Elastic (anti-)neutrino-nucleon scattering in covariant baryon chiral perturbation theory

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第六届强子谱和强子结构研讨会

Aug 29, 2023

Outline

1 Introduction

2 Neutrino Nucleon Scattering in Neutral Current

3 Neutrino Nucleon Scattering in Charged Current

4 Summary and Outlook

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Background

- **Neutrino nucleon(νN) scattering** is an important way to understand the properties of neutrinos.

- There have been large scientific installations at home and abroad to research the properties of neutrinos such as:

  1. The Daya Bay Neutrino experiment in China [F. P. An, et al, PRL 108 (2012), 171803]
  2. The Jiangmen Neutrino experiment [F. An, et al, JPG 43 (2016) 3, 030401]
  3. The Super Kamiokande Neutrino experiment in Japan [Y. Fukuda, 10.1142/9789812704948_0002]

- The study of neutrinos interactions with matter can provide an important input for the accurate determination of neutrino oscillation parameters. See review [Z. Z. Xing and Z. h. Zhao, RPP 84 (2021) 6, 066201] .

Earlier studies

- Early νN scattering was mainly extracted from neutrino nuclear quasielastic scattering experiments [L.A. Ahrens, et al, PRD 35 (1987), 785]

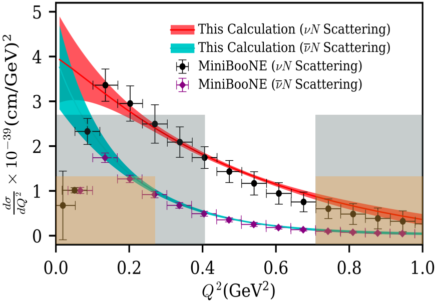
- In 2010 and 2015, the MiniBooNE published $\nu(\bar{\nu})$ -induced neutral current(NC) differential cross section data, concentrated in energy intervals below 2 GeV. [MiniBooNE collaboration, PRD 82 (2010) 092005] [MiniBooNE collaboration, PRD 91 (2015) 012004]

- D. Pelevalov gave experimental data near the threshold [D. Perevalov, FERMILAB-THESIS-2009-47 (2009)] .
  - In the low energy region, the data of νN quasielastic scattering have a complicated varying behavior, which has not been explained in theory.
- G.T. Garvey et al. calculated the strange quark axial vector form factor of the nucleon by analyzing the ratio of proton-to-neutron neutrino-induced [G.T. Garvey et al, PRC C 48 (1993), 1919-1925] , and calculated the strange quark form factor and the axial vector dipole mass of the proton by refitting BNL experimental data on neutrino-proton elastic scattering. [G.T. Garvey et al, PRC 48 (1993), 761-765]

Recent research

- R.S. Sufian et al. determined the NC axial vector form factor $G_A(Q^2)$, and calculated the differential cross section $\frac{d\sigma_{\nu N \to \nu N}}{dQ^2}$ for NC near $Q^2 = 0$ [R. S. Sufian et al, JHEP 01 (2020), 136] .

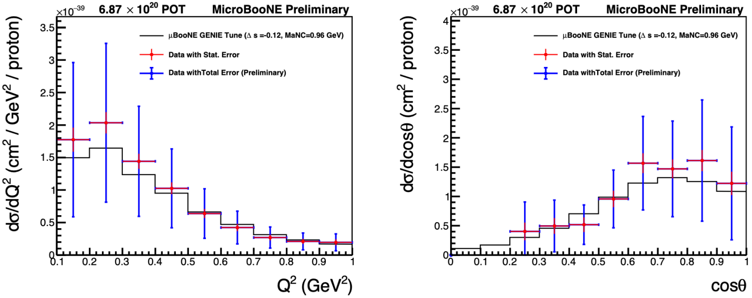
- Dipole parametrization
- In $Q^2 \lesssim 0.15 GeV^2$, the differential cross section predictions of $\nu(\bar{\nu})N$ scattering starts to deviate from MiniBooNE result
  The possible reason:
  1、Pauli blocking effect
  2、Nuclear shadowing



R. S. Sufian et al, JHEP 01 (2020), 136.

Recent results from MicroBooNE

Figure 1: Final differential cross sections $d\sigma/dQ^2$ (left) and $d\sigma/d\cos\theta$ (right) with both statistical and system- atic uncertainties.



Ren Lu, JPS Conf.Proc. 37 (2022) 020309.

- First measurement of muon neutrino neutral current elastic(NCE) scattering from protons ($0.1 < Q^2 < 1 GeV^2$)
- Plan to extract the strange quark contribution to the axial form factor

Our work

- Neutrino nucleon scattering is the most basic process to explore the interactions between neutrinos and matter.
- For neutrino nucleon scattering process, QCD can no longer be used directly for perturbation calculation at low energies due to the color confinement effect.
- Chiral perturbation theory (ChPT) is an effective field theory of QCD at low energy based on chiral symmetry at hadronic level.

