

# Interactions between two heavy mesons within chiral perturbation theory

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# Introduction

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# Hadron spectrum

- Traditional quark model: Meson ( $\bar{q}q$ ), Baryon ( $qqq$ )

Discovery of doubly heavy baryon:  $\Xi_{cc}$

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$D_s(2317)$ : contribution of  $DK$  continuum

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Deuteron: bound state of proton and neutron

$P_c$  states reported at LHCb recently;  $Z_b(10610)$ ,  $Z_b(10650)$

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Deuteron: bound state of proton and neutron

$P_c$  states reported at LHCb recently;  $Z_b(10610)$ ,  $Z_b(10650)$

- other exotic states

X, Y, Z states, debate with different interpretations:

molecules? tetraquark? ordinary charmonium? two diquark? kinetic effects?



# Hadron interactions at low energy

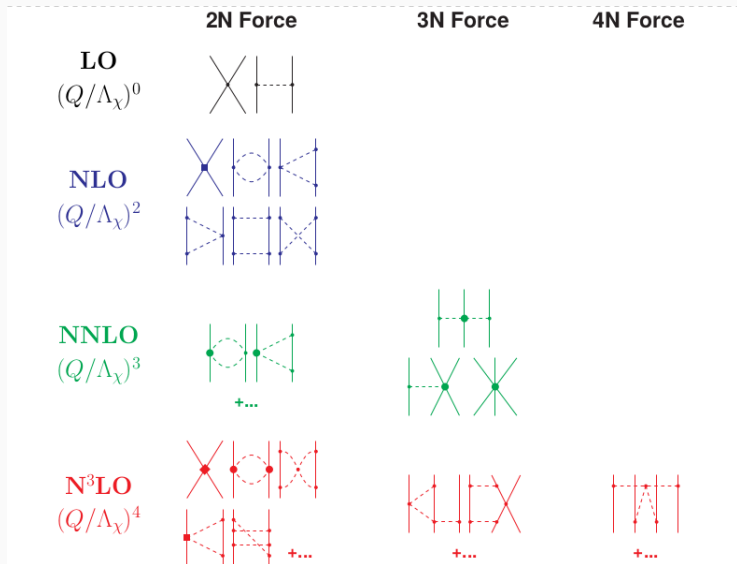
- Spectrum  $\rightleftharpoons$  Interactions
- interactions cannot be perturbatively solved at low energy because of large strong coupling  $\alpha_s$
- Different models and approaches
  - lattice QCD simulation
  - QCD sum rule
  - one-boson-exchange model
  - chiral perturbation theory
  - Hamiltonian effective field theory
  - diquark model
  - ...

# Study of Interactions within chiral perturbation theory (ChPT)

- ChPT with respect on symmetries of QCD
- Power counting
  - NOT in power series:  $\alpha_s$ ,  $\alpha_s^2$ ,  $\alpha_s^3$ , ...
  - expanded with small momentum
  - systematically study, order by order, error controlled
  - check of standard model
- Natural extension

2-body force, 3-body force,...
- Wide applications

# Nucleon-nucleon interaction



# ChPT with heavy hadrons involved

- Dealing systems with light mesons

ChPT results can be expanded as power series of

$$m_\phi/\Lambda_\chi, q/\Lambda_\chi, \dots$$

- Power Counting Breaking (PCB) in systems with heavy hadrons involved

large masses of heavy hadrons make  $q^\mu$  is never small again

power counting can be **recovered** with the help of residual momentum  $\tilde{q}^\mu$

$$\tilde{q}^\mu = q^\mu - m(1, \vec{0}).$$

# Solutions for systems with one heavy hadron

- Heavy hadron effective field theory (EFT)

nonrelativistic reduction at Lagrangian level, breaking of analyticity.

Simple and still correct if not analytically extending results too far away

- Infrared regularization

relativistic Lagrangian, drop PCB terms at regularization

good power counting and analyticity

- Extended on-mass-shell scheme

relativistic Lagrangian, drop PCB terms at final results

good power counting and analyticity

Results with three different schemes will be same if

- being summarized at ALL orders, or
- the mass of heavy hadron becomes infinite.

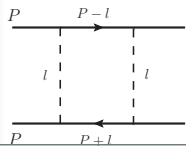
# ChPT with few hadrons involved—new trouble

The amplitude of following 2-Particle-Reducible diagram contains <sup>1</sup>

$$\mathcal{I} \equiv i \int d^4 \rho \frac{i}{\rho^0 + P^0 - \vec{P}^2/(2m_N) + i\epsilon} \frac{i}{-\rho^0 + P^0 - \vec{P}^2/(2m_N) + i\epsilon} \approx \frac{-\pi}{\vec{P}^2/(2m_N) + i\epsilon} \quad (1)$$

- naïve power counting scheme  $\rightarrow \mathcal{I} \sim O(1/|\vec{P}|)$
- eq. (1)  $\rightarrow \mathcal{I} \sim O(m_N/|\vec{P}|^2)$

$\mathcal{I}$  is actually enhanced by a large factor  $m_N/|\vec{P}|$ .



