# Gravitational Form Factors and Internal Nature of the Proton

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based on “Gluon gravitational form factors of protons from charmonium photoproduction". Chin. Phys. C 47, 07410 (2023) and “Gravitational form factors of the proton from near-threshold vector meson photoproduction". arXiv:2308.04644
in collaboration with 王晓云 (兰州理工大学理学院　兰州大学兰州理论物理中心)

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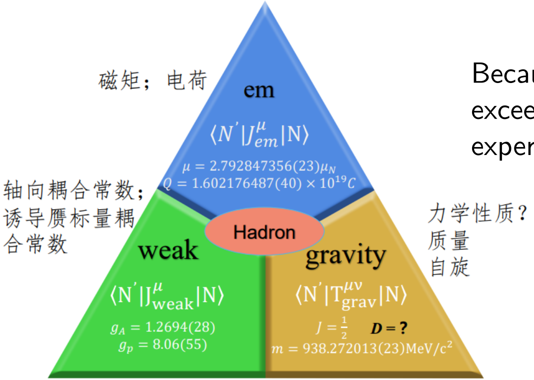
# background

The structure of hadrons is most conveniently probed by exploring the other fundamental forces: electromagnetic, weak, and gravitational interactions.




Because the GFFs is weak! exceeding the limits of experimental measurement.

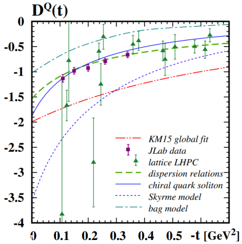
## background

- In the standard model (plus gravity), the direct way to measure its matrix elements is graviton-proton scattering. (Gravitational interaction between a proton and an electron is $10^{-39}$ times weaker than electromagnetic interaction.)
- Indirect experiment: the proton quark D-term extracted from DVCS experiment.
- The information on the proton gluon GFFs is very meager.
- And the GFFs are measurable quantities defined purely for the total system.

cite: V. D. Burkert, L. Elouadrhiri and F. X. Girod, "The pressure distribution inside the proton," Nature 557, 7705, 396-399 (2018).

# introduction—theoretical results

| | |
|---|---|
| chiral symmetry | some calculation: pion kaon and $\eta$ D-term |
| model study (for proton GFFs) | the bag model<br>chiral quark-solition model<br>skyrme model |
| model study (for hadrons GFFs) | light-front constituent quark model<br>diquark approaches<br>holographic AdS/QCD<br>light-cone QCD sum rules |

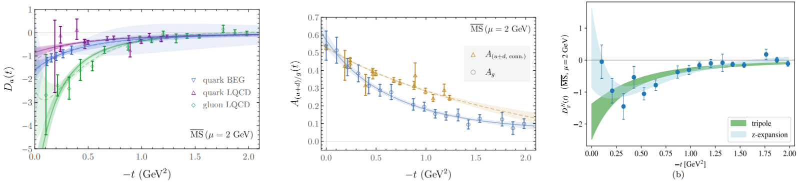



take the quark D-term as an example, the theoretical results vary widely!

cite: M. V. Polyakov and P. Schweitzer, "Forces inside hadrons: pressure, surface tension, mechanical radius, and all that," Int. J. Mod. Phys. A **33**, 1830025 (2018).

# lattice QCD

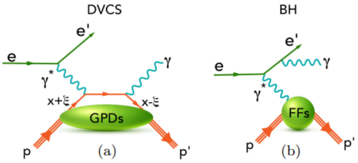
Lattice QCD for GFFs $A_g(t), A_q(t)$ and $D_g(t), D_q(t)$:



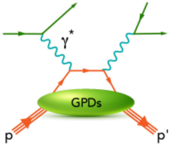

cite:
P. E. Shanahan and W. Detmold, "Pressure Distribution and Shear Forces inside the Proton," Phys. Rev. Lett. **122**, 072003 (2019).
P. E. Shanahan and W. Detmold, "Gluon gravitational form factors of the nucleon and the pion from lattice QCD," Phys. Rev. D **99**, 014511 (2019).
D. A. Pefkou, D. C. Hackett and P. E. Shanahan, "Gluon gravitational structure of hadrons of different spin," Phys. Rev. D **105**, 054509 (2022).

