Searching for hadronic molecular states with quark contents $bc\bar{s}\bar{q}$, $b\bar{c}s\bar{q}$ and $b\bar{c}\bar{s}q$

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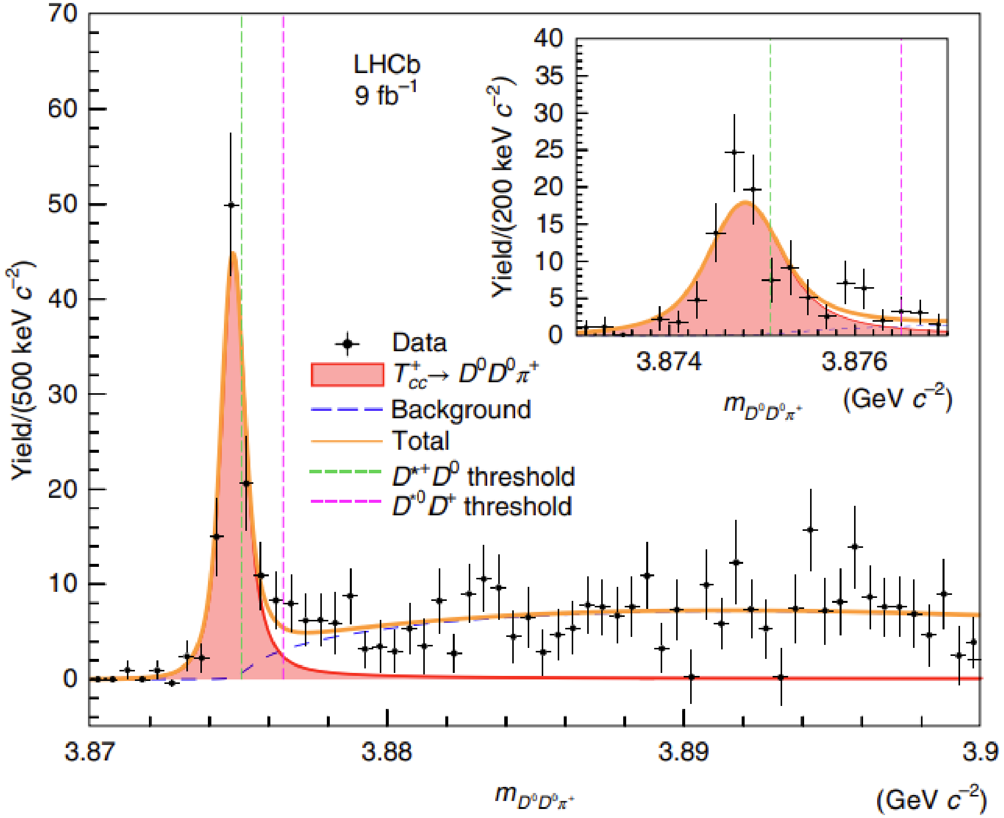
- Result of possible molecular states
- Summary and Outlook

The double charmed tetraquark state T_{cc}



The distribution of the $D_0 D_0 \pi^+$ mass

- The BW mass and width of T_{cc} :

$$M_{\rm BW} = M_{D^{*+}} + M_{D^0} - (273 \pm 61 \pm 5^{+11}_{-14})\ \text{keV},$$
$$\Gamma_{\rm BW} = 410 \pm 165 \pm 43^{+18}_{-38}\ \text{keV}.$$

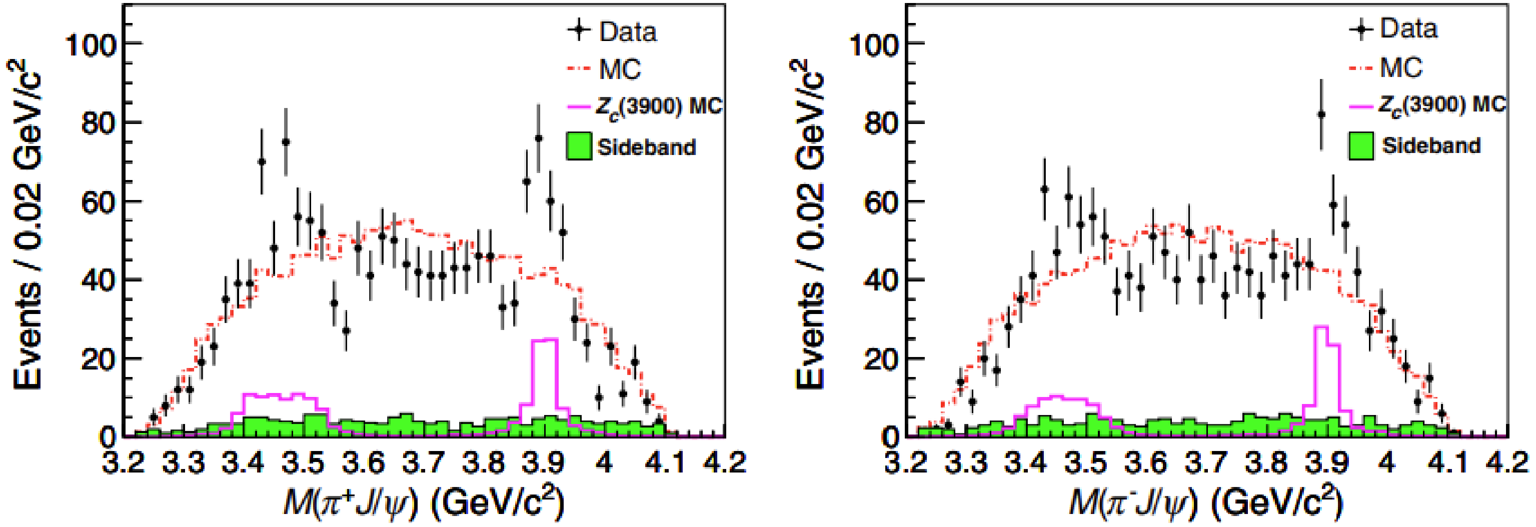
- Considering the experimental resolution produces the resonance :

$$m_{\rm pole} = M_{D^{*+}} + M_{D^0} - (360 \pm 40^{+0}_{-4})\ \text{keV},$$
$$\Gamma_{\rm pole} = 48 \pm 2^{+0}_{-14}\ \text{keV}.$$

- quantum numbers, $(I)J^P = (0)1^+$

LHCb, Nat. Phys. 18, 751 (2022).

$Z_c(3900)$ and its strange partner $Z_{cs}(3985)$



invariant mass distributions of $J/\psi\pi$ in $e^+e^- \to \pi^+\pi^- J/\psi$

- In 2013, a structure in $J/\psi\pi$ mass distribution was reported by BESIII Collaboration：

