東南大學
SOUTHEAST UNIVERSITY

New Look at $\psi(4160)$ and $\psi(4230)$

----Could they be the same state?

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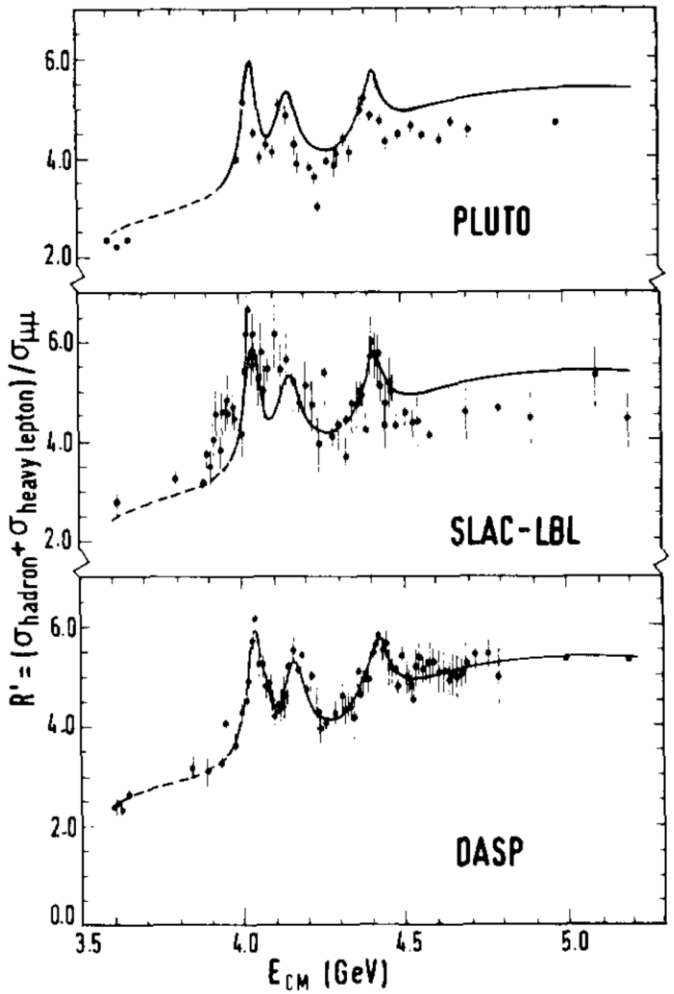
Arxiv: 2304.07052,
in collaboration with Chun-Yong Li and Zhiguang Xiao

2023年8月28日，中国科学院大学，雁栖湖

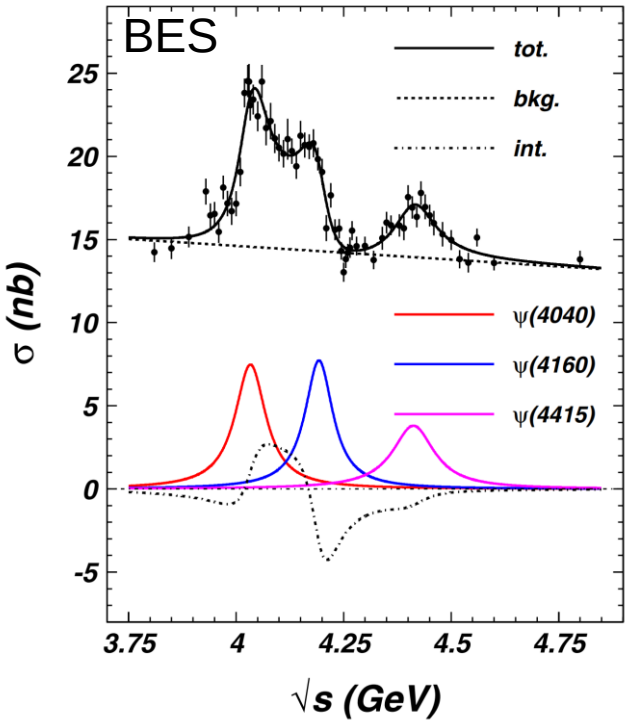
目录

outline

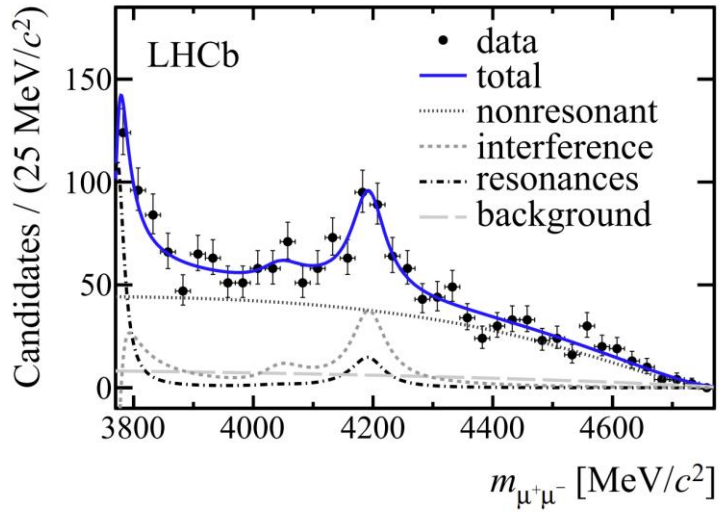
Experiment status of $\psi(4160)$



PLB. 76 (1978) 361



PRD. 82 (2010) 077501

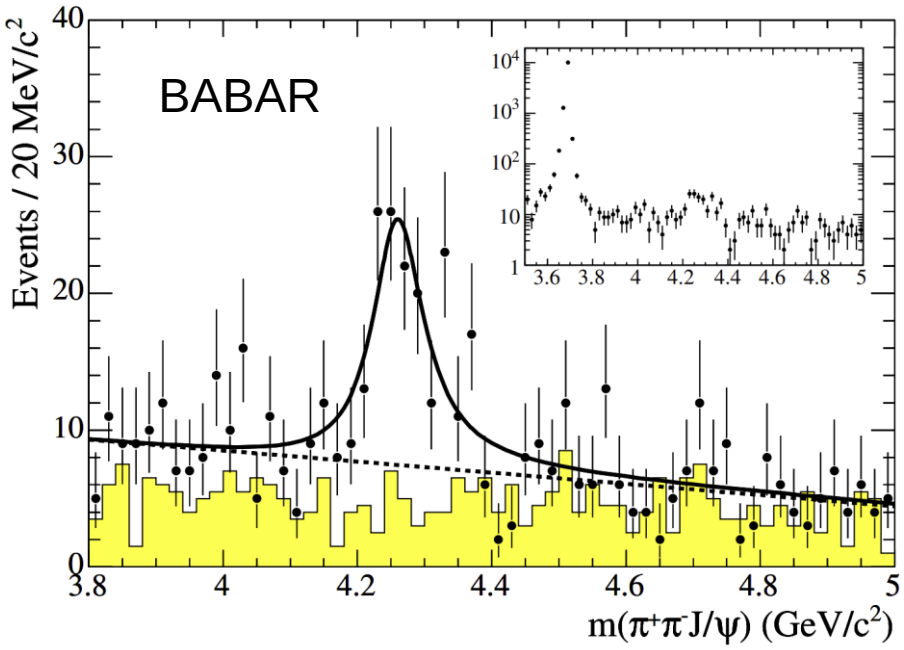


PRL 111 (2013) 11, 112003

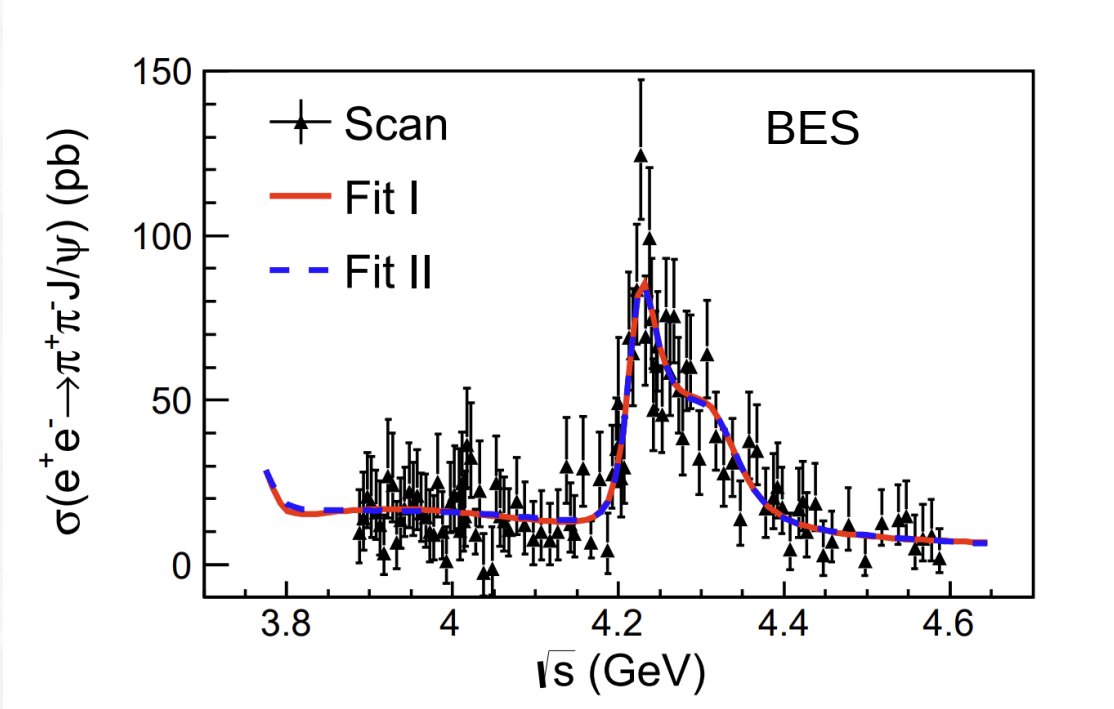
Experiment status of $\psi(4160)$

VALUE (MeV)		*DOCUMENT ID*		*TECN*	*COMMENT*
4191 ± 5	**OUR AVERAGE**				
$4191\ ^{+9}_{-8}$		AAIJ	2013BC	LHCB	$B^+ \to K^+\mu^+\mu^-$
4191.7 ±6.5		[1] ABLIKIM	2008D	BES2	$e^+\,e^- \to$ hadrons
		• • We do not use the following data for averages, fits, limits, etc. • •			
4193 ±7		[2] MO	2010	RVUE	$e^+\,e^- \to$ hadrons
4151 ±4		[3] SETH	2005A	RVUE	$e^+\,e^- \to$ hadrons
4155 ±5		[4] SETH	2005A	RVUE	$e^+\,e^- \to$ hadrons
4159 ±20		BRANDELIK	1978C	DASP	e^+e^-

