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The study of the Sivers asymmetry and Collins asymmetry in the SIDIS process

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Based on Shuailiang Yang, Jianxi Song, Xiaoyu Wang, De-Min Li and Zhun Lu, arXiv:2303.09015.

Shuailiang Yang, Qi Xu, Yateng Zhang and Xiaoyu Wang , Symmetry 2023, 15, 841.

OUTLINE

- Introduction
- Framework
- The study of the Sivers asymmetry in the SIDIS process
- The study of the Collins asymmetry in the SIDIS process
- Summary and Outlook

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Quantum Chromodynamics(QCD)

The Lagrangian of QCD

$$\mathcal{L} = \sum_q \bar{\psi}_{q,a} \left(i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t^C_{ab} A^C_\mu - m_q \delta_{ab} \right) \psi_{q,b} - \frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu}$$

$\psi_{q,a}$ $\longrightarrow$ Quark spinor (The q is the quark flavor and the a is the quark color)

A^C_μ $\longrightarrow$ Gluon field (C shows the type of the gluon)

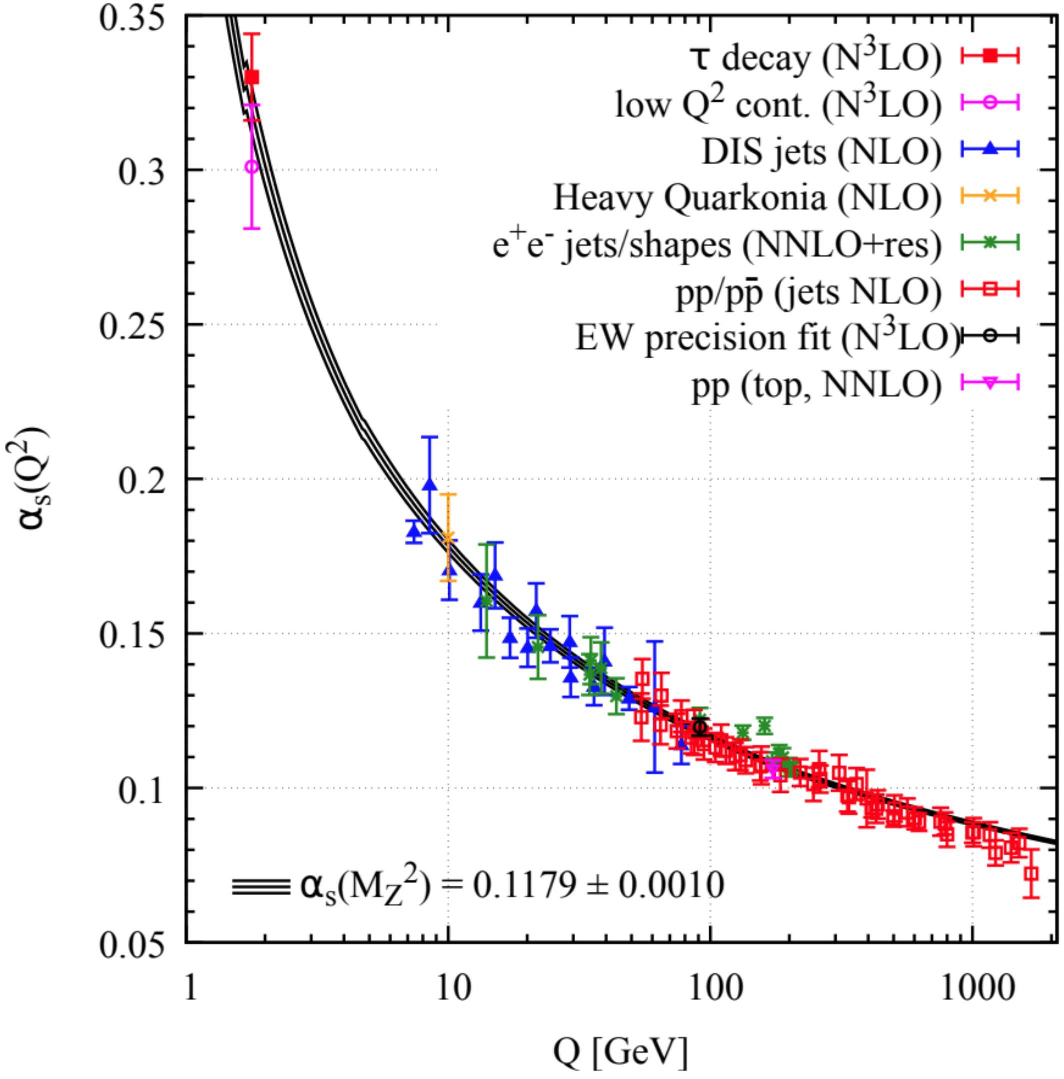
$F^A_{\mu\nu}$ $\longrightarrow$ The gauge field tensor