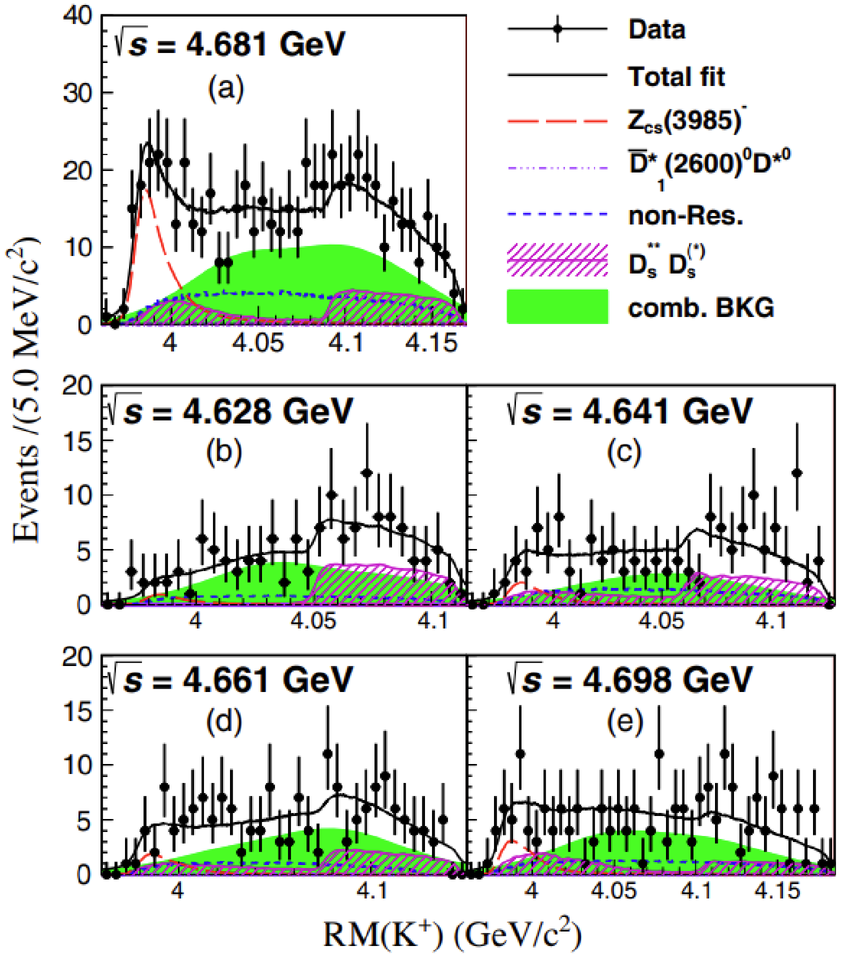
$$M_{Z_c} = 3899.0 \pm 3.6 \pm 4.9$$

$$\Gamma_{Z_c} = 46 \pm 10 \pm 20$$

- $M_{Z_c} - M_{D^*} - M_{\bar{D}} \sim 21$ MeV

BESIII, Phys. Rev. Lett. 110, 252001 (2013).

$Z_c(3900)$ and its strange partner $Z_{cs}(3985)$



K^+ recoil-mass spectra in $e^+e^- \to K^+(D_s^- D^{*0} + D_s^{*-} D^0)$

BESIII, Phys. Rev. Lett. 126, 102001 (2021).

- In 2021, the possible strange partner of $Z_c(3900)$ was reported by BESIII Collaboration :

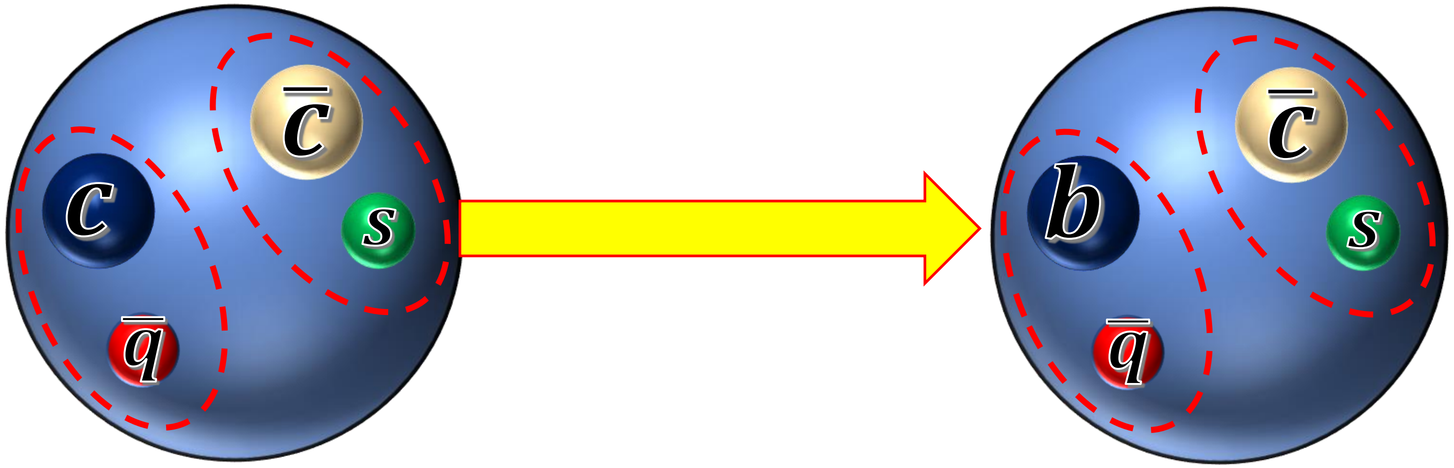
$$M_{Z_{cs}} = 3982.5^{+1.8}_{-2.6} \pm 2.1$$

$$\Gamma_{Z_{cs}} = 12.8^{+5.3}_{-4.4} \pm 3.0$$

- $M_{Z_{cs}} - M_{D_s^*} - M_{\bar{D}} \sim 5$ MeV
- $M_{Z_{cs}} - M_{D_s} - M_{\bar{D}^*} \sim 8$ MeV

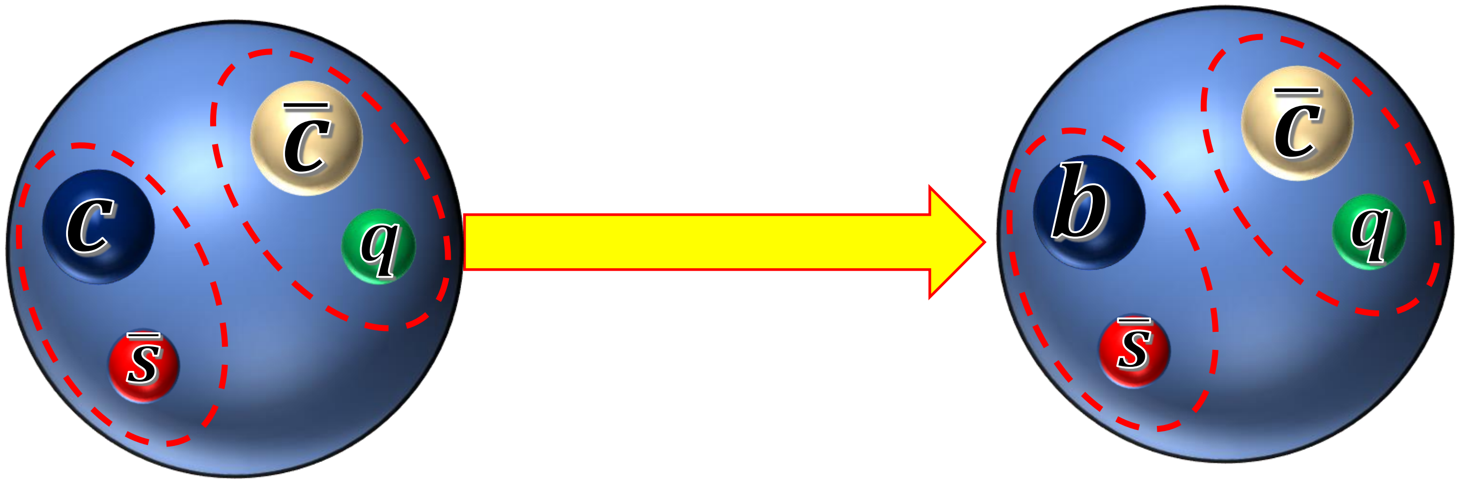
The possible heavy quark partner of Z_{cs}/T_{cc} states

