Quark masses and low energy constants in the continuum from the CLQCD ensembles

Yi-Bo Yang

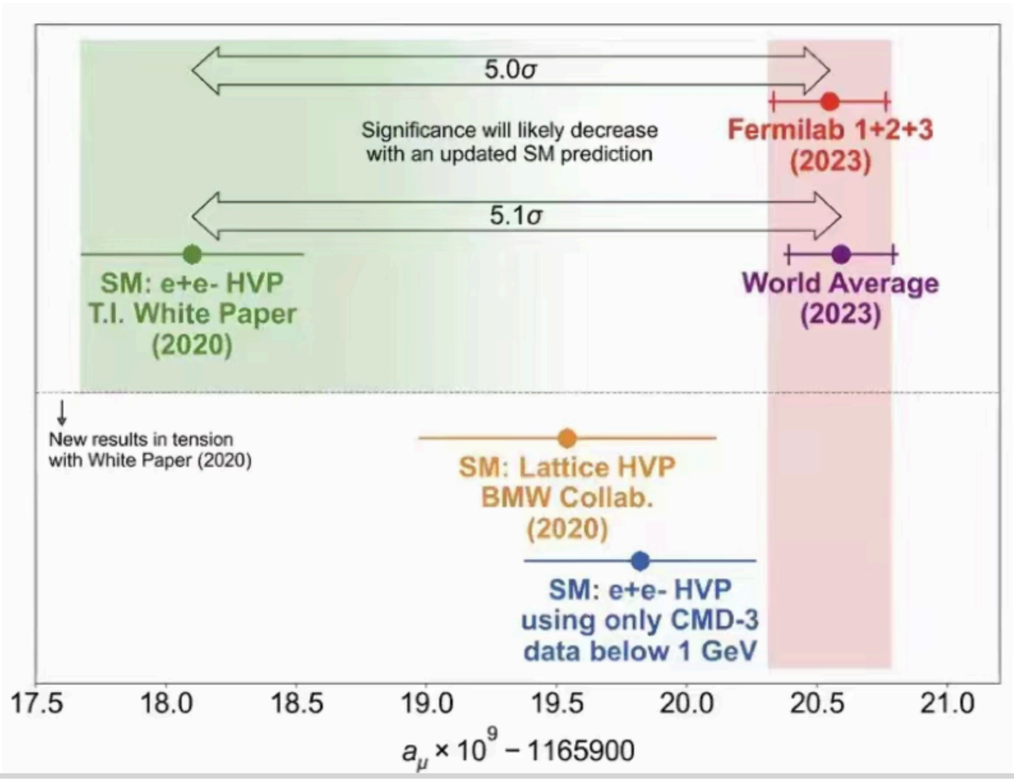
University of Chinese Academy of Sciences

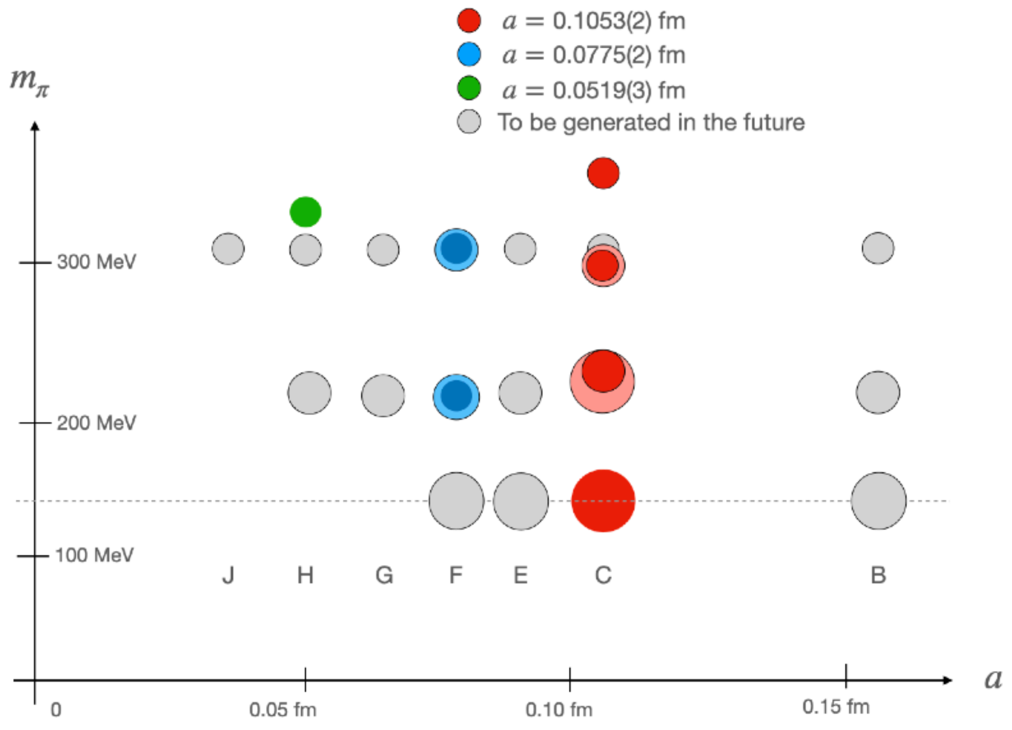
ICTP-AP International Centre for Theoretical Physics Asia-Pacific 国际理论物理中心-亚太地区

Collaborators: Zhi-Cheng Hu, Bo-Lun Hu, Ji-Hao Wang, Ming Gong, Liuming Liu, Peng Sun, Wei Sun, Wei Wang, and Dian-Jun Zhao

