Investigation of the $D_s^+ \to \pi^+\pi^- K^+$ decay

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① $D_s^+(D^+) \to K^+K^-\pi^+$:

P. L. Frabetti et al. [E687], Phys. Lett. B 351, 591-600 (1995).

R. E. Mitchell et al. [CLEO], Phys. Rev. D 79, 072008 (2009).

M. Ablikim et al. [BESIII], Phys. Rev. D 104, 012016 (2021).

② $D_s^+ \to \pi^+\pi^0\eta$:

M. Ablikim et al. [BESIII], Phys. Rev. Lett. 123, 112001 (2019)

③ $D_s^+ \to \pi^+\pi^-\pi^+$:

B. Aubert et al. [BaBar], Phys. Rev. D 79, 032003 (2009).

M. Ablikim et al. [BESIII], Phys. Rev. D 106, 112006 (2022).

④ $D_s^+ \to K_s^0K_s^0\pi^+$:

M. Ablikim et al. [BESIII], Phys. Rev. D 105, L051103 (2022).

⑤ $D_s^+ \to K_s^0K^+\pi^0$:

M. Ablikim et al. [BESIII], Phys. Rev. Lett. 129, 18 (2022).

- Theories：

① $D_s^+ \to K^+K^-\pi^+$:

J. Y. Wang et al. Phys. Lett. B 821, 136617 (2021).

Z. Y. Wang et al. Phys. Rev. D 105, 016025 (2022).

R. Escribano et al. arXiv:2302.03312 [hep-ph].

② $D_s^+ \to \pi^+\pi^0\eta$:

R. Molina et al. Phys. Lett. B 803, 135279 (2020).

③ $D_s^+ \to \pi^+\pi^-\pi^+$:

J. M. Dias et al. Phys. Rev. D 94, 096002 (2016).

N. N. Achasov et al. Phys. Rev. D 107, 056009 (2023).

④ $D_s^+ \to K_s^0K_s^0\pi^+$:

L. R. Dai et al. Eur. Phys. J. C 82, 225 (2022).

⑤ $D_s^+ \to K_s^0K^+\pi^0$:

X. Zhu et al. Phys. Rev. D 107, 034001 (2023).

Background and Motivations.

- $D_s^+ \to \pi^+\pi^- K^+$:

$$\frac{\Gamma\left(D_s^+ \to K^+\pi^+\pi^-\right)}{\Gamma\left(D_s^+ \to K^+K^-\pi^+\right)} = 0.127 \pm 0.007 \pm 0.014\,.$$

J.M. Link et al. [FOCUS Collaboration], Phys. Lett. B 601, 10-19 (2004).

Decay channel	Fit fraction (%)	Phase ϕ_j (degrees)	Amplitude coefficient
$\rho(770)K^+$	$38.83 \pm 5.31 \pm 2.61$	0 (fixed)	1 (fixed)
$K^*(892)\pi^+$	$21.64 \pm 3.21 \pm 1.14$	$161.7 \pm 8.6 \pm 2.2$	$0.747 \pm 0.080 \pm 0.031$
NR	$15.88 \pm 4.92 \pm 1.53$	$43.1 \pm 10.4 \pm 4.4$	$0.640 \pm 0.118 \pm 0.026$
$K^*(1410)\pi^+$	$18.82 \pm 4.03 \pm 1.22$	$-34.8 \pm 12.1 \pm 4.3$	$0.696 \pm 0.097 \pm 0.025$
$K_0^*(1430)\pi^+$	$7.65 \pm 5.0 \pm 1.70$	$59.3 \pm 19.5 \pm 13.2$	$0.444 \pm 0.141 \pm 0.060$
$\rho(1450)K^+$	$10.62 \pm 3.51 \pm 1.04$	$-151.7 \pm 11.1 \pm 4.4$	$0.523 \pm 0.091 \pm 0.020$
C.L. = 5.5%	$\chi^2 = 38.5$	d.o.f. = 43 (#bins) - 17 (#free parameters)	

Medina Ablikim et al. [BESIII Collaboration], JHEP 08, 196 (2022).

Amplitude	Phase ϕ_n (rad)	FF(%)	Statistical significance(σ)
$D_s^+ \to K^+\rho^0$	0.0 (fixed)	$32.5 \pm 3.1 \pm 3.6$	>10
$D_s^+ \to K^+\rho(1450)^0$	$2.72 \pm 0.14 \pm 0.24$	$12.7 \pm 3.2 \pm 2.7$	>10
$D_s^+ \to K^+f_0(500)$	$0.98 \pm 0.17 \pm 0.19$	$7.0 \pm 2.2 \pm 4.0$	6.8
$D_s^+ \to K^+f_0(980)$	$5.02 \pm 0.15 \pm 0.15$	$4.4 \pm 1.3 \pm 1.1$	6.9
$D_s^+ \to K^+f_0(1370)$	$6.03 \pm 0.14 \pm 0.26$	$19.9 \pm 3.1 \pm 2.9$	>10
$D_s^+ \to K^*(892)^0\pi^+$	$3.03 \pm 0.09 \pm 0.04$	$30.3 \pm 1.9 \pm 1.8$	>10
$D_s^+ \to K^*(1410)^0\pi^+$	$5.62 \pm 0.14 \pm 0.09$	$4.7 \pm 2.2 \pm 2.1$	5.2
$D_s^+ \to K_0^*(1430)^0\pi^+$	$1.89 \pm 0.19 \pm 0.18$	$18.9 \pm 2.5 \pm 2.4$	8.6

Intermediate process	BF(10^{-3})	PDG(10^{-3})
$D_s^+ \to K^+\rho^0$	$1.99 \pm 0.20 \pm 0.22$	2.5 ± 0.4
$D_s^+ \to K^+\rho(1450)^0$	$0.78 \pm 0.20 \pm 0.17$	0.69 ± 0.64
$D_s^+ \to K^*(892)^0\pi^+$	$1.85 \pm 0.13 \pm 0.11$	1.41 ± 0.24
$D_s^+ \to K^*(1410)^0\pi^+$	$0.29 \pm 0.13 \pm 0.13$	1.23 ± 0.28
$D_s^+ \to K_0^*(1430)^0\pi^+$	$1.15 \pm 0.16 \pm 0.15$	0.50 ± 0.35
$D_s^+ \to K^+f_0(500)$	$0.43 \pm 0.14 \pm 0.24$	-
$D_s^+ \to K^+f_0(980)$	$0.27 \pm 0.08 \pm 0.07$	-
$D_s^+ \to K^+f_0(1370)$	$1.22 \pm 0.19 \pm 0.18$	-
$D_s^+ \to (K^+\pi^+\pi^-)_{NR}$	-	1.03 ± 0.34

$$\mathcal{B}(D_s^+ \to K^+\pi^+\pi^-) = (6.11 \pm 0.18_{\rm stat.} \pm 0.11_{\rm syst.}) \times 10^{-3}$$

Formalism

$D_s^+ \to \pi^+\pi^- K^+$

- The external and internal W-emission mechanism:

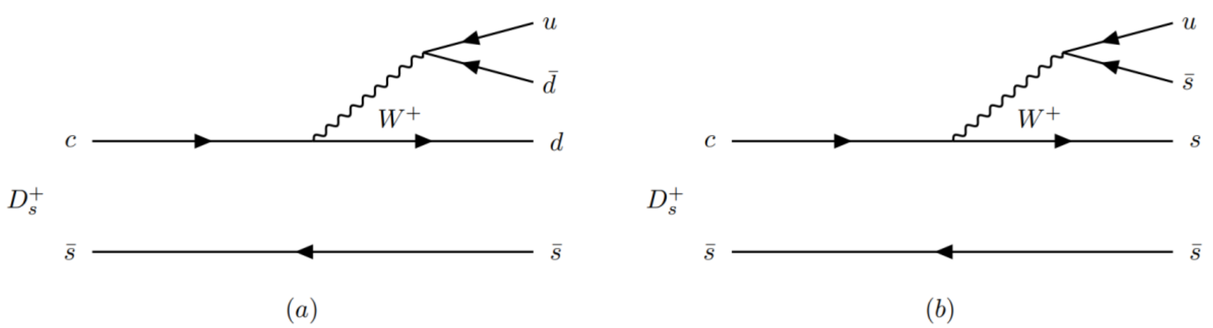


FIG. 1: W-external emission mechanism for the $D_s^+ \to K^+\pi^+\pi^-$ decay.

