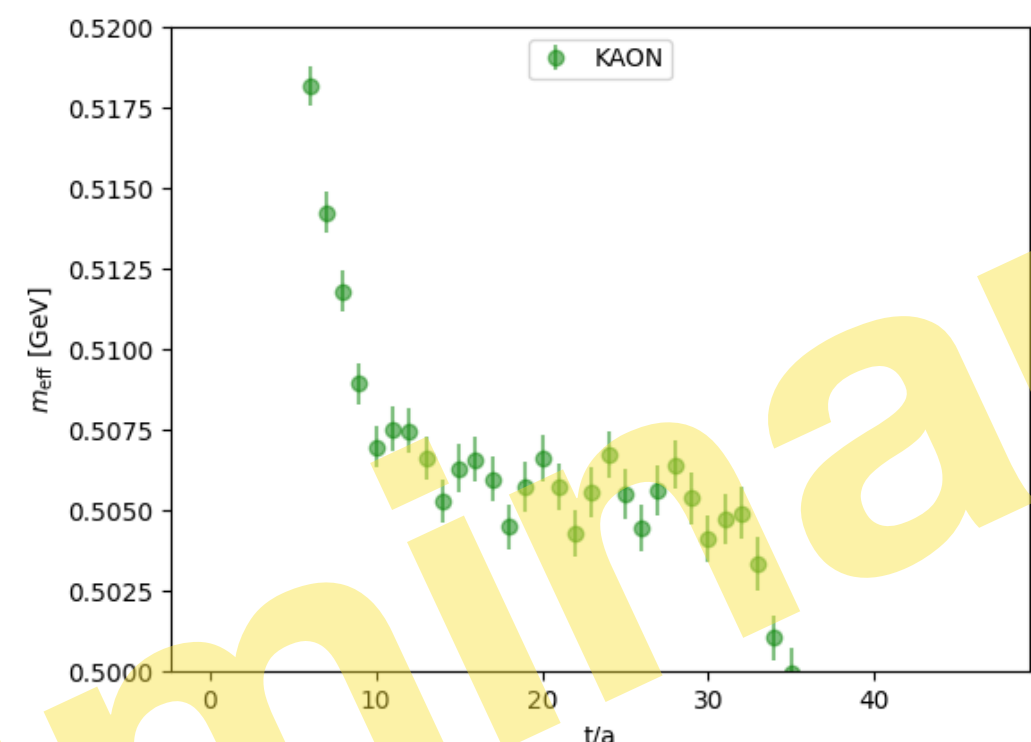
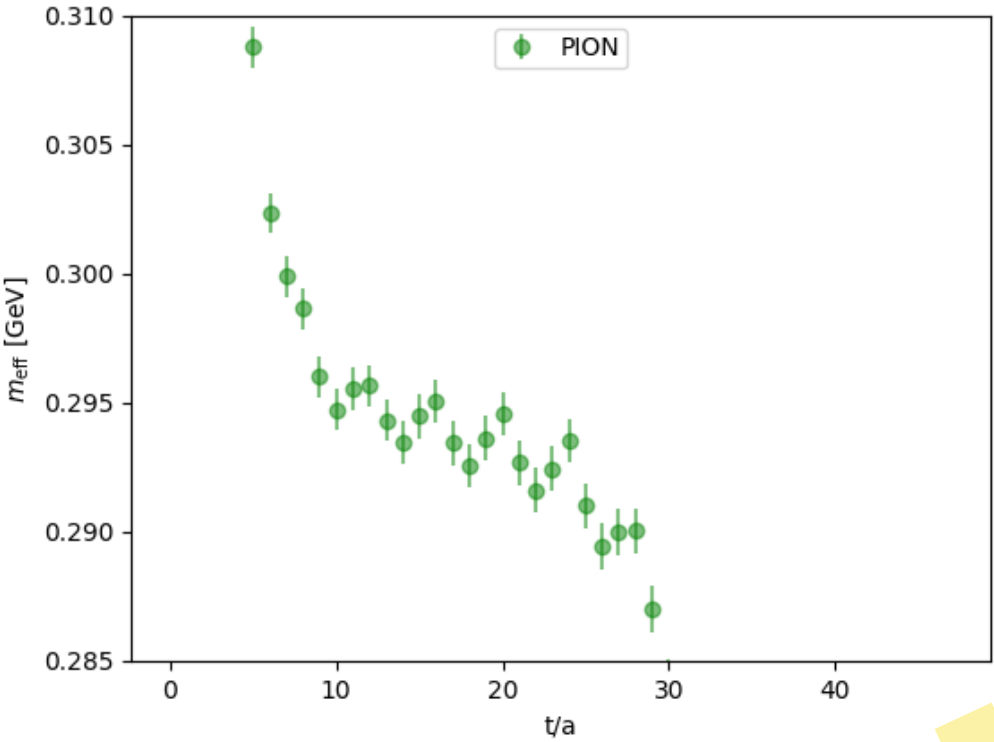
(S, I)	Processes	$\mathcal{O}(p^1)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$		EOMS	HB
				Tree	Loop		
$(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \rightarrow \Omega_{cc}\bar{K}$	-0.27	0.29	-0.11	-0.001	$-0.09^{+0.12}_{-0.13}$	-0.20(1)
$(1, 1)$	$\Xi_{cc}K \rightarrow \Xi_{cc}K$	-0.27	0.27	-0.13	-0.47	-0.60 ± 0.13	-0.25(1)
$(1, 0)$	$\Xi_{cc}K \rightarrow \Xi_{cc}K$	0.27	0.34	0.13	0.30	1.03 ± 0.19	0.92(2)
$(0, \frac{3}{2})$	$\Xi_{cc}\pi \rightarrow \Xi_{cc}\pi$	-0.12	0.04	-0.01	-0.06	-0.16 ± 0.02	-0.10(2)
$(-1, 0)$	$\Xi_{cc}\bar{K} \rightarrow \Xi_{cc}\bar{K}$	0.54	0.24	0.25	0.16	$1.19^{+0.22}_{-0.21}$	2.15(11)
	$\Omega_{cc}\eta \rightarrow \Omega_{cc}\eta$	-0.001	0.37	0.0	$0.05 + 0.55i$	$0.42^{+0.18}_{-0.19} + 0.55i$	$0.57(3) + 0.21i$
$(-1, 1)$	$\Omega_{cc}\pi \rightarrow \Omega_{cc}\pi$	0.0	0.04	0.0	-0.04	-0.01 ± 0.02	-0.002(1)
	$\Xi_{cc}\bar{K} \rightarrow \Xi_{cc}\bar{K}$	0.0	0.31	0.0	$-0.04 + 0.10i$	$0.27^{+0.13}_{-0.13} + 0.10i$	$0.26(1) + 0.19i$
$(0, \frac{1}{2})$	$\Xi_{cc}\pi \rightarrow \Xi_{cc}\pi$	0.25	0.04	0.01	0.04	0.34 ± 0.02	0.36(1)
	$\Xi_{cc}\eta \rightarrow \Xi_{cc}\eta$	-0.001	0.32	0.0	-0.26	$0.06^{+0.14}_{-0.15}$	$0.34(1) + 0.10i$
	$\Omega_{cc}K \rightarrow \Omega_{cc}K$	0.27	0.29	0.11	$-0.01 + 0.55i$	$0.66^{+0.13}_{-0.13} + 0.55i$	$1.18(6) + 0.29i$