Our aim

- In ChPT, calculate the 4 form factors of the neutral current(NC) and charged current(CC)
- We will combine the 4 form factors and the differential cross sections to carry out numerical calculation and compare with experimental results
- Try to explain the discrepancy between theoretical and experimental data in the low energy region

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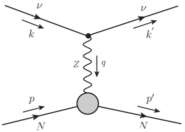
- The process of neutrino nucleon elastic scattering: $\nu(k) + N(p) \rightarrow \nu(k') + N(p')$



Only consider single boson exchange !

- The differential cross section:

$$\frac{d\sigma}{dQ^2} = \frac{|\overline{\mathcal{M}}|^2}{64\pi m_N^2 E_\nu^2}$$

- The scattering amplitude

$$\mathcal{M} = i\frac{G_F}{\sqrt{2}} L_\mu H^\mu$$

$$L_\mu \equiv \bar{\nu}(k')\gamma_\mu(1-\gamma_5)\nu(k)$$

$$H^\mu \equiv \left\langle N(p') \middle| \mathcal{J}_Z^\mu \middle| N(p) \right\rangle$$

- The amplitude squared:

$$|\overline{\mathcal{M}}|^2 = \frac{G_F^2}{2} L_{\mu\nu} H^{\mu\nu}$$

- The leptonic and hadronic tensors:

$$L_{\mu\nu} = 8\Big[k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k \cdot k' \pm i\epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta\Big]$$

$$H_{\mu\nu} = \frac{1}{2} Tr\Big[(\not{p} + m_N)\widetilde{\Gamma}_\mu(\not{p}' + m_N)\Gamma_\nu\Big]$$

with $\widetilde{\Gamma}_\mu = \gamma_0 \Gamma_\mu^\dagger \gamma_0$
where

$$H^\mu = \left\langle N(p') \middle| \mathcal{J}^\mu \middle| N(p) \right\rangle = \bar{u}(\not{p}')\Gamma^\mu u(p)$$

The Form Factors and Differential Cross Section

- Ignore the second class currents

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{i}{2m_N}\sigma^{\mu\nu} q_\nu F_2 - \gamma^\mu \gamma_5 G_A - \frac{q^\mu}{m_N}\gamma_5 G_P$$

where F_1, F_2 , G_A and G_P are the nucleon weak NC Dirac, Pauli, axial and induced pseudoscalar form factors.

- The differential cross section(NC)

$$\frac{d\sigma}{\mathrm{d}Q^2} = \frac{G_F^2 M^2}{8\pi E_\nu^2}\left[A \pm \frac{(s-u)}{M^2} B + \frac{(s-u)^2}{M^4} C\right]$$

with

$$A \equiv \frac{Q^2}{M^2}\left[G_A^2(1+\eta) + 4\eta F_1 F_2 - \left(F_1^2 - \eta F_2^2\right)(1-\eta)\right]$$

$$B \equiv \frac{Q^2}{M^2} G_A\left(F_1 + F_2\right)$$

$$C \equiv \frac{1}{4}\left[G_A^2 + F_1^2 + \eta F_2^2\right]$$

where $(s-u) = 4ME_\nu - Q^2$, $\eta = Q^2/4M^2$

The Form Factors and Differential Cross Section

- Vector current is conserved:

$$F_i = \left(\frac{1}{2} - \sin^2\theta_W\right)\left[F_i^{EM,p} - F_i^{EM,n}\right]\tau_3 - \sin^2\theta_W\left[F_i^{EM,p} + F_i^{EM,n}\right] - \frac{1}{2}F_i^{(s)}, \quad i = 1, 2,$$

- The electric and magnetic Sachs form factors:

$$G_E(Q^2) = F_1^{EM}(Q^2) - \frac{Q^2}{4m_N^2}F_2^{EM}(Q^2),$$
$$G_M(Q^2) = F_1^{EM}(Q^2) + F_2^{EM}(Q^2),$$

- The weak axial vector form factor:

$$G_A(Q^2) = \frac{\tau_3}{2}G_A^v(Q^2) - \frac{1}{2}G_A^{(s)}(Q^2).$$

e.g. dipole parameterization:

$$G_A^v(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}, \quad G_A^{(s)}(Q^2) = \frac{\Delta s}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}$$

- We want to calculate the form factors in ChPT.