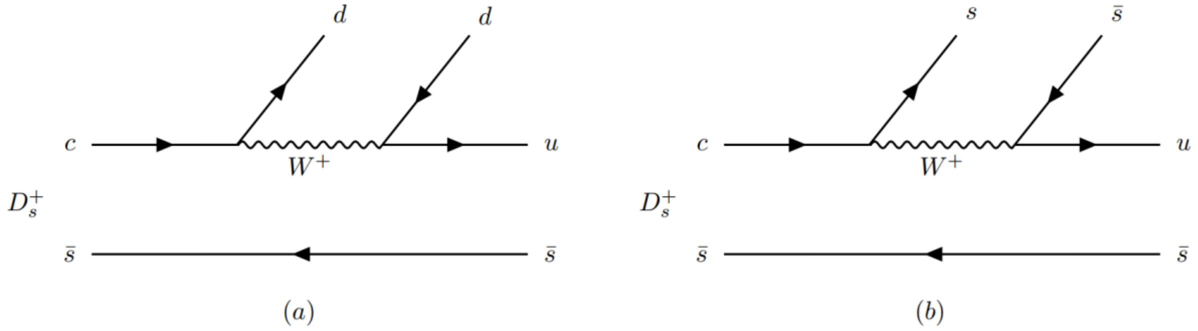


FIG. 2: W-internal emission mechanism for the $D_s^+ \to K^+\pi^+\pi^-$ decay.

$$
\begin{aligned}
H^{(1a)} =& V_P V_{cd} V_{ud} \left\{ \left(u\bar{d} \to \pi^+\right) \left[d\bar{s} \to d\bar{s} \cdot \left(u\bar{u} + d\bar{d} + s\bar{s}\right)\right] \right.\\
&\left. + \left(d\bar{s} \to K^0\right) \left[u\bar{d} \to u\bar{d} \cdot \left(u\bar{u} + d\bar{d} + s\bar{s}\right)\right] \right\}\\
=& V_P V_{cd} V_{ud} \left\{ \left(u\bar{d} \to \pi^+\right) \left[M_{23} \to (M \cdot M)_{23}\right] \right.\\
&\left. + \left(d\bar{s} \to K^0\right) \left[M_{12} \to (M \cdot M)_{12}\right] \right\},
\end{aligned}
$$

$$
\begin{aligned}
H^{(1b)} =& V'_P V_{cs} V_{us} \left\{ \left(u\bar{s} \to K^+\right) \left[\bar{s}\bar{s} \to \bar{s}\bar{s} \cdot \left(u\bar{u} + d\bar{d} + s\bar{s}\right)\right] \right.\\
&\left. + \left(\bar{s}\bar{s} \to -\frac{2}{\sqrt{6}}\eta\right) \left[u\bar{s} \to u\bar{s} \cdot \left(u\bar{u} + d\bar{d} + s\bar{s}\right)\right] \right\}\\
=& V'_P V_{cs} V_{us} \left\{ \left(u\bar{s} \to K^+\right) \left[M_{33} \to (M \cdot M)_{33}\right] \right.\\
&\left. + \left(\bar{s}\bar{s} \to -\frac{2}{\sqrt{6}}\eta\right) \left[M_{13} \to (M \cdot M)_{13}\right] \right\}.
\end{aligned}
$$

$$
|\pi^0\rangle = \frac{1}{\sqrt{2}}|(u\bar{u} - d\bar{d})\rangle, \quad |\eta\rangle = \frac{1}{\sqrt{6}}|(u\bar{u} + d\bar{d} - 2s\bar{s})\rangle.
$$

$$
\begin{aligned}
H^{(2a)} =& \beta V_P V_{cd} V_{ud} \left\{ \left(d\bar{d} \to -\frac{1}{\sqrt{2}}\pi^0\right) \left[u\bar{s} \to u\bar{s} \cdot \left(u\bar{u} + d\bar{d} + s\bar{s}\right)\right] \right.\\
&+ \left(d\bar{d} \to \frac{1}{\sqrt{6}}\eta\right) \left[u\bar{s} \to u\bar{s} \cdot \left(u\bar{u} + d\bar{d} + s\bar{s}\right)\right]\\
&\left. + \left(u\bar{s} \to K^+\right) \left[d\bar{d} \to d\bar{d} \cdot \left(u\bar{u} + d\bar{d} + s\bar{s}\right)\right] \right\}\\
=& \beta V_P V_{cd} V_{ud} \left\{ \left(d\bar{d} \to -\frac{1}{\sqrt{2}}\pi^0\right) \left[M_{13} \to (M \cdot M)_{13}\right] \right.\\
&\left. + \left(d\bar{d} \to \frac{1}{\sqrt{6}}\eta\right) \left[M_{13} \to (M \cdot M)_{13}\right] + \left(u\bar{s} \to K^+\right) \left[M_{22} \to (M \cdot M)_{22}\right] \right\},
\end{aligned}
$$

$$
\begin{aligned}
H^{(2b)} =& \beta V'_P V_{cs} V_{us} \left\{ \left(s\bar{s} \to -\frac{2}{\sqrt{6}}\eta\right) \left[u\bar{s} \to u\bar{s} \cdot \left(u\bar{u} + d\bar{d} + s\bar{s}\right)\right] \right.\\
&\left. + \left(u\bar{s} \to K^+\right) \left[s\bar{s} \to s\bar{s} \cdot \left(u\bar{u} + d\bar{d} + s\bar{s}\right)\right] \right\}\\
=& \beta V'_P V_{cs} V_{us} \left\{ \left(s\bar{s} \to -\frac{2}{\sqrt{6}}\eta\right) \left[M_{13} \to (M \cdot M)_{13}\right] \right.\\
&\left. + \left(u\bar{s} \to K^+\right) \left[M_{33} \to (M \cdot M)_{33}\right] \right\},
\end{aligned}
$$

$D_s^+ \to \pi^+\pi^- K^+$

- The SU(3) matrix M in quark and hadron level:

