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Light-flavor tensor meson nonet in chiral effective field theory

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Outline:

1. Background

2. Tensor mesons in Resonance Chiral Theory

3. Phenomenologies (Exp and Lat)

   (a) Tensor masses

   (b) $T \to$ PP' decays

   (c) $T \to$ Pγ & $T \to \gamma\gamma$

4. Summary

Background

Low lying mesons of QCD

- **light pseudoscalars (Goldstone):** π、K、η (chiral perturbation theory: χPT)

.. Mass Gap ....

- **well established vectors:** ρ , K^*, ω, φ

  (VMD: vector meson dominace (VMD) in many situations)

- **scalars with opaque nature:** σ, κ, $f_0(980)$, $a_0(980)$

- **less focused tensors:** $f_2(1270)$, $f_2'(1525)$, $K_2^*(1430)$, $a_2(1320)$

  (See also disucssions via ρρ scattering: [Geng,Molina,Oset,PRD'08'09] [Du,Gulmez et al.,EPJC'17'18] **)**

After integration of the heavy d.o.f (resonances), low energy constants (LECs) of χPT can be predicted: [Ecker et al., NPB'88] [Cirigliano et al, NPB'06]

$$\mathcal{L}_4 = L_1\langle u_\mu u^\mu \rangle^2 + L_2\langle u_\mu u^\nu \rangle\langle u^\mu u_\nu \rangle + L_3\langle u_\mu u^\mu u_\nu u^\nu \rangle + \ldots\ldots$$

$$L_3 = -\frac{3G_V^2}{4M_V^2} + \frac{c_d^2}{2M_S^2} \quad \text{or} \quad L_3 = 4\pi f^4 \left(\frac{2\Gamma_S^{(0)}}{3M_S^{(0)\,5}} - \frac{9\Gamma_V^{(0)}}{M_V^{(0)\,5}} \right)$$ [ZHG et al., JHEP'07]

❑ **Contributions from tensor resonances are still under debate !**

[Toublan, PRD'96] [Ecker, Zauner, .,EPCJ'07] [ZHG Ph.D thesis, '09]

- **Vectors:** ρ , K^*, ω, φ
- **Tensors:** f_2 , f_2' , K_2^* , a_2

Mixings: vectors *V.S.* tensors

$$\psi_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

$$\psi_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

$$f' = \psi_8 \cos\theta - \psi_1 \sin\theta$$

$$f = \psi_8 \sin\theta + \psi_1 \cos\theta$$

Ideal mixing $\theta_{id} = 35.3^o$

$$f' = -s\bar{s}$$

$$f = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$$

➢ **Vector mixing angle (ω-φ mixing)**

$\theta_V \sim 36.5^o$: 3% away from θ_{id}

[PDG, 2022]

➢ **Teonsr mixing angle (f_2-f_2' mixing)**

$\theta_T \sim (28\sim30)^o$: (15~21)% away from θ_{id}

❑ **Implication:** 1/Nc correction (OZI rule violation) is sizable for the tensor resonances.

Quick view of the light-flavor tensor mesons

[PDG, 2022]

$f_2(1270)$ $I^G(J^{PC}) = 0^+(2^{++})$

Mass $m = 1275.5 \pm 0.8$ MeV
Full width $\Gamma = 186.7^{+2.2}_{-2.5}$ MeV (S = 1.4)

$f_2(1270)$ DECAY MODES	Fraction (Γ_i/Γ)
$\pi\pi$	$(84.2\ ^{+2.9}_{-0.9})$ %
$\pi^+\pi^-2\pi^0$	$(7.7\ ^{+1.1}_{-3.2})$ %
$K\overline{K}$	$(4.6\ ^{+0.5}_{-0.4})$ %
$2\pi^+2\pi^-$	(2.8 ± 0.4) %
$\eta\eta$	$(4.0 \pm 0.8) \times 10^{-3}$
$4\pi^0$	$(3.0 \pm 1.0) \times 10^{-3}$
$\gamma\gamma$	$(1.42 \pm 0.24) \times 10^{-5}$
$\eta\pi\pi$	$< 8 \times 10^{-3}$
$K^0K^-\pi^+$ + c.c.	$< 3.4 \times 10^{-3}$
e^+e^-	$< 6 \times 10^{-10}$

$a_2(1320)$ $I^G(J^{PC}) = 1^-(2^{++})$

Mass $m = 1318.2 \pm 0.6$ MeV (S = 1.2)
Full width $\Gamma = 107 \pm 5$ MeV [i]