Experiment status of $\psi(4230)$



PRL. 95 (2005) 142001



PRL. 118 (2017) 9, 092001

Experiment status of $\psi(4230)$

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
4222.5 ± 2.4	**OUR AVERAGE** Error includes scale factor of 1.7. See the ideogram below.				
4221.4 ±1.5 ±2.0		[1] ABLIKIM	2022AM	BES3	$e^+ e^- \to \pi^+\pi^- J/\psi$
4225.3 ±2.3 ±21.5		[2] ABLIKIM	2022AU	BES3	$e^+ e^- \to K^+K^- J/\psi$
4234.4 ±3.2 ±0.2		[3] ABLIKIM	2021AJ	BES3	$e^+ e^- \to \pi^+\pi^-\psi(2S)$
4216.7 ±8.9 ±4.1		[4] ABLIKIM	2020AG	BES3	$e^+ e^- \to \mu^+\mu^-$
4220.4 ±2.4 ±2.3		[5] ABLIKIM	2020N	BES3	$e^+ e^- \to \pi^0\pi^0 J/\psi$
4218.6 ±3.8 ±2.5		[5] ABLIKIM	2020O	BES3	$e^+ e^- \to \eta J/\psi$
4218.5 ±1.6 ±4.0		[6] ABLIKIM	2019AI	BES3	$e^+ e^- \to \omega\chi_{c0}$
4228.6 ±4.1 ±6.3		ABLIKIM	2019R	BES3	$e^+ e^- \to \pi^+ D^0 D^{*-}$ + c.c.
4200.6 $^{+7.9}_{-13.3}$ ±3.0		[7] ABLIKIM	2019V	BES3	$e^+ e^- \to \gamma\chi_{c1}(3872)$
4218 $^{+5.5}_{-4.5}$ ±0.9		ABLIKIM	2017G	BES3	$e^+ e^- \to \pi^+\pi^- h_c$
	• • We do not use the following data for averages, fits, limits, etc. • •				
4231.9 ±5.3 ±4.9		ABLIKIM	2020N	BES3	$e^+ e^- \to \pi^0 Z_c(3900)^0$, $Z_c^0 \to \pi^0 J/\psi$
4222.0 ±3.1 ±1.4		[8] ABLIKIM	2017B	BES3	$e^+ e^- \to \pi^+\pi^- J/\psi$
4209.5 ±7.4 ±1.4		[9] ABLIKIM	2017V	BES3	$e^+ e^- \to \pi^+\pi^-\psi(2S)$
4209.1 ±6.8 ±7.0		[10] ZHANG	2017B	RVUE	$e^+ e^- \to \pi^+\pi^-\psi(2S)$
4223.3 ±1.6 ±2.5		[11] ZHANG	2017C	RVUE	$e^+ e^- \to \pi^+\pi^- J/\psi$ or $\psi(2S)$
4230 ±8 ±6	180	[12] ABLIKIM	2015C	BES3	$e^+ e^- \to \omega\chi_{c0}$
4258.6 ±8.3 ±12.1		[13] LIU	2013B	BELL	$e^+ e^- \to \gamma\pi^+\pi^- J/\psi$
4245 ±5 ±4		[14] LEES	2012AC	BABR	10.58 $e^+ e^- \to \gamma\pi^+\pi^- J/\psi$
4247 ±12 $^{+17}_{-32}$		[15, 13] YUAN	2007	BELL	10.58 $e^+ e^- \to \gamma\pi^+\pi^- J/\psi$
4284 $^{+17}_{-16}$ ±4	13.6	HE	2006B	CLEO	9.4 – 10.6 $e^+ e^- \to \gamma\pi^+\pi^- J/\psi$
4259 ±8 $^{+2}_{-6}$	125	[16] AUBERT,B	2005I	BABR	10.58 $e^+ e^- \to \gamma\pi^+\pi^- J/\psi$

Experiment status of $\psi(4230)$

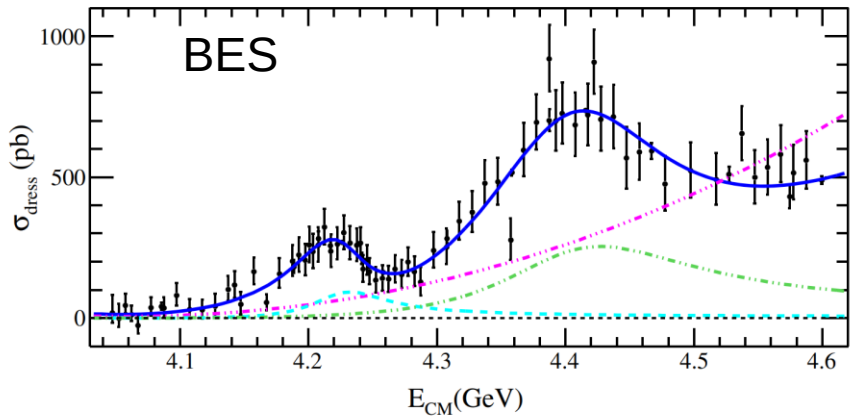


FIG. 2. Fit to the dressed cross section of $e^+e^- \to \pi^+ D^0 D^{*-}$, where the black dots with error bars are the measured cross sections and the blue line shows the fit result. The error bars are statistical only. The pink dashed triple-dot line describes the phase-space contribution, the green dashed double-dot line describes the R_2 contribution, and the light blue dashed line describes the R_1 contribution.

PRL. 122 (2019) 102002

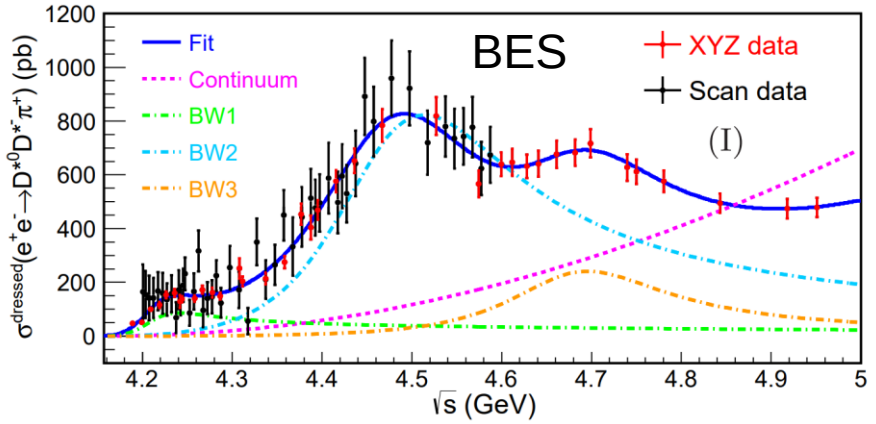


FIG. 3. The fit results (solution I) of the dressed cross section line shape of $e^+e^- \to D^{*0}D^{*-}\pi^+$. The black and red points with error bars are data, including statistical and systematic uncertainties. The blue curve is the total fit. The green, azure and orange dashed curves describe three BW functions, and the pink dashed curve is the three body phase space contribution.

PRL. 130 (2023) 121901

Motivation

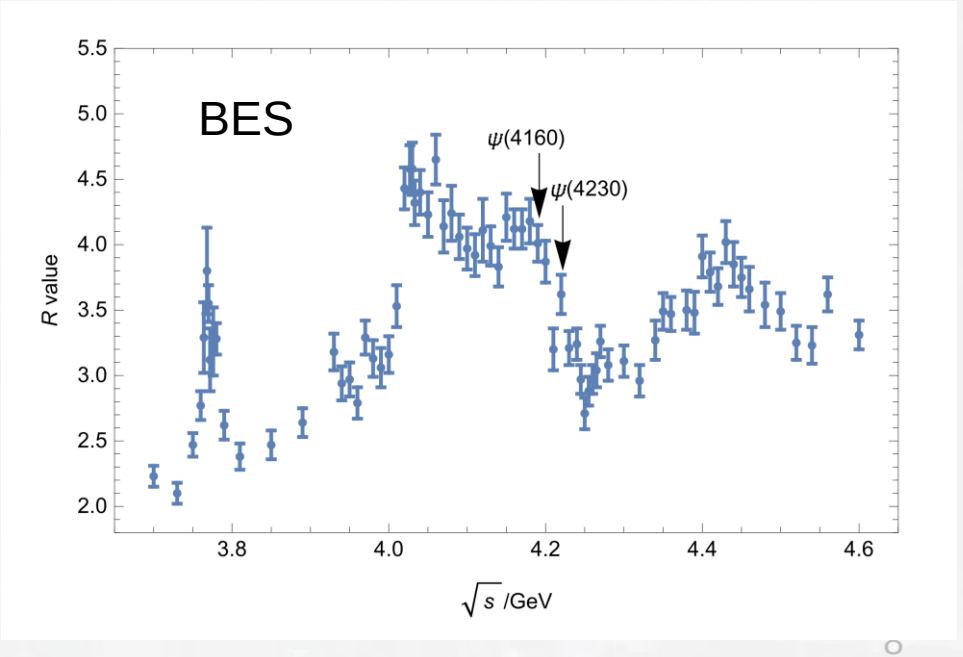
$\psi(4160)$ and $\psi(4230)$

- Both of them are 1^{--} states, small mass splitting, similar widths
- Hardly accommodated in the quark models simultaneously
- Mutually exclusive decay modes

  $\psi(4160)$ mainly open-charm modes

  $\psi(4230)$ mainly hidden-charm modes, except $\pi^+ D^0 D^{*-}$ and $D^{*0} D^{*-} \pi^+$
- No same decay mode has been reported.

	Mass	*Width*
$\psi(4160)$	4191 ± 5 MeV	70 ± 10 MeV
$\psi(4230)$	4222.5 ± 2.4 MeV	48 ± 8 MeV



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Mode		Fraction (Γ_i/Γ)	Scale Factor/ Conf. Level	P(MeV/c)
Γ_1	e^+e^-	$(6.9 \pm 3.3) \times 10^{-6}$		2096
Γ_2	$\mu^+\mu^-$	seen		2093
Γ_3	$D\overline{D}$	seen		956
Γ_4	$D^0\overline{D}^0$	seen		956
Γ_5	D^+D^-	seen		947
Γ_6	$D^*\overline{D}$ + c.c.	seen		798
Γ_7	$D^*(2007)^0\overline{D}^0$ + c.c.	seen		802
Γ_8	$D^*(2010)^+D^-$ + c.c.	seen		792
Γ_9	$D^*\overline{D}^*$	seen		592
Γ_{10}	$D^*(2007)^0\overline{D}^*(2007)^0$	seen		604
Γ_{11}	$D^*(2010)^+D^*(2010)^-$	seen		592
Γ_{12}	$D^0D^-\pi^+$ +c.c. (excl. $D^*(2007)^0\overline{D}^0$ +c.c., $D^*(2010)^+D^-$ +c.c.)	not seen		
Γ_{13}	$D\overline{D}^*\pi$ +c.c. (excl. $D^*\overline{D}^*$)	seen		
Γ_{14}	$D^0D^{*-}\pi^+$ +c.c. (excl. $D^*(2010)^+D^*(2010)^-$)	not seen		
Γ_{15}	$D_s^+D_s^-$	not seen		719
Γ_{16}	$D_s^{*+}D_s^-$ +c.c.	seen		385

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How to describe a resonance

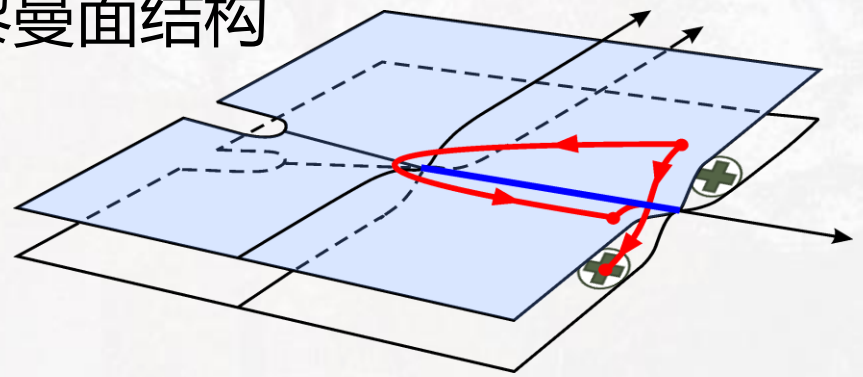
$$\langle p_1' p_2', b \left| iT \right| p_1 p_2, a \rangle = i(2\pi)^4 \delta^4(p_1 + p_2 - p_1' - p_2') \mathcal{M}(p_1, p_2; p_1', p_2')_{ba}$$