Outline

Background

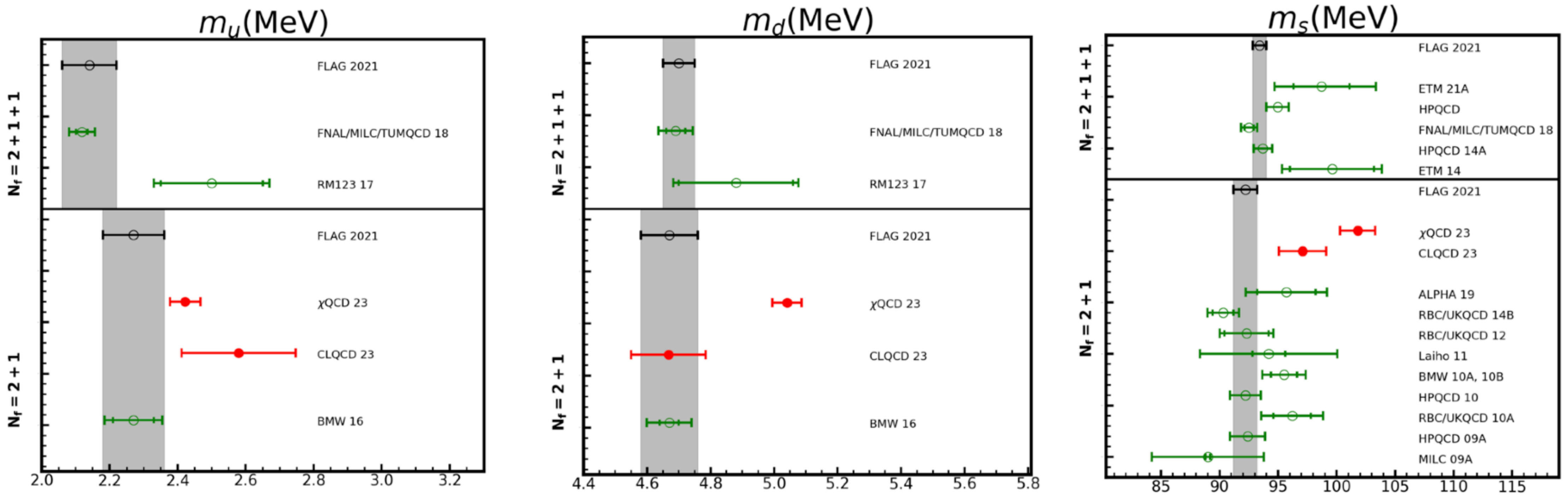




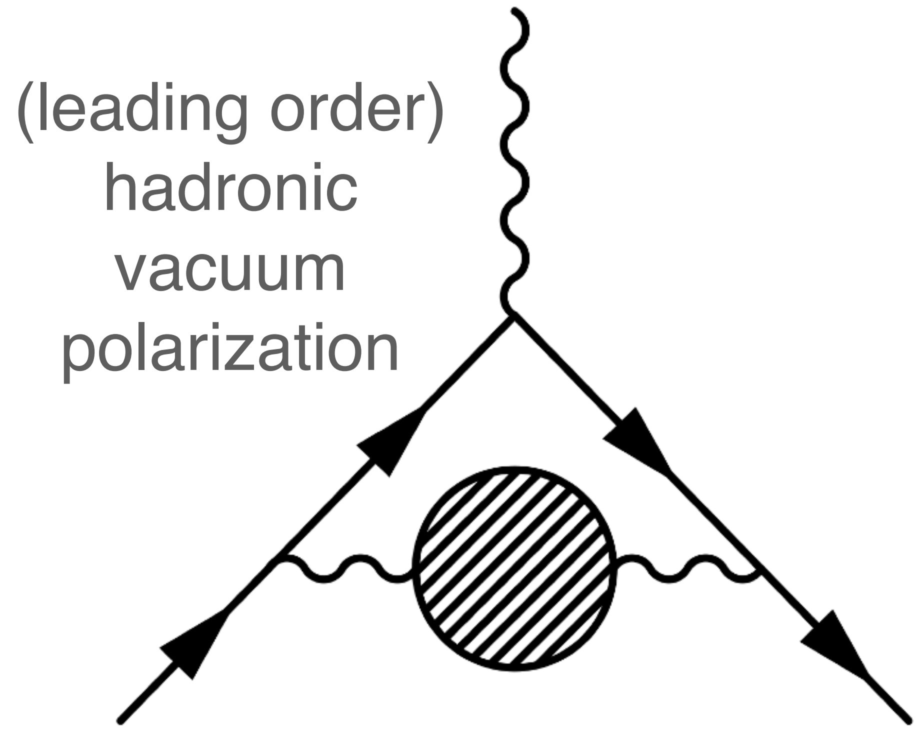
CLQCD ensembles

Renormalization

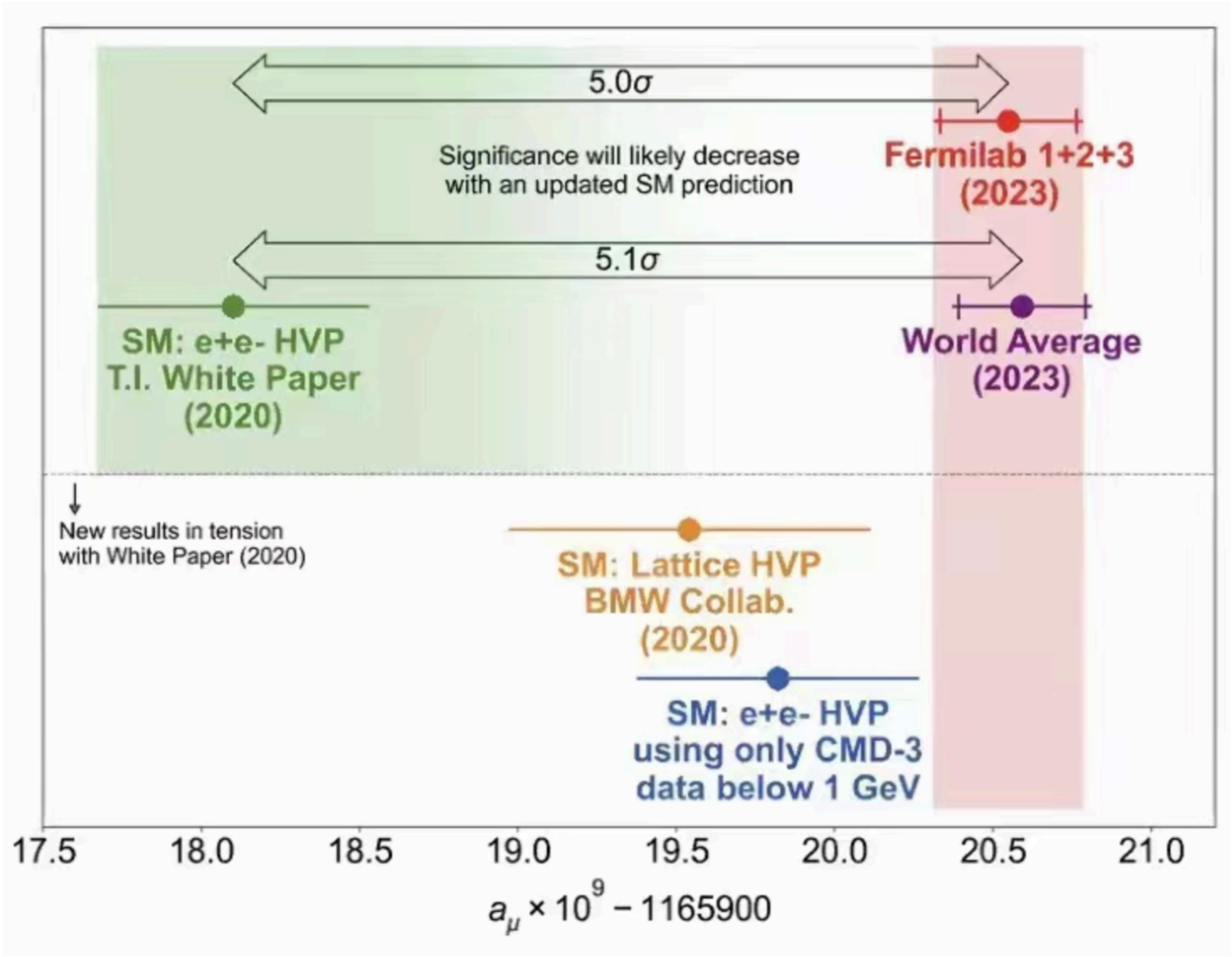
and final results

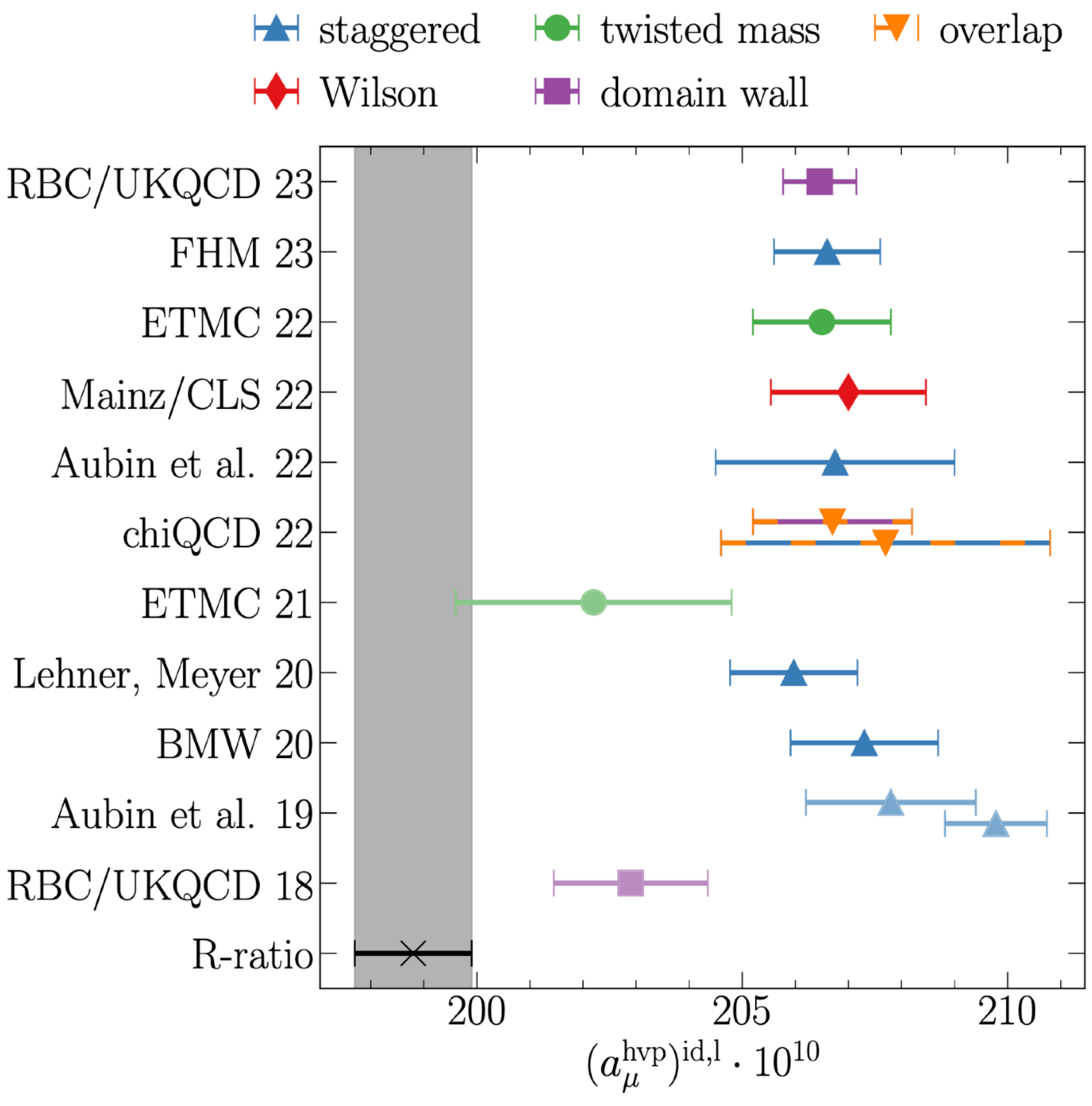


Background



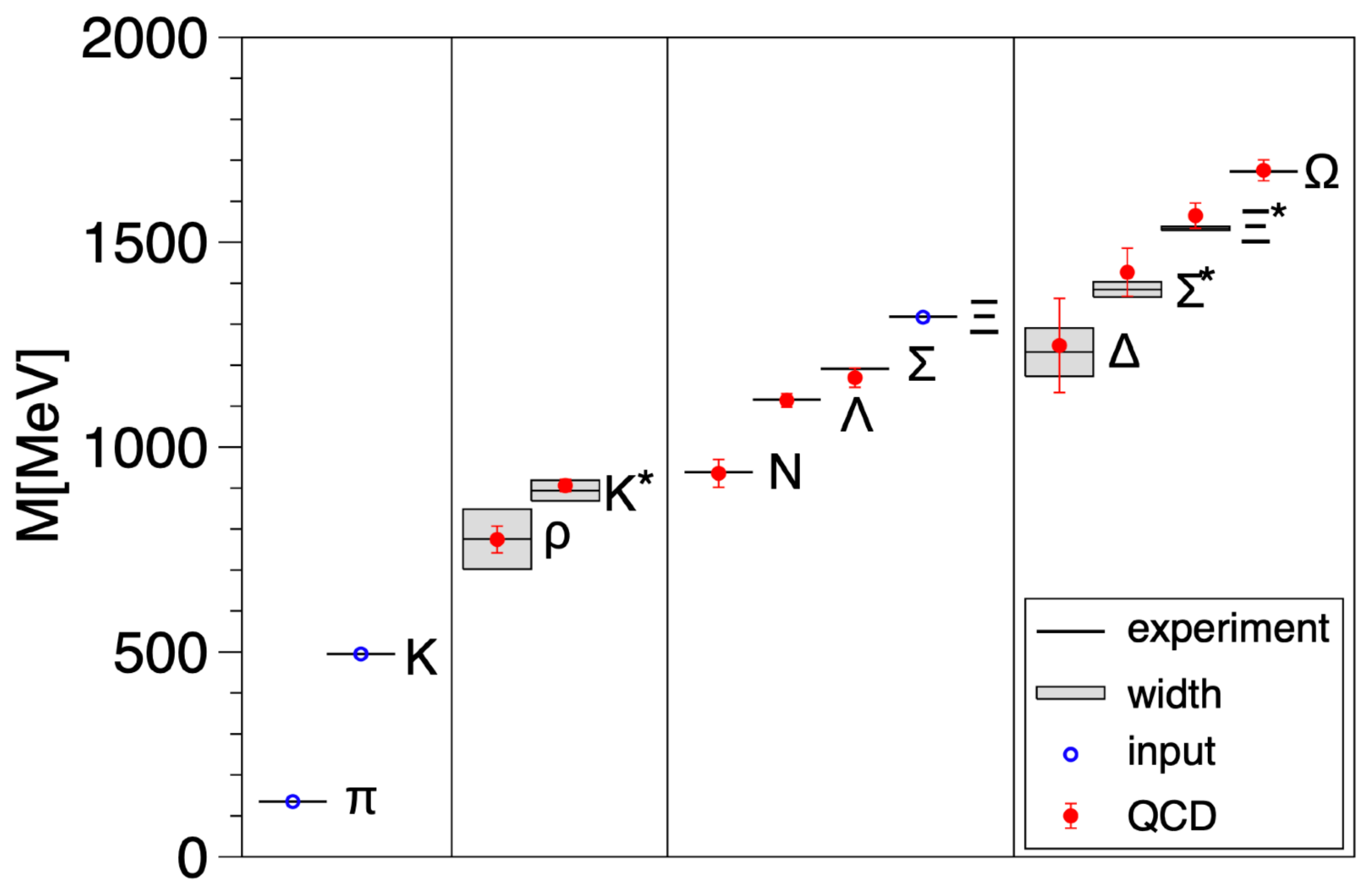
LQCD input for Muon g-2



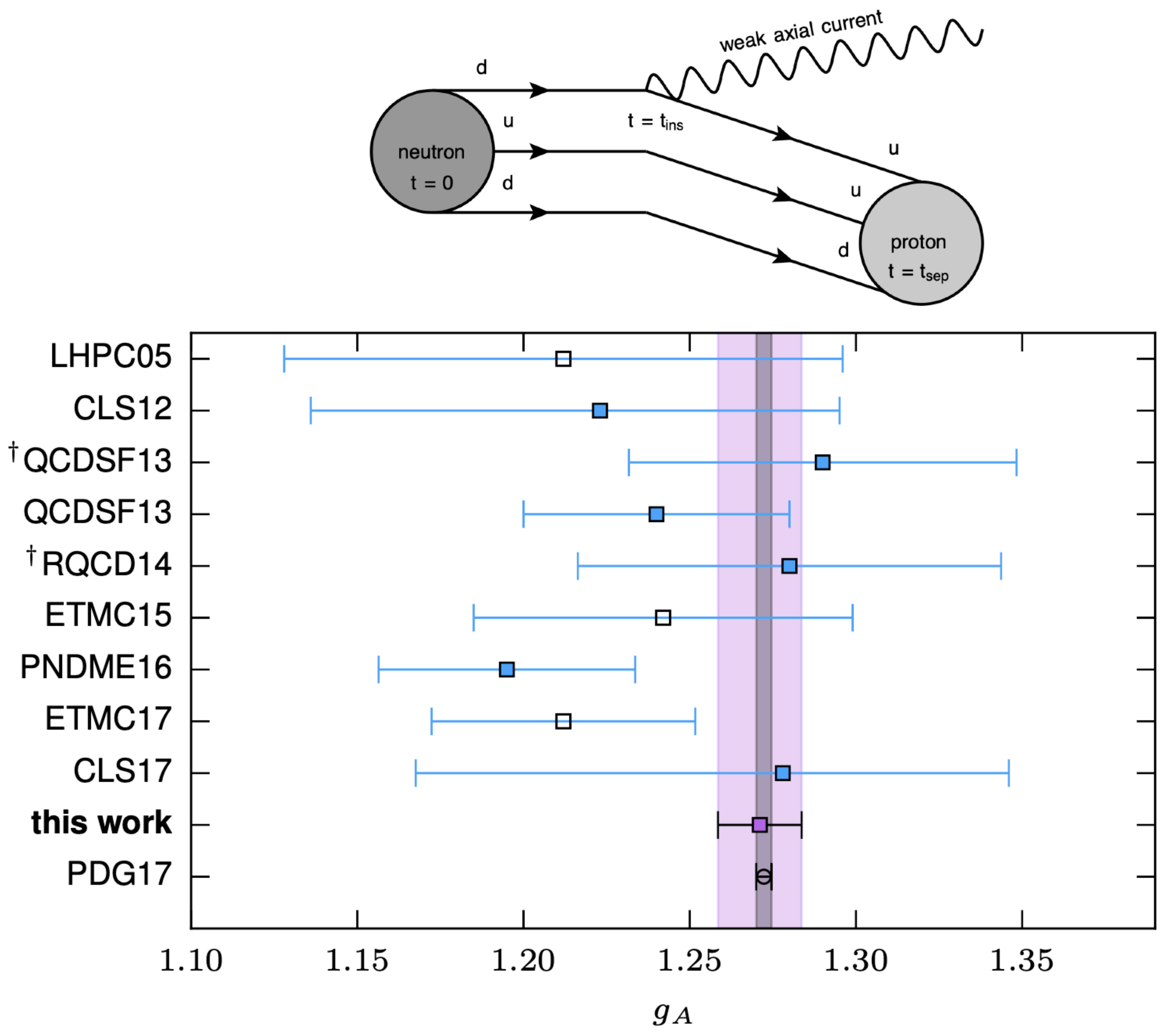


Background

Other high precision LQCD inputs



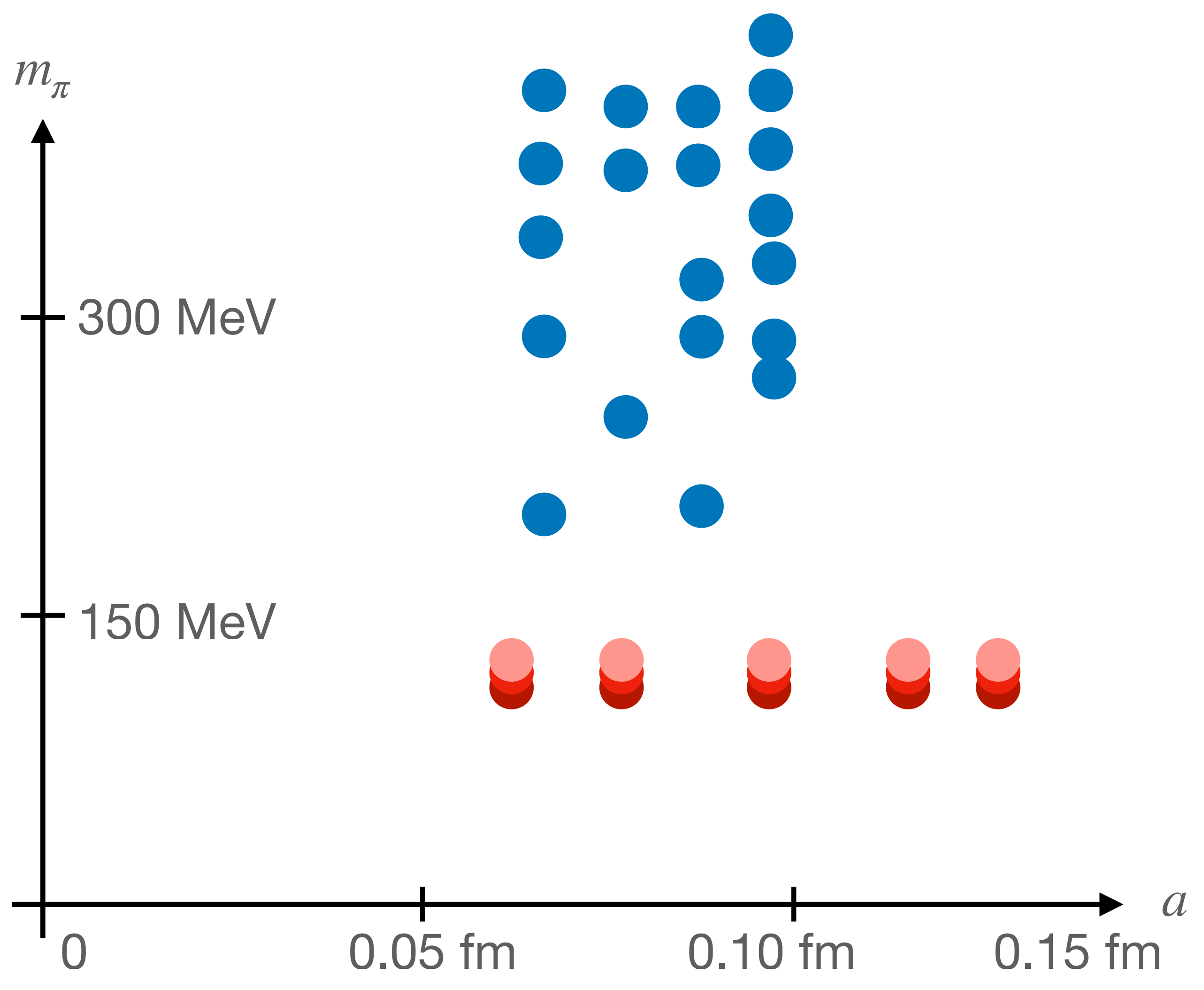
BMWc, Science 322(2008)1224



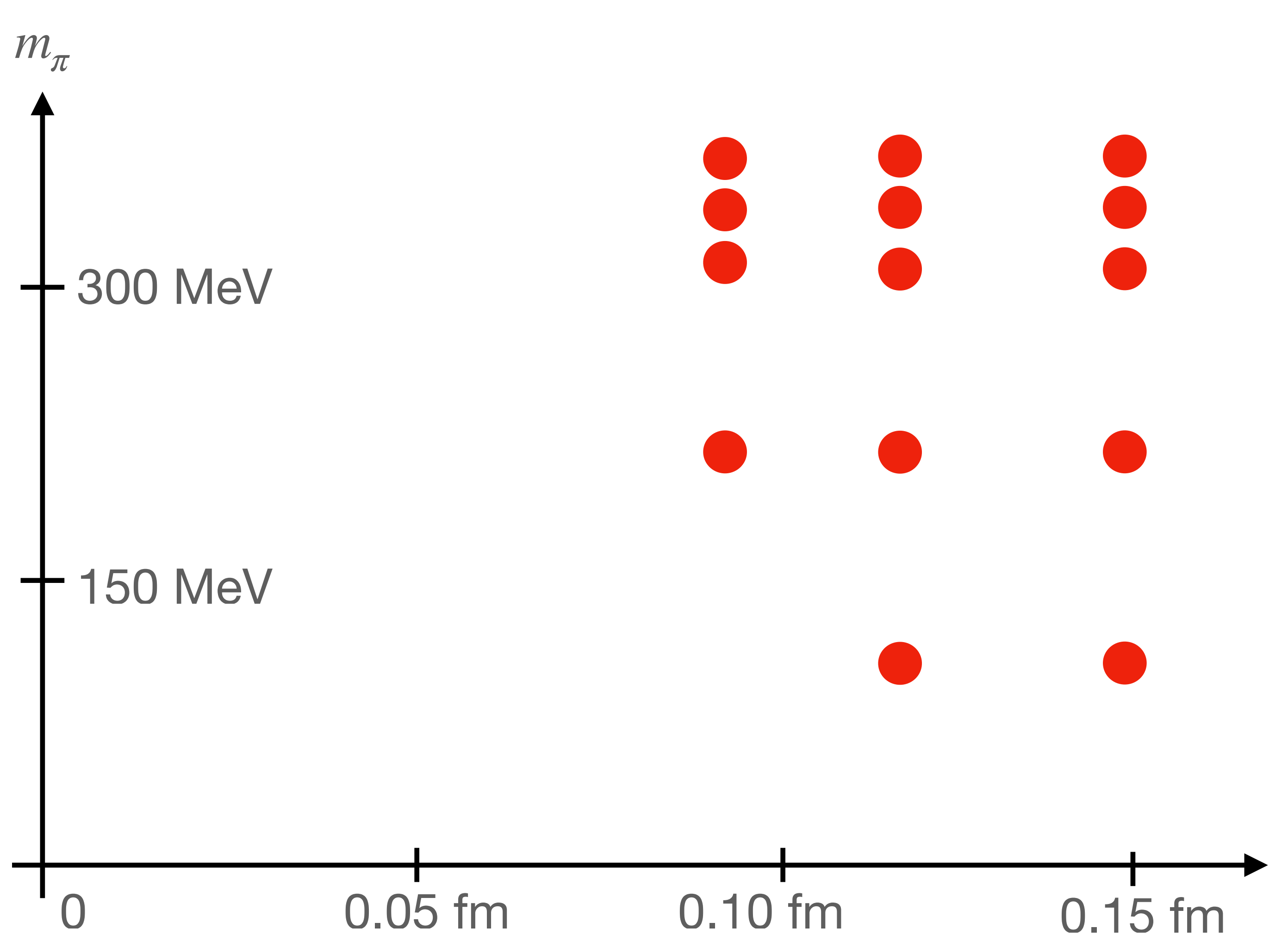
CalLAT, Nature 558(2018)7708,91-94

Background

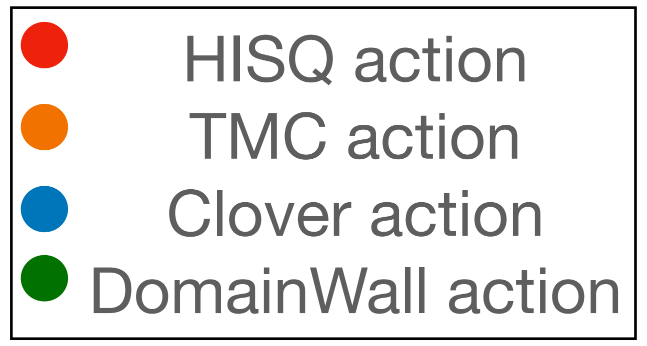
LQCD ensembles used



BMW collaboration



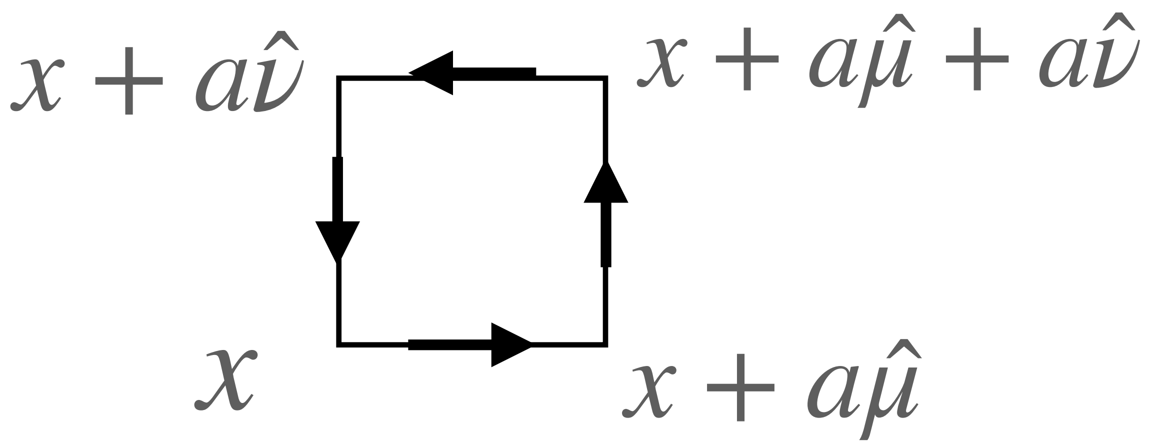
CalLAT (generated by MILC collaboration)

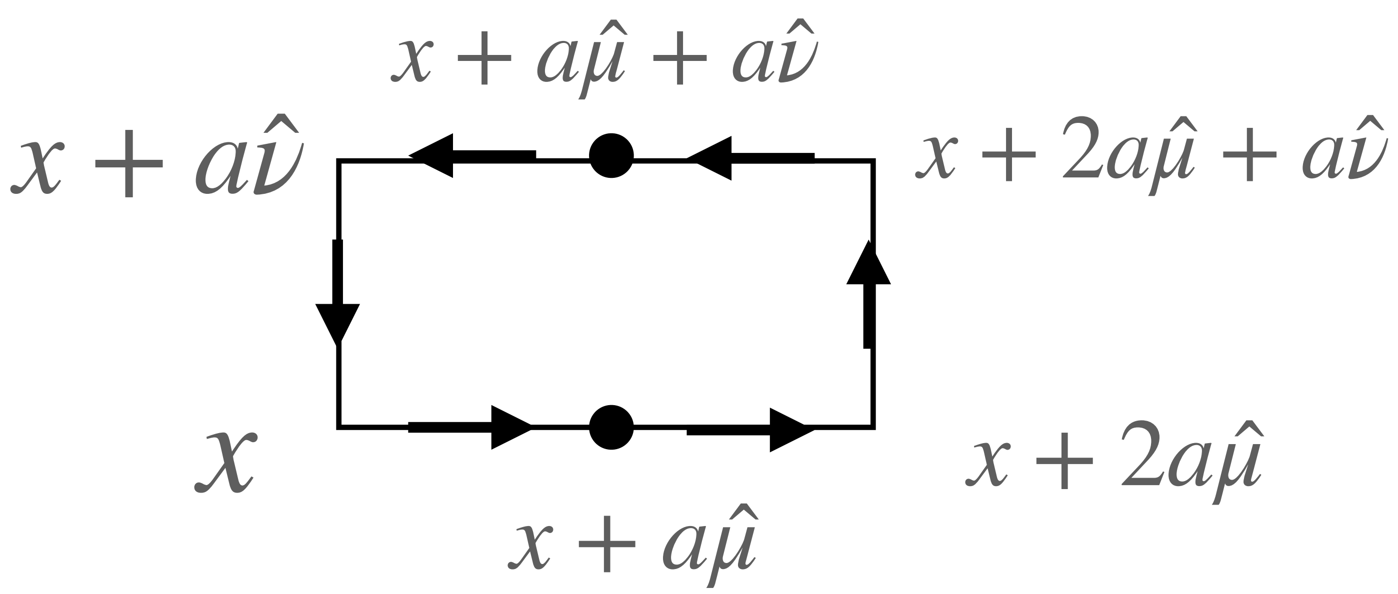


- Several lattice spacings to do the continuum extrapolation;
- Several pion masses to do the chiral extrapolation to physical pion mass;
- Large enough volume or infinite volume extrapolation.

Background

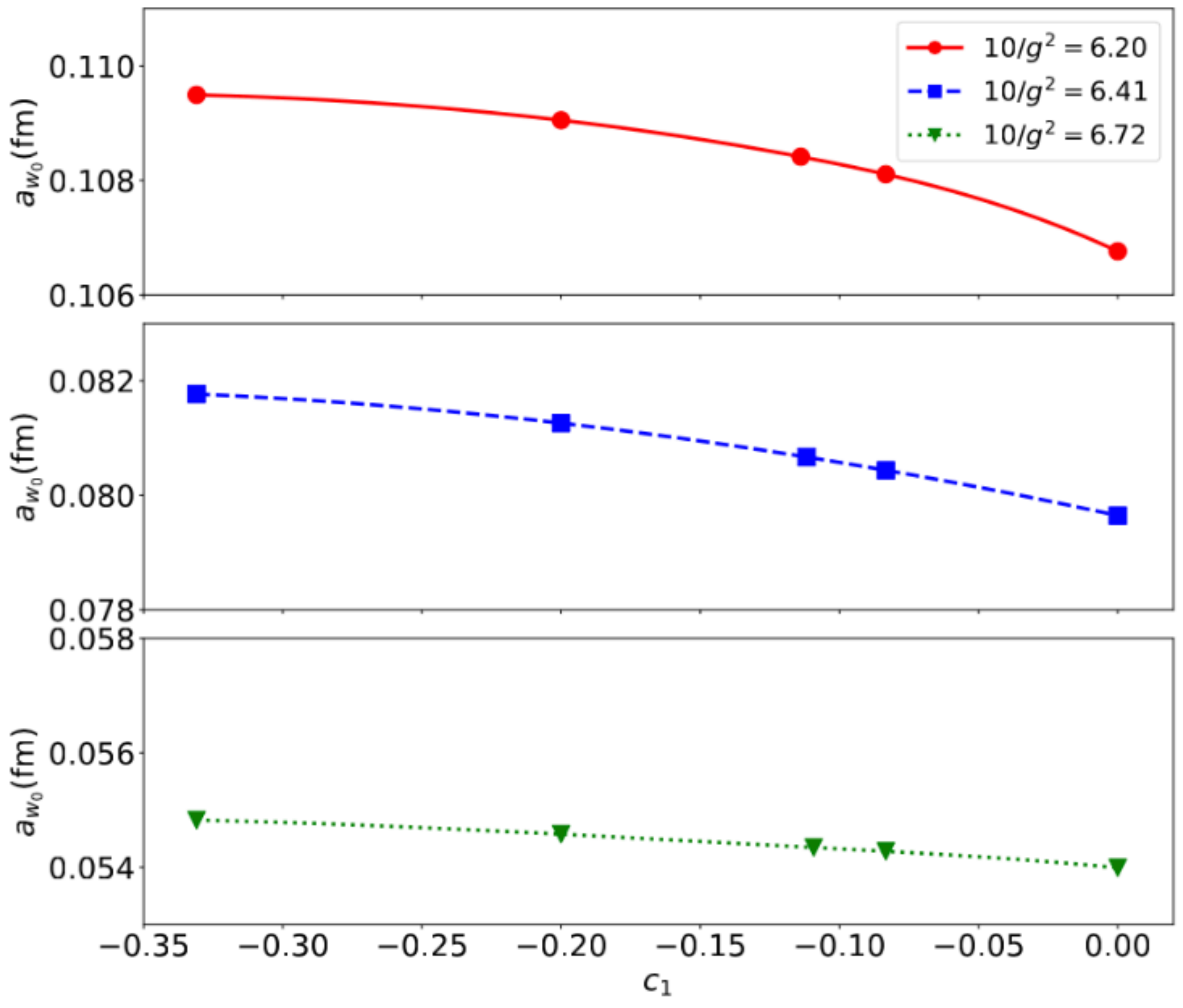
Gauge actions

$$S_g = \sum_{x,\mu,\nu} \Big[\ (1-8c_1)$$ 

$$+ \ c_1$$ 

$$\Big]$$

$$= \int \mathrm{d}^4 x F_{\mu\nu}(x) F^{\mu\nu}(x) + \mathcal{O}(a^2)$$



- Continuum limit should be independent of c_1;
- Larger $|c_1|$ can suppress the discretization error to $\mathcal{O}(a^4)$;
- But enlarge the simulation cost significantly at small a.

Background

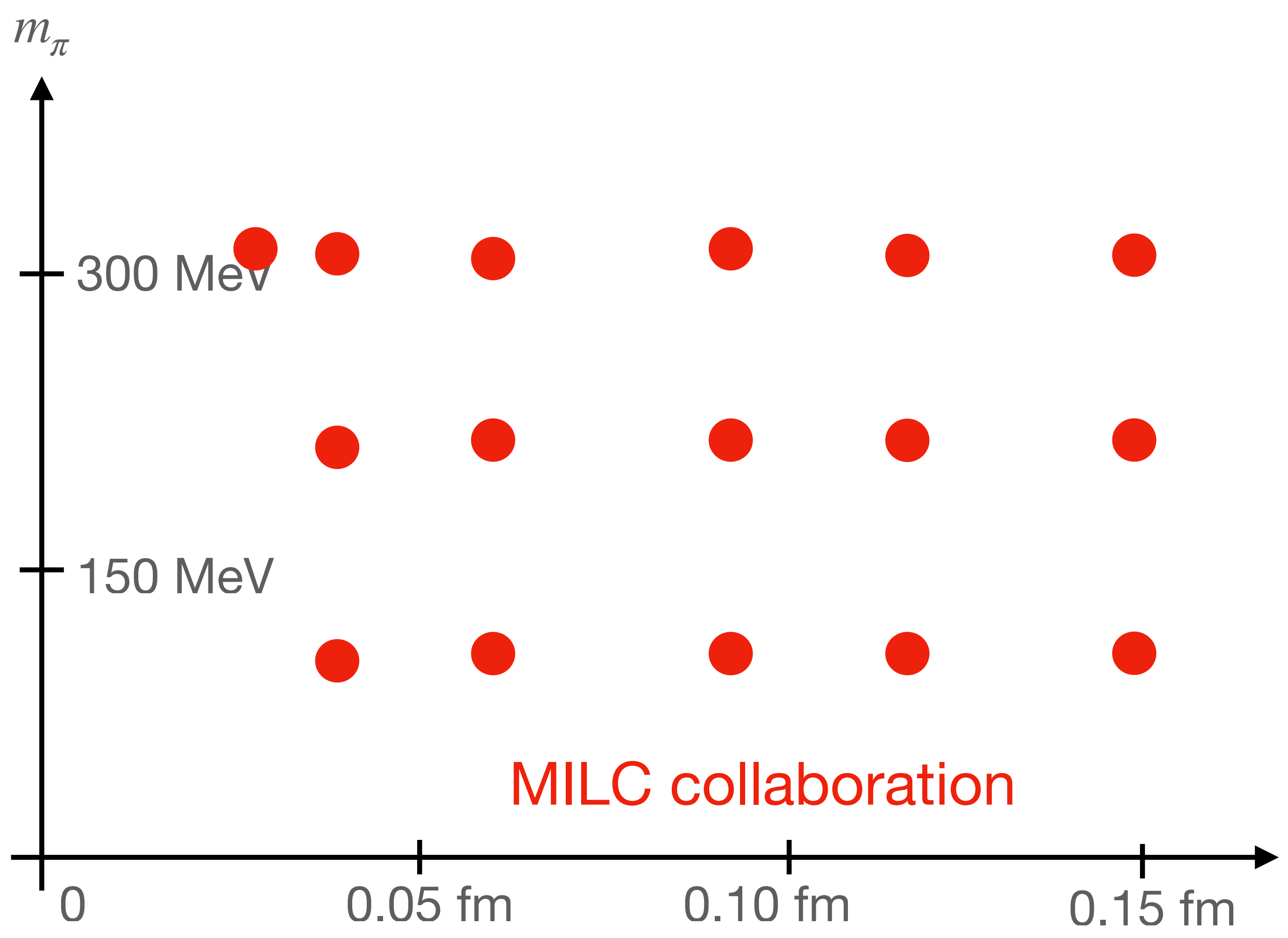
Fermion actions

	Naive	Staggered/HISQ	Wilson/Clover	Twisted-mass	Overlap/Domain wall
Form	$D^{\text{naive}} = \gamma_\mu(\delta_{x,x+\mu} - \delta_{x,x-\mu})$	$D^{\text{st}} = \gamma_\mu^{\text{st}}(x)(\delta_{x,x+\mu} - \delta_{x,x-\mu})$	$D^{\text{clv}} = D + aD^2 + ac_{sw}F_{\mu\nu}\sigma^{\mu\nu}$	$D^{\text{tm}} = D^{\text{clv}} + i\tau_3 m$	$D^{\text{ov}} = [1 + \gamma_5 D(-\rho)/\sqrt{D^\dagger(-\rho)D(-\rho)}]/\rho$
Fermion copies	16	4	1	1	1
Chiral symmetry breaking	N/A	$\mathcal{O}(a^4)$	$\mathcal{O}(\alpha_s/a)$	$\mathcal{O}(\alpha_s)$	N/A
Cost	1	~1/4	~1.1	~1.1	~10-100

