Predictions of the hybrid mesons with exotic quantum numbers $J^{PC} = 2^{+-}$

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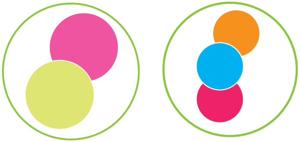
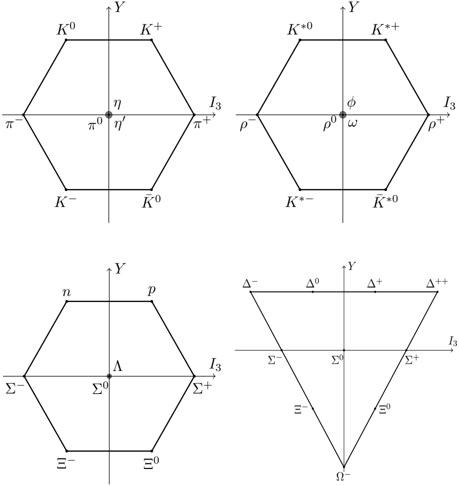
Conventional hadrons

- Quark Model
  - meson: $\bar{q}q$
  - baryon: qqq

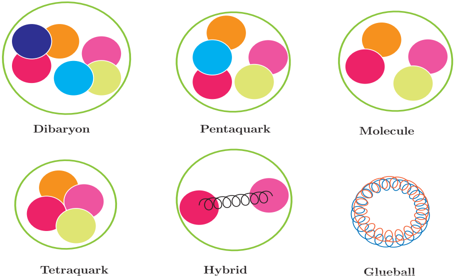


Meson Baryon



Exotics in QCD

- Exotics in QCD



- Exotic quantum numbers

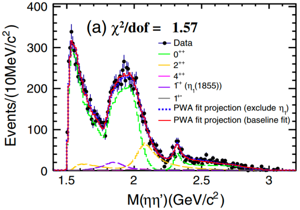
Possible quantum numbers in $q\bar{q}$ system.

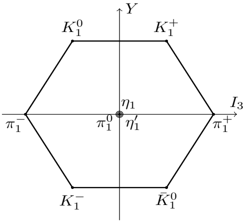
	$S=0$	$S=1$
$L=0$	0^{-+}	1^{--}
$L=1$	1^{+-}	0^{++}, 1^{++},2^{++}
$L=2$	2^{-+}	1^{--},2^{--},3^{--}

  - 1^{-+}, $0^{\pm-}$, 2^{+-}

1^{-+} hybrid candidates

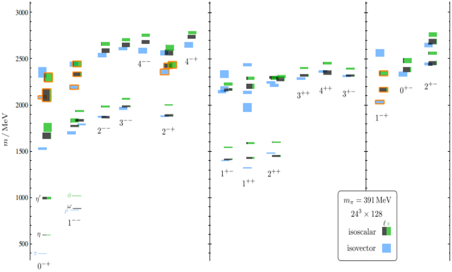
- Observation of $\eta_1(1855)$ PRL129,192002
  - $J/\psi \to \gamma\eta_1(1855) \to \gamma\eta\eta'$
  - $m = 1855 \pm 9^{+6}_{-1}$MeV
  - $\Gamma = 188 \pm 18^{+3}_{-8}$MeV
- Explanations
  - $K\bar{K}_1(1400)$ NPA1030 122571
  - $[1_c]_{\bar{s}s} \otimes [1_c]_{\bar{q}q}$ PRD106,074003
  - dynamically generated state Universe9,109
  - hybrid 2302.06785,2302.07687
- Other hybrid candidates
  - $\pi_1(1400)$
  - $\pi_1(1600)$
  - $\pi_1(2015)$
- Hybrid nonent?