Solid line for nucleon, dashed line for pion.

( $P$  represents the residual momentum)

**Box Diagram.**

<sup>1</sup>we have not listed the parts preserving power counting

- not directly calculate physical observables with perturbation theory
- systematically study effective potentials first (without 2PR contribution)
- solve the dynamical equation to get the physical observables (equivalent to recover the 2PR contributions)

# Effective potentials between two heavy mesons

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With Heavy Meson EFT, we study the systems made up of

- $DD$
- $D^*D$
- $D^*D^*$

Similar for  $B^{(*)}B^{(*)}$  and corresponding anti-meson pair system.

We have not studied systems like  $D\bar{D}$  because there exist annihilation effects.

- Leading order vertex

contact terms:  $D^{(*)} D^{(*)} D^{(*)} D^{(*)}$  vertex

$D^{(*)} D^{(*)} \pi$ ,  $D^{(*)} D^{(*)} \pi\pi$  vertex

- Next-to-leading order vertex

they absorb divergences, provide finite higher-order corrections

$$\begin{aligned}\mathcal{L}_{4H}^{(0)} &= D_a \text{Tr} [H\gamma_\mu \bar{H}] \text{Tr} [H\gamma^\mu \bar{H}] + D_b \text{Tr} [H\gamma_\mu \gamma_5 \bar{H}] \text{Tr} [H\gamma^\mu \gamma_5 \bar{H}] \\ &\quad + E_a \text{Tr} [H\gamma_\mu \lambda^a \bar{H}] \text{Tr} [H\gamma^\mu \lambda_a \bar{H}] + E_b \text{Tr} [H\gamma_\mu \gamma_5 \lambda^a \bar{H}] \text{Tr} [H\gamma^\mu \gamma_5 \lambda_a \bar{H}], \\ \mathcal{L}_{H\phi}^{(1)} &= -\langle (i v \cdot \partial H) \bar{H} \rangle - \langle H v \cdot \Gamma \bar{H} \rangle + g \langle H \psi \gamma_5 \bar{H} \rangle - \frac{1}{8} \Delta \langle H \sigma^{\mu\nu} \bar{H} \sigma_{\mu\nu} \rangle,\end{aligned}$$

- **Leading order vertex**

contact terms:  $D^{(*)}D^{(*)}D^{(*)}D^{(*)}$  vertex

$D^{(*)}D^{(*)}\pi$ ,  $D^{(*)}D^{(*)}\pi\pi$  vertex

- **Next-to-leading order vertex**

they absorb divergences, provide finite higher-order corrections

$$\begin{aligned}\mathcal{L}_{4H}^{(2)} &= D_a^h \text{Tr} [H\gamma_\mu \bar{H}] \text{Tr} [H\gamma^\mu \bar{H}] \text{Tr} (\chi_+) + \dots \\ &\quad + D_a^d \text{Tr} [H\gamma_\mu \tilde{\chi}_+ \bar{H}] \text{Tr} [H\gamma^\mu \bar{H}] + \dots \\ &\quad + D_1^q \text{Tr} [(D^\mu H)\gamma_\mu \gamma_5 (D^\nu \bar{H})] \text{Tr} [H\gamma_\nu \gamma_5 \bar{H}] + \dots\end{aligned}$$

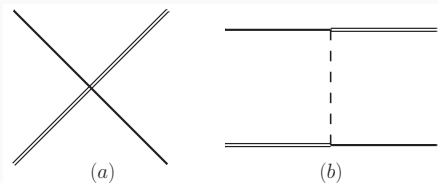
# Diagrams

- Leading order

contact, one-pion exchange

- Next-to-leading order

two-pion exchange, renormalization to  $D^{(*)}D^{(*)}\pi$  coupling, loop corrections to contact term, tree diagrams with NL vertice