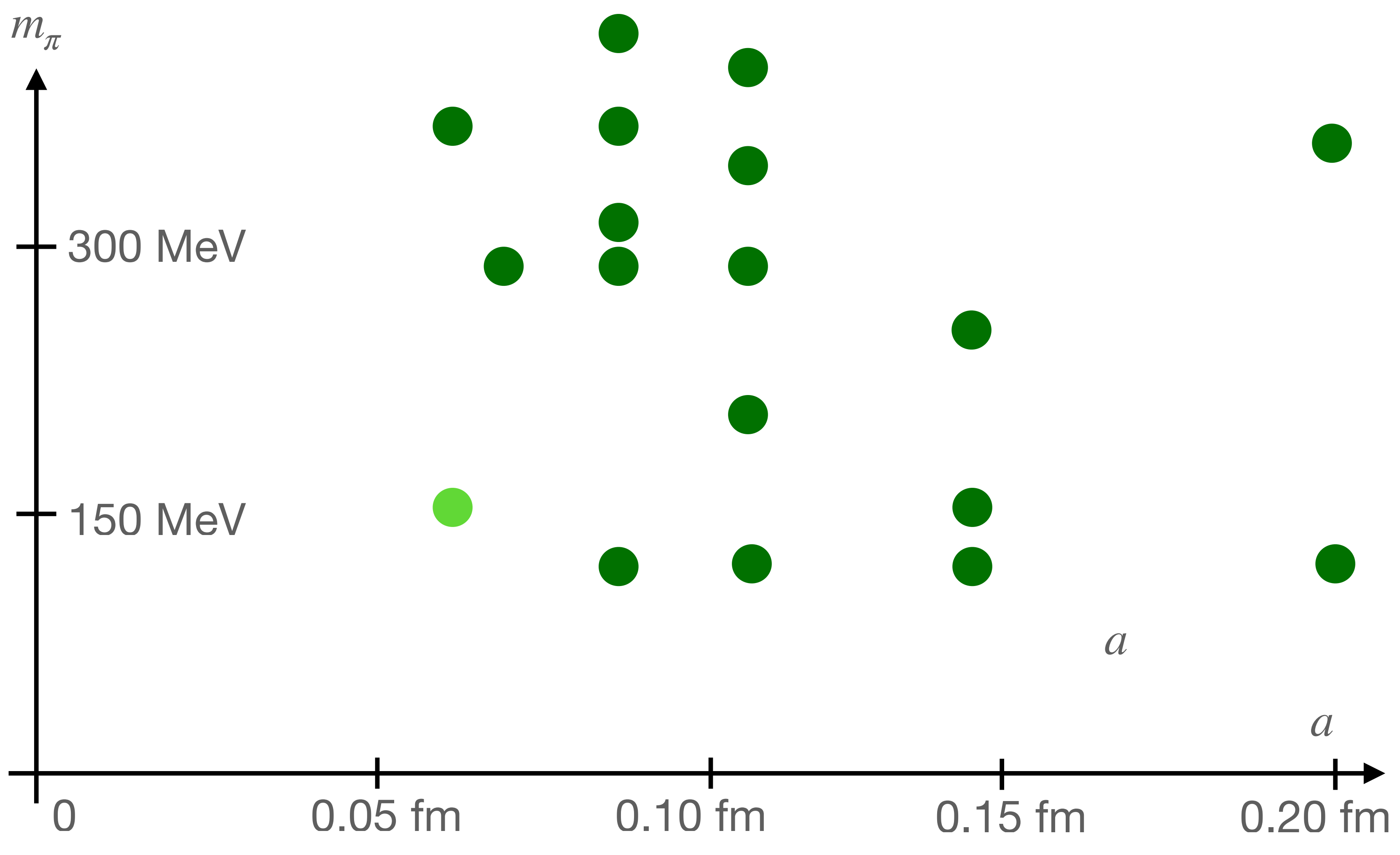
Background

LQCD ensembles worldwide

US



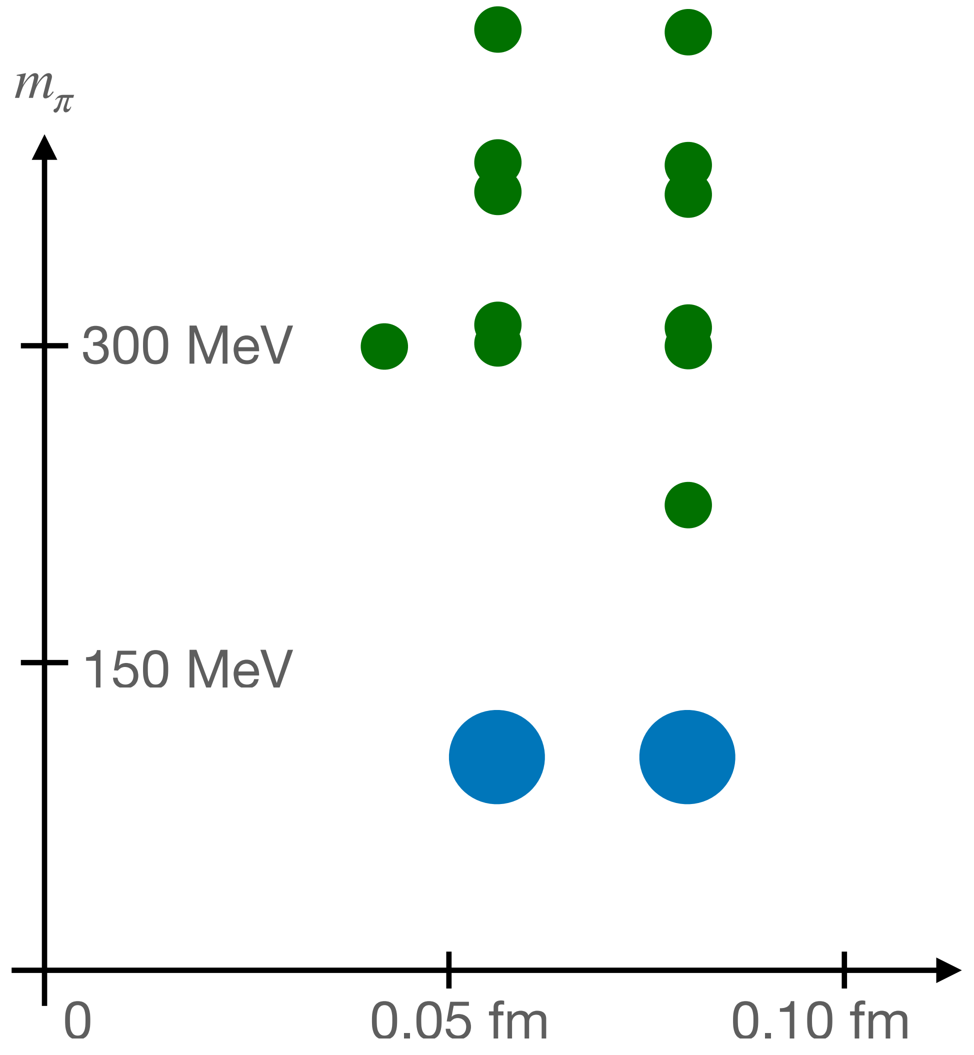
MILC collaboration



RBC/UKQCD collaboration

HISQ action
TMC action
Clover action
DomainWall action

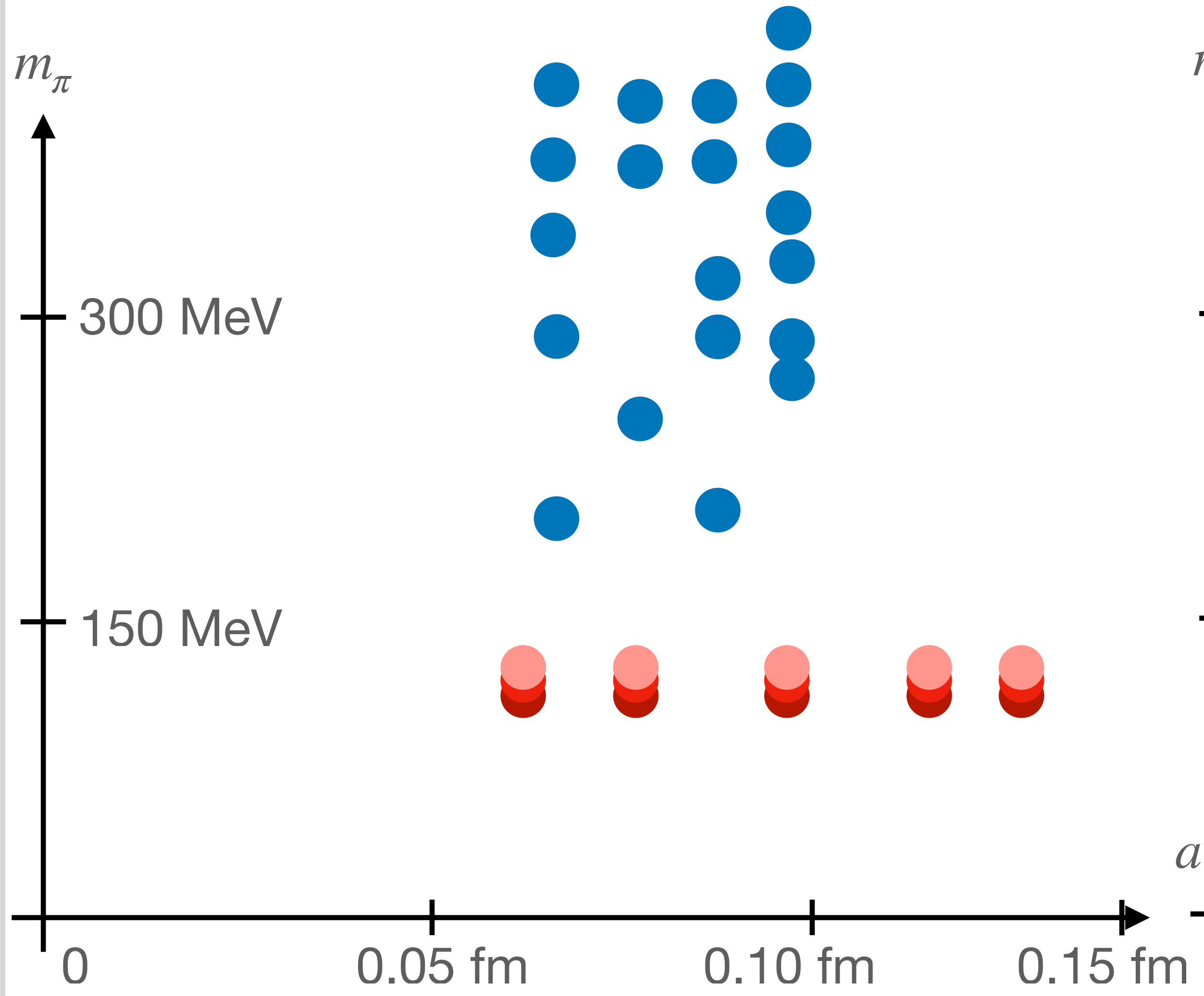
JP



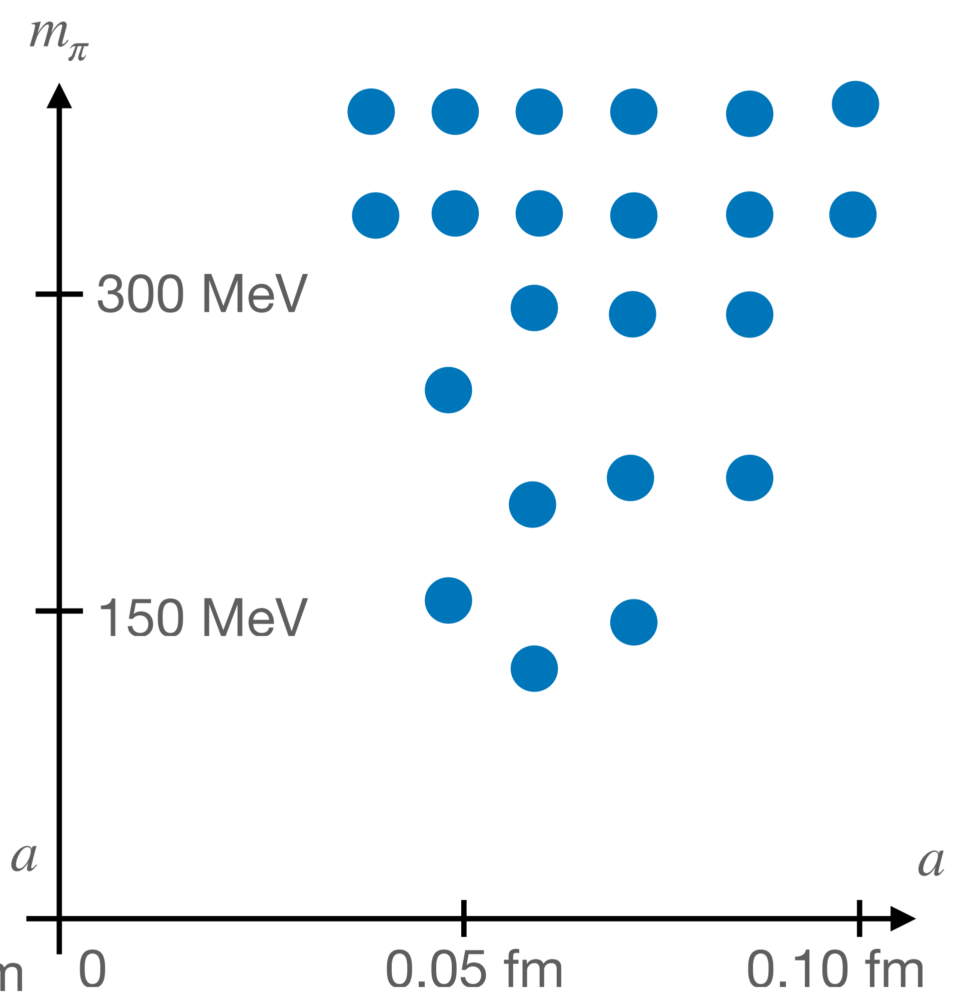
JLQCD collaboration

PACS collaboration

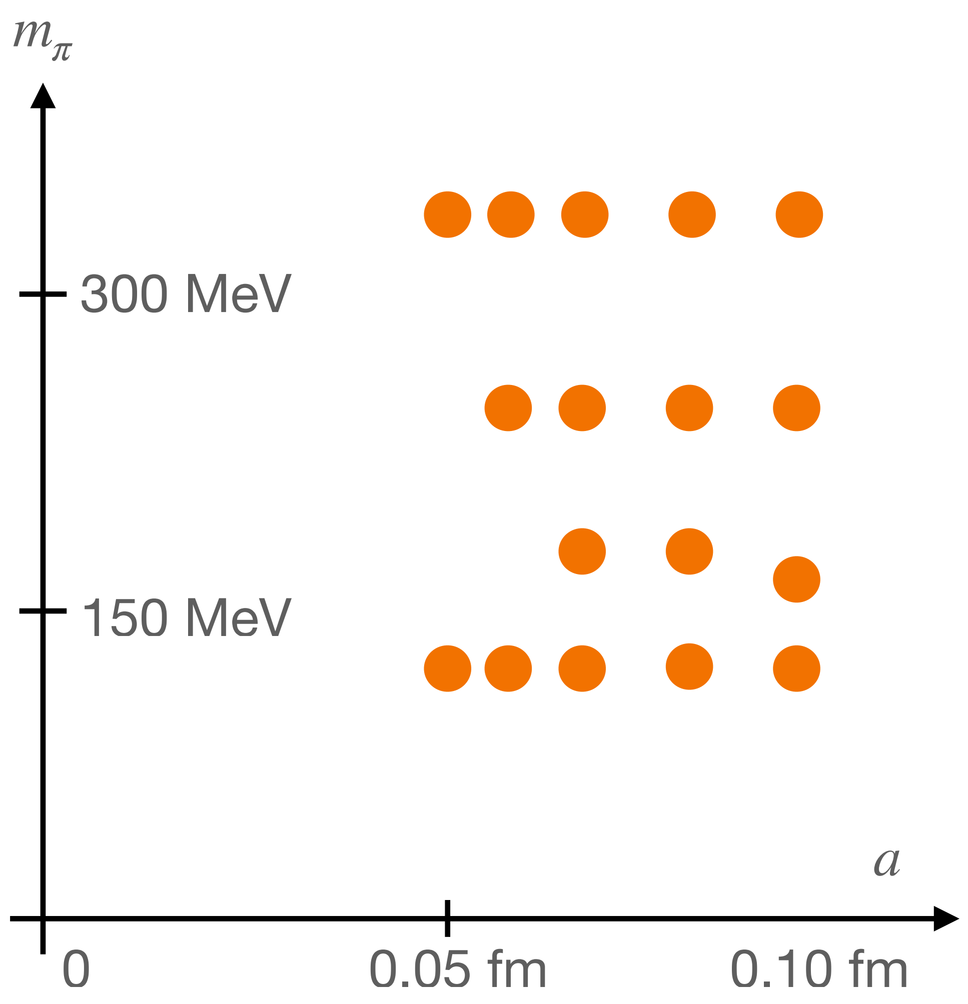
EU



BMW collaboration



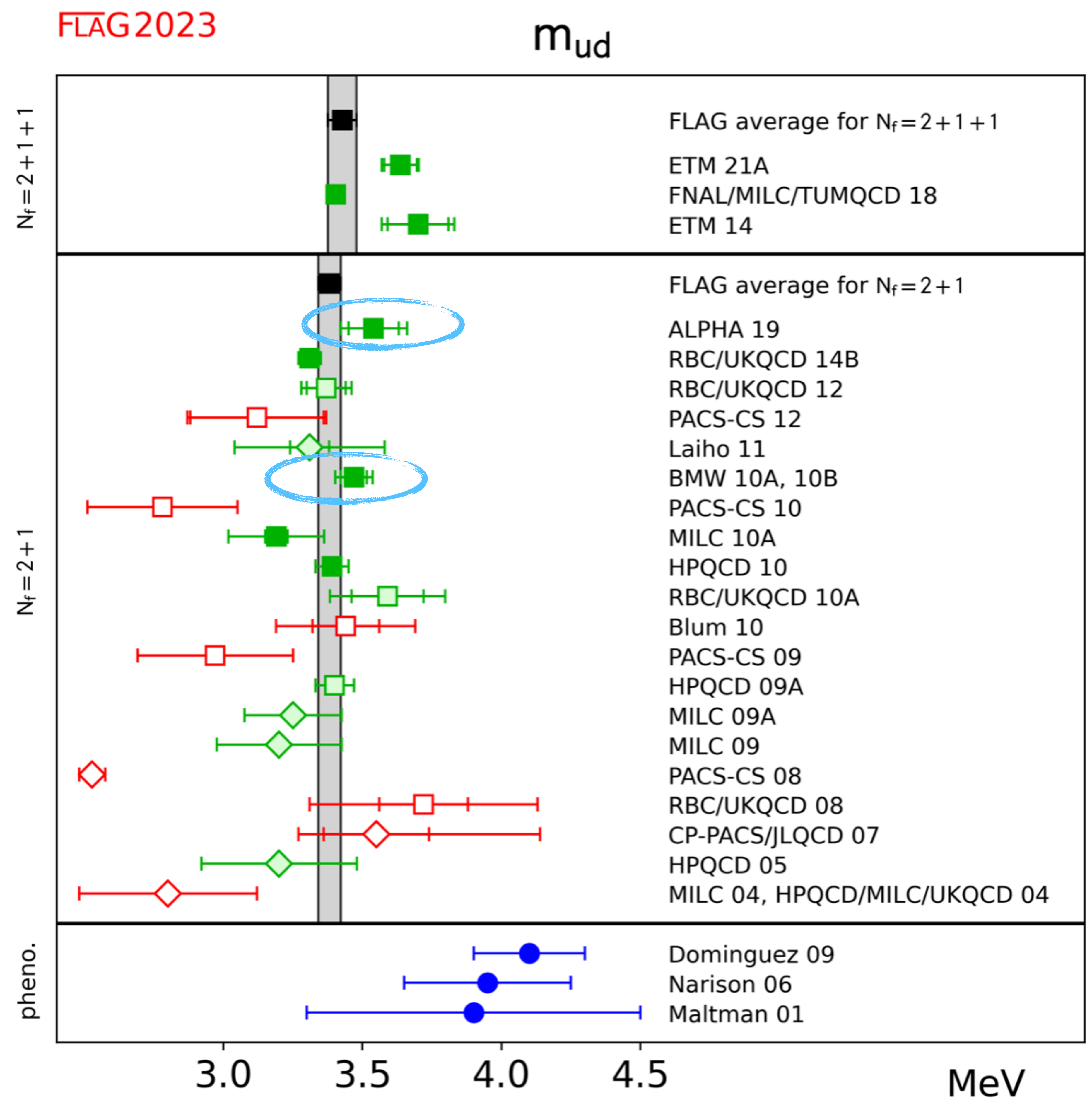
CLS collaboration



ETM collaboration

Background

Quark mass determination



(pdf, bib)

http://flag.unibe.ch/2021/MainPage

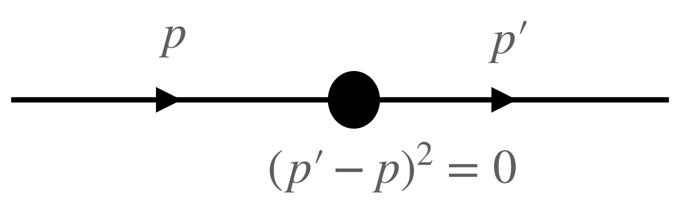
- A natural concern of the clover fermion is, whether its additive chiral symmetry breaking is harmful for the chiral character of QCD, likes the quark mass.
- Only two Clover fermion results are included in the FLAG averages of m_{ud}:
- BMW10A/B: 3.47(05)(05) MeV;

  S. Durr, et.al., BMWc., JEHP08 (2011) 08,148
- ALPHA 19: 3.54(12)(09) MeV.

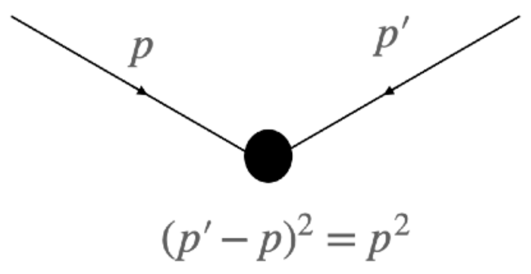
  M. Bruno, et.al., ALPHA, Eur.Phys.J.C80 (2020) 169
- BMW10A/B used the RI/MOM renormalization scheme and claimed 2% uncertainty in total, less than 1% from the renormalization.

Background

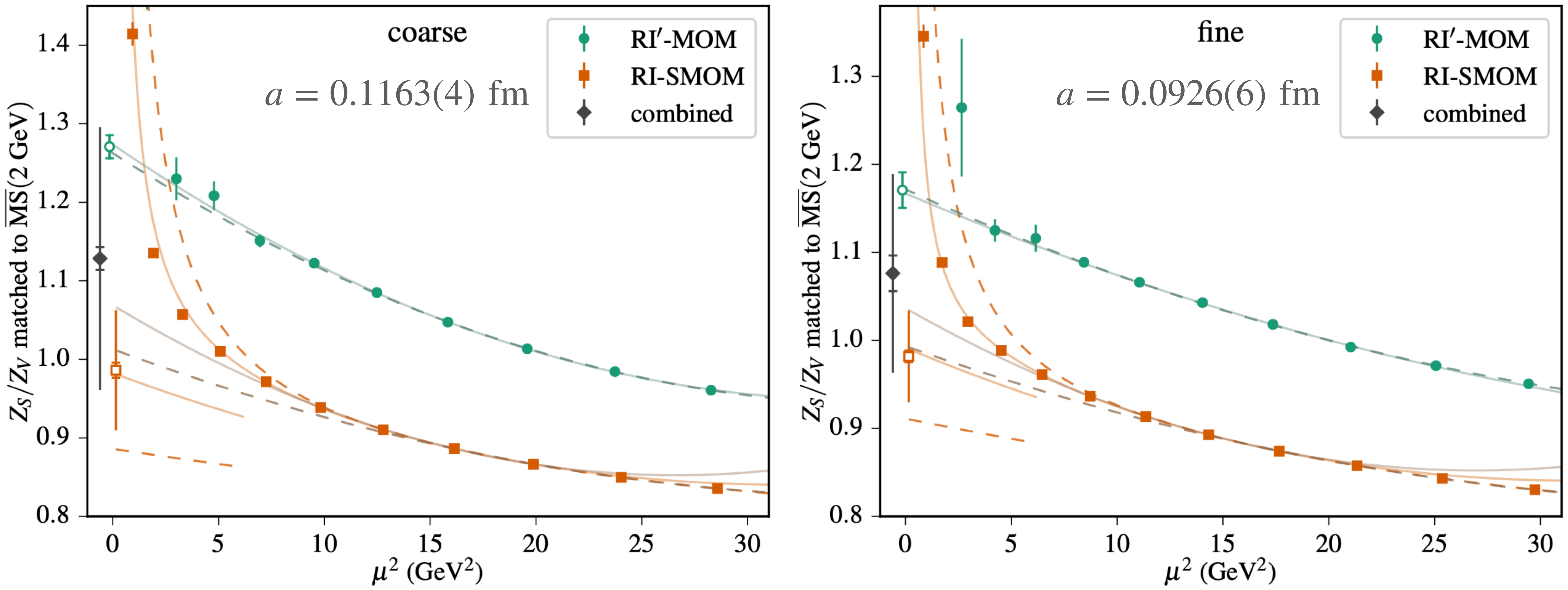
Concerns on the renormalization



MOM G. Martinelli, et.al., NPB445 (1995) 81



SMOM Y. Aoki, et.al., PRD78 (2008) 054510
C. Sturm, et.al., PRD80 (2009) 014501

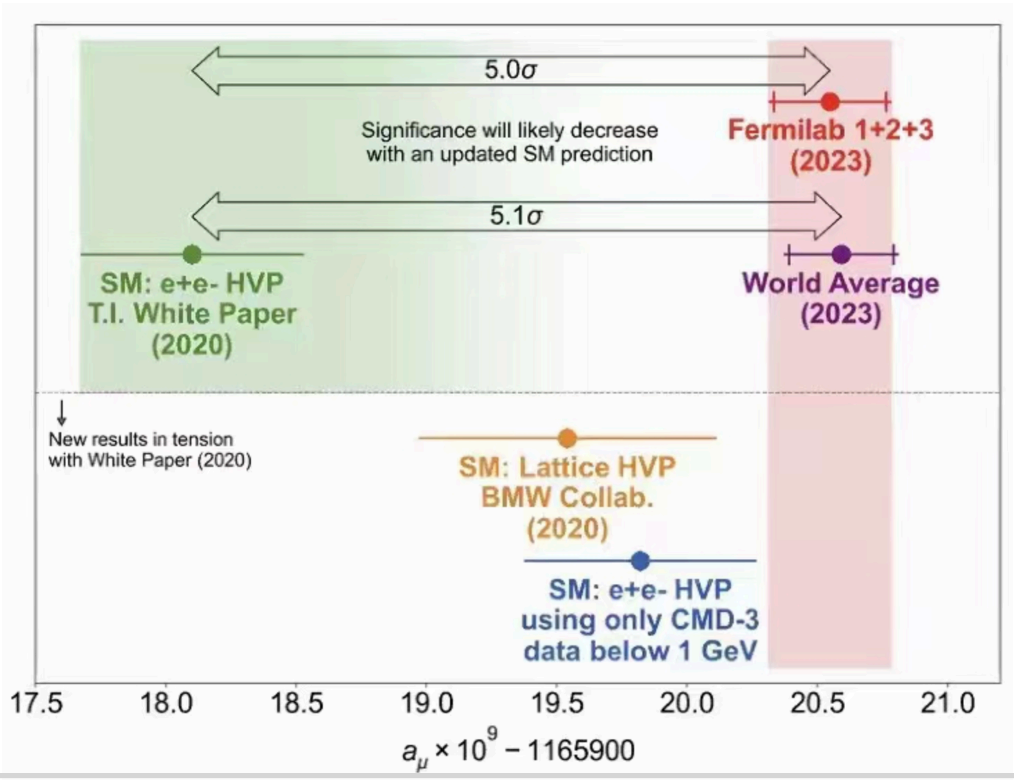


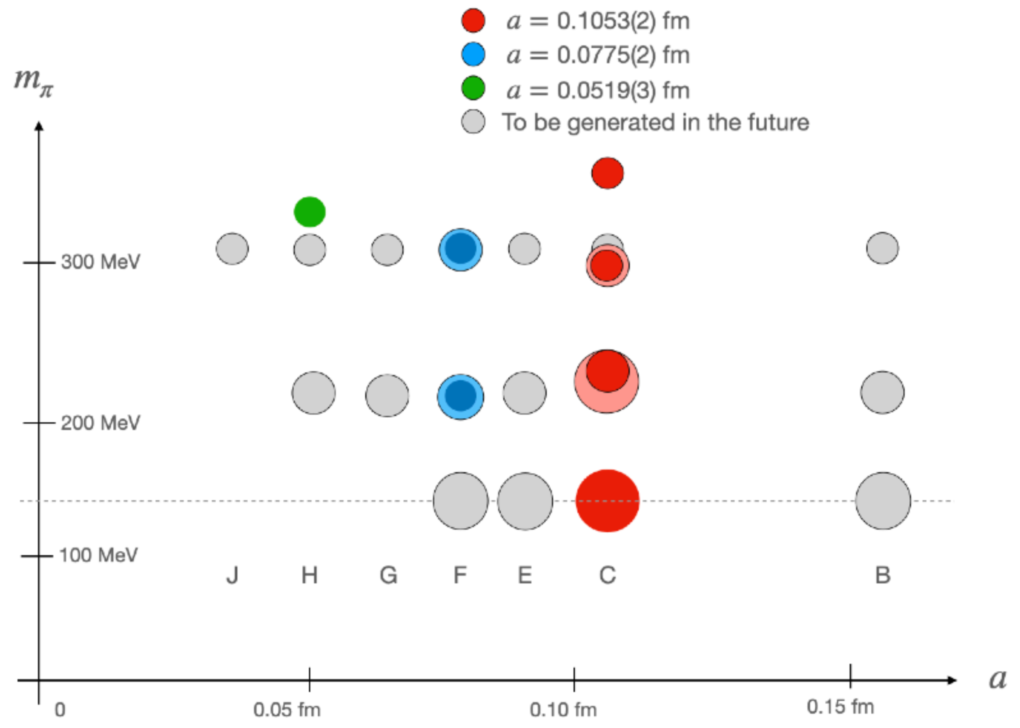
N. Hasan, et.al., Phys.Rev.D99 (2019) 114505

- But more recent study suggests that using different intermediate renormalization scheme (and then convert to $\overline{\mathrm{MS}}$ scheme) can make Z_S to differ by 30% at $a = 0.11$ fm.
- The systematic uncertainty of the renormalization should be rechecked,
- and also the other quantities relate to the chiral symmetry.