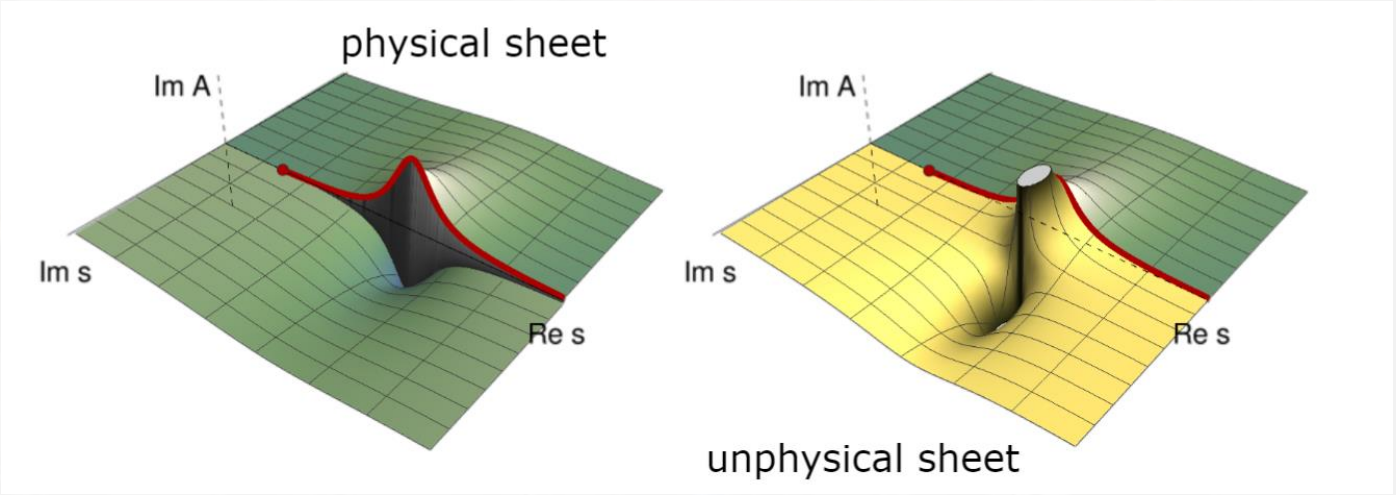
Disc $\mathcal{M}_{ba} = \mathcal{M}_{ba} - \mathcal{M}_{ab}^* = i(2\pi)^4 \sum_c \int d\Phi_c \mathcal{M}_{cb}^* \mathcal{M}_{ca}$

$$Im\mathcal{M}_{ba} = \sum_c \mathcal{M}_{cb}^* \rho_c \mathcal{M}_{ca}$$

Unitarity and analyticity

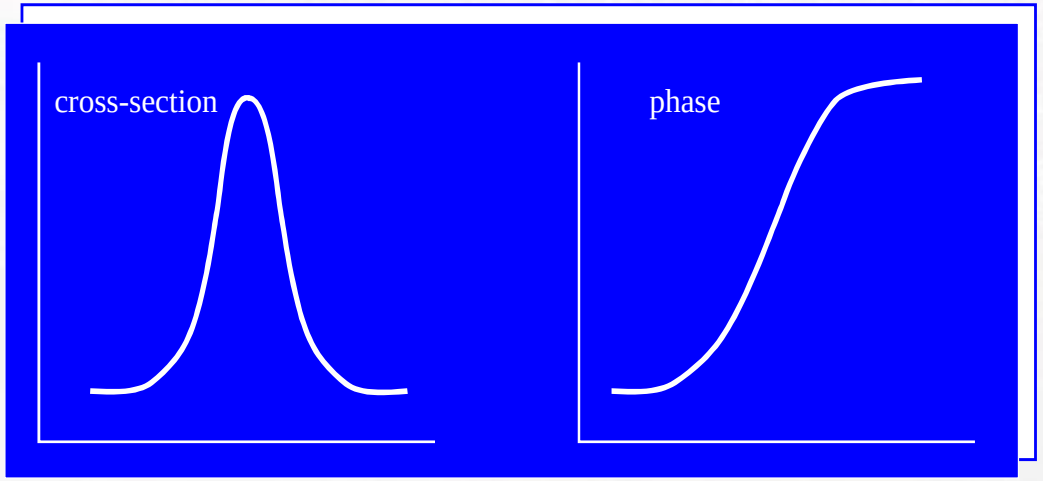
黎曼面结构





PDG review

How to describe a resonance



The simplest Breit-Wigner

$$T = \frac{1}{M^2 - s - i\,M\,\Gamma}$$

Or Breit-Wigner distribution with energy-dependent width

$$T_l(s) = \frac{B_l^2(q)\Gamma}{M^2 - m^2 - i\,\rho B_l^2(q) m_0 \Gamma}$$

$$B_l(q, q_R) = \frac{F_l(q)}{F_l(q_R)},\ \rho_i = \frac{2q_i}{m}$$

Blatt-Weisskopf Barrier Factors

$$F_0(x) = 1$$

$$F_1(x) = \sqrt{\frac{x}{x+1}}$$

$$F_2(x) = \sqrt{\frac{13x^2}{(x-3)^2 + 9x}}$$

……

by Hippel and Quigg (1972)

A lesson from Taylor’s book

The behavior of the partial cross section $\sigma_l(p)$,

$$\sigma_l(p) = \frac{4\pi(2l+1)}{p^2}\sin^2\delta_l(p)$$

near a resonance depends on the value of the background δ_{bg}. Four different possibilities are shown in Fig. 13.3. The first and simplest, shown as the curves below (a), is when the background δ_{bg} is zero, and $\delta(p)$ therefore

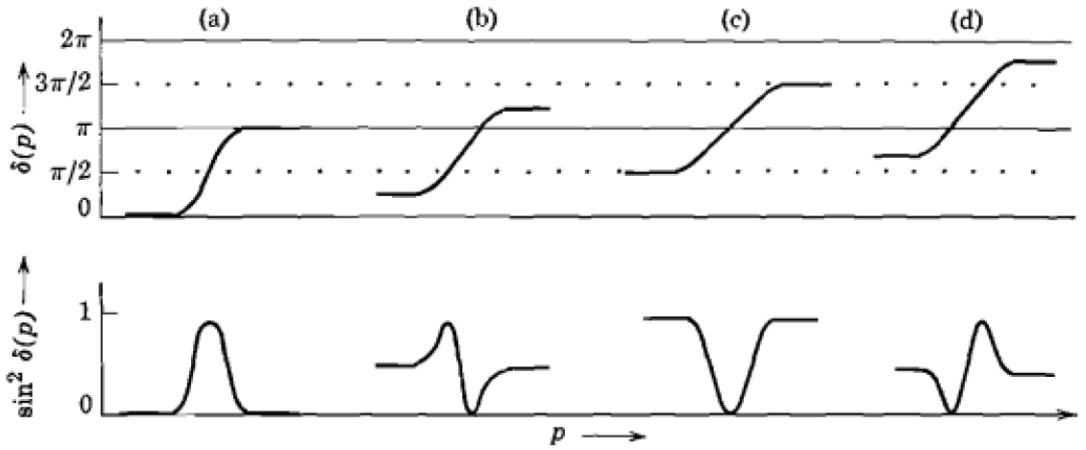


FIGURE 13.3. Four possible resonances. The $\delta(p)$ plots show the resonant phase shifts for $\delta_{bg} = 0$, $\pi/4$, $\pi/2$, and $3\pi/4$. The $\sin^2\delta(p)$ plots show the corresponding behavior of the partial cross section [apart from a factor $4\pi(2l+1)/p^2$].

J.R.Taylor, Scattering theory, P241-242

What we should know about Breit-Wigner distribution in the experimental analysis

- Only distribution, no analyticity
- Multi-solution ambiguity

$n+1$ Breit-Wigner with 2^n solutions

of similar fit quality

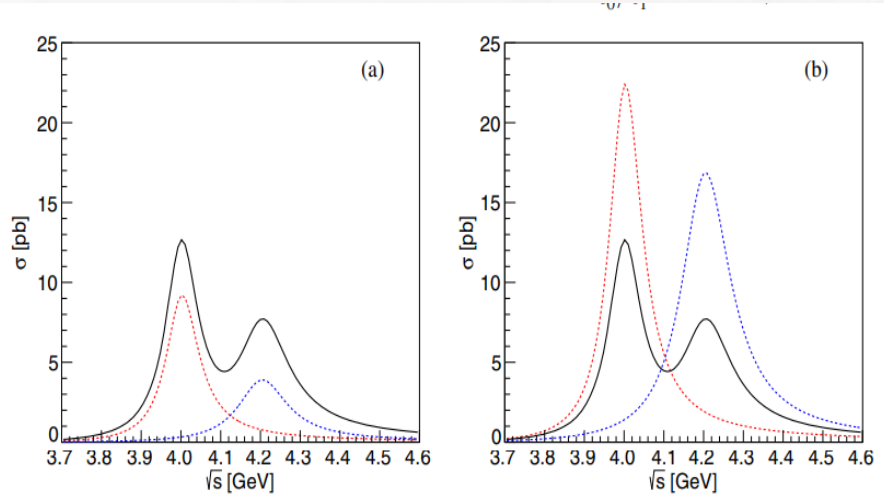


FIG. 2. An example of interference between two Breit-Wigner distributions. The solid curve is the total cross section, and the dashed curves are the individual contributions from the resonances.

K. Zhu et. al., IJMPA 26, 4511 (2011)
A. D. Bukin, arXiv:0710.5627
X. Han and C. P. Shen, CPC 42, 043001 (2018)
Y. Bai and D.Chen, PRD 99, 072007

PRD 99, 072007

What we should know about Breit-Wigner distribution in the experimental analysis

- Only distribution, no analyticity
- Multi-solution ambiguity
- Non-unitrarity

Coupled-channel models with unitarity

- E. Eichten, K. Gottfried, T. Kinoshita, K. Lane, and T.-M. Yan, Phys.Rev. D 21, 203 (1980)
- N. A. Tornqvist, Z. Phys. C 68, 647 (1995)
- H.-B. Li, X.-S. Qin, and M.-Z. Yang, Phys. Rev. D 81, 011501 (2010)
- X. Cao and H. Lenske, Hadron 2019, pp. 433–437, arX-iv:1408.5600
- A. Limphirat, W. Sreethawong, K. Khosonthongkee, and Y. Yan, Phys. Rev. D 89, 054030 (2014)
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- T. V. Uglov, Y. S. Kalashnikova, A. V. Nefediev, G. V. Pakhlova, and P. N. Pakhlov, JETP Lett. 105, 1 (2017)
- T. Wolkanowski, F. Giacosa, and D. H. Rischke, Phys. Rev. D 93, 014002 (2016), arXiv:1508.00372
- Z. Yang, G.-J. Wang, J.-J. Wu, M. Oka, and S.-L. Zhu, Phys. Rev. Lett. 128, 112001 (2022)
- S. X. Nakamura, Q. Huang, J. J. Wu, H. P. Peng, Y. Zhang, and Y. C. Zhu, Phys.Rev.D 107 (2023) 9, L091505
- Z.-L. Zhang, Z.-W. Liu, S.-Q. Luo, F.-L. Wang, B. Wang, and H. Xu, Phys. Rev. D 107, 034036 (2023)
- ……

A method to restore the unitarity----K matrix

$$S = 1 + 2i\,A$$

$$A = K(1 - iK)^{-1}$$

$$K_{ij} = \sum_{\alpha} G_{i\alpha}(s)\frac{1}{M_\alpha^2 - s}G_{j\alpha}(s)$$

$$G_{i\alpha}^2(s) = g_{i\alpha}^2\frac{k_i^{2l_i+1}}{\sqrt{s}}\theta(s - s_i)$$

$$A_{ij} = \sum_{\alpha\beta} G_{i\alpha}(s)P_{\alpha\beta}(s)G_{j\beta}(s)$$

$$\left(P^{-1}(s)\right)_{\alpha\beta} = \left(M_\alpha^2 - s\right)\delta_{\alpha\beta} - i\sum_{m} G_{m\alpha}G_{m\beta}$$

33 free parameters to fit well $e^+e^- \to D\overline{D}, D\overline{D^*}, D^*\overline{D^*}, D\overline{D}\pi$, but no poles' informations could be extracted.