The Chiral Lagrangian

- The effective Lagrangian

$$\mathcal{L}_{eff} = \sum_{i=1}^{2} \mathcal{L}_{\pi\pi}^{(2i)} + \sum_{j=1}^{3} \mathcal{L}_{\pi N}^{(j)}$$

- Meson field is contained in U :

$$U = u^2 = exp\left(i\frac{\Phi}{F}\right)$$

where $\Phi = \vec{\phi} \cdot \vec{\tau} = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$

- πN Lagrangian

$$\begin{aligned}
\mathcal{L}_{\pi N}^{(1)} =& \bar{\Psi}_N \{ i\not{D} - m + \frac{1}{2} g \not{u} \gamma_5 \} \Psi_N \\
\mathcal{L}_{\pi N}^{(2)} =& \frac{1}{8m} c_6 \bar{\Psi}_N \left\{ F_{\mu\nu}^{+} \sigma^{\mu\nu} \right\} \Psi_N + \frac{1}{8m} c_7 \bar{\Psi}_N \left\{ \langle F_{\mu\nu}^{+} \rangle \sigma^{\mu\nu} \right\} \Psi_N + \cdots \\
\mathcal{L}_{\pi N}^{(3)} =& \frac{i}{2m} d_6 \bar{\Psi}_N [D^\mu, f_{\mu\nu}^{+}] D^\nu \Psi_N + h.c. + \frac{2i}{m} d_7 \bar{\Psi}_N (\partial^\mu \hat{v}_{\mu\nu}^{(s)}) D^\nu \Psi_N + h.c. \\
&+ \frac{1}{2} d_{16} \gamma^\mu \gamma_5 \bar{\Psi}_N \langle \chi_+ \rangle u_\mu \Psi_N + \frac{i}{2} d_{18} \gamma^\mu \gamma_5 \bar{\Psi}_N [D_\mu, \chi_-] \Psi_N \\
&+ \frac{1}{2} d_{22} \gamma^\mu \gamma_5 \bar{\Psi}_N [D^\nu, f_{\mu\nu}^{-}] \Psi_N + \cdots
\end{aligned}$$

The Chiral Lagrangian

- $\pi\pi$ Lagrangian

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle$$
$$\mathcal{L}_{\pi\pi}^{(4)} = \frac{1}{8}\ell_4\langle u_\mu u^\mu\rangle\langle\chi_+\rangle + \frac{1}{16}(\ell_3+\ell_4)\langle\chi_+\rangle^2 + \cdots$$

- The chiral blocks

$$f_{\mu\nu}^{\pm} = u f_{\mu\nu}^{L} u^\dagger \pm u^\dagger f_{\mu\nu}^{R} u$$
$$f_{\mu\nu}^{L} = \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu]$$
$$f_{\mu\nu}^{R} = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu]$$
$$\hat{v}_{\mu\nu}^{(s)} = \partial_\mu \hat{v}_\nu^{(s)} - \partial_\nu \hat{v}_\mu^{(s)}$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u$$
$$\chi = diag(M^2, M^2)$$

- The covariant derivative:

$$D_\mu = \partial_\mu + \Gamma_\mu - i\hat{v}_\mu^s$$

where

$$u_\mu = i\left\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger\right\}$$
$$\Gamma_\mu = \frac{1}{2}\left\{u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - i\ell_\mu)u^\dagger\right\}$$

- Introduce the $\mathcal{Z}$ boson

$$\ell_\mu = \left(\frac{g_W}{2\cos\theta_W}\right)\left(-2cos^2\theta_W\right)\mathcal{Z}_\mu\frac{\tau_3}{2}$$
$$r_\mu = \left(\frac{g_W}{2\cos\theta_W}\right)\left(2sin^2\theta_W\right)\mathcal{Z}_\mu\frac{\tau_3}{2}$$
$$\hat{v}_\mu^{(s)} = \left(\frac{g_W}{2\cos\theta_W}\right)\left(2sin^2\theta_W\right)\mathcal{Z}_\mu\frac{\tau_0}{2}$$

Feynman Diagrams

- The hadronic part has a more complicated case due to the strong interactions inside the nucleon.

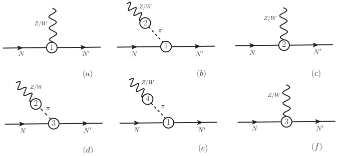


Figure 2: The tree-level diagrams of $\nu(\bar{\nu})N$ elastic scattering

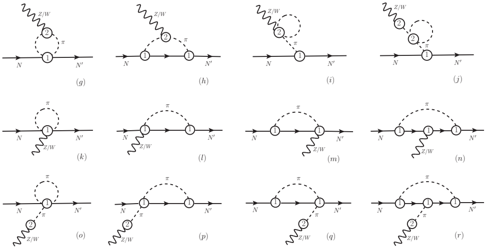


Figure 3: The one-loop diagrams of $\nu(\bar{\nu})N$ elastic scattering

- The pion exchange in hadron vertices mainly contributes to G_P, and G_P is entered into the CC scattering process.