Outline

Background

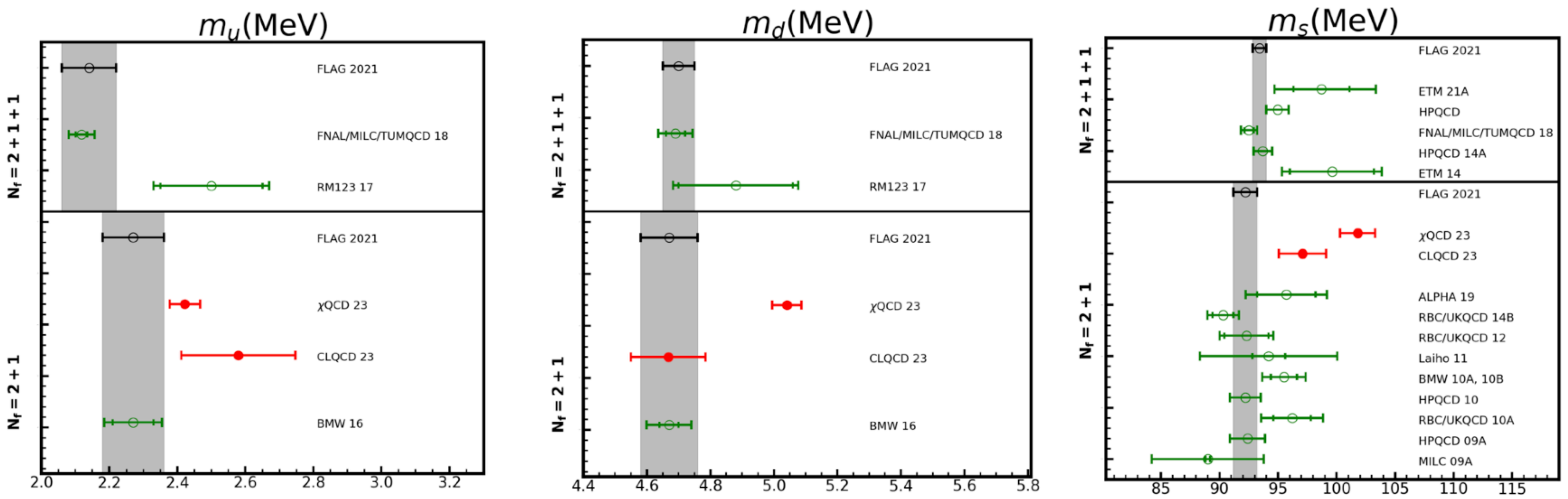




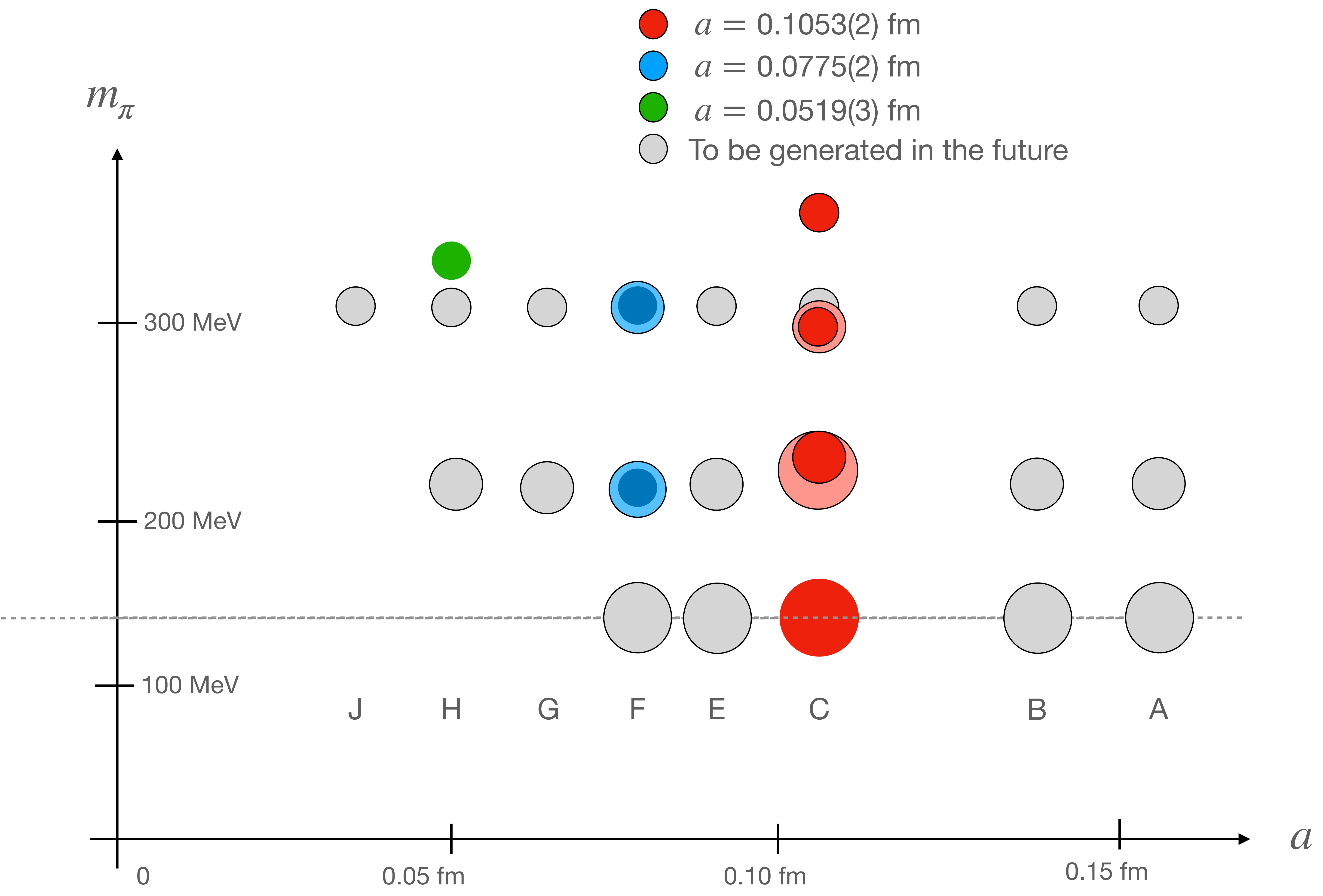
CLQCD ensembles

Renormalization

and final results



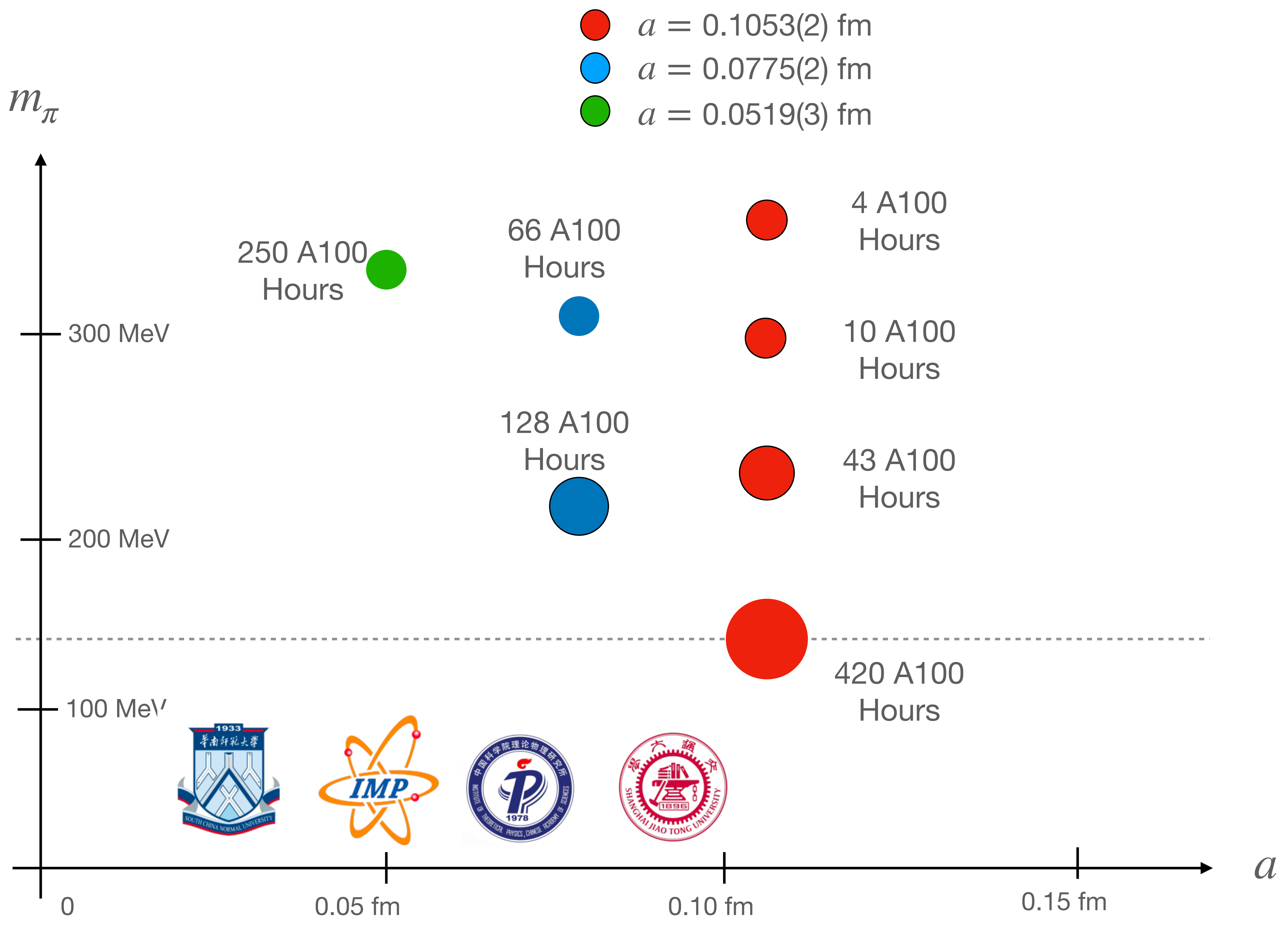
CLQCD ensembles



CLQCD choice and advantages

- Maximum lattice size $48^3 \times 144$:
  - Can reach the FLAG “green star” criteria with lowest cost.
- Clover fermion action with stout smearing:
  - Can reach the physical quark mass at coarse $a \sim$ 0.1053(2) fm;
  - Much cheaper than the OV/DW fermion but free of the fermion doubling;
  - Additive chiral symmetry breaking can be resolved after the continuum extrapolation, with proper renormalization (to be shown later).
- Symanzik gauge action with tadpole improvement:
  - Can reach quite fine $a \sim$ 0.03 fm based on the study of MILC collaboration.
- Similar pion mass and volume at different lattice spacing:
  - Can estimate the discretization error with given pion mass and/or momentum.

CLQCD ensembles

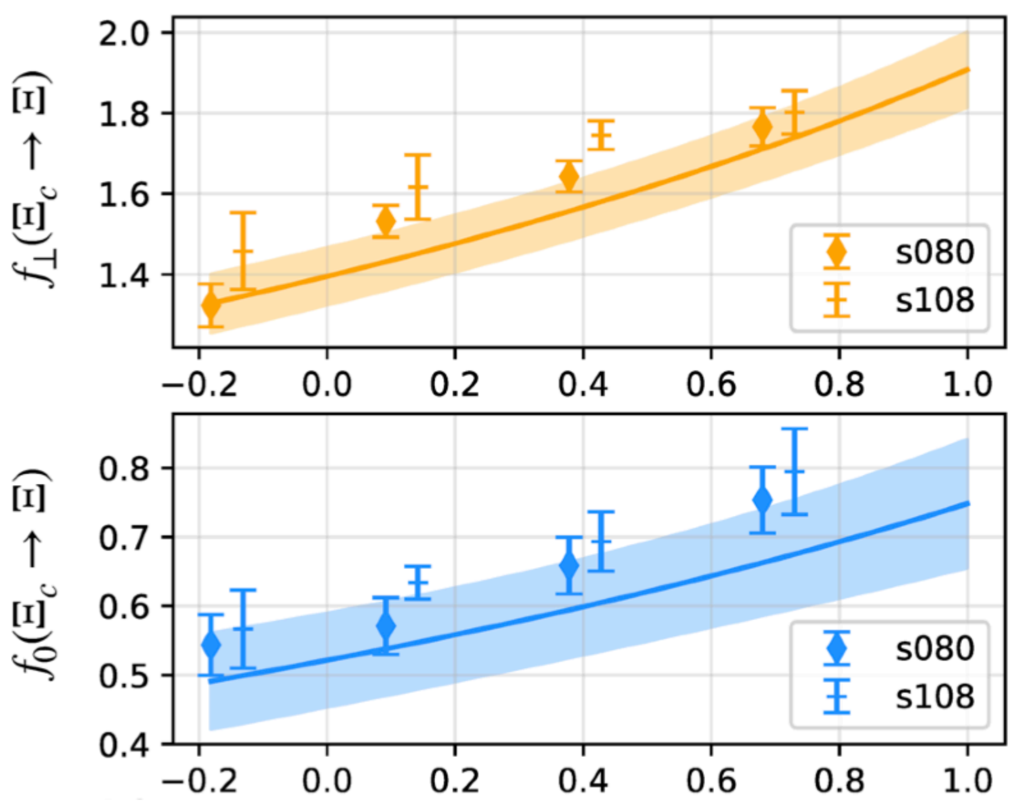


Cost of each ensemble

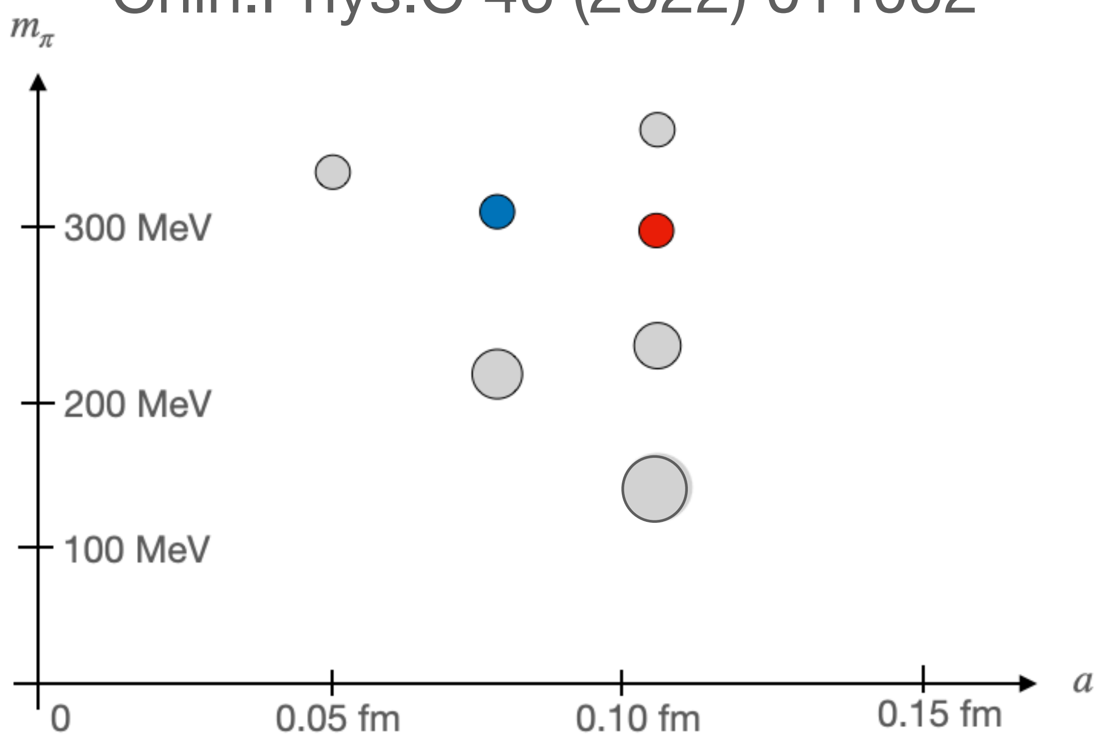
- Cost of an independent configuration (per 10 traj.'s) with $\tau = 1.0$, converted to A100 GPU hours;
- Needs ~1,000 configurations per ensemble;
- Currently used 658k A100 hours, equals to 3.3M Chinese Yuan with the market price.
- Working on the Sugon machines to avoid the embargo of A100 GPU.

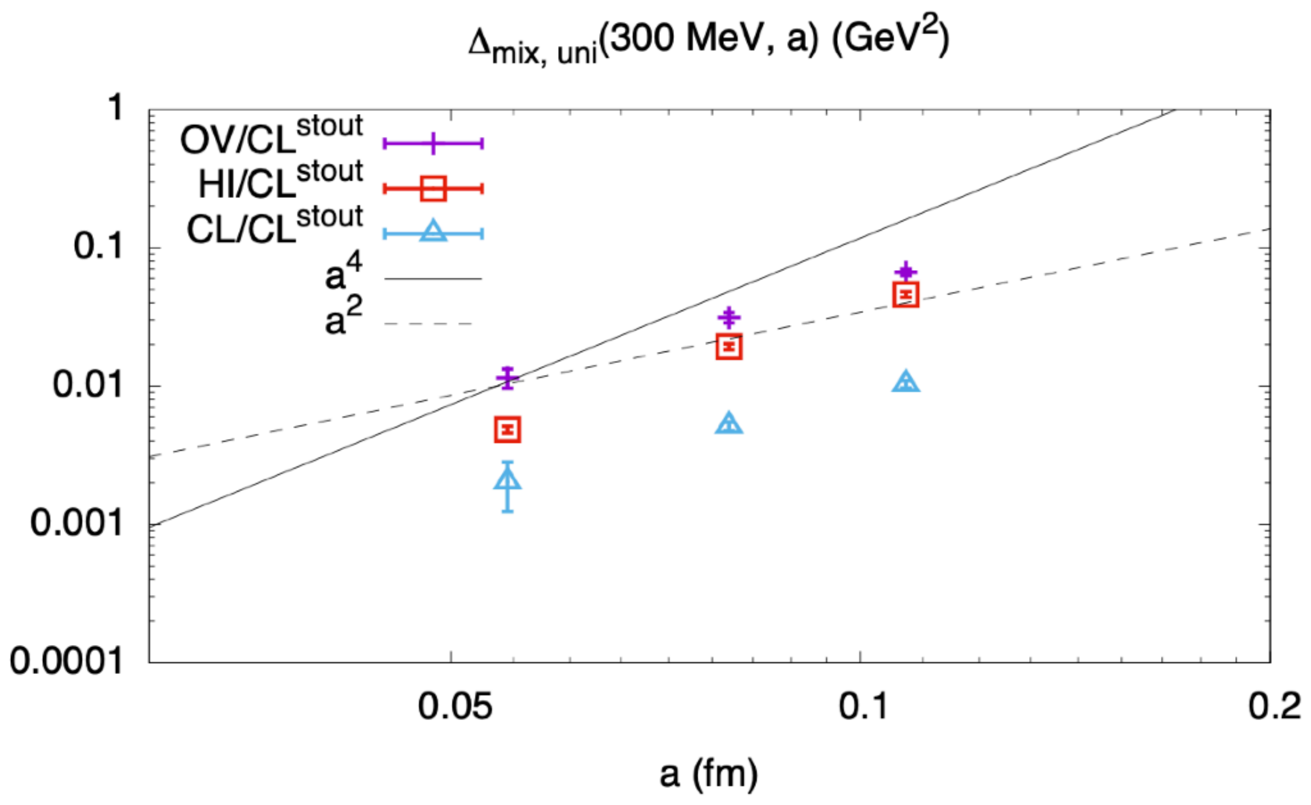
CLQCD ensembles

Published/accepted works with the CLQCD ensembles

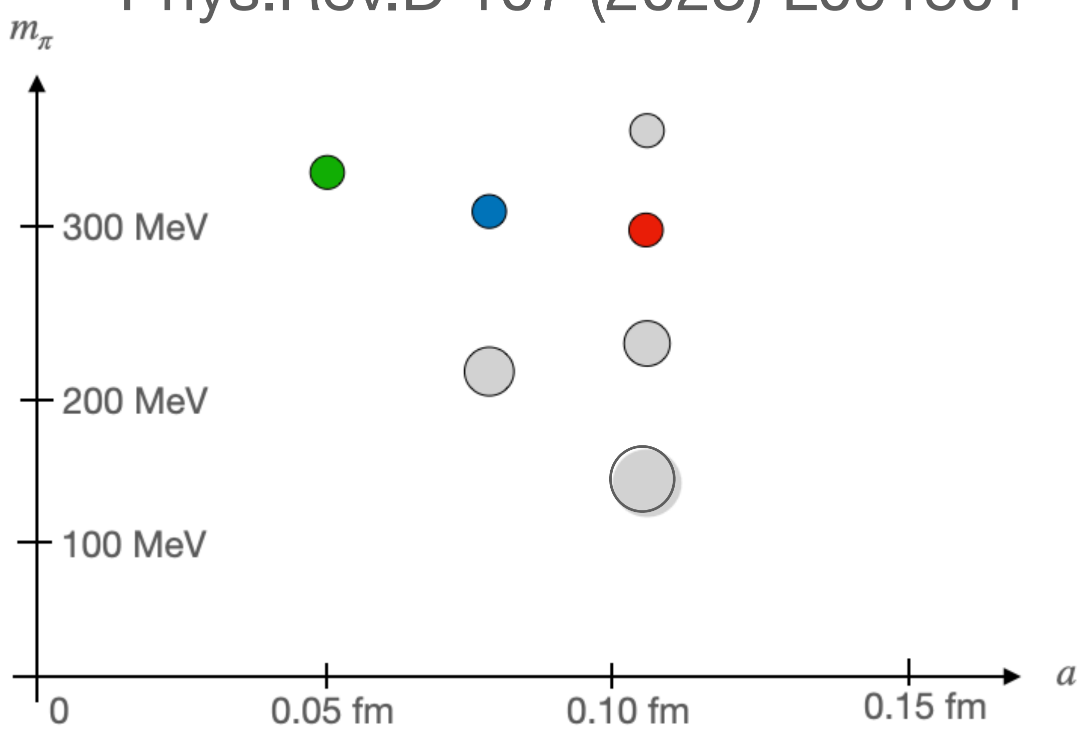


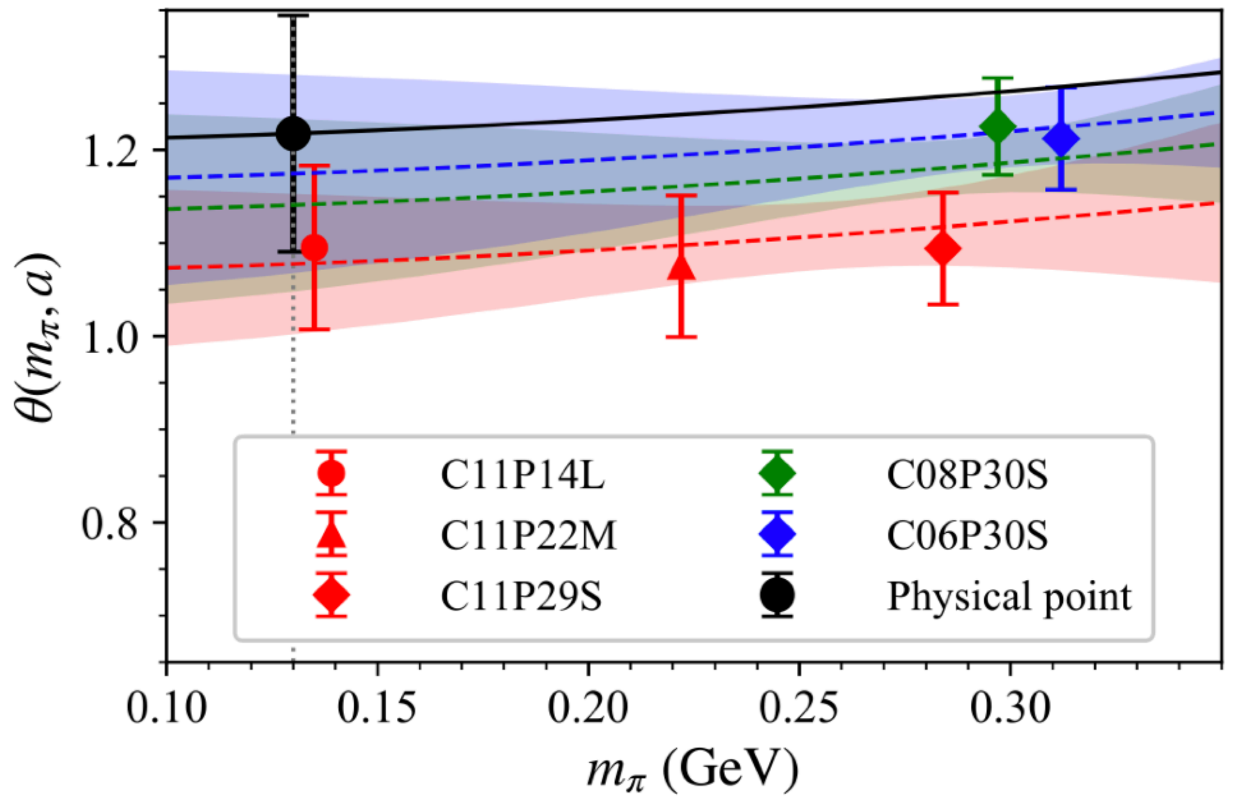
Q.A. Zhang, et.al.,
Chin.Phys.C 46 (2022) 011002



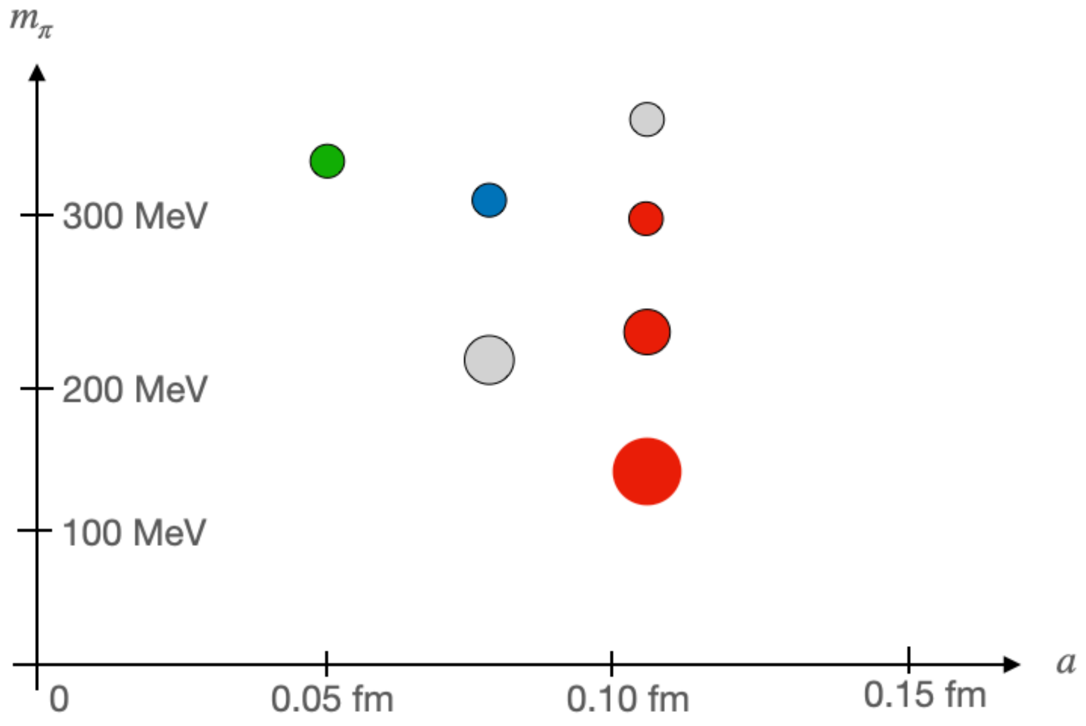


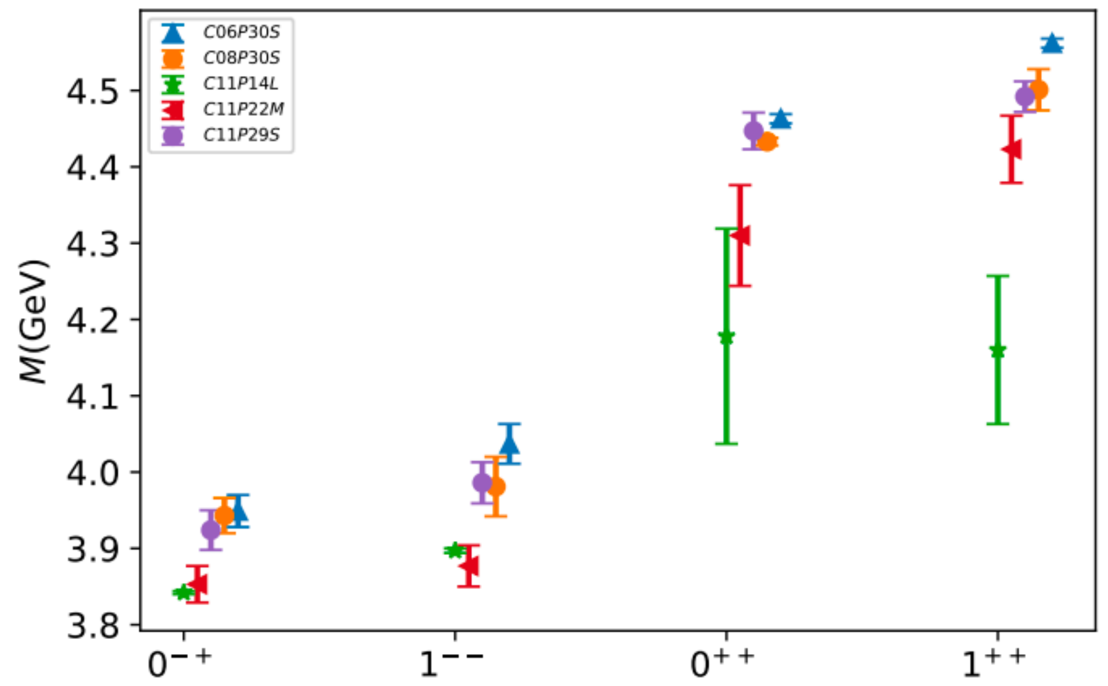
D.J. Zhao, et.al., χQCD,
Phys.Rev.D 107 (2023) L091501