Theoretical predictions on hybrid spectrum

- MIT bag model
  - lightest: 1^{--},$(0, 1, 2)^{-+}$
  - higher: $(0, 2)^{+-}$
  - highest: 0^{--}
- Supported by LQCD calculations



PRD88,094505

- Other studies: Flux tube model, Bethe-Salpeter equation, QCD sum rules
- 2^{+-} hybrid states are predicted to be narrow,less than 10 MeV PRD59,034016

Naming Scheme

- 0^{-+} states

$$I^G(J^{PC}) = 0^+(0^{-+}) \to \eta(\bar{q}q),\ \eta'(\bar{q}q + \bar{s}s)$$
$$I^G(J^{PC}) = 1^-(0^{-+}) \to \pi$$

- 1^{+-} states

$$I^G(J^{PC}) = 0^-(1^{+-}) \to h_1(1170)$$
$$I^G(J^{PC}) = 1^+(1^{+-}) \to b_1(1235)$$

- 2^{+-} states (exotic quantum number)

$$I^G(J^{PC}) = 0^-(2^{+-}) \to h_2(\bar{q}gq),\ h_2'(\bar{s}gs)$$
$$I^G(J^{PC}) = 1^+(2^{+-}) \to b_2(\bar{q}gq)$$

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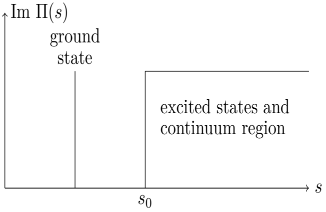
Phenomenology-Hadron level

- Dispersion relation

$$\Pi\left(q^2\right) = \frac{\left(q^2\right)^N}{\pi}\int_<^\infty ds\frac{\mathrm{Im}\,\Pi(s)}{s^N\left(s-q^2-i\varepsilon\right)} + \sum_{n=0}^{N-1}\left(q^2\right)^n b_n\,,$$

- Spectral density

$$\rho(s) = \frac{\mathrm{Im}\,\Pi(s)}{\pi} = f_H^2\delta\left(s-m_H^2\right) + \theta\left(s-s_0\right)\rho^{\text{cont}}(s)\,,$$



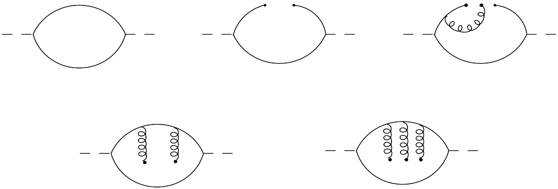
OPE-Quark level

- The correlation function can be calculated using operator product expansion

$$\Pi(q^2) = \sum_{n=0}^{\infty} C_n(q^2) O_n$$

- The operators often considered are

$$\left\langle \bar{q}q \right\rangle, \left\langle \alpha_s GG \right\rangle, \left\langle g_s \bar{q}\sigma Gq \right\rangle, \left\langle g_s^3 G^3 \right\rangle$$



Quark-hadron duality and Borel transform

- The correlation function described in the above two ways is equivalent

$$\Pi^{(Phe)}(q^2) = \Pi^{(OPE)}(q^2)$$

$$\frac{\left(q^2\right)^N}{\pi}\int_<^\infty ds \frac{\operatorname{Im}\Pi(s)}{s^N\left(s-q^2-i\varepsilon\right)} + \sum_{n=0}^{N-1}\left(q^2\right)^n b_n = \sum_{n=0}^{\infty} C_n(q^2) O_n$$

- Borel transform

$$\hat{B} = \lim_{\substack{Q^2, n\to\infty \\ Q^2/n = M_B^2}} \frac{\left(Q^2\right)^{n+1}}{(n)!}\left(-\frac{d}{dQ^2}\right)^n .$$

$$\mathcal{L}_k\left(s_0, M_B^2\right) = f_H^2 m_H^{2k} e^{-m_H^2/M_B^2} = \int_<^{s_0} ds e^{-s/M_B^2}\rho^{\mathrm{OPE}}(s) s^k ,$$