Results

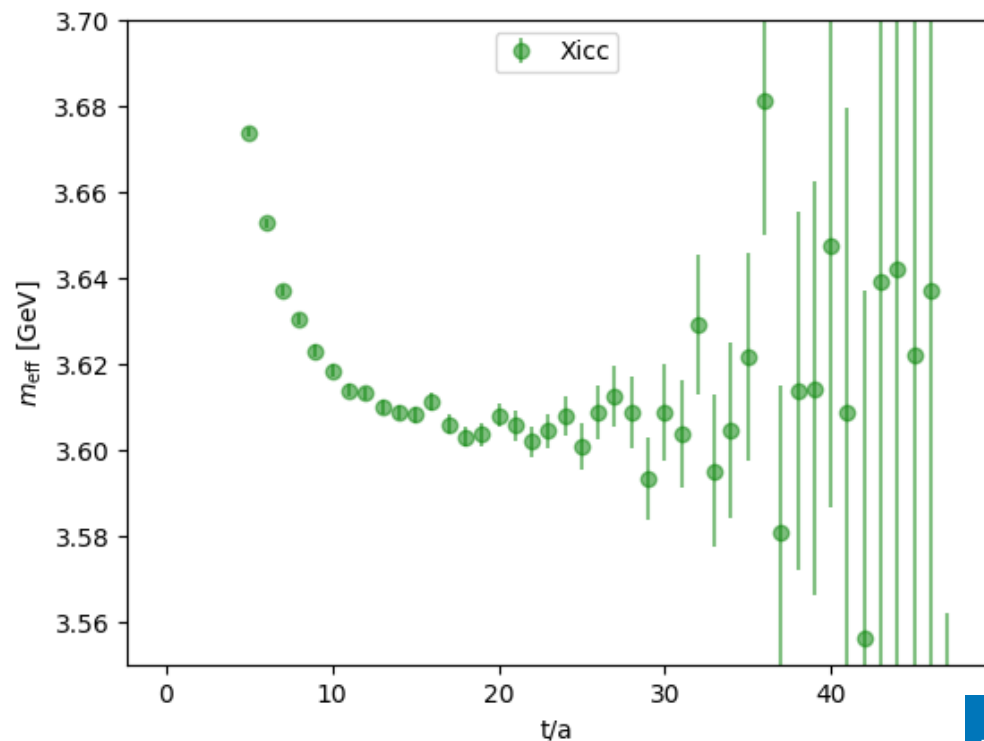
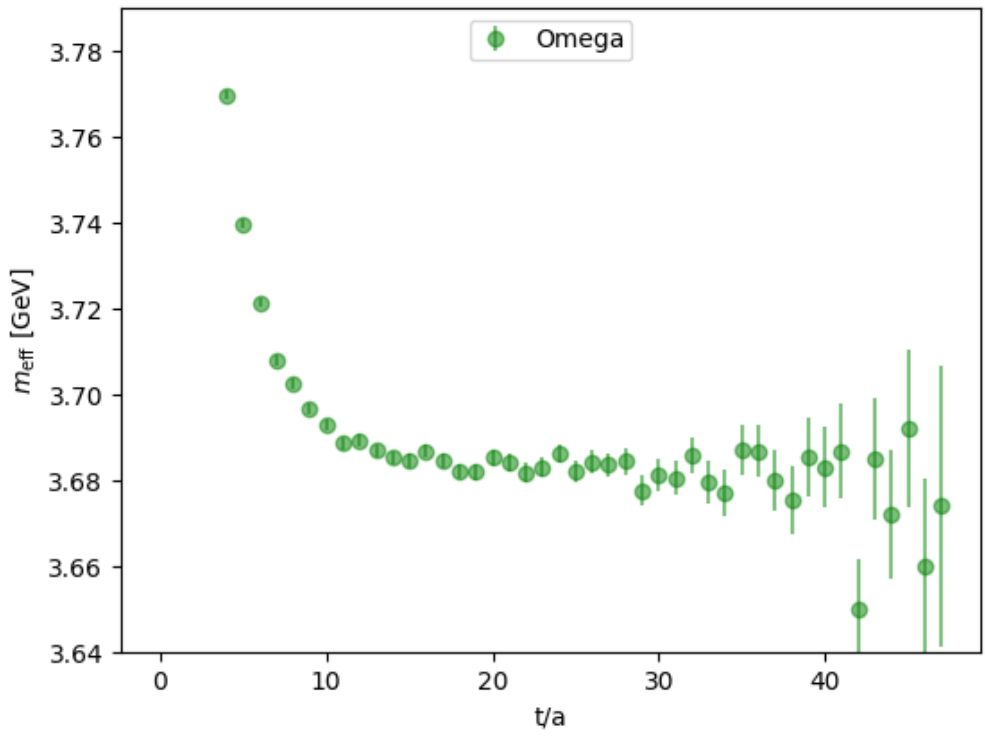
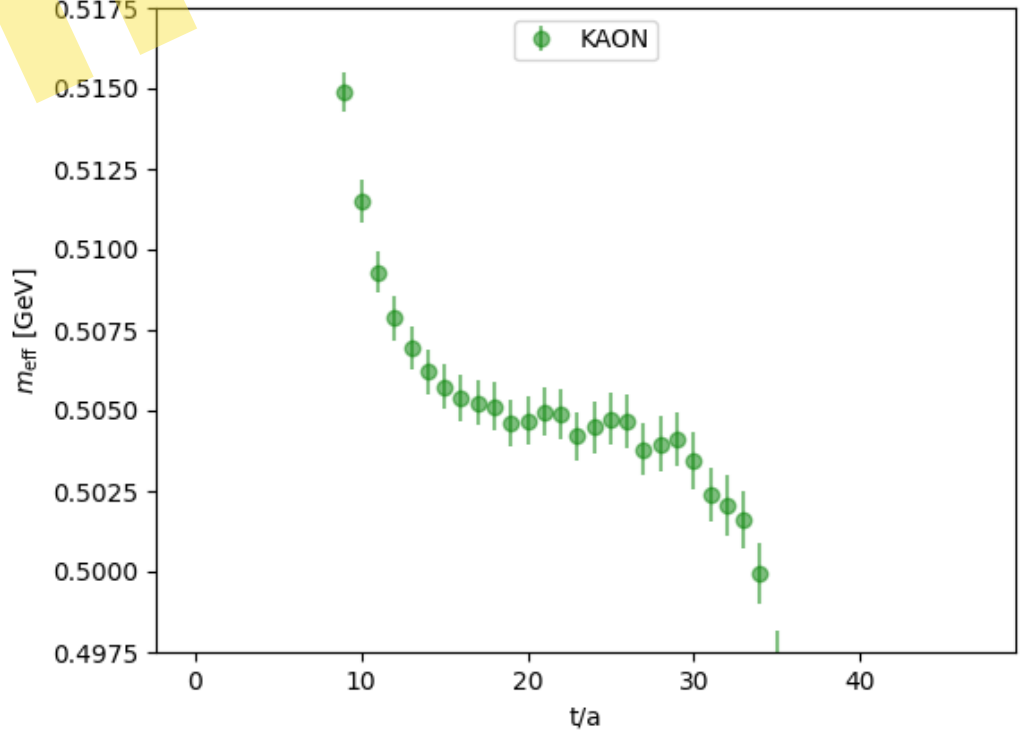
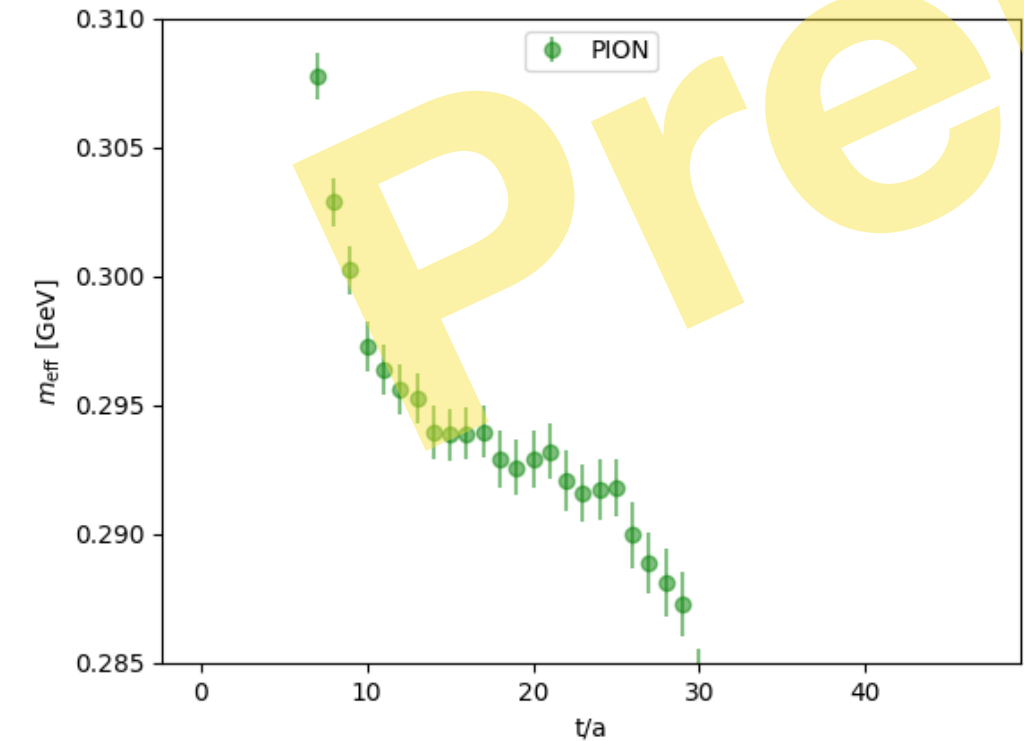
Single particle effective mass

	π	K	Ξ_{cc}	Ω_{cc}
L32	0.2942(9)	0.5052(6)	3.6053(15)	3.6847(9)
L48	0.2942(5)	0.5058(3)	3.6235(10)	3.6996(9)