## experimental measurement—indirect exclusive processes

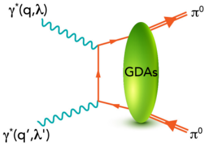
1. Deeply virtual Compton scattering with positron and electron beams ($\gamma p \to p'\gamma^*$)



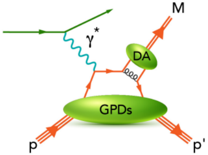

2. Time like Compton scattering and double DVCS ($\gamma p \to p'\gamma^*$ and $\gamma^* \to e^+e^-$ or $\mu^+\mu^-$)




3. electron-positron colliders: $\gamma\gamma^* \to \pi^0\pi^0$



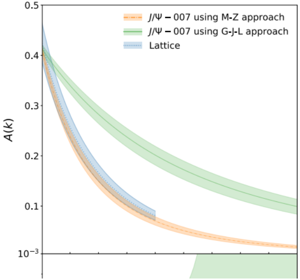

4. Meson production $\gamma p \to Vp$

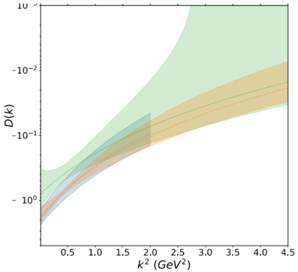



cite: V. D. Burkert, *et al.* "Colloquium: Gravitational Form Factors of the Proton," [arXiv:2303.08347].

## current study

- The near-threshold vector meson photoproduction are used to study the proton mass radius, the trace anomaly contribution, and the $V-p$ scattering length.
- Recently, the near-threshold $J/\psi$ photoproduction offers a superior path to access the gluon GFFs.



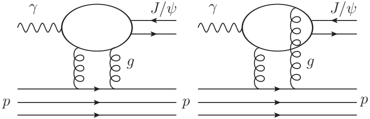

holographic QCD and GPD+VMD approach

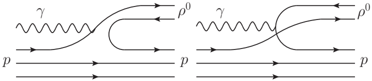
cite: B. Duran, *et al.* "Determining the gluonic gravitational form factors of the proton," Nature **615**, 813-816 (2023).

# our work

The heavy quarkonium (such as $J/\psi$) photoproduction offers a path to access the gluon GFFs, and the light vector meson (such as $\rho^0$) photoproduction mainly reflects the quark GFFs.

- $J/\psi$ photoproduction: The scalar gluon operator is dominant in the production amplitude of heavy quarkonium. The quark exchange is strongly suppressed by the OZI rule.
- $\rho^0$ photoproduction: the exchange between the proton and the meson states are occupied by valence quarks.



## our advantage

- completeness: The separate quark and gluon EMT operators are not conserved.
  Therefore, the individual quark and gluon GFFs studies acquire scale- and scheme-dependence.

- verifiability: DVCS experiment is insensitive to quark D-term.

- experimental operability: The vector meson photoproduction ($J/\psi$ and $\rho^0$) are the most abundant experimental measurement presently.

# GFFs

The $G(t)$ can be written as

$$G(t) = MA(t) - \frac{t}{4M}\left(A(t) - 2J(t) + D(t)\right) \tag{1}$$

$M$ is the proton mass. $A(0) = 1$, $J(0) = \frac{1}{2}$.

The quark and gluon GFFs:

$$G_{q/g}(t) = MA_{q/g}(t) - \frac{t}{4M}\left(-B_{q/g}(t) + D_{q/g}(t)\right) + M\bar{C}_{q/g}(t) \tag{2}$$

assumption: $B(t) = B_q(t) + B_g(t) \approx 0$ and $B_g(t) + D_g(t) \approx 0$

cite: X. B. Tong, J. P. Ma and F. Yuan, "Perturbative calculations of gravitational form factors at large momentum transfer," JHEP 10, 046 (2022).