g_s $\longrightarrow$ Coupling strength

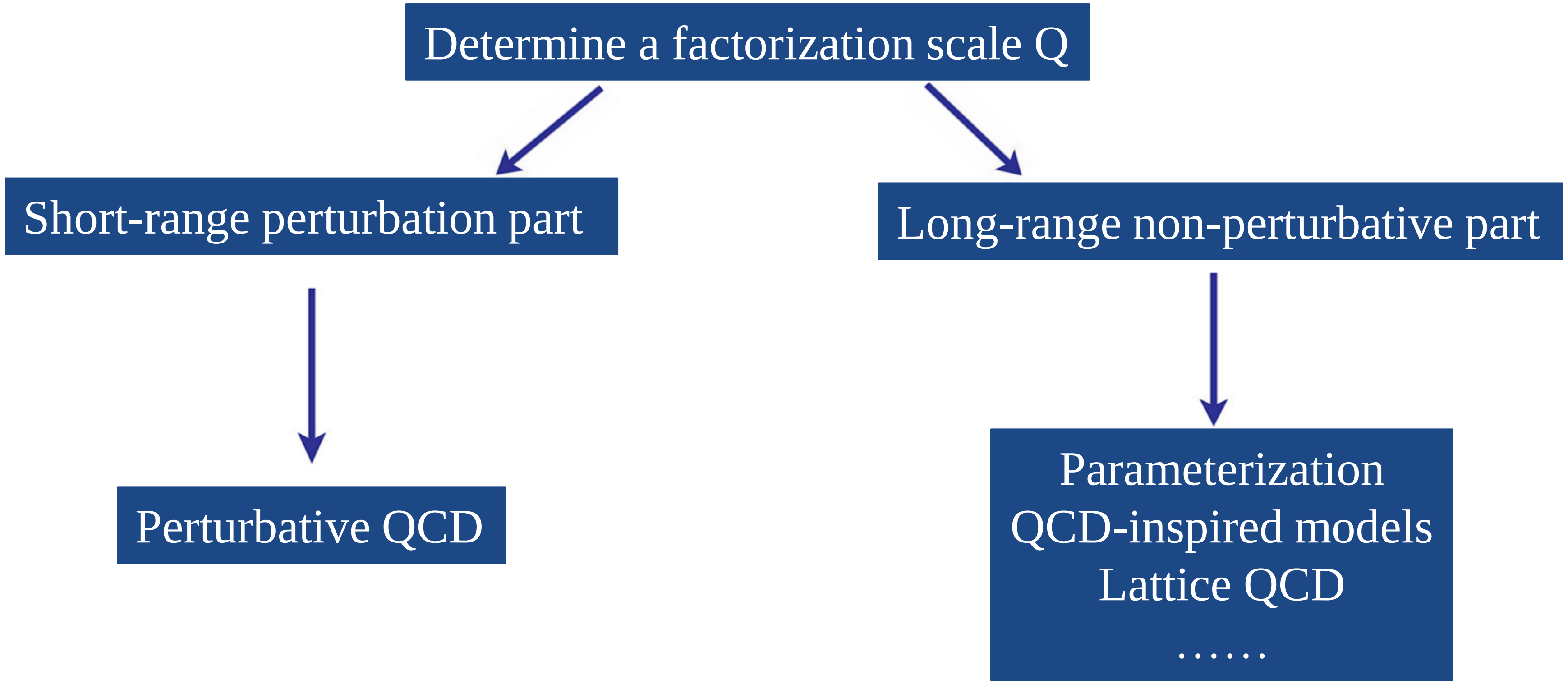
Asymptotic freedom

Run coupling constant $\alpha_s\left(Q^2\right)=\frac{g_s^2}{4\pi}$

$$\alpha_s\left(Q^2\right)=\frac{4\pi}{\left(11-\frac{2}{3}n_f\right)\ln\left(Q^2/\Lambda_{QCD}^2\right)}$$



Factorization formalism



- Collinear factorization formalism

Factorization formalism

- Transverse Momentum Dependent(TMD) factorization formalism.

TMDs		Quark polarization		
		Unpolarized (U)	Longitudinally polarized (L)	Transversely polarized (T)
Nucleon polarization	U	f_1 Unpolarized		$h_1^{\perp}$ Boer–Mulders
	L		g_{1L} Helicity	$h_{1L}^{\perp}$ Longi-transversity
	T	$f_{1T}^{\perp}$ Sivers	g_{1T} Trans-helicity	h_1 Transversity $h_{1T}^{\perp}$ Pretzelosity

Nucleon spin   Quark spin

TMD PDFs at leading-twist

What have we done?

➢ The study of Sivers distribution function

- There is much unknown information about the sea quark Sivers distribution function.
- The Electron Ion Collider provides a good platform for us to study of the sea quark Sivers distribution function.

➢ The study of transversity distribution function

- The current information on the sea quark transversity distribution function is almost blank.
- The information of the sea quark transversity distribution function can be obtained by studying the Collins asymmetry on the Electron Ion Collider.

What have we done?

- We apply the TMD factorization formalism and the TMD evolution effects to predicted the Sivers asymmetry in charged K produced and Λ hyperon produced in SIDIS process at the kinematical configurations of EIC and EicC.

- We apply the TMD factorization formalism to predicted the Collins asymmetry in Λ hyperon produced in SIDIS process at the kinematical configurations of EIC and EicC.

OUTLINE

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- Introduction
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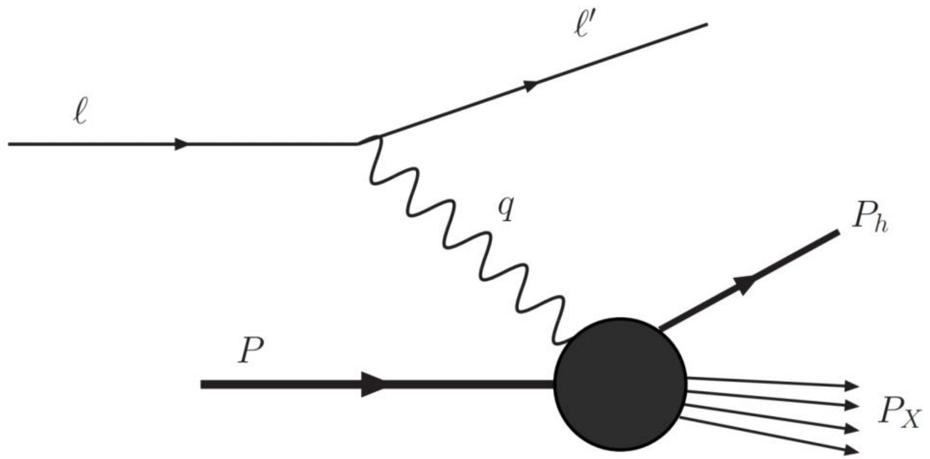
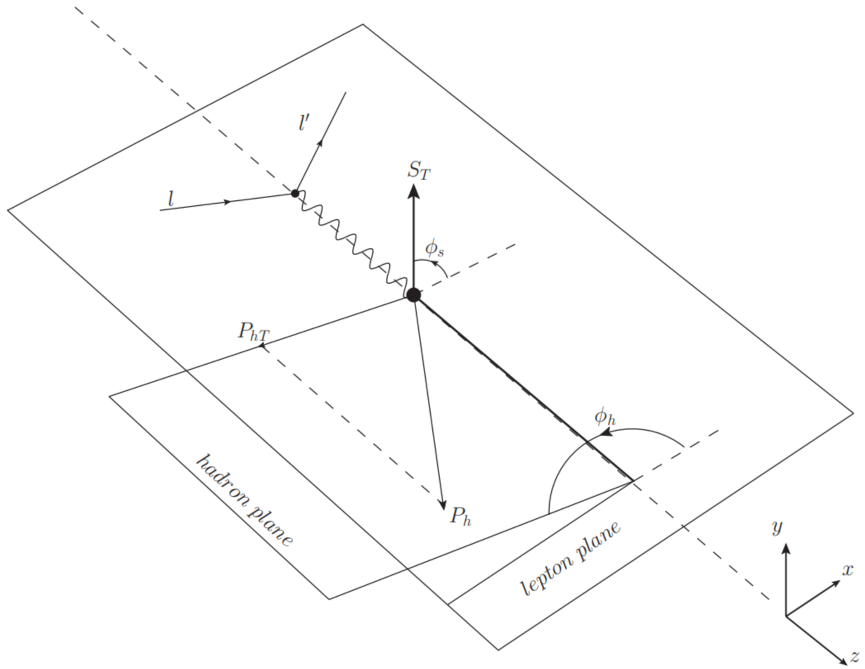
Semi-inclusive deep inelastic scattering(SIDIS)

SIDIS Process

$$l(\ell) + N(P) \to l'\,(\ell') + h\,(P_h) + X\,(P_X)$$



The SIDIS process



The reference frame in SIDIS process

$$s = (P+\ell)^2\,, \quad x = \frac{Q^2}{2P\cdot q}\,, \quad y = \frac{P\cdot q}{P\cdot \ell}\,, \quad z = \frac{P\cdot P_h}{P\cdot q}$$

Semi-inclusive deep inelastic scattering(SIDIS)

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Differential Cross Section in SIDIS Process

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,dz\,dP_T^2\,d\phi_h\,d\psi} \\
&= \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right) \\
&\times\Big\{F_{UU,T}+\epsilon F_{UU,L}+\sqrt{2\epsilon(1+\epsilon)}F_{UU}^{\cos\phi_h}\cos\phi_h+\epsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h+\lambda_e\sqrt{2\epsilon(1-\epsilon)}F_{LU}^{\sin\phi_h}\sin\phi_h \\
&+S_L\Big[\sqrt{2\epsilon(1+\epsilon)}F_{UL}^{\sin\phi_h}\sin\phi_h+\epsilon F_{UL}^{\sin 2\phi_h}\sin 2\phi_h\Big]+\lambda_e S_L\Big[\sqrt{1-\epsilon^2}F_{LL}+\sqrt{2\epsilon(1-\epsilon)}F_{LL}^{\cos\phi_h}\cos\phi_h\Big] \\
&+S_T\Big[\big(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\epsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\big)\sin(\phi_h-\phi_S)+\epsilon F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S)+\epsilon F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S) \\
&\qquad+\sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin\phi_S}\sin\phi_S+\sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin(2\phi_h-\phi_S)}\sin(2\phi_h-\phi_S)\Big] \\
&+\lambda_e S_T\Big[\sqrt{1-\epsilon^2}F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S) \\
&\qquad+\sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos\phi_S}\cos\phi_S+\sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos(2\phi_h-\phi_S)}\cos(2\phi_h-\phi_S)\Big]\Big\}
\end{aligned}
$$

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OUTLINE

Sivers distribution function

Sivers distribution function

- It denotes the asymmetric distribution of unpolarized quarks inside a transversely polarized nucleon.
- T-odd.
- QCD predicted that the Sivers function has opposite sign between SIDIS and Drell-Yan processes.