Z_{cs}:$c\bar{c}s\bar{q}$



Z_{bcs}:$b\bar{c}s\bar{q}$

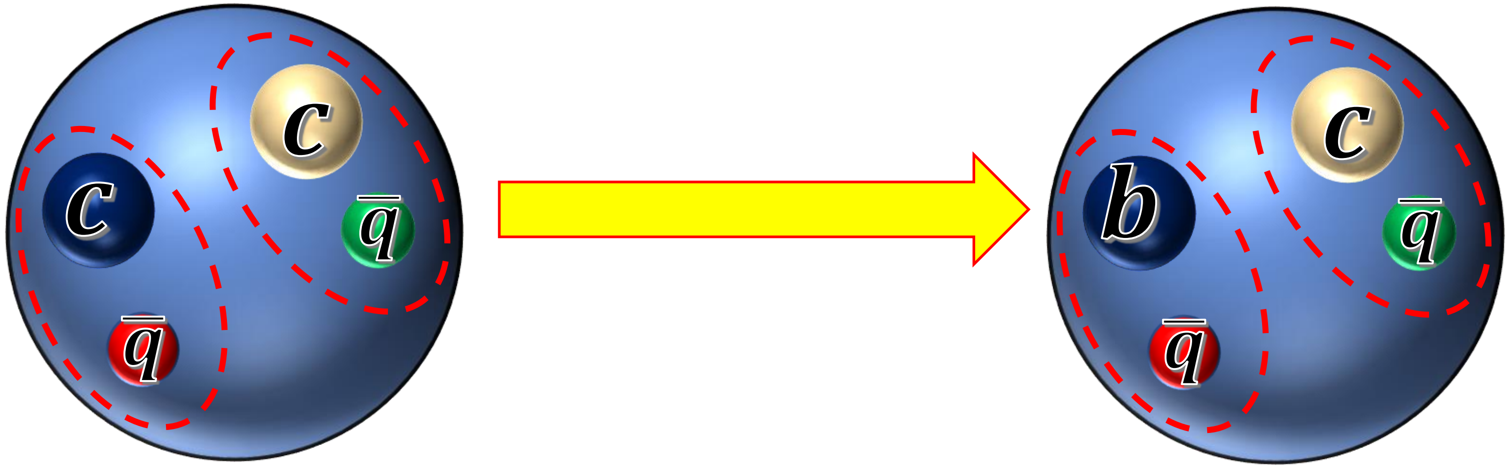
$\bar{Z}_{cs}$:$c\bar{c}q\bar{s}$



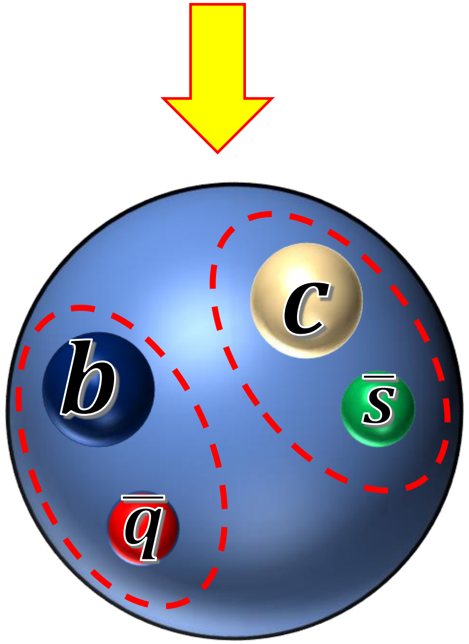
$\bar{Z}_{bcs}$:$b\bar{c}q\bar{s}$

The possible heavy quark partner of Z_{cs}/T_{cc} states

T_{cc} :$cc\bar{q}\bar{q}$



T_{bc} :$bc\bar{q}\bar{q}$

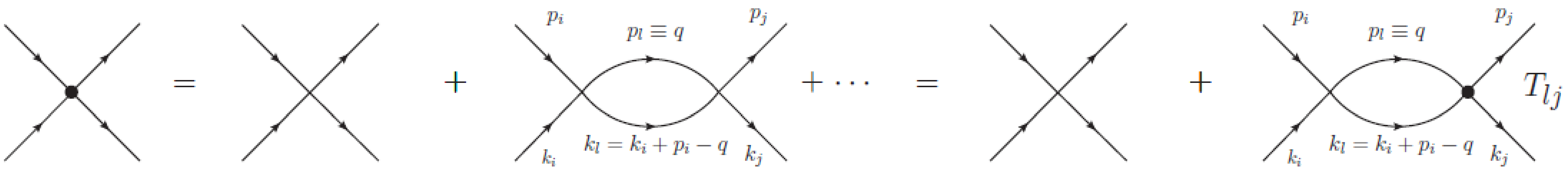


T_{bcs} :$bc\bar{s}\bar{q}$

- In the heavy quark limit, the b/c quark could be considered as a rest color resource, so the two different quark components could have similar properties.

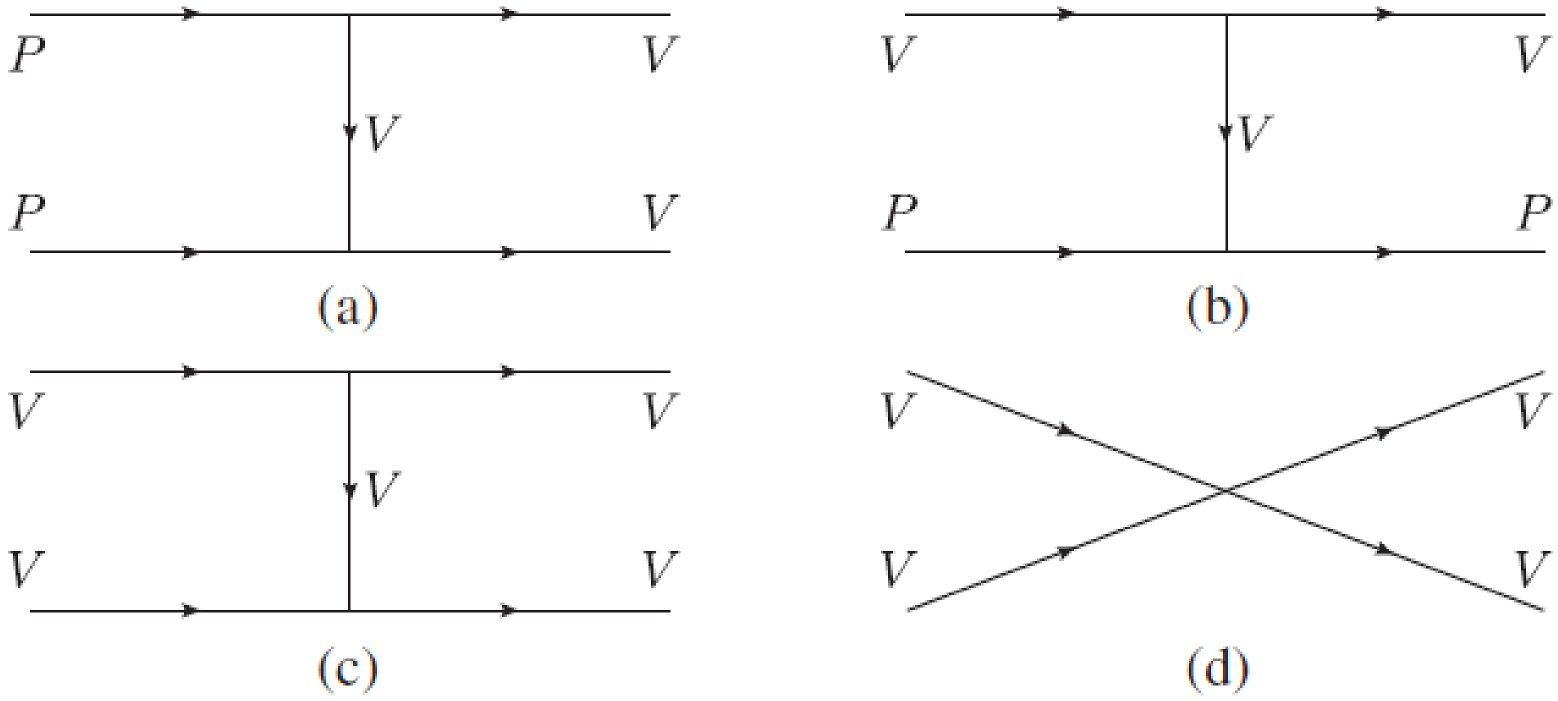
BS-eq within LHG

Scattering matrix solved through the Bethe-Salpeter equation in coupled channels



$$T_{ij} = V_{ij} + \sum_l V_{il} G_{ll} V_{lj} + \cdots = V_{ij} + V_{il} G_{ll} T_{lj}$$