$a_2(1320)$ DECAY MODES	Fraction (Γ_i/Γ)
3π	(70.1 ± 2.7) %
$\eta\pi$	(14.5 ± 1.2) %
$\omega\pi\pi$	(10.6 ± 3.2) %
$K\overline{K}$	(4.9 ± 0.8) %
$\eta'(958)\pi$	$(5.5 \pm 0.9) \times 10^{-3}$
$\pi^\pm\gamma$	$(2.91 \pm 0.27) \times 10^{-3}$
$\gamma\gamma$	$(9.4 \pm 0.7) \times 10^{-6}$
e^+e^-	$< 5 \times 10^{-9}$

$f_2'(1525)$ $I^G(J^{PC}) = 0^+(2^{++})$

Mass $m = 1517.4 \pm 2.5$ MeV (S = 2.8)
Full width $\Gamma = 86 \pm 5$ MeV (S = 2.2)

$f_2'(1525)$ DECAY MODES	Fraction (Γ_i/Γ)
$K\overline{K}$	(87.6 ± 2.2) %
$\eta\eta$	(11.6 ± 2.2) %
$\pi\pi$	$(8.3 \pm 1.6) \times 10^{-3}$
$\gamma\gamma$	$(9.5 \pm 1.1) \times 10^{-7}$

$K_2^*(1430)$ $I(J^P) = \frac{1}{2}(2^+)$

$K_2^*(1430)^\pm$ mass $m = 1427.3 \pm 1.5$ MeV (S
$K_2^*(1430)^0$ mass $m = 1432.4 \pm 1.3$ MeV
$K_2^*(1430)^\pm$ full width $\Gamma = 100.0 \pm 2.1$ MeV
$K_2^*(1430)^0$ full width $\Gamma = 109 \pm 5$ MeV (S =

$K_2^*(1430)$ DECAY MODES	Fraction (Γ_i/Γ)
$K\pi$	(49.9 ± 1.2) %
$K^*(892)\pi$	(24.7 ± 1.5) %
$K^*(892)\pi\pi$	(13.4 ± 2.2) %
$K\rho$	(8.7 ± 0.8) %
$K\omega$	(2.9 ± 0.8) %
$K^+\gamma$	$(2.4 \pm 0.5) \times 10^{-3}$
$K\eta$	$(1.5^{+3.4}_{-1.0}) \times 10^{-3}$
$K\omega\pi$	$< 7.2 \times 10^{-4}$
$K^0\gamma$	$< 9 \times 10^{-4}$

[Lattice: HSC,PRD'18]

m_π=391 MeV

f_2^a	1470(15)	$-\frac{i}{2}160(18)$ MeV
a_2	1505(5)	$-\frac{i}{2}20(3)$ MeV
K_2^*	1577(7)	$-\frac{i}{2}66(7)$ MeV
f_2^b	1602(10)	$-\frac{i}{2}54(14)$ MeV,

$\Gamma_{f_2\to\pi\pi}$	136.0 ± 22.0
$\Gamma_{f_2\to K\bar{K}}$	19.2 ± 7.8
$\Gamma_{f_2'\to\pi\pi}$	4.3 ± 3.1
$\Gamma_{f_2'\to K\bar{K}}$	49.7 ± 20.1
$\Gamma_{a_2\to K\bar{K}}$	7.1 ± 1.9
$\Gamma_{a_2\to\pi\eta}$	13.1 ± 3.0
$\Gamma_{K_2^*\to\pi K}$	62 ± 12

- **Masses, $T\to PP'$, $T\to P\gamma$ will be focused in this talk.**

Tensor mesons in Resonance Chiral Theory

Tensor meson with $J^P = 2^+$: symmetric rank-2 tensor $T_{\mu\nu}$

The on-shell tensor is traceless, i.e. $T_\mu{}^\mu = 0$

[Bellucci et al., NPB'94]
[Toublan, PRD'96]
[Ecker Zauner, EPJC'07]

$$\mathcal{L}_{\rm kin} = -\frac{1}{2}\langle T_{\mu\nu} D^{\mu\nu,\rho\sigma} T_{\rho\sigma}\rangle$$

$$\begin{aligned} D^{\mu\nu,\rho\sigma} = {} & (\Box + M_T^2)\left[\frac{1}{2}(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) - g^{\mu\nu}g^{\rho\sigma}\right] \\ & + g^{\mu\nu}\partial^\rho\partial^\sigma + g^{\rho\sigma}\partial^\mu\partial^\nu \\ & - \frac{1}{2}(g^{\mu\sigma}\partial^\rho\partial^\nu + g^{\mu\rho}\partial^\nu\partial^\sigma + g^{\nu\sigma}\partial^\rho\partial^\mu + g^{\nu\rho}\partial^\mu\partial^\sigma) \end{aligned}$$