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}.$$

$$M \to \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

- The hadonizations at the hadron level:

$$(M\cdot M)_{12} = \frac{2}{\sqrt{6}}\pi^+\eta + K^+\bar{K}^0,$$

$$(M\cdot M)_{13} = \frac{1}{\sqrt{2}}\pi^0 K^+ + \pi^+ K^0 - \frac{1}{\sqrt{6}}\eta K^+,$$

$$(M\cdot M)_{22} = \pi^+\pi^- + \frac{1}{2}\pi^0\pi^0 + \frac{1}{6}\eta\eta - \frac{1}{\sqrt{3}}\pi^0\eta + K^0\bar{K}^0,$$

$$(M\cdot M)_{23} = \pi^- K^+ - \frac{1}{\sqrt{2}}\pi^0 K^0 - \frac{1}{\sqrt{6}}K^0\eta,$$

$$(M\cdot M)_{33} = K^+K^- + K^0\bar{K}^0 + \frac{2}{3}\eta\eta.$$

- The contributions for different mechanism:

$$H^{(1a)} = V_P V_{cd} V_{ud}\left(\pi^+\pi^- K^+ - \frac{1}{\sqrt{2}}\pi^+\pi^0 K^0 + \frac{1}{\sqrt{6}}\pi^+ K^0\eta + K^+K^0\bar{K}^0\right),$$

$$H^{(1b)} = V_P' V_{cs} V_{us}\left(K^+K^+K^- + K^+K^0\bar{K}^0 + K^+\eta\eta - \frac{1}{\sqrt{3}}\pi^0 K^+\eta - \frac{2}{\sqrt{6}}\pi^+ K^0\eta\right),$$

$$H^{(2a)} = \beta \times V_P V_{cd} V_{ud}\left(\pi^+\pi^- K^+ + K^+K^0\bar{K}^0 - \frac{1}{\sqrt{2}}\pi^+\pi^0 K^0 + \frac{1}{\sqrt{6}}\eta\pi^+ K^0\right),$$

$$H^{(2b)} = \beta \times V_P' V_{cs} V_{us}\left(K^+K^+K^- + K^+K^0\bar{K}^0 + \eta\eta K^+ - \frac{1}{\sqrt{3}}\eta\pi^0 K^+ - \frac{2}{\sqrt{6}}\eta\pi^+ K^0\right).$$

- The relationship of the CKM matrix elements :

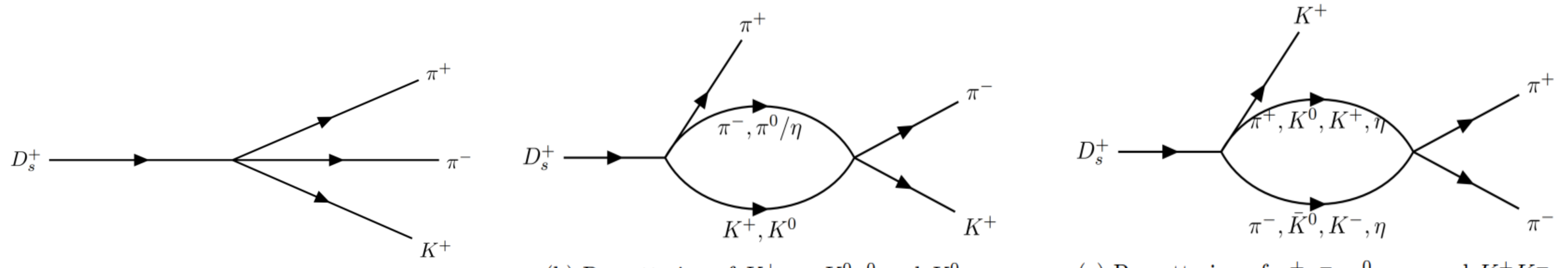
$$V_{cd}V_{ud} = -V_{us}V_{cs}$$

$D_s^+ \to \pi^+\pi^- K^+$

- The total contributions for the decay $D_s^+ \to K^+\pi^+\pi^-$:

$$
\begin{aligned}
H =& H^{(a)} + H^{(b)} + H^{(2a)} + H^{(2b)} \\
=& V_{cd}V_{ud}(1+\beta)\Big[V_P\big(\pi^+\pi^-K^+ - \frac{1}{\sqrt{2}}\pi^+\pi^0K^0 + \frac{1}{\sqrt{6}}\eta\pi^+K^0 + K^+K^0\bar{K}^0\big) \\
& + V_P'\big(-K^+K^+K^- - \eta\eta K^+ + \frac{2}{\sqrt{6}}\eta\pi^+K^0 - K^+K^0\bar{K}^0 + \frac{1}{\sqrt{3}}\eta\pi^0K^+\big)\Big] \\
=& C_1\big(\pi^+\pi^-K^+ - \frac{1}{\sqrt{2}}\pi^+\pi^0K^0 + \frac{1}{\sqrt{6}}\eta\pi^+K^0 + K^+K^0\bar{K}^0\big) \\
& - C_2\big(K^+K^+K^- + \eta\eta K^+ - \frac{2}{\sqrt{6}}\eta\pi^+K^0 + K^+K^0\bar{K}^0\big).
\end{aligned}
$$

- Tree-level production and final state interactions via rescattering mechanism:



(a) Tree-level production. (b) Rescattering of $K^+\pi^-$, $K^0\pi^0$ and $K^0\eta$. (c) Rescattering of $\pi^+\pi^-$, $\pi^0\eta$, $\eta\eta$ and K^+K^-.

$D_s^+ \to \pi^+\pi^- K^+$

- The amplitudes for the decay $D_s^+ \to K^+\pi^+\pi^-$ in the S-wave:

$$
\begin{aligned}
t(s_{12}, s_{23}) = & C_1\Big[1 + G_{\pi^- K^+}(s_{23}) T_{\pi^- K^+ \to \pi^- K^+}(s_{23}) + G_{\pi^+\pi^-}(s_{12}) T_{\pi^+\pi^- \to \pi^+\pi^-}(s_{12}) \\
& - \frac{1}{\sqrt{2}} G_{\pi^0 K^0}(s_{23}) T_{\pi^0 K^0 \to \pi^- K^+}(s_{23}) + \frac{1}{\sqrt{6}} G_{\eta K^0}(s_{23}) T_{\eta K^0 \to \pi^- K^+}(s_{23}) \\
& + G_{K^0 \bar{K}^0}(s_{12}) T_{K^0 \bar{K}^0 \to \pi^+\pi^-}(s_{12})\Big] - C_2\Big[G_{K^+K^-}(s_{12}) T_{K^+K^- \to \pi^+\pi^-}(s_{12}) \\
& + G_{\eta\eta}(s_{12}) T_{\eta\eta \to \pi^+\pi^-}(s_{12}) - \frac{2}{\sqrt{6}} G_{\eta K^0}(s_{23}) T_{\eta K^0 \to \pi^- K^+}(s_{23}) \\
& + G_{K^0 \bar{K}^0}(s_{12}) T_{K^0 \bar{K}^0 \to \pi^+\pi^-}(s_{12})\Big]
\end{aligned}
$$

- The diagonal matrix G is two intermediate meson propagators:

$$
G_{ii}(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m_1^2 + i\varepsilon} \frac{1}{(p_1 + p_2 - q)^2 - m_2^2 + i\varepsilon}.
$$

- The integral is logarithmically divergent, there are two methods to solve this problem:

  ✓ the three-momentum cut off:

$$
G_{ii}(s) = \int_0^{q_{\max}} \frac{q^2 dq}{(2\pi)^2} \frac{\omega_1 + \omega_2}{\omega_1 \omega_2 \left[s - (\omega_1 + \omega_2)^2 + i\varepsilon\right]}
$$

$$
\omega_i = \sqrt{\left(\vec{q}^{\,2} + m_i^2\right)} \qquad s = (p_1 + p_2)^2
$$

  ✓ the dimensional regularization method:

$$
\begin{aligned}
G_{ii}(s) = & \frac{1}{16\pi^2}\{a_\mu + \ln\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s}\ln\frac{m_2^2}{m_1^2} \\
& + \frac{q_{cm}(s)}{\sqrt{s}}[\ln\left(s - \left(m_2^2 - m_1^2\right) + 2q_{cm}(s)\sqrt{s}\right) + \ln\left(s + \left(m_2^2 - m_1^2\right) + 2q_{cm}(s)\sqrt{s}\right) \\
& - \ln\left(-s - \left(m_2^2 - m_1^2\right) + 2q_{cm}(s)\sqrt{s}\right) - \ln\left(-s + \left(m_2^2 - m_1^2\right) + 2q_{cm}(s)\sqrt{s}\right)]\}
\end{aligned}
$$

$D_s^+ \to \pi^+\pi^- K^+$

- For a given μ , these two regularization methods can be related by demanding the same value of the loop function at threshold, which results in a relationship between the free parameters a_μ and $q_{\max}$:

$$a_\mu = 16\pi^2[G^{CO}(s_{thr}, q_{max}) - G^{DR}(s_{thr}, \mu)],$$

Gloria Montaña, Angels Ramos, Laura Tolos, Juan M. Torres-Rincon, Arxiv: 2211.01896 (2022).