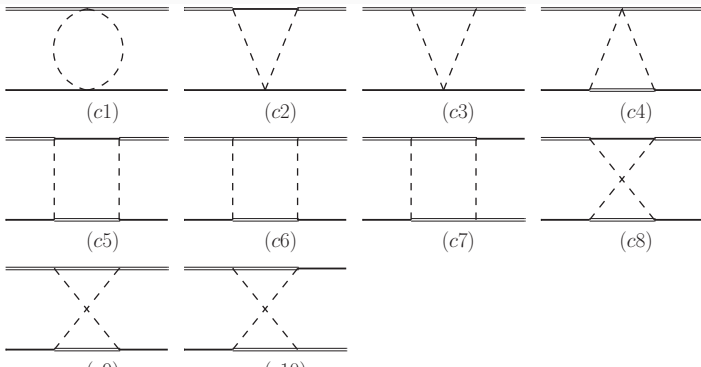
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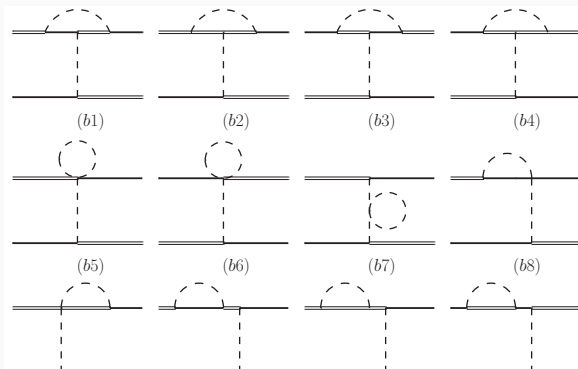
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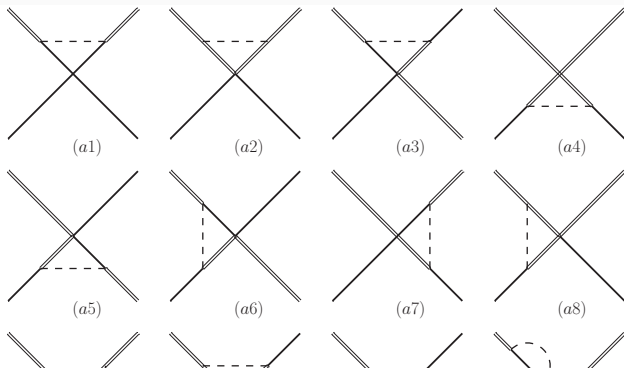
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# Regularization and renormalization

We calculate diagrams with dimension regularization and modified minimal subtraction scheme.

We have checked that the potentials are finite after the renormalization of the wavefunctions and vertex.

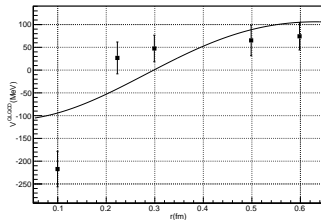
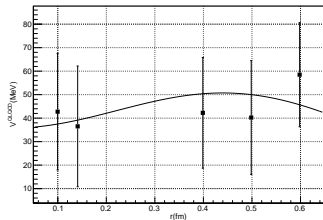


# Determination of low-energy constants

- fit to experimental data
- first principle of QCD
- fit to data of Lattice QCD
- phenomenological models

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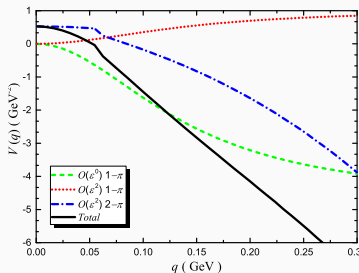
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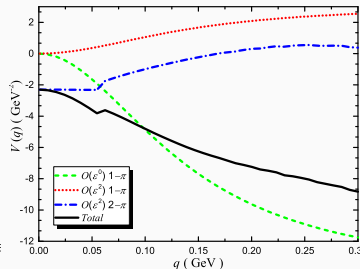
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# Effective potentials in momentum space



$DD^* : l=1$



$DD^* : l=0$

## Possible molecular states

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# Search for new states

- Potentials  $\rightarrow$  partial waves, dynamical equation (momentum space)  
 $\rightarrow$  T matrices  $\rightarrow$  poles
- Potentials  $\rightarrow$  Fourier transform, dynamical equation (coordinate space)  
 $\rightarrow$  eigenvalues of bound states for different partial waves

Taking  $DD^*$  as an example

- $l = 0$ : bound state with around  $E = 21$  MeV.  
 $l = 1$ : no bound state.
- Comparison with one-boson-exchange model  
 $\rho$  contribution is covered not only by the two-pion-exchange part but also by contact terms. <sup>2</sup>

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<sup>2</sup>Similar results as those in Phys. Rev. D 88, 114008 (2013).