L48



L32

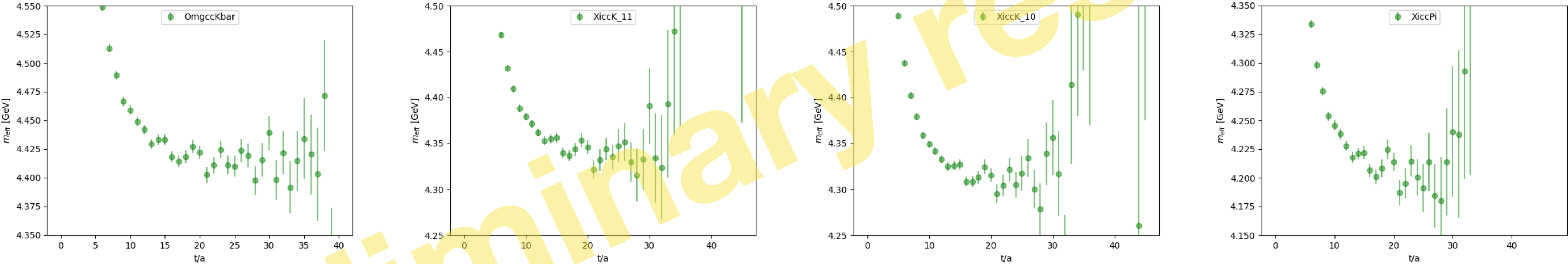


Results

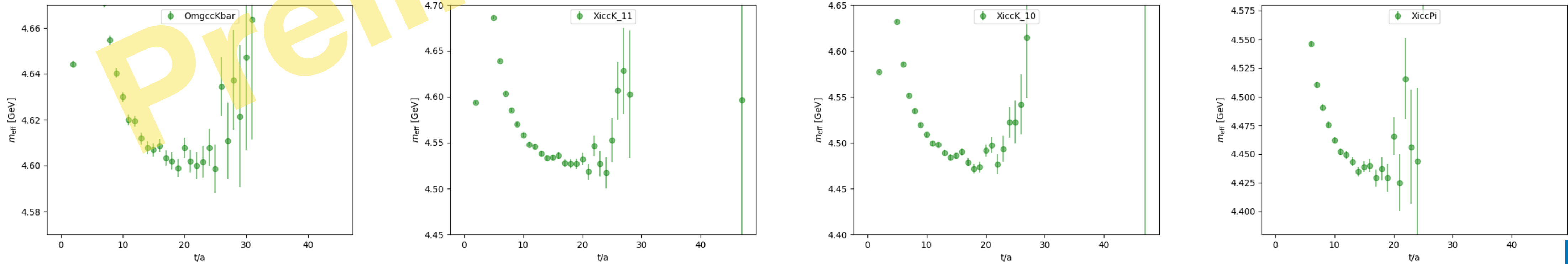
Two particle effective mass

	$\Omega_{cc}\bar{K}(-2,1/2)$	$\Xi_{cc}K(1,1)$	$\Xi_{cc}K(1,0)$	$\Xi_{cc}\pi(0,3/2)$