# GFFs

the gluon and quark GFFs part:

$$G_g(t) \approx \frac{3}{4} M A_g(t) - \frac{t}{4M} D_g(t)$$
$$G_q(t) \approx M A_q(t) + \frac{1}{4} M A_g(t) - \frac{t}{4M} D_q(t) \qquad (3)$$

The gluon contribution $A_g(0) = 0.414$ was obtained from global QCD analysis and LQCD results.And

$$A(t) = \frac{A_q(0)}{(1 - t/m_q^2)^2} + \frac{A_g(0)}{(1 - t/m_g^2)^2}, D(t) = \frac{D_q(0)}{(1 - t/d_q^2)^3} + \frac{D_g(0)}{(1 - t/d_g^2)^3}$$

mechanical properties: the pressure $p(r)$ and shear forces $s(r)$

$$s(r) = -\frac{1}{2} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \widetilde{D}(r), \qquad p(r) = \frac{1}{3} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \widetilde{D}(r) \qquad (4)$$

# three radius from GFFs

- energy radius: The mean square radius of the energy density $T_{00}(r)$

$$\langle R_E^2 \rangle = \frac{\int d^3 r r^2 T_{00}(r)}{\int d^3 r T_{00}(r)} \tag{5}$$

- mechanical radius: The mean square radius of the normal forces distribution $F_n(r) = \frac{2}{3}s(r) + p(r)$

$$\langle R_{mech}^2 \rangle = \frac{\int \mathrm{d}^3 r \ r^2 \left[F_n(r)\right]}{\int \mathrm{d}^3 r \left[F_n(r)\right]} \tag{6}$$

- $A$-term mass radius: the mean square radius of the mass distribution (quark+gluon contribution)

$$\langle R_{A-term}^2 \rangle = \frac{\int d^3 r r^2 A(r)}{\int d^3 r A(r)} \tag{7}$$

cite: M. V. Polyakov and P. Schweitzer, “Forces inside hadrons: pressure, surface tension, mechanical radius, and all that,” Int. J. Mod. Phys. A **33**, 1830025 (2018).

## The GFFs and vector meson photoproduction

The photoproduction cross section of the vector meson:

$$\frac{d\sigma_{\gamma p\to Vp}}{dt} = \frac{1}{64\pi W^2}\frac{1}{|\mathbf{p}_\gamma|^2}\left|\mathcal{M}_{\gamma p\to Vp}\right|^2 \tag{8}$$

The amplitude of light and heavy quarkonium primarily attributes to the quark and gluon part of the EMT:

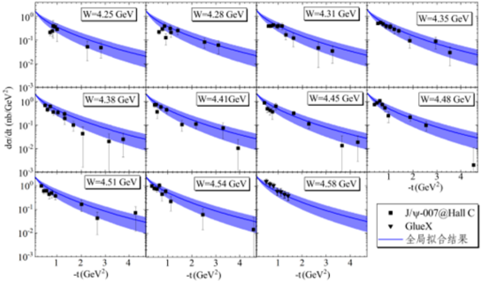
$$\mathcal{M}_{\gamma p\to Vp} = -Q_e c_2 2Mg^2 \left\langle P'\middle| T_{00}^{q(g)} \middle| P\right\rangle \tag{9}$$

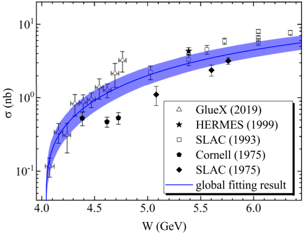
the component of the quark and gluon part EMT become

$$\begin{aligned}
\left\langle P'\middle| T_{00}^{g} \middle| P\right\rangle &= \bar{u}(P')u(P)\left(\frac{3}{4}MA_g(t) - \frac{t}{4M}D_g(t)\right) \\
\left\langle P'\middle| T_{00}^{q} \middle| P\right\rangle &= \bar{u}(P')u(P)\left(MA_q(t) + \frac{1}{4}MA_g(t) - \frac{t}{4M}D_q(t)\right)
\end{aligned} \tag{10}$$

# The gluon GFFs from $J/\psi$ photoproduction

global fitting the near-threshold $J/\psi$ photoproduction data, including the differential and total cross section：

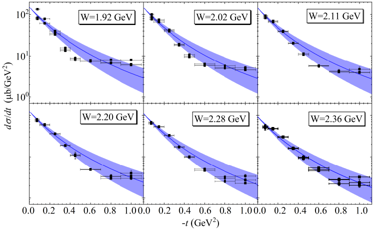


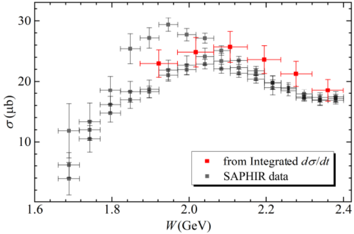




The value of the differential cross section $d\sigma/dt|_{t=0}$ is used with the help of two gluon exchange model.
The parameters $m_g$, $D_g(0)$ and $d_g$ in gluon GFFs are attained.

