

Anti-triplet Charmed Baryon Weak Decays with $SU(3)$ Flavor Symmetry

SU(3)味對稱在三重態粲重子弱衰變的應用

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National Tsing-Hua University
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第九届海峡两岸粒子物理 与宇宙学研讨会

29 June - 2 July, 2019, 新疆大学
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Motivation

- Unknown baryon wave functions
- Failure of the conventional factorization approach

$$\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0)_{EXP} = (1.24 \pm 0.10)\%$$

- Measurements with higher precision in Belle and BESIII

$$\mathcal{B}(\Lambda_c^+ \rightarrow p K^- \pi^+) = (6.23 \pm 0.33)\%$$

- C. Q. Geng, Y. K. Hsiao, C. W. Liu and T. H. Tsai, JHEP **1711**, 147 (2017)
- C. Q. Geng, Y. K. Hsiao, C. W. Liu and T. H. Tsai, Eur. Phys. J. C **78**, 593 (2018)
- C. Q. Geng, Y. K. Hsiao, C. W. Liu and T. H. Tsai, Phys. Rev. D **97**, 073006 (2018)
- C. Q. Geng, C. W. Liu and T. H. Tsai, Phys. Lett. B **790**, 225 (2019)
- C. Q. Geng, C. W. Liu, T. H. Tsai and Y. Yu, Phys. Rev. D **99**, 114022 (2019)

Outline

- 1 Introduction of $SU(3)$ Flavor Symmetry
- 2 Anti-triplet Charmed Baryon Weak Decays
- 3 Numerical Results

Introduction of SU(3) Flavor Symmetry

$$|1\rangle = |u\rangle, |2\rangle = |d\rangle, |3\rangle = |s\rangle$$

$$|^1\rangle = |\bar{u}\rangle, |^2\rangle = |\bar{d}\rangle, |^3\rangle = |\bar{s}\rangle$$

$$\pi^+ = |^2_1\rangle = \delta_{i2} \delta^{j1} |^i_j\rangle = (\pi^+)^j_i |^i_j\rangle$$

$$M = (M)^j_i |^i_j\rangle$$

$$(M)^j_i = \begin{pmatrix} \frac{1}{\sqrt{2}}(\pi^0 + c\phi\eta + s\phi\eta') & \pi^- & K^- \\ \pi^+ & \frac{-1}{\sqrt{2}}(\pi^0 - c\phi\eta - s\phi\eta') & \bar{K}^0 \\ K^+ & K^0 & -s\phi\eta + c\phi\eta' \end{pmatrix}_{ij},$$

where $(c\phi, s\phi) = (\cos\phi, \sin\phi)$ and $\phi = (39.3 \pm 1.0)^\circ$.¹

¹T. Feldmann, P. Kroll and B. Stech, Phys. Rev. D **58**, 114006 (1998); Phys. Lett. B **449**, 339 (1999).

Introduction of SU(3) Flavor Symmetry

Invariant tensor

$$\epsilon^{ijk} |_{ijk}\rangle$$

$$\delta_j^i |^j_i\rangle$$

Introduction of SU(3) Flavor Symmetry

Invariant tensor

$$\epsilon^{ijk} |_{ijk}\rangle \quad \delta_j^i |^j_i\rangle$$

Antisymmetric tensor gives us singlet

$$\frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) ,$$
$$\frac{1}{\sqrt{6}} (|rgb\rangle - |rbg\rangle + |gbr\rangle - |grb\rangle + |brg\rangle - |bgr\rangle)$$

Same thing happens in color states of mesons

$$\frac{1}{\sqrt{3}} (|r\bar{r}\rangle + |g\bar{g}\rangle + |b\bar{b}\rangle)$$

Introduction of SU(3) Flavor Symmetry

Anti-triplet charmed baryons

$$\Lambda_c^+ = |udc\rangle - |duc\rangle = |12\rangle_{B_c} - |21\rangle_{B_c} = \epsilon^{ijk} \delta_{k3} |ij\rangle_{B_c} \sim \delta_{k3} |^k\rangle$$

$$(\mathbf{B}_c)_i = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+)_i$$

Octet Baryons

$$\mathbf{B} = \mathbf{B}^{ijk} |ijk\rangle = (\mathbf{B}_n)_i \epsilon^{ijk} |ijk\rangle$$

$$p = |112\rangle - |121\rangle = (\delta^{i1} \delta^{j1} \delta^{k2} - \delta^{i1} \delta^{j2} \delta^{k1}) |ijk\rangle = (\delta^{i1} \delta_{l3} \epsilon^{ljk}) |ijk\rangle$$

$$(\mathbf{B}_n)_j^i = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}_{ij}$$