$$\mathcal{L}_m^{(0)} = -\frac{M_T^2}{2}\langle T_{\mu\nu} T^{\mu\nu}\rangle$$

$$\sum_\lambda \epsilon_{\mu\nu}(k;\lambda)\epsilon^*_{\rho\sigma}(k;\lambda) = \frac{1}{2}(P_{\mu\rho}P_{\nu\sigma} + P_{\nu\rho}P_{\mu\sigma}) - \frac{1}{3}P_{\mu\nu}P_{\rho\sigma}$$

$$\left(P_{\mu\nu} \equiv g_{\mu\nu} - \frac{k_\mu k_\nu}{M_T^2}\right)$$

Interactions between resonances and pseudo Nambu-Goldstone bosons (pNGBs)

$$\langle R_1 R_2 \cdots R_j \chi^{(n)}(\Phi) \rangle$$

R_i: Resonance fields

chiral building blocks with power counting index n

- 1/Nc counting rule: number of traces

  One additional trace brings one more 1/Nc supppression factor.

- Quark-mass corrections only enter via the operators themselves.

  Couplings are independent of m_q (crucial for chiral extrapolation).

- Flavor assignment for light-flavor tensor nonet

$$T_{\mu\nu} = \begin{pmatrix} \frac{a_2^0}{\sqrt{2}} + \frac{f_2^8}{\sqrt{6}} + \frac{f_2^0}{\sqrt{3}} & a_2^+ & K_2^{*+} \\ a_2^- & -\frac{a_2^0}{\sqrt{2}} + \frac{f_2^8}{\sqrt{6}} + \frac{f_2^0}{\sqrt{3}} & K_2^{*0} \\ K_2^{*-} & \bar{K}_2^{*0} & -\frac{2f_2^8}{\sqrt{6}} + \frac{f_2^0}{\sqrt{3}} \end{pmatrix}_{\mu\nu}$$

Relevant operators for the *T* masses

[Chen, Cheng, Yan, Duan, ZHG, PRD'23]

LO $$\mathcal{L}_m^{(0)} = -\frac{M_T^2}{2}\langle T_{\mu\nu}T^{\mu\nu}\rangle$$

NLO $$\mathcal{L}_m^{(1)} = \lambda_T\langle T_{\mu\nu}T^{\mu\nu}\chi_+\rangle + \lambda'_T\langle T_{\mu\nu}\rangle\langle T^{\mu\nu}\rangle$$

$O(p^2, N_C^0)$ $O(p^0, N_C^{-1})$

O(δ) ~ O(p^2) ~ O($1/N_c$)

NNLO $$\langle T_{\mu\nu}T^{\mu\nu}\chi_+\chi_+\rangle, \quad \langle T_{\mu\nu}T^{\mu\nu}\rangle\langle\chi_+\rangle, \quad \langle T_{\mu\nu}\rangle\langle T^{\mu\nu}\chi_+\rangle$$

(impossible to pin down their coefficients when only focusing masses)

Relevant operators for the *T→PP'* decays

$$\mathscr{L}_{TPP}^{(0)} = g_T\langle T_{\mu\nu}\{u^\mu, u^\nu\}\rangle$$

$$\begin{aligned}\mathscr{L}_{TPP}^{(1)} =& f_T\langle T_{\mu\nu}\{\{u^\mu, u^\nu\}, \chi_+\}\rangle + f'_T\langle T_{\mu\nu}(u^\mu\chi_+u^\nu + u^\nu\chi_+u^\mu)\rangle \\ &+ g'_T\langle T_{\mu\nu}\rangle\langle u^\mu u^\nu\rangle + g''_T\left(\langle T_{\mu\nu}u^\mu\rangle\langle u^\nu\rangle + \langle T_{\mu\nu}u^\nu\rangle\langle u^\mu\rangle\right),\end{aligned}$$