How to access Sivers function

- The transverse single spin asymmetry can be utilized to extract the information of the Sivers function.
- The QCD-inspired models (the spectator model, the light-cone quark model) can be used to study the Sivers function

Definition of Sivers asymmetry

- The 5-fold differential cross section with a transversely polarized target has the following general form

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{hT}^2} = \frac{\alpha_{em}^2(Q)}{Q^2}\frac{y}{2(1-\varepsilon)}\left\{F_{UU,T} + |S_\perp| \sin\left(\phi_h - \phi_S\right) F_{UT,T}^{\sin(\phi_h - \phi_S)} + \ldots\right\}$$

- The Sivers asymmetry can be written in terms of the structure functions

$$A_{UT}^{\sin(\phi_h-\phi_s)} \equiv \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{\sigma_0\left(x,y,Q^2\right) F_{UT}^{\sin(\phi_h-\phi_s)}}{\sigma_0\left(x,y,Q^2\right) F_{UU}}$$

$$F_{UU,T} = \mathcal{C}\left[f_1 D_1\right]$$

$$F_{UT,T}^{\sin(\phi_h-\phi_s)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^\perp D_1\right]$$

$$\mathcal{C}\left[\omega f D\right] = x\sum_q e_q^2 \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{hT}/z\right)\omega(\boldsymbol{p}_T, \boldsymbol{k}_T) f^q(x, p_T^2)\, D^q(z, k_T^2)$$

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The structure functions

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➢ Unpolarized structure function

$$
\begin{aligned}
F_{UU}\left(Q;P_{hT}\right) &= \mathcal{C}\left[f_1 D_1\right] \\
&= x\sum_q e_q^2 \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{hT}/z\right) f_1^q\left(x,p_T^2\right) D_1^q\left(z,k_T^2\right) \\
&= \frac{x}{z^2}\sum_q e_q^2 \int d^2\boldsymbol{p}_T d^2\boldsymbol{K}_T \delta^{(2)}\left(\boldsymbol{p}_T + \boldsymbol{K}_T/z - \boldsymbol{P}_{hT}/z\right) f_1^q\left(x,p_T^2\right) D_1^q\left(z,\frac{K_T^2}{z^2}\right) \\
&= \frac{x}{z^2}\sum_q e_q^2 \int d^2\boldsymbol{p}_T d^2\boldsymbol{K}_T \int \frac{d^2b}{(2\pi)^2} e^{-i(\boldsymbol{p}_T + \boldsymbol{K}_T/z - \boldsymbol{P}_{hT}/z)\cdot\boldsymbol{b}} f_1^q\left(x,p_T^2\right) D_1^q\left(z,\frac{K_T^2}{z^2}\right) \\
&= \frac{x}{z^2}\sum_q e_q^2 \int \frac{d^2b}{(2\pi)^2} e^{i\boldsymbol{P}_{hT}\cdot\boldsymbol{b}/z} \tilde{f}_1^{q/p}\left(x,b\right) \tilde{D}_1^{h/q}\left(z,b\right)
\end{aligned}
$$

$$
\int d^2\boldsymbol{p}_T e^{-i\boldsymbol{p}_T\cdot\boldsymbol{b}} f_1^q(x,p_T^2) = \tilde{f}_1^{q/p}(x,b)
$$

$$
\int d^2\boldsymbol{K}_T e^{-i\boldsymbol{K}_T/z\cdot\boldsymbol{b}} D_1^q\left(z,\frac{K_T^2}{z^2}\right) = \tilde{D}_1^{h/q}\left(z,b\right)
$$

The structure functions

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➢ **Sivers structure function**

$$F_{UT,T}^{\sin(\phi_h-\phi_S)}(Q;P_{hT}) = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}f_{1T}^{\perp}D_1\right]$$

$$= x\sum_q e_q^2\int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\delta^{(2)}\left(\boldsymbol{p}_T-\boldsymbol{k}_T-\boldsymbol{P}_{hT}/z\right)\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}f_{1T}^{\perp}\left(x,p_T^2\right)D_1^q\left(z,k_T^2\right)\right]$$

$$= \frac{x}{z^2}\sum_a e_q^2\int d^2\boldsymbol{p}_T d^2\boldsymbol{K}_T\delta^{(2)}\left(\boldsymbol{p}_T+\boldsymbol{K}_T/z-\boldsymbol{P}_{hT}/z\right)\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}f_{1T}^{\perp}\left(x,p_T^2\right)D_1^q\left(z,\frac{K_T^2}{z^2}\right)\right]$$

$$= \frac{x}{z^2}\sum_q e_q^2\int d^2\boldsymbol{p}_T d^2\boldsymbol{K}_T\int\frac{d^2b}{(2\pi)^2}e^{-i(\boldsymbol{p}_T+\boldsymbol{K}_T/z-\boldsymbol{P}_{hT}/z)\cdot b}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}f_{1T}^{\perp}\left(x,p_T^2\right)D_1^q\left(z,\frac{K_T^2}{z^2}\right)\right]$$

$$= -\frac{x}{2z^2}\sum_q e_q^2\int\frac{d^2b}{(2\pi)^2}e^{i\boldsymbol{P}_h\cdot\boldsymbol{b}/z}i\hat{h}_\alpha b^\alpha T_{q,F}(x,x)\,\tilde{D}_1^{h/q}(z,b)$$

$$f_{1T}^{\perp q(a)}(x,b) = \frac{1}{M}\int d^2p_\perp e^{-ip_\perp\cdot b}p_\perp^\alpha f_{1T}^{\perp q}\left(x,p_\perp^2\right) = \frac{ib^\alpha}{2}T_{q,F}(x,x)$$

$$\tilde{D}_1^{h/q}(z,b) = \int d^2K_T e^{-iK_T\cdot b/z}D_1^q\left(z,\frac{K_T^2}{z^2}\right)$$

TMD evolution

- The TMD evolution equation for the ζ_F dependence is encoded in a Collins-Soper (CS) equation through

$$\frac{\partial \ln \tilde{F}(x, b; \mu, \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \frac{\partial \ln \tilde{D}(z, b; \mu, \zeta_D)}{\partial \ln \sqrt{\zeta_D}} = \tilde{K}(b; \mu)$$