BS-eq within LHG



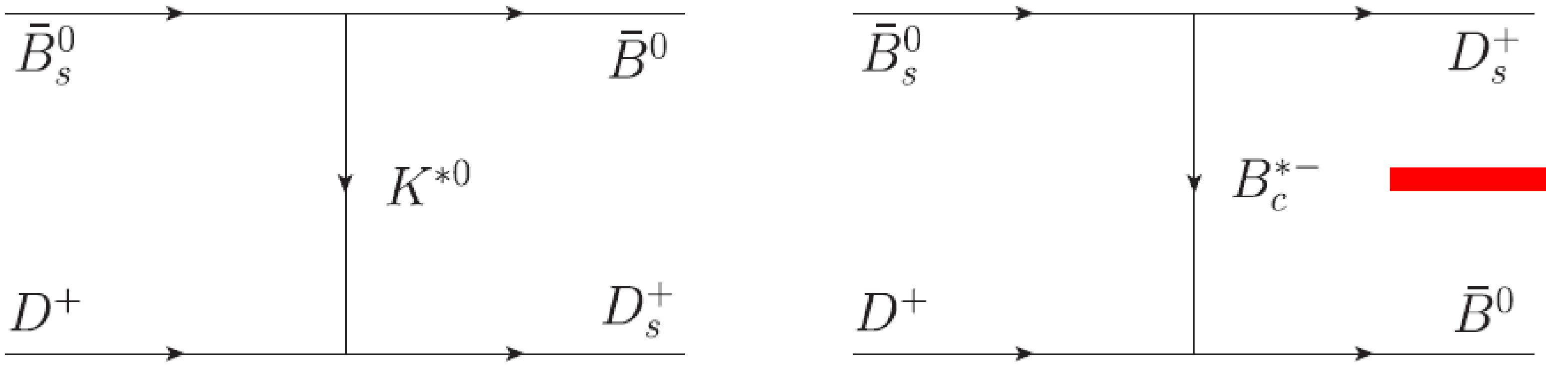
$$\mathcal{L}_{VPP} = -ig\langle [P, \partial_\mu P] V^\mu \rangle,$$

$$\mathcal{L}_{VVV} = ig\langle (V^\mu \partial_\nu V_\mu - \partial_\nu V^\mu V_\mu) V^\nu \rangle,$$

$$\mathcal{L}_{VVVV} = \frac{g^2}{2} \langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu \rangle.$$

P: pseudoscalar meson, V: vector meson

P-P interaction in $bc\bar{s}\bar{q}$ system



Heavy vector exchange negligible

$$V_{PP}(s) = C_{PP} \times g^2(p_1 + p_3)(p_2 + p_4),$$

- Two coupled channels in $bc\bar{s}\bar{q}$ system,

$$\bar{B}_s^0 D^+, \qquad \bar{B}^0 D_s^+.$$

$$C_{PP} = \left(\begin{array}{c|cc} J=0 & \bar{B}_s^0 D^+ & \bar{B}^0 D_s^+ \\ \hline \bar{B}_s^0 D^+ & 0 & \frac{1}{m_{K^*}^2} \\ \bar{B}^0 D_s^+ & \frac{1}{m_{K^*}^2} & 0 \end{array}\right),$$

P-P interaction in $bc\bar{s}\bar{q}$ system

$\bar{B}_s^0 D^+$ 7236.6

$\bar{B}^0 D_s^+$ 7248.0

Similar to forming an isospin channel

➢ mixing of the two channels

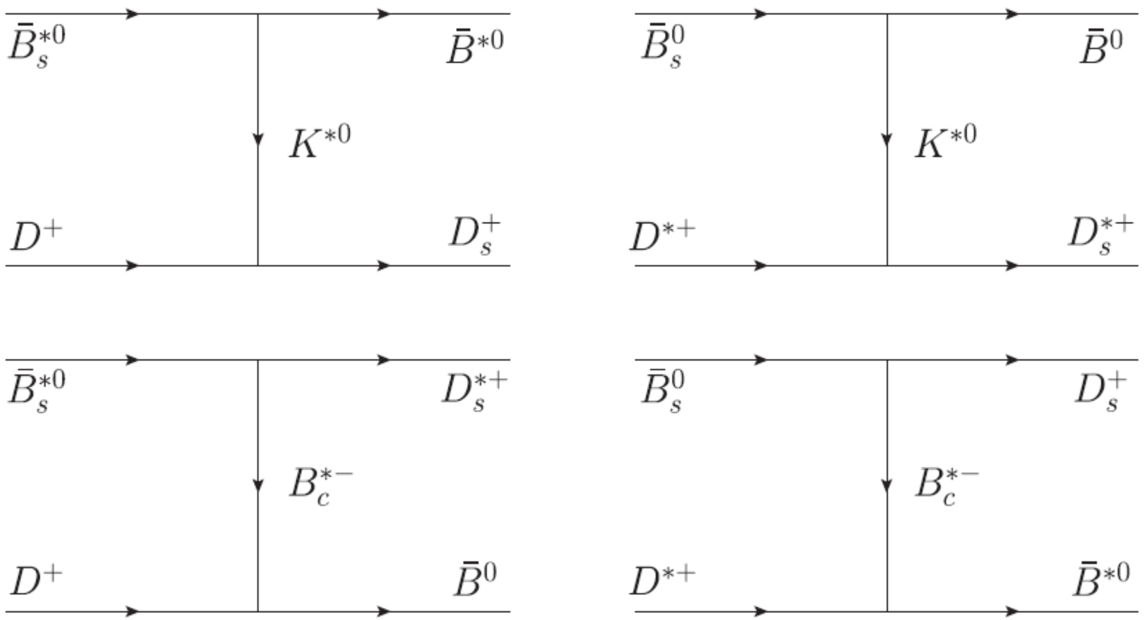
$$|(\bar{B}D)_s^+; J=0\rangle = \frac{1}{\sqrt{2}}(|\bar{B}_s^0 D^+\rangle_{J=0} + |\bar{B}^0 D_s^+\rangle_{J=0}),$$

$$|(\bar{B}D)_s^-; J=0\rangle = \frac{1}{\sqrt{2}}(|\bar{B}_s^0 D^+\rangle_{J=0} - |\bar{B}^0 D_s^+\rangle_{J=0}),$$

$$C_{PP} = \begin{pmatrix} J=0 & \bar{B}_s^0 D^+ & \bar{B}^0 D_s^+ \\ \bar{B}_s^0 D^+ & 0 & \frac{1}{m_{K^*}^2} \\ \bar{B}^0 D_s^+ & \frac{1}{m_{K^*}^2} & 0 \end{pmatrix} \longrightarrow C'_{PP} = \begin{pmatrix} J=0 & (\bar{B}D)_s^+ & (\bar{B}D)_s^- \\ (\bar{B}D)_s^+ & \frac{1}{m_{K^*}^2} & 0 \\ (\bar{B}D)_s^- & 0 & -\frac{1}{m_{K^*}^2} \end{pmatrix}$$

V-P interaction in $bc\bar{s}\bar{q}$ system

$$\bar{B}_s^{*0} D^+, \quad \bar{B}^{*0} D_s^+, \quad \bar{B}_s^0 D^{*+}, \quad \bar{B}^{*0} D_s^{*+}$$



$$|(\bar{B}^* D)_s^+; J=1\rangle = \frac{1}{\sqrt{2}}(|\bar{B}_s^{*0} D^+\rangle_{J=1} + |\bar{B}^{*0} D_s^+\rangle_{J=1})$$

$$|(\bar{B}^* D)_s^-; J=1\rangle = \frac{1}{\sqrt{2}}(|\bar{B}_s^{*0} D^+\rangle_{J=1} - |\bar{B}^{*0} D_s^+\rangle_{J=1})$$