M. Y. Duan, J. Y. Wang, G. Y. Wang, E. Wang, and D. M. Li, Eur. Phys. J. C 80, 1041 (2020).

Hiwa A. Ahmed, Wang, Zhong-Yu, Sun, Zhi-Feng and Xiao, C. W, Eur. Phys.J. C 81:695 (2021).

Wang, Zhong-Yu and Yi, Jing-Yu and Sun, Zhi-Feng and Xiao, C. W, Phys Rev D.105.016025 (2021).

- The value of the parameter:

$$\mu = 0.6\ \text{GeV}$$

$$a_{\pi^+K^-} = -1.57,\ \ a_{\pi^0\overline{K}^0} = -1.57,\ \ a_{\eta\overline{K}^0} = -1.66$$

$$a_{\pi^+\pi^-} = -1.30,\ \ a_{\pi^0\pi^0} = -1.29,\ \ a_{K^+K^-} = -1.63,\ \ a_{K^0\overline{K}^0} = -1.63,\ \ a_{\eta\eta} = -1.68$$

$D_s^+ \to \pi^+\pi^- K^+$

- T is the two-body scattering amplitudes, it can be evaluated by the coupled channel Bethe-Salpeter equation of ChUA:

$$T = [1 - VG]^{-1} V,$$

- The interaction potentials of each coupled channel for different isospin:

➢ I = 0: $\pi^+\pi^-$, $\pi^0\pi^0$, K^+K^-, $K^0\overline{K}^0$, $\eta\eta$

$$V_{11} = -\frac{1}{2f^2}s, \quad V_{12} = -\frac{1}{\sqrt{2}f^2}(s - m_\pi^2), \quad V_{13} = -\frac{1}{4f^2}s,$$

$$V_{14} = -\frac{1}{4f^2}s, \quad V_{15} = -\frac{1}{3\sqrt{2}f^2}m_\pi^2, \quad V_{22} = -\frac{1}{2f^2}m_\pi^2,$$

$$V_{23} = -\frac{1}{4\sqrt{2}f^2}s, \quad V_{24} = -\frac{1}{4\sqrt{2}f^2}s, \quad V_{25} = -\frac{1}{6f^2}m_\pi^2,$$

$$V_{33} = -\frac{1}{2f^2}s, \quad V_{34} = -\frac{1}{4f^2}s,$$

$$V_{35} = -\frac{1}{12\sqrt{2}f^2}(9s - 6m_\eta^2 - 2m_\pi^2), \quad V_{44} = -\frac{1}{2f^2}s,$$

$$V_{45} = -\frac{1}{12\sqrt{2}f^2}(9s - 6m_\eta^2 - 2m_\pi^2),$$

$$V_{55} = -\frac{1}{18f^2}(16m_K^2 - 7m_\pi^2),$$

➢ I = 1/2 : $K^+\pi^-$, $K^0\pi^0$, $K^0\eta$

$$V_{11} = \frac{-1}{6f^2}\left(\frac{3}{2}s - \frac{3}{2s}\left(m_\pi^2 - m_K^2\right)^2\right)$$

$$V_{12} = \frac{1}{2\sqrt{2}f^2}\left(\frac{3}{2}s - m_\pi^2 - m_K^2 - \frac{\left(m_\pi^2 - m_K^2\right)^2}{2s}\right),$$

$$V_{13} = \frac{1}{2\sqrt{6}f^2}\left(\frac{3}{2}s - \frac{7}{6}m_\pi^2 - \frac{1}{2}m_\eta^2 - \frac{1}{3}m_K^2 + \frac{3}{2s}\left(m_\pi^2 - m_K^2\right)\left(m_\eta^2 - m_K^2\right)\right),$$

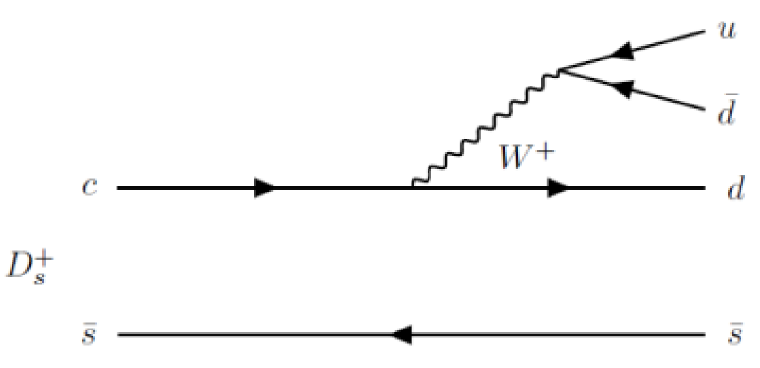
$$V_{22} = \frac{-1}{4f^2}\left(-\frac{s}{2} + m_\pi^2 + m_K^2 - \frac{\left(m_\pi^2 - m_K^2\right)^2}{2s}\right)$$

$$V_{23} = -\frac{1}{4\sqrt{3}f^2}\left(\frac{3}{2}s - \frac{7}{6}m_\pi^2 - \frac{1}{2}m_\eta^2 - \frac{1}{3}m_K^2 + \frac{3}{2s}\left(m_\pi^2 - m_K^2\right)\left(m_\eta^2 - m_K^2\right)\right)$$

$$V_{33} = -\frac{1}{4f^2}\left(-\frac{3}{2}s - \frac{2}{3}m_\pi^2 + m_\eta^2 + 3m_K^2 - \frac{3}{2s}\left(m_\eta^2 - m_K^2\right)^2\right)$$

$D_s^+ \to \pi^+\pi^-K^+$

- The contribution of P-wave:

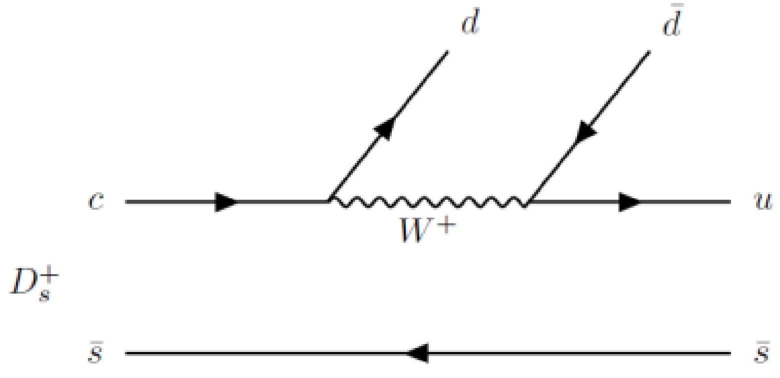


(a)