- μ dependence is encoded in a RG equation through

$$\frac{d\tilde{K}}{d\ln\mu} = -\gamma_K(\alpha_s(\mu))$$
$$\frac{d\ln \tilde{F}(x, b; \mu, \zeta_F)}{d\ln\mu} = \gamma_F\left(\alpha_s(\mu); \frac{\zeta_F^2}{\mu^2}\right)$$
$$\frac{d\ln \tilde{D}(z, b; \mu, \zeta_D)}{d\ln\mu} = \gamma_D\left(\alpha_s(\mu); \frac{\zeta_D^2}{\mu^2}\right)$$

- The solution of the energy dependence for TMDs has the general form as

$$\tilde{F}_{q/p}(x, b; Q) = \mathcal{F} \times e^{-S} \times \tilde{F}_{q/p}(x, b; \mu_B)$$
$$\tilde{D}_{h/q}(z, b; Q) = \mathcal{D} \times e^{-S} \times \tilde{D}_{h/q}(z, b; \mu_B)$$

TMD evolution formalism

➢ TMD evolution

- To combine the information at small b (perturbative region) with that at large b(non-perturbative region), a matching procedure must be introduced, with b_{max} serving the boundary between the two regions and define.

$$b_* = \frac{b}{\sqrt{1 + b^2/b_{\max}^2}}$$

$b_* \approx b$ at low values of b

$b_* \approx b_{\max}$ at large b values.

- At small b region, PDFs/FFs can be written as convolutions of the perturbatively calculable hard coefficients and the corresponding collinear counterparts at fixed energy μ_B.

$$\tilde{F}_{q/p}(x, b; \mu_B) = C_{q\leftarrow i} \otimes F_{i/p}(x, \mu_B)$$

$$\tilde{D}_{h/q}\left(z, b; \mu_B\right) = \hat{C}_{j\leftarrow q} \otimes D_{h/j}\left(z, \mu_B\right)$$

$$C_{q\leftarrow i} \otimes F_{i/p}(x, \mu_B) \equiv \sum_i \int_x^1 \frac{d\xi}{\xi} C_{q\leftarrow i}(\frac{x}{\xi}, \mu_B) F_{i/p}(\xi, \mu_B),$$

$$\hat{C}_{j\leftarrow q} \otimes D_{h/j}(z, \mu_B) \equiv \sum_j \int_z^1 \frac{d\xi}{\xi} \hat{C}_{j\leftarrow q}\left(\frac{z}{\xi}, \mu_B\right) D_{h/j}\left(\xi, \mu_B\right)$$

TMD evolution formalism

TMD evolution

- The Sudakov-like form factor can be separated into a perturbatively calculable part and a nonperturbative part

$$S(Q;b) = S_{\rm pert}(Q;b_*) + S_{\rm NP}(Q;b)$$

- The perturbative part of S being

$$S_{\rm pert}(Q;b_*) = \int_{\mu_b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[A(\alpha_s(\bar{\mu}))\ln(\frac{Q^2}{\bar{\mu}^2}) + B(\alpha_s(\bar{\mu}))\right]$$

$$A = \sum_{n=1}^{\infty} A^{(n)}(\frac{\alpha_s}{\pi})^n \qquad B = \sum_{n=1}^{\infty} B^{(n)}(\frac{\alpha_s}{\pi})^n$$

- The nonperturbative part of S being

$$S_{\rm NP}^{\rm pdf/ff} = b^2\left(g_1^{\rm pdf/ff} + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)$$

$$g_1^{\rm pdf} = \frac{\langle p_\perp^2\rangle_{Q_0}}{4} \qquad g_1^{\rm ff} = \frac{\langle k_\perp^2\rangle_{Q_0}}{4z_h^2}$$

The structure functions

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➢ Unpolarized structure function

$$F_{UU}(Q;P_{hT}) = \frac{x}{z^2}\sum_q e_q^2 \int_0^\infty \frac{bdb}{(2\pi)} \boxed{J_0(\frac{P_{hT}b}{z})} e^{\boxed{-S_{\rm pert}(Q;b_*)-S_{\rm NP}^{\rm SIDIS}(Q;b)}}$$
$$\left(\sum_i \int_x^1 \frac{d\xi}{\xi} \boxed{C_{q\leftarrow i}^{(\rm SIDIS)}}(\frac{x}{\xi},\mu_B) \boxed{f_1^{i/p}(\xi,\mu_B)}\right) \times \left(\sum_j \int_z^1 \frac{d\xi}{\xi} \boxed{\hat{C}_{j\leftarrow q}^{(\rm SIDIS)}}\left(\frac{z}{\xi},\mu_B\right) \boxed{D_1^{h/j}(\xi,\mu_B)}\right)$$

➢ Sivers structure function

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = -\frac{x}{2z^2}\sum_q e_q^2 \int \frac{d^2b}{(2\pi)^2} e^{i\boldsymbol{P}_{hT}/z\cdot\boldsymbol{b}} \hat{\boldsymbol{h}}\cdot b^\alpha T_{q,F}(x,x)\, \tilde{D}_1^{h/q}(z,b)$$
$$= \frac{x}{2z^2}\sum_q e_q^2 \int_0^\infty \frac{b^2db}{2\pi} \boxed{J_1(\frac{P_{hT}b}{z})} e^{\boxed{-S_{\rm pert}(Q;b_*)-S_{\rm NP\ Sivers}^{\rm SIDIS}(Q;b)}}$$
$$\left(\sum_i \int_x^1 \frac{d\xi}{\xi} \boxed{\Delta C_{q\leftarrow i}^{T(DIS)}}\left(\frac{x}{\xi},\mu_B\right) \boxed{T_{i,F}(\xi,\xi;\mu_B)}\right) \times \left(\sum_j \int_z^1 \frac{d\xi}{\xi} \boxed{\hat{C}_{j\leftarrow q}^{(SIDIS)}}\left(\frac{z}{\xi},\mu_B\right) \boxed{D_1^{h/j}(\xi,\mu_B)}\right)$$

Bessel function

Sudakov-like form factor

The QCD perturbation coefficient

Collinear part

Numerical Estimate

- The model results from the diquark spectator model of collinear unpolarized fragmentation function of Λ hyperon

$$D_1^{\Lambda}(z) = \frac{g_s^2}{4(2\pi)^2} \frac{e^{-\frac{2m_q^2}{\Lambda^2}}}{z^4 L^2} \Big\{ z(1-z)\left((m_q + M_\Lambda)^2 - m_D^2\right) \times \exp\left(\frac{-2zL^2}{(1-z)\Lambda^2}\right) + \left((1-z)\Lambda^2 - 2\left((m_q+M_\Lambda)^2 - m_D^2\right)\right) \times \frac{z^2 L^2}{\Lambda^2} \Gamma\left(0, \frac{2zL^2}{(1-z)\Lambda^2}\right) \Big\}$$

PRD 96(3)034010,2017

- The parameterization of Qiu-Sterman (QS) function

$$T_{q,F}(x, x, \mu_B) = N_q \frac{(\alpha_q + \beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}} x^{\alpha_q} (1-x)^{\beta_q} f_1(x, \mu_B)$$