Uglov et al., JETP Lett. 105, 1 (2017), arXiv:1611.07582

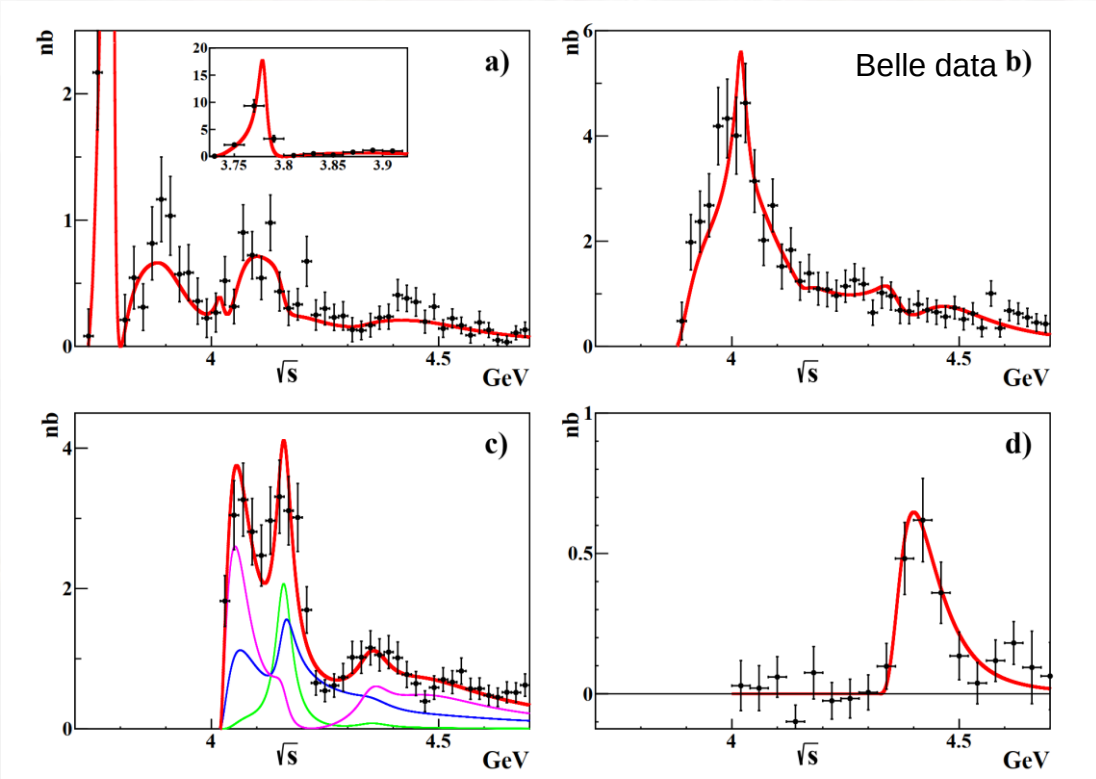


Figure 1. Exclusive cross sections for the $e^+e^- \to D\bar{D}$ ($[e^+e^- \to D^+D^-]+[e^+e^- \to D^0\bar{D}^0]$) (plot (a)), $e^+e^- \to D^+D^{*-}$ (plot (b)), $e^+e^- \to D^{*+}D^{*-}$ (plot (c)), and $e^+e^- \to D\bar{D}\pi$ ($[e^+e^- \to D^0D^-\pi^+] + [e^+e^- \to \bar{D}^0D^+\pi^-]$) (plot (d)). In all plots, the points with the error bars represent the Belle data [41] and the red curves show the fit results. In plot (c), the blue, magenta and green thin curves represent the P-wave with $S = 0$, the P-wave with $S = 2$, and the F-wave with $S = 2$ contributions in $D^{*+}D^{*-}$ final state, respectively.

Background

Gamow态的数学基础

Energy eigenfunction with a complex eigenvalue, proposed by G.Gamow to understand alpha decay. G.Gamow , Z.Phys. 51 (1928) 204-212

However, in Hilbert space, a self-adjoint operator can only admit real eigenvalues. A rigorous treatment to Gamow state require an extension of Hilbert space to Rigged Hilbert space(RHS).

A.Bohm, M.Gadella, Dirac kets, Gamow vectors, and Gelfund Triplets, Springer Lectures Notes in Physics Vol.348, Springer, Berlin

Friedrichs-Lee 模型

A solvable model to demonstrate the properties of Gamow state.

Several variants: K. O. Friedrichs, Communications on Pure and Applied Mathematics 1, 361 (1948).

Lee model T. D. Lee, Phys. Rev. 95, 1329 (1954),

Fano model U. Fano, Phys. Rev. 124, 1866 (1961).

Anderson model P. W. Anderson, Phys. Rev. 124, 41 (1961).

Background

Free Hamiltonian H_0 with a simple continuous spectrum $R^+ \equiv [0,\infty)$, and a discrete eigenstate $|1\rangle$ with the eigenvalue $\omega_0 > 0$.

$$H_0|1\rangle = \omega_0|1\rangle, H_0|\omega\rangle = \omega|\omega\rangle$$

Then the free Hamiltonian is

$$H_0 = \omega_0|1\rangle\langle 1| + \int_0^\infty d\omega\ \omega|\omega\rangle\langle\omega|$$

Normalization :

$$\langle 1|1\rangle = 1, \langle 1|\omega\rangle = \langle\omega|1\rangle = 0, \langle\omega'|\omega\rangle = \delta(\omega' - \omega)$$

Suppose there is an interaction between the continuous and discrete parts

$$V = \lambda\int_0^\infty d\omega\ [f(\omega)|\omega\rangle\langle 1| + f^*(\omega)|1\rangle\langle\omega|]$$

Background

We can solve the eigenstate $|\Psi(x)\rangle$ of the full Hamiltonian $H = H_0 + V$, with eigenvalue x.

$$H\,|\Psi(x)\rangle = x\,|\Psi(x)\rangle$$

Since the $|1\rangle$ and $|\omega\rangle$ form a complete set, $|\Psi(x)\rangle$ could be expressed as

$$|\Psi(x)\rangle = \alpha(x)\,|1\rangle + \int_0^\infty d\omega\ \varphi(x,\omega)\,|\omega\rangle$$

Using

$$V|1\rangle = \lambda f(\omega)\,|\omega\rangle, V|\omega\rangle = \lambda f^*(\omega)\,|1\rangle$$

We have

$$(\omega_0 - x)\alpha(x) + \lambda\int_0^\infty f^*(\omega)\psi(x,\omega)d\omega = 0,$$

$$(\omega - x)\psi(x,\omega) + \lambda f(\omega)\alpha(x) = 0.$$

Background

For Eigenvalue $x > 0$

$$\psi_{\pm}(x,\omega) = -\frac{\lambda\alpha(x) f(\omega)}{\omega - x \pm i\varepsilon} + \gamma_{\pm}(\omega)\delta(\omega - x),$$

$$(\omega_0 - x)\alpha_{\pm}(x) + \lambda f^{*}(x)\gamma_{\pm}(x) - \alpha_{\pm}(x)\lambda^2 \int_0^{\infty} \frac{f(\omega) f^{*}(\omega)}{\omega - x \pm i\varepsilon} d\omega = 0.$$

Define resolvent function

$$\eta^{\pm}(x) = x - \omega_0 - \lambda^2 \int_0^{\infty} \frac{|f(\omega)|^2}{x - \omega \pm i\varepsilon} d\omega$$

Solution:

$$|\Psi(x)\rangle = |x\rangle + \lambda \frac{f^{*}(x)}{\eta^{\pm}(x)} [|1\rangle + \lambda \int_0^{\infty} \frac{f(\omega)}{x - \omega \pm i\varepsilon} |\omega\rangle d\omega]$$

Two-pole structure(双极点结构)

resolvent function

$$\eta(s) = s - \omega_0^2 - \int_{sth} ds' \frac{\rho(s')}{s - s' + i\epsilon}$$

$$\rho(s) \equiv 2\omega_0 \frac{kE_1E_2}{\sqrt{s}} \alpha(k)^2$$

双极点结构：一个裸态产生两个共振极点.

Phys.Rev.D 83 (2011) 014010

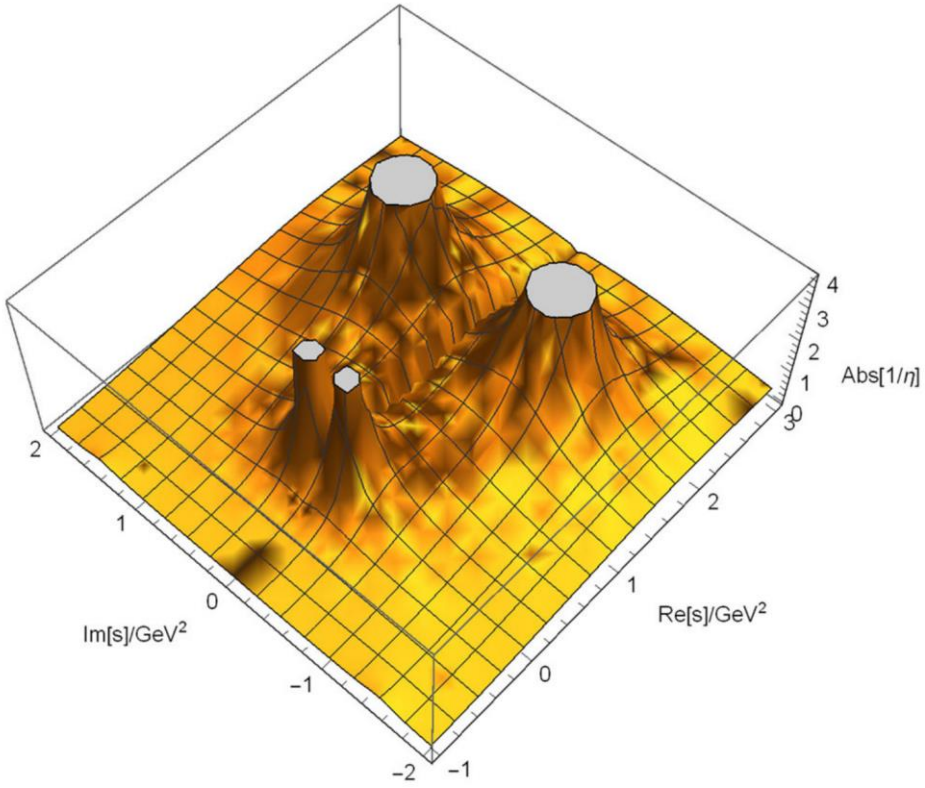
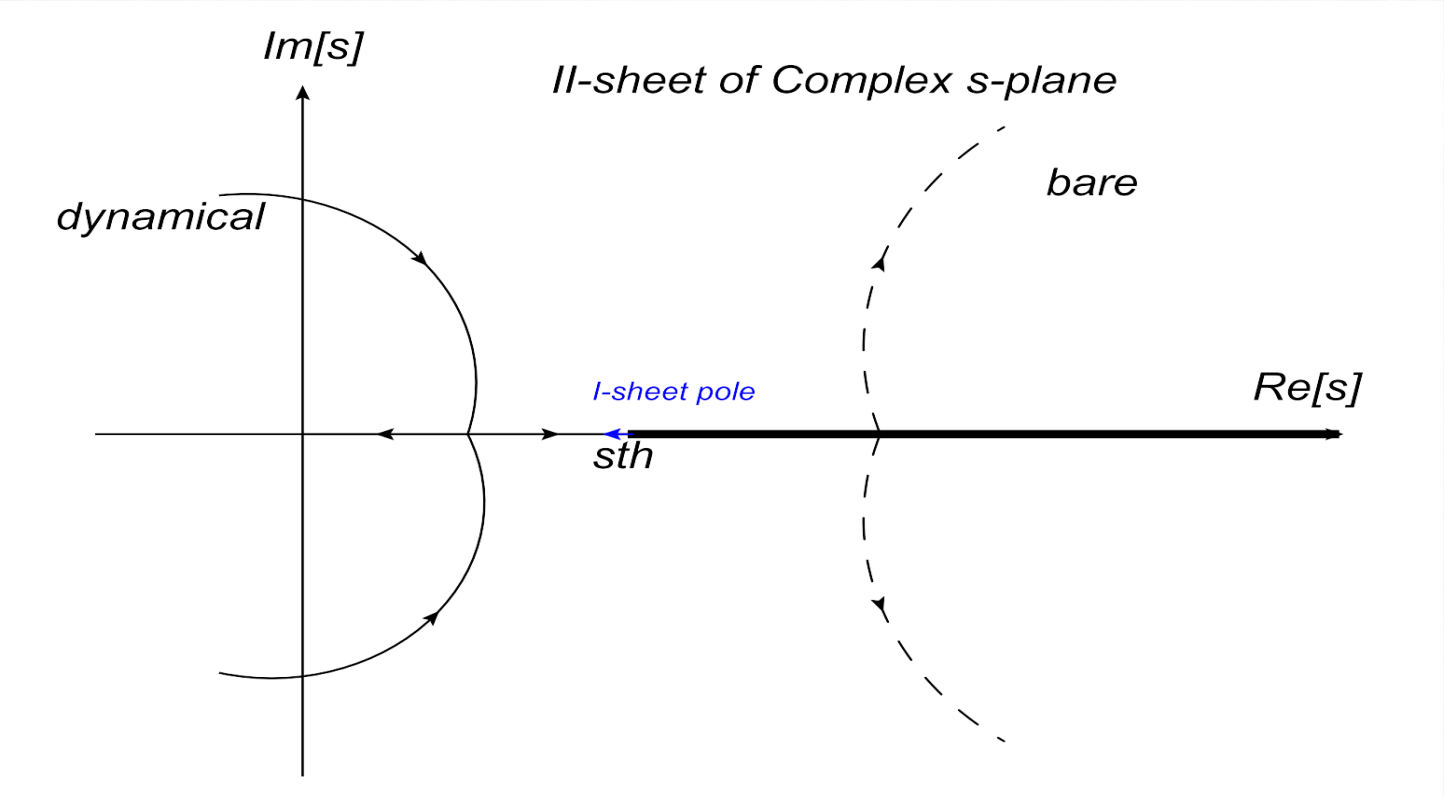