The $d\bar{s}$ quarks are not only in S-wave to form π^-K^+ through hadronization, but also in P-wave to produce the meson $K^*(892)^0$ and $K^*(1430)$, which decays into π^-K^+ finally. The full relativistic amplitude for $D_s^+ \to \pi^+K^* \to \pi^+\pi^-K^+$ can be written as:

$$M_{K^*(892)}(s_{12}, s_{23}) = \frac{D_{K^*(892)} e^{i\alpha_{K^*(892)}}}{s_{23} - m_{K^*(892)}^2 + i m_{K^*(892)} \Gamma_{K^*(892)}} \left[(m_K^2 - m_\pi^2) \frac{m_{D_s^+}^2 - m_\pi^2}{m_{K^*(892)}^2} - s_{13} + s_{12} \right],$$

$$M_{K^*(1430)}(s_{12}, s_{23}) = \frac{D_{K^*(1430)} e^{i\alpha_{K^*(1430)}}}{s_{23} - m_{K^*(1430)}^2 + i m_{K^*(1430)} \Gamma_{K^*(1430)}} \left[(s_{23} - m_K^2 - m_\pi^2) \cdot (s_{13} + s_{12} - m_K^2 - m_\pi^2) \right],$$



(c)

Similarly, the $d\bar{d}$ quarks can form the ρ meson in the P-wave, and then ρ, $f_0(1370)$ and $\rho(1450)$ decays into $\pi^+\pi^-$. The full relativistic amplitude for the these decay processes is given by:

$$M_\rho(s_{12}, s_{23}) = \frac{D_\rho e^{i\alpha_\rho}}{s_{12} - m_\rho^2 + i m_\rho \Gamma_\rho} (s_{23} - s_{13}),$$

$$M_{f_0(1370)}(s_{12}, s_{23}) = \frac{D_{f_0(1370)} e^{i\alpha_{f_0(1370)}}}{s_{12} - m_{f_0(1370)}^2 + i m_{f_0(1370)} \Gamma_{f_0(1370)}} \left[(s_{12} - 2m_\pi^2) \cdot (s_{13} + s_{23} - 2m_\pi^2) \right],$$

- s_{ij} are not independent totally and fulfill the constraint condition,

$$s_{12} + s_{23} + s_{13} = m_{D_s^+}^2 + m_K^2 + m_\pi^2 + m_\pi^2,$$

$D_s^+ \to \pi^+\pi^-K^+$

- The double differential width distribution of three-body decay:

$$\frac{d^2\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3}\frac{1}{32m_{D_s^+}^3}\left(\left|t(s_{12},s_{23}) + M_{K^*(892)} + M_{K^*(1430)} + M_{f_0(1370)} + M_{\rho} + M_{\rho(1450)}\right|^2\right)$$

- For integrating s_{23}, the limits of integration are:

$$(s_{23})_{\max} = (E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} - \sqrt{E_3^{*2} - m_3^2}\right)^2$$

$$(s_{23})_{\min} = (E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} + \sqrt{E_3^{*2} - m_3^2}\right)^2$$

where: $E_2^* = \frac{(s_{12} - m_1^2 + m_2^2)}{2\sqrt{s_{12}}}, E_3^* = \frac{(m_{D_s^+}^2 - s_{12} - m_3^2)}{2\sqrt{s_{12}}}$

- The limits of integral variable for the invariant masses are higher than 1.2 GeV, we need to smoothly extrapolate $G(s)T(s)$ above the energy cut $\sqrt{s} \geq \sqrt{s_{cut}} = 1.1$ GeV :

$$G(s)T(s) = G(s_{\text{cut}})T(s_{\text{cut}})e^{-\alpha(\sqrt{s}-\sqrt{s_{\text{cut}}})}, \quad \text{for } \sqrt{s} > \sqrt{s_{\text{cut}}}$$

- The parameters need to be fitted:

S-wave: C_1, C_2, α

P-wave: $D_\rho, \alpha_\rho, D_{K^*(892)}, \alpha_{K^*(892)}, D_{K^*(1430)}, \alpha_{K^*(1430)}, D_{f_0(1370)}, \alpha_{f_0(1370)}, D_{\rho(1450)}, \alpha_{\rho(1450)},$

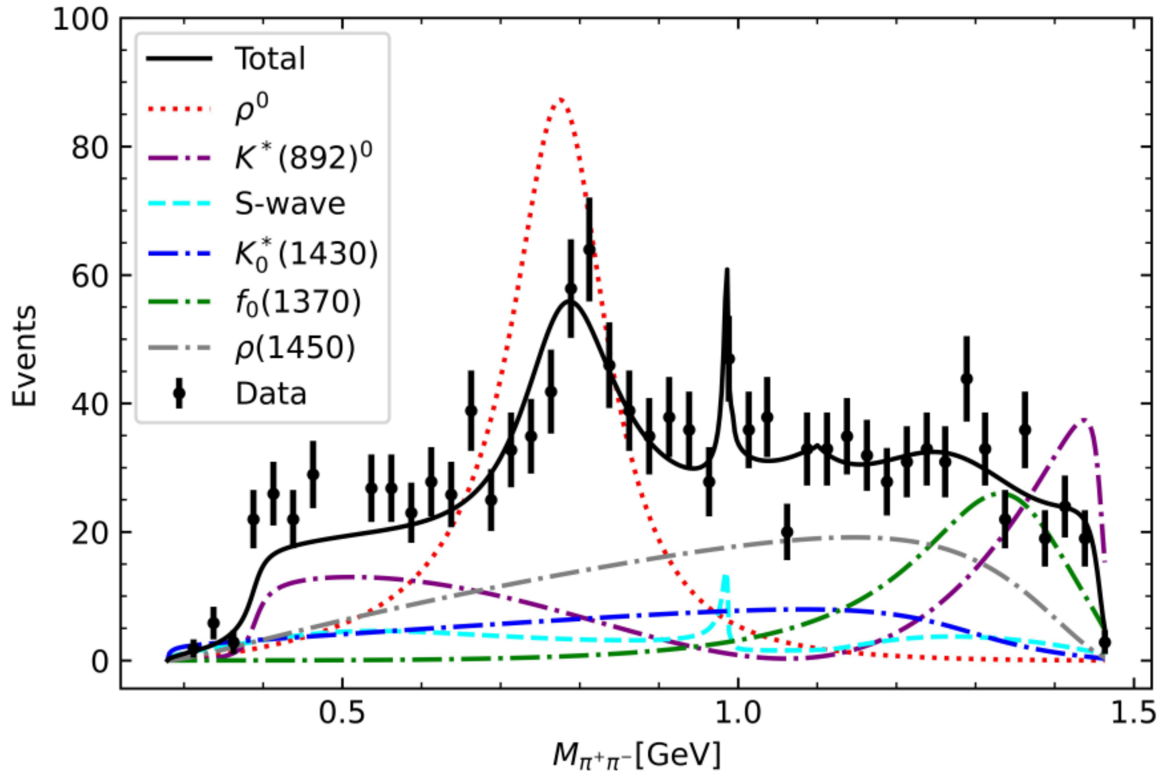
V. R. Debastiani, W. H. Liang, J. J. Xie, and E. Oset, Phys. Lett. B 766, 59 (2017).