L32	4.1955(12)	4.1171(17)	4.0971(19)	3.9073(19)
L48	4.2091(12)	4.1340(12)	4.1276(12)	3.9215(14)

L48

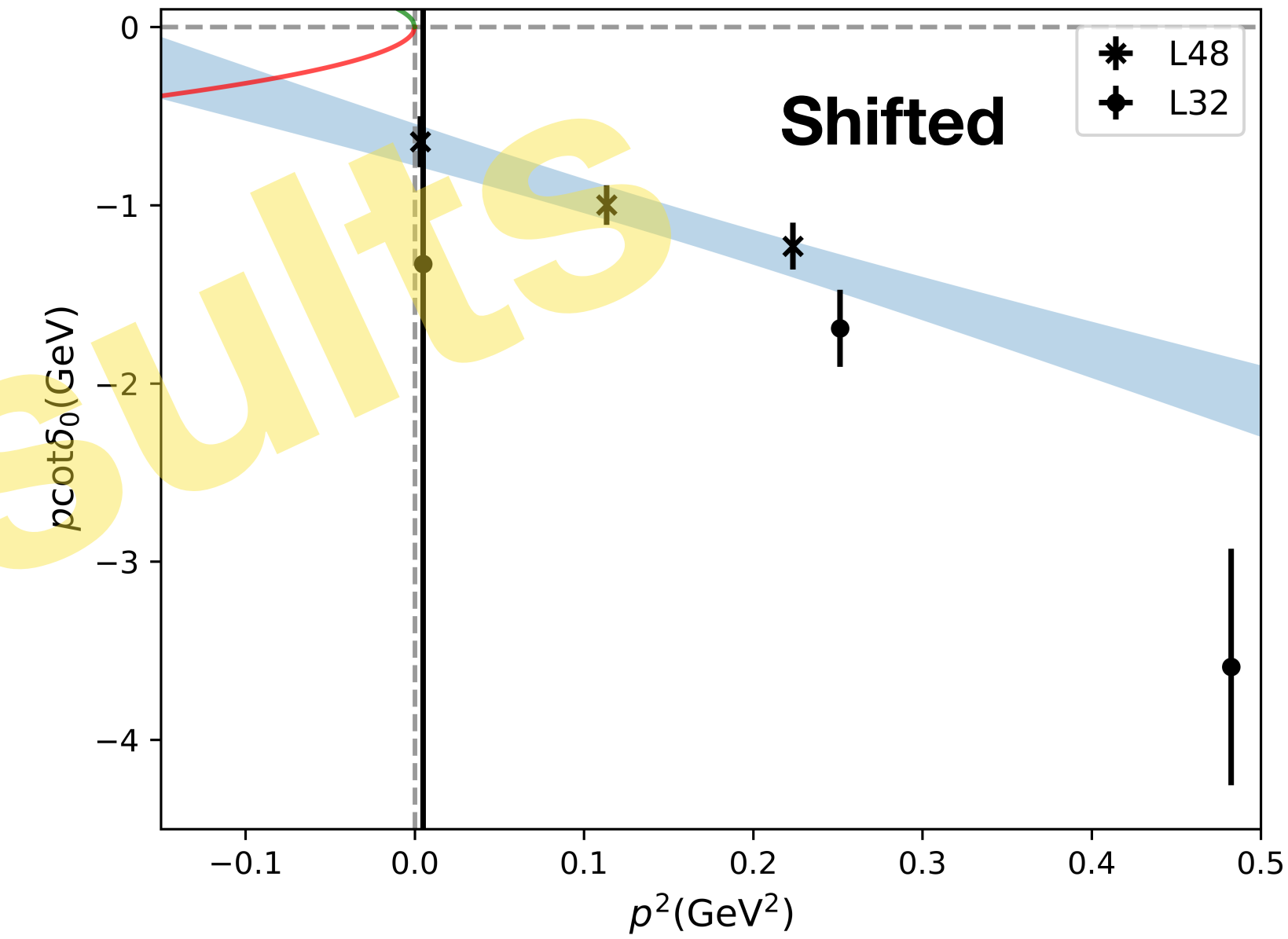
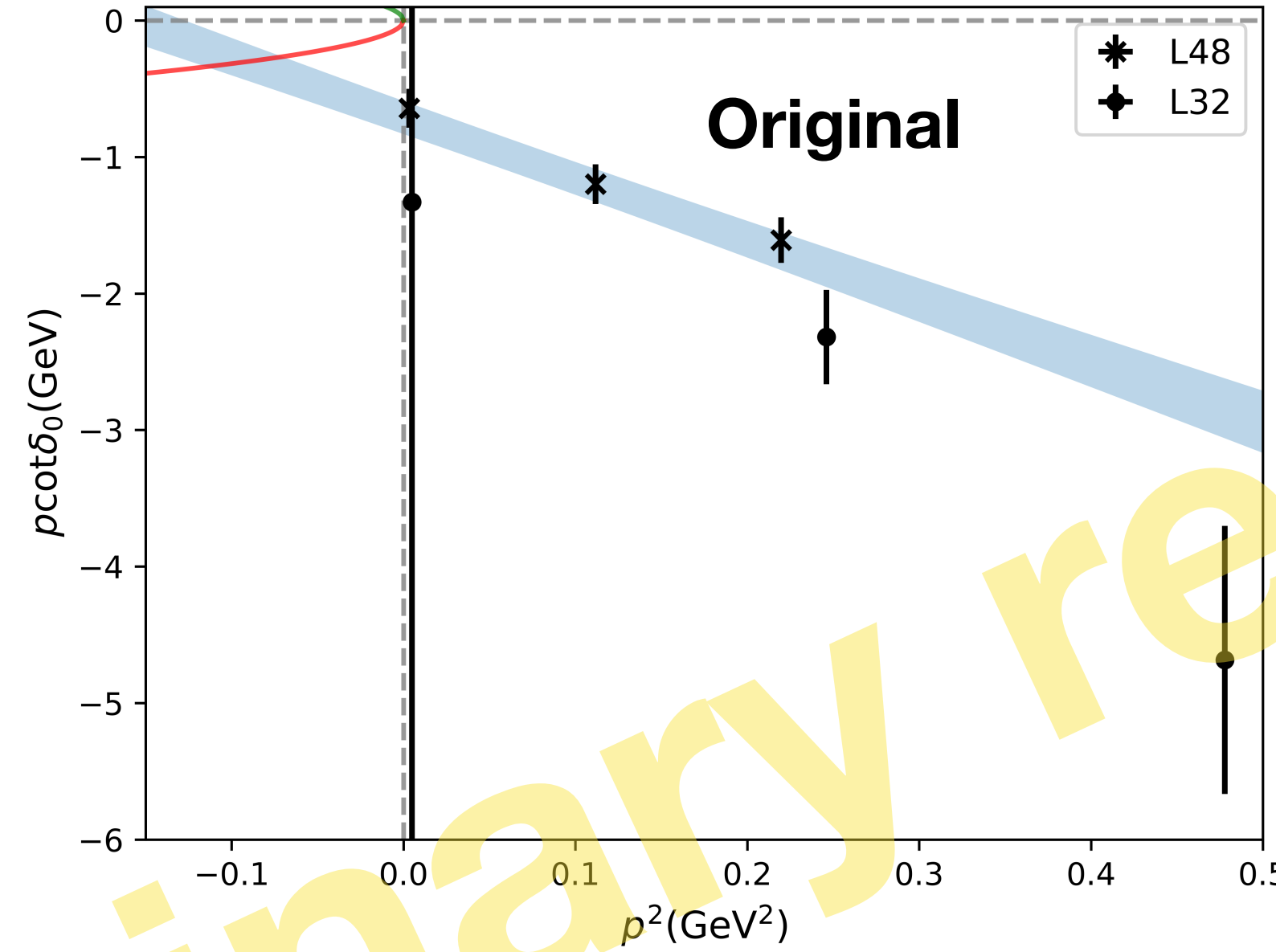
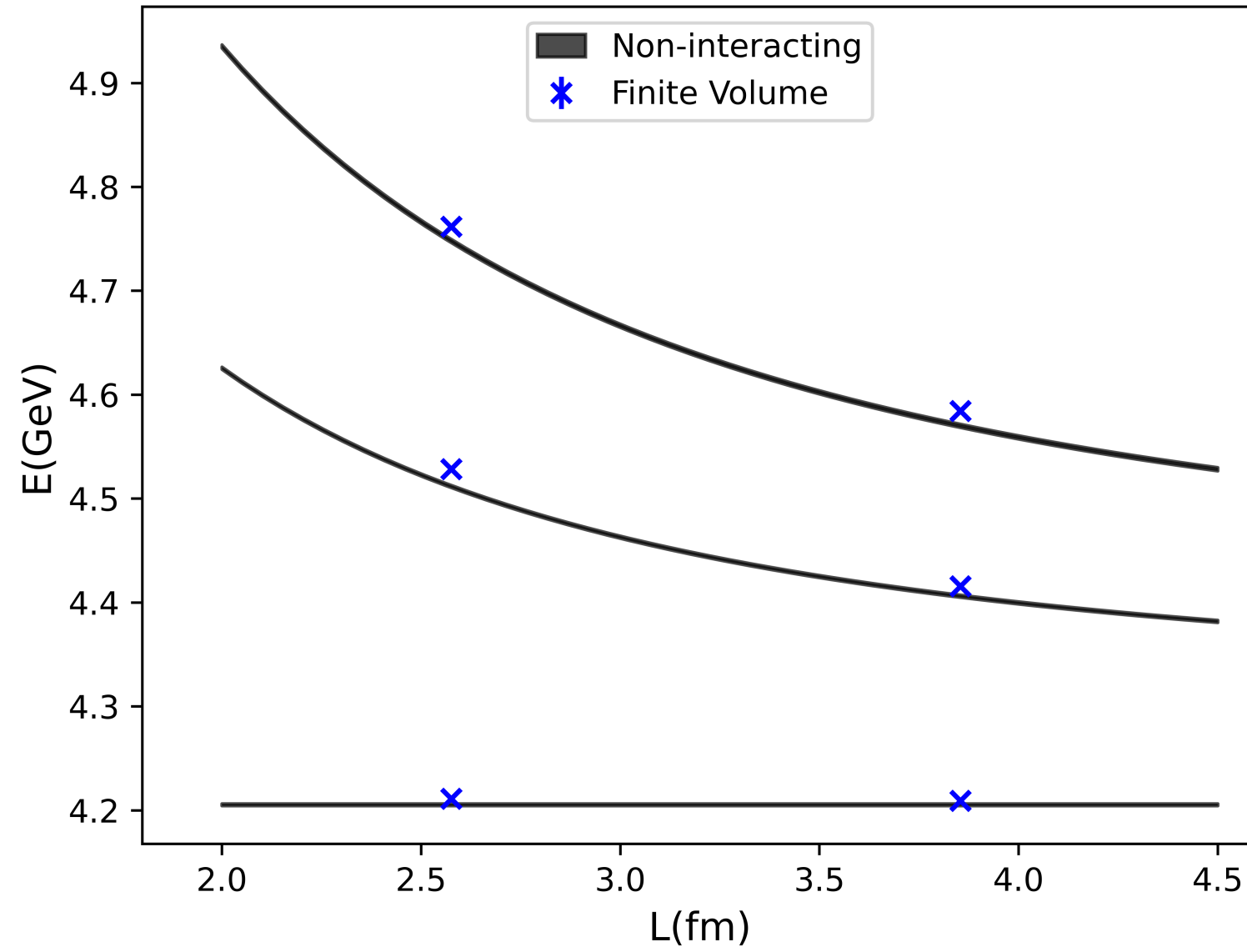