Introduction of SU(3) Flavor Symmetry

Sextet charmed baryons

$$\Sigma_c^+ = |udc\rangle + |duc\rangle = |12\rangle_{B_c} + |21\rangle_{B_c} = (\mathbf{B}'_c)^{ij} |ij\rangle_{B_c}$$

$$(\mathbf{B}'_c)^{ij} = \begin{pmatrix} \Sigma_c^{++} & \frac{1}{\sqrt{2}}\Sigma_c^+ & \frac{1}{\sqrt{2}}\Xi_c'^+ \\ \frac{1}{\sqrt{2}}\Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt{2}}\Xi_c'^0 \\ \frac{1}{\sqrt{2}}\Xi_c'^+ & \frac{1}{\sqrt{2}}\Xi_c'^0 & \Omega_c^0 \end{pmatrix},$$

Decuplet Baryons

$$\psi = \psi(\text{space}) \otimes \psi(\text{spin}) \otimes \psi(\text{flavor}) \otimes \psi(\text{color})$$

$$\mathbf{B}_D = \frac{1}{\sqrt{3}} \left(\begin{pmatrix} \sqrt{3}\Delta^{++} & \Delta^+ & \Sigma'^+ \\ \Delta^+ & \Delta^0 & \frac{\Sigma'^0}{\sqrt{2}} \\ \Sigma'^+ & \frac{\Sigma'^0}{\sqrt{2}} & \Xi'^0 \end{pmatrix}, \begin{pmatrix} \Delta^+ & \Delta^0 & \frac{\Sigma'^0}{\sqrt{2}} \\ \Delta^0 & \sqrt{3}\Delta^- & \Sigma'^- \\ \frac{\Sigma'^0}{\sqrt{2}} & \Sigma'^- & \Xi'^- \end{pmatrix}, \begin{pmatrix} \Sigma'^+ & \frac{\Sigma'^0}{\sqrt{2}} & \Xi'^0 \\ \frac{\Sigma'^0}{\sqrt{2}} & \Sigma'^- & \Xi'^- \\ \Xi'^0 & \Xi'^- & \sqrt{3}\Omega^- \end{pmatrix} \right),$$

Introduction of SU(3) Flavor Symmetry

Sextet charmed baryons

$$\Sigma_c^+ = |udc\rangle + |duc\rangle = |12\rangle_{B_c} + |21\rangle_{B_c} = (\mathbf{B}'_c)^{ij} |ij\rangle_{B_c}$$

$$(\mathbf{B}'_c)^{ij} = \begin{pmatrix} \Sigma_c^{++} & \frac{1}{\sqrt{2}}\Sigma_c^+ & \frac{1}{\sqrt{2}}\Xi_c'^+ \\ \frac{1}{\sqrt{2}}\Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt{2}}\Xi_c'^0 \\ \frac{1}{\sqrt{2}}\Xi_c'^+ & \frac{1}{\sqrt{2}}\Xi_c'^0 & \Omega_c^0 \end{pmatrix},$$