PRD 89:074013,2014

Kinematical region

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- **For EIC：**

$$0.001 < x < 0.4, \quad 0.07 < y < 0.9, \quad 0.2 < z < 0.8,$$

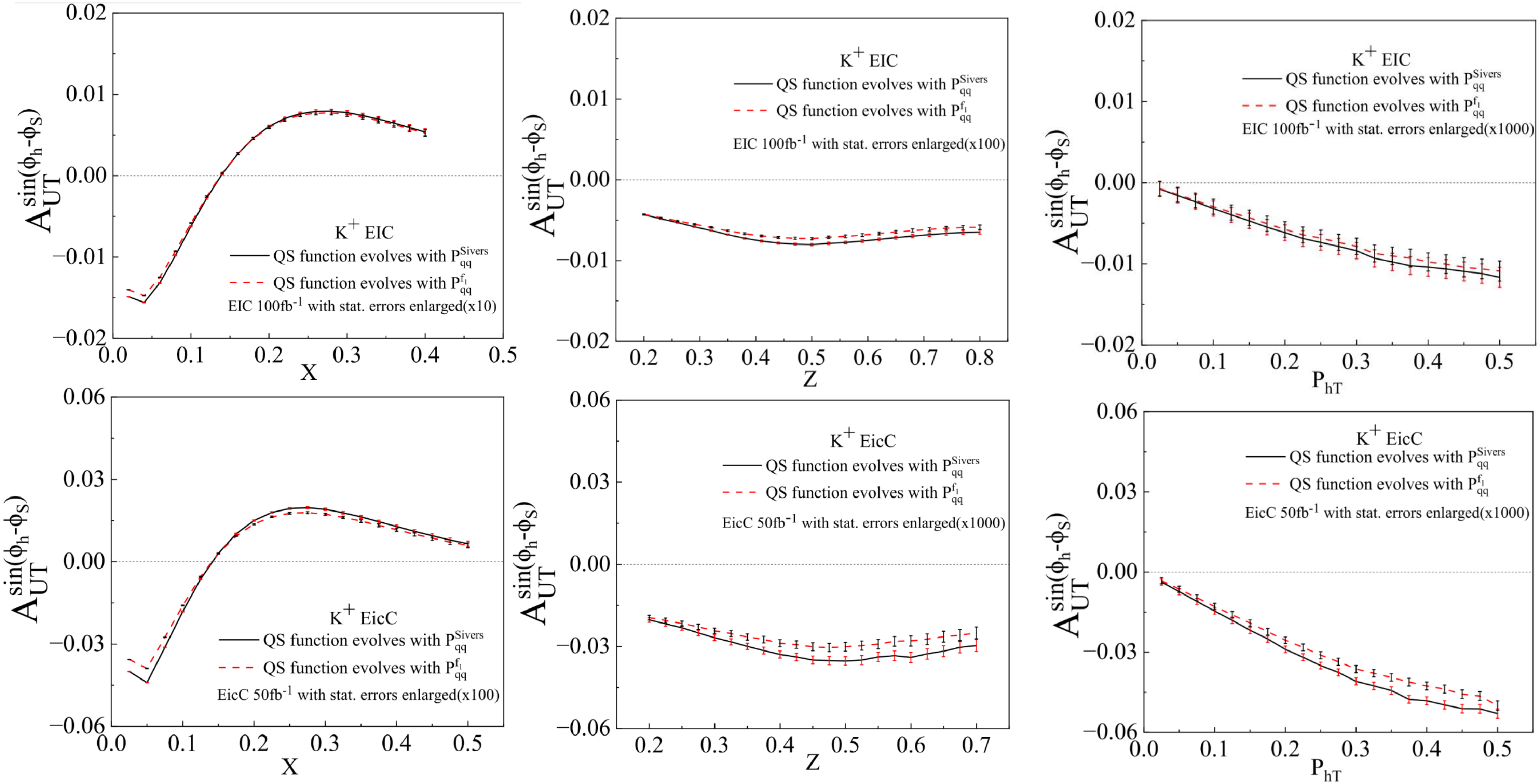
$$1\mathrm{GeV}^2 < Q^2, \quad W > 5\mathrm{GeV}, \quad \sqrt{s} = 100\mathrm{GeV}, \quad P_{hT} < 0.5\mathrm{GeV}$$

- **For EicC：**

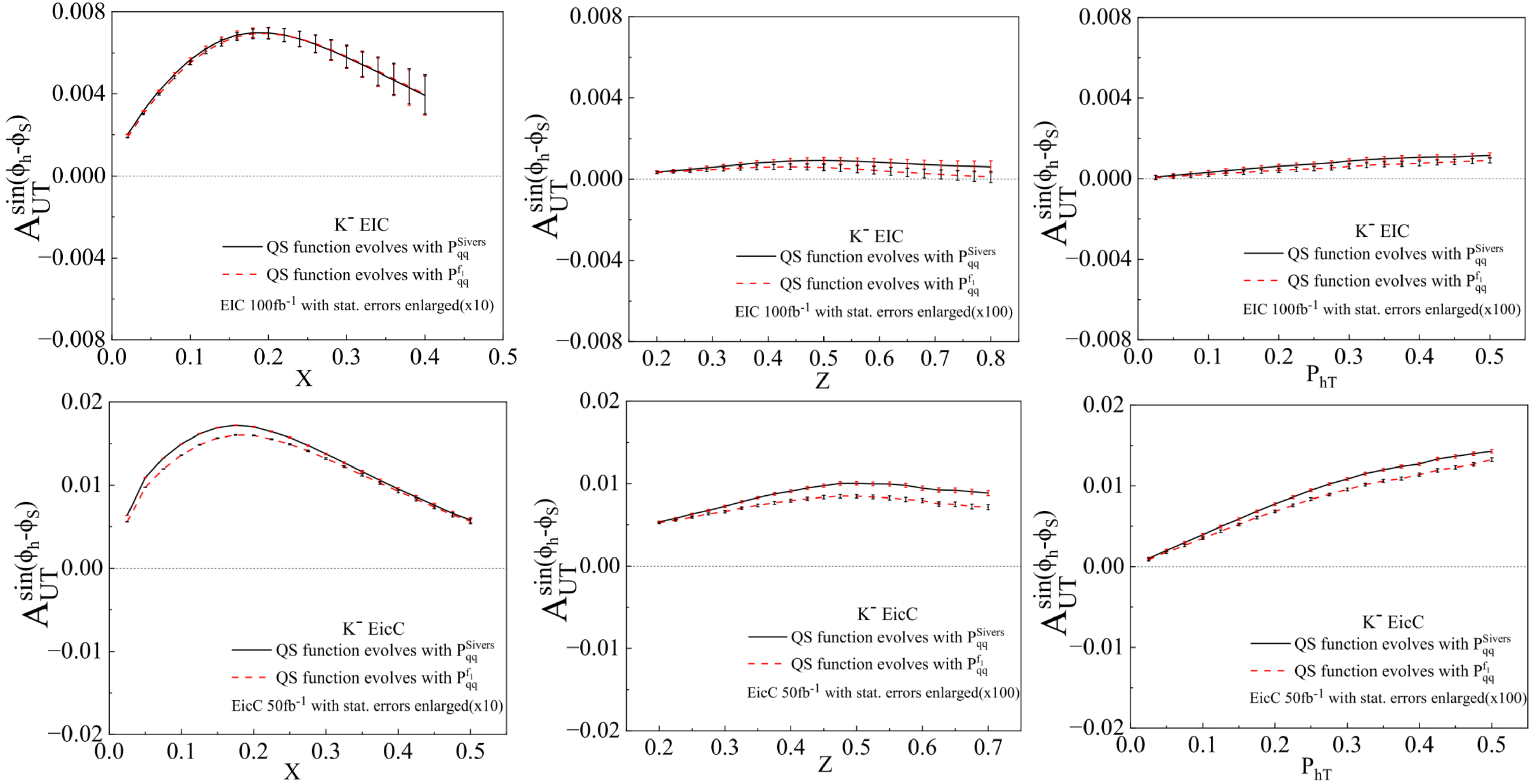
$$0.005 < x < 0.5, \quad 0.07 < y < 0.9, \quad 0.2 < z < 0.7$$

$$1\mathrm{GeV}^2 < Q^2 < 200\mathrm{GeV}^2, \quad W > 2\mathrm{GeV}, \quad \sqrt{s} = 16.7\mathrm{GeV}, \quad P_{hT} < 0.5\mathrm{GeV}$$