Operators for the *T→Pγ* , *γγ* decays

$$\mathscr{L}^{TP\gamma} = i\frac{c_{TP\gamma}}{2}\epsilon_{\mu\nu\alpha\beta}\langle T^{\alpha\lambda}[f_+^{\mu\nu}, \partial^\beta u_\lambda]\rangle$$

$$\mathscr{L}_{T\gamma\gamma}^{(0)} = c_{T\gamma\gamma}\langle T_{\mu\nu}\Theta_\gamma^{\mu\nu}\rangle, \quad \mathscr{L}_{T\gamma\gamma}^{(1)} = d_{T\gamma\gamma}\langle T_{\mu\nu}\Theta_\gamma^{\mu\nu}\chi_+\rangle \qquad \Theta_\gamma^{\mu\nu} = f_{+\alpha}^{\mu}f_+^{\alpha\nu} + \frac{1}{4}g^{\mu\nu}f_+^{\rho\sigma}f_{+\rho\sigma}$$

Phenomenologies

❖ Masses of tensor resonances

$$f_2^8 = \sin\theta_T f_2 + \cos\theta_T f_2' \,, \quad f_2^0 = \cos\theta_T f_2 - \sin\theta_T f_2'$$

Up to NLO

$$M_{f_2}^2 = M_T^2 - 4\lambda_T m_K^2 - 3\lambda_T' - \sqrt{16\lambda_T^2(m_K^2 - m_\pi^2)^2 - 8\lambda_T\lambda_T'(m_K^2 - m_\pi^2) + 9{\lambda_T'}^2}$$

$$M_{f_2'}^2 = M_T^2 - 4\lambda_T m_K^2 - 3\lambda_T' + \sqrt{16\lambda_T^2(m_K^2 - m_\pi^2)^2 - 8\lambda_T\lambda_T'(m_K^2 - m_\pi^2) + 9{\lambda_T'}^2}$$

$$M_{a_2}^2 = M_T^2 - 4\lambda_T m_\pi^2$$

$$M_{K_2^*}^2 = M_T^2 - 4\lambda_T m_K^2$$

$$\tan 2\theta_T = \frac{8\sqrt{2}(m_\pi^2 - m_K^2)\lambda_T}{4(m_\pi^2 - m_K^2)\lambda_T + 9\lambda_T'}$$

Large Nc limit ⟹ $\lambda_T' \to 0$

$$\theta_T = \arctan(2\sqrt{2})/2 = 35.3^\circ$$

(ideal mixing)

Case 1: Exp data only

Exp data

$$M_{f_2}^{\rm Exp} = 1275.5 \pm 0.8, \qquad M_{a_2}^{\rm Exp} = 1318.2 \pm 0.6,$$

$$M_{K_2^*}^{\rm Exp} = 1429.9 \pm 4.1, \qquad M_{f_2'}^{\rm Exp} = 1517.4 \pm 2.5,$$

Fitted parameters

$$M_T = (1308.5 \pm 1.2)\ \text{MeV}, \quad \lambda_T = -0.336 \pm 0.008, \quad \lambda_T' = (25718 \pm 1054)\ \text{MeV}^2$$

Prediction: $\theta_T^{\rm Phy} = (29.1 \pm 0.1)^\circ$

Case 2: Lat data only

Lat data

(m_π=391 MeV, m_K=550 MeV)

$$M_{f_2}^{\rm Lat} = 1470 \pm 15, \qquad M_{a_2}^{\rm Lat} = 1505 \pm 5,$$

$$M_{K_2^*}^{\rm Lat} = 1577 \pm 7, \qquad M_{f_2'}^{\rm Lat} = 1602 \pm 10,$$

Fitted parameters

$$M_T = (1444 \pm 18)\ \text{MeV}, \quad \lambda_T = -0.307 \pm 0.052, \quad \lambda_T' = (27920 \pm 16526)\ \text{MeV}^2,$$

Prediction: $\theta_T^{\rm Lat} = (25.0^{+5.6}_{-4.4})^\circ$

An important lesssson:

- **Mass splitting parameters λ_T、λ_T' from Exp and Lat are compatible, while M_T from the two fits are different.**