Decuplet Baryons (spin-3/2)

$$\psi = \psi(\text{space}) \otimes \psi(\text{spin}) \otimes \psi(\text{flavor}) \otimes \psi(\text{color})$$

$$\mathbf{B}_D = \frac{1}{\sqrt{3}} \left(\begin{pmatrix} \sqrt{3}\Delta^{++} & \Delta^+ & \Sigma'^+ \\ \Delta^+ & \Delta^0 & \frac{\Sigma'^0}{\sqrt{2}} \\ \Sigma'^+ & \frac{\Sigma'^0}{\sqrt{2}} & \Xi'^0 \end{pmatrix}, \begin{pmatrix} \Delta^+ & \Delta^0 & \frac{\Sigma'^0}{\sqrt{2}} \\ \Delta^0 & \sqrt{3}\Delta^- & \Sigma'^- \\ \frac{\Sigma'^0}{\sqrt{2}} & \Sigma'^- & \Xi'^- \end{pmatrix}, \begin{pmatrix} \Sigma'^+ & \frac{\Sigma'^0}{\sqrt{2}} & \Xi'^0 \\ \frac{\Sigma'^0}{\sqrt{2}} & \Sigma'^- & \Xi'^- \\ \Xi'^0 & \Xi'^- & \sqrt{3}\Omega^- \end{pmatrix} \right),$$

Charmed Baryon Weak Decays

Effective Hamiltonian for c transition

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} (c_- \mathcal{O}_- + c_+ \mathcal{O}_+) .$$

$$\mathcal{O}_- = \frac{1}{2} \sum_{q,q'=d,s} V_{uq} V_{cq'}^* ((\bar{u}q)(\bar{q}'c) - (\bar{q}'q)(\bar{u}c)) ,$$

$$\mathcal{O}_+ = \frac{1}{2} \sum_{q,q'=d,s} V_{uq} V_{cq'}^* ((\bar{u}q)(\bar{q}'c) + (\bar{q}'q)(\bar{u}c)) ,$$

where $(\bar{q}q') \equiv \bar{q}^\alpha \gamma_\mu (1 - \gamma_5) q_\alpha$.

Charmed Baryon Weak Decays

Effective Hamiltonian for c transition

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left(c_- H(6)_{lk} (\epsilon^{ijl}/2) O_{ij}^k + c_+ H(\overline{15})_k^{ij} O_{ij}^k \right)$$

$$O_{ij}^k = \frac{1}{2} (\bar{q}_i q_k)_{V-A} (\bar{q}_j c)_{V-A} \quad s_c \equiv V_{cd}$$

$$H(6)_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & -2s_c \\ 0 & -2s_c & 2s_c^2 \end{pmatrix}_{ij},$$

$$H(\overline{15})_k^{ij} = \left(\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij}, \begin{pmatrix} 0 & s_c & 1 \\ s_c & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}_{ij}, \begin{pmatrix} 0 & -s_c^2 & -s_c \\ -s_c^2 & 0 & 0 \\ -s_c & 0 & 0 \end{pmatrix}_{ij} \right)_k$$

Charmed Baryon Weak Decays

The process $B_c \rightarrow B_D(\text{spin-3/2}) M$

$$\mathbf{M} = \langle \mathbf{B}_D M | \mathcal{H}_{\text{eff}} | \mathbf{B}_c \rangle = i q_\mu \bar{w}_{\mathbf{B}_D}^\mu (P - D \gamma_5) u_{\mathbf{B}_c},$$

$$\langle \mathbf{B}_D M | \mathcal{H}_{\text{eff}} | \mathbf{B}_c \rangle_{L=1} = (\overline{\mathbf{B}_D})_{ijk} \overline{M}_l^m H_p^{no}(\mathbf{B}_c)_q \langle (ijk)_{\mathbf{B}_D} (l^m)_M | O_{no}^p | (q)_{\mathbf{B}_c} \rangle_{L=1}$$

Charmed Baryon Weak Decays

The process $B_c \rightarrow B_D(\text{spin-3/2}) M$

$$\mathbf{M} = \langle \mathbf{B}_D M | \mathcal{H}_{\text{eff}} | \mathbf{B}_c \rangle = iq_\mu \bar{w}_{\mathbf{B}_D}^\mu (P - D\gamma_5) u_{\mathbf{B}_c},$$

$$\langle \mathbf{B}_D M | \mathcal{H}_{\text{eff}} | \mathbf{B}_c \rangle_{L=1} = (\overline{\mathbf{B}_D})_{ijk} \overline{M}_l^m H_p^{no}(\mathbf{B}_c)_q \langle (ijk)_{\mathbf{B}_D} (l^m)_M | O_{no}^p | (q)_{\mathbf{B}_c} \rangle_{L=1}$$