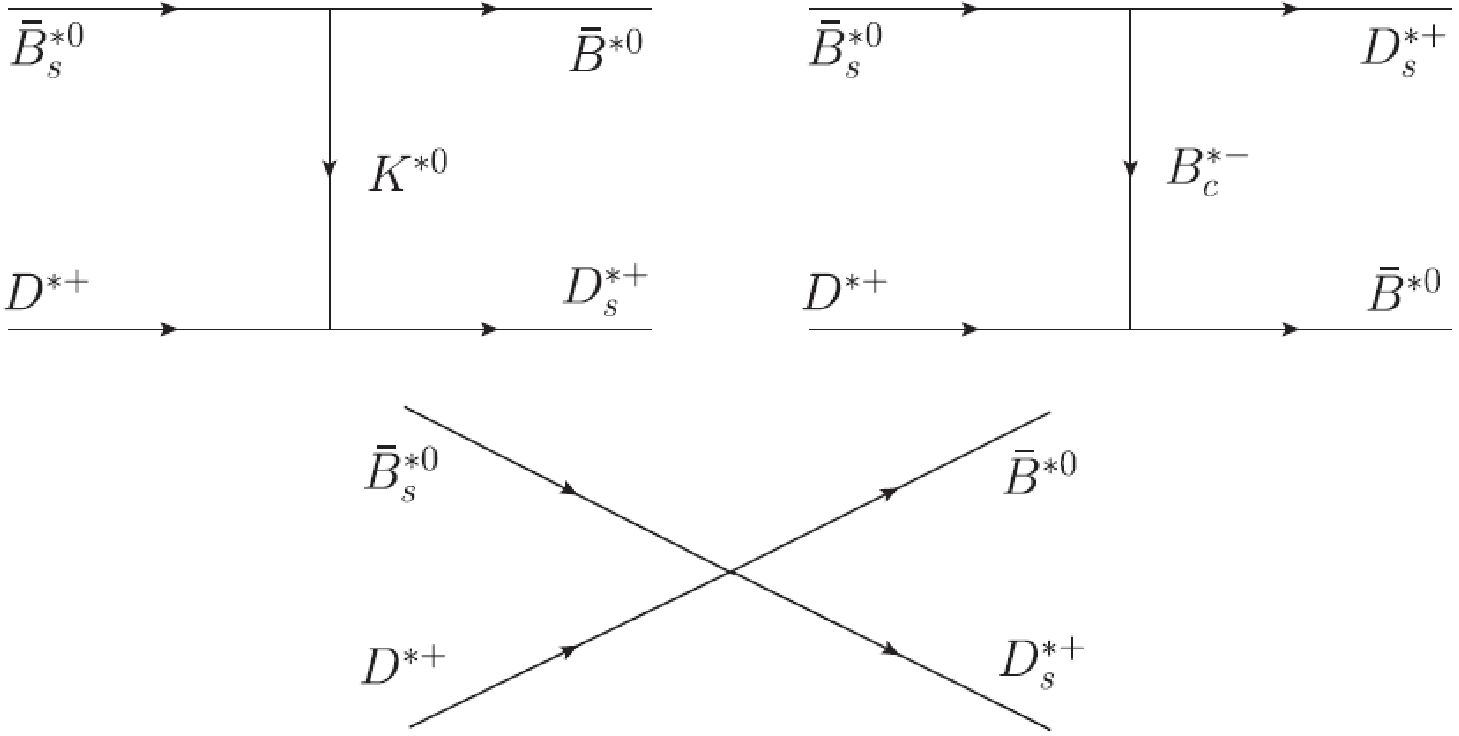
$$|(\bar{B} D^*)_s^+; J=1\rangle = \frac{1}{\sqrt{2}}(|\bar{B}_s^0 D^{*+}\rangle_{J=1} + |\bar{B}^0 D_s^{*+}\rangle_{J=1})$$

$$|(\bar{B} D^*)_s^-; J=1\rangle = \frac{1}{\sqrt{2}}(|\bar{B}_s^0 D^{*+}\rangle_{J=1} - |\bar{B}^0 D_s^{*+}\rangle_{J=1})$$

V-P interaction in $bc\bar{s}\bar{q}$ system

$$C_{VP} = \begin{pmatrix} J=1 & \bar{B}_s^{*0}D^+ & \bar{B}^{*0}D_s^+ & \bar{B}_s^0D^{*+} & \bar{B}^0D_s^{*+} \\ \hline \bar{B}_s^{*0}D^+ & 0 & \frac{1}{m_{K^*}^2} & 0 & 0 \\ \bar{B}^{*0}D_s^+ & \frac{1}{m_{K^*}^2} & 0 & 0 & 0 \\ \bar{B}_s^0D^{*+} & 0 & 0 & 0 & \frac{1}{m_{K^*}^2} \\ \bar{B}^0D_s^{*+} & 0 & 0 & \frac{1}{m_{K^*}^2} & 0 \end{pmatrix} \Rightarrow C'_{VP} = \begin{pmatrix} J=1 & (\bar{B}^*D)_s^+ & (\bar{B}^*D)_s^- & (\bar{B}D^*)_s^+ & (\bar{B}D^*)_s^- \\ \hline (\bar{B}^*D)_s^+ & \frac{1}{m_{K^*}^2} & 0 & 0 & 0 \\ (\bar{B}^*D)_s^- & 0 & -\frac{1}{m_{K^*}^2} & 0 & 0 \\ (\bar{B}D^*)_s^+ & 0 & 0 & \frac{1}{m_{K^*}^2} & 0 \\ (\bar{B}D^*)_s^- & 0 & 0 & 0 & -\frac{1}{m_{K^*}^2} \end{pmatrix}$$

V-V interaction in $bc\bar{s}\bar{q}$ system



- Additional four-vector contact terms are taken into account from :

$$\mathcal{L}_{VVVV} = \frac{g^2}{2}\langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu \rangle$$

Loop function

- Bethe-Salpeter equation:

$$T_{PP/VP/VV}(s) = \frac{V_{PP/VP/VV}(s)}{1 - V_{PP/VP/VV}(s)G(s)}$$

- Loop function:

$$G_{ii}(s) = i\int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_1^2 + i\epsilon} \frac{1}{(p-q)^2 - m_2^2 + i\epsilon}$$

Cutoff regularization

$$G_{ii}(s) = \int_0^{q_{\max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\omega_1\omega_2} \frac{1}{s - (\omega_1 + \omega_2)^2 + i\epsilon}$$

Loop function

- Looking for poles on complex plane,

$$G_{ii}^{II}(s) = G_{ii}(s) + i\frac{k}{4\pi\sqrt{s}}, \qquad k(s) = \sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}/(2\sqrt{s})$$

- Coupling constants are defined as the residue of the amplitude at the poles:

$$T_{ij}(s) = \frac{g_i g_j}{s - s_p^2} \qquad g_i^2 = \lim_{\sqrt{s}\to s_p} (s - s_p^2) T_{ii}(s)$$

Results

Content: $bc\bar{s}\,\bar{d}$	$I(J^P)$	E_B (MeV)	Channel	$\lvert g_i\rvert$ (GeV)
$\lvert(\bar{B}D)_s^-; J=0\rangle$	$\frac{1}{2}(0^+)$	15.7	$\bar{B}_s^0 D^+$	19
			$\bar{B}^0 D_s^+$	21
$\lvert(\bar{B}^* D)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	17.3	$\bar{B}_s^{*0} D^+$	20
			$\bar{B}^{*0} D_s^+$	21
$\lvert(\bar{B} D^*)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	16.4	$\bar{B}_s^0 D^{*+}$	20
			$\bar{B}^0 D_s^{*+}$	23
$\lvert(\bar{B}^* D^*)_s^-; J=0\rangle$	$\frac{1}{2}(0^+)$	13.6	$\bar{B}_s^{*0} D^{*+}$	19
			$\bar{B}^{*0} D_s^{*+}$	21
$\lvert(\bar{B}^* D^*)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	18.2	$\bar{B}_s^{*0} D^{*+}$	21
			$\bar{B}^{*0} D_s^{*+}$	23
$\lvert(\bar{B}^* D^*)_s^-; J=2\rangle$	$\frac{1}{2}(2^+)$	20.5	$\bar{B}_s^{*0} D^{*+}$	22
			$\bar{B}^{*0} D_s^{*+}$	24

- Six bound state in $bc\bar{s}\bar{q}$ system with cutoff parameter $q_{max} = 600$ MeV.
- 1 pole generated from the P-P interaction, 2 poles generated from the V-P interaction and 3 poles generated from the V-V interaction.
- No width.
  - Not consider the width of the initial and final states.
  - Not consider the box diagrams with pion exchange.