L32



Results

$$\Omega_{cc}\bar{K} \rightarrow \Omega_{cc}\bar{K} \ (-2, 1/2)$$



$$E'_p = E_p + \left(M_{\phi(p)}^{cont.} + M_{B_{cc}(p)}^{cont.} \right) - \left(M_{\phi(p)}^{lat.} + M_{B_{cc}(p)}^{lat.} \right)$$

Chisq/dof: 1.56
Scattering length: $-0.278(47)$
Effective range: $-1.76(17)$

Chisq/dof: 1.18
Scattering length: $-0.299(53)$
Effective range: $-1.14(20)$

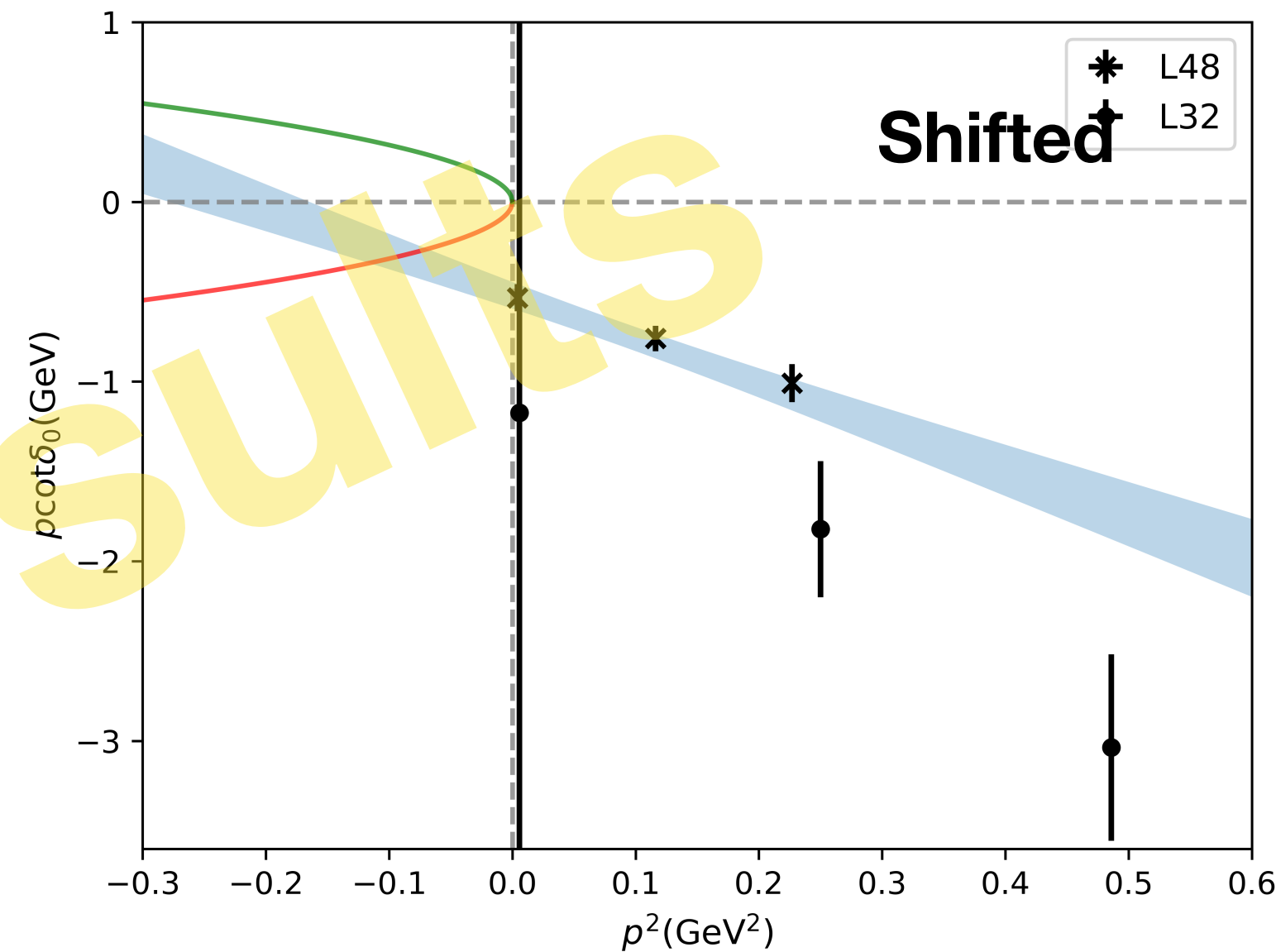
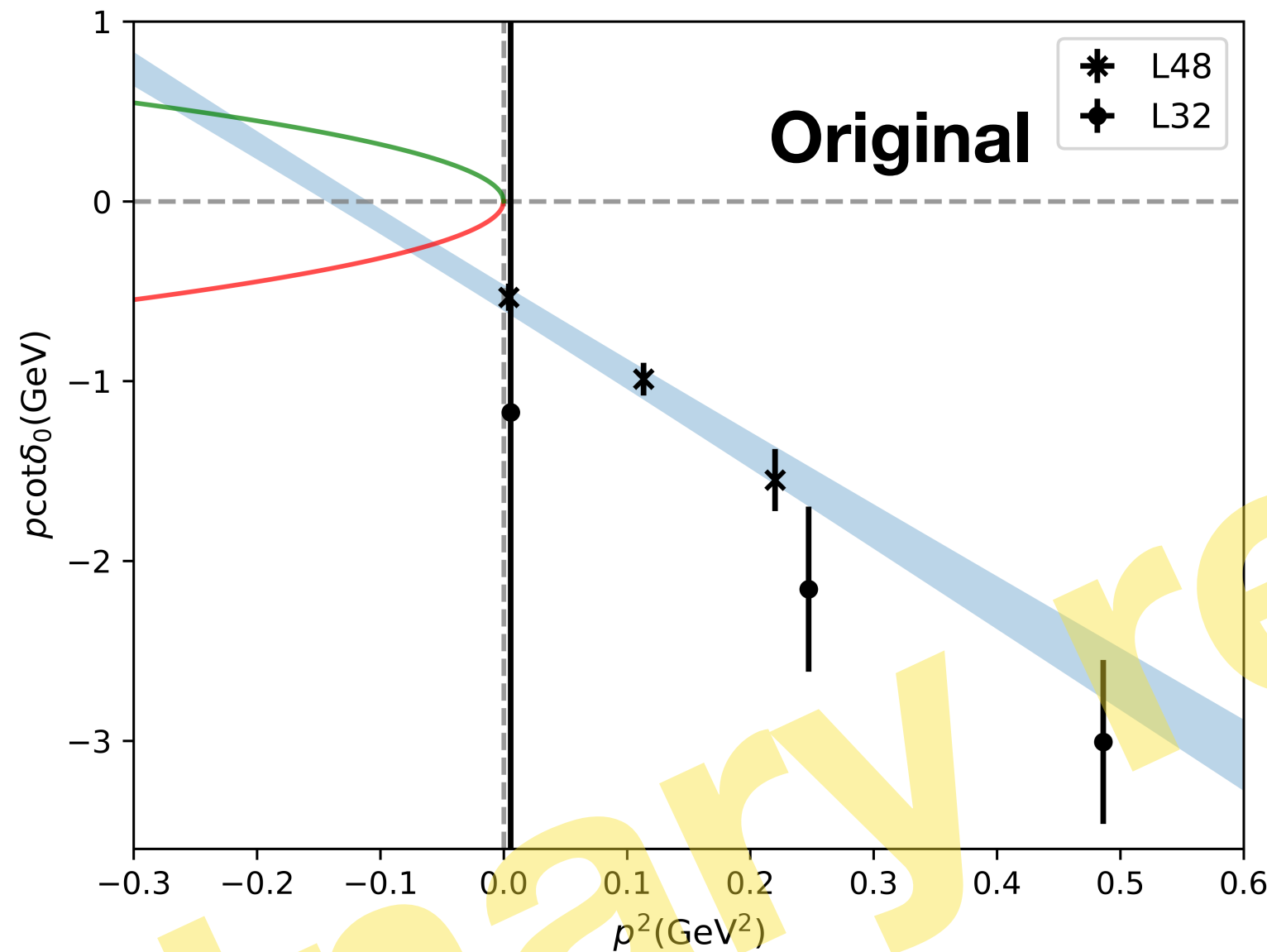
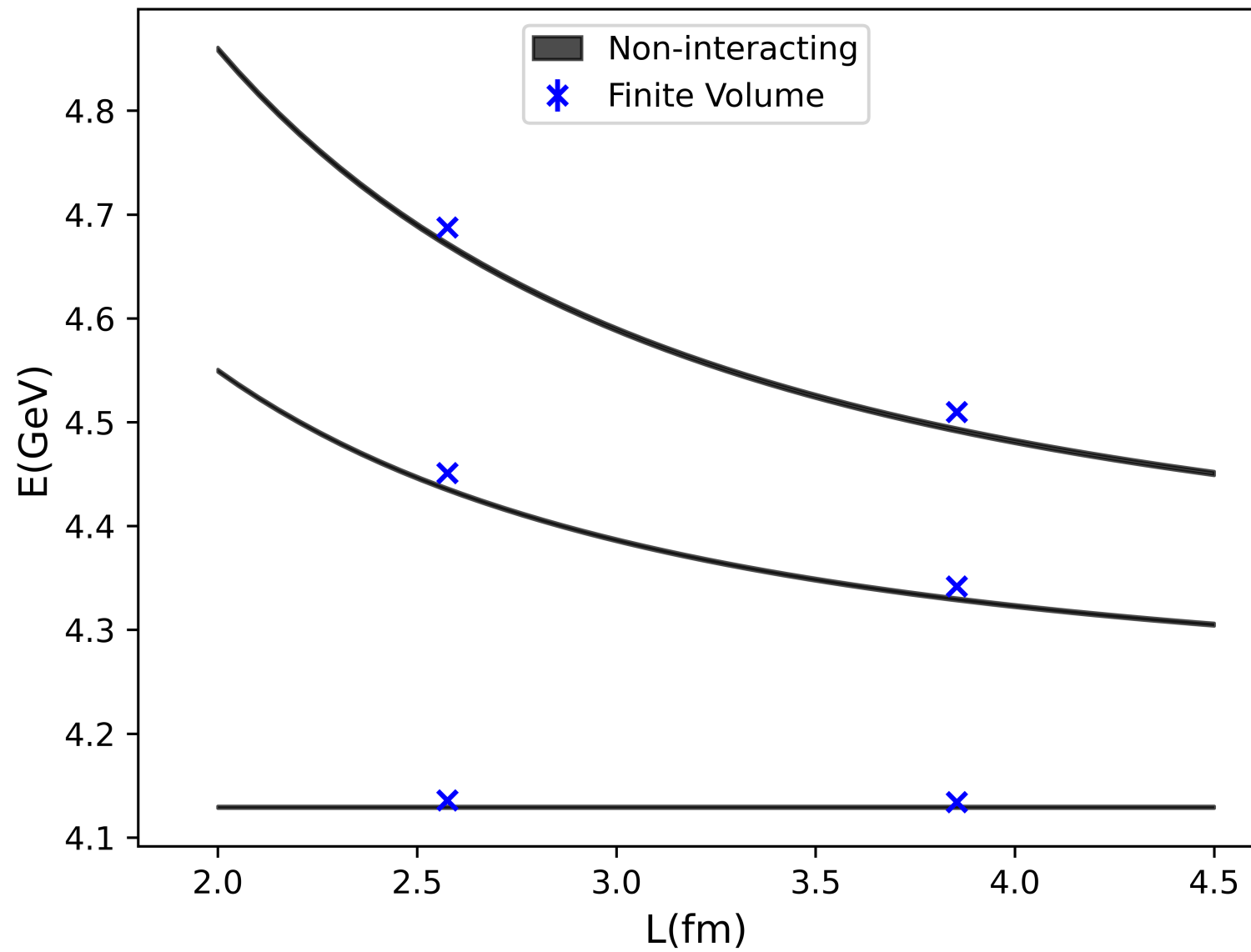
(S, I)	Processes	$\mathcal{O}(p^1)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$		EOMS	HB
				Tree	Loop		
$(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \rightarrow \Omega_{cc}\bar{K}$	-0.27	0.29	-0.11	-0.001	$-0.09^{+0.12}_{-0.13}$	$-0.20(1)$
$(1, 1)$	$\Xi_{cc}K \rightarrow \Xi_{cc}K$	-0.27	0.27	-0.13	-0.47	-0.60 ± 0.13	$-0.25(1)$
$(1, 0)$	$\Xi_{cc}K \rightarrow \Xi_{cc}K$	0.27	0.34	0.13	0.30	1.03 ± 0.19	$0.92(2)$
$(0, \frac{3}{2})$	$\Xi_{cc}\pi \rightarrow \Xi_{cc}\pi$	-0.12	0.04	-0.01	-0.06	-0.16 ± 0.02	$-0.10(2)$

$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0 p^2$$

$$t_l^{(J)} \sim \frac{1}{p \cot \delta_l^{(J)} - ip}$$

Results

$$\Xi_{cc}K \rightarrow \Xi_{cc}K (1, 1)$$



$$E'_p = E_p + \left(M_{\phi(p)}^{cont.} + M_{B_{cc}(p)}^{cont.} \right) - \left(M_{\phi(p)}^{lat.} + M_{B_{cc}(p)}^{lat.} \right)$$

Chisq/dof: 0.42
 Scattering length: -0.368(50)
 Effective range: -1.67(11)

Chisq/dof: 1.27
 Scattering length: -0.379(54)
 Effective range: -0.96(16)