Fig. 2 $|1/\eta(s)|$ for the scalar $(u\bar{u} + d\bar{d})/\sqrt{2}$ on the complex s plane of the second Riemann sheet with $\gamma = 4.3$ GeV

Eur.Phys.J.C 80 (2020) 12, 1191
Eur.Phys.J.C 81 (2021) 6, 551

Two-pole structure(双极点结构)



The general trajectories of two pairs of poles for the two-pole structures on the second Riemann sheet of the s-plane as γ increases.

Eur.Phys.J.C 80 (2020) 12, 1191
Eur.Phys.J.C 81 (2021) 6, 551

Sum rule:

This kind of two-pole structure contribute a total phase shift of 180° in the single channel approximation

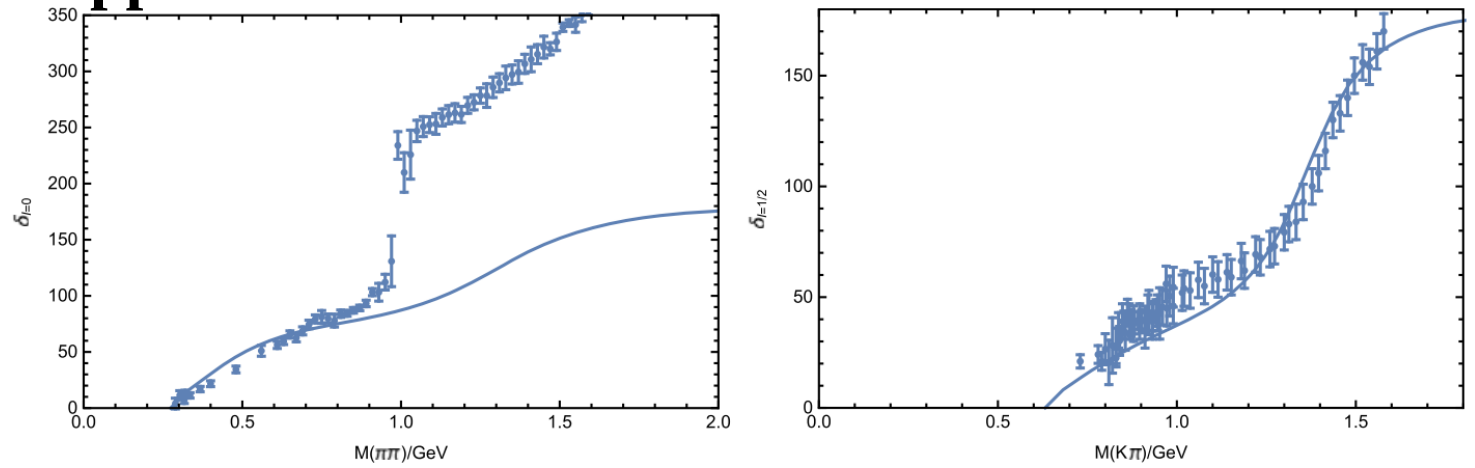


Fig. 3 Comparisons of experimental phase shifts and the theoretical calculations when $\gamma = 4.3\,\mathrm{GeV}$. The left one is that of $IJ = 00\ \pi\pi$ scattering and the right one is that of $IJ = \frac{1}{2}0\ \pi K$ scattering. The solid line is the contribution of two-pole structure

Eur.Phys.J.C 80 (2020) 12, 1191
Eur.Phys.J.C 81 (2021) 6, 551

猜测：双极点结构的普遍性

An ambitious prediciton with only a few parameters in this scheme
(1) Some Bare Masses (2) a global coupling γ

Table 1 Correspondence of the discrete states and the continuum states as the parameter $\gamma = 4.3$ GeV. The values in the fourth column are the input mass of bare states. Unit is GeV

"Discrete"	"Continuum"	GI mass	Input	Poles	Experiment states	PDG values [19]
$\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}(1^3P_0)$	$(\pi\pi)_{I=0}$	1.09	1.3	$\sqrt{s_{r1}} = 1.34 - 0.29i$	$f_0(1370)$	$1.35^{\pm0.15} - 0.2^{\pm0.05}i$
				$\sqrt{s_{r2}} = 0.39 - 0.26i$	$f_0(500)$	$0.475^{\pm0.075} - 0.275^{\pm0.075}i$
$u\bar{s}(1^3P_0)$	$(\pi K)_{I=\frac{1}{2}}$	1.23	1.42	$\sqrt{s_{r1}} = 1.41 - 0.17i$	$K_0^*(1430)$	$1.425^{\pm0.05} - 0.135^{\pm0.04}i$
				$\sqrt{s_{r2}} = 0.66 - 0.34i$	$K_0^*(700)$	$0.68^{\pm0.05} - 0.30^{\pm0.04}i$
$s\bar{s}(1^3P_0)$	$K\bar{K}$	1.35	1.68	$\sqrt{s_{r1}} = 1.71 - 0.16i$	$f_0(1710)$	$1.704^{\pm0.012} - 0.062^{\pm0.009}i$
				$\sqrt{s_b} = 0.98, \sqrt{s_v} = 0.19$	$f_0(980)$	$0.99^{\pm0.02} - 0.028^{\pm0.023}i$
$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}(1^3P_0)$	$\pi\eta$	1.09	1.3	$\sqrt{s_{r1}} = 1.26 - 0.14i$	$a_0(1450)$	$1.474^{\pm0.019} - 0.133^{\pm0.007}i$
				$\sqrt{s_{r2}} = 0.70 - 0.42i$	$a_0(980)$	$0.98^{\pm0.02} - 0.038^{\pm0.012}i$
$c\bar{u}(1^3P_0)$	$D\pi$	2.4	2.4	$\sqrt{s_{r1}} = 2.58 - 0.24i$	$D_0^*(2300)$	$2.30^{\pm0.019} - 0.137^{\pm0.02}i$
				$\sqrt{s_{r2}} = 2.08 - 0.10i$		
$c\bar{s}(1^3P_0)$	DK	2.48	2.48	$\sqrt{s_{r1}} = 2.80 - 0.23i$		
				$\sqrt{s_b} = 2.24, \sqrt{s_v} = 1.8$	$D_{s0}^*(2317)$	$2.317^{\pm0.0005} - 0.0038^{\pm0.0038}i$
$b\bar{u}(1^3P_0)$	$\bar{B}\pi$	5.76	5.76	$\sqrt{s_{r1}} = 6.01 - 0.21i$		
				$\sqrt{s_{r2}} = 5.56 - 0.07i$		
$b\bar{s}(1^3P_0)$	$\bar{B}K$	5.83	5.83	$\sqrt{s_{r1}} = 6.23 - 0.17i$		
				$\sqrt{s_b} = 5.66, \sqrt{s_v} = 5.3$		
$c\bar{c}(2^3P_1)$	$D\bar{D}^*$	3.95	3.95	$\sqrt{s_{r1}} = 4.01 - 0.049i$	$X(3940)$	
				$\sqrt{s_b} = 3.785$	$X(3872)$	$3.871^{\pm0.0005}$

Eur.Phys.J.C 80 (2020) 12, 1191
Eur.Phys.J.C 81 (2021) 6, 551

猜测：双极点结构的普遍性

An ambitious prediciton with only a few parameters in this scheme
(1) Some Bare Masses (2) a global coupling γ

$$results\ of\ \gamma = 3.0\text{GeV}$$

$c\bar{u}(1^3P_0)$	$\sqrt{s_{r1}} = 2.39 - 0.18i$	$\sqrt{s_{r2}} = 2.21 - 0.28i$	$D_0^*(2300)$
$c\bar{s}(1^3P_0)$	$\sqrt{s_{r1}} = 2.68 - 0.26i$	$\sqrt{s_b} = 2.32, \sqrt{s_v} = 1.9$	$D_{s0}^*(2317)$
$b\bar{u}(1^3P_0)$	$\sqrt{s_{r1}} = 5.85 - 0.26i$	$\sqrt{s_{r2}} = 5.62 - 0.13i$	
$b\bar{s}(1^3P_0)$	$\sqrt{s_{r1}} = 6.11 - 0.22i$	$\sqrt{s_b} = 5.72, \sqrt{s_v} = 5.4$	
$c\bar{c}(2^3P_1)$	$\sqrt{s_r} = 3.99 - 0.05i$	$\sqrt{s_b} = 3.84$	$X(3872)$

Notice that it is a prediction with only one parameter γ.

Eur.Phys.J.C 80 (2020) 12, 1191
Eur.Phys.J.C 81 (2021) 6, 551

多粒子多道的Friedrichs-Lee-QPC框架

Hamiltonian

$$H_0 = \sum_{\rho=1}^{D} M_\rho |\rho\rangle\langle\rho| + \sum_{i=1}^{C} \int_{a_i}^{\infty} \mathrm{d}\omega\, E|E;i\rangle\langle E;i|,$$

$$V = \sum_{\rho=1}^{D}\sum_{i=1}^{C} \int_{a_i}^{\infty} \mathrm{d}E[f^*_{\rho i}(E)|\rho\rangle\langle E;i| + f_{\rho i}(E)|E;i\rangle\langle\rho|].$$

Scattering matrix

$$S_{i,j} = \delta_{ij} - 2\pi i \sum f^*_{\rho j}(E)[\eta^{-1}]_{\rho\lambda} f_{\lambda i}(E)$$

$$[\eta]_{\rho\lambda} \equiv (E - M_\rho)\delta_{\rho\lambda} - \sum_{i=1}^{C} \int_{a_i}^{\infty} \mathrm{d}E' \frac{f^*_{\rho i}(E') f_{\lambda i}(E')}{(E - E')}$$

ψ coupling to hadron pairs

ψ coupling to e^+e^-

QPC model

$$\mathcal{M}(\psi_\rho \to e^+e^-) = \frac{e g_{e\rho}}{M_\rho^2}(\bar{u}\gamma_\mu v)\epsilon^\mu,$$

Cross section of $e^+e^- \to$ *hadron pairs*

$$\sigma_i(E) = \frac{32\pi^5\alpha}{E^5} |\sum_{\rho\lambda} g_{e\rho}[\eta^{-1}]_{\rho\lambda} f_{\lambda i}|^2$$