Results

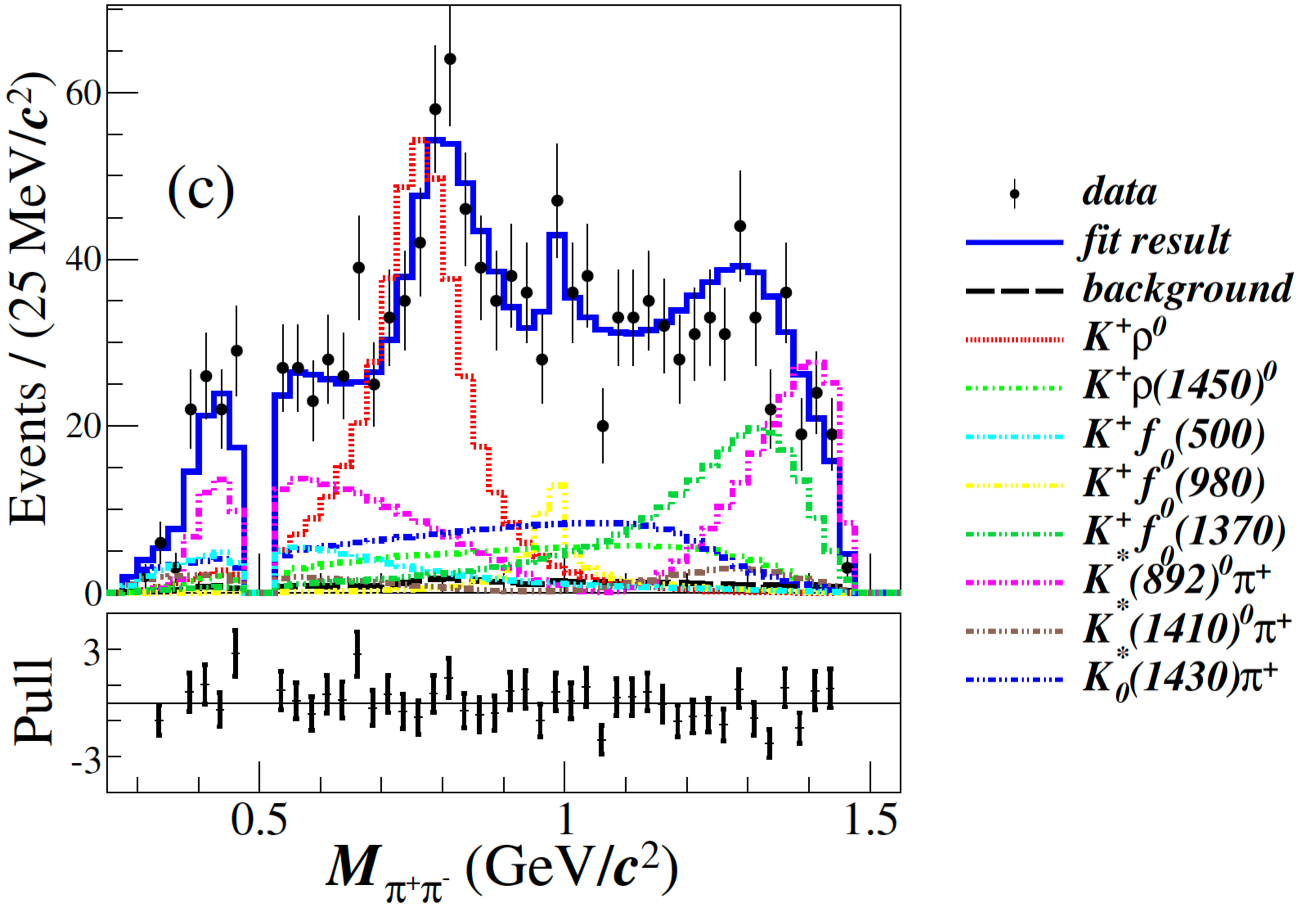
Fitting results (Combined fitting)

- $\pi^+\pi^-$ $\chi^2/dof = 183.37/128 = 1.43$

Our Results:



BESIII Experiment:

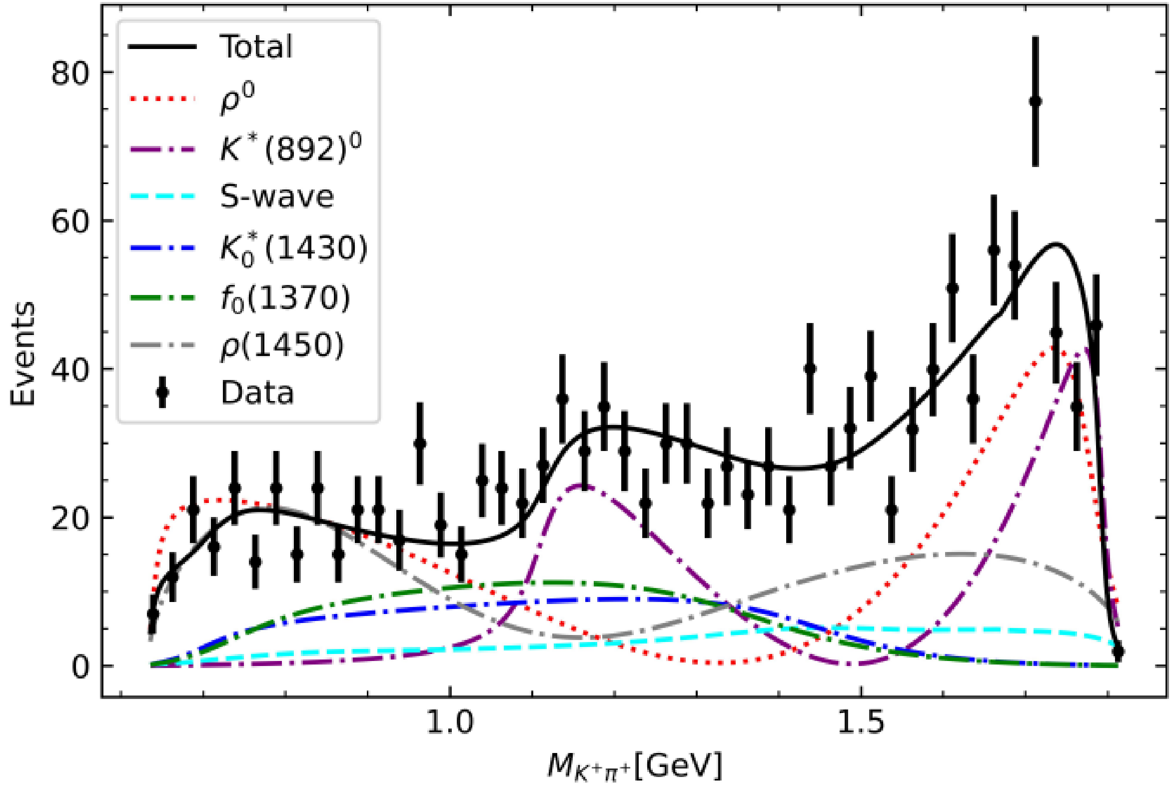


Parameters	C_1	C_2	α	D_ρ	α_ρ	$D_{K^*(892)}$	$\alpha_{K^*(892)}$	$D_{K^*(1430)}$	$\alpha_{K^*(1430)}$	$D_{f_0(1370)}$	$\alpha_{f_0(1370)}$	$D_{\rho(1450)}$	$\alpha_{\rho(1450)}$	$\chi^2/dof.$
Fit	263.74	-63.08	12.34	80.77	0.18	62.99	3.54	-62.50	1.21	-60.24	3.07	-456.70	0.93	1.43