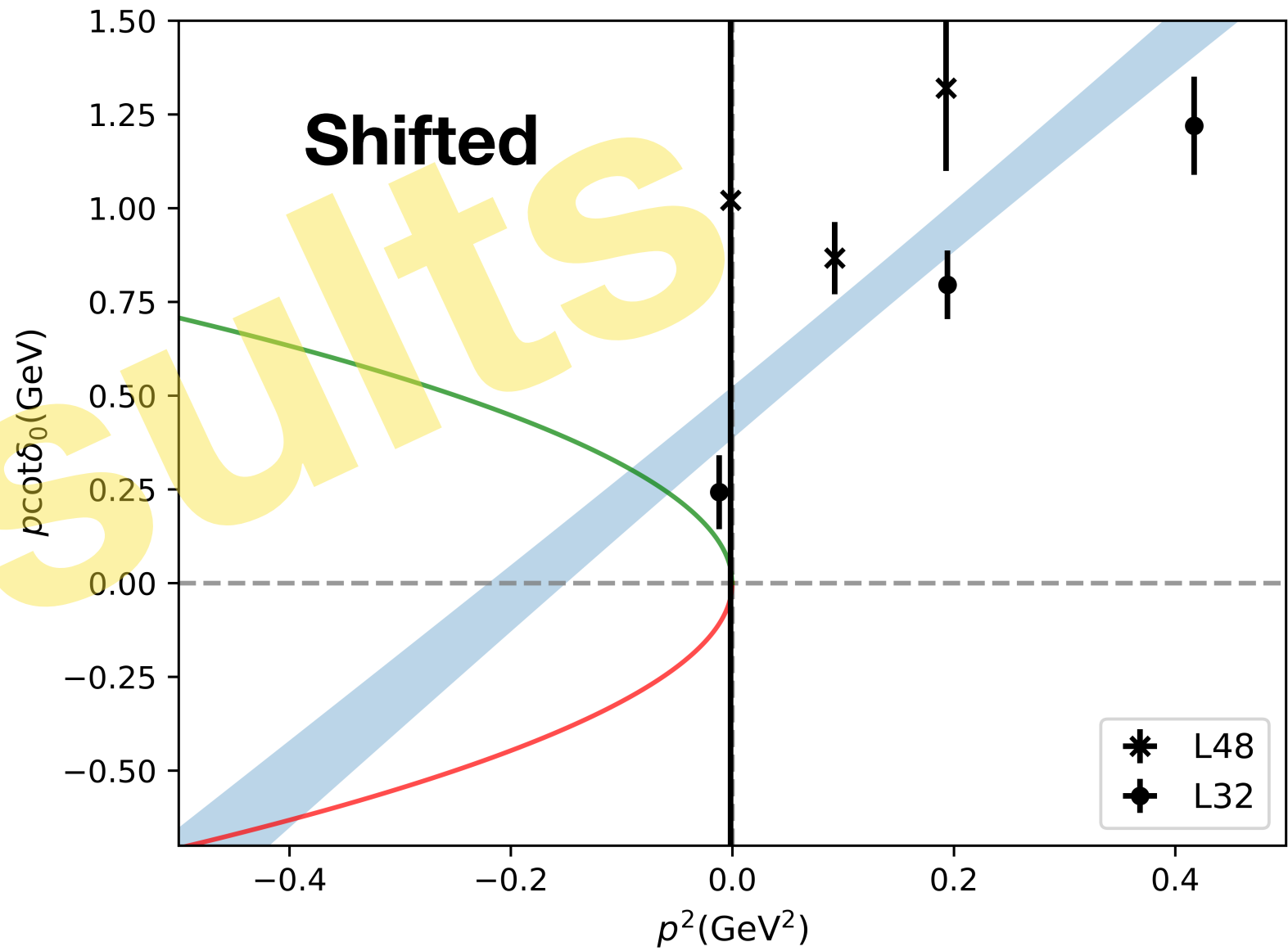
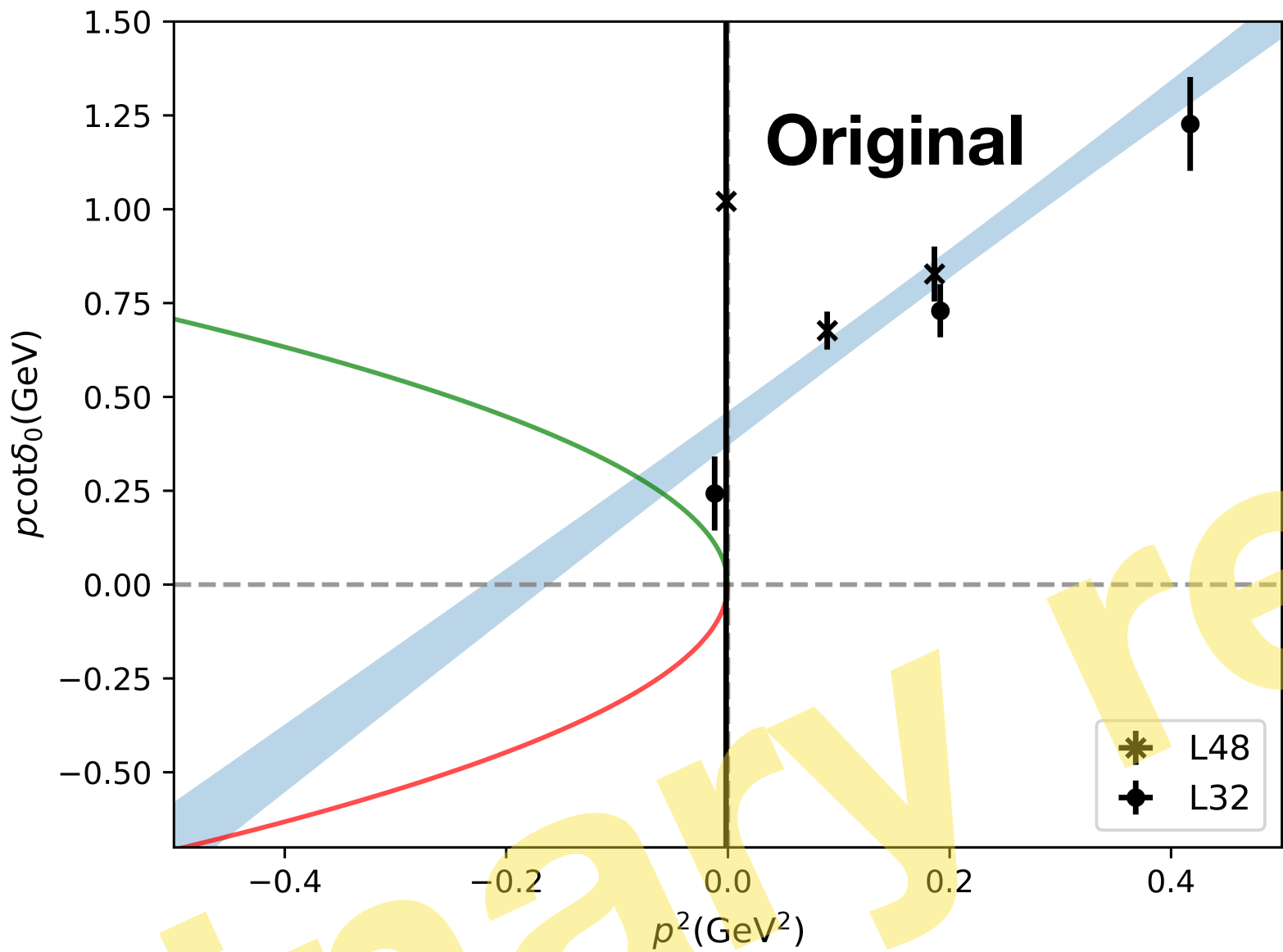
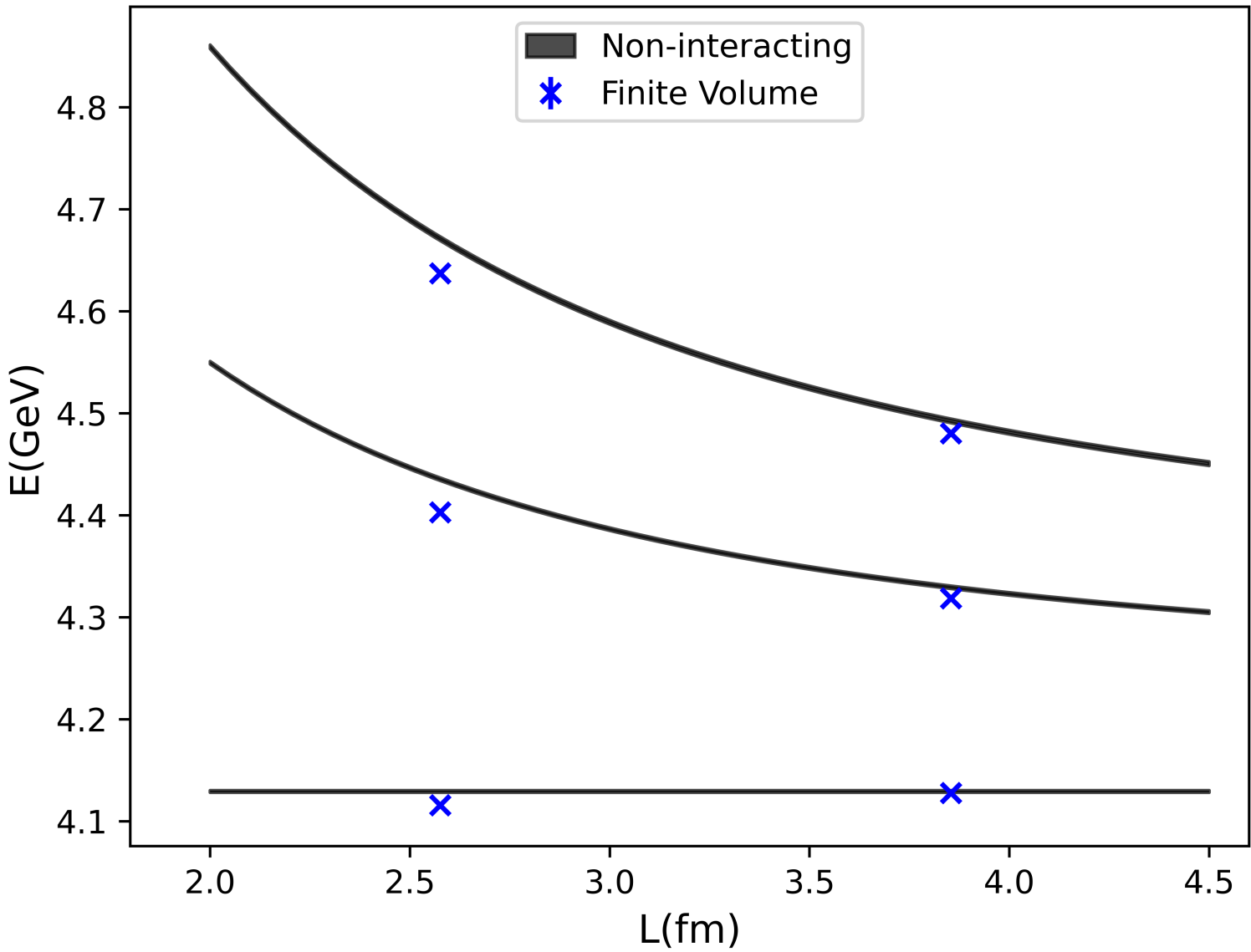
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(1, 1)	$\Xi_{cc}K \rightarrow \Xi_{cc}K$	-0.27	0.27	-0.13	-0.47	-0.60 ± 0.13	-0.25(1)
(1, 0)	$\Xi_{cc}K \rightarrow \Xi_{cc}K$	0.27	0.34	0.13	0.30	1.03 ± 0.19	0.92(2)
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$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0 p^2$$