多粒子多道的Friedrichs-Lee-QPC框架拟合

4 channels

$$e^+e^- \to D\bar{D}, D\bar{D}^*, D^*\bar{D}^*, D\bar{D}\pi$$

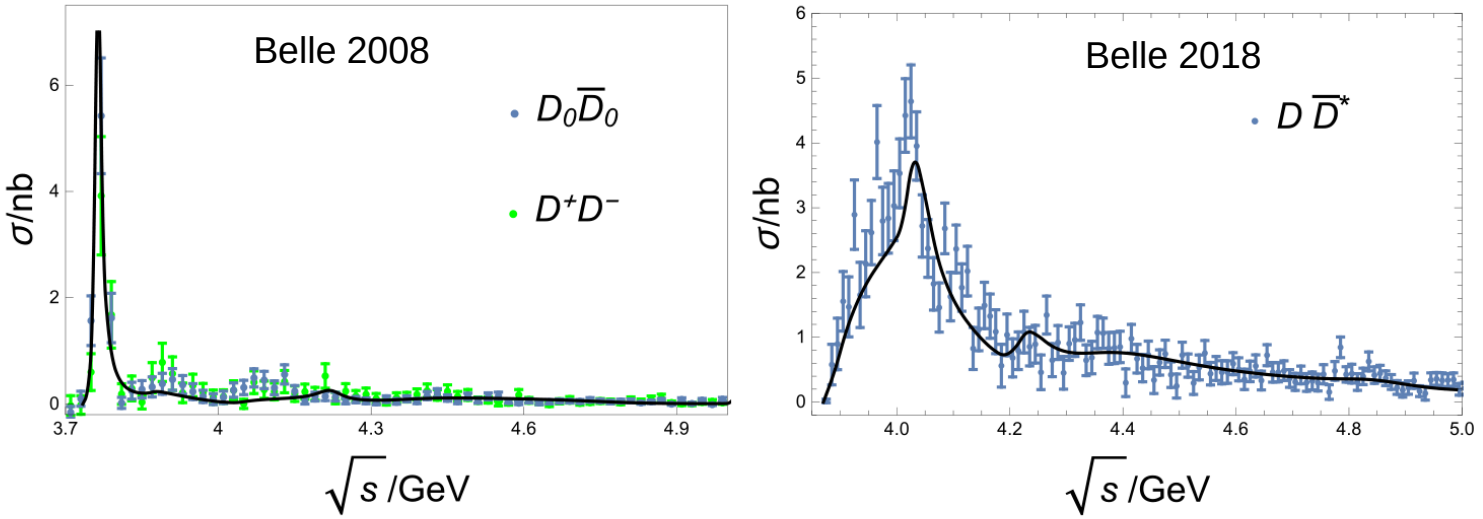
4 bare discrete states are supposed

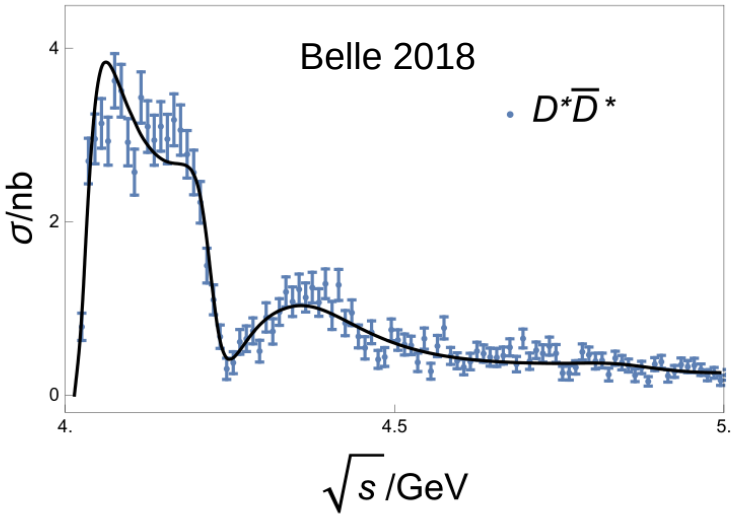
$$\boldsymbol{\psi(1D), \psi(3S), \psi(2D), \psi(4S)}$$

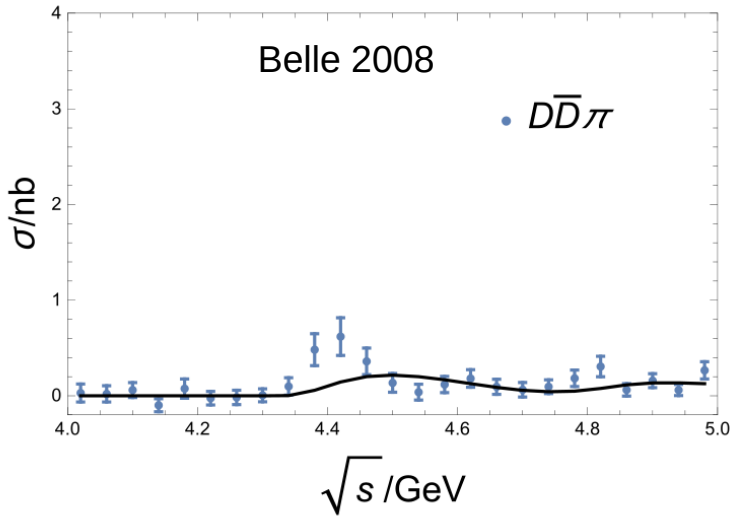
15 parameters

	Background	$\psi(1D)$	$\psi(3S)$	$\psi(2D)$	$\psi(4S)$
Bare mass	1.83	4.09	4.36	4.62	5.35
$g_{e\rho}$	26.4	1.82	5.32	-3.68	-6.12
$\phi_{e\rho}/^\circ$	0(fixed)	35.2	-31.6	64.6	129.3
γ	4.48				

Combined fit of $e^+e^- \to D\bar{D}, D\bar{D}^*, D^*\bar{D}^*, D\bar{D}\pi$







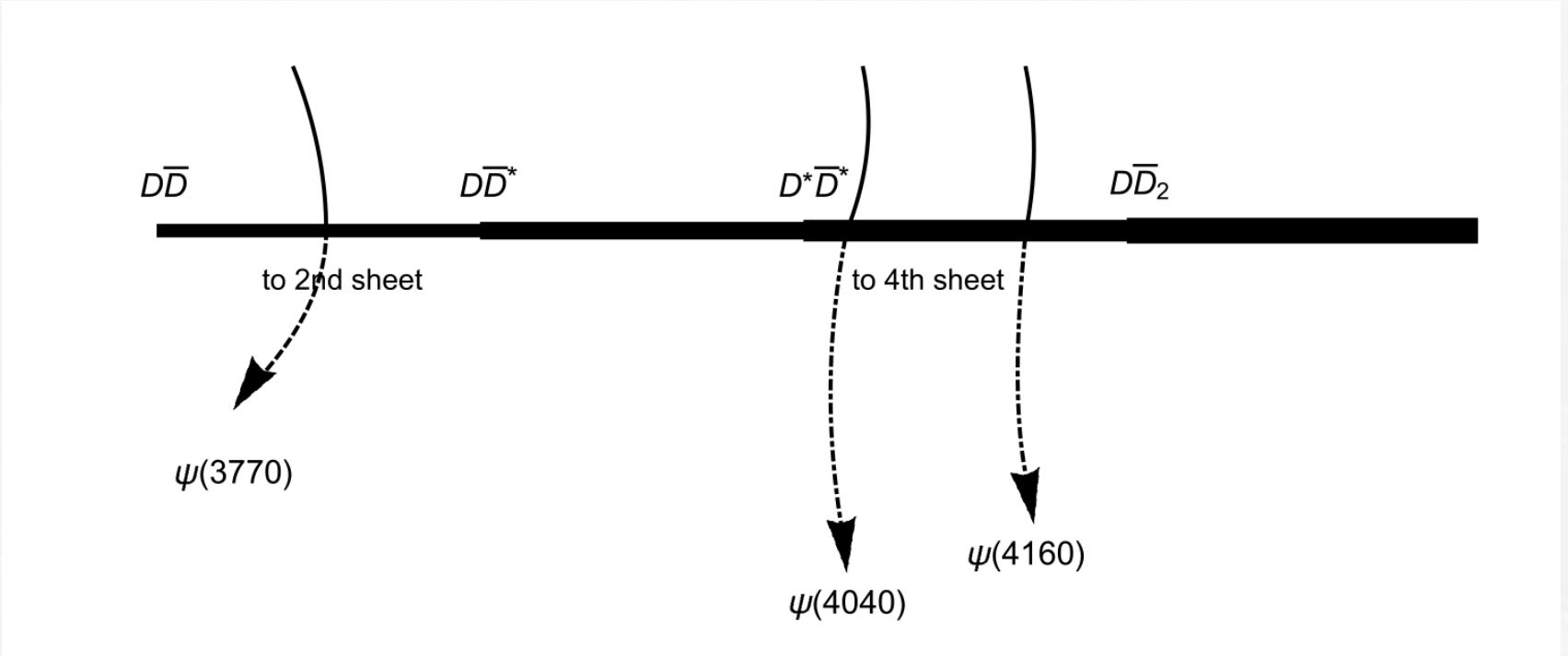
$$\frac{\chi^2}{d.o.f} = \frac{379}{293-15} \sim 1.36$$