$$\langle (ijk)_{\mathbf{B}_D} (l^m)_M | O_{no}^p | (q)_{\mathbf{B}_c} \rangle_{L=1} \sim P_1 \delta_m^i \delta_n^j \delta_o^q \epsilon^{klp}$$

$$\begin{aligned} P(\mathcal{O}_{\overline{15}}) &= P_1(\mathbf{B}'_n)_{ijk}(M)_l^i H(\overline{15})_m^n(\mathbf{B}_c)_n \epsilon^{klm} + P_2(\mathbf{B}'_n)_{ijk}(M)_l^i H(\overline{15})_m^{jk}(\mathbf{B}_c)_n \epsilon^{lmn} \\ &\quad + P_3(\mathbf{B}'_n)_{ijk}(M)_m^l H(\overline{15})_l^{ij}(\mathbf{B}_c)_n \epsilon^{kmn}, \\ P(\mathcal{O}_6) &= P_0(\mathbf{B}'_n)_{ijk}(\mathbf{B}_c)_l H_{nm}(6)(M)_o^i \epsilon^{jln} \epsilon^{kmo}. \end{aligned}$$

Example

$$P(\Lambda_c^+ \rightarrow \Delta^{++} K^-) = -2P_0 - P_1$$

Charmed Baryon Weak Decays

We omit O_+ for two reasons

- Factorizable part
 $\langle \mathbf{B}_D | j_{(A)}^\mu | \mathbf{B}_c \rangle = 0$
flavor symmetry
- Nonfactorizable part:
 $\langle \mathbf{B}_2 | O_+ | \mathbf{B}_p \rangle = 0$
color symmetry

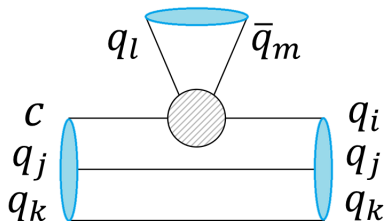


Figure: Factorizable diagram

Charmed Baryon Weak Decays

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- Factorizable part
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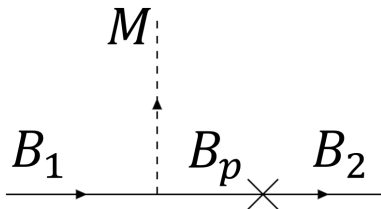


Figure: Baryon pole

Charmed Baryon Weak Decays

The process $B_c \rightarrow B_D(\text{spin-3/2}) M$

$$\mathbf{M} = \langle \mathbf{B}_D M | \mathcal{H}_{\text{eff}} | \mathbf{B}_c \rangle = i q_\mu \bar{w}_{\mathbf{B}_D}^\mu (P - D \gamma_5) u_{\mathbf{B}_c},$$

$$\langle \mathbf{B}_D M | \mathcal{H}_{\text{eff}} | \mathbf{B}_c \rangle_{L=1} = (\overline{\mathbf{B}_D})_{ijk} \overline{M}_l^m H_p^{no}(\mathbf{B}_c)_q \langle (ijk)_{\mathbf{B}_D} (l^m)_M | O_{no}^p | (q)_{\mathbf{B}_c} \rangle_{L=1}$$

$$\langle (ijk)_{\mathbf{B}_D} (l^m)_M | O_{no}^p | (q)_{\mathbf{B}_c} \rangle_{L=1} \sim P_1 \delta_m^i \delta_n^j \delta_o^q \epsilon^{klp}$$

$$P(\mathcal{O}_{\overline{15}}) = P_1(\mathbf{B}'_n)_{ijk}(M)_l^i H(\overline{15})_m^{jn}(\mathbf{B}_c)_n \epsilon^{klm} + P_2(\mathbf{B}'_n)_{ijk}(M)_l^i H(\overline{15})_m^{jk}(\mathbf{B}_c)_n \epsilon^{lmn} \\ + P_3(\mathbf{B}'_n)_{ijk}(M)_l^i H(\overline{15})_m^{ij}(\mathbf{B}_c)_n \epsilon^{kmn},$$

$$P(\mathcal{O}_6) = P_0(\mathbf{B}'_n)_{ijk}(\mathbf{B}_c)_l H_{nm}(6)(M)_o^i \epsilon^{jln} \epsilon^{kmo} \equiv P_0 f_{\mathbf{B}_c \mathbf{B}_D M}.$$