Results

Content: $bc\bar{s}\bar{d}$	$I(J^P)$	E_B (MeV)	Channel	$\lvert g_i \rvert$ (GeV)
$\lvert(\bar{B}D)_s^-; J=0\rangle$	$\frac{1}{2}(0^+)$	15.7	$\bar{B}_s^0 D^+$	19
			$\bar{B}^0 D_s^+$	21
$\lvert(\bar{B}^* D)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	17.3	$\bar{B}_s^{*0} D^+$	20
			$\bar{B}^{*0} D_s^+$	21
$\lvert(\bar{B}D^*)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	16.4	$\bar{B}_s^0 D^{*+}$	20
			$\bar{B}^0 D_s^{*+}$	23
$\lvert(\bar{B}^* D^*)_s^-; J=0\rangle$	$\frac{1}{2}(0^+)$	13.6	$\bar{B}_s^{*0} D^{*+}$	19
			$\bar{B}^{*0} D_s^{*+}$	21
$\lvert(\bar{B}^* D^*)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	18.2	$\bar{B}_s^{*0} D^{*+}$	21
			$\bar{B}^{*0} D_s^{*+}$	23
$\lvert(\bar{B}^* D^*)_s^-; J=2\rangle$	$\frac{1}{2}(2^+)$	20.5	$\bar{B}_s^{*0} D^{*+}$	22
			$\bar{B}^{*0} D_s^{*+}$	24

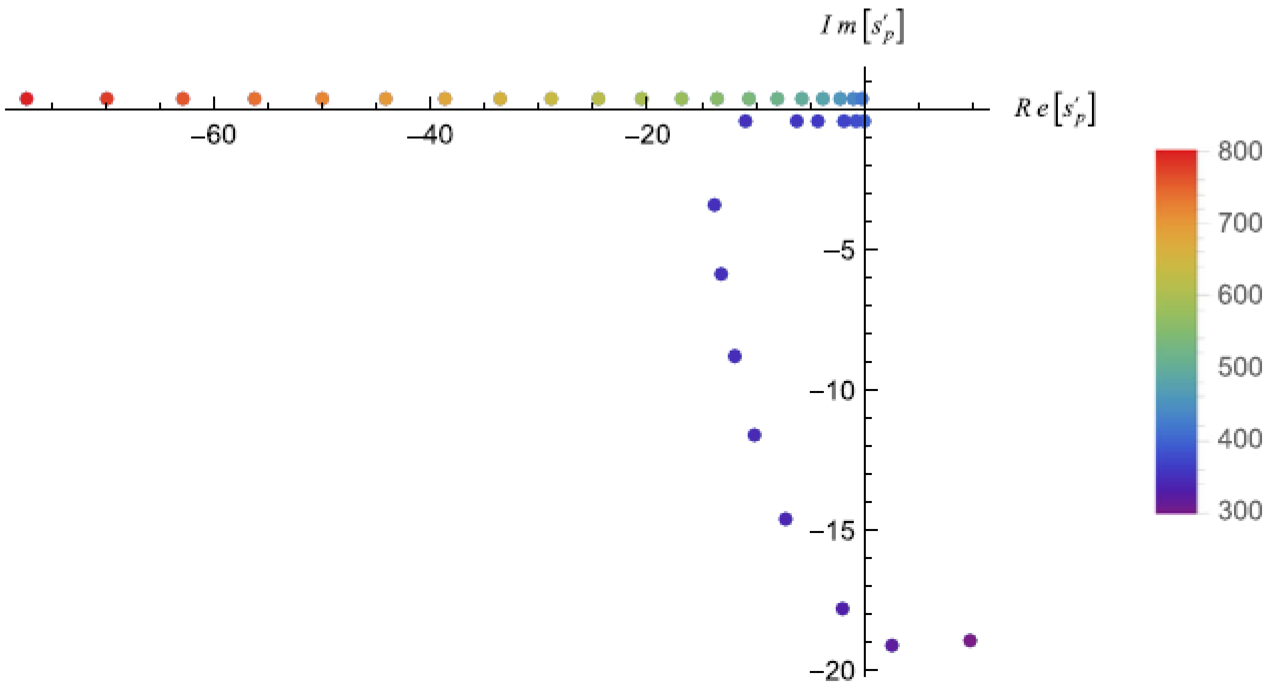
- While the heavy meson exchange is neglected, this bound state only couples to the $\bar{B}_s^0 D^{*+}$ and $\bar{B}^{*0} D_s^+$ channels. It could not decay into the $\bar{B}_s^{*0} D^+$ and $\bar{B}^0 D_s^{*+}$ channels, even if it lies above the threshold of the lighter coupled channels.

Results

Content: $bc\bar{s}\bar{d}$	$I(J^P)$	Pole	Channel	Threshold
$\|(\bar{B}D)_s^-; J=0\rangle$	$\frac{1}{2}(0^+)$	$7235.2+i0$	$\bar{B}_s^0 D^+$	7236.6
			$\bar{B}^0 D_s^+$	7248.0
$\|(\bar{B}^* D)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	$7284.9+i0$	$\bar{B}_s^{*0} D^+$	7285.1
			$\bar{B}^{*0} D_s^+$	7293.1
$\|(\bar{B}D^*)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	$7375.7+i0$	$\bar{B}_s^0 D^{*+}$	7377.2
			$\bar{B}^0 D_s^{*+}$	7391.9
$\|(\bar{B}^* D^*)_s^-; J=0\rangle$	$\frac{1}{2}(0^+)$	$7423.0+i0$	$\bar{B}_s^{*0} D^{*+}$	7425.7
			$\bar{B}^{*0} D_s^{*+}$	7436.9
$\|(\bar{B}^* D^*)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	$7425.4+i0$	$\bar{B}_s^{*0} D^{*+}$	7425.7
			$\bar{B}^{*0} D_s^{*+}$	7436.9
$\|(\bar{B}^* D^*)_s^-; J=2\rangle$	$\frac{1}{2}(2^+)$	$7425.6+i0$	$\bar{B}_s^{*0} D^{*+}$	7425.7
			$\bar{B}^{*0} D_s^{*+}$	7436.9

- No bound state pole on the first Riemann sheet.
- Six virtual state on the second Riemann sheet (-+) in $bc\bar{s}\bar{q}$ system has been found with cutoff parameter $q_{max} = 400$ MeV.