K^+

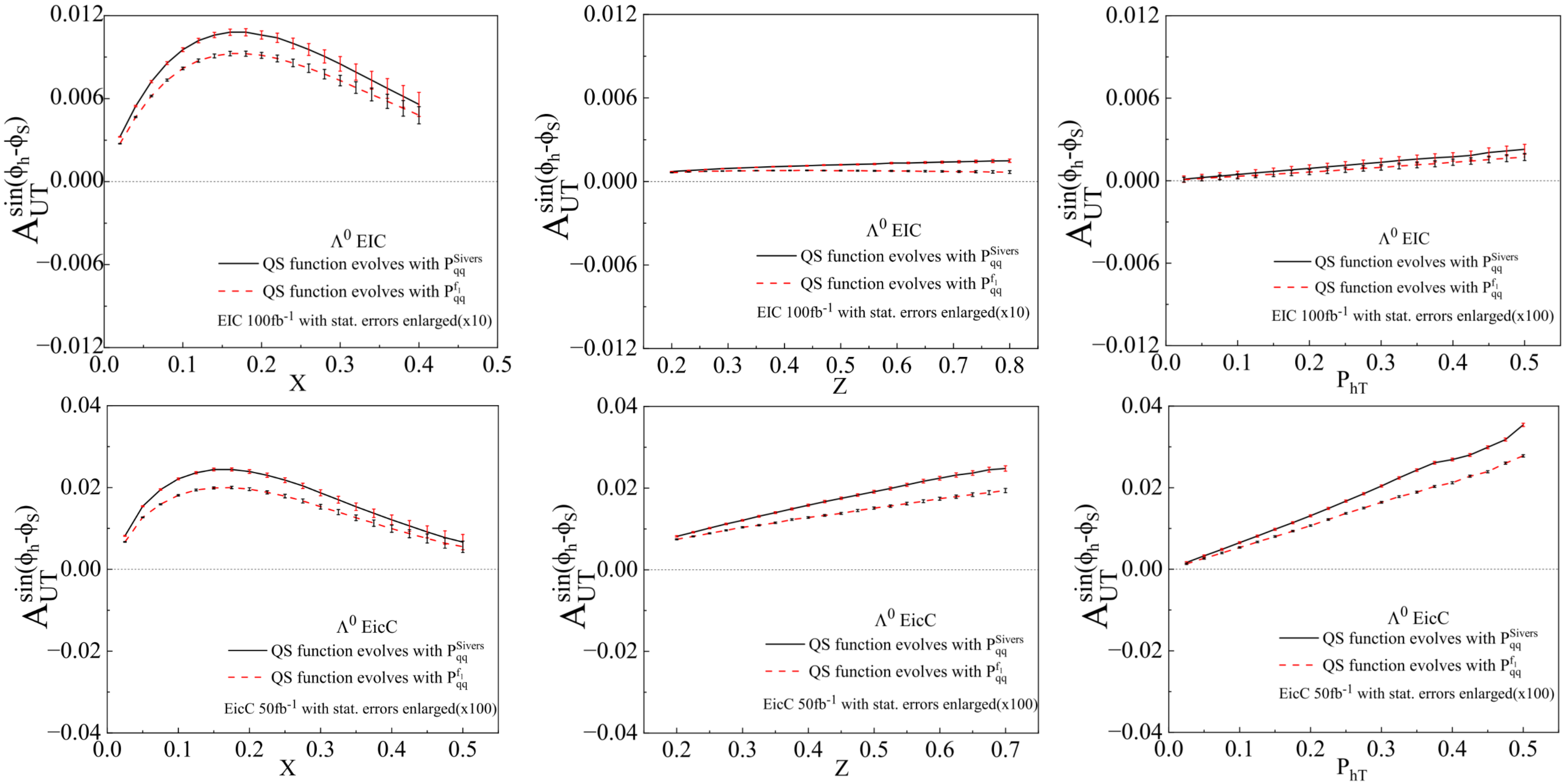

K+ EIC
QS function evolves with P_qq^Sivers
QS function evolves with P_qq^f1
EIC 100fb-1 with stat. errors enlarged(x10)
EIC 100fb-1 with stat. errors enlarged(x100)
EIC 100fb-1 with stat. errors enlarged(x1000)
K+ EicC
EicC 50fb-1 with stat. errors enlarged(x100)
EicC 50fb-1 with stat. errors enlarged(x1000)
A_UT^sin(φh-φS)
X
Z
P_hT

K⁻



K⁻ EIC
QS function evolves with P_{qq}^{Sivers}
QS function evolves with $P_{qq}^{f_1}$
EIC 100fb^{-1} with stat. errors enlarged(x10)
EIC 100fb^{-1} with stat. errors enlarged(x100)
K⁻ EicC
EicC 50fb^{-1} with stat. errors enlarged(x10)
EicC 50fb^{-1} with stat. errors enlarged(x100)
$A_{UT}^{\sin(\phi_h-\phi_S)}$
X
Z
P_{hT}

Λ



Λ^0 EIC
QS function evolves with P_{qq}^{Sivers}
QS function evolves with $P_{qq}^{f_1}$
EIC 100fb^{-1} with stat. errors enlarged(x10)
EIC 100fb^{-1} with stat. errors enlarged(x100)
Λ^0 EicC
EicC 50fb^{-1} with stat. errors enlarged(x100)
$A_{UT}^{\sin(\phi_h-\phi_S)}$
X
Z
P_{hT}

Discussion

- Sivers asymmetry is sizable at EIC and EicC.
- The Sivers asymmetry in EicC is larger than in EIC.
- The future higher precision EICs may provide a unique opportunity to extract the proton Sivers function of valence quark and sea quark from the K^+ production, to analyze the flavor dependence of the Sivers distribution function among sea quarks from K^- produced SIDIS process, to investigate the flavor dependence from Λ hyperon produced SIDIS process.
- The Sivers asymmetry obtained using the evolution kernel of QS function is greater than that obtained using the unpolarized distribution function evolution kernel.

Conclusion

- The Sivers asymmetry in $K^{\pm}$ and Λ hyperon production in SIDIS process can serve as a tool to extract the information of sea quark Sivers function as well as to constrain the flavor dependence of Sivers function by utilizing future high energy and high luminosity EICs.
- The DGLAP evolution of the Qiu-Sterman function in the TMD evolution schemes will play a role in the phenomenological calculation, which should be considered in the future interpretation of experimental data as well as theoretical studies.