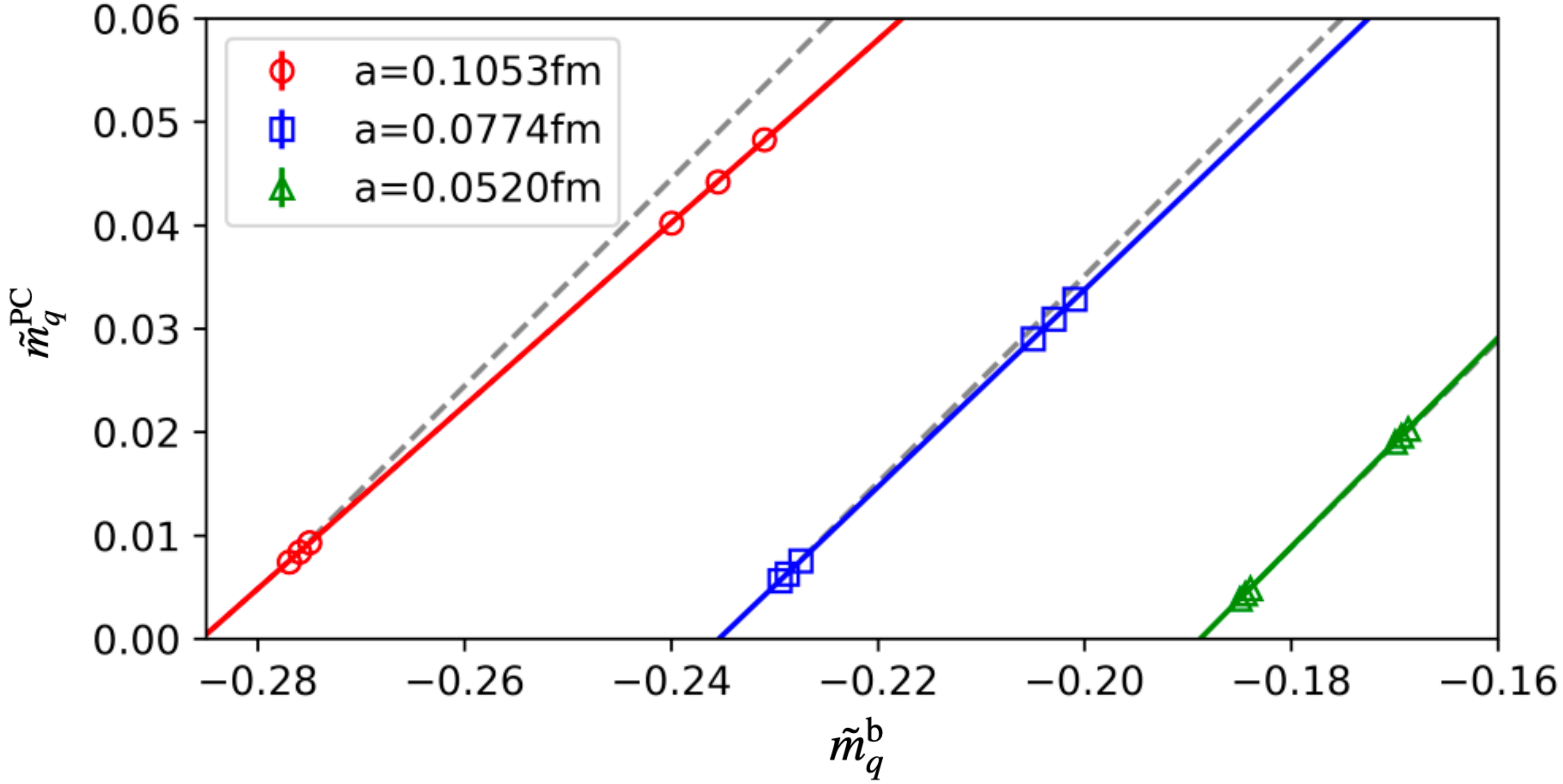
Q.A. Zhang, et.al.,
Phys.Lett.B 841 (2023) 137941





H. Liu, et.al.,
2207.00183, accepted by SCPMA

CLQCD ensembles



Quark mass through PCAC

- Due to the additive α_s/a correction, the dimensionless bare quark mass $\tilde{m}_q^{\rm b} = m_q^{\rm b} a$ is negative.
- The renormalized quark mass should be defined as $m_q^R = Z_m(m_q^{\rm b} - m_{\rm crti})$, where $m_{\rm crti}$ is defined as the $m_q^{\rm b}$ which vanishes the pion mass.
- One can avoid this difficulty by defining the quark mass through PCAC relation:

$$\langle 0 | \partial_4 A_4 | \mathrm{PS} \rangle = (m_q^{\rm PC} + m_{\bar{q}}^{\rm PC}) \langle 0 | P | \mathrm{PS} \rangle$$

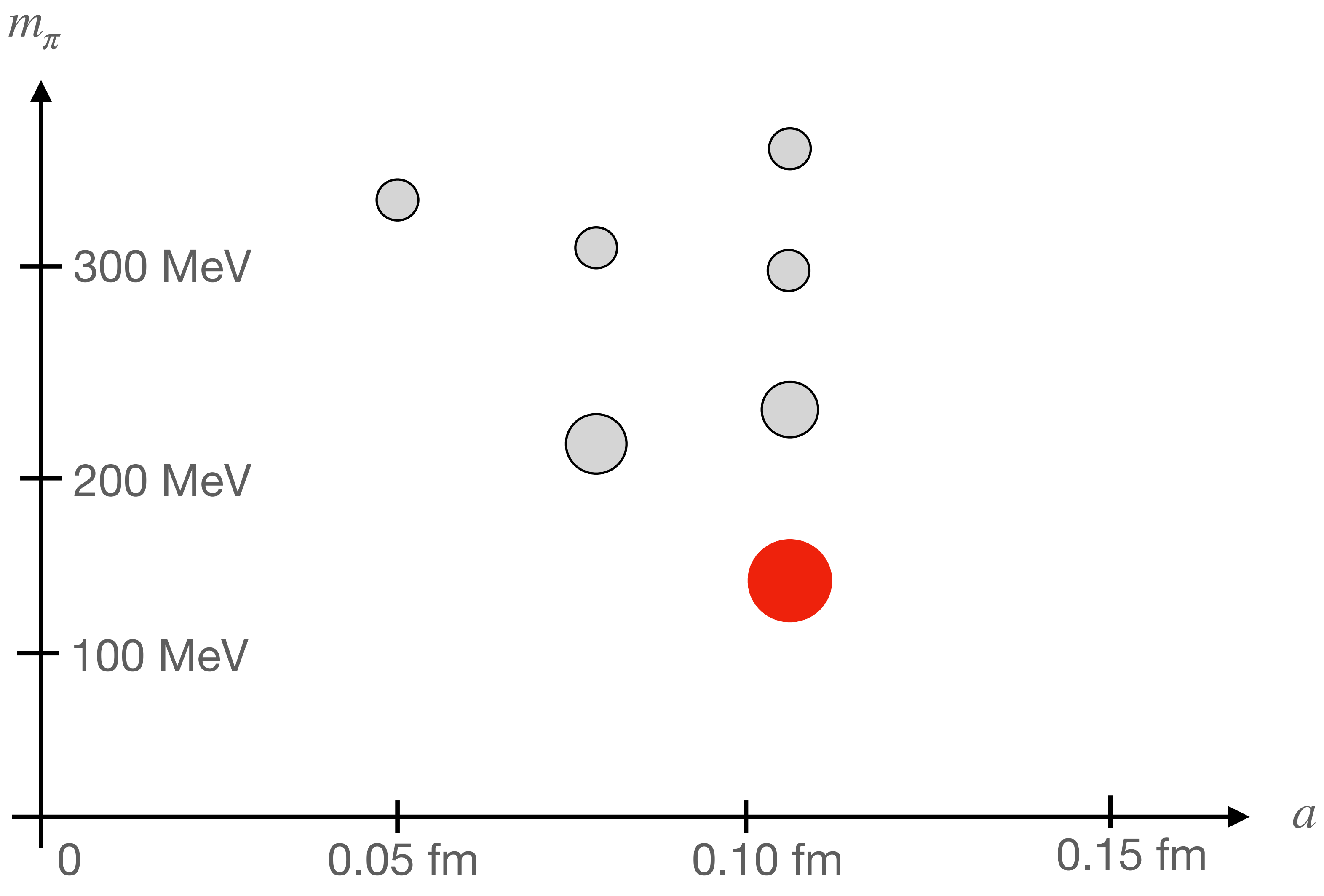
T. Ishikawa, et.al., JLQCD, Phys.Rev.D78 (2008) 011502

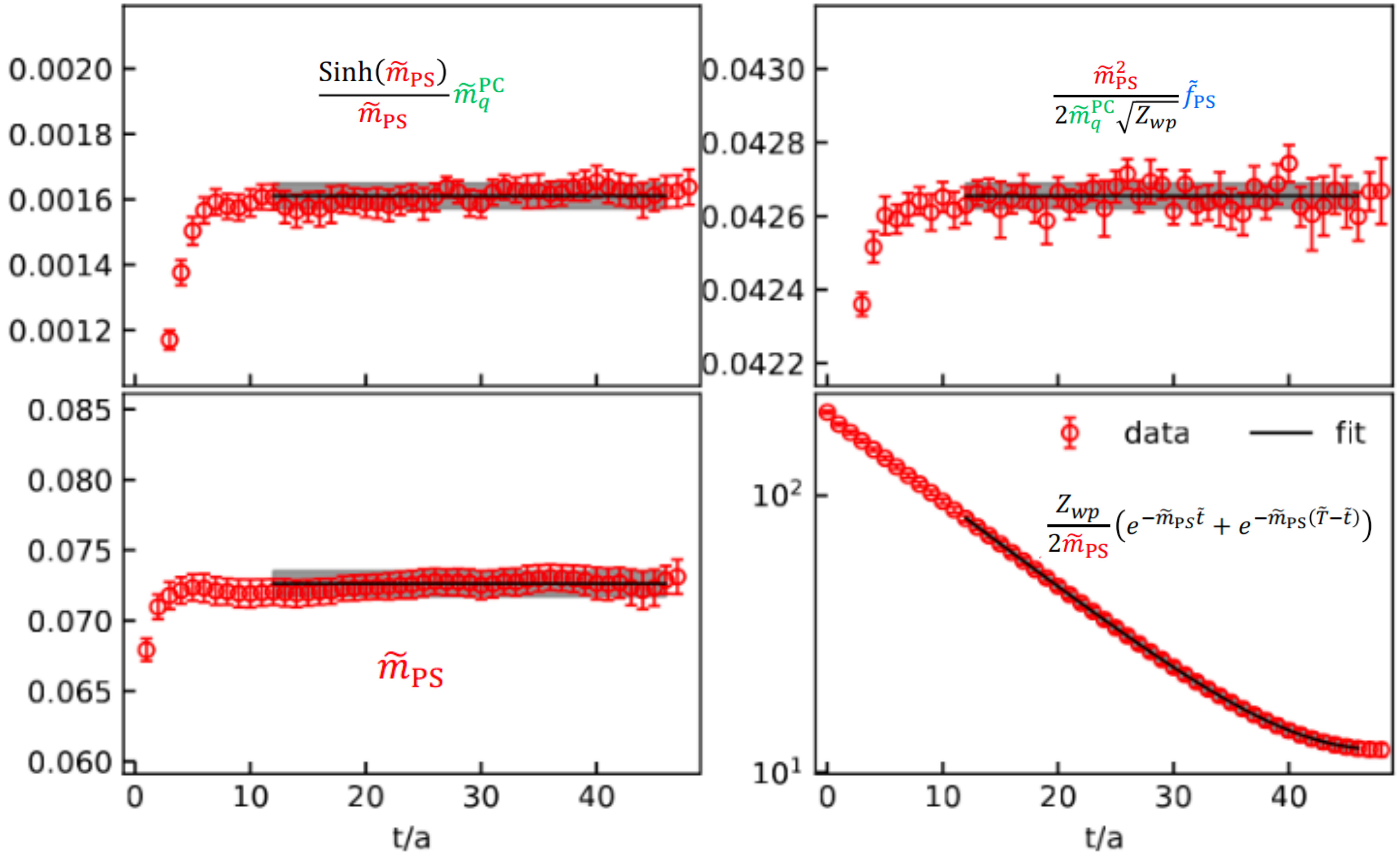
- And then $m_q^{\rm PC}$ is always positive and can be renormalized as $m_q^R = Z_P/Z_A m_q^{\rm PC}$.

CLQCD ensembles

Joint fit of pion correlators

- Joint fit of $\tilde{m}_q^{\mathrm{PC}} = m_q^{\mathrm{PC}} a$, $\tilde{f}_{\mathrm{PS}} = f_{\mathrm{PS}} a$, and $\tilde{m}_{\mathrm{PS}} = m_{\mathrm{PS}} a$, with several 2pt at $a^{-1} \sim 2$ GeV and physical pion mass;
- Used 48 measurements on each of 203 configurations.





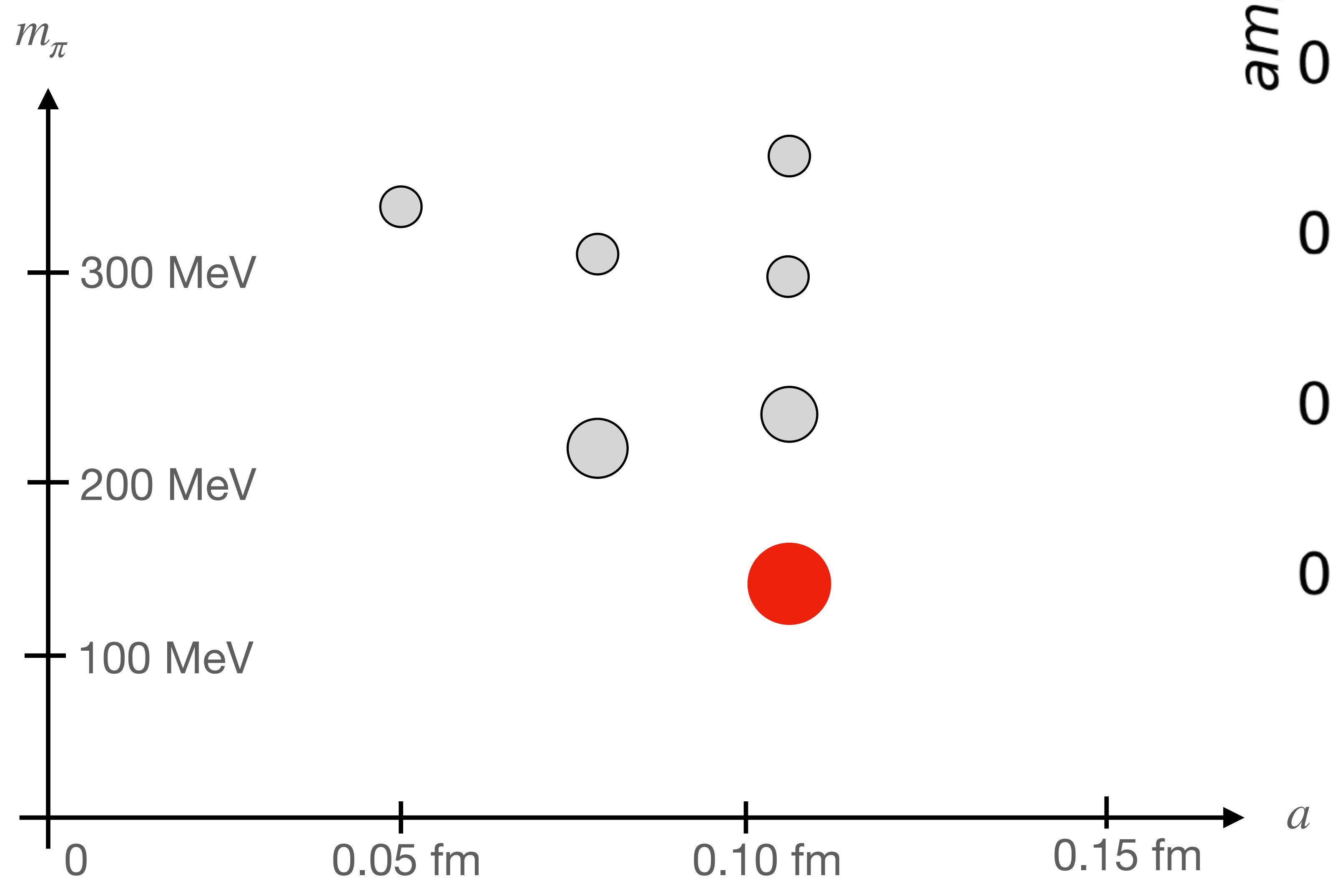
CLQCD ensembles

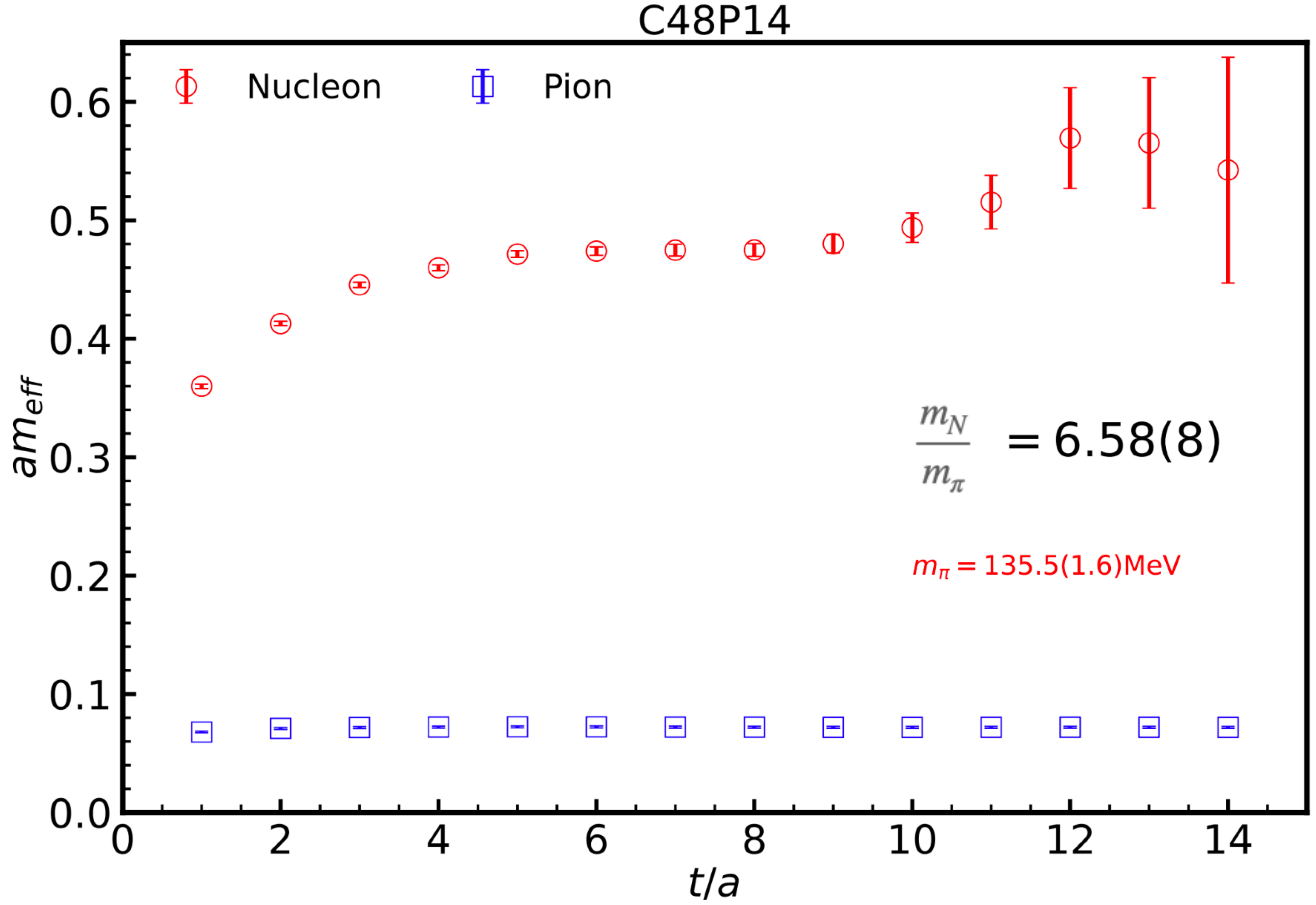
Nucleon mass v.s. pion mass

- With the same quark propagator, the ratio between the nucleon mass and pion mass is

  0.475(6)/0.0723(9)=6.58(8).

- Which is quite close to the physical value 0.939/0.135=6.96.





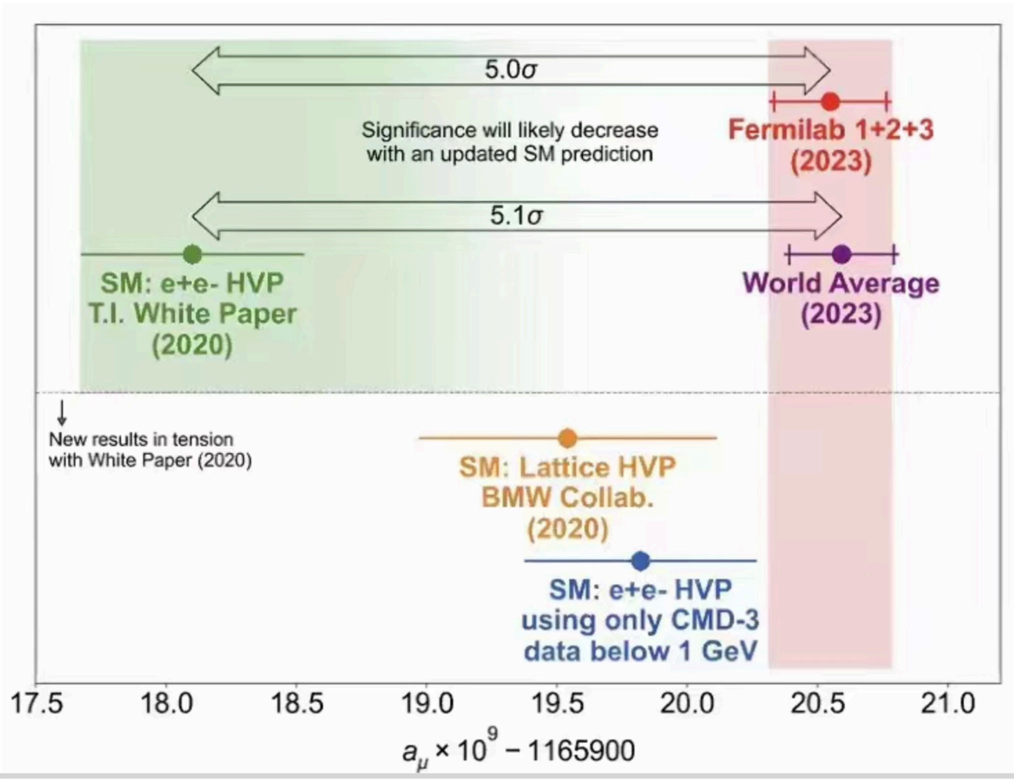
- Using the lattice spacing determined from the gradient flow, we have

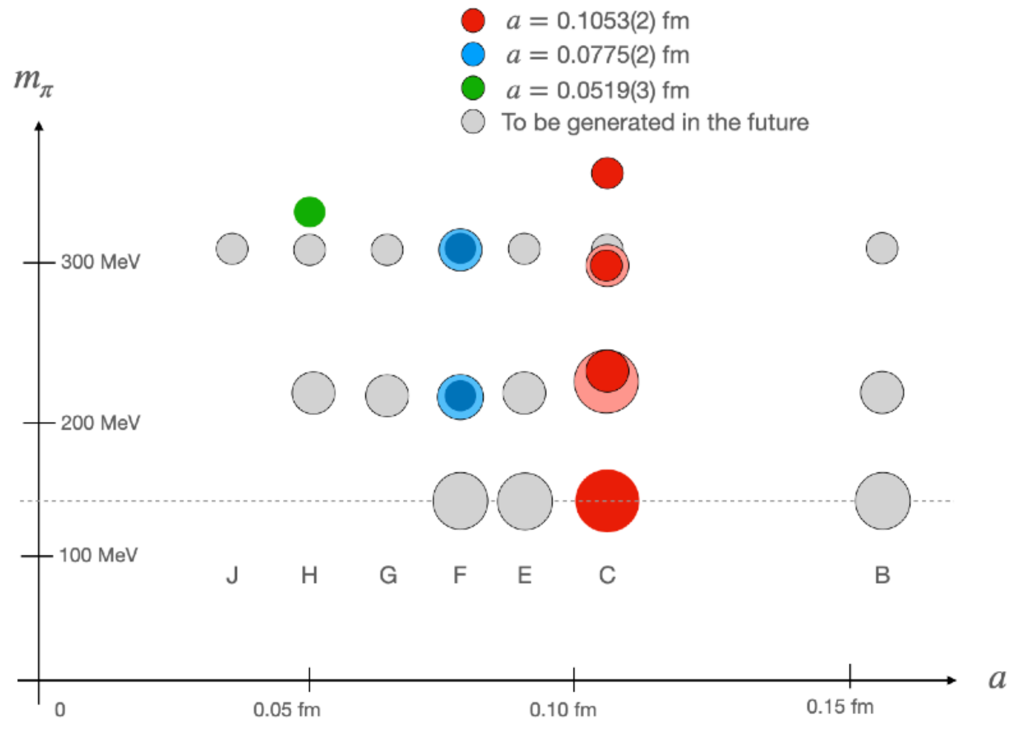
$$m_\pi = 135.5(1.6)\ \mathrm{MeV}, \quad m_N = 890(10)\ \mathrm{MeV}.$$

- m_N are ~5% smaller than the physical value, and can be a discretization effect based on the lattice spacing dependence of f_π.