$$t_l^{(J)} \sim \frac{1}{p \cot \delta_l^{(J)} - ip}$$

$$\Xi_{cc}K \rightarrow \Xi_{cc}K (1, 0)$$

Results



$$E'_p = E_p + \left(M_{\phi(p)}^{cont.} + M_{B_{cc}(p)}^{cont.} \right) - \left(M_{\phi(p)}^{lat.} + M_{B_{cc}(p)}^{lat.} \right)$$

Chisq/dof: 0.87
 Scattering length: 0.475(53)
 Effective range: 0.867(56)

Chisq/dof: 1.57
 Scattering length: 0.434(66)
 Effective range: 0.978(72)

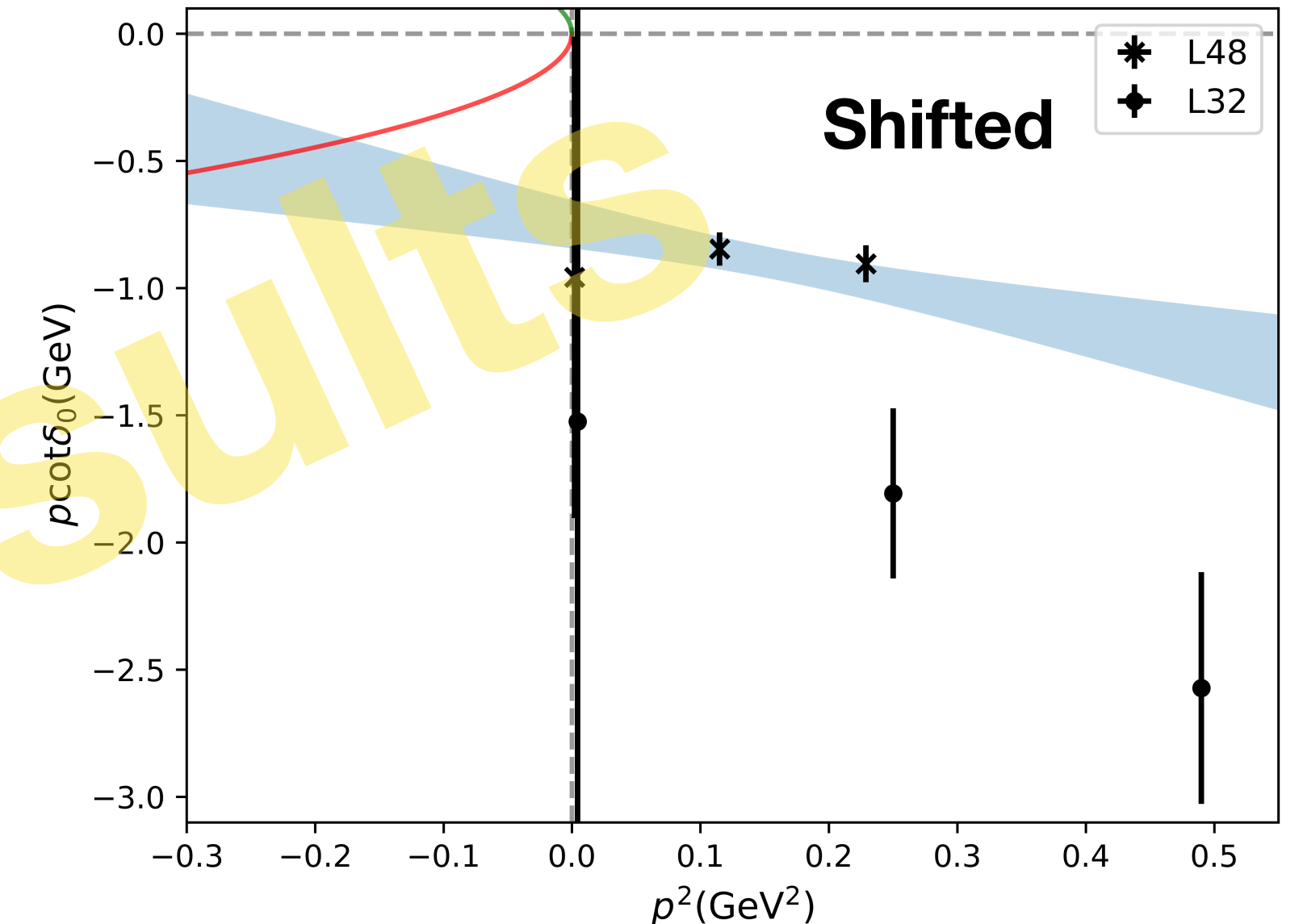
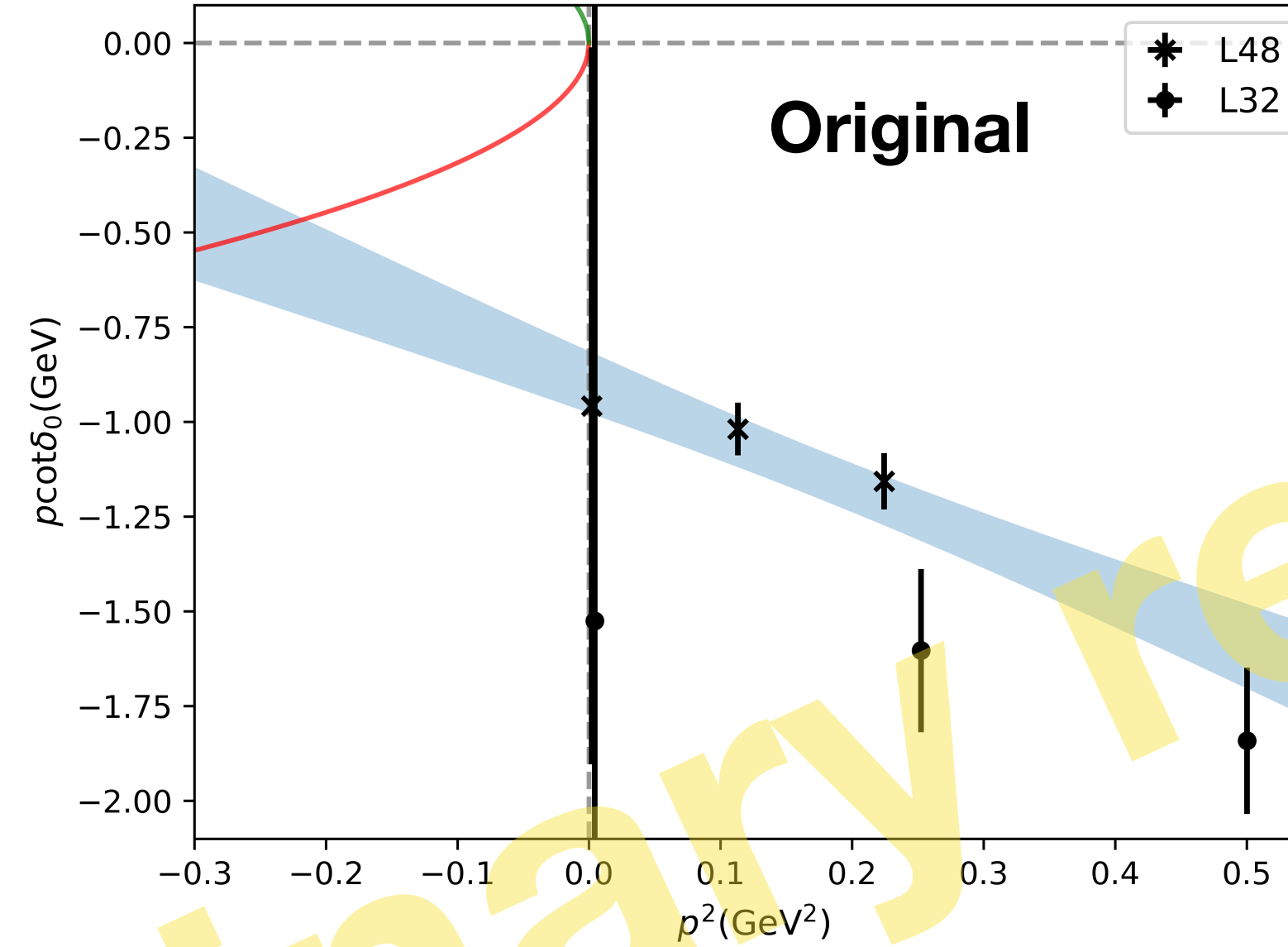
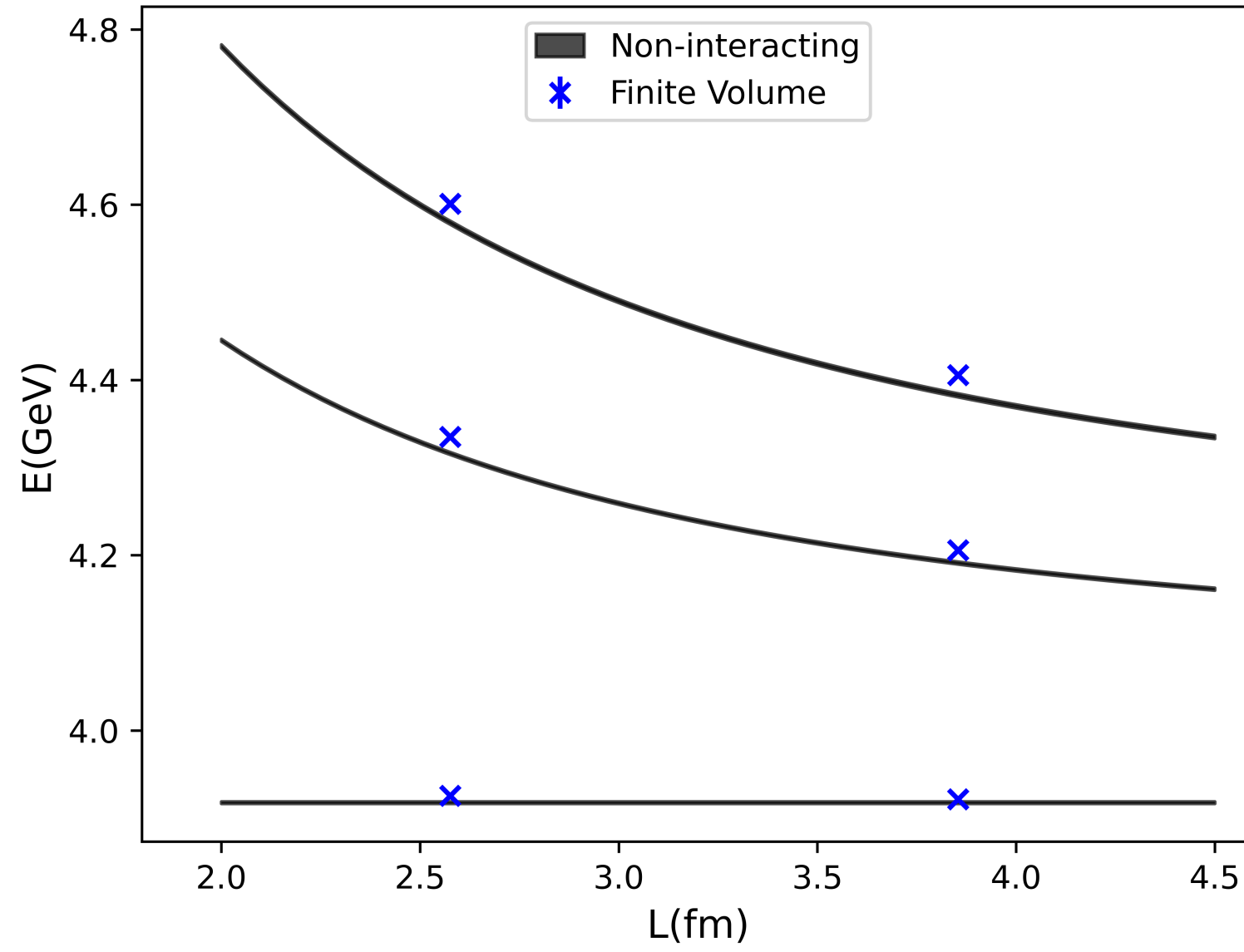
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(1, 1)	$\Xi_{cc}K \rightarrow \Xi_{cc}K$	-0.27	0.27	-0.13	-0.47	-0.60 ± 0.13	-0.25(1)
(1, 0)	$\Xi_{cc}K \rightarrow \Xi_{cc}K$	0.27	0.34	0.13	0.30	1.03 ± 0.19	0.92(2)
$(0, \frac{3}{2})$	$\Xi_{cc}\pi \rightarrow \Xi_{cc}\pi$	-0.12	0.04	-0.01	-0.06	-0.16 ± 0.02	-0.10(2)