Amplitudes of NC

- The amplitude: $\mathcal{M} = i\frac{G_F}{\sqrt{2}} L_\mu H^\mu$

$$
\begin{aligned}
H^\mu =& H^\mu_{tree} + H^\mu_{loop} = H^\mu_a + H^\mu_b + H^\mu_c + H^\mu_d + H^\mu_e + H^\mu_f + H^\mu_{loop} \\
H^\mu_a =& \bar{u}(p')\Big\{\Big(\cos 2\theta_W \frac{\tau_3}{2} - \sin^2\theta_W\Big)\gamma^\mu - g_A \frac{\tau_3}{2}\gamma^\mu\gamma_5\Big\}u(p) \\
H^\mu_b =& \bar{u}(p')\Big\{\frac{g_A m_N}{t - m_\pi^2}\tau_3 q^\mu \gamma_5\Big\}u(p) \\
H^\mu_c =& \bar{u}(p')\Big\{\Big[c_6 \cos 2\theta_W \frac{\tau_3}{2} - (c_6 + 2c_7)\sin^2\theta_W\Big]\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}\Big\}u(p) \\
H^\mu_d =& \bar{u}(p')\Big\{\frac{2m_N m_\pi^2(2d_{16} - d_{18})}{t - m_\pi^2}\tau_3 q^\mu\gamma_5\Big\}u(p) \\
H^\mu_e =& \bar{u}(p')\Big\{\frac{2g_A m_N m_\pi^2 \ell_4}{F^2(t - m_\pi^2)}\tau_3 q^\mu\gamma_5\Big\}u(p) \\
H^\mu_f =& \bar{u}(p')\Big\{\big(4d_7 t\sin^2\theta_W - d_6 t\cos 2\theta_W\tau_3\big)\gamma^\mu + \big(d_6 t\cos 2\theta_W \tau_3 - 4d_7 t\sin^2\theta_W\big)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N} \\
& - \big(4d_{16}m_\pi^2 + d_{22}t\big)\frac{\tau_3}{2}\gamma^\mu\gamma_5 + d_{22} m_N \tau_3 q^\mu\gamma_5\Big\}u(p) \\
H^\mu_{loop} =& \cdots
\end{aligned}
$$

- The constants marked in red are the low energy constants(LECs).

Renormalization of NC and Parameter Values

- The ultraviolet(UV) divergences $\Longrightarrow$ $\overline{MS}-1$(or $\widetilde{MS}$) scheme.
  The UV divergences from loops are subtracted by the infinite parts in the bare paramaters g, c_6 ,c_7, d_6, d_7, d_{16}, d_{18}, d_{22}, ℓ_4, respectively.

- The power counting breaking(PCB) terms $\Longrightarrow$ the EOMS scheme.

	Power Counting	Relativistic	Analyticity
$\overline{MS}-1$	×	✓	✓
HB	✓	×	×
IR	✓	✓	×
EOMS	✓	✓	✓

- The extended-on-mass-shell (EOMS) ✓
  The PCB terms in loops can be properly canceled by the LECs in the chiral Lagrangians.

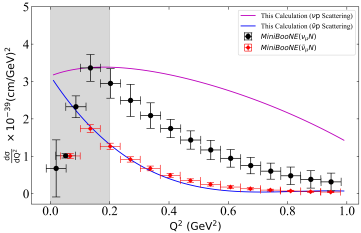
- The parameter values

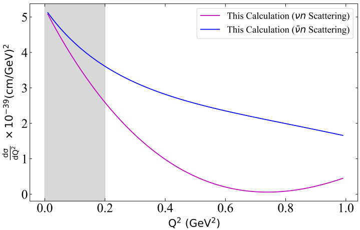
m_N	m_π	$\mathring{g}_A$	F_π	E_ν	$E_{\bar{\nu}}$
0.939	0.138	1.13	0.0924	0.80	0.65

c_6	c_7	d_6	d_7	d_{16}	d_{18}	d_{22}
1.35	-2.68	0	-0.49	-0.83	-0.20	0.96

[D. L. Yao et al, PRD 98 (2018) 7, 076004] [D. L. Yao et al, PLB 794 (2019), 109–113]

Preliminary Numerical Results of NC





- Preliminary numerical results of NC. Left: $\nu(\bar{\nu})p$ -NC ; Right: $\nu(\bar{\nu})n$ -NC

- The figures above show the preliminary numerical results in the NC, the values of those paratemers are in need of further improvement
- The shaded gray indidates the ChPT valid region
- The $d\sigma/dQ^2$ for $\bar{\nu}p$ scattering agree well with the MiniBooNE
- The $d\sigma/dQ^2$ for νp scattering is far different from the MiniBooNE
- Our preliminary results still don't agree with the MiniBooNE data in $Q^2 \lesssim 0.15\,GeV^2$

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The Relationship between NC and CC

- The Lagrangian of electromagnetic and weak interaction in SM

$$\mathcal{L}_{EW} = -e\mathcal{J}^{\mu}_{EM}A_{\mu} - \frac{g}{2\cos\theta_W}\mathcal{J}^{\mu}_{NC}\mathcal{Z}_{\mu} - \frac{g}{2\sqrt{2}}\mathcal{J}^{\mu}_{CC}W^{\dagger}_{\mu} + h.c.$$

where $\mathcal{J}^{\mu}_{EM}$、 $\mathcal{J}^{\mu}_{NC}$ and $\mathcal{J}^{\mu}_{CC}$ are the electromagnetic (EM) current, weak neutral current (NC) and charged current (CC), respectively.