Outline

Background

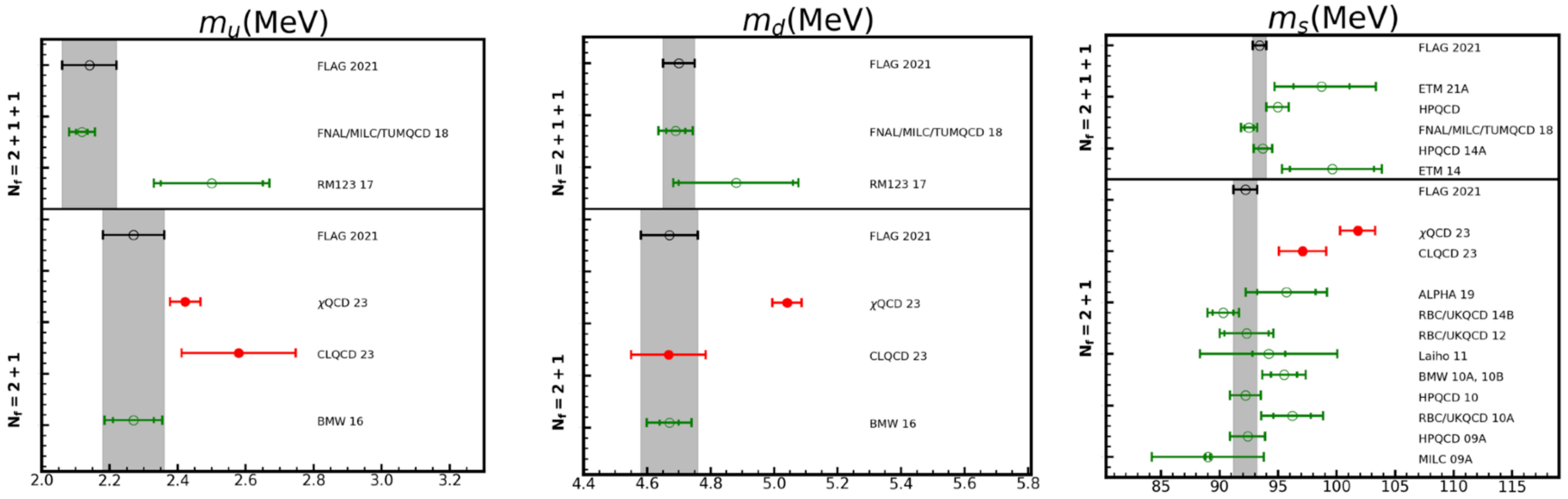




CLQCD ensembles

Renormalization

and final results

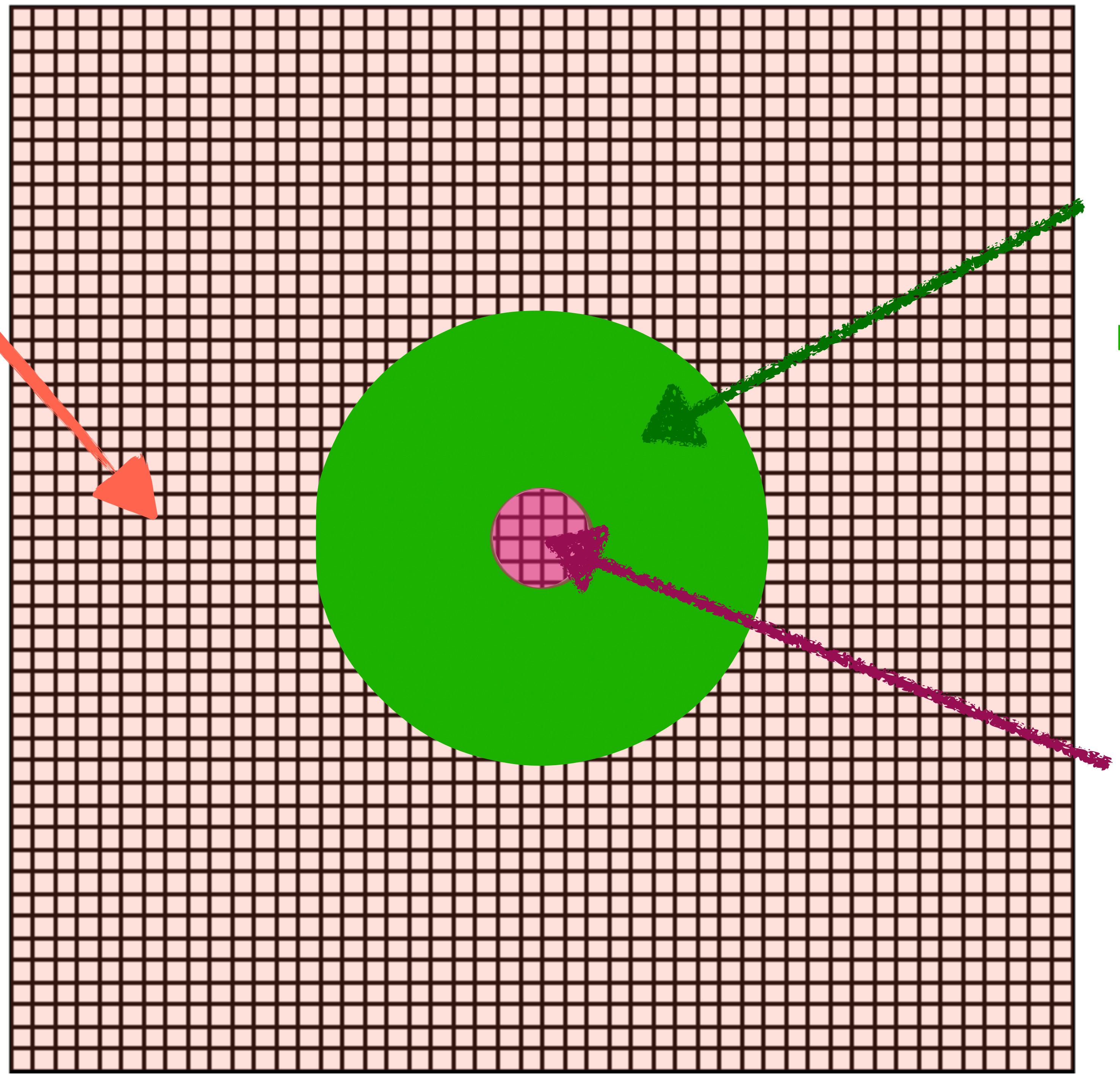


Renormalization and final results

Renormalization through intermediate scheme

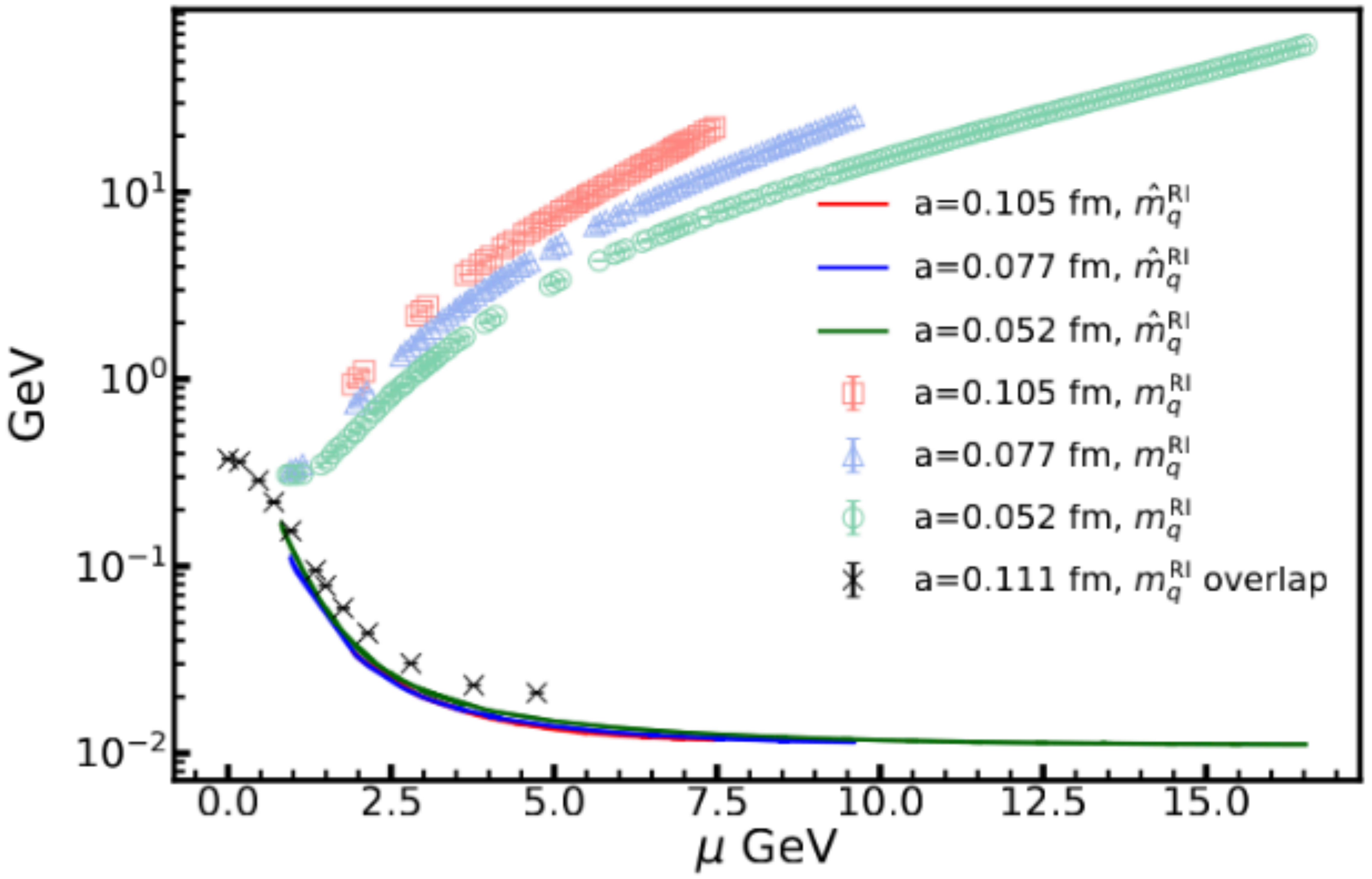
$$m_q^{\overline{\mathrm{MS}}}(\mu) = \frac{Z_m^{\mathrm{MOM,Lat}}(Q, 1/a)}{Z_m^{\mathrm{MOM,Dim}}(Q, \mu, \epsilon)} Z_m^{\overline{\mathrm{MS}},\mathrm{Dim}}(\epsilon) m_q^{\mathrm{Lat}}(1/a) + \mathcal{O}(a^m, \alpha_s^n)$$

- The RI/MOM renormalization targets to cancel the $\alpha_s \log(a)$ divergences using the off-shell quark matrix element;
- **Up to the $\mathcal{O}(a^2p^2)$ correction which can be eliminated by the $a^2p^2 \to 0$ extrapolation.**



G. Martinelli, et.al, NPB445(1995)81, arXiv: hep-lat/9411010

Renormalization and final results

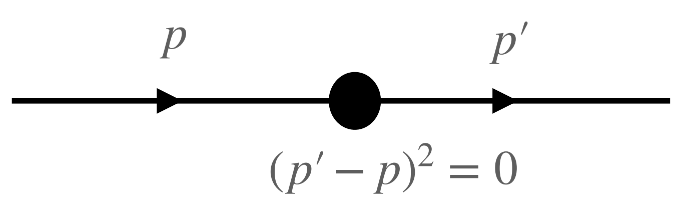


two definitions of quark mass

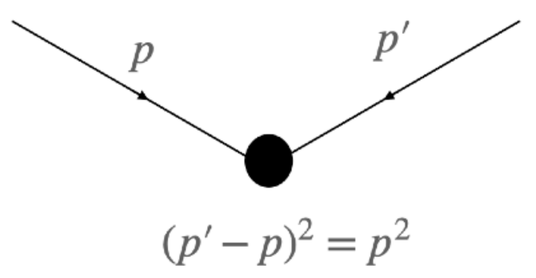
- $m_q^{\mathrm{RI}} = \dfrac{\frac{1}{12}\mathrm{Tr}[S^{-1}(p)]\,|_{p^2=\mu^2}}{Z_q^{\mathrm{RI}}(\mu)}$ is the natural of the regularization independent (RI) quark mass and equals to $\hat{m}_q^{\mathrm{RI}} = \dfrac{Z_A m_q^{\mathrm{PC}}}{\hat{Z}_P^{\mathrm{MOM}}(\mu)}$ for the chiral fermion.
- But m_q^{RI} (data points) of the clover fermion suffer from huge lattice artifacts and diverges at large μ.
- $\hat{m}_q^{\mathrm{RI}}$ of the clover fermion has much smaller discretization error and its μ dependence is similar to that of the overlap fermion.

Renormalization and final results

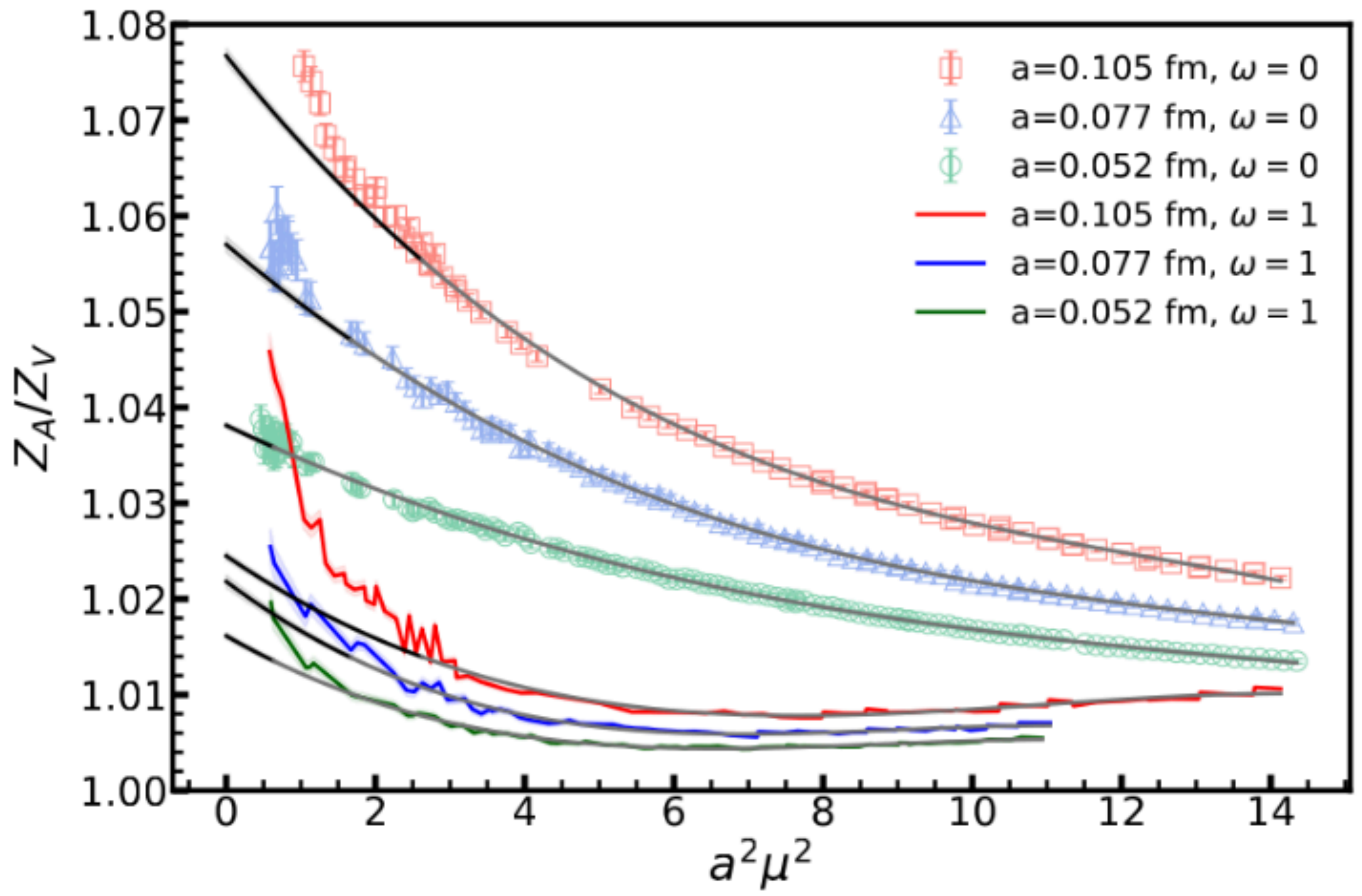
Z_A with MOM and SMOM schemes

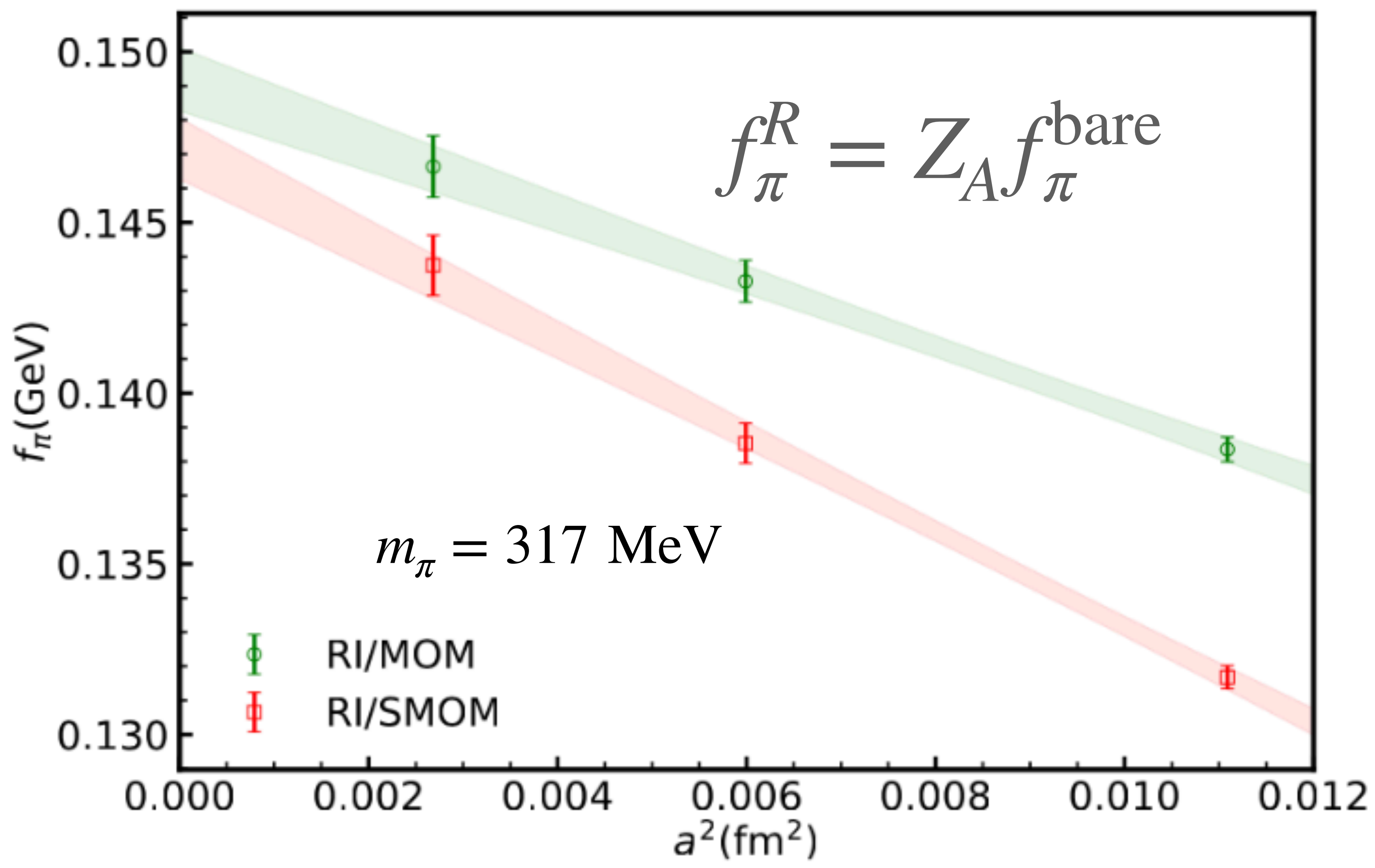


MOM G. Martinelli, et.al., NPB445 (1995) 81



SMOM Y. Aoki, et.al., PRD78 (2008) 054510
C. Sturm, et.al., PRD80 (2009) 014501

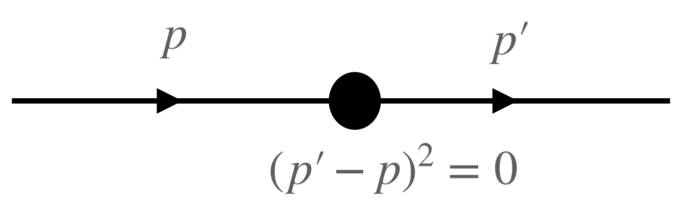




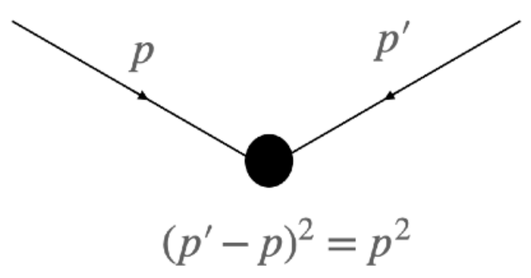
- Clover fermion introduce additional chiral symmetry breaking between Z_V and Z_A, while the effect suppress at smaller lattice spacing.
- The difference between MOM and SMOM also suppresses after the continuum extrapolation, while the lattice spacing dependence of the MOM scheme is smaller.

Renormalization and final results

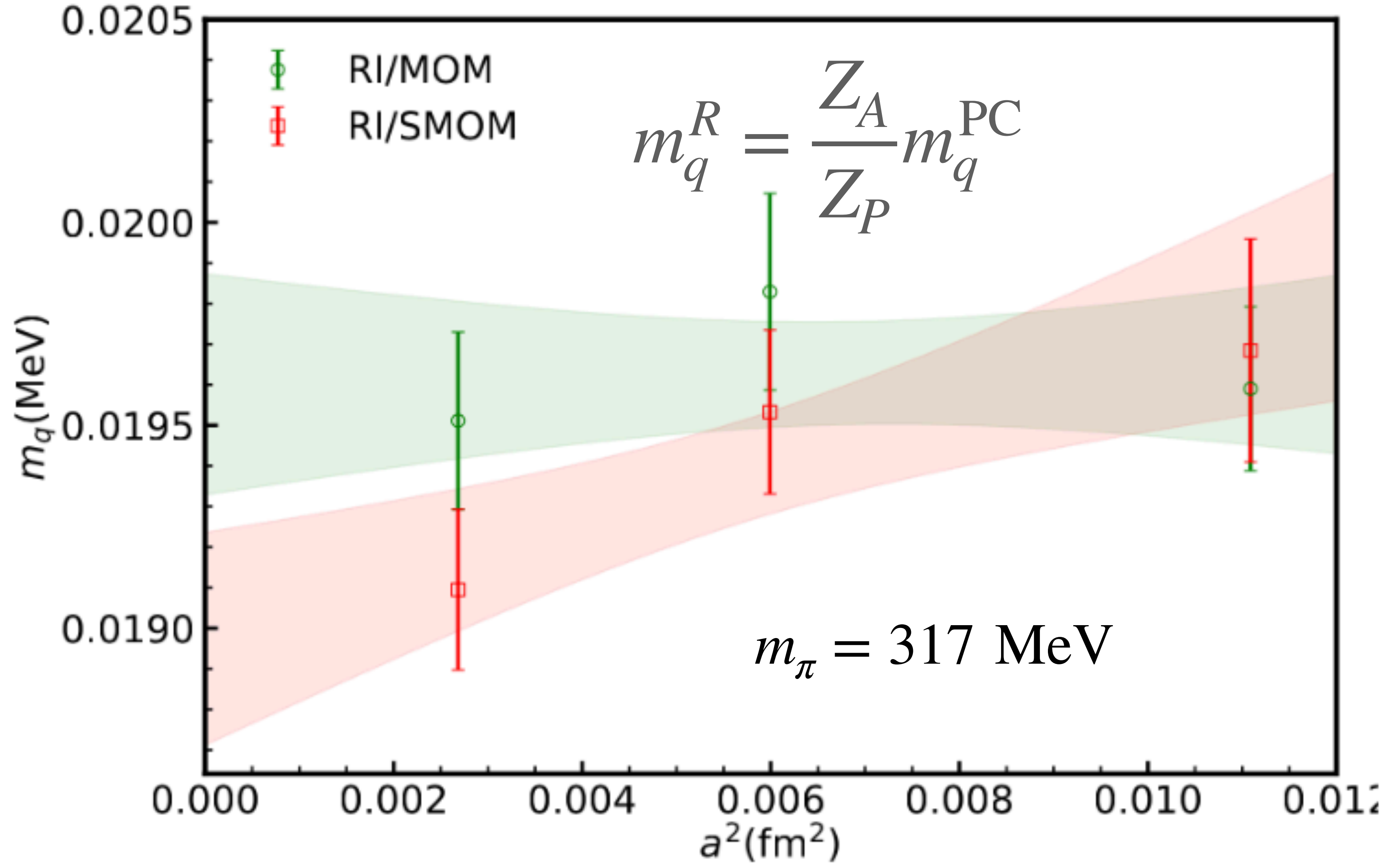
Z_P with MOM and SMOM schemes



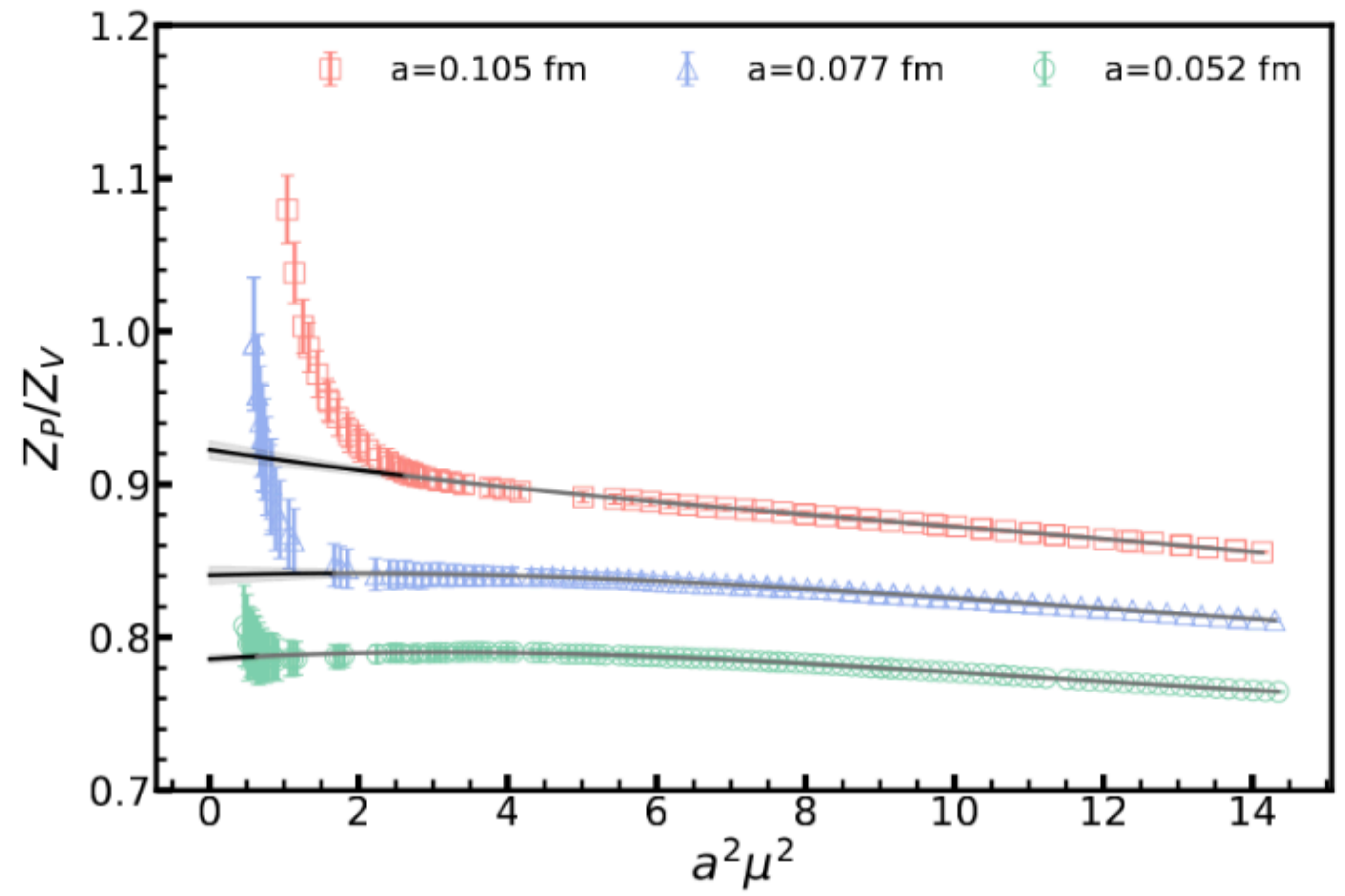
MOM G. Martinelli, et.al., NPB445 (1995) 81