We use similar approaches to study the system of  $\bar{B}^{(*)}\bar{B}^{(*)}$  in S wave

- $\bar{B}\bar{B}$ :  $I(J^P) = 1(0^+)$
- $\bar{B}\bar{B}^*$ :  $I(J^P) = 1(1^+)$ ,  $I(J^P) = 0(1^+)$
- $\bar{B}^*\bar{B}^*$ :  $I(J^P) = 1(0^+)$ ,  $I(J^P) = 1(2^+)$ ,  $I(J^P) = 0(1^+)$

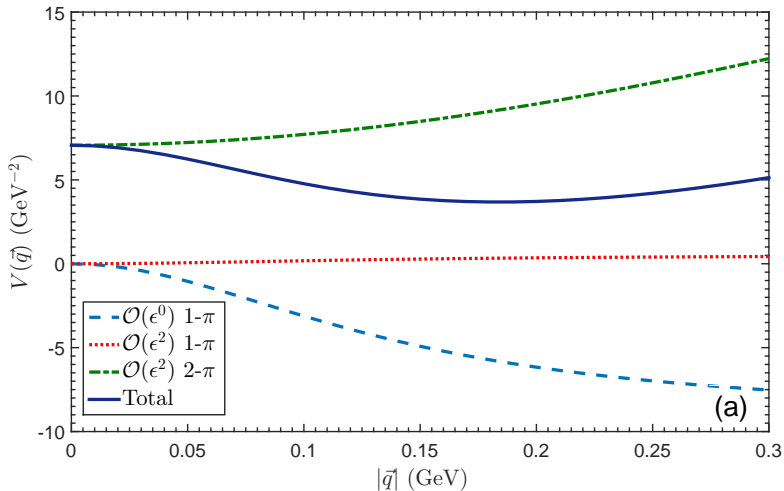
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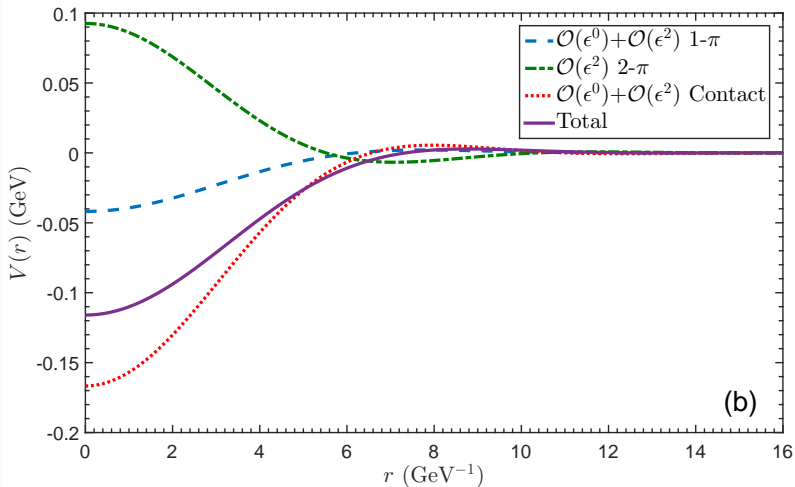
## Example:

potentials for  $\bar{B}^* \bar{B}^*$  with  $I(J^P) = 0(1^+)$  in momentum space



Example:

potentials for  $\bar{B}^* \bar{B}^*$  with  $I(J^P) = 0(1^+)$  in coordinate space



## Results for $\bar{B}^{(*)}\bar{B}^{(*)}$ systems

we find two bound states in the channels of  $0(1^+) \bar{B}\bar{B}^*$  and  $\bar{B}^*\bar{B}^*$

- binding energies:  $\Delta E_{\bar{B}\bar{B}^*} \simeq -12.6^{+9.2}_{-12.9}$  MeV,  $\Delta E_{\bar{B}^*\bar{B}^*} \simeq -23.8^{+16.3}_{-21.5}$  MeV

masses:  $m_{\bar{B}\bar{B}^*} \simeq 10591.4^{+9.2}_{-12.9}$  MeV,  $m_{\bar{B}^*\bar{B}^*} \simeq 10625.5^{+16.3}_{-21.5}$  MeV

- strong decays is forbidden because of phase space  
they can be searched in  $\bar{B}\bar{B}\gamma$  or  $\bar{B}\bar{B}\gamma\gamma$

**Table 1:** The binding energies of  $0(1^+) \bar{B}\bar{B}^*$  and  $\bar{B}^*\bar{B}^*$  states obtained with different strategies in units of MeV.

Binding energy	No $\mathcal{O}(\epsilon^2)$ LECs	Strategy A	Strategy B
$\Delta E_{\bar{B}\bar{B}^*}$	$-12.6^{+9.2}_{-12.9}$	$-10.4^{+7.2}_{-9.7}$	$-15.9^{+9.7}_{-12.7}$
$\Delta E_{\bar{B}^*\bar{B}^*}$	$-23.8^{+16.3}_{-21.5}$	$-20.1^{+14.5}_{-20.0}$	$-28.2^{+18.6}_{-23.6}$

# Summary

We have studied the potentials upto one-loop level between two heavy mesons within chiral perturbation theory.

By solving the Schrodinger equations, we found some bound states in some channels.

Thanks!

Thanks!

## this report is mainly based on the following articles

- Phys.Rev. D99 (2019) no.3, 036007
- Phys.Rev. D99 (2019) no.1, 014027
- Phys.Rev. D89 (2014) no.7, 074015