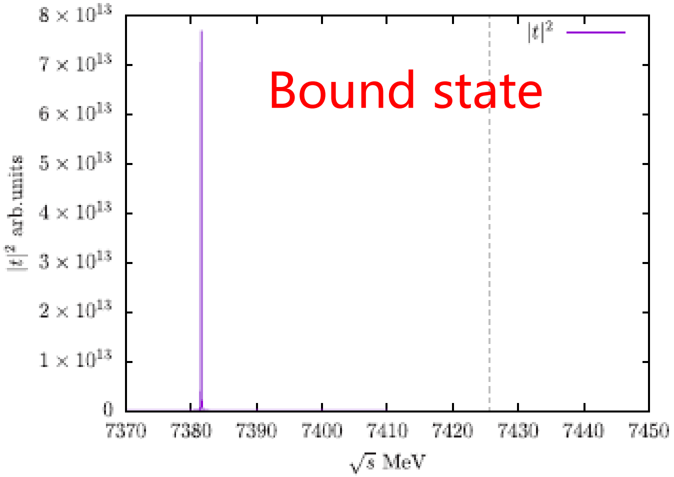
Results



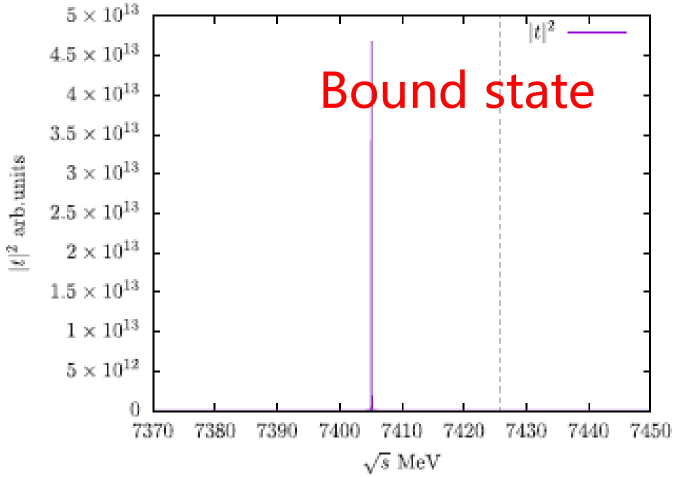
The pole position $s'_p = s_p - m_{thr}$ of the combination $|(\bar{B}^* D^*)_s^-; J = 2 >$ as a function of the cutoff momentum $q_{max} = 300 - 800$ MeV.

- When $q_{max} > 410$ MeV , a bound state pole appears, while it becomes a virtual state when $q_{max} < 410$ MeV.