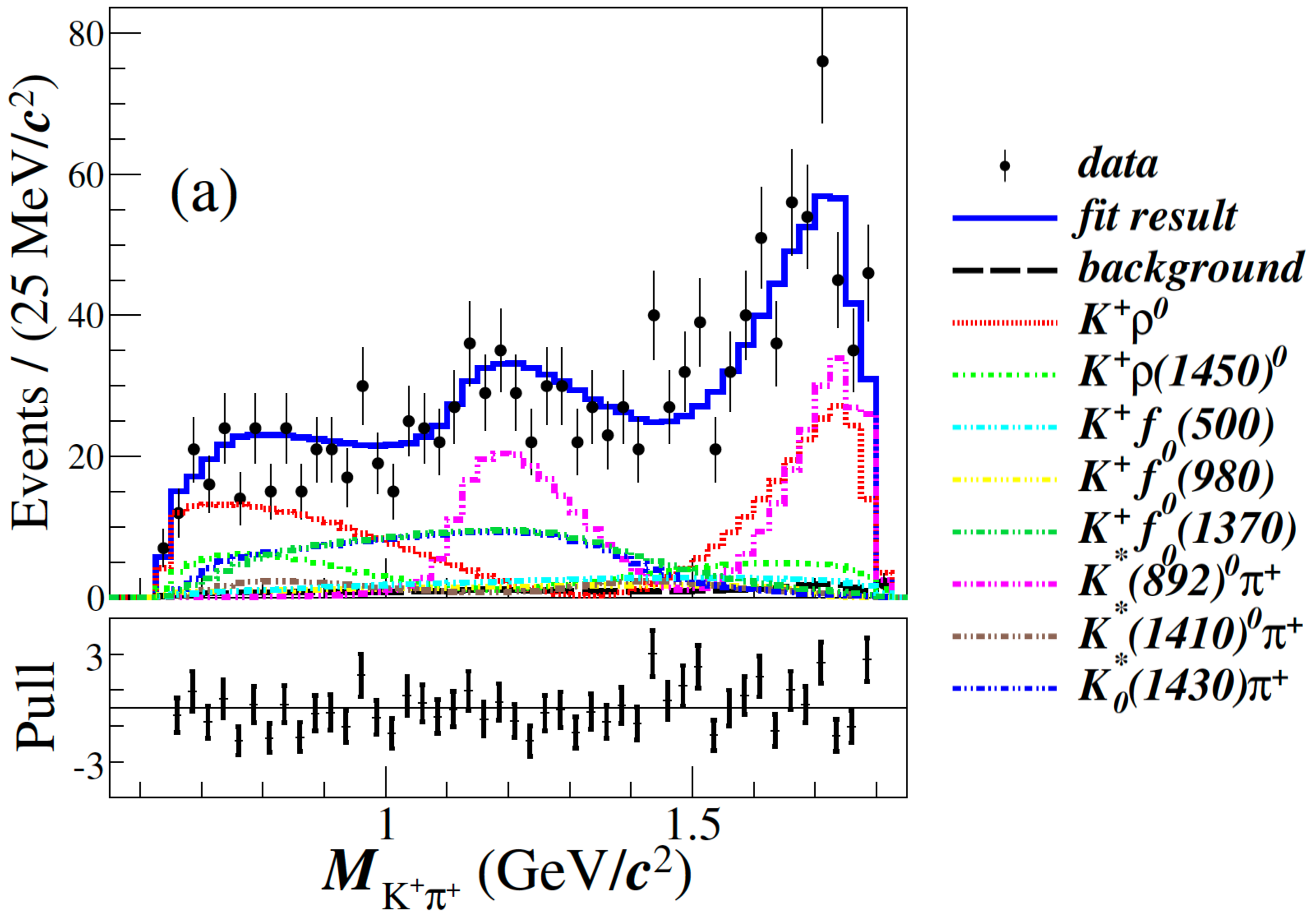
Fitting results (Combined fitting)

- $K^+\pi^+$ $\chi^2/dof = 183.37/128 = 1.43$

Our Results:



BESIII Experiment:

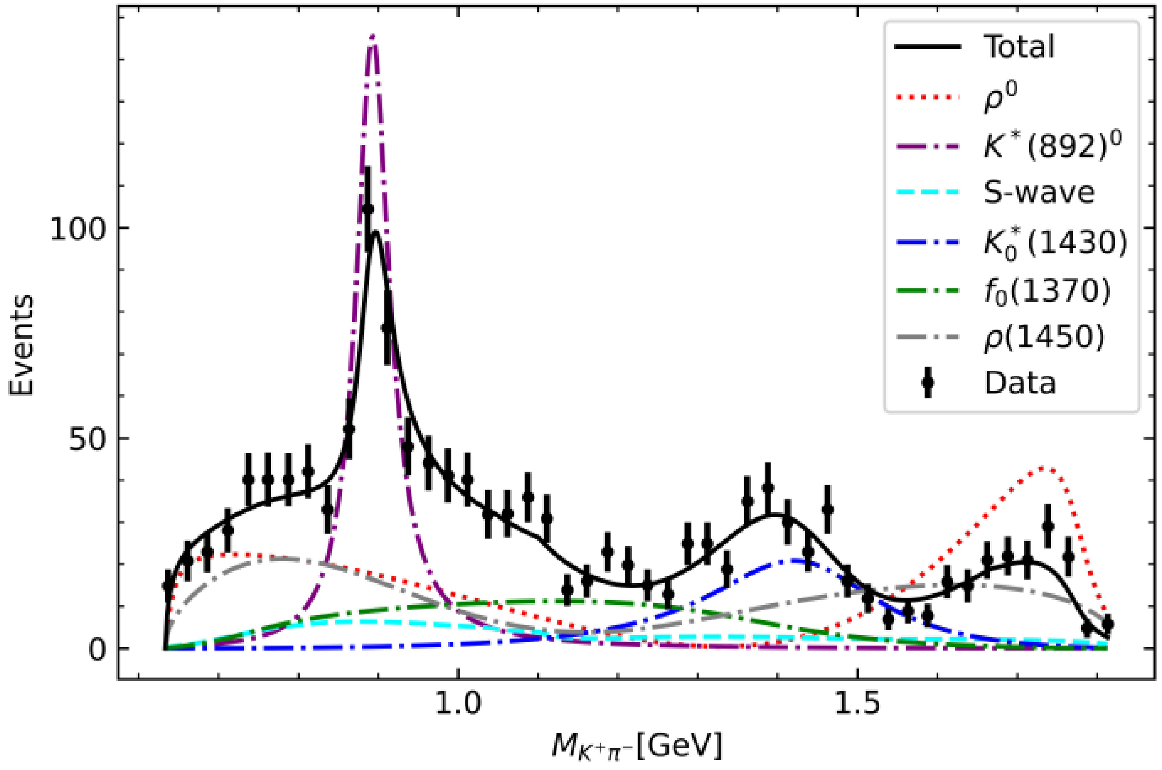


Parameters	C_1	C_2	α	D_ρ	α_ρ	$D_{K^*(892)}$	$\alpha_{K^*(892)}$	$D_{K^*(1430)}$	$\alpha_{K^*(1430)}$	$D_{f_0(1370)}$	$\alpha_{f_0(1370)}$	$D_{\rho(1450)}$	$\alpha_{\rho(1450)}$	$\chi^2/dof.$
Fit	263.74	-63.08	12.34	80.77	0.18	62.99	3.54	-62.50	1.21	-60.24	3.07	-456.70	0.93	1.43