OUTLINE

Transversity distribution function

➢ Transversity distribution function

- It describes the asymmetric distribution of the transversely polarized quarks in a transversely polarized nucleon
- Chiral-odd

➢ How to access transversity function

- (TMD) factorization frame in SIDIS ——Collins function NPB 396, 161 (1993)
- Collinear factorization in SIDIS——twist-3 fragmentation function PRD 93, 074009 (2016)、PRD.103.114011(2021)
- Collinear factorization in SIDIS——dihadron fragmentation function PRL 107, 012001(2011)
- Drell-Yan process——the antiquark transversity Phys.Rept. (2002) 1-168

Definition of Collins asymmetry

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- The 5-fold differential cross section with a transversely polarized target has the following general form

$$\frac{d^5\sigma\left(S_T\right)}{dx_B dy dz_h d^2\boldsymbol{P}_{hT}} = \sigma_0\left(x_B, y, Q^2\right)\left[F_{UU} + \sin\left(\phi_h+\phi_s\right)\frac{2(1-y)}{1+(1-y)^2}F_{UT}^{\sin(\phi_h+\phi_s)} + \ldots\right]$$

$$\sigma_0 = \frac{2\pi\alpha_{\mathrm{cm}}^2}{Q^2}\frac{1+(1-y)^2}{y}$$

- The Collins asymmetry can be written in terms of the structure functions

$$A_{UT}^{\sin(\phi_h+\phi_s)} = \frac{\sigma_0\left(x_B, y, Q^2\right)}{\sigma_0\left(x_B, y, Q^2\right)}\frac{2(1-y)}{1+(1-y)^2}\frac{F_{UT}^{\sin(\phi_h+\phi_s)}}{F_{UU}}$$

$$F_{UU,T} = \mathcal{C}\left[f_1 D_1\right]$$

$$F_{UT}^{\sin(\phi_h+\phi_s)} = \mathcal{C}\left[\frac{-\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}h_1 H_1^{\perp}\right]$$

$$\mathcal{C}[\omega f D] = x\sum_q e_q^2\int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\delta^{(2)}\left(\boldsymbol{p}_T-\boldsymbol{k}_T-\boldsymbol{P}_{hT}/z\right)\omega(\boldsymbol{p}_T,\boldsymbol{k}_T)f^q(x,p_T^2)\,D^q(z,k_T^2)$$

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The structure functions

➢ Collins structure functions

$$F_{UT}^{\sin(\phi_h+\phi_S)}(Q;P_{hT}) = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}h_1H_1^{\perp}\right]$$

$$= x\sum_q e_q^2\int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\delta^{(2)}\left(\boldsymbol{p}_T-\boldsymbol{k}_T-\boldsymbol{P}_{hT}/z\right)\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}h_1^q\left(x,p_T^2\right)H_1^{\perp,q}\left(z,k_T^2\right)\right]$$

$$= \frac{x}{z^2}\sum_a e_q^2\int d^2\boldsymbol{p}_T d^2\boldsymbol{K}_T\delta^{(2)}\left(\boldsymbol{p}_T+\boldsymbol{K}_T/z-\boldsymbol{P}_{hT}/z\right)\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{K}_T}{M_h\cdot z}h_1^q\left(x,p_T^2\right)H_1^{\perp,q}\left(z,\frac{K_T^2}{z^2}\right)\right]$$

$$= \frac{x}{z^2}\sum_q e_q^2\int d^2\boldsymbol{p}_T d^2\boldsymbol{K}_T\int\frac{d^2b}{(2\pi)^2}e^{-i(\boldsymbol{p}_T+\boldsymbol{K}_T/z-\boldsymbol{P}_{hT}/z)\cdot\boldsymbol{b}}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{K}_T}{M_h\cdot z}h_1^q\left(x,p_T^2\right)H_1^{\perp,q}\left(z,\frac{K_T^2}{z^2}\right)\right]$$

$$= \frac{x}{z^3}\sum_q e_q^2\int\frac{d^2b}{(2\pi)^2}e^{i\boldsymbol{P}_{hT}\cdot\boldsymbol{b}/z}\hat{h}_\alpha\tilde{h}_1^{q/p}(x,b)\widetilde{H}_{1,h/q}^{\perp}(z,b)$$

$$\tilde{h}_1^{q/p}(x,b;Q) = \int d^2\boldsymbol{p}_\perp e^{-i\boldsymbol{p}_\perp\cdot\boldsymbol{b}}h_1^q\left(x,p_\perp^2\right)$$

$$\widetilde{H}_{1,h/q}^{\perp}(z,b;Q) = \int d^2\boldsymbol{K}_T e^{-i\boldsymbol{K}_T\cdot\boldsymbol{b}/z}\frac{K_T^\alpha}{M_h}H_1^{\perp,q}(z,\frac{K_T^2}{z^2})$$

The structure functions

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➢ Collins structure functions

$$
\begin{aligned}
F_{UT}^{\sin(\phi_h+\phi_S)}(Q;P_{hT}) &= \frac{x}{z^3}\sum_q e_q^2 \int \frac{d^2b}{(2\pi)^2} e^{i\boldsymbol{P}_{hT}\cdot\boldsymbol{b}/z}\hat{h}_\alpha \tilde{h}_1^{q/p}(x,b)\widetilde{H}_{1,h/q}^{\perp}(z,b) \\
&= -\frac{x}{2z^2}\sum_q e_q^2 \int_0^\infty \frac{b^2 db}{2\pi} \boxed{J_1\left(\frac{P_{hT}b}{z}\right)} e^{\boxed{-S_{\text{pert}}(Q;b_*)-S_{NP\ Collins}^{\text{SIDIS}}(Q;b)}} \\
&\left(\sum_i \int_x^1 \frac{d\xi}{\xi} \boxed{\delta C_{q\leftarrow i}^{(\text{SIDIS})}}\left(\frac{x}{\xi},\mu_B\right)\boxed{h_1^{i/p}(\xi,\mu_B)}\right) \times \left(\sum_j \int_z^1 \frac{d\xi}{\xi} \boxed{\delta\hat{C}_{j\leftarrow q}^{(\text{SIDIS})}}\left(\frac{z}{\xi},\mu_B\right)\boxed{\hat{H}_{h/j}^{(3)}(\xi,\mu_B)}\right)
\end{aligned}
$$

Bessel function

Sudakov-like form factor

The QCD perturbation coefficient

Collinear part

Numerical Estimate

- The model results from the diquark spectator model of collinear unpolarized fragmentation function of Λ hyperon

$$D_1^{\Lambda}(z) = \frac{g_s^2}{4(2\pi)^2}\frac{e^{-\frac{2m_q^2}{\Lambda^2}}}{z^4 L^2}\Big\{z(1-z)\left((m_q+M_\Lambda)^2-m_D^2\right)\times\exp\left(\frac{-2zL^2}{(1-z)\Lambda^2}\right) + \left((1-z)\Lambda^2 - 2\left((m_q+M_\Lambda)^2-m_D^2\right)\right)\times\frac{z^2L^2}{\Lambda^2}\Gamma\left(0,\frac{2zL^2}{(1-z)\Lambda^2}\right)\Big\}$$

PRD 96(3)034010,2017

- The parameterization of transversity function

$$h_1^{q/p}(x,Q_0) = N_q^h x^{a_q}(1-x)^{b_q}\frac{(a_q+b_q)^{(a_q+b_q)}}{a_q^{a_q}b_q^{b_q}}\frac{1}{2}\left(f_1^{q/p}(x,Q_0)+g_1^{q/p}(x,Q_0)\right)$$

PRD 93, 014009 (2016)