Example

$$P(\Lambda_c^+ \rightarrow \Delta^{++} K^-) = -2P_0 - P_1$$

Numerical Results

Decay width and up-down asymmetry

$$\Gamma(B_i \rightarrow B_f + P) = \frac{|p_c|^3 m_i (m_f + E_f)}{6\pi m_f^2} \{ |P|^2 + |\overline{D}|^2 \}$$

$$\alpha = \frac{2 \operatorname{Re}(P \overline{D}^*)}{(|P|^2 + |\overline{D}|^2)}$$

Table: Experimental data of the up-down asymmetries and branching ratios.

channel	$10^3 \mathcal{B}_{exp}$	α_{exp}
$\Lambda_c^+ \rightarrow \Delta^{++} K^-$	10.7 ± 2.5	
$\Lambda_c^+ \rightarrow \Sigma'^+ \eta$	9.1 ± 2.0	
$\Lambda_c^+ \rightarrow \Xi'^0 K^+$	5.02 ± 1.04	-1.00 ± 0.34
$\Xi_c^0 \rightarrow \Omega^- K^+$	4.2 ± 1.0	
$\Xi_c^+ \rightarrow \Sigma'^+ \overline{K}^0$	1.0 ± 0.5^{23}	
$\Xi_c^+ \rightarrow \Xi'^0 \pi^+$	$< 0.1^{23}$	

²J. M. Link *et al.* [FOCUS Collaboration], Phys. Lett. B **571**, 139 (2003)

³measured relative to $\mathcal{B}(\Xi_c^+ \rightarrow \Xi^- 2\pi^+)$

Numerical Results

Fitting Results

$$P = P_0 f_{\mathbf{B}_c \mathbf{B}_D M}, \quad D = D_0 f_{\mathbf{B}_c \mathbf{B}_D M}$$

$$(P_0, D_0)_{pm} = (3.2 \pm 0.4, -5.1 \pm 2.5) 10^{-2} G_F \text{GeV}, \quad R_{P_0 D_0} = 0.70$$

Table: Our results of the up-down asymmetries and branching ratios.

channel	$10^3 \mathcal{B}_{exp}$	α_{exp}	$10^3 \mathcal{B}_{SU(3)}$	$\alpha_{SU(3)}$
$\Lambda_c^+ \rightarrow \Delta^{++} K^-$	10.7 ± 2.5		15.3 ± 2.4	$-0.86^{+0.44}_{-0.14}$
$\Lambda_c^+ \rightarrow \Sigma'^+ \eta$	9.1 ± 2.0		3.1 ± 0.6	$-0.97^{+0.43}_{-0.03}$
$\Lambda_c^+ \rightarrow \Xi'^0 K^+$	5.02 ± 1.04	-1.00 ± 0.34	1.0 ± 0.2	$-1.00^{+0.34}_{-0.00}$
$\Xi_c^0 \rightarrow \Omega^- K^+$	4.2 ± 1.0		3.2 ± 0.7	$-1.00^{+0.34}_{-0.00}$
$\Xi_c^+ \rightarrow \Sigma'^+ \bar{K}^0$	1.0 ± 0.5^{23}		0	-
$\Xi_c^+ \rightarrow \Xi'^0 \pi^+$	$< 0.1^{23}$		0	-

²J. M. Link *et al.* [FOCUS Collaboration], Phys. Lett. B **571**, 139 (2003)

³measured relative to $\mathcal{B}(\Xi_c^+ \rightarrow \Xi^- 2\pi^+)$

Summary

- There is only two parameters in $\mathbf{B}_c \rightarrow \mathbf{B}_D M$.
- We find out

$$\alpha(\Lambda_c^+ \rightarrow \Delta^{++} K^-) = -0.86_{-0.14}^{+0.44},$$

which could be tested by the future experiment in BESSIII and Belle.

- Through the experimental data, we predict the non-leptonic decays branching ratios.