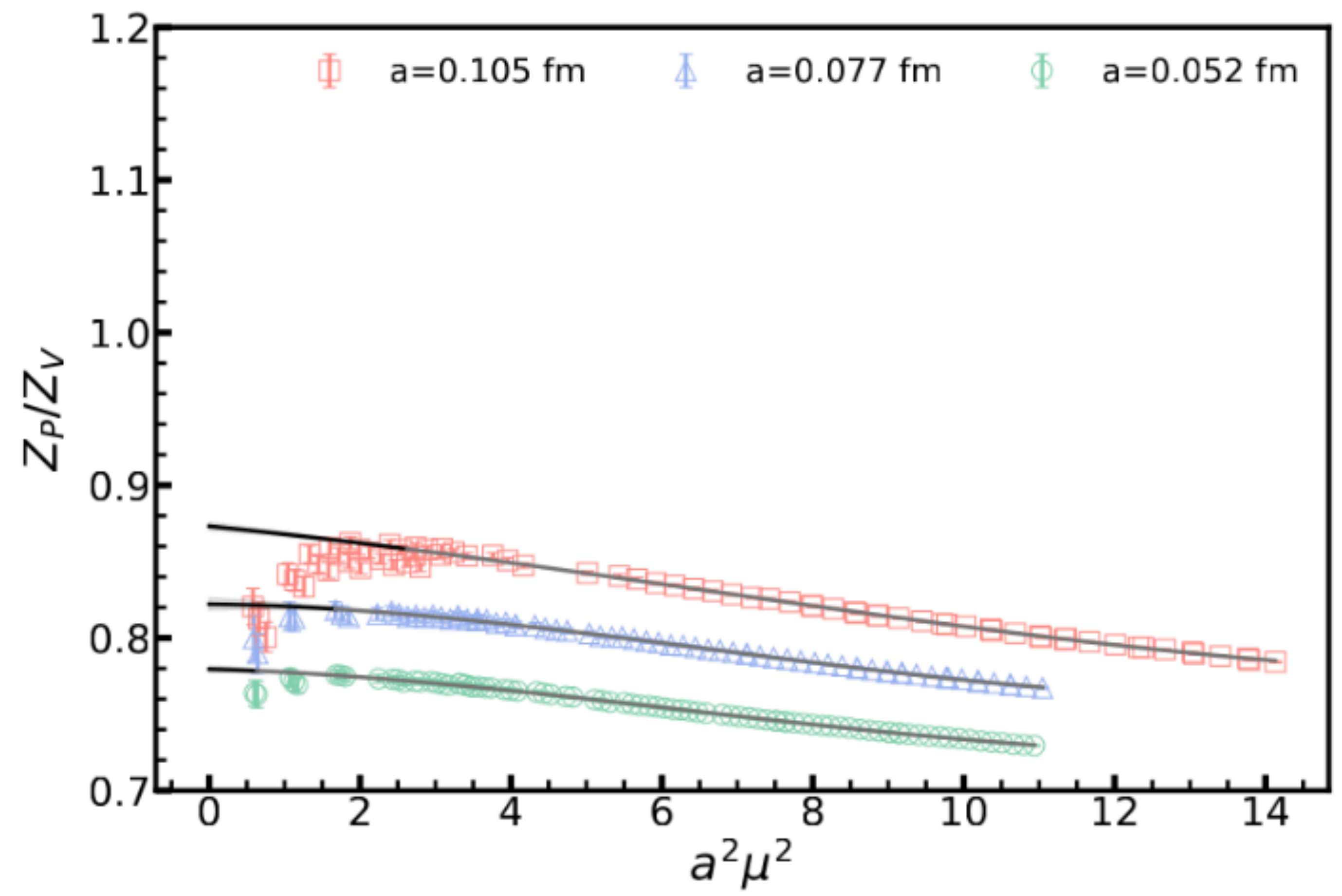


SMOM Y. Aoki, et.al., PRD78 (2008) 054510
C. Sturm, et.al., PRD80 (2009) 014501

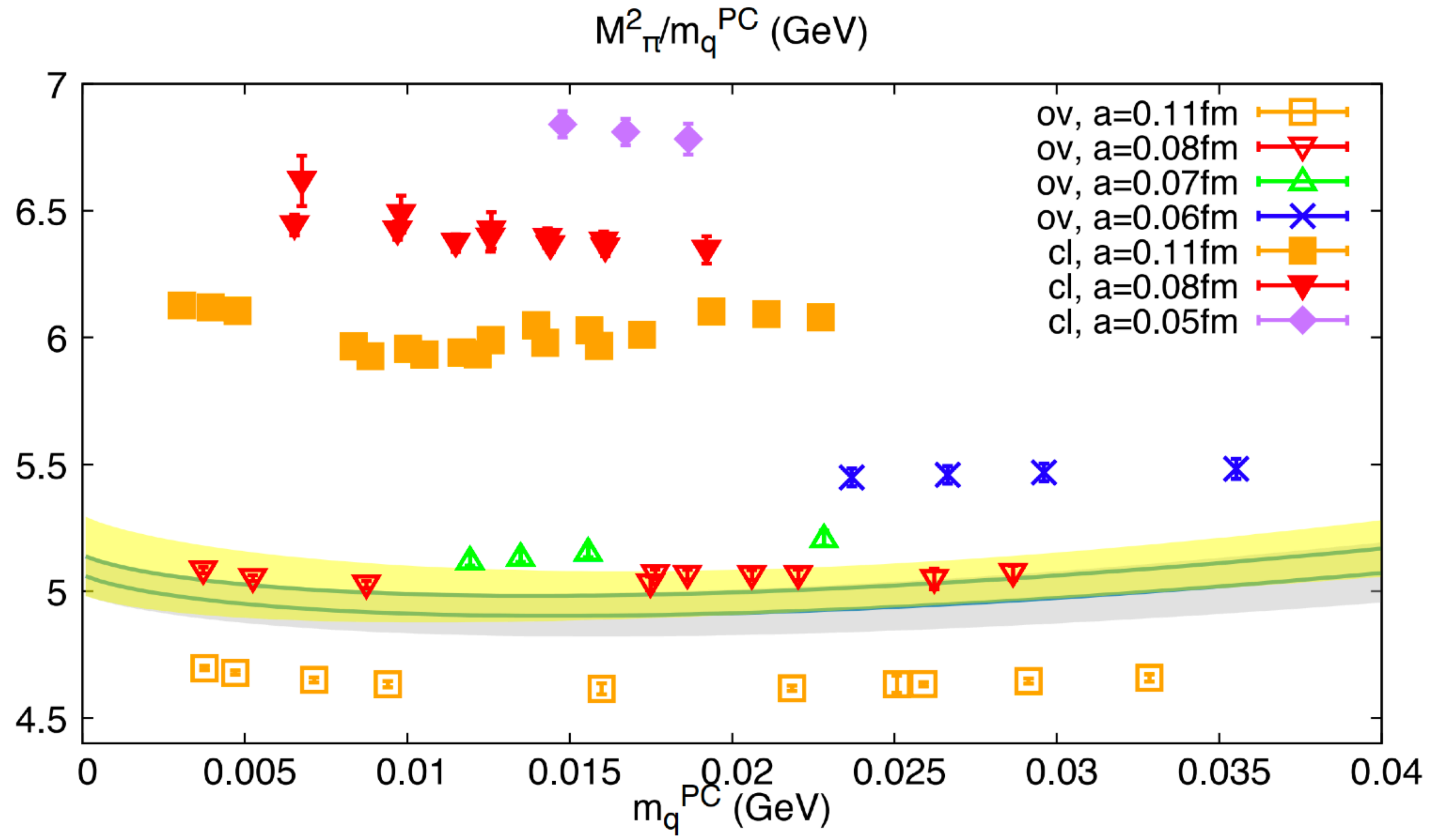


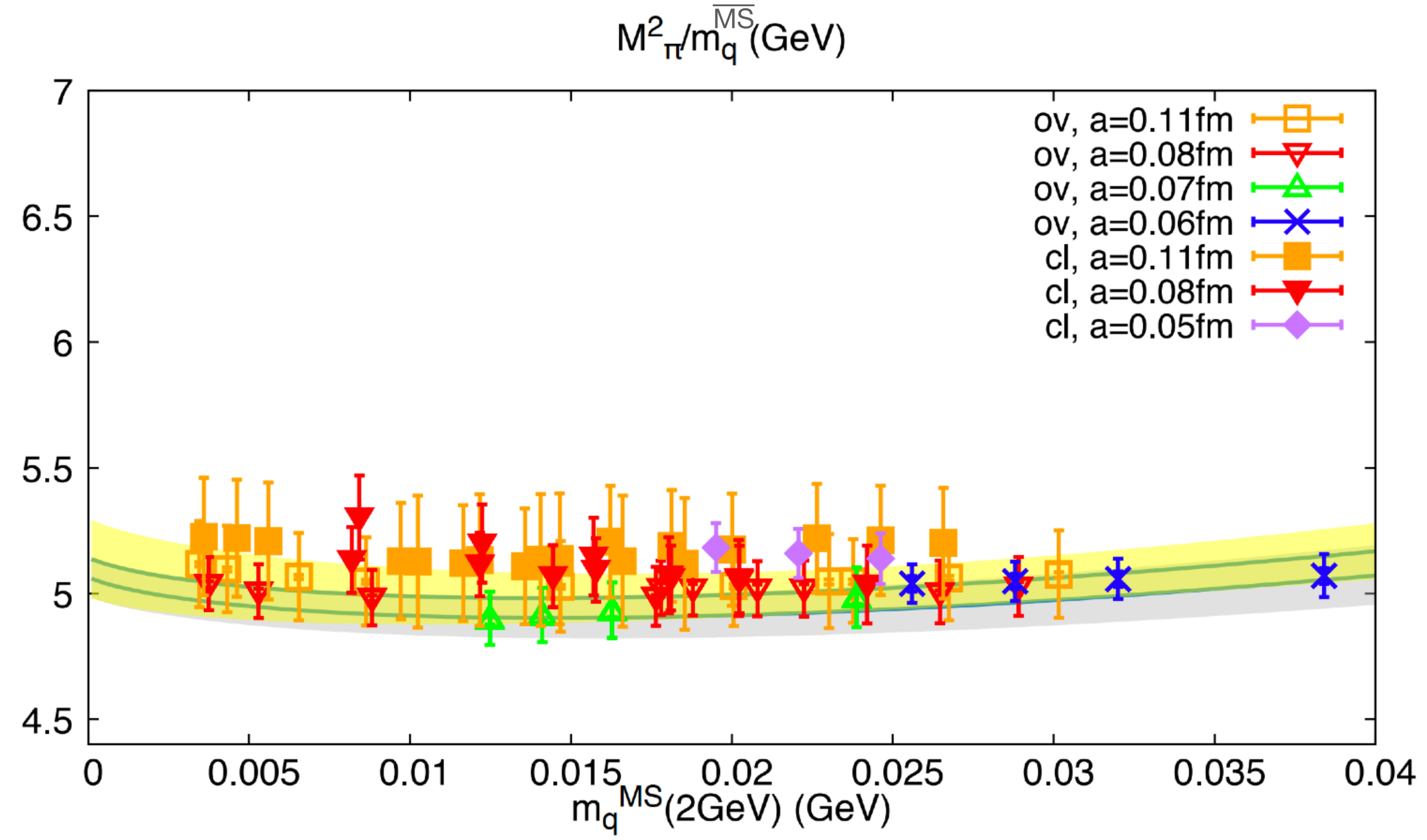
- Difference between MOM and SMOM is relatively small at finite lattice spacing, while would be larger after the continuum extrapolation.
- Lattice spacing dependence of the renormalized quark mass is mild.





Renormalization and final results





Renormalized quark mass

- Non-perturbative renormalization to $\overline{\text{MS}}$ 2 GeV eliminates the regularization scale $1/a$ dependence of m_π^2/m_q.
- m_π^2/m_q using the clover fermion also turns out to be consistent with that using the overlap fermion.
- The large uncertainty of the renormalized m_π^2/m_q majorly comes from the missing higher order effect of the perturbative matching

$$\frac{Z_P^{\overline{\text{MS}}}}{Z_P^{\text{MOM}}} = 1 + 0.4244\alpha_s + 1.007\alpha_s^2 + 2.722\alpha_s^3 + 8.263\alpha_s^4 + \mathcal{O}(\alpha_s^5)$$

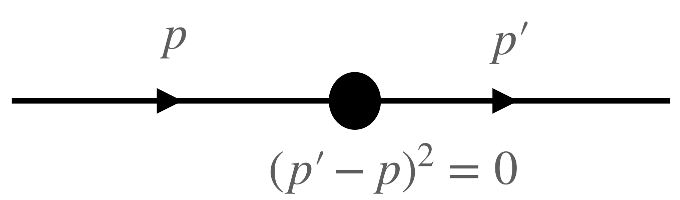
$$= \frac{1 - 2.611\alpha_s - 0.2813\alpha_s^2 - 0.3349\alpha_s^3}{1 - 3.036\alpha_s} + \mathcal{O}(\alpha_s^5),$$

J.A. Gracey, Eur.Phys.J.C83 (2023) 181

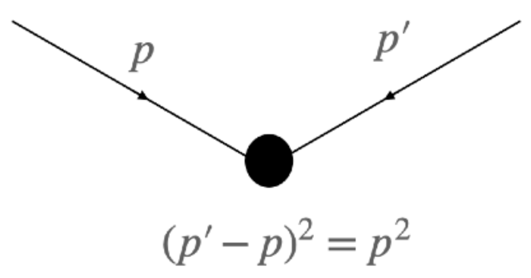
- and can be highly suppressed after the continuum extrapolation.

Renormalization and final results

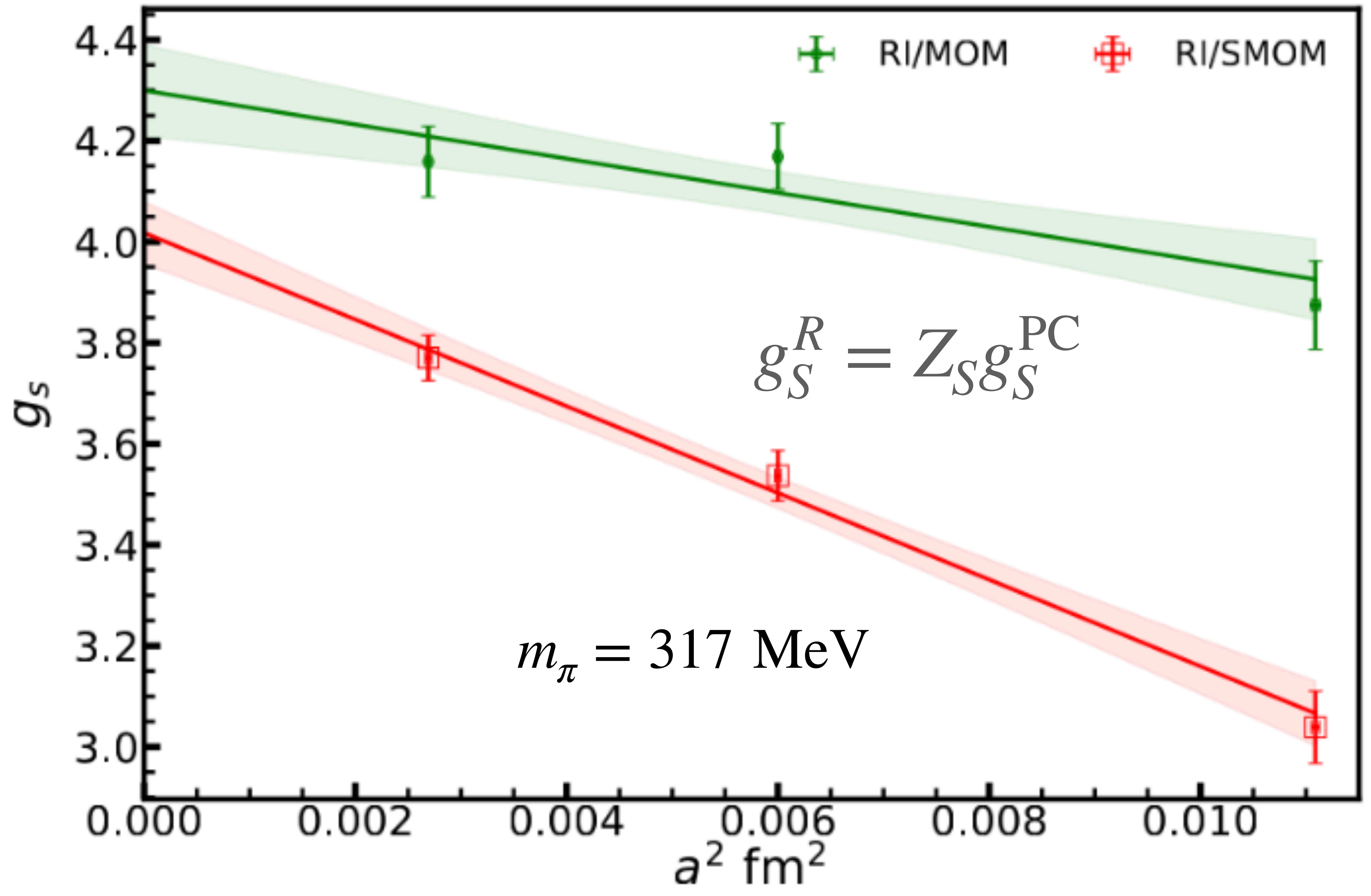
Z_S with MOM and SMOM schemes



MOM G. Martinelli, et.al., NPB445 (1995) 81

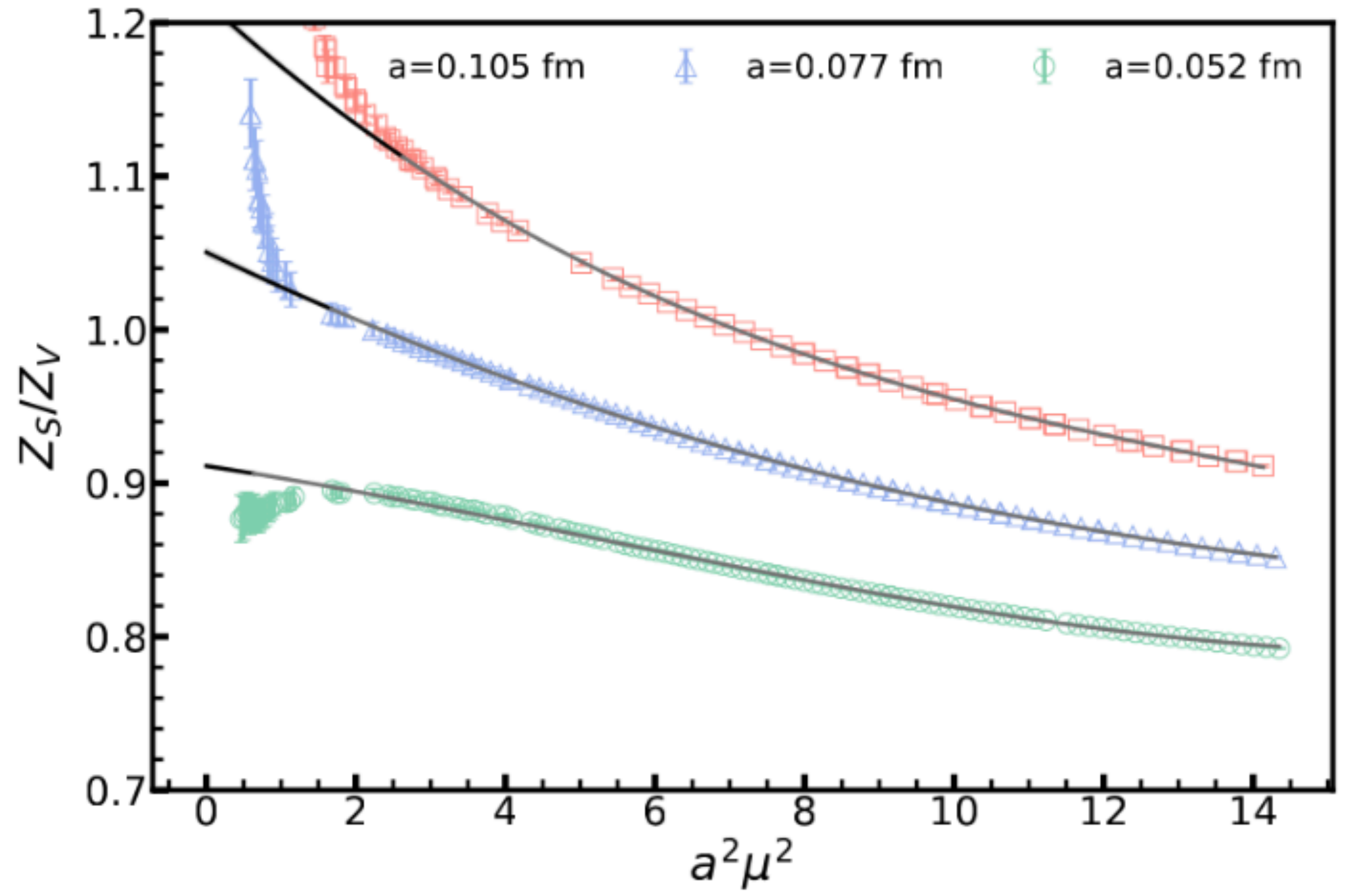


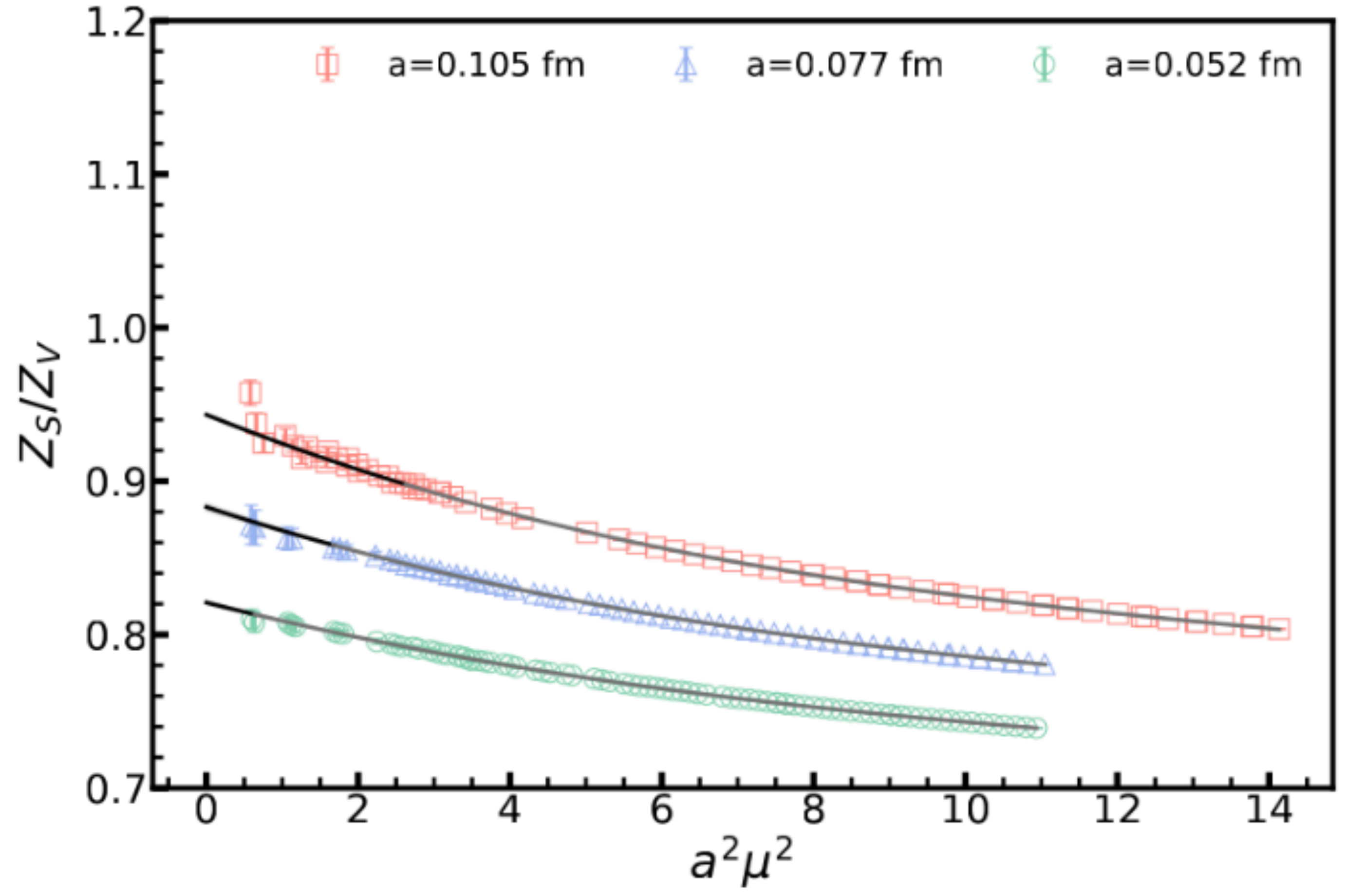
SMOM Y. Aoki, et.al., PRD78 (2008) 054510
C. Sturm, et.al., PRD80 (2009) 014501



- Difference between MOM and SMOM in Z_S is much larger than the Z_P case, while becomes smaller at smaller lattice spacing.

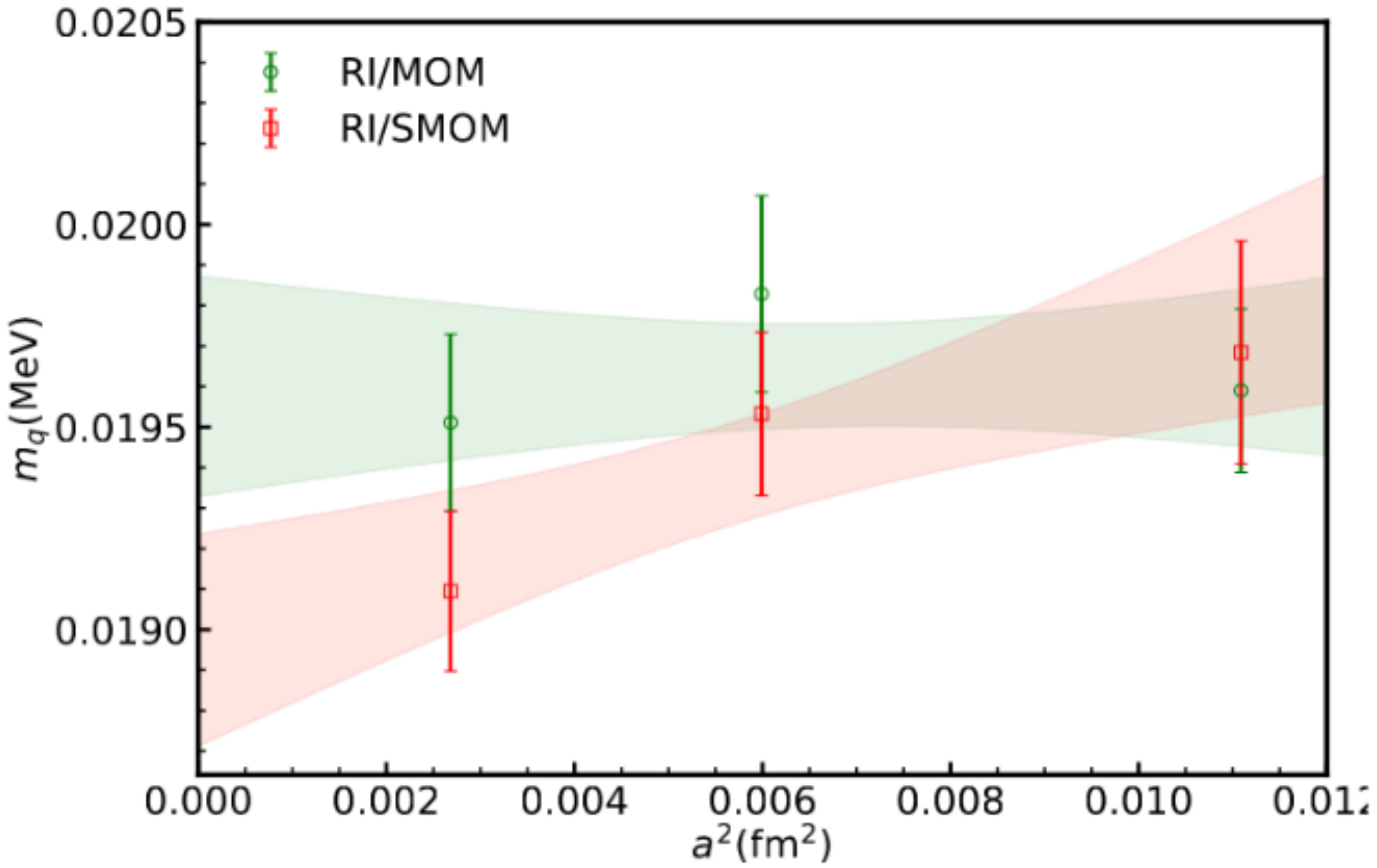
- Lattice spacing dependence using the MOM scheme is milder than the SMOM case.