- Hadron mass

$$m_H\left(s_0, M_B^2\right) = \sqrt{\frac{\mathcal{L}_1\left(s_0, M_B^2\right)}{\mathcal{L}_0\left(s_0, M_B^2\right)}} .$$

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Couplings

- Following currents cannot couple to spin-2 states since two indices are antisymmetric.

$$\bar{q} g_s G_{\mu\nu} q, \quad \bar{q} g_s \tilde{G}_{\mu\nu} q, \quad \bar{q} g_s \gamma_5 G_{\mu\nu} q, \quad \bar{q} g_s \gamma_5 \tilde{G}_{\mu\nu} q$$

- Following currents cannot couple to 2^{+-} states.

$$\begin{aligned} &\bar{q} g_s \sigma_\mu^{\ \alpha} G_{\alpha\nu} q, && 2^{++}, \\ &\bar{q} g_s \sigma_\mu^{\ \alpha} \tilde{G}_{\alpha\nu} q, && 2^{-+}, \\ &\bar{q} g_s \gamma_5 \sigma_\mu^{\ \alpha} G_{\alpha\nu} q, && 2^{-+}, \\ &\bar{q} g_s \gamma_5 \sigma_\mu^{\ \alpha} \tilde{G}_{\alpha\nu} q, && 2^{++}, \end{aligned}$$

- Two ways to obtain 2^{+-} states
  - Two indices currents with nonlocal operators with derivatives

$$\bar{q} g_s \gamma_5 \gamma_\mu \overleftrightarrow{D}^\alpha \tilde{G}_{\alpha\nu} q$$

  - Currents with three and even more indices ? $J_{\alpha\beta\gamma}$

Couplings

- Couplings between physical states and 3-indices currents

$$\langle 0|J_{\alpha\beta\gamma}|0^{(-P)C}(p)\rangle = Z_1^0 p_\alpha g_{\beta\gamma} + Z_2^0 p_\beta g_{\alpha\gamma} + Z_3^0 p_\gamma g_{\alpha\beta} + Z_4^0 p_\alpha p_\beta p_\gamma \,,$$
$$\langle 0|J_{\alpha\beta\gamma}|0^{PC}(p)\rangle = Z_5^0 \varepsilon_{\alpha\beta\gamma\tau} p^\tau \,,$$
$$\langle 0|J_{\alpha\beta\gamma}|1^{PC}(p)\rangle = Z_1^1 \epsilon_\alpha g_{\beta\gamma} + Z_2^1 \epsilon_\beta g_{\alpha\gamma} + Z_3^1 \epsilon_\gamma g_{\alpha\beta} + Z_4^1 \epsilon_\alpha p_\beta p_\gamma + Z_5^1 \epsilon_\beta p_\alpha p_\gamma + Z_6^1 \epsilon_\gamma p_\alpha p_\beta \,,$$
$$\langle 0|J_{\alpha\beta\gamma}|1^{(-P)C}(p)\rangle = Z_7^1 \varepsilon_{\alpha\beta\gamma\tau}\epsilon^\tau + Z_8^1 \varepsilon_{\alpha\beta\tau\lambda}\epsilon^\tau p^\lambda p_\gamma + Z_9^1 \varepsilon_{\alpha\gamma\tau\lambda}\epsilon^\tau p^\lambda p_\beta \,,$$
$$\langle 0|J_{\alpha\beta\gamma}|2^{(-P)C}(p)\rangle = Z_1^2 \epsilon_{\alpha\beta} p_\gamma + Z_2^2 \epsilon_{\alpha\gamma} p_\beta + Z_3^2 \epsilon_{\beta\gamma} p_\alpha \,,$$
$$\langle 0|J_{\alpha\beta\gamma}|2^{PC}(p)\rangle = Z_4^2 \varepsilon_{\alpha\beta\tau\theta}\epsilon_\gamma{}^\tau p^\theta + Z_5^2 \varepsilon_{\alpha\gamma\tau\theta}\epsilon_\beta{}^\tau p^\theta \,,$$
$$\langle 0|J_{\alpha\beta\gamma}|3^{PC}(p)\rangle = Z_1^3 \epsilon_{\alpha\beta\gamma} \,,$$