- In the quark sector (only consider u and d) :

$$\mathcal{J}^{\mu}_{EM} = \frac{1}{3}V^{\mu}_0 + V^{\mu}_3$$

$$\mathcal{J}^{\mu}_{NC} = (1 - 2\sin^2\theta_W)V^{\mu}_3 - A^{\mu}_3 - \frac{2}{3}\sin^2\theta_W V^{\mu}_0$$

$$\mathcal{J}^{\mu}_{CC} = \cos\theta_C\left[(V^{\mu}_1 - A^{\mu}_1) + i(V^{\mu}_2 - A^{\mu}_2)\right]$$

and $\mathcal{J}^{\mu}_{CC}W^{\dagger}_{\mu} + h.c. = \sum_{a=1}^{2}\mathcal{J}^{\mu}_{CC,a}W^{a}_{\mu}$
where

$$\mathcal{J}^{\mu}_{CC,a} = \sqrt{2}\cos\theta_C(V^{\mu}_a - A^{\mu}_a)$$

- The vector and axial-vector currents

$$V^{\mu}_a \equiv \bar{q}\gamma^{\mu}\frac{\tau_a}{2}q\ , \quad a = 0,1,2,3$$

$$A^{\mu}_a \equiv \bar{q}\gamma^{\mu}\gamma_5\frac{\tau_a}{2}q\ , \quad a = 1,2,3$$

where $q = (q_u, q_d)^T$

The Relationship between NC and CC

- Isospin decomposition:

$$H_\mu = \langle N'(p')|\mathcal{J}_\mu|N(p)\rangle = \chi_f^\dagger \left[\frac{\tau^a}{2}\mathcal{H}_\mu^v + \frac{\tau^0}{2}\mathcal{H}_\mu^s\right]\chi_i \,, \quad a = 1,2,3$$

- NC(a=3): $\mathcal{H}_\mu^v = (1-2\sin^2\theta_W)\mathcal{V}_\mu^v - \mathcal{A}_\mu^v \,, \quad \mathcal{H}_\mu^s = -2\sin^2\theta_W \mathcal{V}_\mu^s$
- CC(a=1,2): $\mathcal{H}_\mu^v = \cos\theta_C(\mathcal{V}_\mu^v - \mathcal{A}_\mu^v) \,, \quad \mathcal{H}_\mu^s = 0$

$$\begin{aligned}
V_\mu^a =& \bar{u}(p')\left[\gamma^\mu \mathcal{F}_1^v + \frac{i}{2m}\sigma^{\mu\nu}q_\nu \mathcal{F}_2^v\right]\frac{\tau^a}{2}u(p) = \mathcal{V}_\mu^v \frac{\tau^a}{2} \,, \quad a = 1,2,3 \\
A_\mu^a =& \bar{u}(p')\left[\gamma^\mu \gamma_5 \mathcal{G}_A^v + \frac{q^\mu}{m}\gamma_5 \mathcal{G}_P^v\right]\frac{\tau^a}{2}u(p) = \mathcal{A}_\mu^v \frac{\tau^a}{2} \,, \quad a = 1,2,3 \\
V_\mu^0 =& \bar{u}(p')\left[\gamma^\mu \mathcal{F}_1^s + \frac{i}{2m}\sigma^{\mu\nu}q_\nu \mathcal{F}_2^s\right]\frac{\tau^0}{2}u(p) = \mathcal{V}_\mu^s \frac{\tau^0}{2} \\
\mathcal{J}_\mu^{NC} =& (1-2\sin^2\theta_W)\, V_\mu^3 - A_\mu^3 - 2\sin^2\theta_W V_\mu^0 \\
\mathcal{J}_\mu^{CC} =& \cos\theta_C\left[(V_\mu^1 - A_\mu^1) + i(V_\mu^2 - A_\mu^2)\right]
\end{aligned}$$

There are 6 unkown form factors ($\mathcal{F}_{1,2}^v$, $\mathcal{F}_{1,2}^s$, $\mathcal{G}_{A,P}$)

- The relationship between NC and CC:
  NC and CC have the FFs ($\mathcal{F}_i^v, \mathcal{G}_i^v$) in common!

The Relationship between NC and CC

- Definition of FFs

$$H^\mu = \langle N(p')|\mathcal{J}^\mu|N(p)\rangle = \bar{u}(\not{p}')\Gamma^\mu u(p)$$

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{i}{2m_N}\sigma^{\mu\nu} q_\nu F_2 - \gamma^\mu \gamma_5 G_A - \frac{q^\mu}{m_N}\gamma_5 G_P$$

- Physical amplitudes in NC:

$$H^\mu(\nu N \to \nu N') = \chi^\dagger_{N'} H^\mu_{NC} \chi_N$$

$$F_i\,(\nu N \to \nu N') = \chi^\dagger_{N'}\left[(1-2\sin^2\theta_W)\mathcal{F}^v_i \frac{\tau_3}{2} - 2\sin^2\theta_W \mathcal{F}^s_i \frac{\tau_0}{2}\right]\chi_N\ , \quad i = 1,2$$

$$G_i\,(\nu N \to \nu N') = \chi^\dagger_{N'}\left[\mathcal{G}^v_i \frac{\tau_3}{2}\right]\chi_N\ , \quad i = A, P$$