Case 3: Joint fit to both data from Exp and Lat

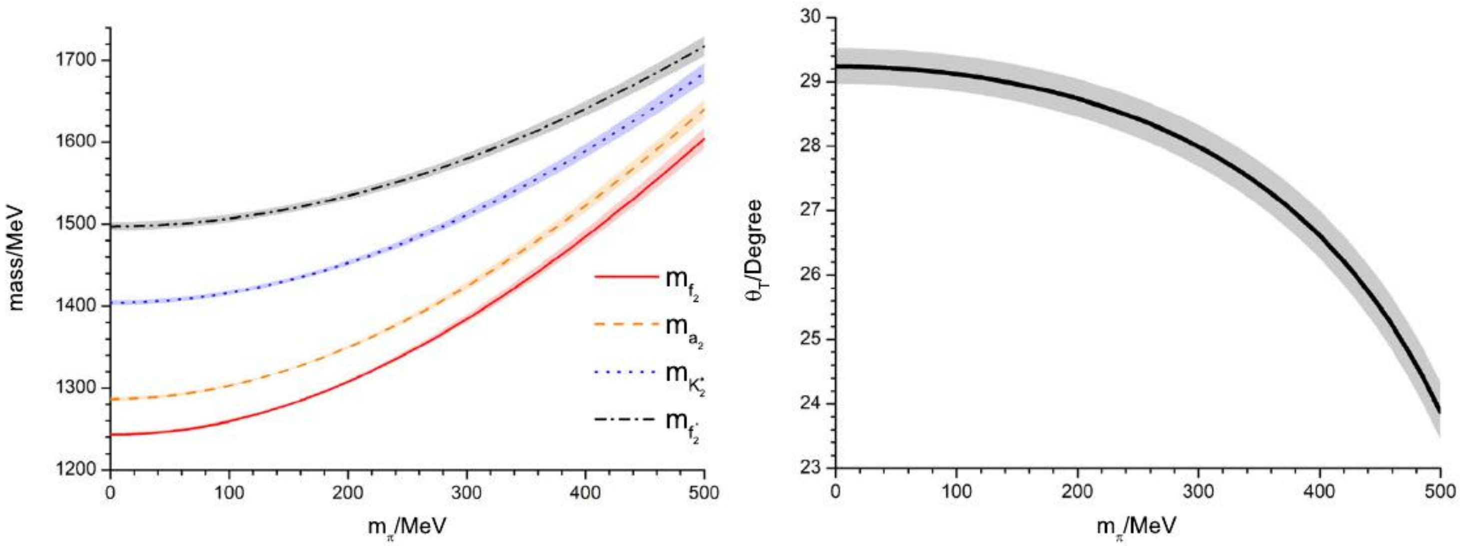
[Chen,Cheng,Yan,Duan,ZHG,PRD'23]

To reconcile M_T from Exp and Lat: $\lambda_T''\langle T_{\mu\nu}T^{\mu\nu}\rangle\langle\chi_+\rangle$ **(NNLO)**

Parameters from joint fit

$$M_T = (998.7 \pm 26.4)\ \text{MeV}, \qquad \lambda_T = -0.335 \pm 0.009,$$
$$\lambda_T' = (25732 \pm 1216)\ \text{MeV}^2, \qquad \lambda_T'' = -0.350 \pm 0.026,$$

	Experimental	Theoretical	Lattice	Theoretical
M_{f_2} (MeV)	1275.5 ± 0.8	1275.5 ± 1.7	1470 ± 15	1466 ± 8
M_{a_2} (MeV)	1318.2 ± 0.6	1318.2 ± 1.3	1505 ± 5	1505 ± 8
$M_{K_2^*}$ (MeV)	1429.9 ± 4.1	1428.8 ± 2.8	1577 ± 7	1570 ± 8
$M_{f_2'}$ (MeV)	1517.4 ± 2.5	1517.2 ± 5.1	1602 ± 10	1620 ± 8
$\theta_T(^\circ)$	$\cdots$	29.0 ± 0.4	$\cdots$	26.4 ± 0.3



❖ $T\to P\,P'$ decays ($P=\pi,K,\eta,\eta'$)

Two-mixing-angle formalism for *η-η'*

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \frac{1}{F}\begin{pmatrix} F_8\cos\theta_8 & -F_0\sin\theta_0 \\ F_8\sin\theta_8 & F_0\cos\theta_0 \end{pmatrix}\begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

Examples of decay width formulas

$$\Gamma_{f_2\to\pi\pi} = \left[\frac{4\cos\theta_T + 2\sqrt{2}\sin\theta_T}{F_\pi^2} g_T + \frac{(16\cos\theta_T + 8\sqrt{2}\sin\theta_T)m_\pi^2}{F_\pi^2} f_T + \frac{6\cos\theta_T}{F_\pi^2} g'_T + \frac{(8\cos\theta_T + 4\sqrt{2}\sin\theta_T)m_\pi^2}{F_\pi^2} f'_T\right]^2 \frac{p^5(m_{f_2}, m_\pi, m_\pi)}{30\pi m_{f_2}^2},$$