Extract the nearby poles

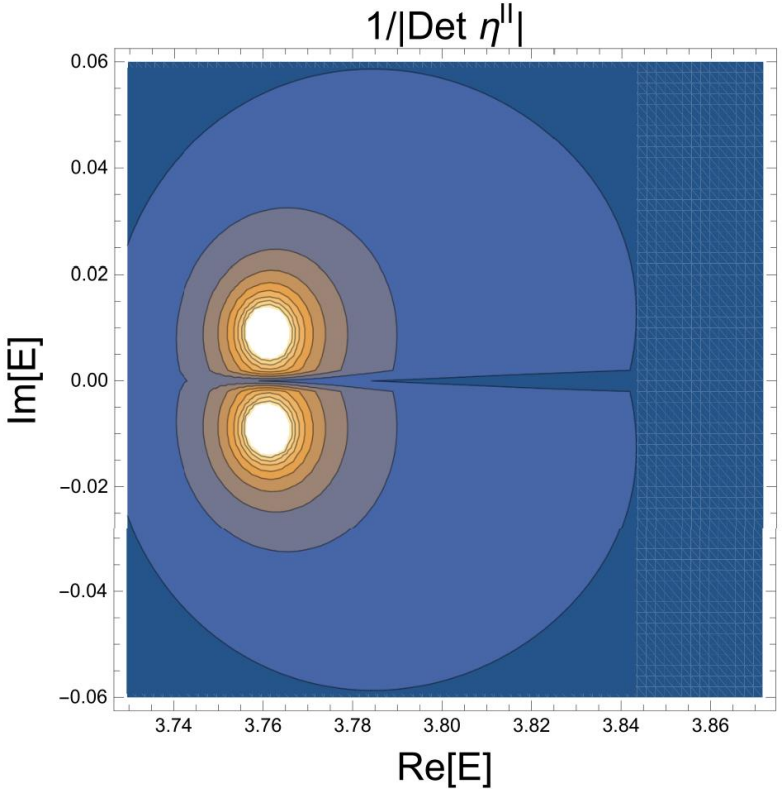
4 channels

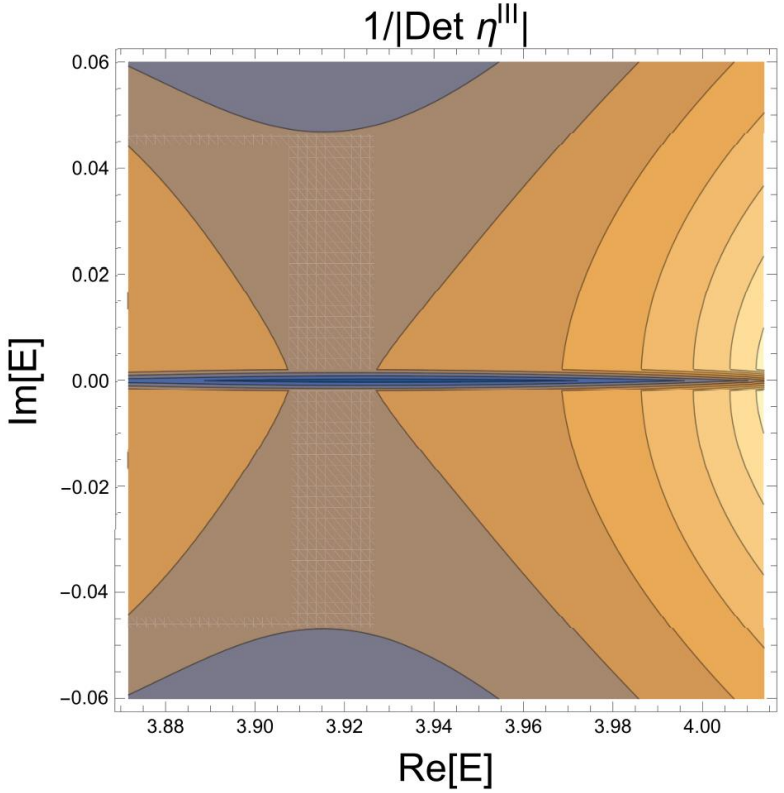
16 Riemann sheets

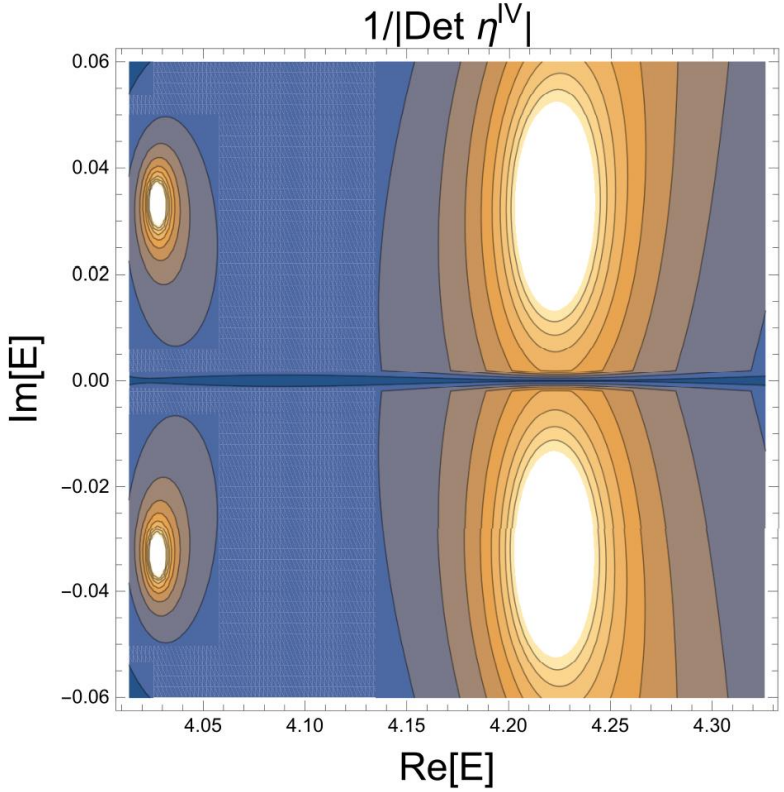
Extract the nearby poles by find the solution of $det\left[\eta^{(N-sheet)}\right] = 0$



Contour plots of $1/det[\eta]$ on unphysical Riemann sheets







$z_0^{II} = 3.762 - \frac{0.020}{2} i$ GeV

$\psi(3770)$

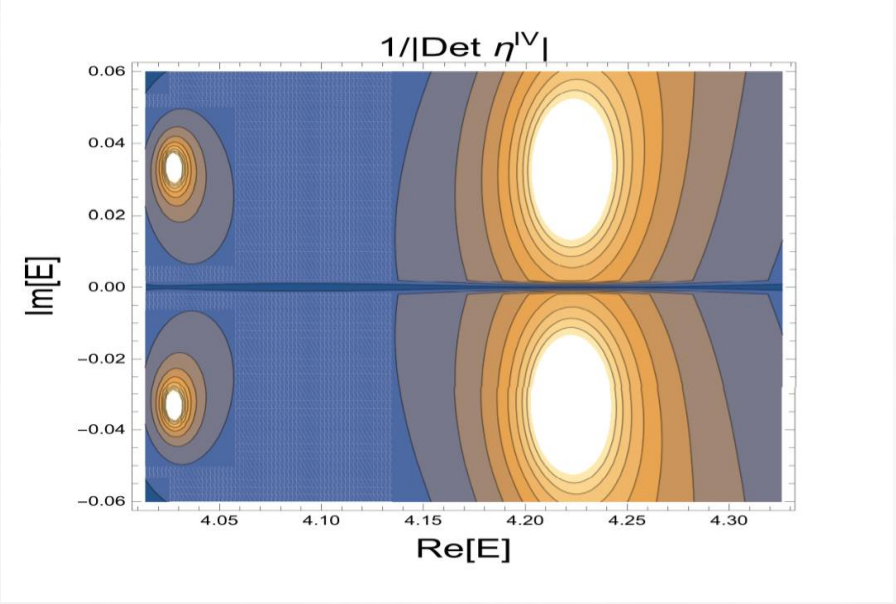
A shadow pole above $D^*\bar{D}^*$, $\psi(4008)???$

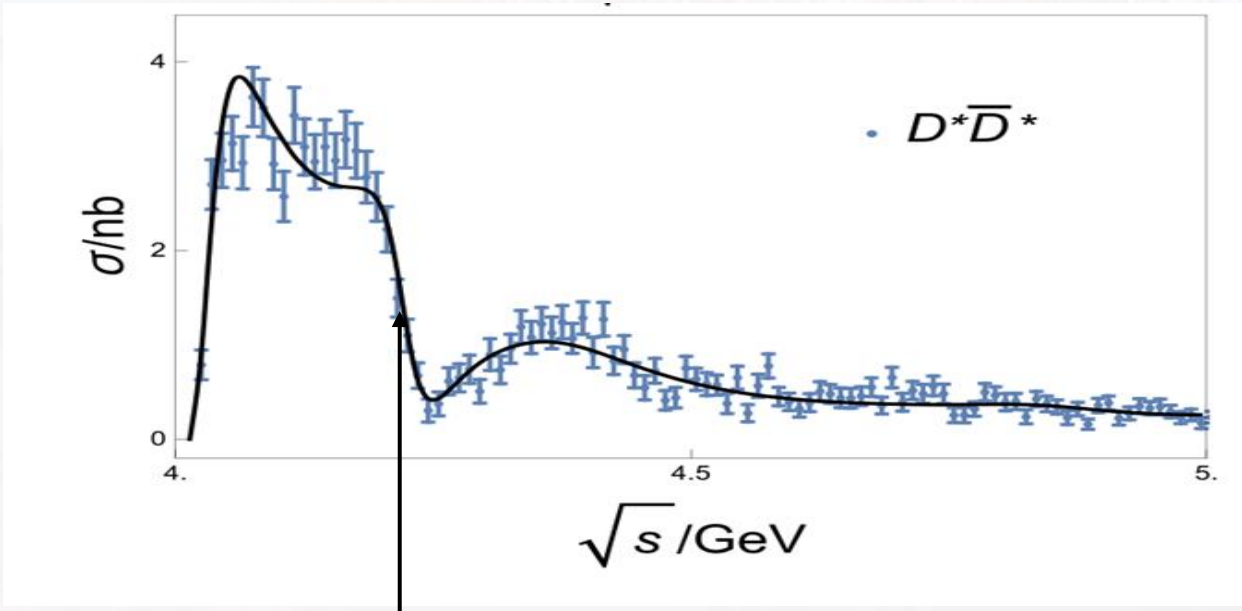
$z_{0,1}^{IV} = 4.028 - \frac{0.068}{2} i$ GeV $\psi(4040)$

$z_{0,2}^{IV} = 4.222 - \frac{0.064}{2} i$ GeV

$\psi(4160)/\psi(4230)$

Contour plots of $1/det[\eta]$ on 4-th sheet



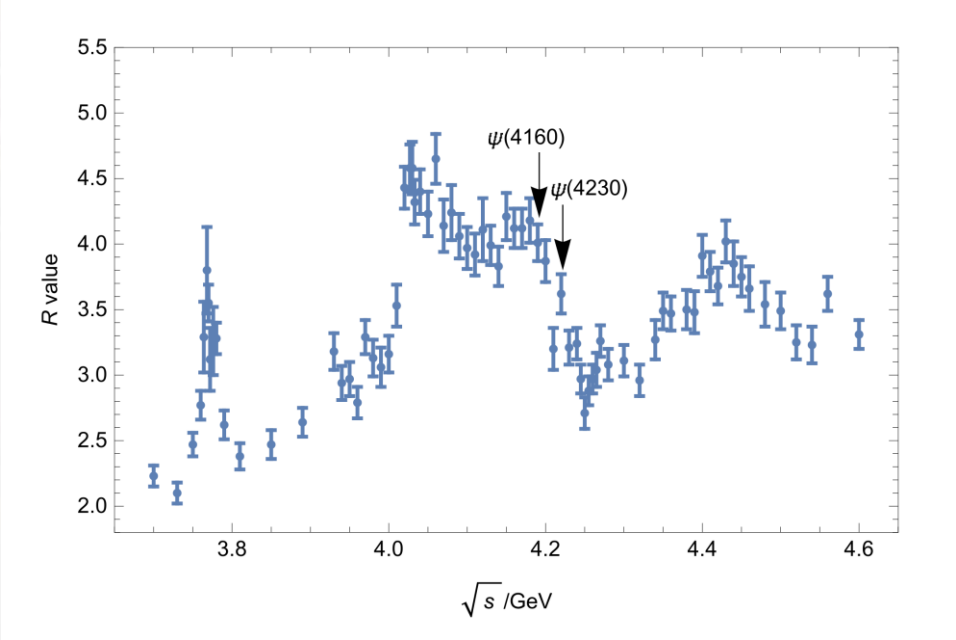


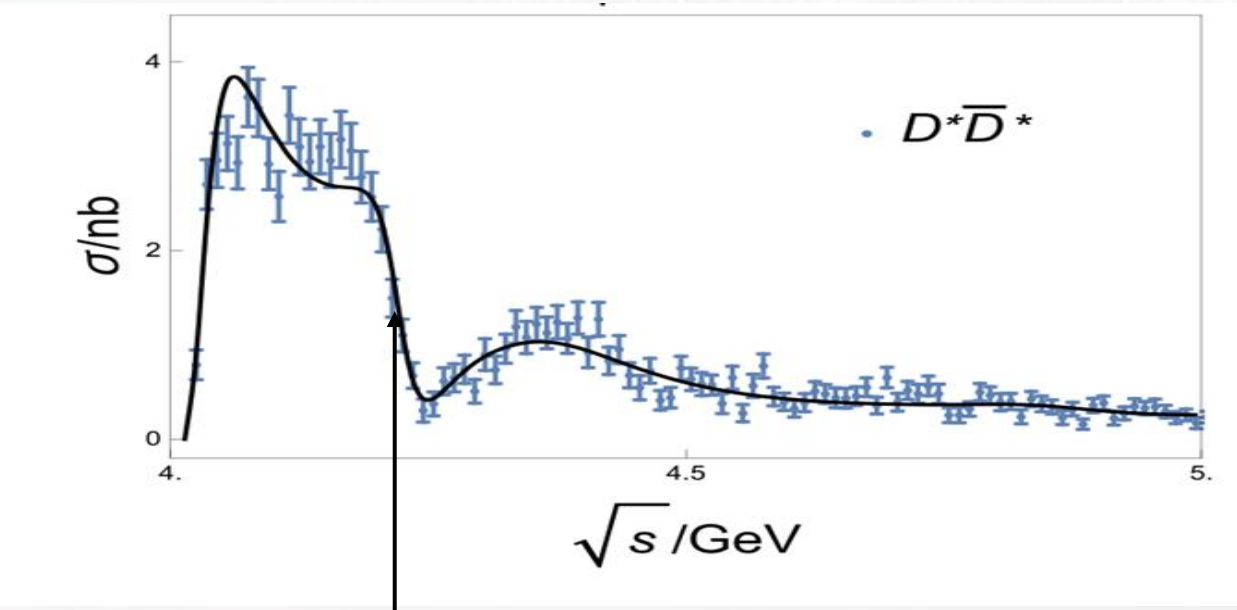
$$z_{0,2}^{IV} = 4.222 - \frac{0.064}{2} i \text{ GeV}$$