- Physical amplitudes in CC:

$$H^\mu(\nu_\ell n \to \ell^- p) = \chi^\dagger_p H^\mu_{CC} \chi_n$$

$$F_i\,(\nu_\ell n \to \ell^- p) = \chi^\dagger_p\left[\cos\theta_C \cdot \mathcal{F}^v_i \frac{\tau_i}{2}\right]\chi_n\ , \quad i = 1,2$$

$$G_i\,(\nu_\ell n \to \ell^- p) = \chi^\dagger_p\left[\cos\theta_C \cdot \mathcal{G}^v_i \frac{\tau_i}{2}\right]\chi_n\ , \quad i = A, P$$

The Differential Cross Section of CC

- The differential cross section(CC)

$$\frac{d\sigma}{\mathrm{d}Q^2} = \frac{G_F^2 M^2 |V_{ud}|^2}{8\pi E_\nu^2}\left[A \pm \frac{(s-u)}{M^2}B + \frac{(s-u)^2}{M^4}C\right]$$

with

$$A \equiv \frac{(m^2+Q^2)}{M^2}\left\{(1+\eta)\,G_A^2 + 4\eta F_1 F_2 - (1+\eta)\left(F_1^2 - \eta F_2^2\right)\right.$$
$$\left. - \frac{m^2}{4M^2}\left[(F_1+F_2)^2 + (G_A+2G_P)^2 - \left(\frac{Q^2}{M^2}+4\right)G_P^2\right]\right\}$$
$$B \equiv \frac{Q^2}{M^2} G_A\left(F_1+F_2\right)$$
$$C \equiv \frac{1}{4}\left[G_A^2 + F_1^2 + \eta F_2^2\right]$$

where $(s-u) = 4ME_\nu - Q^2 - m^2$, $\eta = Q^2/4M^2$, M and m are the nucleon and the lepton mass, respectively.

Preliminary Numrical Results of CC

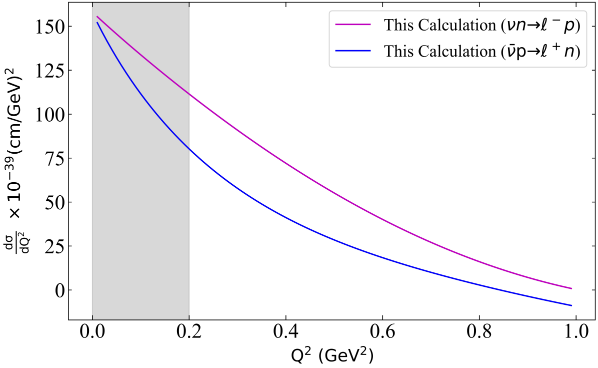


Figure 4: Preliminary numerical results of CC

- The shaded gray indidates the ChPT valid region
- Detailed results of CC are ongoing!

Interactions with Δ

- Considered the Δ-resonance: $\Delta(1232)$, a state of spin-$\frac{3}{2}$
  The chiral effective Lagrangian

$$\mathcal{L}_{eff} = \sum_{i=1}^{2} \mathcal{L}_{\pi\pi}^{(2i)} + \sum_{j=1}^{3} \mathcal{L}_{\pi N}^{(j)} + \sum_{k=1}^{2} [\mathcal{L}_{\pi\Delta}^{(k)} + \mathcal{L}_{\pi N\Delta}^{(k)}]$$

- The effective Lagrangian for pion-Δ and pion-nucleon-Δ interactions

$$\begin{aligned}
\mathcal{L}_{\pi\Delta}^{(1)} &= \bar{\Psi}^{i,\mu} \xi_{ij}^{\frac{3}{2}} \left(i\gamma_{\mu\nu\alpha} D^{\alpha,jk} - m_\Delta \gamma_{\mu\nu} \delta^{jk} \right) \xi_{kl}^{\frac{3}{2}} \Psi^{l,\nu} \\
\mathcal{L}_{\pi\Delta}^{(2)} &= \bar{\Psi}^{i,\mu} \xi_{ij}^{\frac{3}{2}} \left(a_1 Tr[\chi_+] \delta^{jk} g_{\mu\nu} \right) \xi_{kl}^{\frac{3}{2}} \Psi^{l,\nu}
\end{aligned}$$