$$\begin{aligned}\Gamma_{f_2\to\eta\eta} = &\\ &\left\{\frac{2}{3\sqrt{3}F_0^2F_8^2\cos^2(\theta_0-\theta_8)}\{-\sin\theta_T[\sqrt{2}F_0^2[3g_T + 2(2f_T + f'_T)(8m_K^2 - 5m_\pi^2)]\cos^2\theta_0\right.\\ &+ 2F_0F_8[6g_T + 8f_T(4m_K^2 - m_\pi^2)]\cos\theta_0\sin\theta_8 + 8\sqrt{2}F_8^2(2f_T + f'_T)(m_K^2 - m_\pi^2)\sin^2\theta_8]\\ &+ \cos\theta_T[F_0^2[6g_T + 8f_T(4m_K^2 - m_\pi^2)]\cos^2\theta_0 + 16\sqrt{2}F_0F_8(2f_T + f'_T)(m_K^2 - m_\pi^2)\cos\theta_0\sin\theta_8\\ &\left.+ F_8^2(6g_T + 9g'_T + 16f_Tm_K^2 + 8f'_Tm_K^2 + 8f_Tm_\pi^2 + 4f'_Tm_\pi^2)\sin^2\theta_8]\}\right\}^2 \frac{p^5(m_{f_2}, m_\eta, m_\eta)}{30\pi m_{f_2}^2},\end{aligned}$$

For others channels, see

[Chen,Cheng,Yan,Duan,ZHG,PRD'23]

Fit I: One-mixing-angle description for η-η' ($F_8=F_0=F$, $\theta_8=\theta_0=\theta$)

Parameters from the joint fit to the widths from Exp and Lat

$$g_T = (16.6 \pm 1.1)\ \text{MeV}, \qquad f_T = (3.9 \pm 1.4) \times 10^{-6}\ \text{MeV}^{-1},$$

$$g'_T = (4.8 \pm 0.6)\ \text{MeV}, \qquad f'_T = (-3.1 \pm 2.9) \times 10^{-6}\ \text{MeV}^{-1}, \qquad \theta^{\text{Phy}} = (-9.0 \pm 5.5)^\circ,$$

Important: $F_\pi=F_K=F$ **must be taken to obtain reasonable fit, which is consistent with the one-mixing-angle description.**

Fit II: Two-mixing-angle description for η-η'

$$g_T = (19.9 \pm 1.5)\ \text{MeV}, \qquad f_T = (1.2 \pm 0.2) \times 10^{-5}\ \text{MeV}^{-1},$$

$$g'_T = (6.3 \pm 0.7)\ \text{MeV}, \qquad f'_T = (7.5 \pm 5.3) \times 10^{-6}\ \text{MeV}^{-1}, \qquad \theta_8^{\text{Phy}} = (-17.3 \pm 6.3)^\circ,$$