- Possible currents

$$J^{(1)}_{\alpha\beta\gamma} = \bar{q} g_s \gamma_\alpha \gamma_5 G_{\beta\gamma} q, \qquad J^{PC} = (0\,,1\,,2)^{\pm-} \,,$$
$$J^{(2)}_{\alpha\beta\gamma} = \bar{q} g_s \gamma_\alpha \gamma_5 \tilde{G}_{\beta\gamma} q, \qquad J^{PC} = (0\,,1\,,2)^{\pm-} \,,$$
$$J^{(3)}_{\alpha\beta\gamma} = \bar{q} g_s \gamma_\alpha G_{\beta\gamma} q, \qquad J^{PC} = (0\,,1\,,2)^{\pm+} \,,$$
$$J^{(4)}_{\alpha\beta\gamma} = \bar{q} g_s \gamma_\alpha \tilde{G}_{\beta\gamma} q, \qquad J^{PC} = (0\,,1\,,2)^{\pm+} \,,$$

Currents

- Two currents can couple to 2^{+-} states

$$J^{(1)}_{\alpha\beta\gamma} = \bar{q} g_s \gamma_\alpha \gamma_5 G_{\beta\gamma} q, \qquad J^{PC} = (0\,,1\,,2)^{\pm -}\,,$$
$$J^{(2)}_{\alpha\beta\gamma} = \bar{q} g_s \gamma_\alpha \gamma_5 \tilde{G}_{\beta\gamma} q, \qquad J^{PC} = (0\,,1\,,2)^{\pm -}\,,$$

- Each current can couple to only one 2^{+-} state

$$\left\langle 0 \left| J^{(1)}_{\alpha\beta\gamma} \right| 2^{+-}(p) \right\rangle = f^{-} \left(\varepsilon_{\alpha\beta\tau\theta} \epsilon_\gamma{}^\tau p^\theta - \varepsilon_{\alpha\gamma\tau\theta} \epsilon_\beta{}^\tau p^\theta \right)\,,$$
$$\left\langle 0 \left| J^{(2)}_{\alpha\beta\gamma} \right| 2^{+-}(p) \right\rangle = f^{-\prime} (\epsilon_{\alpha\beta} p_\gamma - \epsilon_{\alpha\gamma} p_\beta)\,,$$

The above two states are the same state.

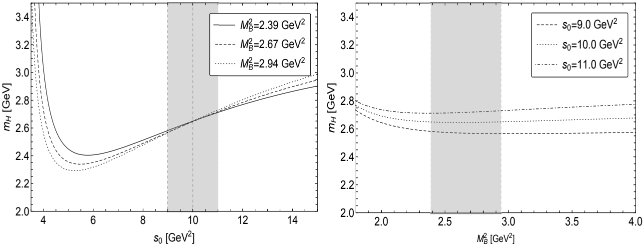
- Projector constructed

$$\mathbb{P}_{\alpha_1,\beta_1,\gamma_1,\alpha_2,\beta_2,\gamma_2} = \frac{1}{80p^2} \sum \left(\varepsilon_{\alpha_1\beta_1\tau_1\theta_1} \epsilon_{\gamma_1}{}^{\tau_1} p^{\theta_1} - \varepsilon_{\alpha_1\gamma_1\tau_1\theta_1} \epsilon_{\beta_1}{}^{\tau_1} p^{\theta_1} \right) \times \left(\varepsilon_{\alpha_2\beta_2\tau_2\theta_2} \epsilon_{\gamma_2}{}^{\tau_2 *} p^{\theta_2} - \varepsilon_{\alpha_2\gamma_2\tau_2\theta_2} \epsilon_{\beta_2}{}^{\tau_2 *} p^{\theta_2} \right)\,.$$

- Correlation function of 2^{+-} state can be extracted

$$\Pi_2(p^2) = \mathbb{P}^{\alpha_1,\beta_1,\gamma_1,\alpha_2,\beta_2,\gamma_2} \Pi_{\alpha_1\beta_1\gamma_1,\alpha_2\beta_2\gamma_2}(p^2)\,.$$

Numerical analysis



Variations of hadron mass with s_0 and M_B^2 for the $\bar{q}gq$ light hybrid meson.