# The quark GFFs from $\rho^0$ photoproduction

global fitting the near-threshold $\rho^0$ photoproduction data:

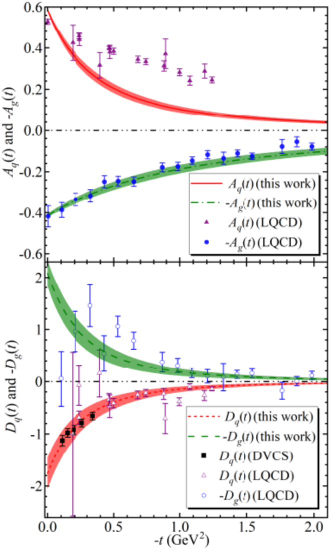



The total cross sections calculated by integrating the functions fitted to the differential cross sections for the six beam energy regions.

The parameters $m_q$, $D_q(0)$ and $d_q$ in quark GFFs are extracted.

# The result of the quark and gluon GFFs




For quark GFFs:

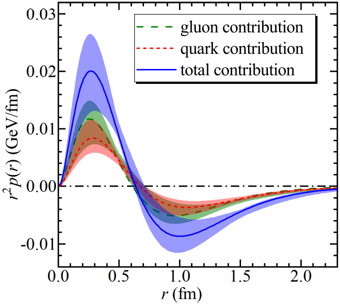
the quark $D$-term exhibits a notable agreement with the DVCS experiment data and LQCD determinations.
$A$-term is approximately half smaller than the LQCD results.

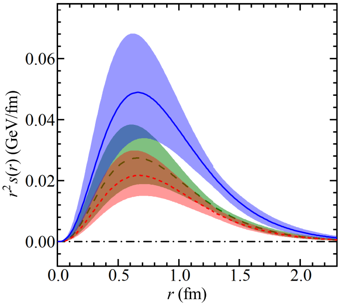
For gluon GFFs:

our statement obtained for gluon $A$ and $D$-term are approximately comparable with the LQCD results.

# The obtained internal nature of the proton

The pressure and shear forces distributions inside the proton:




| $A$-term mass radius | mechanical radius | energy radius |
|---|---|---|
| $0.69 \pm 0.04$ fm | $0.77 \pm 0.05$ fm | $0.85 \pm 0.06$ fm |
| $p(0)$ (GeV/fm$^3$) | $T_{00}(0)$ (GeV/fm$^3$) | — |
| $1.49^{+0.87}_{-0.60}$ | $5.40^{+2.54}_{-2.76}$ | — |

## summary

- A groundbreaking analysis of the complete proton GFFs from the NTVMP are investigated.
  The obtained GFFs are compared with the DVCS and LQCD.
- This work employ a first-ever definition about the mean square radius of the mass distribution.
- The energy radius derived from GFFs can also be written as

$$\langle R_E^2 \rangle = \frac{\int d^3 r r^2 T_{00}(r)}{\int d^3 r T_{00}(r)} = -\frac{6}{G(0)} \frac{dG(t)}{dt}\bigg|_{t=0}$$

  which bears a striking resemblance to proton charge and magnetic radius defined from the electric and magnetic form factor.

## relevant experiment

- Experimental information on vector meson photoproduction is essential to gain insight into the quark and gluon GFFs of the proton. (including $\rho^0, \omega$ and $\phi, J/\psi, \Upsilon$)
- The precise DVCS measurement on quark GFFs $A_q(t), D_q(t)$ can be realized in the JLab and EICs.
- Domestic experiment:
  1.Electron-Positron Colliders (BEPC II): an opportunity the study the structure of unstable hadrons like pions that are not available as targets.
  2. High Energy Photon Source (HEPS): electron energy from 500 MeV to 6 GeV in booster.

*Thanks!*