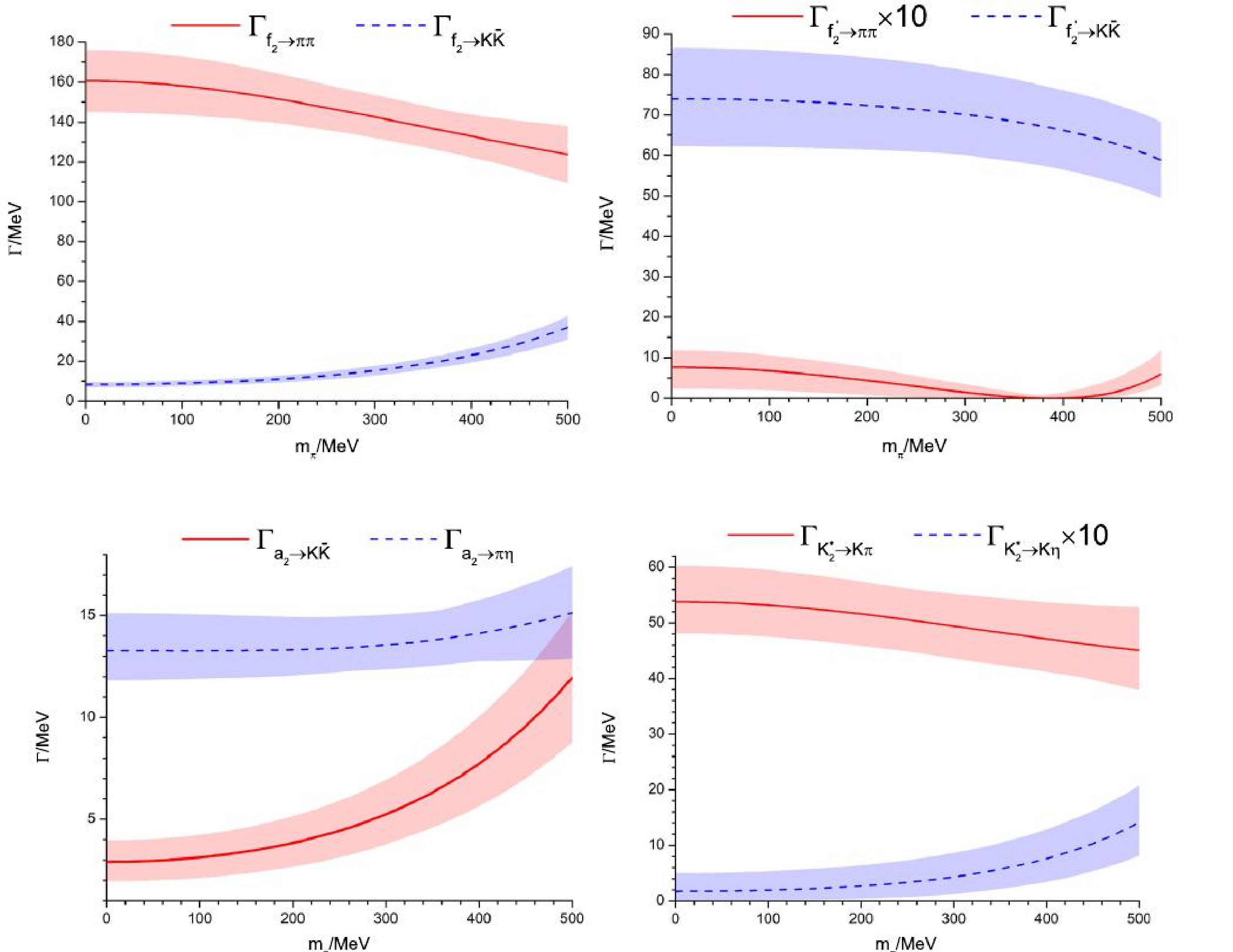
Important: **we need to distinguish** F_π、F_K **in the decay widths, which is consistent with the two-mixing-angle description that receives from higher order chiral corrections.**

	[PDG, 2022]	Fit I	Fit II
$\Gamma_{f_2\to\pi\pi}$	157.2 ± 7.3	156.9 ± 14.3	157.2 ± 15.5
$\Gamma_{f_2\to K\bar{K}}$	8.6 ± 1.1	9.4 ± 1.5	8.6 ± 1.6
$\Gamma_{f_2\to\eta\eta}$	0.7 ± 0.2	1.0 ± 0.2	0.8 ± 0.3
$\Gamma_{f_2'\to\pi\pi}$	0.7 ± 0.2	0.6 ± 0.5	0.7 ± 0.4
$\Gamma_{f_2'\to K\bar{K}}$	75.3 ± 6.3	74.1 ± 12.4	70.2 ± 12.3
$\Gamma_{f_2'\to\eta\eta}$	10.0 ± 2.6	9.7 ± 3.5	7.5 ± 2.9
$\Gamma_{a_2\to K\bar{K}}$	5.2 ± 1.1	3.3 ± 1.2	4.0 ± 1.3
$\Gamma_{a_2\to\pi\eta}$	15.5 ± 2.1	13.3 ± 2.9	13.2 ± 3.4
$\Gamma_{a_2\to\pi\eta'}$	0.6 ± 0.1	0.6 ± 0.1	0.6 ± 0.1
$\Gamma_{K_2^*\to\eta K}$	$0.2^{+0.4}_{-0.2}$	$0.2^{+0.4}_{-0.2}$	$0.1^{+0.4}_{-0.1}$
$\Gamma_{K_2^*\to\pi K}$	52.1 ± 6.1	53.5 ± 6.4	59.9 ± 7.1

	[HSC, PRD'15 '18]	Fit I	Fit II
$\Gamma_{f_2\to\pi\pi}$	136.0 ± 22.0	132.0 ± 10.8	117.0 ± 10.2
$\Gamma_{f_2\to K\bar{K}}$	19.2 ± 7.8	24.4 ± 3.7	20.9 ± 3.3
$\Gamma_{f_2'\to\pi\pi}$	4.3 ± 3.1	$(1.2^{+16.5}_{-1.2})\times10^{-2}$	0.2 ± 0.2
$\Gamma_{f_2'\to K\bar{K}}$	49.7 ± 20.1	59.7 ± 9.2	53.4 ± 7.8
$\Gamma_{a_2\to K\bar{K}}$	7.1 ± 1.9	8.0 ± 2.3	9.2 ± 2.2
$\Gamma_{a_2\to\pi\eta}$	13.1 ± 3.0	13.8 ± 2.5	17.9 ± 2.0
$\Gamma_{K_2^*\to\pi K}$	62 ± 12	47.4 ± 6.3	48.4 ± 6.4