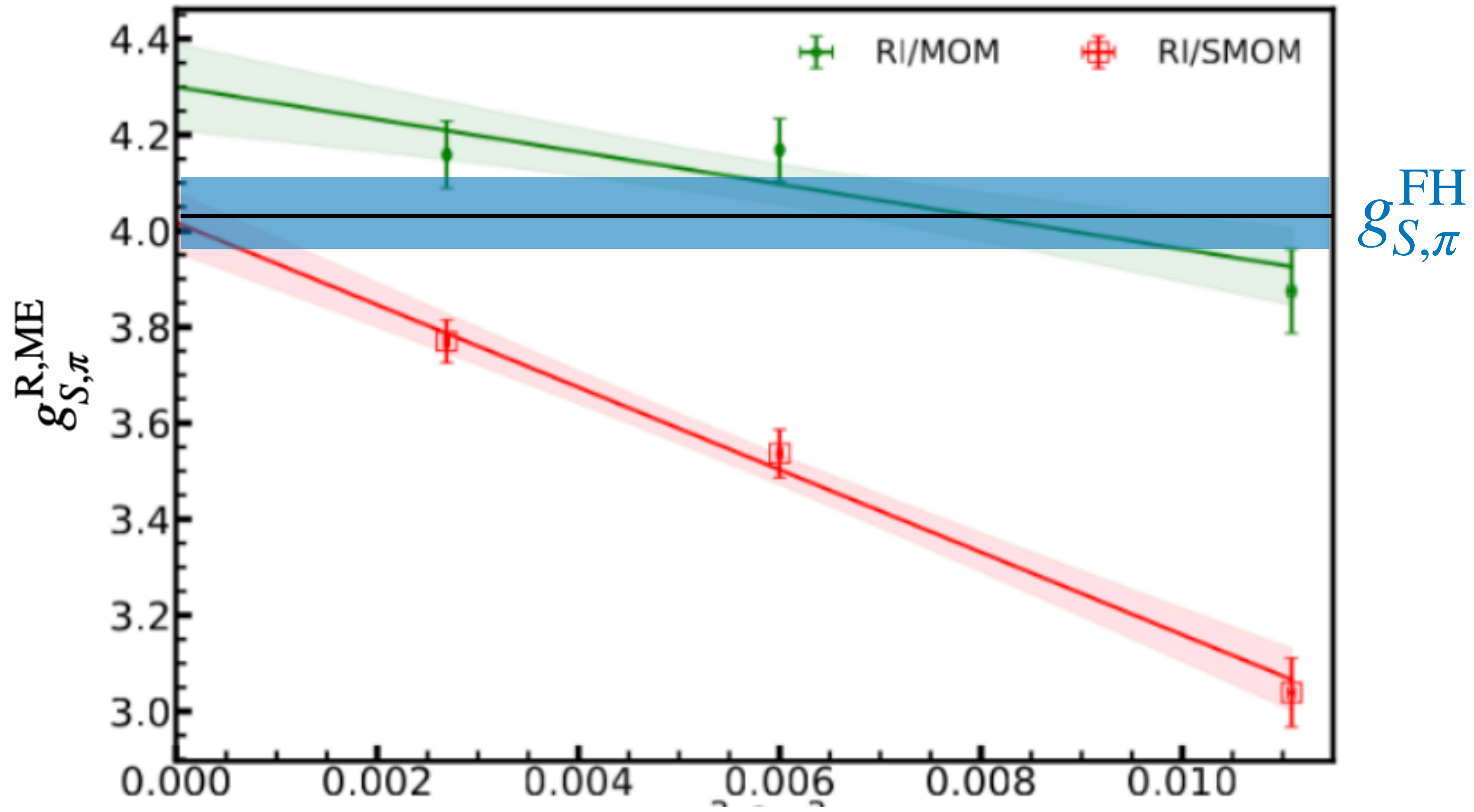




Renormalization and final results

Restore of chiral symmetry in the continuum





- Renormalized quark mass $m_q^R = Z_A/Z_P m_q^{\mathrm{PC}}$ with 317 MeV pion mass at three lattice spacings:
  - The intermediate renormalization scheme dependence is 3.1(1.5)%.
  - RI/MOM scheme has smaller discretization error.
  - Feynman-Hellman theorem can extract $g_{S,\pi}$ as

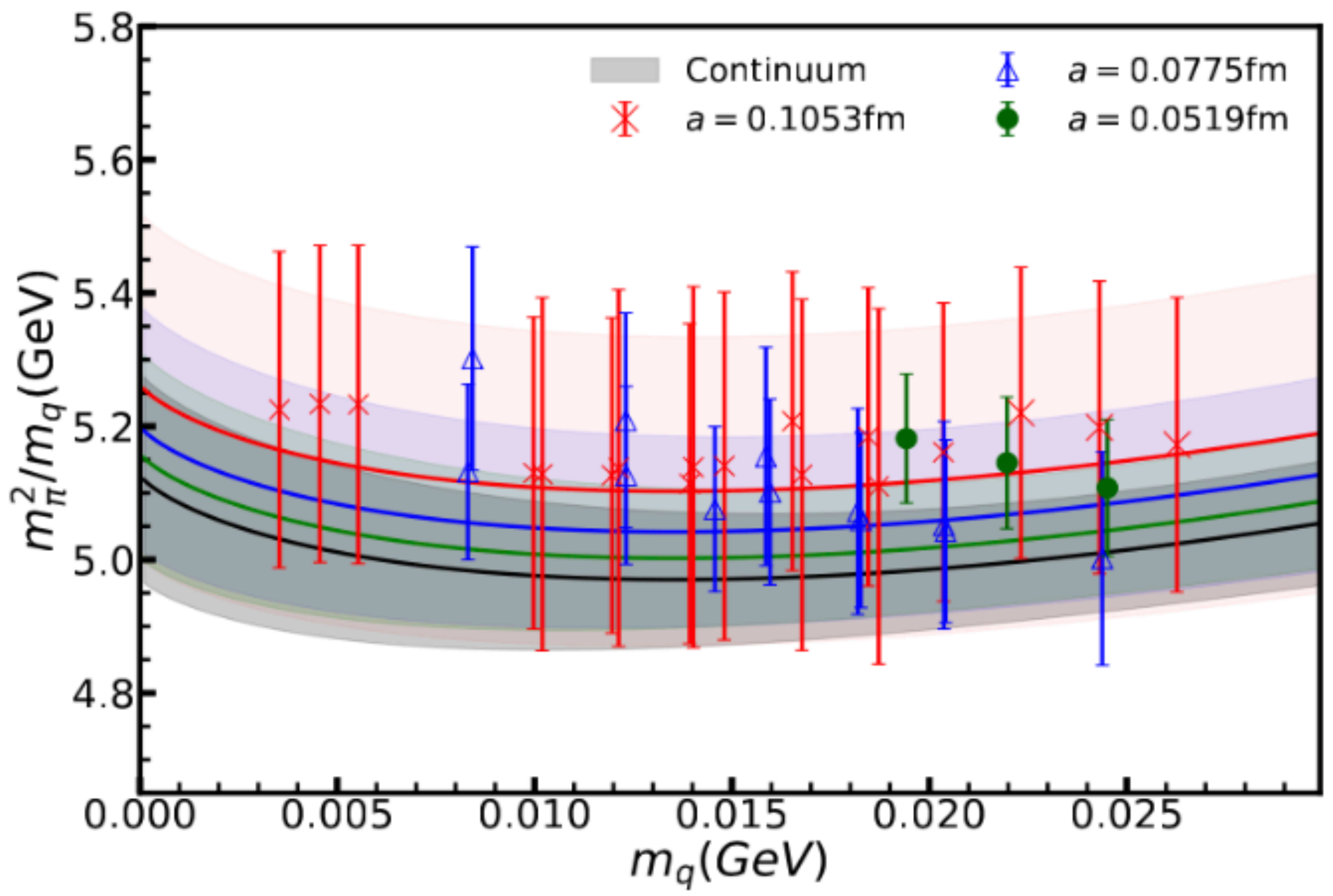
$$g_{S,\pi}^{\mathrm{FH}} = \frac{1}{2}\frac{\partial m_\pi(m_q)}{\partial m_q} \simeq \frac{m_\pi}{4m_q} + \mathcal{O}(m_q, a^2)$$

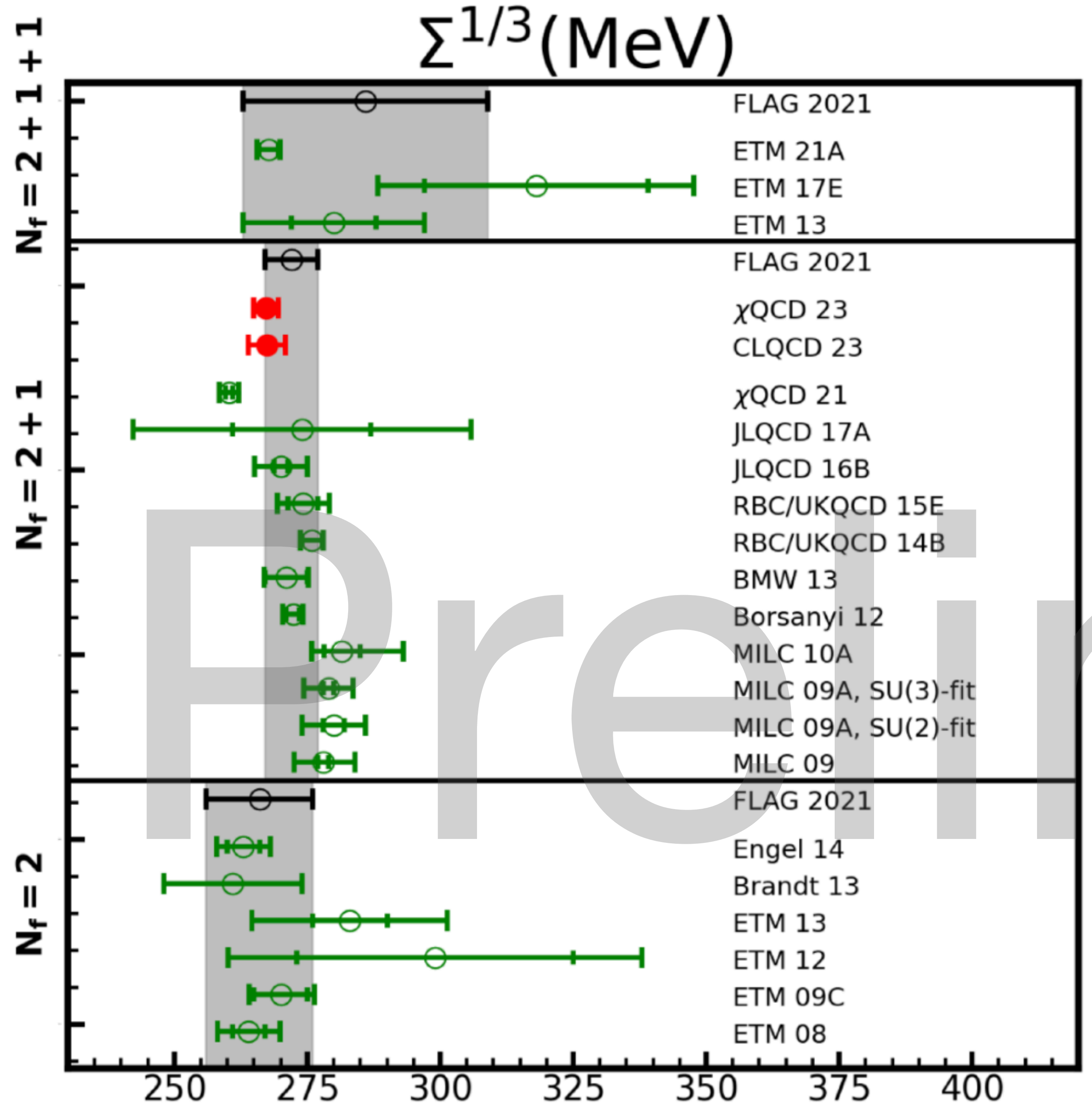
  which is 4.04(6)(12) for $m_\pi = 317\ \mathrm{MeV}$ in the continuum.

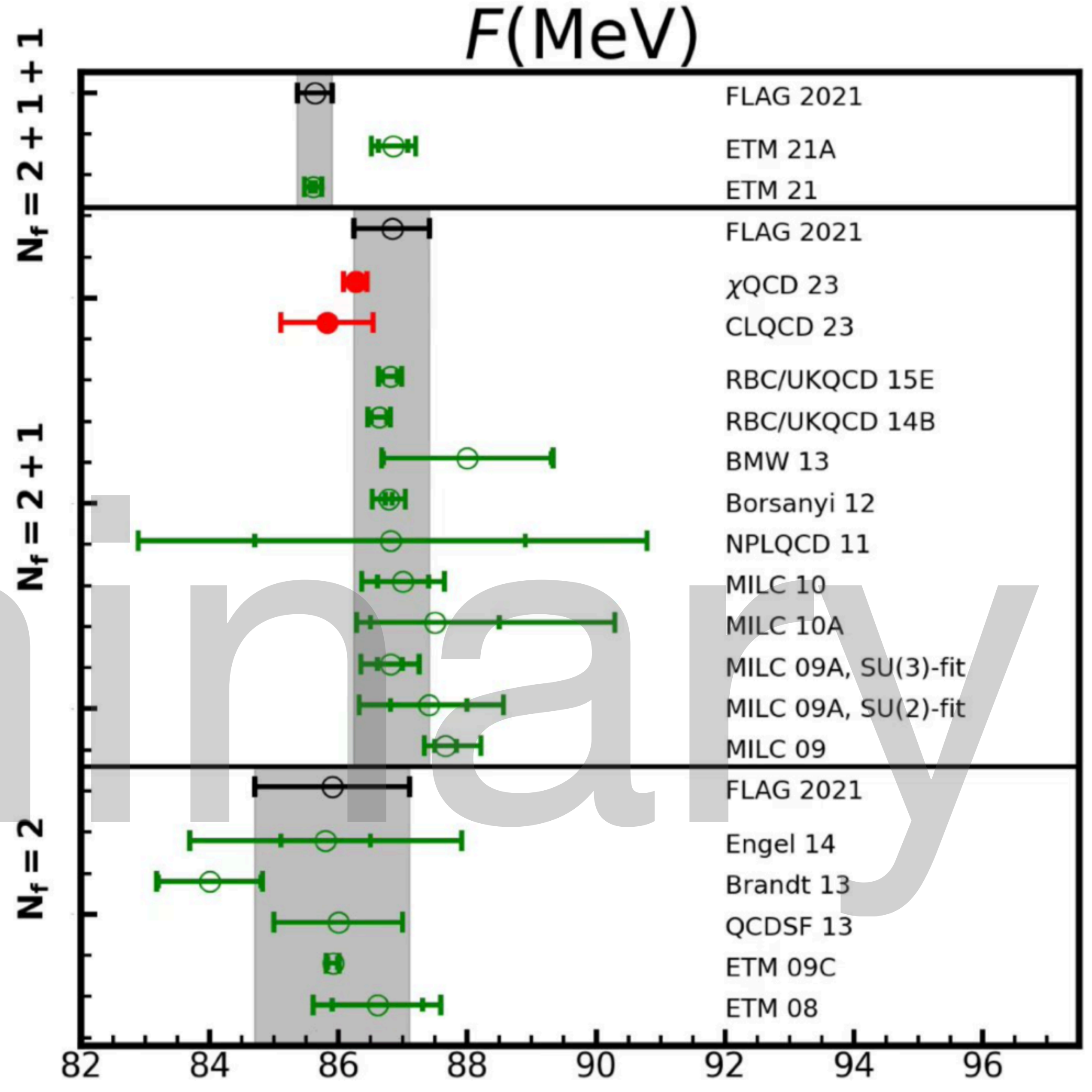
- Renormalized $g_{S,\pi}^{\mathrm{R,ME}} = Z_S \frac{\langle H|S|H\rangle_{\mathrm{conn}}}{\langle H|H\rangle}$ based on the direct calculation:
  - The intermediate renormalization scheme dependence is 6.6(2.4)%.
  - $g_{S,\pi}^{\mathrm{ME}}$ using RI/MOM scheme has smaller discretization error, and agree with $g_{S,\pi}^{\mathrm{R,FH}}$ within 2σ at all the lattice spacings.

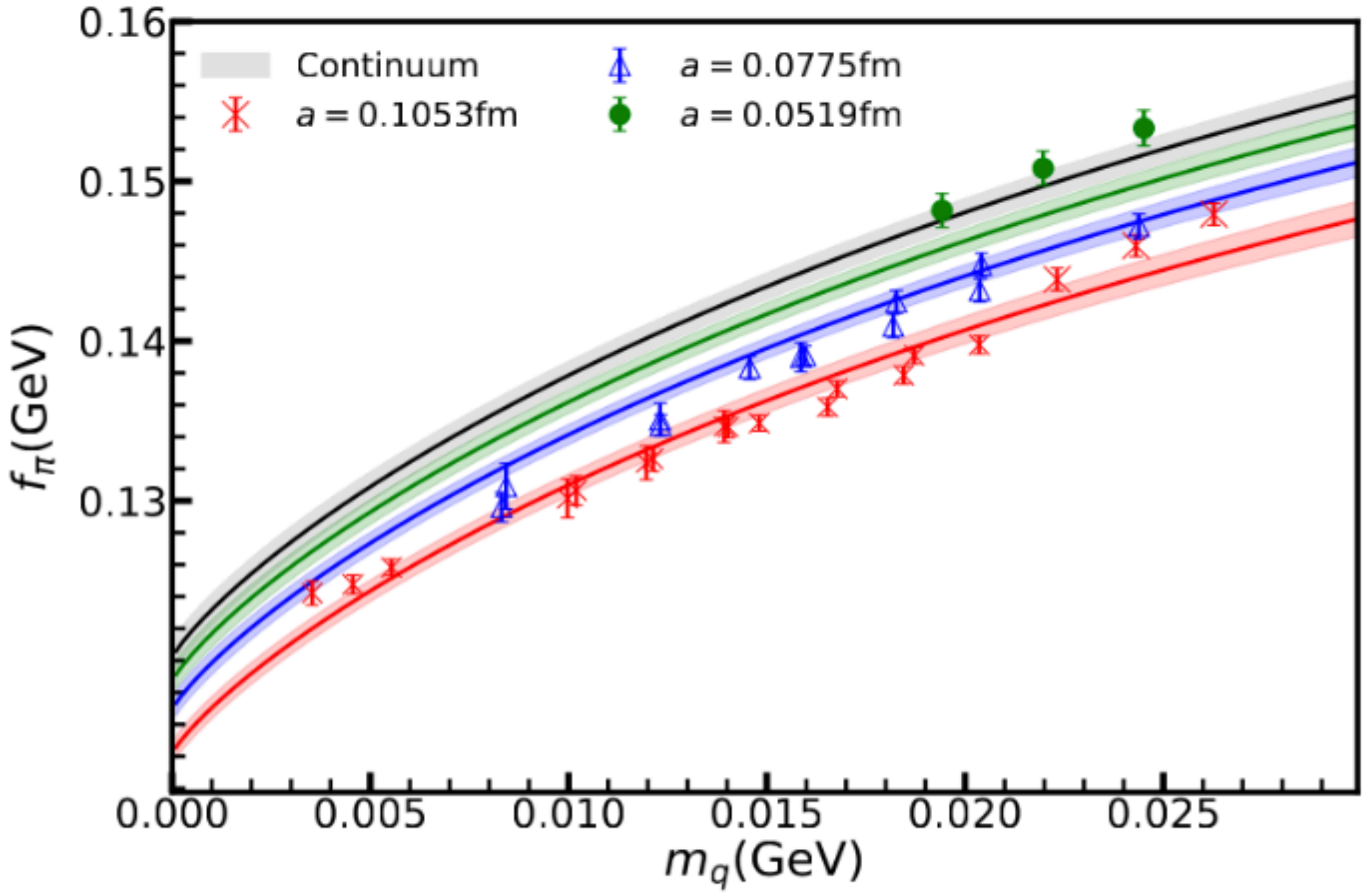
Renormalization and final results

Global fit of the pion mass and decay constant









$$m^2_{\pi,\mathrm{vv}} = \Lambda^2_\chi 2y_\mathrm{v}\{1+\frac{2}{N_f}[(2y_\mathrm{v}-y_\mathrm{s})\ln(2y_\mathrm{v})+(y_\mathrm{v}-y_\mathrm{s})]$$
$$+2y_\mathrm{v}(2\alpha_8-\alpha_5)+2y_\mathrm{s}N_f(2\alpha_6-\alpha_4)\}$$
$$(1+c_{m,a}a^2+c_{m,l}e^{-m_\pi L}),$$
$$F_{\pi,\mathrm{vv}} = F(1-\frac{N_f}{2}(y_\mathrm{v}+y_\mathrm{s})\ln(y_\mathrm{v}+y_\mathrm{s})+y_\mathrm{v}\alpha_5+y_\mathrm{s}N_f\alpha_4)$$
$$(1+c_{f,a}a^2+c_{f,l}e^{-m_\pi L})$$

- Global fit of all the ensembles to obtain the quark mass dependence of m_π and f_π in the continuum and infinite volume limit, which allows us to extract the χPT low energy constants.

Renormalization and final results

QED effects in the pesudoscalar masses

P.Zyla et,al, PTEP(2020)083C01 (PDG2020):

- $m_p = 938.27\ \mathrm{MeV} = m_{p,\mathrm{QCD}} + 1.00(16)\ \mathrm{MeV} + \ldots;$
- $m_n = 939.57\ \mathrm{MeV};$
- $\boxed{m_\pi^0 = 134.98\ \mathrm{MeV};}$
- $m_\pi^+ = 139.57\ \mathrm{MeV} = m_\pi^0 + 4.53(6)\ \mathrm{MeV} + \ldots;$

X. Feng, et,al. Phys.Rev.Lett.128(2022)062003

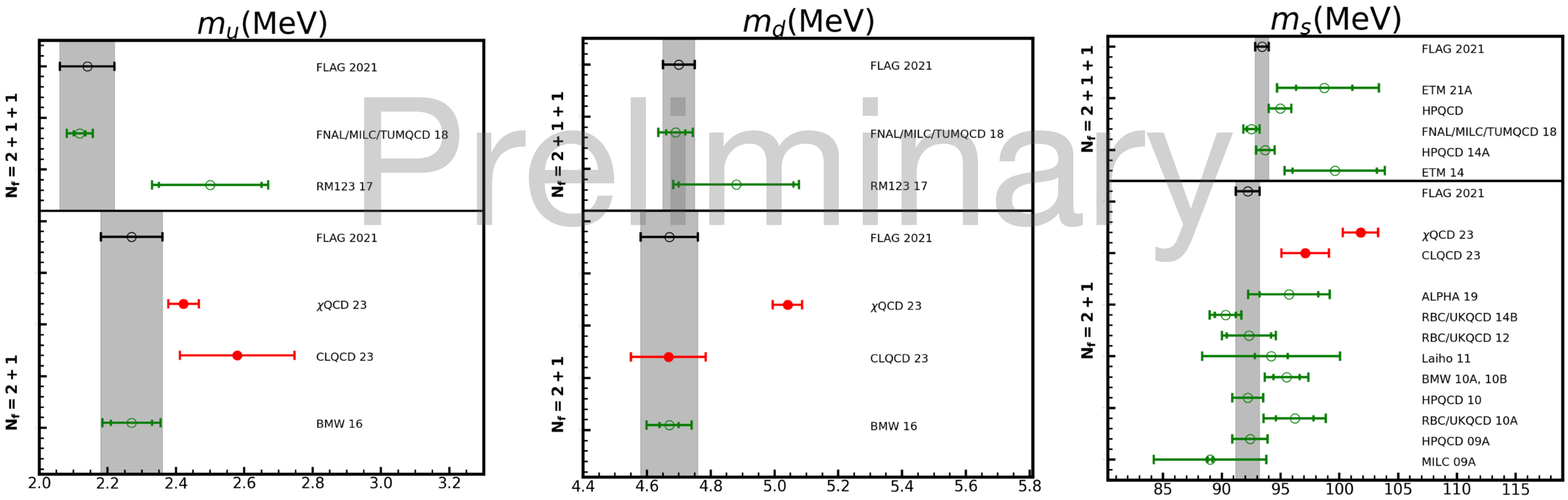
- $m_K^0 = 497.61(1)\ \mathrm{MeV} = m_{K,\mathrm{QCD}}^0 + 0.17(02)\ \mathrm{MeV} + \ldots;$
- $m_K^+ = 493.68(2)\ \mathrm{MeV} = m_{K,\mathrm{QCD}}^+ + 2.24(15)\ \mathrm{MeV} + \ldots$

D. Giusti, et,al. PRD95(2017)114504

$$\boxed{\alpha_s^{\mathrm{bare}}} \rightarrow \boxed{\text{Lattice spacing } a} \rightarrow \boxed{(m_u^{\mathrm{bare}} + m_d^{\mathrm{bare}})/2} \rightarrow \boxed{m_u^{\mathrm{bare}},\ m_d^{\mathrm{bare}} \text{ and } m_s^{\mathrm{bare}}}$$

Renormalization and final results

Quark mass of three light flavors



Summary

- Lattice QCD ensembles at multiple lattice spacing, pion mass and volume are the foundation of the SM high accuracy prediction and test;

- We chose the clover fermion and Symanzik gauge actions to generate the ensembles, and figured out the proper renormalization to restore the chiral symmetry at 5% level.

- Current prediction of quark masses and low energy constants agree with the lattice averages within 5-10%, and more accurate studies are on-going.