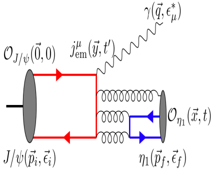
$$
\begin{aligned}
m_{b_2/h_2} &= 2.65 \pm 0.09 \text{ GeV}\,, \\
m_{h_2'} &= 2.74 \pm 0.09 \text{ GeV}\,,
\end{aligned}
$$

- Lattice calculation

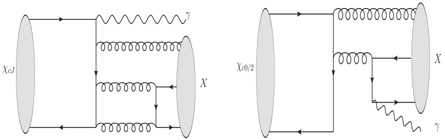
$$m_{b_2/h_2^{(\prime)}} = 2.4 - 2.8 \text{ GeV}\,,$$

Production

- The production mechanisms of $\eta_1(1855)$ from J/ψ radiative decays. PRD107,054511



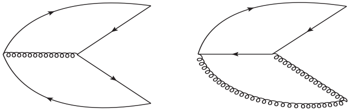
- Two possible production mechanisms of 2^{+-} light hybrids from χ_{cJ} radiative decays.



- BESIII experiments: $\psi(3686) \to \gamma\chi_{cJ} \to \gamma\gamma X$

Decays

- Some possible two-meson decay mechanism of hybrid meson



- Two-meson decay modes $I^G J^{PC} = 0^- 2^{+-}$ (h_2 and h_2') and $1^+ 2^{+-}$ (b_2).

$I^G(J^{PC})$	$0^-(2^{+-})$	$1^+(2^{+-})$
S-wave	$K_0^* \bar{K}_2^*$ (both columns)	
	$a_1 b_1, f_1 h_1, f_2 h_1,$ $\eta_1 \omega, \pi_1 \rho$	$a_1 h_1, f_1 b_1, f_2 b_1,$ $\pi_1 \omega, \eta_1 \rho$
P-wave	$K\bar{K}_1, K\bar{K}_2^*, K^*\bar{K}_0^*, K^*\bar{K}_1, K^*\bar{K}_2^*$ (both columns)	
	$h_1\eta, b_1\pi, f_0\omega,$ $f_1\omega, f_2\omega, a_0\rho, a_1\rho$	$b_1\eta, h_1\pi, f_0\rho,$ $f_1\rho, f_2\rho, a_0\omega, a_1\pi, a_2\pi$
D-wave	$K\bar{K}^*$ (both columns)	
	$\rho\pi, \omega\eta^{(\prime)}$	$\omega\pi, \rho\eta^{(\prime)}$

- Experimental search
  - $b_2 \to \omega/a_1/h_1/a_2\pi \to 4\pi$
  - $h_2 \to \rho\pi \to 3\pi$ and $h_2 \to b_1\pi \to 5\pi$
  - $\pi\gamma,\ \eta^{(\prime)}\gamma$

Content

Summary

- Non-strange and strangeonium light hybrid mesons with 2^{+-} by using the method of QCD sum rules.
- $m_{b_2/h_2} = 2.65 \pm 0.09$ GeV $\quad m_{h_2'} = 2.74 \pm 0.09$ GeV.
- Generated through two-gluon and three-gluon emission processes in radiative decays of χ_{cJ}.
- BESIII experiments: $\psi(3686) \to \gamma\chi_{cJ} \to \gamma\gamma X$
- Experimental search: $b_2 \to 4\pi,\ h_2 \to 3\pi, 5\pi$

Thank You !