Predictions to the trajectories of decay widths by varying m_π

[Chen,Cheng,Yan,Duan,ZHG,PRD'23]



❖ $T \to P\gamma$

$$\mathcal{L}^{TP\gamma} = i\frac{c_{TP\gamma}}{2}\epsilon_{\mu\nu\alpha\beta}\langle T^{\alpha\lambda}[f_+^{\mu\nu}, \partial^\beta u_\lambda]\rangle$$

$$\Gamma^{\rm Exp}_{a_2^\pm \to \pi^\pm\gamma} = (0.31 \pm 0.04)\ {\rm MeV},$$
$$\Gamma^{\rm Exp}_{K_2^{*\pm} \to K^\pm\gamma} = (0.24 \pm 0.06)\ {\rm MeV},$$

$$c_{TP\gamma} = (5.4 \pm 0.5) \times 10^{-5}\ {\rm MeV}^{-1}$$
$$\Gamma^{\rm Theo}_{a_2^\pm \to \pi^\pm\gamma} = (0.30 \pm 0.04)\ {\rm MeV},$$
$$\Gamma^{\rm Theo}_{K_2^{*\pm} \to K^\pm\gamma} = (0.25 \pm 0.03)\ {\rm MeV},$$

Lesson: **the LO $c_{TP\gamma}$ is already enough to describe the available $T \to P\gamma$ data.**

$T \to \gamma\gamma$ decays $\Gamma^{\rm Exp}_{f_2\to\gamma\gamma} = 2.7 \pm 0.5,\quad \Gamma^{\rm Exp}_{f_2'\to\gamma\gamma} = 0.082 \pm 0.015,\quad \Gamma^{\rm Exp}_{a_2\to\gamma\gamma} = 1.0 \pm 0.1$

Case 1: **only take LO** $\mathcal{L}^{(0)}_{T\gamma\gamma} = c_{T\gamma\gamma}\langle T_{\mu\nu}\Theta_\gamma^{\mu\nu}\rangle$

- **Fix $\theta_T = 29.0^o$ from the mass determination, an overall description to all the three channels can not be satisfactorily achieved.**
- **To free θ_T, one would obtain $\theta_T = 27.2 \pm 1^o$, which disagrees with $\theta_T = 29.0 \pm 0.4^o$ from the mass determination.**

Case 2: **LO + NLO** $\mathcal{L}^{(0)}_{T\gamma\gamma} = c_{T\gamma\gamma}\langle T_{\mu\nu}\Theta_\gamma^{\mu\nu}\rangle \quad \mathcal{L}^{(1)}_{T\gamma\gamma} = d_{T\gamma\gamma}\langle T_{\mu\nu}\Theta_\gamma^{\mu\nu}\chi_+\rangle$

Fix $\theta_T = 29.0^o$, the fit gives

$$c_{T\gamma\gamma} = (2.4 \pm 0.1) \times 10^{-4}\ {\rm MeV}^{-1},$$
$$d_{T\gamma\gamma} = (-3.2 \pm 1.5) \times 10^{-11}\ {\rm MeV}^{-3},$$

$$\Gamma^{\rm Theo}_{f_2\to\gamma\gamma} = 2.6 \pm 0.2,\qquad \Gamma^{\rm Theo}_{f_2'\to\gamma\gamma} = 0.082 \pm 0.015,$$
$$\Gamma^{\rm Theo}_{a_2\to\gamma\gamma} = 1.0 \pm 0.1,$$

Summary

- **Quark-mass and 1/Nc corrections are systematically incorpoated for the tensor mesons within Resonance Chiral Theory.**

- **Tensor masses and $T \to P P'$ decay widths from Exp and Lat are well described. The f_2-f_2' mixing angle and the pion-mass dependence of tensor masses and decay widths are predicted.**

- **Satisfactory descripitons of two types of radiative decays: $T \to P\gamma$ & $T \to \gamma\gamma$, are obtained.**

- **This work provides useful inputs to the future study of the tensor contributions to the $P P' \to P P'$ and $P\gamma \to P'\gamma$ and $\gamma\gamma \to \gamma\gamma$ processes.**

谢谢大家！