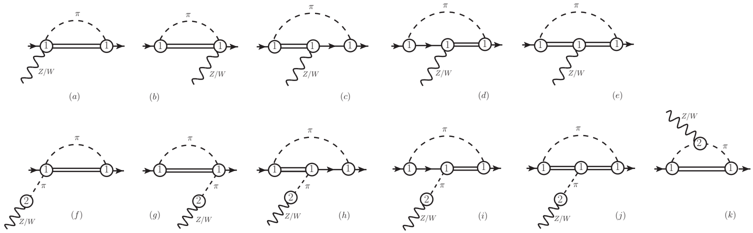
$$\begin{aligned}
\mathcal{L}_{\pi N\Delta}^{(1)} =& h_A \bar{\Psi}^{i,\alpha} \xi_{ij}^{\frac{3}{2}} \omega_\alpha^j \Psi_N + H.c. \\
\mathcal{L}_{\pi N\Delta}^{(2)} =& \bar{\Psi}^{i,\alpha} \xi_{ij}^{\frac{3}{2}} \Big\{ - i\frac{b_1}{2} F_{\alpha\beta}^{+,j} \gamma_5 \gamma^\beta + ib_2 F_{\alpha\beta}^{-,j} \gamma^\beta + ib_3 \omega_{\alpha\beta}^j \gamma^\beta \\
& + i\frac{b_7}{m} F_{\alpha\beta}^{-,j} iD^\beta + i\frac{b_8}{m} \omega_{\alpha\beta}^j iD^\beta \Big\} \Psi_N + H.c.
\end{aligned}$$

- The covariant derivative: $\mathcal{D}_{\mu,ij} = (\partial_\mu + \Gamma_\mu)\delta_{ij} - i\epsilon_{ijk} Tr[\tau^k \Gamma_\mu]$

Interactions with Δ

- The projection operator: $\xi_{ij}^{\frac{3}{2}} = \delta_{ij} - \frac{1}{3}\tau_i\tau_j$
- The Dirac matrices: $\gamma_{\mu\nu\alpha} = \frac{1}{4}\{[\gamma_\mu,\gamma_\nu],\gamma_\alpha\}$, $\quad \gamma_{\mu\nu} = \frac{1}{2}[\gamma_\mu,\gamma_\nu]$
- The chiral blocks:

$$F^{\pm,i}_{\mu\nu} = \frac{1}{2}Tr[\tau^i F^{\pm}_{\mu\nu}] \ , \ \omega^i_\mu = \frac{1}{2}Tr[\tau^i u_\mu] \ , \ \omega^i_{\mu\nu} = \frac{1}{2}Tr[\tau^i[D_\mu, u_\nu]]$$



- The calculation of the Feynman digrams with Δ-resonance is on-going .

Outline

1 Introduction

2 Neutrino Nucleon Scattering in Neutral Current

3 Neutrino Nucleon Scattering in Charged Current

4 Summary and Outlook

Summary and Outlook

Summary

- In ChPT, we calculated the scattering amplitudes up to $\mathcal{O}(p^3)$ in NC (tree diagrams and loop diagrams) and extracted 4 form factors
- Obtained the relationship: NC and CC have the FFs $(\mathcal{F}_i^v, \mathcal{G}_i^v)$ in common
- We have obtained the preliminary numerical results of the NC and CC

Next step

- We will continue to try to explain the complicated varying behavior of the MiniBooNE in the low energy region. If it's still unexplained, then Pauli blocking and nuclear shadowing effect have a great influence.
- Because our partial preliminary numerical results of NC are not consistent with MiniBooNE data, we will consider the Δ-resonance and the contribution of strange quark in later work.

Thank you very much for your patience!

The 6 physical processes of $\nu\bar{\nu}N$ scattering

- 4 physical processes in NC:

$$\begin{aligned}
\nu(k) + p(p) &\rightarrow \nu(k') + p(p') \\
\nu(k) + n(p) &\rightarrow \nu(k') + n(p') \\
\bar{\nu}(k) + p(p) &\rightarrow \bar{\nu}(k') + p(p') \\
\bar{\nu}(k) + n(p) &\rightarrow \bar{\nu}(k') + n(p')
\end{aligned}$$

- 2 physical processes in CC:

$$\begin{aligned}
\nu_\ell(k) + n(p) &\rightarrow \ell^-(k') + p(p') \\
\bar{\nu}_\ell(k) + p(p) &\rightarrow \ell^+(k') + n(p')
\end{aligned}$$

The Study of G_A、G_E and G_M

Electromagnetic form-factors of the nucleon in relativistic baryon chiral perturbation theory

Thomas Fuchs (Mainz U., Inst. Kernphys.), Jambul Gegelia (Mainz U., Inst. Kernphys. and Tbilisi State U.), Stefan Scherer (Mainz U., Inst. Kernphys.)
May, 2003

25 pages
Published in: *J.Phys.G* 30 (2004) 1407-1426
e-Print: nucl-th/0305070 [nucl-th]
DOI: 10.1088/0954-3899/30/10/008
Report number: MKPH-T-03-7
View in: ADS Abstract Service

pdf cite claim reference search 75 citations

Extraction of nucleon axial charge and radius from lattice QCD results using baryon chiral perturbation theory

De-Liang Yao (Valencia U. and Valencia U., IFIC), Luis Alvarez-Ruso (Valencia U. and Valencia U., IFIC), Manuel J. Vicente-Vacas (Valencia U. and Valencia U., IFIC)
Aug 29, 2017

11 pages
Published in: *Phys.Rev.D* 96 (2017) 11, 116022
Published: Dec 28, 2017
e-Print: 1708.08776 [hep-ph]
DOI: 10.1103/PhysRevD.96.116022
View in: ADS Abstract Service

pdf cite claim reference search 32 citations

The G_E and G_M are calculated at order q^4 in a manifestly Lorentz-invariant formulation of baryon chiral perturbation theory [Fuchs et al, J. Phys. G 30 (2004) , 1407-1426] .