- The model results of Collins function of Λ hyperon

$$\widehat{H}_{h/q}^{(3)} = 2M_\Lambda H_1^{\perp(1)}(z) = z^2\int d^2k_T \frac{k_T^2}{M_\Lambda}H_1^{\perp}\left(z,z^2k_T^2\right)$$

$$H_1^{\perp(q)}\left(z,k_T^2\right) = \frac{\alpha_s g_D'^2 C_F}{(2\pi)^4}\frac{e^{\frac{-2k^2}{\lambda^2 z^2(1-z))^\beta}}}{z^2(1-z)}\frac{1}{(k^2-m_q^2)}\left(H_{1(a)}^{\perp(q)}\left(z,k_T^2\right)+H_{1(b)}^{\perp(q)}\left(z,k_T^2\right)+H_{1(c)}^{\perp(q)}\left(z,k_T^2\right)+H_{1(d)}^{\perp(q)}\left(z,k_T^2\right)\right)$$

PRD 97, 114015 (2018)

Kinematical region

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- **For EIC：**

$$0.001 < x < 0.4, \quad 0.07 < y < 0.9, \quad 0.2 < z < 0.8,$$

$$1\mathrm{GeV}^2 < Q^2, \quad W > 5\mathrm{GeV}, \quad \sqrt{s} = 100\mathrm{GeV}, \quad P_{hT} < 0.5\mathrm{GeV}$$

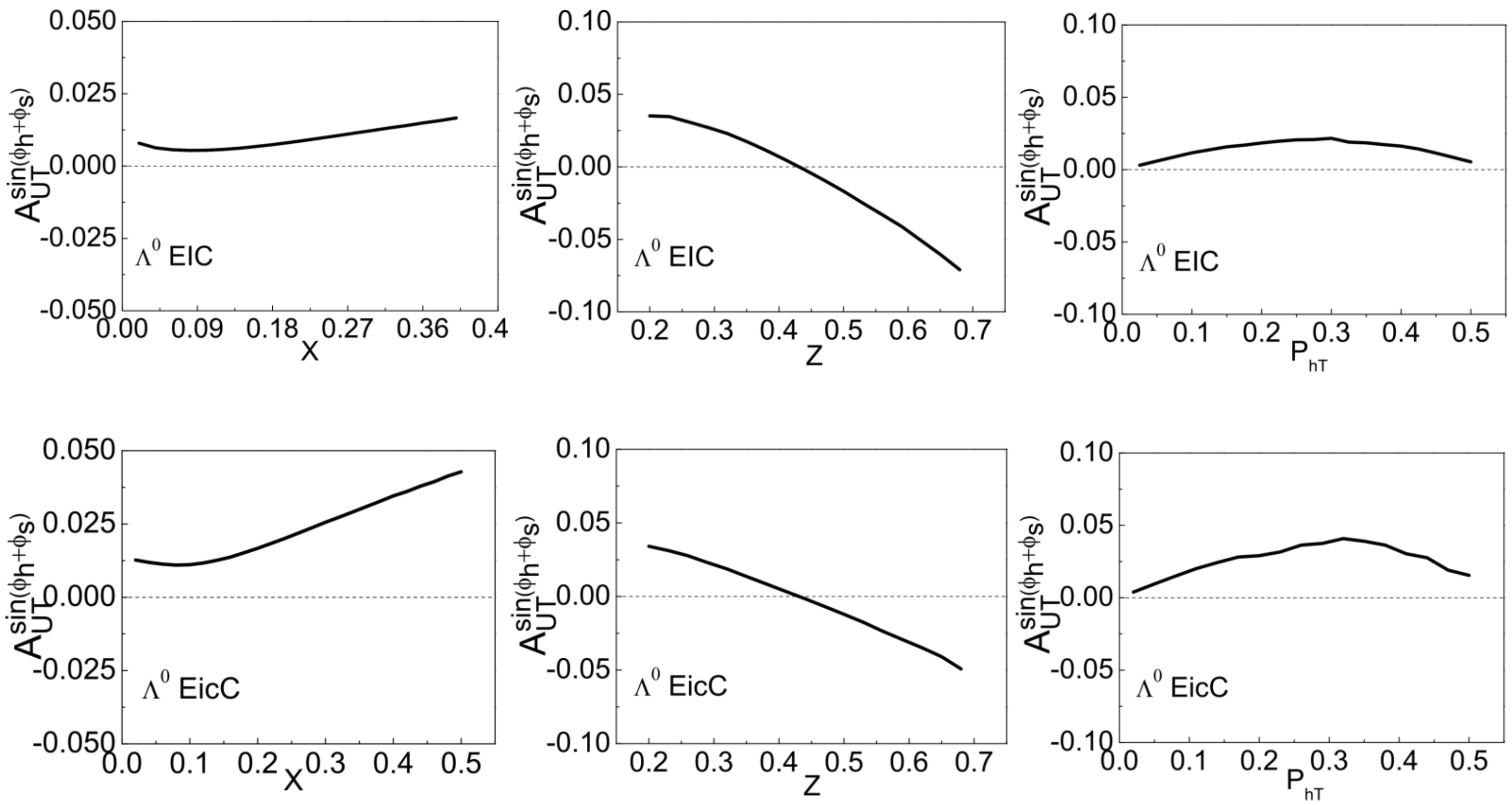
- **For EicC：**

$$0.005 < x < 0.5, \quad 0.07 < y < 0.9, \quad 0.2 < z < 0.7$$

$$1\mathrm{GeV}^2 < Q^2 < 200\mathrm{GeV}^2, \quad W > 2\mathrm{GeV}, \quad \sqrt{s} = 16.7\mathrm{GeV}, \quad P_{hT} < 0.5\mathrm{GeV}$$

Result

- Both sizable at EIC and EicC.
- The Collins asymmetry in EicC is larger than in EIC.



Conclusion

- The measurement of the Collins asymmetry of semi-inclusive Λ production at future electron–ion colliders can provide useful constraints on the sea quark transversity distribution function as well as the flavor dependence of the TMD transversity distribution function.

- There is still no parametric extraction of Λ hyperon Collins function, which indicates the importance of future precise measurement data to extract the parameterization of the Λ Collins function combining with e^+e^- annihilation data.

OUTLINE

- Introduction
- Framework
- The study of the Sivers asymmetry in the SIDIS process
- The study of the Collins asymmetry in the SIDIS process
- **Summary and Outlook**

Summary and Outlook

➢ Summary

- The asymmetry is a very powerful tool for studying the hadronic structure and contributes to understanding the non-perturbative properties of QCD.
- The higher precision EICs are the most promising experimental setup for studying the internal structure of nucleons, in the future.
- We apply the TMD factorization formalism to predict the Sivers asymmetry of charged Kaon production and Λ hyperon production in SIDIS process are sizable at the kinematics of EIC and EicC, which can serve as a tool to extract the information of sea quark Sivers function as well as to constrain the flavor dependence of Sivers function.
- The measurement of the Collins asymmetry of semi-inclusive Λ production at future electron–ion colliders can provide useful constraints on the sea quark transversity distribution function as well as the flavor dependence of the TMD transversity distribution function.

Summary and Outlook

Outlook

- The DGLAP evolution of the Qiu-Sterman function in the TMD evolution schemes will play a role in the phenomenological calculation, which should be considered in the future interpretation of experimental data as well as theoretical studies.

- There is still no parametric extraction of Λ hyperon Collins function, which indicates the importance of future precise measurement data to extract the parameterization of the Λ Collins function combining with e^+e^- annihilation data.

Thank you!