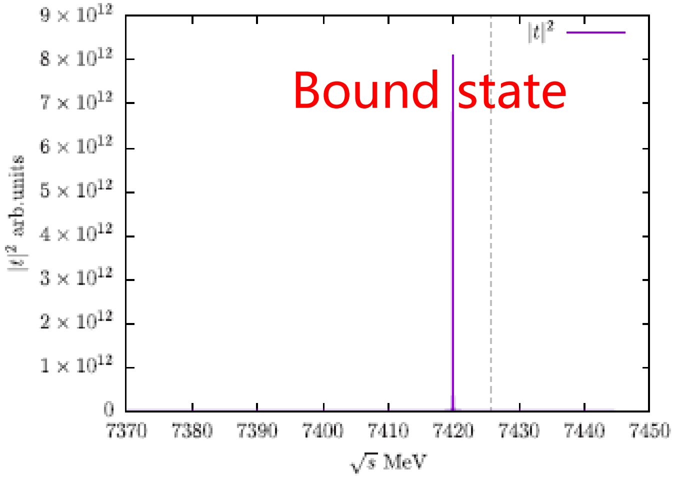
Results



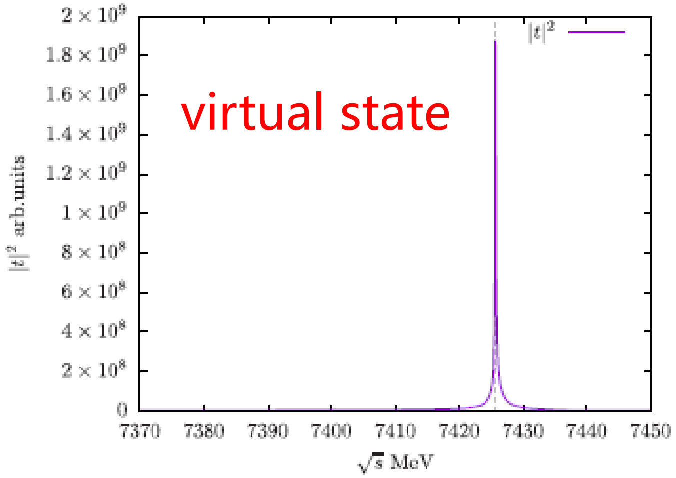
(a) $|t|^2$ with $q_{\max} = 700$



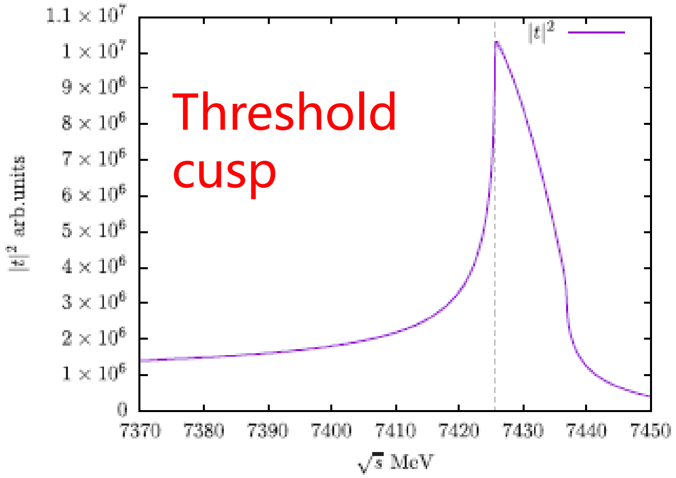
(b) $|t|^2$ with $q_{\max} = 600$



(c) $|t|^2$ with $q_{\max} = 500$



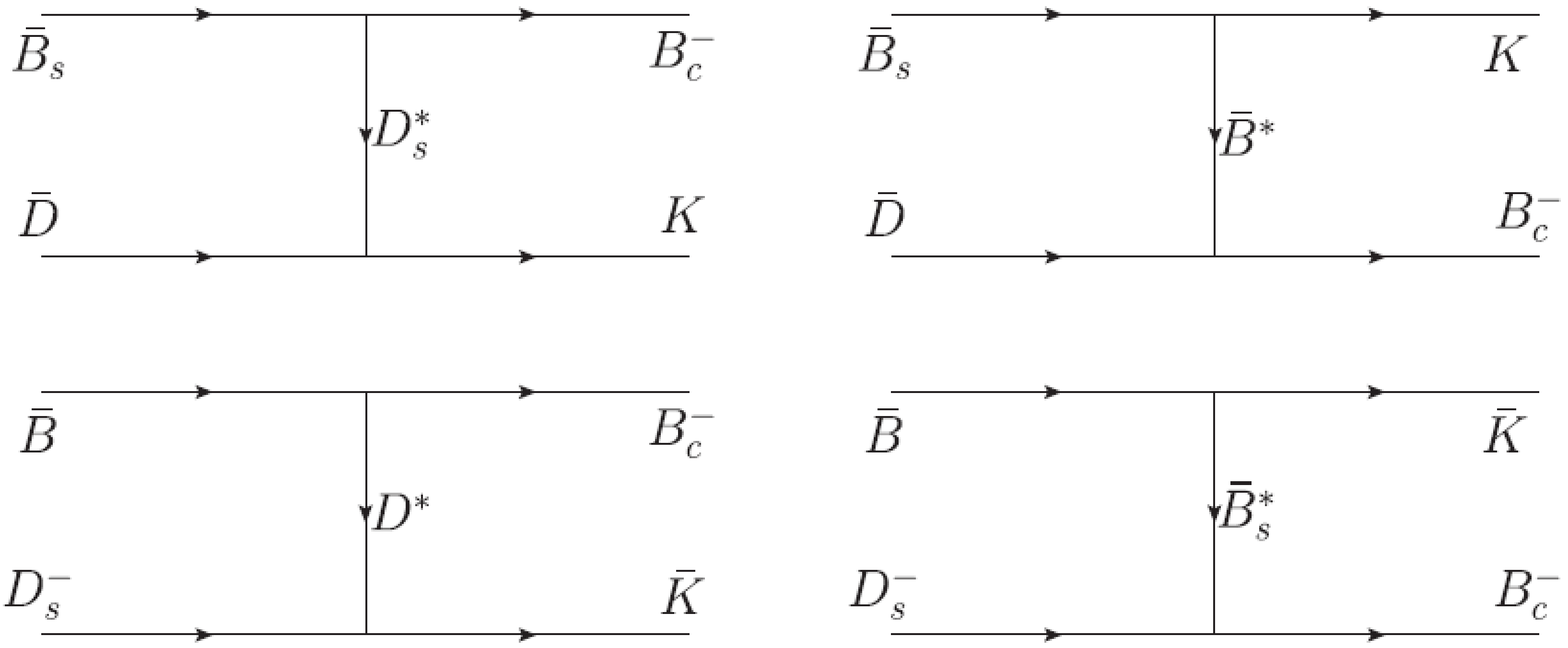
(d) $|t|^2$ with $q_{\max} = 400$



(e) $|t|^2$ with $q_{\max} = 300$

Results

Interactions in the $b\bar{c}s\bar{q}$ and $b\bar{c}\bar{s}q$ system



- No light vector meson exchange

Summary and Outlook

- Six bound states in $bc\bar{s}\bar{q}$ system with the binding energies about 10-20 MeV has been found while cutoff parameter $q_{max} = 600$ MeV has been taken, those bound states change to virtual states while cutoff parameter $q_{max} = 400$ MeV .

- No deeply bound pole has been found in the $b\bar{c}s\bar{q}$ and $b\bar{c}\bar{s}q$ system, for there is no light vector exchange.

- The corresponding structure could be seen in the following procedure:

  - $|(\bar{B}^*D^*)_{\bar{s}}^-; J=2>$ through its D-wave two-body decay patterns $|(\bar{B}^*D^*)_{\bar{s}}^-; J=2> \to \bar{B}_s D/\bar{B}D_s$.
  - $|(\bar{B}^*D^*)_{\bar{s}}^-; J=1>$ through its P-wave three-body decay patterns $|(\bar{B}^*D^*)_{\bar{s}}^-; J=1> \to \bar{B}_s D\pi/\bar{B}D_s\pi$.

- Further experiments are needed.

Thanks

Back-up

BS-eq within LHG

$$P = \begin{pmatrix} \frac{\eta}{\sqrt{3}}+\frac{\eta'}{\sqrt{6}}+\frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ & \bar{D}^0 & B^+ \\ \pi^- & \frac{\eta}{\sqrt{3}}+\frac{\eta'}{\sqrt{6}}-\frac{\pi^0}{\sqrt{2}} & K^0 & D^- & B^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}}+\sqrt{\frac{2}{3}}\eta' & D_s^- & B_s^0 \\ D^0 & D^+ & D_s^+ & \eta_c & B_c^+ \\ B^- & \bar{B}^0 & \bar{B}_s^0 & B_c^- & \eta_b \end{pmatrix}, \quad V = \begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} & B^{*+} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} & D^{*-} & B^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} & B_s^{*0} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi & B_c^{*+} \\ B^{*-} & \bar{B}^{*0} & \bar{B}_s^{*0} & B_c^{*-} & \Upsilon \end{pmatrix}.$$

A flavor SU(5) symmetry is assumed

BS-eq within LHG

$$V_{VV}(s)^{co} = m_{K^*}^2 \cdot C_{VV} g^2 (-2\epsilon_\mu \epsilon^\mu \epsilon_\nu \epsilon^\nu + \epsilon_\mu \epsilon_\nu \epsilon^\mu \epsilon^\nu + \epsilon_\mu \epsilon_\nu \epsilon^\nu \epsilon^\mu).$$

$$C_{VV} = \left(\begin{array}{c|cc} J=0,1,2 & \bar{B}_s^{*0} D^{*+} & \bar{B}^{*0} D_s^{*+} \\ \hline \bar{B}_s^{*0} D^{*+} & 0 & \frac{1}{m_{K^*}^2} \\ \bar{B}^{*0} D_s^{*+} & \frac{1}{m_{K^*}^2} & 0 \end{array} \right),$$

$$\mathcal{P}^{(0)} = \frac{1}{3} \epsilon_\mu \epsilon^\mu \epsilon_\nu \epsilon^\nu$$

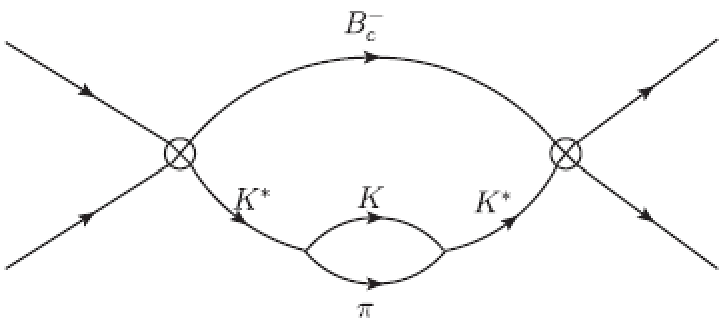
$$\mathcal{P}^{(1)} = \frac{1}{2} (\epsilon_\mu \epsilon_\nu \epsilon^\mu \epsilon^\nu - \epsilon_\mu \epsilon_\nu \epsilon^\nu \epsilon^\mu)$$

$$\mathcal{P}^{(2)} = \frac{1}{2} (\epsilon_\mu \epsilon_\nu \epsilon^\mu \epsilon^\nu + \epsilon_\mu \epsilon_\nu \epsilon^\nu \epsilon^\mu) - \frac{1}{3} \epsilon_\mu \epsilon^\mu \epsilon_\nu \epsilon^\nu,$$

BS-eq within LHG

$$V_{VV}(s)^{co} = m_{K^*}^2 \cdot C_{VV} \times \begin{cases} -4g^2 & \text{for } J = 0, \\ 0 & \text{for } J = 1, \\ 2g^2 & \text{for } J = 2. \end{cases} \qquad C_{VV} = \left(\begin{array}{c|cc} J = 0,1,2 & \bar{B}_s^{*0} D^{*+} & \bar{B}^{*0} D_s^{*+} \\ \hline \bar{B}_s^{*0} D^{*+} & 0 & \frac{1}{m_{K^*}^2} \\ \bar{B}^{*0} D_s^{*+} & \frac{1}{m_{K^*}^2} & 0 \end{array} \right),$$

Results



$$G(s) = \int_0^{q_{\max}} \frac{q^2 dq}{4\pi^2} \frac{\omega_{B_c^{(*)}} + \omega_{K^*}}{\omega_{B_c^{(*)}} \omega_{K^*}} \frac{1}{\sqrt{s} + \omega_{B_c^{(*)}} + \omega_{K^*}} \times \frac{1}{\sqrt{s} - \omega_{K^*} - \omega_{B_c^{(*)}} + i \frac{\sqrt{s'}}{2\omega_{K^*}} \Gamma_{K^*}(s')},$$

where $s' = (\sqrt{s} - \omega_{B_c^{(*)}})^2 - \vec{q}^2$ and

$$\Gamma_{K^*}(s') = \Gamma_{K^*}(m_{K^*}^2) \frac{m_{K^*}^2}{s'} \left(\frac{p_\pi(s')}{p_\pi(m_{K^*}^2)} \right)^3 \times \Theta(\sqrt{s'} - m_K - m_\pi),$$

A. Feijoo, W. H. Liang and E. Oset, Phys. Rev. D 104, no.11, 114015 (2021)