$\psi(4160)/\psi(4230)$

at the half hillside of sharp declined cross section, neither at the shoulder nor at the dip

Contour plots of $1/det[\eta]$ on 4-th sheet





$$z_{0,2}^{IV} = 4.222 - \frac{0.064}{2} i \text{ GeV}$$

$$\psi(4160)/\psi(4230)$$

at the half hillside of sharp declined cross section, neither at the shoulder nor at the dip

If $\psi(4160)$ and $\psi(4230)$ are the same $\psi(2^3D_1)$ state

benefits

- Quark model expectation
- Explanation of the mutually exclusive decay modes of $\psi(4160)$ and $\psi(4230)$
- Reliability of coupled channel method analysis

Challenges faced in verifying this conjecture

- A fit with more bare discrete states and more decay model

$$D\bar{D}, D\bar{D}^*, D^*\bar{D}^*, D\bar{D}_1, D\bar{D}_2^*, D_s^+D_s^-, D_s^+D_s^{*-}, D_s^{*+}D_s^{*-}, \cdots\cdots$$

- More accurate experiment data

Expected experiment explorations

More precise $J/\psi\eta$ data

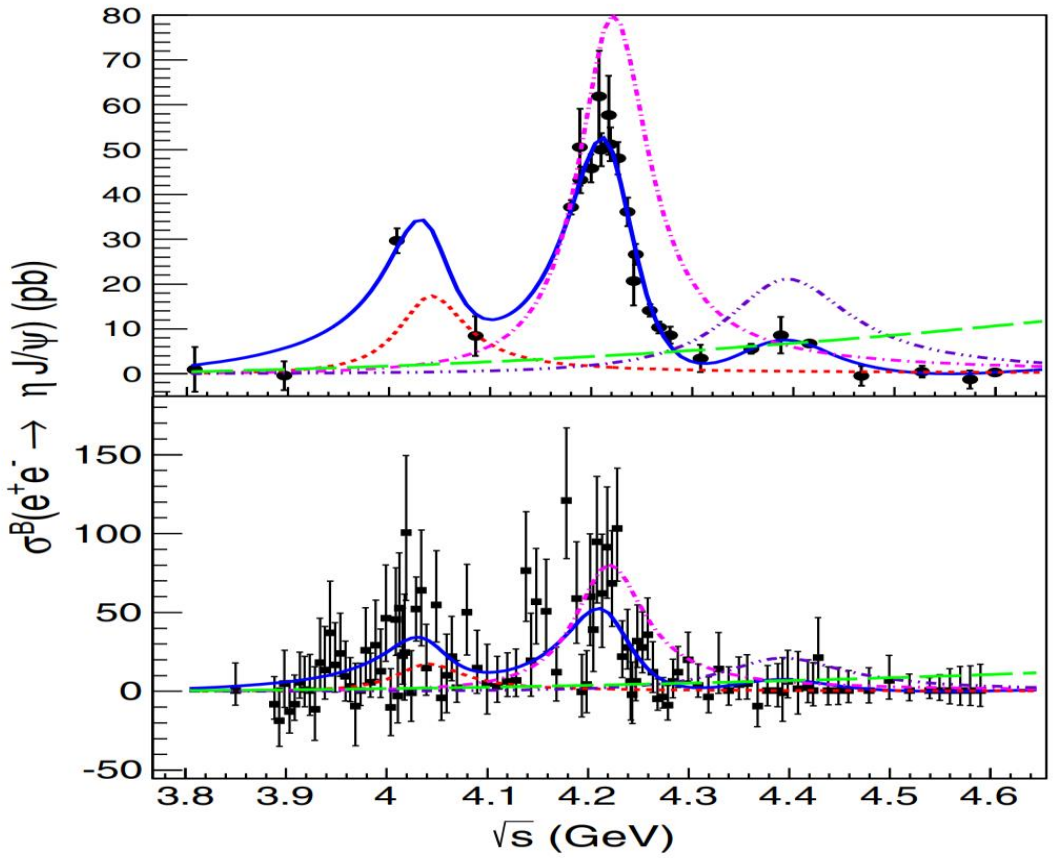


FIG. 2. Top: Cross section and fits of $e^+e^- \to \eta J/\psi$ for XYZ data. Bottom: Same for the scan data. Dots with error bars are data. The solid (blue) curves represent the fit results of the

BES,
PRD 102, 031101

Expected experiment explorations

More precise $D_s^{(*)+} D_s^{(*)-}$ data

Belle, PRD 83, 011101(R) (2011)

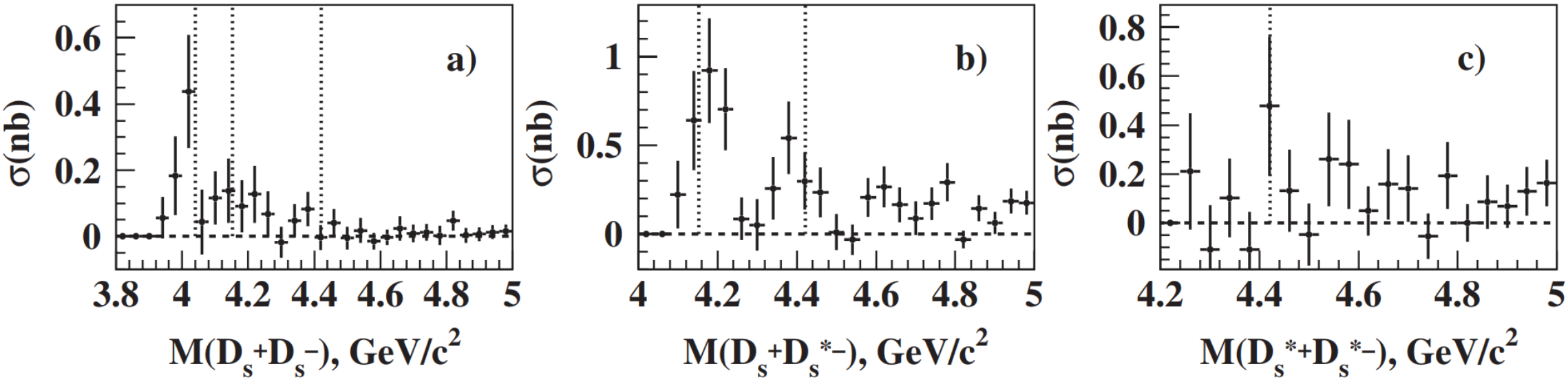
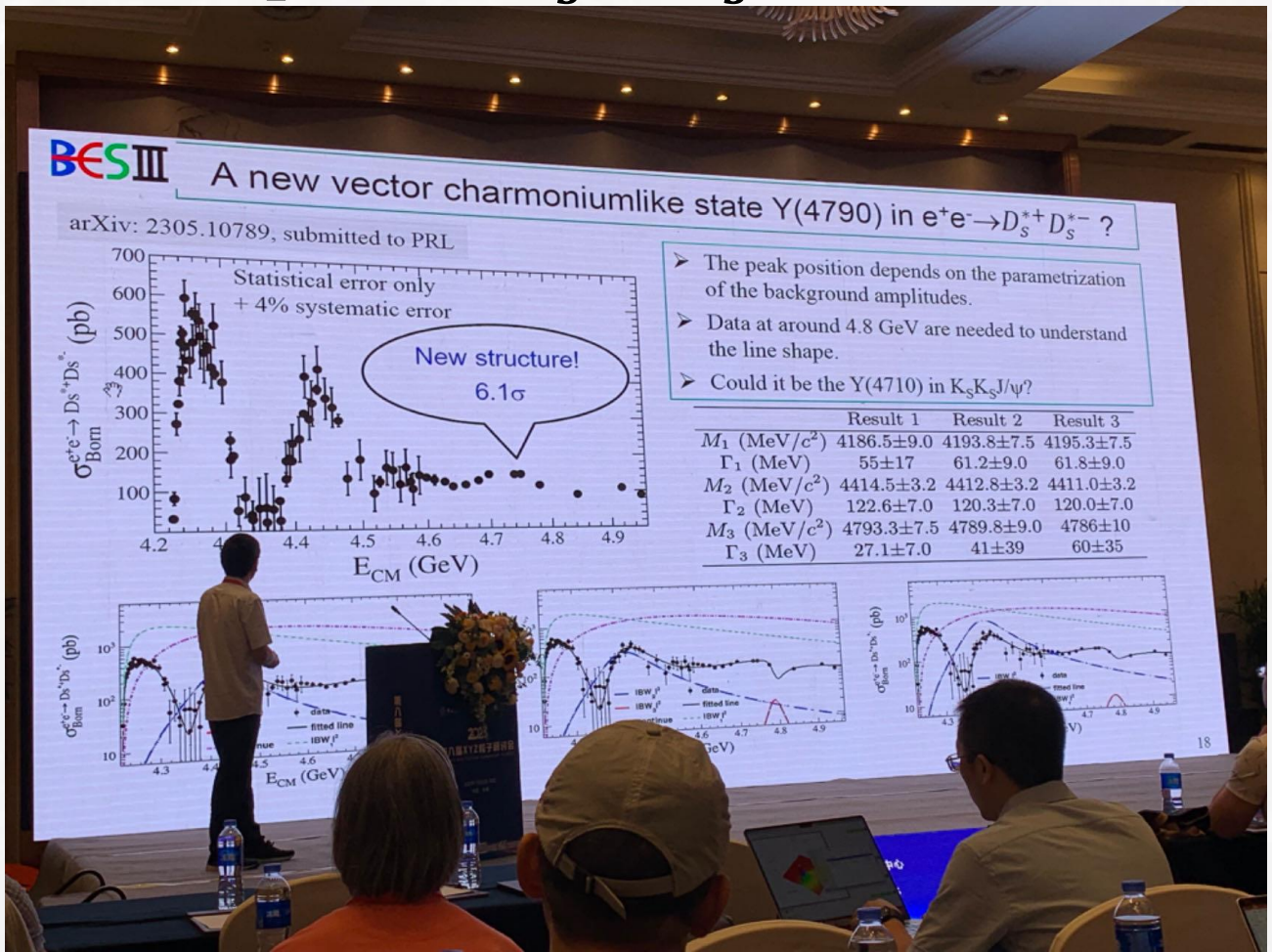


FIG. 4. The cross section averaged over the bin width for (a) the $e^+e^- \to D_s^+ D_s^-$ process, (b) the $e^+e^- \to D_s^+ D_s^{*-}$ + c.c. process, and (c) the $e^+e^- \to D_s^{*+} D_s^{*-}$ process. Error bars show statistical uncertainties only. There is a common systematic uncertainty for all measurements, 11% for $D_s^+ D_s^-$, 17% for $D_s^+ D_s^{*-}$, and 31% for $D_s^{*+} D_s^{*-}$. This uncertainty is described in the text. The dotted lines show masses of the $\psi(4040)$, $\psi(4160)$, and $\psi(4415)$ states [18].

Expected experiment explorations

More precise $D_s^{(*)+}D_s^{(*)-}$ data



Prof. Yuan's talk 2023.7.26

BES, arxiv: 2305.10789

Summary

- $\psi(4160)$ and $\psi(4230)$ might be the same $\psi(c\bar{c}\,2^3D_1)$ state.
- More experiments such as $J/\psi\eta$ and $D_s^{(*)}\bar{D}_s^{(*)}$ are required to verify or falsify the conjecture.
- We present a coupled-channel scheme with unitarity and analyticity, which could be used to study several resonances and several decay modes simultaneously.
- This scheme could be rewritten into the form used in the experimental analysis.

Thanks for your patience!