$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0p^2$$

$$t_l^{(J)} \sim \frac{1}{p \cot \delta_l^{(J)} - ip}$$

Results

$$\Xi_{cc}\pi \rightarrow \Xi_{cc}\pi \ (0, 3/2)$$



$$E'_p = E_p + \left(M_{\phi(p)}^{cont.} + M_{B_{cc}(p)}^{cont.} \right) - \left(M_{\phi(p)}^{lat.} + M_{B_{cc}(p)}^{lat.} \right)$$

Chisq/dof: 0.55
Scattering length: $-0.220(20)$
Effective range: $-0.55(11)$

Chisq/dof: 1.85
Scattering length: $-0.263(33)$
Effective range: $-0.39(18)$

(S, I)	Processes	$\mathcal{O}(p^1)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$		EOMS	HB
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(1, 0)	$\Xi_{cc}K \rightarrow \Xi_{cc}K$	0.27	0.34	0.13	0.30	1.03 ± 0.19	0.92(2)
$(0, \frac{3}{2})$	$\Xi_{cc}\pi \rightarrow \Xi_{cc}\pi$	-0.12	0.04	-0.01	-0.06	-0.16 ± 0.02	-0.10(2)

$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0 p^2$$

$$t_l^{(J)} \sim \frac{1}{p \cot \delta_l^{(J)} - ip}$$

Summary and outlook

Summary

- Better understanding of hadron spectroscopy.
- Determination of LECs in BChPT.
- Examination of the validity of HDAS.
- A possible bound state in $(S, I) = (1, 0)$ channel.

Outlook

- More interpolators.
- Coupled channels.
- Lighter pion mass ~ 220 MeV.
- Chiral extrapolation.

(S, I)	Scattering channel	Connected(C)/ Disconnected(D)	(S, I)	Relation to $D\phi$ scattering
✓ $(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \rightarrow \Omega_{cc}\bar{K}$	C	$(2, \frac{1}{2})$	$D_s K \rightarrow D_s K$
$(-1, 1)$	$\Omega_{cc}\pi \rightarrow \Omega_{cc}\pi$ $\Xi_{cc}\bar{K} \rightarrow \Xi_{cc}\bar{K}$ $\Omega_{cc}\pi \rightarrow \Xi_{cc}\bar{K}$	C	$(1, 1)$	$D_s \pi \rightarrow D_s \pi$ $DK \rightarrow DK$ $D_s \pi \rightarrow DK$
$(-1, 0)$	$\Xi_{cc}\bar{K} \rightarrow \Xi_{cc}\bar{K}$ $\Omega_{cc}\eta \rightarrow \Omega_{cc}\eta$ $\Xi_{cc}\bar{K} \rightarrow \Omega_{cc}\eta$	D	$(1, 0)$	$DK \rightarrow DK$ $D_s \eta \rightarrow D_s \eta$ $DK \rightarrow D_s \eta$
✓ $(1, 0)$	$\Xi_{cc}K \rightarrow \Xi_{cc}K$	C	$(-1, 0)$	$D\bar{K} \rightarrow D\bar{K}$
✓ $(1, 1)$	$\Xi_{cc}K \rightarrow \Xi_{cc}K$	C	$(-1, 1)$	$D\bar{K} \rightarrow D\bar{K}$
$(0, \frac{1}{2})$	$\Xi_{cc}\pi \rightarrow \Xi_{cc}\pi$ $\Xi_{cc}\eta \rightarrow \Xi_{cc}\eta$ $\Omega_{cc}K \rightarrow \Omega_{cc}K$ $\Xi_{cc}\pi \rightarrow \Xi_{cc}\eta$ $\Xi_{cc}\pi \rightarrow \Omega_{cc}K$ $\Xi_{cc}\eta \rightarrow \Omega_{cc}K$	D	$(0, \frac{1}{2})$	$D\pi \rightarrow D\pi$ $D\eta \rightarrow D\eta$ $D_s \bar{K} \rightarrow D_s \bar{K}$ $D\pi \rightarrow D\eta$ $D\pi \rightarrow D_s \bar{K}$ $D\eta \rightarrow D_s \bar{K}$
✓ $(0, \frac{3}{2})$	$\Xi_{cc}\pi \rightarrow \Xi_{cc}\pi$	C	$(0, \frac{3}{2})$	$D\pi \rightarrow D\pi$

Thank you for your attention!