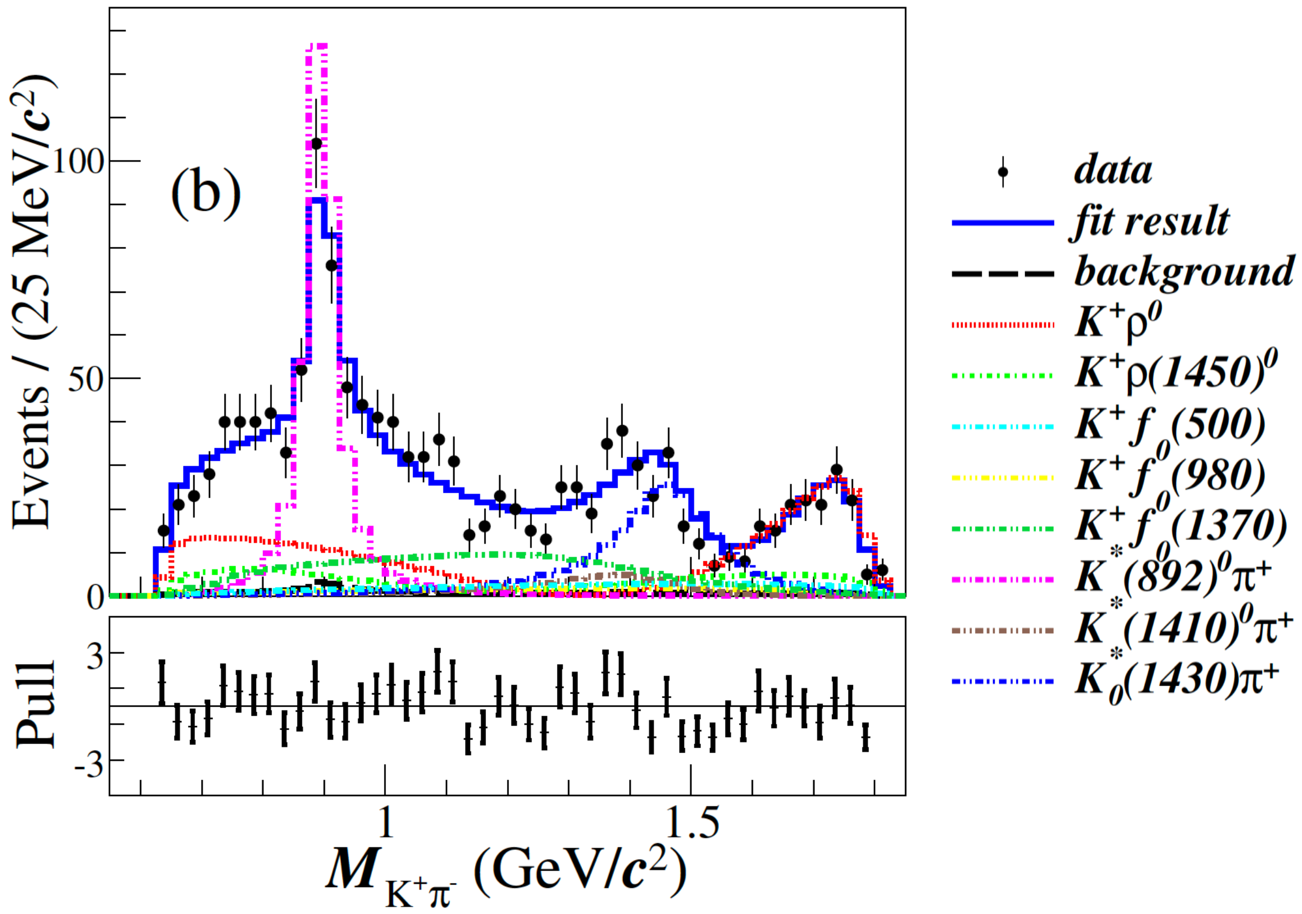
Fitting results (Combined fitting)

- $K^+\pi^-$ $\chi^2/dof = 183.37/128 = 1.43$

Our Results:



BESIII Experiment:



Parameters	C_1	C_2	α	D_ρ	α_ρ	$D_{K^*(892)}$	$\alpha_{K^*(892)}$	$D_{K^*(1430)}$	$\alpha_{K^*(1430)}$	$D_{f_0(1370)}$	$\alpha_{f_0(1370)}$	$D_{\rho(1450)}$	$\alpha_{\rho(1450)}$	$\chi^2/dof.$
Fit	263.74	-63.08	12.34	80.77	0.18	62.99	3.54	-62.50	1.21	-60.24	3.07	-456.70	0.93	1.43

Branching fractions

- The ratios of the branching fractions between different resonances :

$$\frac{\mathcal{B}\left[D_S^+ \to K^+ f_0(500) \to K^+\pi^+\pi^-\right]}{\mathcal{B}\left[D_S^+ \to K^*(892)^0\pi^+ \to K^+\pi^+\pi^-\right]} = 0.20^{+0.02}_{-0.02},$$

$$\frac{\mathcal{B}\left[D_S^+ \to K^+ f_0(980) \to K^+\pi^+\pi^-\right]}{\mathcal{B}\left[D_S^+ \to K^*(892)^0\pi^+ \to K^+\pi^+\pi^-\right]} = 0.06^{+0.02}_{-0.02},$$

$$\frac{\mathcal{B}\left[D_S^+ \to K^+ \rho \to K^+\pi^+\pi^-\right]}{\mathcal{B}\left[D_S^+ \to K^*(892)^0\pi^+ \to K^+\pi^+\pi^-\right]} = 1.59^{+0.02}_{-0.03},$$

$$\frac{\mathcal{B}\left[D_S^+ \to f_0(1370) K^+ \to K^+\pi^+\pi^-\right]}{\mathcal{B}\left[D_S^+ \to K^*(892)^0\pi^+ \to K^+\pi^+\pi^-\right]} = 0.58^{+0.06}_{-0.11},$$

$$\frac{\mathcal{B}\left[D_S^+ \to K^+ \rho(1450) \to K^+\pi^+\pi^-\right]}{\mathcal{B}\left[D_S^+ \to K^*(892)^0\pi^+ \to K^+\pi^+\pi^-\right]} = 1.28^{+0.02}_{-0.05},$$

- The branching ratios for intermediate :

$$B(D_s^+ \to K^*(892)\pi^+, K^*(892) \to K^+\pi^-) = (1.85 \pm 0.13 \pm 0.11) \times 10^{-3}$$

Decay process	Ours (10^{-3})	BESIII (10^{-3})	PDG (10^{-3})
$D_s^+ \to K^+ f_0(500)$	$0.38 \pm 0.03^{+0.03}_{-0.03}$	$0.43 \pm 0.14 \pm 0.24$	-
$D_s^+ \to K^+ f_0(980)$	$0.11 \pm 0.01^{+0.04}_{-0.04}$	$0.27 \pm 0.08 \pm 0.07$	-
$D_s^+ \to K^+ \rho^0$	$2.94 \pm 0.27^{+0.03}_{-0.05}$	$1.99 \pm 0.20 \pm 0.22$	2.5 ± 0.4
$D_s^+ \to K^+ f_0(1370)$	$1.07 \pm 0.10^{+0.11}_{-0.20}$	$1.22 \pm 0.19 \pm 0.18$	-
$D_s^+ \to K_0^*(1430)^0 \pi^+$	$1.06 \pm 0.10^{+0.01}_{-0.02}$	$1.15 \pm 0.16 \pm 0.15$	0.50 ± 0.35
$D_s^+ \to K^+ \rho(1450)^0$	$2.38 \pm 0.22^{+0.04}_{-0.09}$	$0.78 \pm 0.20 \pm 0.17$	0.69 ± 0.64

Conclusions

Conclusions

- Based on the BESIII Collaboration measurements for the decay $D_s^+ \to \pi^+\pi^-K^+$, we investigate this decay process within the chiral unitary approach, where $f_0(500)$ and $f_0(980)$ are dynamically generated in the coupled channel interactions. Besides, the contributions of the intermediate resonances $K^*(892)$, $K^*(1430)$, ρ, $\rho(1450)$ and $f_0(1370)$ in the P wave are also considered.

- Considering the coherent effects between the S and P waves, we obtain the free parameters by combined fitting the experimental data, and then figure out the contributions of the S- and P-waves intermediate resonances. The results show that the $\pi^+\pi^-$, $K^+\pi^-$ and $K^+\pi^+$ invariant mass distributions are consistent with experiments. However, the contributions of ρ and $\rho(1450)$ in the coherent fit is significantly bigger than experiments, also see their obtained branching fractions.

- We also calculate the branching fractions of the dominant decay channels, it is obvious that the theoretical results are almost in good agreement with the experimental measurements and PDG within the uncertainties, except for the branching ratio of the ρ and ρ(1450) is quite large and the one of the $f_0(980)$ became half as small.

Thank you!