The nucleon axial form factor is calculated up to the leading one-loop order in a covariant chiral effective field theory with the $\Delta(1232)$ resonance as an explicit degree of freedom. [D.L. Yao et al, Phys. Rev. D 96 (2017) 11, 116022] .

The MiniBooNE results

- The NCE scattering sample consists of three different processes: scattering off free protons in hydrogen(H), bound protons in carbon, and bound neutrons in carbon(C).
- The result is the flux-averaged NCE differential cross-section on(CH_2), averaged over these processes. The MiniBooNE $\nu(\bar{\nu})N \to \nu(\bar{\nu})N$ cross-section is expressed as: [R. S. Sufian et al, JHEP 01 (2020), 136] .

$$
\begin{aligned}
\frac{d\sigma_{\nu(\bar{\nu})N\to\nu(\bar{\nu})N}}{dQ^2} = & \frac{1}{7}\eta_{\nu(\bar{\nu})p,H}\frac{d\sigma_{\nu(\bar{\nu})p,H\to\nu(\bar{\nu})p,H}}{dQ^2} \\
& + \frac{3}{7}\eta_{\nu(\bar{\nu})p,C}\frac{d\sigma_{\nu(\bar{\nu})p,C\to\nu(\bar{\nu})p,C}}{dQ^2} \\
& + \frac{3}{7}\eta_{\nu(\bar{\nu})n,C}\frac{d\sigma_{\nu(\bar{\nu})n,C\to\nu(\bar{\nu})n,C}}{dQ^2}
\end{aligned}
$$

  where The efficiency corrections $\eta_{\nu(\bar{\nu})p,H}$, $\eta_{\nu(\bar{\nu})p,C}$ and $\eta_{\nu(\bar{\nu})n,C}$ for each type of NCE scattering process, are estimated from Monte Carlo simulation as a function of Q^2.
- The NCE flux-averaged differential cross-section is shown in Figure. 5

The MiniBooNE results

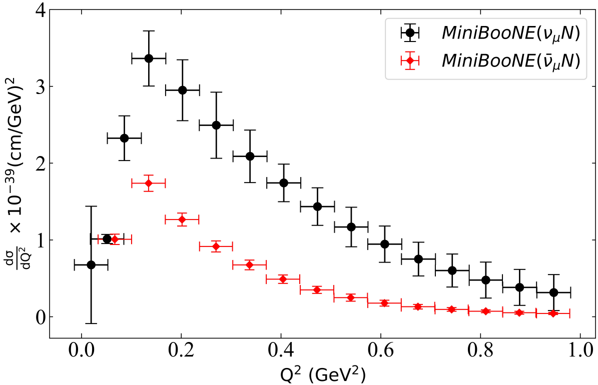


Figure 5: The NCE flux-averaged differential cross-section

reaction	replace $\mathcal{F}_i$ in Eq. (4.7) with
$\ell^- p \to \ell^- p$	F_i^p
$\ell^- n \to \ell^- n$	F_i^n
$\nu n \to \ell^- p$	$F_i^V = F_i^p - F_i^n$
$\nu p \to \nu p$	$\tilde{F}_i^p = (\frac{1}{2} - 2\sin^2\theta_W)F_i^p - \frac{1}{2}F_i^n - \frac{1}{2}F_i^s$
$\nu n \to \nu n$	$\tilde{F}_i^n = (\frac{1}{2} - 2\sin^2\theta_W)F_i^n - \frac{1}{2}F_i^p - \frac{1}{2}F_i^s$

Leitner: 2009zz

- The relation of vector form factors for EM、NC and CC

reaction	replace $\mathcal{F}_A$ in Eq. (4.8) with	replace $\mathcal{F}_P$ in Eq. (4.8) with
$\ell^- p \to \ell^- p$	-	-
$\ell^- n \to \ell^- p$	-	-
$\nu n \to \ell^- p$	F_A	$\frac{2M_N^2}{Q^2+m_\pi^2}F_A$
$\nu p \to \nu p$	$\tilde{F}_A^p = \frac{1}{2}F_A + \frac{1}{2}F_A^s$	-
$\nu n \to \nu n$	$\tilde{F}_A^n = -\frac{1}{2}F_A + \frac{1}{2}F_A^s$	-

Leitner: 2009zz

- The relation of axial vector form factors for EM、NC and CC